Tutorial 4 – 60016 Operations Research

Two-Phase Simplex Method

Exercise 1 *Solve the following LP problem*[1]*:*

$$\max\ y = 4x_1 + 3x_2$$

subject to:

$$\begin{array}{rcrcr} 3x_1 & + & 4x_2 & \leq & 12 \\ 3x_1 & + & 3x_2 & \leq & 10 \\ 4x_1 & + & 2x_2 & \leq & 8 \end{array}$$

and

$$x_1 \geq 0, x_2 \geq 0.$$

Exercise 2 *Consider the following LP similar to the problem in Exercise 1:*

$$\max\ y = 4x_1 + 3x_2$$

subject to:

$$\begin{array}{rcrcr} 3x_1 & + & 4x_2 & \leq & 12 \\ 3x_1 & + & 3x_2 & \leq & 10 \\ 4x_1 & + & 2x_2 & \leq & 8 \\ x_1 & + & x_2 & \geq & 1 \end{array}$$

and

$$x_1 \geq 0, x_2 \geq 0.$$

Exercise 3 *Solve the following LP problem:*

$$\max\ y = 4x_1 + 3x_2$$

subject to:

$$\begin{array}{rcrcr} 3x_1 & + & 4x_2 & \leq & 12 \\ 5x_1 & + & 2x_2 & \leq & 8 \\ x_1 & + & x_2 & \geq & 5 \end{array}$$

and

$$x_1 \geq 0, x_2 \geq 0.$$

[1] This is a problem that can be solved also with the standard simplex method, we will later compare in Exercise 2 the solution against the one obtained with the two-phase algorithm.

Exercise 4 (Exam 2018, Q1a) *Consider the following linear programming (LP) problem.*

$$\max\ y = 4x_1 + 8x_2$$

subject to

$$\begin{aligned} x_1 + 2x_2 &\leq 3 \\ 3x_1 + x_2 &\leq 8 \\ x_1 + x_2 &\geq 2 \end{aligned}$$

and

$$x_1 \geq 0, x_2 \geq 0$$

Solve the LP using the two-phase simplex method, justifying at each step the choice of the variable that leaves the basis.

Exercise 5 (Exam 2017, Q1) *You are given the following linear programming (LP) problem:*

$$\min z = 3x_1 + x_2$$

subject to

$$\begin{aligned} 5x_1 + 5x_2 &\geq 15 \\ -2x_1 - x_2 &\geq -3 \\ x_1 + x_2 &= 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Using Phase 1 of the simplex algorithm, determine an initial basic feasible solution for this LP. The write the initial tableau for Phase-2 and state if the initial basic feasible solution found is degenerate or non-degenerate.