GLPK Case Study 1 - 60016 Operations Research

The case study teaches how to use GLPK to solve a basic resource allocation problem inspired to a realistic situation and explains how to read the glpsol output files. Consider a company producing three brands of fertilisers:

SulphurGrower	(S)
EssarGrower	(E)
NitroGrower	(N).

The raw materials required to produce each of these are:

SulphurGrower	EssarGrower	NitroGrower
15% Sulphur	10% Sulphur	5% Sulphur
5% Nitrate	10% Nitrate	10% Nitrate
10% Potash	10% Potash	15% Potash

During any given period the company can obtain up to 1000 kg of Sulphur, 1600 kg Nitrate, and 2500 kg Potash. During the same period the company has contracts to supply at least 1000 kg of SulphurGrower (S), and 500 kg of EssarGrower (E). There is no upper limit on the volumes of fertilisers that the company can sell. The profits for each fertiliser are:

SulphurGrower	£3/kg
EssarGrower	£5/kg
NitroGrower	£5/kg

1. Write the LP for maximising the profit.
2. Using GMPL, write a corresponding LP model for GLPK.
3. Using glpsol, determine the optimal solution of this model.
4. If the optimal solution is implemented by the company, what raw materials will be the limiting factors for the production?

Solution 1. Denote the variables in the problem by:

x_S	Production of SulphurGrower in kg,
x_E	Production of EssarGrower in kg,
x_N	Production of NitroGrower in kg.

The LP for maximising the profit is then:

$$\max\ y = 3x_S + 5x_E + 5x_N$$

subject to:

$$\begin{array}{ccccccc} 0.15x_S & + & 0.10x_E & + & 0.05x_N & \leq & 1000 \\ 0.05x_S & + & 0.10x_E & + & 0.10x_N & \leq & 1600 \\ 0.10x_S & + & 0.10x_E & + & 0.15x_N & \leq & 2500 \end{array}$$

$$x_S \geq 1000,\ x_E \geq 500,\ x_N \geq 0.$$

2. Using GMPL, we can specify the above LP as follows:

Listing 1: growers.mod

```
set Growers;
set RawMaterials;
param Profit {g in Growers};
param RawSupply {r in RawMaterials};
param Contracts {g in Growers};
param Ratio {r in RawMaterials, g in Growers}; /* percentage of r within g */
```

Row First Column Second

```
var x {g in Growers}, >= 0; /* production of grower g */

maximize y : sum {g in Growers} Profit[g] * x[g];
```

Iterate by row: For each raw material, the used amount has to be smaller than the Raw Supply

```
s.t.
c1 {r in RawMaterials} : sum {g in Growers} Ratio[r,g] * x[g] <= RawSupply[r];
c2 {g in Growers}: x[g] >= Contracts[g];

data;

set RawMaterials := Sulphur Nitrate Potash ;
set Growers := SulphurGrower EssarGrower NitroGrower ;

param Profit :=
SulphurGrower 3
EssarGrower 5
NitroGrower 5 ;
```

As defined in previous section: {g in Growers}, the list has to use Growers components as index

```
param RawSupply :=
Sulphur 1000
Nitrate 1600
Potash  2500 ;

param Contracts :=
SulphurGrower 1000
EssarGrower 500
NitroGrower 0 ;

param Ratio : SulphurGrower   EssarGrower    NitroGrower :=
Sulphur           0.15            0.10           0.05
Nitrate           0.05            0.10           0.10
Potash            0.10            0.10           0.15 ;

end;
```

3. Saving the GMPL model in growers.mod, we can then run glpsol -m growers.mod -o growers.out and examine the output file:

Listing 2: growers.out

```
Problem:    growers
Rows:       7
Columns:    3
Non-zeros:  15
Status:     OPTIMAL
Objective:  y = 80700 (MAXimum)

   No.   Row name   St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 y            B          80700
     2 c1[Sulphur]  NU          1000                        1000             4
     3 c1[Nitrate]  NU          1600                        1600            48
     4 c1[Potash]   B           2410                        2500
     5 c2[SulphurGrower]
                    B           1400          1000
     6 c2[EssarGrower]
                    NL           500           500                        -0.2
     7 c2[NitroGrower]
                    B          14800            -0

   No. Column name  St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 x[SulphurGrower]
                    B           1400             0
     2 x[EssarGrower]
                    B            500             0
     3 x[NitroGrower]
                    B          14800             0

... [Ignore the rest]
```

Activity = Optimal Values for Decision Variables

With 1 more unit of resources, how much the objective function (i.e., profit) will increase

The output file shows two tables:

- a *rows table* that shows in the `Activity` column the value of the objective (row `y`) and how the optimal solution satisfies the constraints (rows `c1[Sulphur]` to `c2[NitroGrower]`)

- a *columns table* (rows `x[SulphurGrower]` to `x[NitroGrower]`), which shows in the `Activity` column the values of the decision variables in the optimal solution.

According to growers.out, the optimal solution is $x^* = (x^*_S, x^*_E, x^*_N) = (1400, 500, 14800)$ and the objective function has an optimal value $y^* = 80700$ (`Objective` row).

4. Inspecting growers.out, we see that the left-hand sides of `c1[Sulphur]` and `c1[Nitrate]` are limited by the upper bound, as visible in the `Activity` column for the *rows table*. If the plan is implemented, the company will use 1000 kg of sulphur and 1600 kg of nitrate, and 2410 kg of potash. Therefore, sulphur and nitrate will be the scarce materials that limit the production. Potash is not a limiting factor since up to 2500 kg can be obtained, but we require just 2410 kg.