

# Python Implementation

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# 1 Modeling Sudoku Variations in Python

The implementation was based on the paper *"An Integer Programming Model for the Sudoku Problem"*.

We are going to be using the **PuLP** python library, in order to model the given problems in terms of Linear Programming.

## 1.1 The Classic Sudoku Solver class

```
self.matrix = matrix 1
self.n = len(matrix) 2
self.m = int(sqrt(self.n)) 3

super().__init__( 4
    name=f"{type(self).__name__}_solver_{self.n}_x_{self.n}".lower(), 5
    sense=LpMinimize 6
) 7
8
```

Listing 1: Initializing the **LpProblem** super class as a minimization problem

```
self.x = [ 1
    [ 2
        [ 3
            LpVariable( 4
                f"x_{i + 1:02d}_{j + 1:02d}_{k + 1:02d}", cat=LpBinary) 5
            for k in range(self.n) 6
        ] for j in range(self.n) 7
    ] for i in range(self.n) 8
] 9
```

Listing 2: Declaring our variables

```
self += 0 1
```

Listing 3: Declaring the objective function

```

for j in range(self.n):
    for k in range(self.n):
        self += lpSum([self.x[i][j][k]
                        for i in range(self.n)]) == 1, f"in column {j +
1:02d} only one {k + 1:02d}"

```

Listing 4: Declaring that there should only be one  $k$  in each column

```

for i in range(self.n):
    for k in range(self.n):
        self += lpSum([self.x[i][j][k]
                        for j in range(self.n)]) == 1, f"in row {i +
1:02d} only one {k + 1:02d}"

```

Listing 5: Declaring that there should only be one  $k$  in each row

```

for k in range(self.n):
    for p in range(self.m):
        for q in range(self.m):
            self += lpSum([
                lpSum([
                    self.x[i][j][k]
                    for i in range(self.m * p, self.m * (p + 1))
                ])
                for j in range(self.m * q, self.m * (q + 1))
            ]) == 1, f"in submatrix {p + 1:02d} {q + 1:02d} only one {k
+ 1:02d}"

```

Listing 6: Declaring that there should only be one  $k$  in each submatrix

```

for i in range(self.n):
    for j in range(self.n):
        self += lpSum([self.x[i][j][k]
                        for k in range(self.n)]) == 1, f"cell {i +
1:02d} {j + 1:02d} must be assigned exactly one value"

```

Listing 7: Declaring that there should only be one  $k$  in each cell

```

1  for i in range(self.n):
2      for j in range(self.n):
3          if self.matrix[i][j]:
4              self += self.x[i][j][self.matrix[i][j] -
5                  1] == 1, f"cell {i + 1:02d} {j +
1:02d} has an initial value of {self.matrix[i][j]:02d}"

```

Listing 8: Declaring that some cells have already received some initial values

```

1  for i in range(self.n):
2      for j in range(self.n):
3          if not self.matrix[i][j]:
4              for value in self.illegal_values(i, j):
5                  self += (
6                      self.x[i][j][value - 1] == 0,
7                      f"cell {i + 1:02d} {j + 1:02d} cannot be assigned a
value of {value:02d}"
8                      )

```

Listing 9: Performing some preprocessing and invalidating some of the candidate values of each cell

```

1  def solve(self, solver=None, **kwargs):
2
3      super().solve(solver=solver, **kwargs)
4
5      if LpStatus[self.status] != "Optimal":
6          raise ValueError(
7              f"Solver failed with status '{LpStatus[self.status]}'")
8
9      for i in range(self.n):
10         for j in range(self.n):
11             if not self.matrix[i][j]:
12                 self.matrix[i][j] = [
13                     self.x[i][j][k].varValue for k in range(self.n)
14                 ].index(1) + 1

```

Listing 10: Calling the LpProblem solve method and reflecting the result on the two-dimensional array "matrix"

```
def illegal_values(self, row, col):  
    values = set()  
  
    for j in range(self.n):  
        if self.matrix[row][j] is not None:  
            values.add(self.matrix[row][j])  
  
    for i in range(self.n):  
        if self.matrix[i][col] is not None:  
            values.add(self.matrix[i][col])  
  
    p, q = row // self.m, col // self.m  
  
    for i in range(self.m * p, self.m * (p + 1)):  
        for j in range(self.m * q, self.m * (q + 1)):  
            if self.matrix[i][j] is not None:  
                values.add(self.matrix[i][j])  
  
    return values
```

Listing 11: Checking for invalid candidate values before even calling `LpProblem.solve`

