FINAL

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2018

MEng Honours Degree in Electronic and Information Engineering Part IV
MEng Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
MSc in Computing Science
MSc in Computing Science (Specialist)
MRes in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C422

COMPUTATIONAL FINANCE

Wednesday 21 March 2018, 14:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 **Term structure of interest rates.**

a Use an arbitrage argument to explain how forward rates are derived from the spot-rate curve.

b If the spot rates for 1 and 2 years are $s_1 = 6.3\%$ and $s_2 = 6.9\%$ what is the forward rate ($f_{1,2}$) for years 1 to 2?

c Show that if the spot rate curve is flat, (i.e. $s(t) = r$ for all t) then all forward rates are also equal to r.

d Consider two 5-year bonds: one has a 9% coupon and sells for £101, the other has a 7% coupon and sells for £93.20. Both have a face value of £100. What is the fair price for a five year zero-coupon bound with a face value of £100?

e Describe how duration and convexity are used to reduce the risks associated with bonds?

The five parts carry, respectively, 15%, 10%, 25%, 25%, and 25% of the marks.

2 **The Capital Asset Pricing Model (CAPM).**

a Explain the difference between the capital market line, and the security market line.

b Consider a market (M) in which there are only two risk assets, A and B, and a risk free asset F. The two risky assets have equal market capitalisation. Find the variance of the market σ_M^2, the covariance of the market with A ($\sigma_{A,M}$) and B $\sigma_{B,M}$. Also compute the β of asset A and B.

c Assume that the expected rate of return on the market portfolio is 23% and the risk free rate is 7%. Assume that the market portfolio is efficient.

i) Specify the equation for the capital market line.

ii) If you invest £300 in the risk free asset, and £700 in the market portfolio, how much money should you expect to have at the end of the year?

iii) Suppose you have £1000 to invest, specify the investment position that will provide an expected return of 39%.

iv) According to CAPM what is the standard- deviation of the portfolio above?

(The four sub-parts carry equal marks.)

d Briefly explain how the CAPM model can be used to perform net-present-value (NPV) calculations.

The four parts carry, respectively, 20%, 20%, 40%, and 20% of the marks.

3 **General Principles of Risk.**

a Explain the difference between Type A and Type B arbitrage.

b Explain why utility functions are assumed to be increasing and concave.

c Suppose that an investor has the following utility function,

$$U(x) = x - 0.04x^2.$$

Two investment possibilities are available. The first one is risk free and has a £5 payoff, the second one is based on a toss of a (fair) coin. When the outcome of the coin toss is heads, the payoff is £10, and when the outcome is tails, the payoff is £0.

i) Evaluate the expected utility of the two alternatives. Which one is to be preferred?

ii) Find the certainty equivalent to the risky investment.

(The two sub-parts carry equal marks.)

d $U(x)$ is a utility function with Arrow-Pratt *absolute* risk aversion coefficient $a(x)$. Let $V(x) = c + bU(x)$. What is the risk aversion coefficient of $V(x)$?

e The Arrow-Pratt *relative* risk aversion coefficient is defined by

$$\mu(x) := -\frac{xU''(x)}{U'(x)}$$

Show that this coefficient is constant for the utility functions $U(x) = \ln(x)$ and $U(x) = \gamma x^\gamma$.

The five parts carry, respectively, 10%, 20%, 30%, 20%, and 20% of the marks.

4 **Option Pricing.**
Consider a family of call options on a non-dividend-paying stock. All options are identical except from the strike price. The value of the call with strike price K is denoted by C(K). Prove the following three general relations in a market with no arbitrage opportunities.

a $K_2 > K_1$ implies $C(K_1) \geq C(K_2)$.

b $K_2 > K_1$ implies $K_2 - K_1 \geq C(K_1) - C(K_2)$.

c $K_3 > K_2 > K_1$ implies,

$$C(K_2) \leq \frac{K_3 - K_2}{K_3 - K_1} C(K_1) + \frac{K_2 - K_1}{K_3 - K_1} C(K_3)$$

The three parts carry, respectively, 20%, 30%, and 50% of the marks.