COMPUTATIONAL FINANCE: 422

Course Outline

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(Slides courtesy of Daniel Kuhn)

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General Information

- Lecturer: Panos Parpas (Huxley Building, Room 357, email: p.parpas@imperial.ac.uk) or MS-Teams.
- Tutorial Helpers:
  - Francesco Borderi (f.borderi17@imperial.ac.uk)
  - Conor Mcmeel (c.mcmeel18@imperial.ac.uk)
- Lecture slides and tutorials are available on CATE/Materials
- There will be weekly tutorials and one assessed coursework; active participation is strongly encouraged!

Approx = 6h Lectures, 2h tutorials.
This week: 2h Lect. Today
2h Lect Friday

Prerequisites

Required:

- 233 - Computational techniques

  for computing students; a similar background is required for the other engineering students

Recommended:

- 343 - Operations research

  Students who have not taken this course are assumed to be familiar with mathematical programming (LP/QP)

Aims of the Course

~3 Nobel Prizes Models.

After this course, students should

- understand the basic concepts of quantitative finance and financial engineering;
- be aware of the major decision, hedging, and valuation problems in finance, know how to formulate these problems as mathematical models, and know several computational techniques to solve the arising models;
- be able to read the technical literature in computational finance and to undertake independent self-study (or research) in the future.

investment decisions reducing risk.

General Remarks

Please note that

- this course does not prepare you for a typical IT job in the financial industry;
- this course discusses tools that would be useful for a quantitative analyst;
- although every effort is made to present the concepts in an intuitive manner, this course may not be suitable for people who experience a discomfort when exposed to mathematical formalism.

Recommended Books I

- D.G. Luenberger, *Investment Science*, Oxford University Press, 1998.
  - Extremely well written; everyone should own that book.
- D.J. Higham, *An Introduction to Financial Option Valuation*, Cambridge University Press, 2004.
  - Very good intro to computational methods; Matlab code available from `http://personal.strath.ac.uk/d.j.higham/option_book.html`
- J. Hull, *Options, Futures, and other Derivatives*, Prentice Hall, 2012.
  - A classic text on derivatives.
- P. Wilmott, *Derivatives: the Theory and Practice of Financial Engineering*, Wiley, 1998.
  - Also a classic text on derivatives.

Recommended Books II

- P. Boyle and F. Boyle, *Derivatives: the Tools that Changed Finance*, Risk Books, 2001.
  - Very good introductory text; available freely from `www.thederivativesbook.com`.
- D. Duffie, *Dynamic Asset Pricing Theory*, Princeton University Press, 2001.
  - Standard text for doctoral students and researchers; more difficult to read than the other books in this list.
- T. Crack, *Heard on the Street: Quantitative Questions from Wall Street Job Interviews*, 2009.
  - Just for fun or to prepare for a job interview in a bank.

This course is mainly based on the books by Luenberger and Higham.

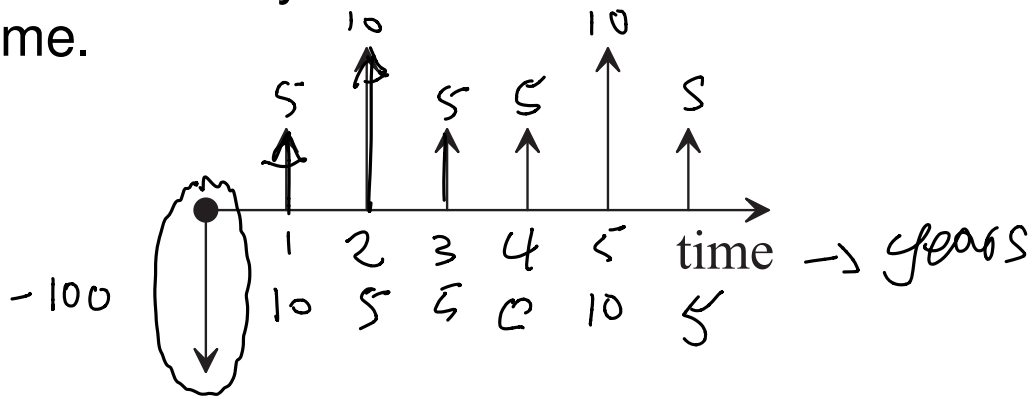
(Planned) Course Outline

- Introduction → Today
- Mathematical Preliminaries } Look at slides, Attempt Tutorial
- The Basic Theory of Interest
- Fixed-Income Securities
- Mean-Variance Portfolio Theory → Investment decisions (Markowitz Model)
- The Capital Asset Pricing Model ← CAPM
- General Principles ~ Risk
- Asset Price Dynamics
- Basic Options Theory
- Additional Options Topics

] Option Valuation.

