Coursework:
60016 OPERATIONS RESEARCH

INSTRUCTIONS

Please read carefully:

1. The coursework consists of three questions. You are asked to answer to all of them.
2. You can do the coursework **individually or in group (max two students per group)**.
3. Make sure that each answer sheet has your name(s) and indicates the number of the question you are answering to.
4. Submit your answers in **electronic copy** before the deadline.
5. The deadline is on **19 November 2020 at 19:00**.
6. It is fine to complete the coursework by hand, scan it into a PDF, and upload it to CATE. You can alternatively decide to produce a latex or Word version of the coursework answer, provided that at the end you upload it in PDF format.

Question 1 (25% of the marks) *You are given the following linear program (LP)*

$$\max y = -2x_1 - x_2 - 6x_3 + x_4$$

subject to

$$\begin{aligned} 3x_1 + x_2 - x_5 &= 2 \\ x_2 + 4x_3 &= 4 \\ x_3 + 4x_4 &\leq 5 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

Consider two index sets $I = \{2, 4, 5\}$ *and* $I' = \{1, 2, 5\}$.

1.a) *Determine if* I *has an associated basic feasible solution. Give details of your calculations.*

1.b) *How would the answer at point 1.a) change for* I'*? Justify.*

1.c) *Suppose now that the objective function is modified to be* $\max y' = 9x_1 + 3x_2 - 3x_5$*, while constraints remain unchanged. Does the new LP have a unique optimal solution? Explain.*

Question 2 (25% of the marks) *Consider the following linear program (LP)*

$$\max y = 3x_1 + 2x_2$$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 3 \\ 3x_1 + x_2 &\leq 7 \\ x_1 - 2x_2 &\leq 0 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2.a) *Draw the feasible set on the (x_1, x_2) plane. Then, determine graphically the optimal solution and the optimal value for this LP.*

2.b) *Verify the optimal solution determined at point 2.a) using the simplex method.*

2.c) *Using pivoting operations on the optimal tableau obtained at point 2.b), determine if the optimal solution is associated to more than one index set.*

Question 3 (50% of the marks) *OilStreamOR produces oil at two refineries and ships it to an oil terminal in Finnart owned by a customer. The company refinery at Cairo can ship up to 1.2 million barrels/week, while the refinery in Haifa can ship up to 2.0 million barrels/week.*

Oil tankers departing from Cairo and Haifa must take an intermediate stop at either Malta or Gibraltar. A tanker first stopping in Malta can also stop a second time in Gibraltar, before reaching its destination. During the stops in Malta or Gibraltar, the oil cannot be transferred between tankers and cannot remain on site when the stop is over.

Shipments from Malta and Gibraltar can travel directly to Finnart, where they unload all of their oil. The oil demand in Finnart is estimated to be 2.8 million barrels/week.

Assume that the cost of transferring oil with a tanker between directly connected locations is £10 per barrel, if shipped with a tanker starting from Cairo, and £15 per barrel, if shipped with a tanker starting from Haifa. The sale price of a barrel is £40.

3.a) *Draw a directed graph to illustrate the allowed routes between refineries, intermediate stops, and the Finnart terminal.*

3.b) *Formulate a LP to maximize profit of OilStreamOR in meeting the oil demand at Finnart. (Do* not *solve the linear program. Also, note that since barrels do not need to be entirely full, a number of barrels can be treated as a continuous quantity.).*