Tutorial 7 – 60016 Operations Research

Cutting Plane Methods, Branch & Bound

Exercise 1

Solve the following problem with the Gomory cutting plane approach

$$\begin{aligned} \underset{x_1,x_2}{\text{maximise}} \quad & y = x_1 + 4x_2 \\ \text{subject to} \quad & 2x_1 + 4x_2 \leq 7 \\ & 10x_1 + 3x_2 \leq 14 \\ & x_1, x_2 \geq 0,\ x_1, x_2 \in \mathbb{N}_0 \end{aligned}$$

Exercise 2

Consider the knapsack constraint set:

$$\begin{aligned} 11x_1 + 6x_2 + 6x_3 + 5x_4 + 5x_5 + 4x_6 + x_7 \leq 19 \\ \mathbf{x} \in \{0,1\}^7. \end{aligned}$$

For each of the following inequalities, identify whether or not they are valid knapsack cover cuts, and explain why. For all valid knapsack covers, identify whether or not they are minimal.

- $x_4 + x_5 + x_6 \leq 2$
- $x_1 + x_2 + x_6 \leq 2$
- $x_2 + x_3 + x_6 + x_7 \leq 3$
- $x_2 + x_4 + x_5 + x_6 \leq 3$
- $x_1 + x_3 + x_4 + x_5 \leq 3$
- $x_2 + x_3 + x_4 + x_5 + x_6 \leq 4$

Exercise 3

Solve the following problem using the branch and bound method

$$\begin{aligned} \underset{x_1,x_2}{\text{maximise}} \quad & y = x_1 + 2x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \leq 7 \\ & -x_1 + x_2 \leq 3 \\ & x_1, x_2 \geq 0,\ x_1, x_2 \in \mathbb{N}_0 \end{aligned}$$

Exercise 4

Solve the following problem with the Gomory cutting plane approach

$$\begin{aligned}
\underset{x_1,x_2}{\text{maximise}} \quad & y = 3x_1 + 4x_2 \\
\text{subject to} \quad & \tfrac{2}{5}x_1 + x_2 \leq 3 \\
& \tfrac{2}{5}x_1 - \tfrac{2}{5}x_2 \leq 1 \\
& x_1, x_2 \geq 0,\ x_1, x_2 \in \mathbb{N}_0
\end{aligned}$$

Exercise 5

Solve the following problem with the Gomory cutting plane approach. Note that both the coefficients and right-hand sides are not integral.

$$\begin{aligned}
\underset{x_1,x_2}{\text{maximise}} \quad & y = 5x_1 + 6x_2 \\
\text{subject to} \quad & 0.2x_1 + 0.3x_2 \leq 1.8 \\
& 0.2x_1 + 0.1x_2 \leq 1.2 \\
& 0.3x_1 + 0.3x_2 \leq 2.4 \\
& x_1, x_2 \geq 0,\ x_1, x_2 \in \mathbb{N}_0
\end{aligned}$$

Exercise 6

Consider the following problem

$$\begin{aligned}
\underset{x_1,x_2}{\text{maximise}} \quad & y = 5x_1 + x_2 \\
\text{subject to} \quad & -x_1 + 2x_2 \leq 4 \\
& x_1 - x_2 \leq 1 \\
& 4x_1 + x_2 \leq 12 \\
& x_1, x_2 \geq 0,\ x_1, x_2 \in \mathbb{N}_0
\end{aligned}$$

- Solve this problem graphically.
- Solve LP relaxation. Round this solution to the nearest integer solution and check whether it is feasible. Then enumerate all the rounded solutions, check them for feasibility and calculate y for those that are feasible. Are any of these feasible rounded solutions optimal for the IP problem?