Cash Flow Streams I

- An investment is the current commitment of resources (e.g. money) in order to achieve later benefits (hopefully more money).
- In most situations, the amount of money to be obtained later is uncertain.
- Broader interpretation: an investment is defined in terms of its resulting cash flow stream, that is, the amounts of money that will flow to and from an investor over time.



Cash Flow Streams II

- Which of two given cash flow streams should I prefer?
- How much would I be willing to pay for a given stream?
- Are two streams together worth more to me than the sum of their individual values?
- Given a collection of several cash flow streams, what is the most favorable combination of them?

Sometimes, the timing and the amounts of the cash flows in a stream are not fixed, but can be influenced by the investor.

$\Rightarrow$ Determination of suitable management strategies is also part of investment science.

$\Rightarrow$ One can view investment science as the tailoring of cash flow streams.

Investments and the Market

- Investment analysis is the process of examining alternatives and deciding which alternative is the most preferable.
- Investment problems differ from other decision problems in an important respect: they are carried out within the framework of a financial market.
- The financial market provides a basis for comparison.
- Important aspects are:
  - the comparison principle;
  - arbitrage;
  - dynamics;
  - risk aversion.

Example: Financial Option Pricing

Aim of this example:

- Provide a (preliminary) understanding of the basic principles of option pricing.

Learning outcomes:

- Students will be able to describe the characteristics of a European call option and
- to calculate its fair value by using on a binomial lattice model.

Betting on Coin Tosses

Basic proposition #1: You pay £1. I flip a coin.

- If it is heads, you get £3.
- If it is tails, you get nothing.

Basic proposition #2: You pay £1. I flip a coin.

- If it is heads, you get £1.
- If it is tails, you get £1, as well.

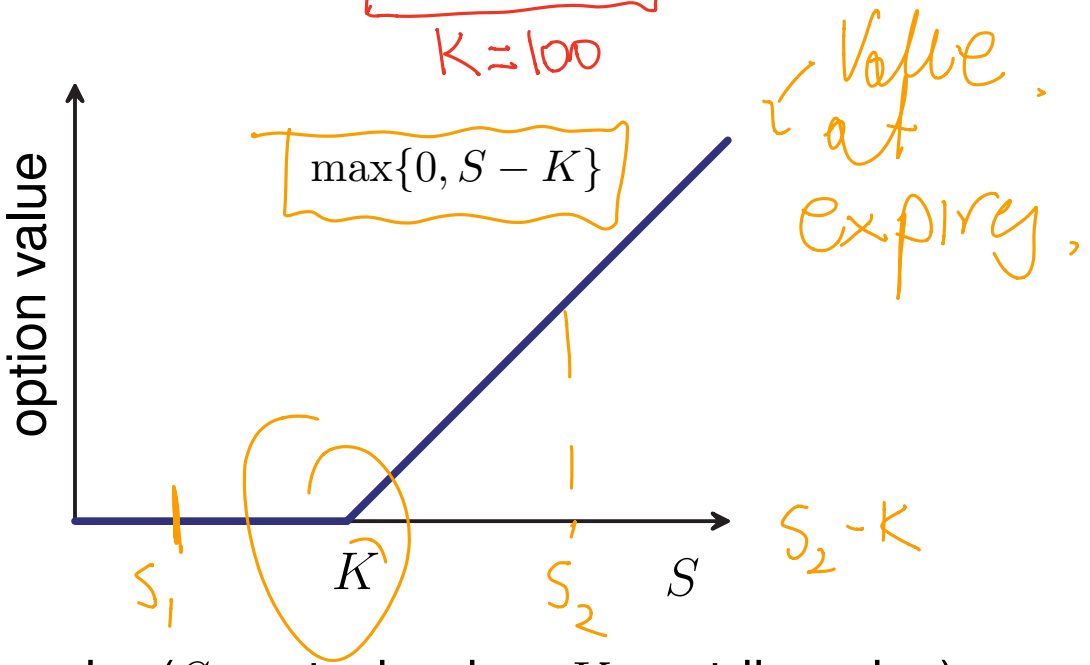
New proposition: I flip the coin twice.

