We use the simplex method for the following linear programming problem:

```
        maximize 10 x1 - 57 x2 - 9 x3 - 24 x4
        subject to
                1/2 x1 - 11/2 x2 - 5/2 x3 + 9 x4  <= 0
                1/2 x1 -  3/2 x2 - 1/2 x3 +   x4  <= 0
                    x1                       <= 1
        x1,x2,x3,x4 >= 0
```

Here is the initial simplex tableau:

```
          x1      x2      x3      x4      x5      x6      x7
    x5    0.5    -5.5    -2.5      9       1       0       0       0
    x6    0.5    -1.5    -0.5      1       0       1       0       0
    x7      1       0       0      0       0       0       1       1
   obj   -10      57       9     24       0       0       0       0
```

The only possible entering variable is x1. There are two choices for departing variable: x5 and x6. I choose x5. The next simplex tableau is:

```
          x1      x2      x3      x4      x5      x6      x7
    x1      1     -11      -5     18       2       0       0       0
    x6      0       4       2     -8      -1       1       0       0
    x7      0      11       5    -18      -2       0       1       1
   obj      0     -53     -41    204      20       0       0       0
```

There are two possible entering variables: x2 and x3. I choose x2. The only possible choice for departing variable is x6. The next simplex tableau is:

```
          x1      x2      x3      x4      x5      x6      x7
    x1      1       0     0.5     -4   -0.75    2.75       0       0
    x2      0       1     0.5     -2   -0.25    0.25       0       0
    x7      0       0    -0.5      4    0.75   -2.75       1       1
   obj      0       0   -14.5     98    6.75   13.25       0       0
```

The only possible entering variable is x3. There are two choices for departing variable: x1 and x2. I choose x1. The next simplex tableau is:

```
          x1      x2      x3      x4      x5      x6      x7
    x3      2       0       1     -8    -1.5     5.5       0       0
    x2     -1       1       0      2     0.5    -2.5       0       0
    x7      1       0       0      0       0       0       1       1
   obj     29       0       0    -18     -15      93       0       0
```

There are two possible entering variables: x4 and x5. I choose x4. The only possible choice for departing variable is x2. The next simplex tableau is:

```
          x1      x2      x3      x4      x5      x6      x7
    x3     -2       4       1      0     0.5    -4.5       0       0
    x4   -0.5     0.5       0      1    0.25   -1.25       0       0
    x7      1       0       0      0       0       0       1       1
   obj     20       9       0      0   -10.5    70.5       0       0
```

The only possible entering variable is x5. There are two choices for departing variable: x3 and x4. I choose x3. The next simplex tableau is:

```
          x1      x2      x3      x4      x5      x6      x7
    x5     -4       8       2      0       1      -9       0       0
    x4    0.5    -1.5    -0.5      1       0       1       0       0
    x7      1       0       0      0       0       0       1       1
   obj    -22      93      21      0       0     -24       0       0
```

There are two possible entering variables: x1 and x6. I choose x6.
The only possible choice for departing variable is x4. The next
simplex tableau is:

	x1	x2	x3	x4	x5	x6	x7	
x5	0.5	-5.5	-2.5	9	1	0	0	0
x6	0.5	-1.5	-0.5	1	0	1	0	0
x7	1	0	0	0	0	0	1	1
obj	-10	57	9	24	0	0	0	0

This is precisely where we started! This shows that an infinite loop
is possible!

Now I'll start over from the beginning (which is where we are right
now) but this time I'll apply Bland's rule whenever there is no choice
of entering variable that will increase the objective function.
(Translation: if all possible choices of entering variables would lead
to no increase of the objective function then I will use Bland's rule
to choose the entering and departing variables...)

