C422: Computational Finance
Coursework

Exercise 1. Dividend Discount Model. The dividend discount model is a way of valuing a company based on the theory that a stock is worth the discounted sum of all of its future dividend payments. In other words, it is used to value stocks based on the net present value of the future dividends.

Assume that each stock of the company pays a dividend D_t at the end of every year, $t = 1, 2, 3, \ldots$, and denote by $r > 0$ the constant annual interest rate (use yearly compounding).

a

What is the price of the stock if the dividends have constant growth rate $g > 0$ (i.e., $D_{t+1} = (1+g)D_t$ for each $t \in \mathbb{N}$)? Your result should only depend on D_1, r and g and no longer contain an infinite sum. Hint: Calculate the net present value of the dividend stream.

b

What is the price of the stock if the dividends increase by a constant amount I every year (i.e., $D_{t+1} = D_t + I$ for each $t \in \mathbb{N}$)? Your result should only depend on D_1, r and I and no longer contain an infinite sum. Hint: Derive first a formula for the expression $\sum_{t=1}^{\infty} tx^t$ for $-1 < x < +1$. Note that this expression can be reduced to a geometric series by using the identity $tx^t = x\frac{\mathrm{d}}{\mathrm{d}x}x^t$.

20 marks and the two parts carry equal weight

Exercise 2. Fixed-Income Securities.

Consider a 10% coupon bond with a face value of £100 that matures in 2 years from now. Assume that there is one coupon payment per year and that the bond is absolutely risk free. The current spot rate curve is given in the following table (yearly compounding).

s_1	s_2	s_3
4%	6%	8%

a

What is the price of this bond?

b

Find an analytical formula for the yield of this bond and use your formula to compute the bond's yield numerically.

c

Compute the bond's Macauley duration.

d

Compute the bond's price and yield in one year from now assuming that the spot rates will evolve according to expectation dynamics. Assume further that the first coupon payment is still to be received.

20 marks and the four parts carry equal weight

Exercise 3. Portfolio Optimisation. Consider a market that contains n risky assets and one risk-free asset. Denote by $\bar{\mathbf{r}} = (\bar{r}_1, \ldots, \bar{r}_n)$ the vector of expected returns and by

$$\Sigma = \begin{pmatrix} \mathrm{cov}(r_1, r_1) & \cdots & \mathrm{cov}(r_1, r_n) \\ \vdots & \ddots & \vdots \\ \mathrm{cov}(r_n, r_1) & \cdots & \mathrm{cov}(r_n, r_n) \end{pmatrix}$$

the covariance matrix of the risky assets, respectively. Moreover, let r_f be the risk-free rate of return. Assume that Σ is strictly positive definite and therefore invertible.

a

What is the expected return $\bar{r}_P$ and the variance σ_P^2 of a portfolio that assigns weights $\mathbf{w} = (w_1, \ldots, w_n)$ to the risky assets and weight w_0 to the risk-free asset?

b

Consider an investor who wishes to maximise $\bar{r}_P - \frac{a}{2}\sigma_P^2$, where the constant $a > 0$ is called the risk-aversion parameter. Assume that short-selling is allowed. Formulate this investment problem as a quadratic program without constraints. Hint: Use the budget constraint to eliminate w_0.

c

Derive the optimality conditions for the optimisation problem formulated in part b and solve it analytically, that is, find an expression for the problem's optimal value and the optimal portfolio weights in terms of r_f, $\bar{\mathbf{r}}$ and Σ.

d

Characterise the optimal objective value and the optimal portfolio when the risk-aversion parameter a tends to $+\infty$.

e

Assume now that $\bar{\mathbf{r}}$ cannot be estimated exactly. Instead, only an unbiased estimator $\hat{\bar{\mathbf{r}}}$ is known, which satisfies $\mathrm{E}(\hat{\bar{\mathbf{r}}}) = \bar{\mathbf{r}}$. Show that an investor who uses $\hat{\bar{\mathbf{r}}}$ to approximate $\bar{\mathbf{r}}$ will—on average—overestimate the optimal value of the investment problem from part b. Assume that r_f and Σ are still known exactly. Hint: use Jensen's inequality, which states that $\mathrm{E}(f(\mathbf{X})) \geq f(\mathrm{E}(\mathbf{X}))$ for any convex function f and random vector $\mathbf{X}$.

The five parts carry, respectively, 2, 6, 6, 2, and 4 marks