- If at least one flip is heads, you get £9.
- If no flip is heads, you get nothing.

How much is this proposition worth?

European Call Options

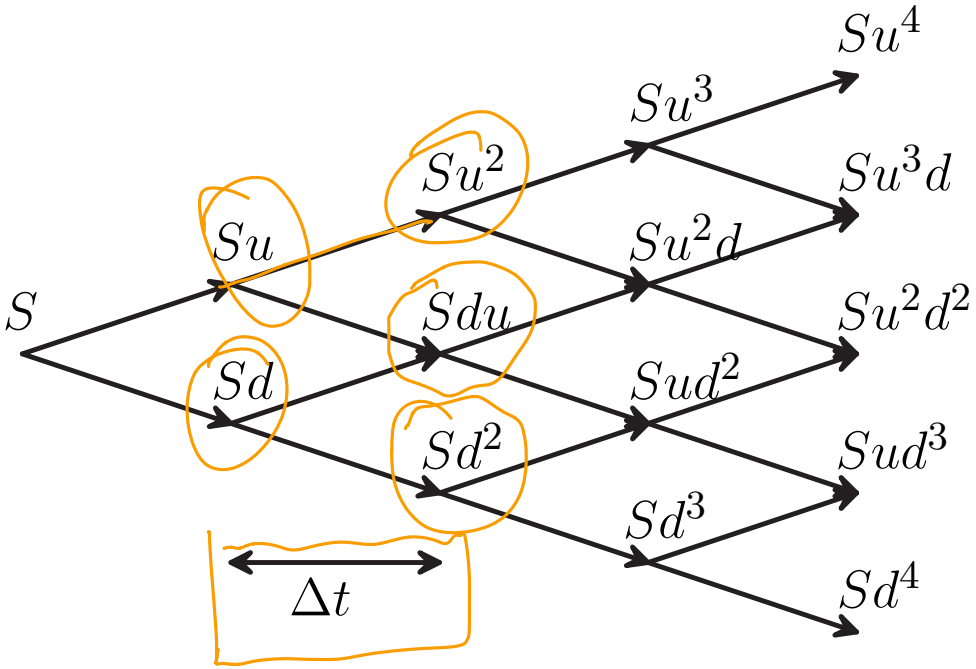
Definition: A European call option gives its holder the right (but not the obligation) to purchase form the writer a specific stock for a prescribed strike price at a future time.



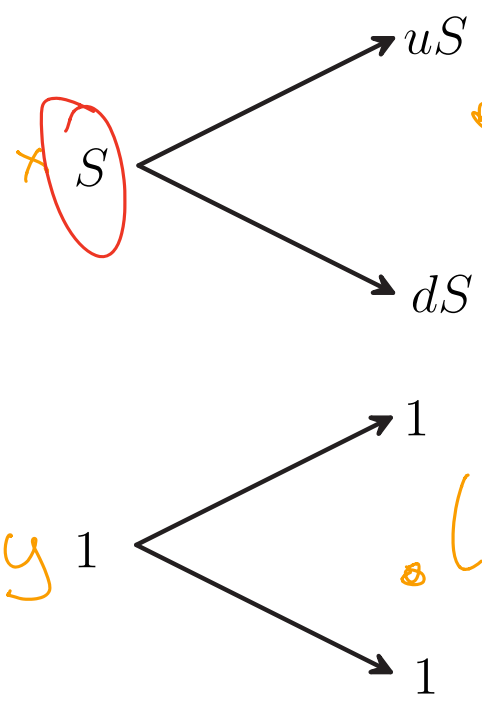
Payoff at expiry ($S =$ stock price, $K =$ strike price)

Binomial Lattice Model

Over a period of length Δt the stock price moves either up (with probability p) or down (with probability $1-p$).