The only possible entering variable is x1 and it won't increase the
objective function. So I'll apply Bland's rule. There are two
choices for departing variable: x5 and x6. By Bland's rule, I choose
the departing variable with lower index. Since 5 < 6, I choose x5.
The next simplex tableau is:

	x1	x2	x3	x4	x5	x6	x7	
x1	1	-11	-5	18	2	0	0	0
x6	0	4	2	-8	-1	1	0	0
x7	0	11	5	-18	-2	0	1	1
obj	0	-53	-41	204	20	0	0	0

There are two possible entering variables: x2 and x3 and neither will
lead to an increase of the objective function. So I apply Bland's
rule and I choose x2. The only possible choice for departing variable
is x6. The next simplex tableau is:

	x1	x2	x3	x4	x5	x6	x7	
x1	1	0	0.5	-4	-0.75	2.75	0	0
x2	0	1	0.5	-2	-0.25	0.25	0	0
x7	0	0	-0.5	4	0.75	-2.75	1	1
obj	0	0	-14.5	98	6.75	13.25	0	0

The only possible entering variable is x3 and it won't lead to an
increase of the objective function. So I apply Bland's rule. There
are two choices for departing variable: x1 and x2. By Bland's rule, I
choose the departing variable with lower index. Since 1 < 2, I choose
x1. The next simplex tableau is:

	x1	x2	x3	x4	x5	x6	x7	
x3	2	0	1	-8	-1.5	5.5	0	0
x2	-1	1	0	2	0.5	-2.5	0	0
x7	1	0	0	0	0	0	1	1
obj	29	0	0	-18	-15	93	0	0

There are two possible entering variables: x4 and x5 and neither will
lead to an increase of the objective function. So I apply Bland's
rule and I choose x4. The only possible choice for departing variable
is x2. The next simplex tableau is:

	x1	x2	x3	x4	x5	x6	x7	
x3	-2	4	1	0	0.5	-4.5	0	0
x4	-0.5	0.5	0	1	0.25	-1.25	0	0
x7	1	0	0	0	0	0	1	1
obj	20	9	0	0	-10.5	70.5	0	0

The only possible entering variable is x5 and it won't lead to an
increase of the objective function. So I apply Bland's rule. There
are two choices for departing variable: x3 and x4. By Bland's rule, I

choose the departing variable with lower index. Since 3 < 4, I choose
x3. The next simplex tableau is:

```
        x1      x2      x3      x4      x5      x6      x7
  x5    -4       8       2       0       1      -9       0       0
  x4   0.5    -1.5    -0.5       1       0       1       0       0
  x7     1       0       0       0       0       0       1       1
  obj  -22      93      21       0       0     -24       0       0
```

There are two possible entering variables: x1 and x6 and neither will
lead to an increase of the objective function. So I apply Bland's
rule and I choose x1. The only possible choice for departing variable
is x4. (NOTE: THIS IS THE FIRST TIME WE DID SOMETHING DIFFERENTLY
FROM THE INFINITE LOOP CASE.) The next simplex tableau is:

```
        x1      x2      x3      x4      x5      x6      x7
  x5     0      -4      -2       8       1      -1       0       0
  x1     1      -3      -1       2       0       2       0       0
  x7     0       3       1      -2       0      -2       1       1
  obj    0      27      -1      44       0      20       0       0
```

The only possible entering variable is x3 and it will lead to an
increase of the objective function. The departing variable is x7.
The next simplex tableau is:

```
        x1      x2      x3      x4      x5      x6      x7
  x5     0       2       0       4       1      -5       2       2
  x1     1       0       0       0       0       0       1       1
  x3     0       3       1      -2       0      -2       1       1
  obj    0      30       0      42       0      18       1       1
```

And we've found an optimal solution! (Blands rule!) x1 = 1, x2 = 0, x3 = 1, x4 = 0,
x5 = 2, x6 = 0, x7 = 0, objective function = 1

Here's another example where an infinite loop is possible.

```
        maximize - x1 + 7 x2 + x3 + 2 x4
        subject to
                x1 +        x2 +      x3 +     x4  <= 1
              1/2 x1 - 11/2 x2 - 5/2 x3 +   9 x4  <= 0
              1/2 x1 -  3/2 x2 - 1/2 x3 +     x4  <= 0
        x1,x2,x3,x4 >= 0
```

