IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2017

MEng Honours Degree in Electronic and Information Engineering Part IV
MEng Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
MSc in Computing Science
MSc in Computing Science (Specialist)
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C422

COMPUTATIONAL FINANCE

Thursday 23 March 2017, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 **Portfolio Optimisation.** Consider a market that contains only risky assets, and assume that short-selling is allowed. The minimum variance portfolio of risky assets (portfolio A) has mean rate of return $\bar{r}_A = 10\%$ and standard deviation $\sigma_A = 10\%$. Moreover, there is another portfolio on the efficient frontier (portfolio B) with mean rate of return $\bar{r}_B = 40\%$ and standard deviation $\sigma_B = 40\%$.

a Suppose that the covariance of the rates of return of A and B is $\sigma_{AB} = -0.01$. What is the portfolio of A and B which has a mean rate of return of 20% (portfolio C)?

b Does portfolio C have minimum variance for its mean rate of return of 20%? Justify your answer.

c Assume that a risk-free asset with rate of return $r_f = 4\%$ is introduced. Find the one portfolio of risky assets with the property that any efficient portfolio can be constructed as a combination of it and the risk-free asset.

d Draw a portfolio diagram and describe its shape for the following three cases:

- The correlation between A and B is 0.
- The correlation between A and B is 1.
- The correlation between A and B is -1.

The four parts carry, respectively, 25%, 15%, 45%, and 15% of the marks.

2 **Risk Management and Fixed Income Securities.**

a Explain how Macaulay duration and convexity can be used to reduce the risk associated with fixed income securities.

b Suppose that the spot rate curve is given as follows:

Year	Spot (%)
1	7.67
2	8.27
3	8.81
4	9.31
5	9.75
6	10.16
7	10.52
8	10.85
9	11.15
10	11.42
11	11.67
12	11.89

Suppose that there are two bonds available for investment. Bond B_1 is a 12-year 6% bond and is currently traded at £65.95. Bond B_2 is a 5-year 10% bond and is currently traded at £101.66. The coupons for both bonds are paid annually, and both bonds have a face value of £100.00.

- Are the bond prices consistent with the spot rate curve? Justify your answer.
- Calculate the modified duration for both bonds.

c Suppose that we have a £1 million obligation payable at the end of 5 years and we wish to invest enough money today to meet this future obligation. We wish to do this in a way that provides a measure of protection against interest rate risk. Using the results from part (b) find a portfolio consisting of bonds B_1 and B_2 that will reduce the risks against a parallel shift in the spot rate curve (i.e. has the same modified duration as our obligation).

The three parts carry, respectively, 15%, 60%, and 25% of the marks.

3 **Option Pricing.**

a An investor who believes a stock will rise in the future can construct a bull spread for that stock. One way to construct such a spread is to buy a call with strike price K_1 and sell a call with the same expiration date but with a strike price of $K_2 > K_1$. Is the initial cost of the spread positive or negative? Justify your answer.

b Suppose that over the period 0 to T a certain stock pays a dividend whose present value is D. Derive a put-call parity relation for European options at time $t = 0$, expiring at T for this stock.

c Consider an options contract with a payoff function at time T equal to

$$\max(0.5S, S - K)$$

where S is the price of a stock and K is a fixed strike price. Let P be the price of the stock at time $t = 0$ and let C_1 and C_2 be the prices of ordinary calls with strike prices K and $2K$, respectively. Find a fair price for this options contract.

d Show that European and American call options have the same value.

e Under what conditions does a European and American put option have the same value?

The five parts carry equal marks.

4 **Utility Theory.** The following game is proposed to you. A fair coin will be tossed repeatedly until a tail appears (the first tail will end the game). The pot starts at £1 and is doubled every time a head appears. You win whatever is in the pot when the game ends. Thus, you win £1 if a tail appears on the first toss, £2 if on the second, £4 if on the third, £8 if on the fourth, etc. In short, you win £2^{k-1} if the coin is tossed k times until the first tail appears.

A reasonable way to analyse the value of this game is by calculating the certainty equivalent of the game's random payoff. Calculate the expected utility and the certainty equivalent of the game in the following situations:

a risk-neutral utility function: $U(x) = x$ (hint: note that the probability of the first tail appearing in the k-th toss is $\frac{1}{2^k}$);

b power utility function: $U(x) = bx^b$ with $b = -1$;

c logarithmic utility function: $U(x) = \log_2(x)$ (hint: use the formula $\sum_{k=1}^{\infty}(k-1)/2^k = 1$).

The three parts carry, respectively, 40%, 30%, and 30% of the marks.