If we now print an instance of the previously defined class we get the following output.

```

sudokulp_solver_9_x_9: 1

MINIMIZE 2
0*__dummy + 0 3
4
5

SUBJECT TO 6
in_column_01_only_one_01: x_01_01_01 + x_02_01_01 + x_03_01_01 + 7
x_04_01_01
+ x_05_01_01 + x_06_01_01 + x_07_01_01 + x_08_01_01 + x_09_01_01 = 1 8
... 9
in_column_09_only_one_09: x_01_09_09 + x_02_09_09 + x_03_09_09 + 10
x_04_09_09
+ x_05_09_09 + x_06_09_09 + x_07_09_09 + x_08_09_09 + x_09_09_09 = 1 11
12
in_row_01_only_one_01: x_01_01_01 + x_01_02_01 + x_01_03_01 + x_01_04_01 13
+ x_01_05_01 + x_01_06_01 + x_01_07_01 + x_01_08_01 + x_01_09_01 = 1 14
... 15
in_row_09_only_one_09: x_09_01_09 + x_09_02_09 + x_09_03_09 + x_09_04_09 16
+ x_09_05_09 + x_09_06_09 + x_09_07_09 + x_09_08_09 + x_09_09_09 = 1 17
18
in_submatrix_01_01_only_one_01: x_01_01_01 + x_01_02_01 + x_01_03_01 19
+ x_02_01_01 + x_02_02_01 + x_02_03_01 + x_03_01_01 + x_03_02_01 + 20
x_03_03_01
= 1 21
... 22
in_submatrix_03_03_only_one_09: x_07_07_09 + x_07_08_09 + x_07_09_09 23
+ x_08_07_09 + x_08_08_09 + x_08_09_09 + x_09_07_09 + x_09_08_09 + 24
x_09_09_09
= 1 25
26
cell_01_01_must_be_assigned_exactly_one_value: x_01_01_01 + x_01_01_02 27
+ x_01_01_03 + x_01_01_04 + x_01_01_05 + x_01_01_06 + x_01_01_07 + 28
x_01_01_08
+ x_01_01_09 = 1 29
... 30
cell_09_09_must_be_assigned_exactly_one_value: x_09_09_01 + x_09_09_02 31
+ x_09_09_03 + x_09_09_04 + x_09_09_05 + x_09_09_06 + x_09_09_07 + 32
x_09_09_08
+ x_09_09_09 = 1 33
34
cell_01_08_has_an_initial_value_of_02: x_01_08_02 = 1 35
... 36
cell_09_02_has_an_initial_value_of_01: x_09_02_01 = 1 37
38
cell_01_01_cannot_be_assigned_a_value_of_02: x_01_01_02 = 0 39
... 40
cell_09_09_cannot_be_assigned_a_value_of_09: x_09_09_09 = 0 41

```

Listing 12: The objective function and constraints of SudokuLP

VARIABLES	1
__dummy = 0 Continuous	2
0 <= x_01_01_01 <= 1 Integer	3
...	4
0 <= x_09_09_09 <= 1 Integer	5

Listing 13: The variables of SudokuLP

## 1.2 The Sudoku X Solver class

```
super().__init__(matrix)
```

1

Listing 14: Initializing the **SudokuLP** super class

```
for k in range(self.n):  
  
    self += lpSum([  
        self.x[r][r][k] for r in range(self.n)  
    ]) == 1, f"in the diagonal only one {k + 1}"
```

1

2

3

4

5

Listing 15: Declaring that there should only be one  $k$  in the positive diagonal

```
for k in range(self.n):  
  
    self += lpSum([  
        self.x[r][self.n - 1 - r][k] for r in range(self.n)  
    ]) == 1, f"in the anti diagonal only one {k + 1}"
```

1

2

3

4

5

Listing 16: Declaring that there should only be one  $k$  in the negative diagonal



### 1.3 The Four Square Sudoku Solver class

```
super().__init__(matrix)
```

1

Listing 17: Initializing the **SudokuLP** super class

```
for i in [1, self.n - self.m - 1]:           1
    for j in [1, self.n - self.m - 1]:       2
        for k in range(self.n):              3
            self += lpSum([                  4
                [                             5
                    lpSum([                  6
                        self.x[r][c][k]      7
                        for c in range(j, j + self.m) 8
                    ])                       9
                ]                             10
            for r in range(i, i + self.m)    11
        ]) == 1, f"in square {i + 1:02d} {j + 1:02d} {i + 12
self.m:02d} {j + self.m:02d} only one {k + 1:02d}"
```

