Tutorial 2 - 60016 Operations Research

Basic Solutions and Basic Representations

Exercise 1 *Find the basic solutions (BS) of the system of equations:*

$$\begin{array}{ccccccc} x_1 & + & x_2 & + & x_3 & = & 3 \\ 3x_1 & + & 2x_2 & + & 4x_3 & = & 10 \end{array}$$

Exercise 2. Consider the following optimisation problem:

$$\max y = \; x_1 + 3x_2 \tag{1a}$$

subject to

$$2x_1 + x_2 \leq 4 \tag{1b}$$
$$x_1 + 2x_2 \leq 4 \tag{1c}$$
$$x_1, x_2 \geq 0. \tag{1d}$$

(a) Bring the problem into standard form by introducing slack variables s_1 and s_2.

(b) For the problem in standard form, determine all basic solutions. Which of these solutions are feasible, and what are their objective values?

(c) Draw the feasible region of problem (1) in the (x_1, x_2)-plane. Where are the basic solutions from part (b)? Which feasible solutions satisfy $s_1 = 0$? Which feasible solutions satisfy $s_2 = 0$?

Exercise 3. Consider the basic solution from Exercise 2 (b) that has x_1 and x_2 as basic variables.

(a) Determine the basic representation for this basic solution.

(b) Is this basic solution optimal? Justify your answer both graphically (see Exercise 2 (c)) and from the basic representation!

(c) Find a non-basic variable such that increasing its value improves the objective value. How much can we increase the value of this variable without leaving the feasible region? Which is the resulting basic solution? Is this solution optimal?

Exercise 4. Entrepreneur S. McDuck runs a 24h supermarket. Since the number of customers per hour varies with the time of the day, the number of required staff does so, too. McDuck decides to establish three shifts that cover different time periods: midnight to noon, 6.00am to 6.00pm and noon to midnight. His estimates for the staff required for each shift are as follows.

	shift			
time period	1	2	3	required staff
midnight – 6.00am	X			2
6.00am – noon	X	X		4
noon – 6.00pm		X	X	5
6.00pm – midnight			X	4

McDuck is planning to pay £48 per day and employee for shift 1, while he has to pay £72 per day and employee for the other two shifts.

McDuck wants to determine a personnel schedule (i.e., how many staff to hire for which shift) that satisfies the stated minimum staff requirements at the lowest daily cost. Formulate the corresponding linear program! For simplicity, assume that employees are 'continuously divisible.'

Exercise 5 *(Exam 2015, Q1b).* Consider a set $\mathcal{S} = \{x_1, x_2, \ldots, x_V\}$ of V points in $\mathbb{R}^n$. We say that a point $x_i \in \mathcal{S}$ is a *convex combination* of the other points if there exist weights $\lambda_j \geq 0$, $j \neq i$, such that

$$x_i = \sum_{\substack{j=1..V \\ j \neq i}} \lambda_j x_j, \qquad \sum_{\substack{j=1..V \\ j \neq i}} \lambda_j = 1.$$

A point $x_i \in \mathcal{S}$ that is **not** a convex combination of the other points is called an *extreme point.*

Assume that you know all the points in $\mathcal{S}$. Write a linear program to decide if a given point $x_i \in \mathcal{S}$ is an extreme point or not, explaining how to interpret the optimal solution. (You cannot use binary or integer variables.)