Performance Engineering Tutorial
Scaling, Forecasting & Control

Exercise 1. Using Amdhal's law, determine the parallel speedup for the following programs:

- Program A: 40% serial execution, 60% parallel execution
- Program B: 20% serial execution, 80% parallel execution
- Program C: 0% serial execution, 100% parallel execution

Assume two scenarios:

- Vertical scaling from a machine with a single core to one with $n = 4$ cores.
- Vertical scaling from a machine with a single core to one with $n = 8$ cores.

Exercise 2. An autoscaling controller predicts the number of request arrivals A_t in timeslot t using an autoregressive AR(1) process. Assume A_t to be stationary. Suppose that the monitoring system reports the following statistics for A_t: mean $E[A_t] = 1$, variance $Var[A_t] = 4$, and lag-1 auto-covariance $K_1 = 2$. Fit the AR(1) process parameters to the data.

Exercise 3. For a stationary AR(1) process:
Question 3.1 Show that the variance of the time series is given by

$$Var[A_t] = \frac{\sigma_\epsilon^2}{1-\phi^2}, \qquad \forall t$$

Question 3.2 Show that the variance of the one-step ahead prediction is given by

$$Var[A_{t+1}|A_t] = \sigma_\epsilon^2$$

Exercise 4. An autoscaling controller predicts the number of request arrivals A_t in timeslot t using an autoregressive AR(1) process. Assume A_t to be stationary. Suppose that the monitoring system reports the following statistics for A_t: mean $E[A_t] = 1$, variance $Var[A_t] = 2$, and lag-1 auto-covariance $K_1 = 0.8$. Fit the AR(1) process parameters to the data.

Exercise 5. A network element incurs at time t a packet loss rate y_t. The loss can be adjusted using the sliding window size parameter u_t. The temporal correlations between the two signals are captured by the input-output model

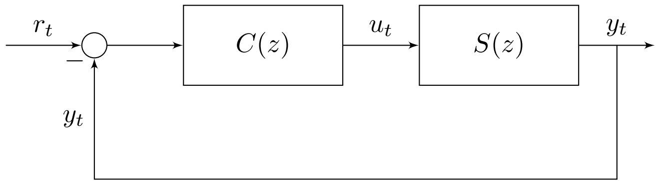
$$y_t = 0.4y_{t-1} + y_{t-2} - u_{t-1} - 2u_{t-2}$$

Let $Y(z) = \mathcal{Z}[y_t]$ and $U(z) = \mathcal{Z}[u_t]$ be the z-transforms for y_t and u_t, respectively. Determine the transfer function $H(z) = Y(z)/U(z)$ for the network protocol.

Exercise 6. A closed-loop controller can periodically adjust a server configuration option u_t, at discrete time instants $t = 1, 2, \ldots$. This option allows to control the server queue-length. Let r_t be the target queue-length level at time t and let y_t be the monitored queue-length level in the server. Denote by $S(z)$ the transfer function of the server and by $C(z)$ the transfer function of the controller. Assume the following assignments

$$S(z) = \frac{1}{4z+1} \qquad\qquad C(z) = \frac{2z}{3z-1}$$

and a block diagram



where $e_t = r_t - y_t$ is an error signal.

Question 6.1 Determine the transfer function for the entire system.

Question 6.2 State if the system is stable and, if so, estimate its settling time.

Question 6.3 Determine the long-term ratio between output signal y_t and input signal r_t.