Listing 18: Declaring that there should only be one  $k$  in each shaded square

## 1.4 The Four Pyramid Sudoku Solver class

```
super().__init__(matrix)
```

1

Listing 19: Initializing the **SudokuLP** super class

```
for k in range(1, self.n + 1):
    self += lpSum([
        lpSum([
            self.x[r - 1][c - 1][k - 1]
            for c in range(self.m + r, self.n - r + 1)
        ]) for r in range(1, self.m + 1)
    ]) == 1, f"in pyramid 1 only one {k + 1}"
```

1

2

3

4

5

6

7

Listing 20: Declaring that there should only be one  $k$  in the first pyramid-shaped pyramid region

```
for k in range(1, self.n + 1):
    self += lpSum([
        lpSum([
            self.x[r - 1][c - 1][k - 1]
            for r in range(1 + c, self.n - self.m + 1 - c + 1)
        ]) for c in range(1, self.m + 1)
    ]) == 1, f"in pyramid 2 only one {k + 1}"
```

1

2

3

4

5

6

7

Listing 21: Declaring that there should only be one  $k$  in the second pyramid-shaped pyramid region

---

```

for k in range(1, self.n + 1):
    self += lpSum([
        lpSum([
            self.x[r - 1][c - 1][k - 1]
            for c in range(self.n + self.m - 1 - r, r - self.m + 1)
        ]) for r in range(self.n - self.m + 1, self.n + 1)
    ]) == 1, f"in pyramid 3 only one {k + 1}"

```

---

Listing 22: Declaring that there should only be one  $k$  in the third pyramid-shaped pyramid region

---

```

for k in range(1, self.n + 1):
    self += lpSum([
        lpSum([
            self.x[r - 1][c - 1][k - 1]
            for r in range(self.n + self.m + 1 - c, c - 1 + 1)
        ]) for c in range(self.n - self.m + 1, self.n + 1)
    ]) == 1, f"in pyramid 4 only one {k + 1}"

```

---

Listing 23: Declaring that there should only be one  $k$  in the fourth pyramid-shaped pyramid region

---

```

options = {
    "sdk": SudokuLP,
    "sdkx": SudokuXLP,
    "sdkfs": FourSquareSudokuLP,
    "sdkfp": FourPyramidSudokuLP
}

extension = path.splitext(args.load)[1][1:]

matrix = load(args.load)
problem = options[extension](matrix)

```

---

Listing 24: Example Usage

## 2 Generating Sudoku Puzzles

```
def transpose(matrix):  
    return list(map(list, [*zip(*matrix)]))
```

1  
2

Listing 25: Generating a new Sudoku Puzzle by transposing the supplied one

```
def relabel(matrix):  
    replacements = {  
        original: replacement  
        for original, replacement in zip(  
            range(1, len(matrix) + 1),  
            np.random.permutation(range(1, len(matrix) + 1))  
        )  
    }  
  
    for i in range(len(matrix)):  
        for j in range(len(matrix)):  
            if matrix[i][j] is not None:  
                matrix[i][j] = replacements[matrix[i][j]]  
  
    return matrix
```

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15

Listing 26: Generating a new Sudoku Puzzle by relabeling the values in the supplied one

---

<code>def reorder(matrix):</code>	1
<code>def swap_rows(matrix):</code>	2
	3
<code>m = int(sqrt(len(matrix)))</code>	4
	5
<code>for p in range(m):</code>	6
<code>for q in range(m):</code>	7
<code>for r in range(m):</code>	8
<code>i1 = randint(m * p, m * (p + 1) - 1)</code>	9
<code>i2 = randint(m * p, m * (p + 1) - 1)</code>	10
<code>matrix[i1], matrix[i2] = matrix[i2], matrix[i1]</code>	11
	12
<code>return matrix</code>	13
	14
<code>return transpose(swap_rows(transpose(swap_rows(matrix))))</code>	15
	16

---

Listing 27: Generating a new Sudoku Puzzle by block reordering the rows and columns of the supplied one

---

<code>methods = {</code>	1
<code>"transpose": transpose,</code>	2
<code>"relabel": relabel,</code>	3
<code>"reorder": reorder</code>	4
<code>}</code>	5
	6
<code>matrix = load(args.load)</code>	7
<code>matrix = methods[args.method](matrix)</code>	8
	9
<code>dump(matrix, args.save)</code>	10

