GLPK Case Study 7 - 60016 Operations Research

The Battle of Red Cliffs[1] was a decisive battle fought at the end of the Han dynasty and immediately prior to the Three Kingdoms period in ancient China. It was fought in the winter between the allied forces of the southern warlords Liu Bei and Sun Quan (Allies), and the numerically superior forces of the northern warlord Cao Cao (Northeners). The allied victory at Red Cliffs ensured the survival of the Allies, frustrated Northeners' effort to conquer the land south of the Yangtze River, and reunite the territory of the Eastern Han dynasty.

Assuming that before the battle, the Northeners issued different attacking configurations, where the strength is similar in all scenarios, but the generals are different. The only real hope for the Allies to stop the invasion was to guess or learn the invasion configuration, and to issue the corresponding defensive plan secretly in advance. Six possible attacking configurations (1 to 6) by the Northeners and six possible defensive plans (A to F) by the Allies were guessed and evaluated, 36 guesses in all. The following table gives the estimated value to the Allies of each hypothetical battle in some numerical units.

	A	B	C	D	E	F
1	13	29	8	12	16	23
2	18	22	21	22	29	31
3	18	22	31	31	27	37
4	11	22	12	21	21	26
5	18	16	19	14	19	28
6	23	22	19	23	30	34

Assuming this is a matrix of a six by six zero-sum game, this assignment problem can be formulated using GMPL.

RedCliffs Dataset

The dataset is given in the file RedCliffs.dat, which provides the following GMPL data structures:

- ROWS: a set of attacking configurations;
- COLS: a set of defensive strategies;
- payoff: a matrix of payoffs.

(a) Write the GMPL model for the linear programming problem from the Northeners' perspective, and find its optimal solution with `glpsol` to maximise the payoff.

(b) Write the GMPL model for the linear programming problem from the Allies' perspective, and find its optimal solution with `glpsol` to minimize the loss.

(c) Compare the result of (a) and (b), what do you observe? Explain your observation. To show the behavior of a "textboox" simplex algorithm implementation on this problem, run glpsol with the --norelax --nosteep --noscale --nopresol parameters. Then comment on the number of iterations performed by the solver.

[1] `http://en.wikipedia.org/wiki/Battle_of_Red_Cliffs`