GLPK Case Study 5 - 60016 Operations Research

A company wishes to build 3 distribution centers in London for distributing its products to all 33 London boroughs (see Figure 1).
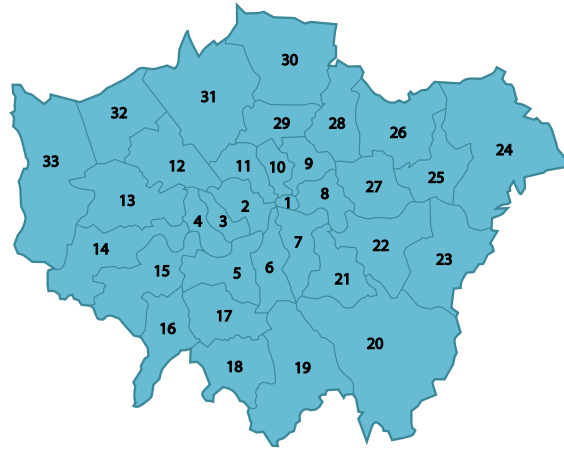

Figure 1. London boroughs

The selected locations should minimize the total transportation cost from the distribution centers to the remaining boroughs. You are given a GLPK data file ('supply_chain.dat') containing the transportation costs for all pairwise boroughs and you need to define the GMPL model that will be used to generate the location of the distribution centers and the assignment from the distribution centers to the boroughs. The integer programming formulation must take into account the following requirements:

- The decision variables are $x \in \{0,1\}^{33}$ and $Y \in \{0,1\}^{33 \times 33}$. The i-th element of x is used to indicate whether a distribution center is built in the i-th borough. the (i,j)-th element of Y is used to indicate whether an assignment is made from the i-th borough to the j-th borough.

- At most 3 distribution centers can be built.

- There must be one assignment to each borough from any of the three distribution centers.

- The objective is to minimize the total cost of the chosen assignment (*i.e.*, min $\sum_{ij} \text{COST}_{ij} Y_{ij}$, where COST is the given cost matrix).

(a) Write the GMPL model for the integer programming problem and find its optimal solution with `glpsol`.

(b) The company has decided to build the distribution centers in boroughs 1, 18, and 30. Using the given GLPK data file 'supply_chain_assignment.dat', write the GMPL model for the assignment problem and find its optimal solution with `glpsol`.

(c) Rerun `glpsol` for the assignment problem (b) with the `--nomip` parameter which relaxes all integer variables as continuous variables. What is the optimal objective value now? Try also with different combinations of the location of distribution centers. What do you observe? Why?