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1. Hitung dan buat matrix kovarian dan korelasi dari data dibawah ini:

X_1	9	2	6	5	8
X_2	12	8	6	4	10
X_3	3	4	0	2	1

2. Hitung eigen value dan eigenvektor dari matrix kovarian dan korelasi dari nomor 1.

jawab :

1. Matrix A :

$$A = \begin{bmatrix} 9 & 12 & 3 \\ 2 & 8 & 4 \\ 6 & 6 & 0 \\ 5 & 4 & 2 \\ 8 & 10 & 1 \end{bmatrix}$$

Untuk menghitung matrix kovarian, gunakan 2 metode :

a. Menggunakan matrix dekomposisi (deviasi).

$$A = \begin{bmatrix} 9 & 12 & 3 \\ 2 & 8 & 4 \\ 6 & 6 & 0 \\ 5 & 4 & 2 \\ 8 & 10 & 1 \end{bmatrix} \Rightarrow y_1 = \begin{bmatrix} 9 \\ 2 \\ 6 \\ 5 \\ 8 \end{bmatrix} ; y_2 = \begin{bmatrix} 12 \\ 8 \\ 6 \\ 4 \\ 10 \end{bmatrix} ; y_3 = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \bar{x}_1 = \frac{9+2+6+5+8}{5} = 6 \; ; \; \bar{x}_2 = \frac{12+6+8+4+10}{5} = 8 \; ; \bar{x}_3 = \frac{3+4+0+2+1}{5} = 2 .$$

maka:

$$d_1 = y_1 - \bar{x}_1 1 = \begin{bmatrix} 9 \\ 2 \\ 6 \\ 5 \\ 8 \end{bmatrix} - \begin{bmatrix} 6 \\ 6 \\ 6 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 0 \\ -1 \\ 2 \end{bmatrix}$$

$$d_2 = y_2 - \bar{x}_2 1 = \begin{bmatrix} 12 \\ 8 \\ 6 \\ 4 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \\ 8 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -2 \\ -4 \\ 2 \end{bmatrix}$$

$$d_3 = y_3 - \bar{x}_3 1 = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 0 \\ -1 \end{bmatrix}$$

$$d_i = \begin{bmatrix} 3 & 4 & 1 \\ -4 & 0 & 2 \\ 0 & -2 & -2 \\ -1 & -4 & 0 \\ 2 & 2 & -1 \end{bmatrix}$$

dgn $i = 1, 2, 3$ (indeks kolom).

$$\Rightarrow d_1'd_1 = [3\ -4\ 0\ -1\ 2]\begin{bmatrix}3\\-4\\0\\-1\\2\end{bmatrix} = 30 = 5s_{11} \Rightarrow s_{11} = {}^{30}/_{5} = 6$$

$$d_2'd_2 = [4\ 0\ -2\ -4\ 2]\begin{bmatrix}4\\0\\-2\\-4\\2\end{bmatrix} = 40 = 5s_{22} \Rightarrow s_{22} = {}^{40}/_{5} = 8$$

$$d_3'd_3 = [1\ 2\ -2\ 0\ -1]\begin{bmatrix}1\\2\\-2\\0\\-1\end{bmatrix} = 10 = 5s_{33} \Rightarrow s_{33} = {}^{10}/_{5} = 2.$$

$$d_1'd_2 = [3\ -4\ 0\ -1\ 2]\begin{bmatrix}3\\-4\\0\\-1\\2\end{bmatrix} = 20 = 5s_{12} \Rightarrow s_{12} = {}^{20}/_{5} = 4$$

$$d_1'd_3 = [3\ -4\ 0\ -1\ 2]\begin{bmatrix}1\\2\\-2\\0\\-1\end{bmatrix} = -7 = 5s_{13} \Rightarrow s_{13} = -{}^{7}/_{5}$$

$$d_2'd_3 = [4\ 0\ -2\ -4\ 2]\begin{bmatrix}4\\0\\-2\\-4\\2\end{bmatrix} = 6 = 5s_{23} \Rightarrow s_{23} = {}^{6}/_{5}$$

$$\Rightarrow S_n = \begin{bmatrix}6 & 4 & -7/5\\ 4 & 8 & 6/5\\ -7/5 & 6/5 & 2\end{bmatrix} \Rightarrow S = \frac{n}{n-1} S_n = \frac{5}{4}\begin{bmatrix}6 & 4 & -7/5\\ 4 & 8 & 6/5\\ -7/5 & 6/5 & 2\end{bmatrix} = \begin{bmatrix}7{,}5 & 5 & -1{,}75\\ 5 & 10 & 1{,}5\\ -1{,}75 & 1{,}5 & 2{,}5\end{bmatrix}$$

b. Menggunakan rumus kovarian.

