IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2019

MEng Honours Degree in Electronic and Information Engineering Part IV
MEng Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
MSc in Computing Science
MSc in Computing Science (Specialist)
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C422

COMPUTATIONAL FINANCE

Friday 22nd March 2019, 14:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 **Fixed-Income Securities.**

a Consider a zero coupon bond with face value F, maturity date T, and yield y. Calculate the bond's value, duration, and convexity under continuous compounding.

b Consider n different fixed-income securities with prices $P_1, P_2, \ldots, P_n$ and convexities $C_1, C_2, \ldots, C_n$, respectively. Assume that all securities have the same yield. Prove that the convexity C of the portfolio consisting of these securities is given by

$$C = \sum_{i=1}^{n} \frac{P_i}{P} C_i \quad \text{where} \quad P = \sum_{i=1}^{n} P_i \,.$$

c You have an obligation to pay £2,000 in exactly 2 years from now. There are three different bonds in which you can invest (short selling is allowed). Relevant data about these bonds is provided in the table below. Find the portfolio that immunizes the obligation against first and second order changes in yield if continuous compounding is used.

	Obligation	Bond 1	Bond 2	Bond 3
Face Value	£2,000	£100	£100	£100
Coupon Rate	0%	0%	0%	0%
Maturity Date	2	1	3	5
Yield	10 %	10%	10%	10%

Hint: In addition to matching the prices and durations, you should also match the convexities.

The three parts carry, respectively, 25%, 25%, and 50% of the marks.

2 **Portfolio Optimization.**

a Consider a simple market consisting of two risky assets. Relevant data about the risky assets is provided in the following table.

	Price/Share	Mean Return	No. of Shares	StDev. of Return
Asset 1	£40	35%	2,000	12.5%
Asset 2	£20	25%	500	12.5%

The correlation between risky assets' rates of return amounts to 60%. Assume short selling is allowed and all investors are Markowitz mean-variance optimizers. The investment period is one year.

Determine the portfolio weights of the one efficient fund (on the capital market line) that contains only risky assets.

b Suppose that there are n assets in the market, and they are uncorrelated with each other. The rate of return of each of these assets has mean m and variance σ^2. Suppose that a portfolio is constructed by taking equal portions of n of these assets, i.e. the weight w_i of asset i is given by $1/n$. What is the overall return and variance of the portfolio?

c As in part (b) assume that there are n assets in the market, and that each asset has a rate of return with mean m and variance σ^2. But the assets are not uncorrelated, instead the covariance of the returns is given by $\text{cov}(r_i, r_j) = 0.3\sigma^2$ for $i \neq j$. What is the variance of the equally weighted portfolio?

The three parts carry, respectively, 50%, 25%, and 25% of the marks.

3 **Leasing an Oil Well.** Assume that the price for oil (in pounds per barrel) evolves according to the following binomial lattice model.

0	1	2	3
100	110	121	133.1
	90	99	108.9
		81	89.1
			72.9

Shown are the prices in n years from now, where $n = 0, 1, 2, 3$. From any non-terminal cell in the lattice we can either move to the right (by a single cell) with probability $p = 0.4$. Or we can move to the right and down (by a single cell) with probability $1 - p = 0.6$.

Consider an oil well that produces at most 1,000,000 barrels per year. The production of oil costs £95 per barrel. We assume that oil extracted from the well in a given year is sold at the price at the beginning of the year, but the corresponding cash flow occurs at the end of the year. Moreover, we assume that the yearly interest rate is constant at 5%.

a What is the fair value of a 2 year lease of the oil well?

b Suppose that it is possible to enhance the productivity of the oil well to 2,000,000 barrels per year. This enhancement would cost £3,000,000, and it would increase the production costs to £100 per barrel. This enhancement can be installed at the beginning of any year, and once in place it applies to all future years. The enhancement becomes the property of the oil well's original owner at the end of the lease. What is the fair value of the lease in the presence of this enhancement option?

The two parts carry, respectively, 40% and 60% of the marks.

4 **Random Walks.** Consider an additive random walk process over N periods of length Δt, which is defined through the recursion

$$z(t_{k+1}) = z(t_k) + \epsilon(t_k)(\Delta t)^{\alpha}, \quad t_k = k \times \Delta t, \quad 0 \le k < N,$$

where α is a positive constant. Assume that $z(0) = 0$ and that the disturbances $\epsilon(t_k)$ are independent and standard normally distributed.

a Choose any $j, k \in \{0, \ldots, N\}$ with $j < k$. Calculate the expected value of $z(t_k) - z(t_j)$.

b Calculate the variance of $z(t_k) - z(t_j)$.

c Describe the full distribution of $z(t_k) - z(t_j)$.

d Set $\Delta t = 1/N$ and consider the limit of the random variable $z(t_N)$ when N tends to infinity (that is, Δt tends to zero). Describe the limiting distribution and calculate its mean and variance.

The four parts carry equal marks.