Single-Period Binomial Options Pricing



$u = 1.1$
$d = 0.9$
$S = 100, K = 100$

Risky Asset

Prop #3

C

$C_u = \max\{uS - K, 0\} = 10$

Prop#3

$C_d = \max\{dS - K, 0\} = 0$

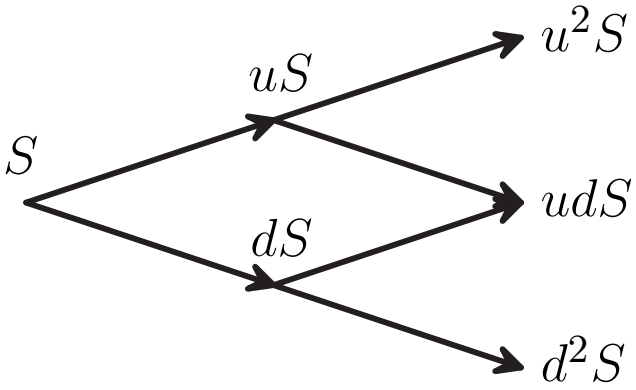
(Bank)

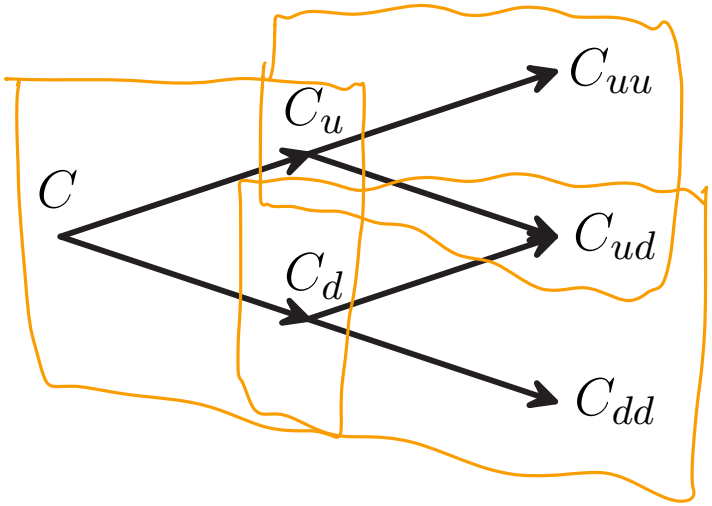
Prop #2

$$\left.\begin{aligned} ux + y &= C_u \\ dx + y &= C_d \end{aligned}\right\} \Rightarrow C = x + y = \frac{1-d}{u-d}C_u + \frac{u-1}{u-d}C_d$$

Multiperiod Binomial Options Pricing

The one-period solution can be extended to multiperiod options by working backward one step at a time.



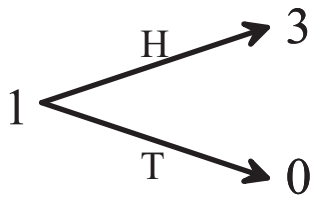


Terminal condition:

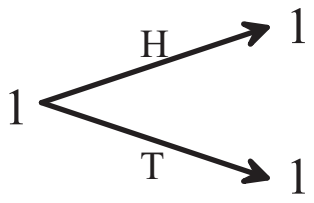
$$\begin{aligned} C_{uu} &= \max\{u^2S - K, 0\} \\ C_{ud} &= \max\{udS - K, 0\} \\ C_{dd} &= \max\{d^2S - K, 0\} \end{aligned}$$

Betting on Coin Tosses (Revisited)

Proposition #1



Proposition #2

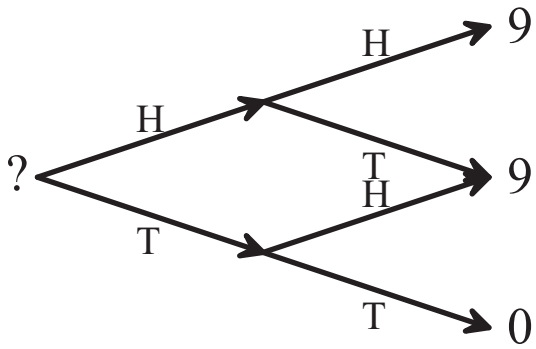


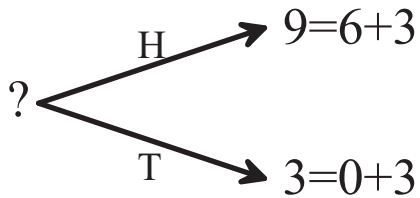
① Math. Prelim. → Review

② Attempt Tutorial 1

③ Review slides Understand example prop#3

New proposition





The value of the new proposition is £5.