---

Listing 28: Example Usage

### 3 The Sudoku (.sdk\*) file format

For anyone interested in the details of our implementation, we now present how sudoku puzzles are being represented internally as well as how one could load or save a two-dimensional array in .sdk\* format.

```
9          # +---+---+---+---+---+---+---+---+---+ 1
1, 1, 5    # | 5 | 3 |   |   | 7 |   |   |   |   | 2
1, 2, 3    # +---+---+---+---+---+---+---+---+---+ 3
1, 5, 7    # | 6 |   |   | 1 | 9 | 5 |   |   |   | 4
2, 1, 6    # +---+---+---+---+---+---+---+---+---+ 5
2, 4, 1    # |   | 9 | 8 |   |   |   |   | 6 |   | 6
2, 5, 9    # +---+---+---+---+---+---+---+---+---+ 7
2, 6, 5    # | 8 |   |   |   | 6 |   |   |   | 3 | 8
3, 2, 9    # +---+---+---+---+---+---+---+---+---+ 9
3, 3, 8    # | 4 |   |   | 8 |   | 3 |   |   | 1 | 10
3, 8, 6    # +---+---+---+---+---+---+---+---+---+ 11
4, 1, 8    # | 7 |   |   |   | 2 |   |   |   | 6 | 12
4, 5, 6    # +---+---+---+---+---+---+---+---+---+ 13
4, 9, 3    # |   | 6 |   |   |   |   | 2 | 8 |   | 14
5, 1, 4    # +---+---+---+---+---+---+---+---+---+ 15
5, 4, 8    # |   |   |   | 4 | 1 | 9 |   |   | 5 | 16
5, 6, 3    # +---+---+---+---+---+---+---+---+---+ 17
5, 9, 1    # |   |   |   |   | 8 |   |   | 7 | 9 | 18
6, 1, 7    # +---+---+---+---+---+---+---+---+---+ 19
6, 5, 2    20
6, 9, 6    21
7, 2, 6    22
7, 7, 2    23
7, 8, 8    24
8, 4, 4    25
8, 5, 1    26
8, 6, 9    27
8, 9, 5    28
9, 5, 8    29
9, 8, 7    30
9, 9, 9    31
```

Listing 29: A .sdk\* file example

### 3.1 Loading Sudoku Puzzles

```
lines = file.readlines() 1
lines = map(lambda line: sub(r"#.*", "", line), lines) 2
lines = map(lambda line: sub(r"\s+", "", line), lines) 3
lines = enumerate(lines) 4
lines = filter(lambda data: len(data[1]) > 0, lines) 5
```

Listing 30: Removing any useless characters

```
try: 1
    index, line = next(lines) 2
    3
    size = int(line) 4
    5
    if size <= 0: 6
        raise ValueError 7
    8
except ValueError: 9
    raise ParseError( 10
        index, line, "is not a valid size specifier") 11
```

Listing 31: Determining the size of the matrix

```
_sqrt = sqrt(size) 1
2
if _sqrt != int(_sqrt): 3
    raise ParseError( 4
        index, line, f"{size} is not a perfect square") 5
```

Listing 32: Performing some sanity checks corresponding to the size of the matrix

```

matrix = [[None for _ in range(size)] for _ in range(size)]
1
2
for index, line in lines:
3
    try:
4
        x, y, z = tuple(map(int, line.split(',')))
5
6
        if x < 0 or y < 0 or z < 0 or z > size:
7
            raise IndexError
8
9
        if matrix[x - 1][y - 1] is not None:
10
            raise ParseError(
11
                index, line,
12
                f"the cell has already been assigned")
13
14
        matrix[x - 1][y - 1] = z
15
16
    except IndexError:
17
        raise ParseError(
18
            index, line,
19
            f"is not a valid entry for a puzzle of size {size}")
20
21
    except ParseError as parse_error:
22
        raise parse_error
23
24
    except:
25
        raise ParseError(
26
            index, line, "Malformed entry")
27
28
return matrix
29

```

Listing 33: Creating and populating a two-dimensional matrix while performing some sanity checks corresponding to each matrix entry



## 3.2 Dumping Sudoku Puzzles

```
file.write(f"{len(matrix)}\n") 1
2
for i in range(len(matrix)): 3
    for j in range(len(matrix)): 4
        if matrix[i][j] is not None: 5
            file.write(f"{i + 1}, {j + 1}, {matrix[i][j]}\n") 6
```

Listing 34: Dumping a two-dimensional matrix corresponding to a Sudoku Puzzle to a file