$$\cancel{S = \sum_{i=1}^{p}\sum_{j=1}^{q} (}$$

$$s_{ij} = \frac{\sum_{i=1}^{n}(x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{n-1}.$$

$$\Rightarrow s_{11} = \frac{\sum_{i=1}^{5}(x_{i1} - \bar{x}_1)^2}{4} = \frac{(3)^2 + (-4)^2 + 0 + (-1)^2 + (2)^2}{4} = 7{,}5$$

$$\Rightarrow s_{12} = \frac{\sum_{i=1}^{5}(x_{i1} - \bar{x}_1)(x_{i2} - \bar{x}_2)}{4} = \frac{(9-6)(12-8) + (2-6)(8-8) + (6-6)(6-8) + (5-6)(4-8) + (8-6)(10-8)}{4}$$

$$= {}^{20}/_{4} = 5.$$

$$S_{13} = \frac{\sum_{i=1}^{5}(x_{i1}-\bar{x}_1)(x_{i3}-\bar{x}_3)}{n} = \frac{(9-6)\cancel{()}^{(3-2)} + (2-6)\cancel{()}^{(4-2)} + (6-6)(0-2) + (5-6)(2-2) + (8-6)(1-2)}{4}$$

$$= -7/4 = -1{,}75.$$

$$S_{22} = \frac{\sum_{i=1}^{5}(x_{i2}-\bar{x}_2)^2}{4} = \frac{(12-8)^2+(8-8)^2+(6-8)^2+(4-8)^2+(10-8)^2}{4}$$

$$= 40/4 = 10$$

$$S_{23} = \frac{\sum_{i=1}^{5}(x_{i2}-\bar{x}_2)^2(x_{i3}-\bar{x}_3)^2}{4} = \frac{(12-8)(3-2)+(8-8)(4-2)+(6-8)(0-2)+(4-8)(2-2)+(10-8)(1-2)}{4}$$

$$= 6/4 = 1{,}5$$

$$S_{33} = \frac{\sum_{i=1}^{5}(x_{i3}-\bar{x}_3)^2}{4} = \frac{(3-2)^2+(4-2)^2+(0-2)^2+(2-2)^2+(1-2)^2}{4}$$

$$= 10/4 = 2{,}5.$$

Maka:

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} 7{,}5 & 5 & -1{,}75 \\ 5 & 10 & 1{,}5 \\ -1{,}75 & 1{,}5 & 2{,}5 \end{bmatrix}$$

Setelah mendapat vektor kovarian, kita hitung matrix korelasi dgn memanfaatkan matrix kovarian, dimana:

$$R = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{21} & 1 & \rho_{23} \\ \rho_{31} & \rho_{32} & 1 \end{bmatrix}, \text{ dimana: } \rho_{ij} = \frac{S_{ij}}{\sqrt{S_{ii}}\sqrt{S_{jj}}}.$$

maka:

$$\rho_{12} = \frac{S_{12}}{\sqrt{S_{11}}\sqrt{S_{22}}} = \frac{5}{\sqrt{7{,}5}\cdot\sqrt{10}} = 0{,}5773503 \; ; \; \rho_{13} = \frac{S_{13}}{\sqrt{S_{11}}\cdot\sqrt{S_{33}}} = -\frac{1{,}75}{\sqrt{7{,}5}\sqrt{2{,}5}} = -0{,}4041452$$

$$\rho_{23} = \frac{S_{23}}{\sqrt{S_{22}}\sqrt{S_{33}}} = \frac{1{,}5}{\sqrt{10}\cdot\sqrt{2{,}5}} = 0{,}3 \; ; \text{ sehingga: } R = \begin{bmatrix} 1 & 0{,}5773503 & -0{,}4041452 \\ 0{,}5773503 & 1 & 0{,}3 \\ -0{,}4041452 & 0{,}3 & 1 \end{bmatrix}$$

2. Menghitung eigenvalue dan eigenvektor dari :

a. Matrix kovarian.