Here is the initial simplex tableau:

```
        x1      x2      x3      x4      x5      x6      x7
  x5     1       1       1       1       1       0       0       1
  x6   0.5    -5.5    -2.5       9       0       1       0       0
  x7   0.5    -1.5    -0.5       1       0       0       1       0
  obj    1      -7      -1      -2       0       0       0       0
```

my choice of entering variable: x4
my choice of departing variable: x7

```
        x1      x2      x3      x4      x5      x6      x7
  x5   0.5     2.5     1.5       0       1       0      -1       1
  x6    -4       8       2       0       0       1      -9       0
  x4   0.5    -1.5    -0.5       1       0       0       1       0
  obj    2     -10      -2       0       0       0       2       0
```

my choice of entering variable: x3
my choice of departing variable: x6

```
        x1      x2      x3      x4      x5      x6      x7
  x5   3.5    -3.5       0       0       1   -0.75    5.75       1
  x3    -2       4       1       0       0     0.5    -4.5       0
  x4  -0.5     0.5       0       1       0    0.25   -1.25       0
```

```
obj     -2      -2       0       0       0       1      -7       0
```

my choice of entering variable: x2
my choice of departing variable: x4

```
         x1      x2      x3      x4      x5      x6      x7
  x5      0       0       0       7       1       1      -3       1
  x3      2       0       1      -8       0    -1.5     5.5       0
  x2     -1       1       0       2       0     0.5    -2.5       0
 obj     -4       0       0       4       0       2     -12       0
```

my choice of entering variable: x1
my choice of departing variable: x3

```
         x1      x2      x3      x4      x5      x6      x7
  x5      0       0       0       7       1       1      -3       1
  x1      1       0     0.5      -4       0   -0.75    2.75       0
  x2      0       1     0.5      -2       0   -0.25    0.25       0
 obj      0       0       2     -12       0      -1      -1       0
```

my choice of entering variable: x7
my choice of departing variable: x2

```
         x1      x2      x3      x4      x5      x6      x7
  x5      0      12       6     -17       1      -2       0       1
  x1      1     -11      -5      18       0       2       0       0
  x7      0       4       2      -8       0      -1       1       0
 obj      0       4       4     -20       0      -2       0       0
```

my choice of entering variable: x6
my choice of departing variable: x1

```
         x1      x2      x3      x4      x5      x6      x7
  x5      1       1       1       1       1       0       0       1
  x6    0.5    -5.5    -2.5       9       0       1       0       0
  x7    0.5    -1.5    -0.5       1       0       0       1       0
 obj      1      -7      -1      -2       0       0       0       0
```

This is precisely where we started! This shows that an infinite loop
is possible.

Now I'll start over from the beginning (which is where we are right
now) but this time I'll apply Bland's rule whenever there is no choice
of entering variable that will increase the objective function.
(Translation: if all possible choices of entering variables would lead
to no increase of the objective function then I will use Bland's rule
to choose the entering and departing variables...)

There are three possible entering variables: x2, x3, and x4. If x2
enters then x5 will depart and the objective function will increase by
7. If x3 enters then x5 will depart and the objective function will
increase by 1. If x4 enters then I have a choice of departing
variables (x6 or x7) both of which would lead to no increase of the
objective function.

Since x2 will lead to the largest increase, I choose x2 as entering
and x5 as departing. (Note that I didn't actually use Bland's rule
since Bland's rule only applies when there's no choice of entering
variable that will lead to an increase in the objective function.)
The next tableau is:

```
         x1      x2      x3      x4      x5      x6      x7
  x2      1       1       1       1       1       0       0       1
  x6      6       0       3    14.5     5.5       1       0     5.5
  x7      2       0       1     2.5     1.5       0       1     1.5
 obj      8       0       6       5       7       0       0       7
```

And we've found an optimal solution! x1 = 0, x2 = 1, x3 = 0, x4 = 0,
x5 = 0, x6 = 11/2, x7 = 3/2, objective function = 7.