

Tutorial 3 - 60016 Operations Research

Simplex Algorithm

Exercise 1. A plant manufactures three types of vehicles: automobiles, trucks and vans, on which the company makes a profit of £ 4000, £ 6000, and £ 3000, respectively, per vehicle.

The plant has three main departments: parts, assembly and finishing operating 120, 100, and 80 hours, respectively, each two-week period. It takes 50, 40, and 30 hours, respectively, to manufacture the parts for automobiles, trucks and vans. Assembly takes 40, 30, and 20 hours, respectively, for an automobile, truck and van. Finishing takes 20, 40, and 10 hours, respectively, for an automobile, truck and van.

How many of each type should the company manufacture in each two-week period to maximise its profits? Formulate a linear program to answer this question and solve it using the Simplex Algorithm.

Exercise 2. Consider the following LP problem:

$$\max y = 2x_1 + 3x_2$$

subject to:

$$\begin{array}{rrcr} x_1 & + & x_2 & \leq & 4 \\ -x_1 & + & 2x_2 & \geq & -1 \end{array}$$

and

$$x_1 \geq 0, x_2 \geq 0.$$

Solve this problem using the Simplex Algorithm.

Exercise 3. Solve the following LP problem using the simplex algorithm:

$$\max y = 2x_1 - 3x_2 + x_3$$

subject to:

$$\begin{array}{rrrrcr} 3x_1 & + & 6x_2 & + & x_3 & \leq & 6 \\ 4x_1 & + & 2x_2 & + & x_3 & \leq & 4 \\ x_1 & - & x_2 & + & x_3 & \leq & 3 \end{array}$$

and

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Exercise 4. Consider the linear programming problem:

$$\max y = 10x_1 - 57x_2 - 9x_3 - 24x_4$$

subject to

$$\begin{array}{rcccccccl} 0.5x_1 & - & 5.5x_2 & - & 2.5x_3 & + & 9x_4 & \leq & 0 \\ 0.5x_1 & - & 1.5x_2 & - & 0.5x_3 & + & x_4 & \leq & 0 \\ -x_1 & & & & & & & \geq & -1 \end{array}$$

and

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0,$$

1. Write the problem in standard form.
2. Write the simplex tableau for an initial basis composed by all slack variables.
3. Starting from the all-slack basis, the simplex algorithm with standard pivoting rules arrives after 4 iterations to the following intermediate tableau:

<i>BV</i>	x_1	x_2	x_3	x_4	x_5	x_6	x_7	<i>RHS</i>
z	-20	-9			10.50	-70.50		0
x_3	-2	4	1		0.50	-4.50		0
x_4	-0.50	0.50		1	0.25	-1.25		0
x_7	1						1	1

Continue the simplex algorithm and show that the algorithm cycles.

4. Starting from the same intermediate tableau used at the previous question, solve the problem using the Simplex algorithm with Bland's rule.
5. Does index set $I = \{2, 3, 4\}$ define a valid basis for this LP?