IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2017

BEng Honours Degree in Computing Part III
BEng Honours Degree in Electronic and Information Engineering Part III
MEng Honours Degree in Electronic and Information Engineering Part III
MEng Honours Degree in Mathematics and Computer Science Part IV
BEng Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degrees in Computing Part III
MSc in Computing Science (Specialist)
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C343

OPERATIONS RESEARCH

Friday 16 December 2016, 14:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 a You are given the following linear programming (LP) problem:

$$\min z = 3x_1 + x_2$$

subject to

$$\begin{aligned} 5x_1 + 5x_2 &\geq 15 \\ -2x_1 - x_2 &\geq -3 \\ x_1 + x_2 &= 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

i) Using Phase 1 of the simplex algorithm, determine an initial basic feasible solution for this LP.

ii) Write the initial tableau for Phase 2. (You do *not* need to solve the Phase 2 problem.)

iii) State if the initial basic feasible solution found in point i) is degenerate. Use both algebraic and geometric considerations to justify your answer.

b i) Discuss the maximum number of iterations for the standard simplex algorithm for a general problem with m rows and n columns. Include in your answer a comment on the significance of the Klee-Minty cube.

ii) Prove finite termination of the standard simplex algorithm for linear programs without degenerate basic feasible solutions. Then, give an example of Bland's rule.

The two parts carry, respectively, 55% and 45% of the marks.

2 a PetrolInc extracts oil at two sites and ships it to customers in London and Belfast. Site 1 can ship up to 120,000 barrels/month, and Site 2 up to 200,000 barrels/month.

Shipments must take an intermediate stop at either Malta or Gibraltar. The only exception to this rule is that Site 2 is also allowed to ship directly to Belfast without stops.

Shipments from Malta and Gibraltar can travel directly to London or Belfast, or travel through both cities in any order. In the last case, they unload half of the oil in each city. Otherwise, all the oil is unloaded at the final destination.

The oil demand is estimated to be at least 160,000 barrels/month in London and at least 140,000 barrels/month in Belfast. Assume that the cost of shipping between directly connected locations is £10/barrel and sale price is £40/barrel.

i) Draw a directed graph to illustrate the possible shipping routes for the oil.

ii) Formulate a linear program to maximize profit in meeting the oil demands of London and Belfast. (Do *not* solve the linear program).

b Formulate the following optimization problem as a linear program in standard form (you are *not* required to solve it).

$$\min \frac{10x_1 + 20x_2 - 10}{5 + x_1 + x_2}$$

subject to

$$\begin{aligned} |x_1 + 2x_2| &\leq 10 \\ x_1, x_2 &\geq 0 \end{aligned}$$

The two parts carry, respectively, 60% and 40% of the marks.

3 Game Theory & Duality

a Consider the payoff matrix of a two-player zero-sum game. The two players are Hawkeye (row player) and Black Widow (column player). The elements correspond to the reward of the row player (Hawkeye).

		Black Widow		
		1	2	3
Hawkeye	1	2	4	6
	2	3	1	5

i) Determine whether any of the column or row strategies are dominated. Delete these columns and/or rows from the payoff matrix.

ii) Derive the linear programming problem for Black Widow's strategy. You do not need to solve this LP.

iii) Derive the linear programming problem for Hawkeye's strategy. You do not need to solve this LP.

iv) Does this game have a Nash equilibrium in mixed strategies? Do not determine any equilibrium, but justify your answer.

b For a given payoff matrix A, suppose we add a constant c to every element in the matrix A. Denote the new game matrix by A'.

i) Show that A and A' have the same optimal strategies for both the row and column player.

ii) Show that the optimal value of A is equal to the optimal value of A' plus c:

$$V_{A'}^* = V_A^* + c,$$

where V_A^* and $V_{A'}^*$ are the optimal solutions to the row player's linear program for payoff matrix A and A', respectively.

The two parts carry, respectively, 60% and 40% of the marks.

4 **Integer Programming Formulations & Branch & Bound**

a Express the following constraints using integer variables.

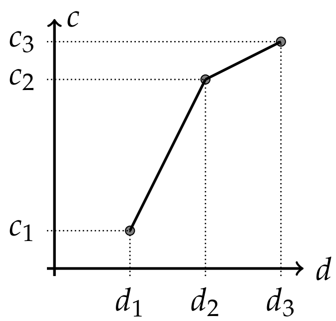
i) Satisfy at least k of m constraints:

$$a_1^{\mathrm{T}} x \leq b_1,\ a_2^{\mathrm{T}} x \leq b_2,\ \ldots,\ a_m^{\mathrm{T}} x \leq b_m$$

ii) If one constraint evaluates to true, force a second constraint to also evaluate as true.

$$a_1^{\mathrm{T}} x \leq b_1 \implies a_2^{\mathrm{T}} x \leq b_2$$

iii) Suppose that we are given the three data points (d_1, c_1), (d_2, c_2), (d_3, c_3) illustrated in the figure. Using integer variables, develop constraints restricting the feasible set to the piecewise linear function shown in the figure.



b Solve the following integer programming problem using the branch-and-bound method.

$$\begin{array}{ll} \min & x_0 = 4x_1 - 6x_2 \\ \text{s.t.} & \quad -x_1 + x_2 \leq 1 \\ & \quad x_1 + 3x_2 \leq 9 \\ & \quad 3x_1 + x_2 \leq 15 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{N}_0 \end{array}$$

Hint: The optimal solution of the LP without the integer constraints is $x_1 = 1.5, x_2 = 2.5, x_0 = -9$

The two parts carry, respectively, 60% and 40% of the marks.