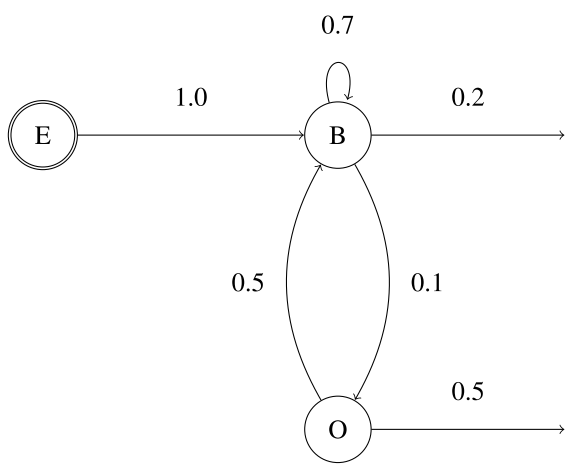
60017 Performance Engineering Tutorial: Demands and Bottlenecks

Exercise 1. Suppose that the following user behaviour graph models the interactions of a set of users with an IT system:



where B represents the *Browse catalog* service and S corresponds to the *Send order* service.

Question 1.1 Determine the average number of visits to each page and the average session length.
Question 1.2 Assume that the total arrival rate of session to the site is λ sessions per second, each following the UBG defined above. Assume that the site is deployed on a two-tier architecture based on a front server tier, composed of machines that act as both web servers and application servers, and a database tier. The front tier is composed of n identical nodes labelled $F_1, \ldots, F_n$, with identical mean service time $S_{f,c}$ for requests of class $c = B, O$ (as usual, we assume each class to represent a page within the UBG). Similarly, the database tier is composed of a single database node labelled B with service demand $D_{b,c}$ for requests of class c.

Give formulas for the total utilization U_f at an arbitrary node within the front tier and for the total utilization U_b of the node within the database tier.
Question 1.3 Assume that $n = 2$, $S_{f,B} = 1$, $S_{f,O} = 2$, $D_{d,B} = 3$, $D_{d,O} = 0$. What is the maximum value of λ that the server can sustain before becoming unstable?

Exercise 2. For a given IT system, we can collect in a demand matrix $D = [D_{i,c}]$, all the service demands at each node i for each service class c. The matrix D therefore shows resources as rows (we here considered resource labels $i = A, B, C, D$) and classes as columns (class labels $c = 1, 2, 3$).

For each of the demand matrices shown below,

- The class bottlenecks, for each class.
- The resources that can never be a bottleneck, i.e., the *dominated* resources.
- The resources that require a linear program (LP) to determine if they are potential bottlenecks.

Question 2.1

$$D = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} 1 & \;2 \end{array} \\ \begin{bmatrix} 10 & 9 \\ 5 & 5 \end{bmatrix} \end{array}$$

Question 2.2

$$D = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 10 & 5 \\ 5 & 9 \end{bmatrix} \end{array}$$

Question 2.3

$$D = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 10 & 5 & 9 \\ 4 & 7 & 1 \\ 3 & 0 & 10 \\ 0 & 10 & 0 \end{bmatrix} \end{array}$$

Question 2.4

$$D = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 10 & 10 & 10 \\ 4 & 2 & 1 \\ 3 & 0 & 9 \\ 0 & 2 & 0 \end{bmatrix} \end{array}$$

Question 2.5

$$D = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 10 & 3 \\ 4 & 7 \\ 3 & 0 \\ 0 & 9 \end{bmatrix} \end{array}$$

Question 2.6

$$D = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 10 & 10 \\ 4 & 7 \\ 3 & 0 \\ 0 & 10 \end{bmatrix} \end{array}$$

Question 2.7 Choose any of the examples above where you needed to use a linear program (LP) to establish if a resource was a potential bottleneck. Write down the LP formulation. Then explain how would you determine the mix that saturates the potential bottleneck from the optimal LP solution, if one exists.