Tutorial 6 – 60016 Operations Research

Game theory

Exercise 1. Consider the following setting where a man and a woman want to decide if they will go to the opera, to the football match or stay home. This game is known as the "battle of the sexes" game.

		Man				
		Opera	football	home		
	Opera	5	-10	3		
Woman	football	9	4	5		
	home	4	2	3		

For the game,

- 1. Is there a dominant strategy equilibrium? If so, find it.
- 2. Is there a pure Nash equilibrium? If so, find it.
- 3. Write down the linear program that the row player (woman) has to solve in order to find her best mixed strategy Nash equilibrium. Do not solve the linear program.

Exercise 2. Consider the following two-person zero-sum game

		Column Player			
		A	В	С	
Row Player	A	1	-1	17	
	В	-1	15	-3	
	С	5	9	9	

1. Does this game have a dominant strategy equilibrium? If so, find it. If not, explain why.

2. Does the game have a pure Nash equilibrium? If so, find it. If not, explain why.

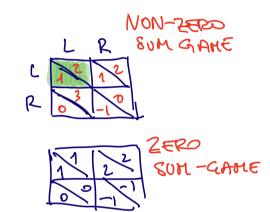
Column Player

3. Formulate the linear program that the column player must solve in order to find his best mixed strategy (you are not required to solve this problem).

BACK@11:30

COHNENT on OR MENTI. COH 9181452 Exercise 3. Consider the following Two-Player Zero-Sum game.

			c_1				
MAX		r_1	-4	0	5	-1	2
	Row Player	r_2	4	9	-5	1	-5
1 111/2		r_3	3	-3	0	-7	5
		r_4	7	2	6	0	5
		r_5	-8	0 9 -3 2 -4	8	-5	9



- 1. Is there a dominant strategy equilibrium? If so, find it. If not, can we at least remove dominated strategies from the problem?
- 2. Is there a pure Nash equilibrium? If so, find it.
- 3. Write down the linear program that the row player has to solve. Do not solve the problem.
- 4. Write down the linear program that the column player has to solve. Do not solve the problem.

Bonus exercise. During Monday's lecture we have discussed the minimax theorem. A consequence of this result is that every two-players zero-sum game with finite actions admits a mixed Nash equilibrium. Prove this statement.

[Hint: Let A be the matrix of payoffs/costs. To prove the claim it is instrumental to consider a pair of probability distributions $(p_1^*, \ldots, p_m^*, q_1^*, \ldots, q_n^*) = (p^*, q^*)$ constructed as follows

$$p^* \in \arg\max_{p \in \Delta_m} \left(\min_{q \in \Delta_n} V(p,q) \right), \qquad q^* \in \arg\min_{q \in \Delta_n} \left(\max_{p \in \Delta_m} V(p,q) \right),$$

where $\Delta_m = \{(p_1, \dots, p_m) \text{ s.t. } \sum_{i=1}^m p_i = 1, \text{ and } p_i \geq 0 \ \forall i = 1, \dots, m\}$ (similarly for Δ_n), and where $V(p,q) = \sum_{i=1}^m \sum_{j=1}^n p_i q_j a_{ij}$. The statement is shown, if you can prove that (p^*, q^*) is a mixed Nash equilibrium, i.e., if you prove that $V(p^*, q^*) \leq V(p^*, q^*)$ for all alternative distributions p, q.]

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C2 donninoted by C4, so revove C2

Ils dominated by ILs, so remove Is

Reduced payoff (costs motivis

looked et Best-worst cose choice -> J24, C4 Q: 10 this a pure NE?

A; now player would wout to devicte =) (24, C4)
Us not a PNE.

Az: look at volue for Ployer row player column

Since VolueRp + VolueCp => 7 PNE

COMMENT "if Volules = Volume Cp =)] PNE"?

statement for notes

7 PNE => Voluer ore Sound