A Functional Dependency Exercise

from Joe Celko's *SQL for Smarties*

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In Joe Celko's book *SQL for Smarties* he shows how to determine a minimal and sufficient database schema by analysis of *functional dependencies*. To illustrate the method, he presents as an example ten functional dependencies (FD's) that must be satisfied by a database. Using the ten FD's the exercise is to determine *minimal* subsets of the ten FD's from which the remaining FD's can be derived.

Celko presents five subsets of the ten FD's as solutions. This article shows, however, that four of his five solutions are incorrect.

Functional Dependencies

First a quick explanation of functional dependencies. A functional dependency is a relationship between two lists of database columns such that when the columns in the first list are given unique values the columns in the second list must all have unique values. The values in the first list *determine* the values in the second list. A notation for this is the names of the columns in the first list followed by an arrow and the names of the columns in the second list.

For example, in an airline reservation system the following functional dependencies may occur:

flight → destination
flight → hour

which indicate that if you know the flight then you can determine the destination and hour of the flight. You could also have those combined as:

flight → destination, hour

You can also have multiple columns specified on the left:

day, flight $\rightarrow$ gate

which indicates that if you know the day and flight you can determine the gate of departure.

The exercise Celko presents in his book consists of ten functional dependencies for an airline reservation system:

1) flight $\rightarrow$ destination
2) flight $\rightarrow$ hour
3) day, flight $\rightarrow$ gate
4) day, flight $\rightarrow$ pilot
5) day, hour, pilot $\rightarrow$ gate
6) day, hour, pilot $\rightarrow$ flight
7) day, hour, pilot $\rightarrow$ destination
8) day, hour, gate $\rightarrow$ pilot
9) day, hour, gate $\rightarrow$ flight
10) day, hour, gate $\rightarrow$ destination

The exercise is to determine minimum subsets of the ten FD's from which all of the remaining FD's can be derived.

Deriving Functional Dependencies

In the above list of ten FD's, note that if we know day, hour, and gate then we can determine the flight (see FD 9). And if we know the flight then we know the destination (see FD 1). In other words, if we know day, hour, and gate then we can determine the destination, or:

day, hour, gate $\rightarrow$ destination

In effect, from the two FD's:

day, hour, gate $\rightarrow$ flight

flight $\rightarrow$ destination

we have derived the FD:

day, hour, gate $\rightarrow$ destination

Armstrong's Axioms

There are specific rules for how FD’s can be combined to give other FD’s. Known as the *Armstrong Axioms,* these are:

Reflexive:	$X \rightarrow X$
Augmentation:	if $X \rightarrow Y$ then $XZ \rightarrow Y$
Union:	if ($X \rightarrow Y$ and $X \rightarrow Z$) then $X \rightarrow YZ$
Decomposition:	if $X \rightarrow Y$ and Z a subset of Y, then $X \rightarrow Z$
Transitivity:	if ($X \rightarrow Y$ and $Y \rightarrow Z$) then $X \rightarrow Z$
Pseudo-transitivity:	if ($X \rightarrow Y$ and $YZ \rightarrow W$) then $XZ \rightarrow W$

An additional rule that can be derived from the above is:

Left-reduction: if $XX \rightarrow Y$ then $X \rightarrow Y$

The Exercise

The exercise that Celko presents is to determine the smallest subsets of the ten FD’s for the airline reservation system from which all the remaining FD’s can be derived, using Armstrong’s Axioms. If a subset of FD’s can be used to derive all the remaining FD’s then the subset is said to *cover* the complete set of FD’s. If the subset does not itself have a subset that is also a cover, then it is a *minimal cover.*

To reduce unnecessary detail and emphasize overall patterns we will use symbols in representing the ten FD's for the exercise:

F	flight	H	hour
D	day	P	pilot
G	gate	X	destination

Using these symbols the ten FD's for the exercise become:

1) $F \rightarrow X$
2) $F \rightarrow H$
3) $D, F \rightarrow G$
4) $D, F \rightarrow P$
5) $D, H, P \rightarrow G$
6) $D, H, P \rightarrow F$
7) $D, H, P \rightarrow X$
8) $D, H, G \rightarrow P$
9) $D, H, G \rightarrow F$
10) $D, H, G \rightarrow X$

Of these FD's, the first two cannot be derived from any others, so any minimal subset that covers all ten of the FD's must include 1) and 2) .

To begin, we observe that each FD other than 1) and 2) can be derived from the others in the list. For example to derive FD 3) $D, F \rightarrow G$:

We obtain $D, F \rightarrow G$ using 2), 4), and 5) as follows:

a.	$D, H, P \rightarrow G$	from 5)
b.	$D, H, D, F \rightarrow G$	from a, 4) and Pseudo-transitivity
c.	$D, F, D, F \rightarrow G$	from b, 2) and Pseudo-transitivity
d.	$D, F \rightarrow G$	c and Left-reduction

To indicate that 3) can be derived from 2), 4), and 5) we write:

