IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2019-2020

MSc in Artifical Intelligence
MSc in Computing Science (Specialist)
MSc in Advanced Computing
MEng Honours Degrees in Computing Part IV
MEng Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degree in Electronic and Information Engineering Part IV
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C422

COMPUTATIONAL FINANCE

Wednesday 11th December 2019, 14:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 **Price Dynamics.**

a Describe the additive model for asset price dynamics and state its disadvantages as a model for asset prices. Explain how the multiplicative model can address these shortcomings.

b A stock with current value $S(0) = 100$ has an expected growth rate of its logarithm of $\nu = 12\%$ and a volatility of that growth rate of $\sigma = 20\%$. Find suitable parameters of a binomial lattice representing this stock with basic period length 3 months. Draw the lattice for a period of 6 months (thus comprising two basic periods) and calculate the stock prices in all the nodes. Also, calculate the probabilities of attaining the various end nodes of the lattice.

c A stock price S is governed by the model

$$\ln S(k+1) = \ln S(k) + w(k)$$

where the period length is 1 month. Let $\nu = \mathrm{E}[w(k)]$ and $\sigma^2 = \mathrm{var}[w(k)]$ for all k. Now suppose the basic period length is changed to 1 year. Then the model is

$$\ln S(K+1) = \ln S(K) + W(K)$$

where each movement in K corresponds to 1 year. What is the natural definition of $W(K)$? Show that $\mathrm{E}[W(K)] = 12\nu$ and $\mathrm{var}[W(K)] = 12\sigma^2$.

The three parts carry, respectively, 25%, 35%, and 40% of the marks.

2 **Derivative Pricing.**

a Show that European and American call options have the same price. Explain why the same argument cannot be used for European and American put options.

b A forward contract on an asset is a financial contract to purchase or sell a specific amount of the asset at a specific price and time in the future. Use an arbitrage argument to derive the arbitrage free forward price F of an asset for delivery at time T given that the price of the asset today is S.

The two parts carry equal marks.

3 **General Principles of Risk.**

a The quadratic utility function is given by,

$$U(x) = ax - \frac{1}{2}bx^2,$$

For what range of x, a, and b is $U(x)$ a valid utility function and why? You may assume that x and a are positive.

b Using the quadratic utility function above explain how it models risk-averse, risk-neutral and risk-seeking investors.

c Explain the relationship between the mean-variance approach to investment and the quadratic utility function.

The three parts carry, respectively, 25%, 25%, and 50% of the marks.

4 **Zero-Beta Assets.** Consider a market of n risky assets where short-selling is allowed. Let $\mathbf{w}_0 \in \mathbb{R}^n$ be the vector of portfolio weights corresponding to the minimum variance point in the feasible region. Let $\mathbf{w}_1 \in \mathbb{R}^n$ be any other portfolio on the efficient frontier. Define r_0 and r_1 to be the corresponding (random) returns.

a There is a formula for the covariance σ_{01} of r_0 and r_1 of the form $\sigma_{01} = A\sigma_0^2$. Find the constant A.

Hint: Consider the portfolios $(1-\alpha)\mathbf{w}_0 + \alpha\mathbf{w}_1$, *and consider small variations of the variance of such portfolios near* $\alpha = 0$.

b Corresponding to the portfolio $\mathbf{w}_1$ there is a portfolio $\mathbf{w}_z$ on the minimum-variance set that has zero beta with respect to $\mathbf{w}_1$. In other words, its return is uncorrelated with that of $\mathbf{w}_1$, that is, $\sigma_{1,z} = 0$. This portfolio can be expressed as $\mathbf{w}_z = (1-\alpha)\mathbf{w}_0 + \alpha\mathbf{w}_1$. Find the value of α.

c Describe qualitatively where the portfolio $\mathbf{w}_z$ is situated on the minimum-variance set.

d If there is no risk-free asset, it can be shown that assets satisfy the generalised CAPM formula is,

$$\bar{r}_i - \bar{r}_z = \beta_{iM}(\bar{r}_M - \bar{r}_z),$$

where the subscript M denotes the market portfolio and $\bar{r}_z$ is the expected rate of return on the portfolio that has zero beta with the market portfolio. Suppose that the expected returns on the market and the zero-beta portfolio are 15% and 9%, respectively. Suppose that stock i has a correlation coefficient with the market of 0.5. Assume also that the standard deviation of the returns of the market and stock i are 15% and 50%, respectively. Find the expected return of stock i.

The four parts carry, respectively, 30%, 30%, 20%, and 20% of the marks.