$$S = \begin{bmatrix} 7,5 & 5 & -1,75 \\ 5 & 10 & 1,5 \\ -1,75 & 1,5 & 2,5 \end{bmatrix}$$

$$\Rightarrow \det(S - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 7,5 & 5 & -1,75 \\ 5 & 10 & 1,5 \\ -1,75 & 1,5 & 2,5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 7,5-\lambda & 5 & -1,75 \\ 5 & 10-\lambda & 1,5 \\ -1,75 & 1,5 & 2,5-\lambda \end{bmatrix}\right) = 0$$

$$\Rightarrow [(7,5-\lambda)(10-\lambda)(2,5-\lambda) + (5)(1,5)(-1,75) + (-1,75)(5)(1,5)] - [(2,5-\lambda)(5)(5) + (1,5)(1,5)(7,5-\lambda) + (-1,75)(10-\lambda)(-1,75)]$$

$$-\lambda^3 + 20\lambda^2 - 118,75\lambda + 161,25 - (110 - 30,3125) = 0$$
$$-\lambda^3 + 20\lambda^2 - 88,4375\lambda + 51,25 = 0$$
$$\lambda^3 - 20\lambda^2 + 88,4375\lambda - 51,25 = 0$$

Dengan menggunakan kalkulator, didapatkan :

$\lambda_1 \approx 13,90490087384 - 15,5431223448 \times 10^{-16} i$

$\lambda_2 \approx 5,41436335062 + 17,763568394 \times 10^{-16} i$

$\lambda_3 \approx 0,68073577554 - 8,881784197 \times 10^{-16} i$.

Dengan menggunakan Newton-Raphson :

$$\boxed{\begin{aligned} \lambda_1 &\approx 13,9049 \\ \lambda_2 &\approx 5,4143 \\ \lambda_3 &\approx 0,6807 \end{aligned}}$$

* utk $\lambda \approx 13,9049$

$$(S - \lambda I)(V) = 0$$

$$\left(\begin{bmatrix} 7,5 & 5 & -1,75 \\ 5 & 10 & 1,5 \\ -1,75 & 1,5 & 2,5 \end{bmatrix} - \begin{bmatrix} 13,9049 & 0 & 0 \\ 0 & 13,9049 & 0 \\ 0 & 0 & 13,9049 \end{bmatrix}\right)\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -6,4049 & 5 & -1,75 \\ 5 & -3,9049 & 1,5 \\ -1,75 & 1,5 & -11,4049 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

⇒ Menggunakan metode Eliminasi Gauss - Jordan :

$$\begin{bmatrix} 1 & 0 & -63,4506 \\ 0 & 1 & -81,6290 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_1 - 63,4506\, V_3 = 0 \Rightarrow V_1 = 63,4506\, V_3$$
$$V_2 - 81,6290\, V_3 = 0 \qquad V_2 = 81,629\, V_3$$

$$\eta = \begin{bmatrix} 63,4506 V_3 \\ 81,629 V_3 \\ V_3 \end{bmatrix} \text{; misal } V_3 = 1 \text{, maka :}$$

$$e_1 = \begin{bmatrix} 63,4506 \\ 81,629 \\ 1 \end{bmatrix}$$

* utk $\lambda_2 = 5,4143$

$(S - \lambda I)(V) = 0$

$$\left(\begin{bmatrix} 7,5 & 5 & -1,75 \\ 5 & 10 & 1,5 \\ -1,75 & 1,5 & 2,5 \end{bmatrix} - \begin{bmatrix} 5,4143 & 0 & 0 \\ 0 & 5,4143 & 0 \\ 0 & 0 & 5,4143 \end{bmatrix} \right) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2,0856 & 5 & -1,75 \\ 5 & 4,5856 & 1,5 \\ -1,75 & 1,5 & -2,9143 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

⇒ Menggunakan metode Eliminasi Gauss-Jordan:

$$\begin{bmatrix} 1 & 0 & 1,0057 \\ 0 & 1 & -0,7695 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_1 + 1,0057\, V_3 = 0 \Rightarrow V_1 = -1,0057\, V_3$$
$$V_2 - 0,7695\, V_3 = 0 \qquad V_2 = 0,7695\, V_3$$

$$\eta = \begin{bmatrix} -1{,}0057 V_3 \\ 0{,}7695 V_3 \\ V_3 \end{bmatrix} \text{; misal } V_3 = 1 \text{, maka: } e_2 = \begin{bmatrix} -1{,}0057 \\ 0{,}7695 \\ 1 \end{bmatrix}$$