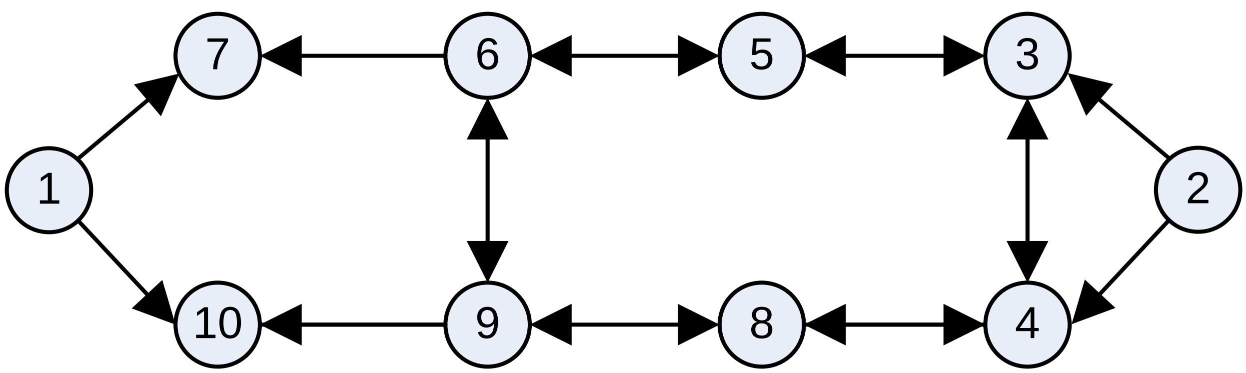
$$2), 4), 5) \Rightarrow 3)$$

A Helpful Diagram

The derivations of all the FD's from 3) to 10) are as follows:

A. 2), 4), 5) $\Rightarrow$ 3)
B. 2), 3), 8) $\Rightarrow$ 4)
C. 3), 6) $\Rightarrow$ 5)
D. 9), 5) $\Rightarrow$ 6)
E. 1), 6) $\Rightarrow$ 7)
F. 4), 9) $\Rightarrow$ 8)
G. 6), 8) $\Rightarrow$ 9)
H. 1), 9) $\Rightarrow$ 10)

Now we are in a position to create a diagram of the derivation relationships between all of the 10 FD's. In the diagram below each numbered circle represents an FD, and the arrows pointing to a numbered circle indicate the set of FD's needed to derived that FD. For example 1) and 6) are required to derive 7), as indicated in the diagram. In comparing the list of derivations above observe that all of the derivations are represented in the diagram.



Notice in the diagram that there are some "dyads" such as 8) and 9) where an arrows points from 8) to 9) and also from 9) to 8). This shows that 8) and 9) are *mutually dependent* FD's. 9) cannot be derived without 8), and 8) cannot be derived without 9). Therefore any minimal cover of the 10 FD's must include one the two FD's in a mutually dependent dyad or it will be impossible to derive either one.

Here are the six dyads from the diagram:

a) $3 \leftrightarrow 4$ b) $5 \leftrightarrow 6$ c) $6 \leftrightarrow 9$
d) $3 \leftrightarrow 5$ e) $8 \leftrightarrow 9$ f) $4 \leftrightarrow 8$

Any cover of the 10 FD's must include at least one number from each dyad.

Here are Celko's five solutions. Do they all include at least one of the two dyad FD's?

Set I

1) $F \rightarrow X$
2) $F \rightarrow H$
9) $D, H, G \rightarrow F$
8) $D, H, G \rightarrow P$
5) $D, H, P \rightarrow G$

Set I does not include 3) or 4). Not a solution.

Set II

1) $F \rightarrow X$
2) $F \rightarrow H$
8) $D, H, G \rightarrow P$
6) $D, H, P \rightarrow F$
5) $D, H, P \rightarrow G$

Set II does not include 3) or 4). Not a solution.

Set III

1) $F \rightarrow X$
2) $F \rightarrow H$
3) $D, F \rightarrow G$
4) $D, F \rightarrow P$
9) $D, H, G \rightarrow F$

Set I does not include 5) or 6). Not a solution.

Set IV

1) $F \rightarrow X$
2) $F \rightarrow H$
3) $D, F \rightarrow G$
8) $D, H, G \rightarrow P$
6) $D, H, P \rightarrow F$

Set IV is a solution.

Set V

1) $F \rightarrow X$
2) $F \rightarrow H$
4) $D, F \rightarrow P$
9) $D, H, G \rightarrow F$
5) $D, H, P \rightarrow G$
6) $D, H, P \rightarrow F$

Set V is not a solution because it is not minimal. 6) can be derived from 5) and 9), as can be seen from the diagram.

Consulting the diagram it is easy to verify that Set I is not a cover because it does not include either FD in the dyad of 3) and 4). Therefore it is impossible to derive either 3) D, F $\rightarrow$ G or 4) D, F $\rightarrow$ P from Set I. 3) and 4) are missing from Set II as well. In the case of Set III neither FD in the dyad of 5) and 6) is included. Therefore it is impossible to derive either 5) D, H, P $\rightarrow$ G or 6) D, H, P $\rightarrow$ F from Set III.

Set IV is a correct solution.

Set V includes the additional FD: 6) D, H, P $\rightarrow$ F. As the diagram shows, FD 6) can be derived from FD's 5) and 9) which are already included in Set V. Therefore Set V is not a *minimum* cover and so is not a correct solution.

In the next article we take a fresh look at functional dependencies. Is there a straight-forward way to find minimum covers?