Algorithmic Operation Research

Homework 5
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Exercise 1. Consider the linear programming problem

$$\begin{aligned}
\min \quad & x_1 - x_2 \\
s.t. \quad & 2x_1 + 3x_2 - x_3 + x_4 \leq 0 \\
& 3x_1 + x_2 + 4x_3 - 2x_4 \geq 3 \\
& -x_1 - x_2 + 2x_3 + x_4 = 6 \\
& x \leq 0 \\
& x_2, x_3 \geq 0
\end{aligned}$$

Write down the corresponding dual problem.

Exercise 2. Consider the primal problem

$$\begin{aligned}
\min \quad & c'x \\
s.t. \quad & Ax \geq b \\
& x \geq 0
\end{aligned}$$

Form the dual problem and convert it into an equivalent minimization problem. Derive a set of conditions on the matrix A and the vectors b,c under which the dual is identical to the primal.

Exercise 3. The purpose of this exercise is to show that solving linear programming problems is no harder than solving systems of linear inequalities. Suppose that we are given a subroutine which, given a system of linear inequalities either produces a solution or decides that no solution exists. Construct a simple algorithm that uses a single call to this subroutine and which finds an optimal solution to any linear programming problem that has an optimal solution.

Exercise 4. Let A be a symmetric matrix. Consider the linear program

$$\begin{aligned}
\min \quad & c'x \\
s.t. \quad & Ax \geq c \\
& x \geq 0
\end{aligned}$$

Prove that if x^* satisfies $Ax^* = c$ and $x^* \geq 0$ then x^* is an optimal solution.

Exercise 5. Write down the proof of complimentary slackness theorem.