* utk $\lambda_3 = 0{,}6873$

$(S - \lambda I)(\vec{v}) = 0$

$$\left(\begin{bmatrix} 7{,}5 & 5 & -1{,}75 \\ 5 & 10 & 1{,}5 \\ -1{,}75 & 1{,}5 & 2{,}5 \end{bmatrix} - \begin{bmatrix} 0{,}6873 & 0 & 0 \\ 0 & 0{,}6873 & 0 \\ 0 & 0 & 0{,}6873 \end{bmatrix} \right) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6{,}8192 & 5 & -1{,}75 \\ 5 & 9{,}3192 & 1{,}5 \\ -1{,}75 & 1{,}5 & 1{,}8192 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

· Menggunakan metode Eliminasi Gauss - Jordan:

$$\begin{bmatrix} 1 & 0 & -0{,}6175 \\ 0 & 1 & 0{,}4923 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} V_1 - 0{,}6175 V_3 &= 0 \\ V_2 + 0{,}4923 V_3 &= 0 \end{aligned} \Rightarrow \begin{aligned} V_1 &= 0{,}6175 V_3 \\ V_2 &= -0{,}4923 V_3 \end{aligned}$$

$$\eta = \begin{bmatrix} 0{,}6175 V_3 \\ -0{,}4923 V_3 \\ V_3 \end{bmatrix} \text{; misal } V_3 = 1 \text{, maka: } e_3 = \begin{bmatrix} 0{,}6175 \\ -0{,}4923 \\ 1 \end{bmatrix}$$

Sehingga, eigenvektor dari matrix S adalah: $\begin{bmatrix} 63{,}4506 \\ 81{,}629 \\ 1 \end{bmatrix}, \begin{bmatrix} -1{,}0057 \\ 0{,}7695 \\ 1 \end{bmatrix}, \begin{bmatrix} 0{,}6175 \\ -0{,}4923 \\ 1 \end{bmatrix}$

b. Matrix Korelasi

$$R = \begin{bmatrix} 1 & 0{,}57735 & -0{,}40414 \\ 0{,}57735 & 1 & 0{,}3 \\ -0{,}40414 & 0{,}3 & 1 \end{bmatrix}$$

$\Rightarrow \det(R - \lambda I) = 0$

$$\det\left(\begin{bmatrix}1 & 0{,}57735 & -0{,}40414\\ 0{,}57735 & 1 & 0{,}3\\ -0{,}40414 & 0{,}3 & 1\end{bmatrix} - \begin{bmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix}1-\lambda & 0{,}57735 & -0{,}40414\\ 0{,}57735 & 1-\lambda & 0{,}3\\ -0{,}40414 & 0{,}3 & 1-\lambda\end{bmatrix}\right) = 0$$

$$\Rightarrow [(1-\lambda)(1-\lambda)(1-\lambda) + (0{,}57735)(0{,}3)(-0{,}40414) + (-0{,}40414)(0{,}57735)(0{,}3)] - [(1-\lambda)(0{,}57735)^2 + (1-\lambda)(0{,}3)^2 + (1-\lambda)(-0{,}40414)^2]$$

$$\Rightarrow (-\lambda^3 + 3\lambda^2 - \lambda + 1 - 0{,}1399981) - (0{,}5866621 - 0{,}5866621\lambda) = 0$$

$$-\lambda^3 + 3\lambda^2 - 2{,}4133379\lambda + 0{,}2733398 = 0$$

Dengan menggunakan kalkulator, didapatkan:

$\lambda_1 = 1{,}5916362192 - 4{,}996003610 \times 10^{-16} i$

$\lambda_2 = 1{,}27351223241 + 3{,}885780586 \times 10^{-16} i$

$\lambda_3 = 0{,}13485154862 - 1{,}38777878078 \times 10^{-16} i$

Dengan menggunakan metode Newton - Raphson:

$$\boxed{\begin{aligned}\lambda_1 &= 1{,}59163\\ \lambda_2 &= 1{,}27351\\ \lambda_3 &= 0{,}13485\end{aligned}}$$

* utk $\lambda_1 = 1{,}59163$

$(S - \lambda I)(V) = 0$

$$\left(\begin{bmatrix}1 & 0{,}57735 & -0{,}40414\\ 0{,}57735 & 1 & 0{,}3\\ -0{,}40414 & 0{,}3 & 1\end{bmatrix} - \begin{bmatrix}1{,}59163 & 0 & 0\\ 0 & 1{,}59163 & 0\\ 0 & 0 & 1{,}59163\end{bmatrix}\right)\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}-0{,}59163 & 0{,}57735 & -0{,}40414\\ 0{,}57735 & -0{,}59163 & 0{,}3\\ -0{,}40414 & 0{,}3 & -0{,}59163\end{bmatrix}\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

