IMPERIAL COLLEGE LONDON

TIMED REMOTE ASSESSMENTS 2020-2021

MEng Honours Degree in Electronic and Information Engineering Part IV
MEng Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc Advanced Computing
MSc Artificial Intelligence
MSc in Computing (Specialism)
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant assessments for the Associateship of the City and Guilds of London Institute

PAPER COMP70006=COMP97025=COMP97026

COMPUTATIONAL FINANCE

Thursday 10 December 2020, 10:00
Duration: 140 minutes
Includes 20 minutes for access and submission

Answer ALL THREE questions
Open book assessment

By completing and submitting work for this assessment, candidates confirm that the submitted work is entirely their own and they have not (i) used the services of any agency or person(s) providing specimen, model or ghostwritten work in the preparation of the work they have submitted for this assessment, (ii) given assistance in accessing this paper or in providing specimen, model or ghostwritten answers to other candidates submitting work for this assessment.

Paper contains 3 questions

1 **Estimation of Expected Returns**

a Suppose that we have n samples of the monthly returns of a stock. The expected returns can be estimated as follows:

$$\hat{\bar{r}} = \frac{1}{n}\sum_{i=1}^{n} r_i, \tag{1}$$

where r_i denotes an i.i.d sample with mean $\bar{r}$ and variance σ^2. What is the standard deviation of the estimate in (1)?

b Suppose that $\bar{r} = 1\%$ and $\sigma = 4\%$ and that you have 12 months of data (i.e. n=12 in (1)). Your colleague suggests that the estimate in (1) will be a good estimate for the true returns of this stock. By good estimate your colleague means that the variance of the estimate is low compared to the expected returns. Do you agree with your colleague? Justify your answer.

c Another colleague suggests to increase the number of sample points to 24 by taking overlapping samples. To achieve this your colleague suggests to take a sample to cover Jan 1st to Feb 1st, the second sample from Jan 15th, to Feb 15th and so on. Your colleague claims that this idea will reduce the error in the monthly estimate of the returns. Do you agree with your colleague? Justify your answer.

Hint: You may ignore missing half-month terms at the end of year.

d Suppose that there are n independent and identically distributed assets in the market with mean $\bar{r}$ and variance σ^2. Compare the expected return and variance of a portfolio with equal weights in each asset and a portfolio where everything is invested in the first asset.

The four parts carry, respectively, 15%, 15%, 35%, and 35% of the marks.

2 **Option Pricing**

a An investor who believes the price of a stock will rise wants to construct a combination of options with a non-negative initial cost. Suggest a combination of European call or put options with the same expiry date that will meet the investors requirements. Also draw the pay-off diagram and explain why the initial cost of the proposed combination is non-negative.

b Suppose that over the period $[0, T]$ a certain stock pays a dividend whose present value is D. Suppose that you have implemented the binomial pricing model for call options on this stock. Show that you can derive the price of a put option on the stock (with the same strike) without running the binomial pricing algorithm. Justify your answer.

c Show that an American Call option has the same value as a European call option. Is an American Put option more valuable than a European put option? Justify your answer. You may assume that the market is free of arbitrage and that the stock price dynamics are modeled by a standard binomial lattice.

The three parts carry, respectively, 30%, 30%, and 40% of the marks.

3 **Fixed Income Securities & Portfolio Optimization**

a A coupon bond pays out 3% every year, with a principal of £1 and a maturity of five years. Decompose the coupon bond into a set of zero coupon bonds.

b A zero-coupon bond has a principal of £100 and matures in 4 years. The market price for the bond is £72. Calculate the yield to maturity, duration and convexity for the bond.

c Zero-coupon bonds are available with principal of £1 and the following maturities:

- 1 year with present value £0.93
- 2 years with present value £0.82
- 3 years with present value £0.74

Calculate the yield to maturities for the three bonds. Use an arbitrage argument to derive the forward rates that should apply between years 1 and 2 and years 2 and 3.

d Show that the variance of the optimal Markowitz portfolio is given by,

$$\text{var}(m) = \frac{Am^2 - 2Bm + C}{AC - B^2},$$

where m is the target return, and

$$A = 1^\top \Sigma^{-1} 1,$$
$$B = \mu^\top \Sigma^{-1} 1$$
$$C = \mu^\top \Sigma^{-1} \mu$$

where Σ is the covariance matrix, μ is the vector of expected returns and 1 is a vector of ones of appropriate dimension. Use the result above to show that the covariance of the minimum variance portfolio given by,

$$\min_m \ \text{var}(m)$$

with any other portfolio is $1/A$.

The four parts carry, respectively, 10%, 15%, 15%, and 60% of the marks.