Menggunakan metode Eliminasi Gauss-Jordan:

$$\begin{bmatrix}1 & 0 & 3{,}94591\\ 0 & 1 & 3{,}34859\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{aligned} V_1 + 3{,}94594 V_3 &= 0 \\ V_2 + 3{,}34359 V_3 &= 0 \end{aligned} \Rightarrow \begin{aligned} V_1 &= -3{,}94594 V_3 \\ V_2 &= -3{,}34359 V_3 \end{aligned}$$

$$\eta = \begin{bmatrix} -3{,}94594 V_3 \\ -3{,}34359 V_3 \\ V_3 \end{bmatrix}, \text{ misal } V_3 = 1, \text{ maka: } e_1 = \begin{bmatrix} -3{,}94594 \\ -3{,}34359 \\ 1 \end{bmatrix}$$

*. utk $\lambda_2 = 1{,}27351$

$(S - \lambda I)(V) = 0$

$$\left(\begin{bmatrix} 1 & 0{,}57735 & -0{,}40414 \\ 0{,}57735 & 1 & 0{,}3 \\ -0{,}40414 & 0{,}3 & 1 \end{bmatrix} - \begin{bmatrix} 1{,}27351 & 0 & 0 \\ 0 & 1{,}27351 & 0 \\ 0 & 0 & 1{,}27351 \end{bmatrix} \right) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -0{,}27351 & 0{,}57735 & -0{,}40414 \\ 0{,}57735 & -0{,}27351 & 0{,}3 \\ -0{,}40414 & 0{,}3 & -0{,}27351 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Menggunakan metode Eliminasi Gauss-Jordan :

$$\begin{bmatrix} 1 & 0 & 0{,}24240 \\ 0 & 1 & -0{,}58515 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} V_1 + 0{,}24240 V_3 &= 0 \\ V_2 \cancel{+} -0{,}58515 V_3 &= 0 \end{aligned} \Rightarrow \begin{aligned} V_1 &= -0{,}2424 V_3 \\ V_2 &= 0{,}58515 V_3 \end{aligned}$$

$$\eta = \begin{bmatrix} -0{,}2424 V_3 \\ 0{,}58515 V_3 \\ V_3 \end{bmatrix}; \text{ misal } V_3 = 1, \text{ maka: } e_2 = \begin{bmatrix} -0{,}2424 \\ 0{,}58515 \\ 1 \end{bmatrix}$$

*. utk $\lambda_3 = 0{,}13485$

$(S - \lambda I)(V) = 0$

$$\left(\begin{bmatrix} 1 & 0{,}57735 & -0{,}40414 \\ 0{,}57735 & 1 & 0{,}3 \\ -0{,}40414 & 0{,}3 & 1 \end{bmatrix} - \begin{bmatrix} 0{,}13485 & 0 & 0 \\ 0 & 0{,}13485 & 0 \\ 0 & 0 & 0{,}13485 \end{bmatrix} \right) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0,86514 & 0,57735 & -0,40414 \\ 0,57735 & 0,86514 & 0,3 \\ -0,40414 & 0,3 & 0,86514 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Menggunakan metode Eliminasi Gauss-Jordan :

$$\begin{bmatrix} 1 & 0 & -1,08958 \\ 0 & 1 & 1,18722 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow V_1 - 1,08958 = 0 \quad \Rightarrow V_1 = 1,08958 V_3$$
$$V_2 + 1,18722 = 0 \quad\quad V_2 = -1,18722 V_3$$

$$\eta = \begin{bmatrix} 1,08958 V_3 \\ -1,18722 V_3 \\ V_3 \end{bmatrix} \text{; misal } V_3 = 1 \text{, maka: } e_3 = \begin{bmatrix} 1,08958 \\ -1,18722 \\ 1 \end{bmatrix}$$

Sehingga, eigenvektor dari matrix R adalah : $\begin{bmatrix} -3,94594 \\ -3,34359 \\ 1 \end{bmatrix}, \begin{bmatrix} -0,2424 \\ 0,58515 \\ 1 \end{bmatrix}, \begin{bmatrix} 1,08958 \\ -1,18722 \\ 1 \end{bmatrix}$