

```
clear variables
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In rotating co-ordinate system aligned with pole:

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syms t r(t) v(t) a(t) rho(t) theta(t) urho(t) utheta(t)
```

```
r = rho(t) * urho
```

$$r(t) = \rho(t) \text{urho}(t)$$

```
v = subs(diff(r, t), ...  
        {diff(urho(t), t), diff(utheta(t), t)}, ...  
        {utheta(t) * diff(theta(t), t), -diff(theta(t), t) * urho(t)} ...  
        )
```

$$v(t) =$$
$$\text{urho}(t) \frac{\partial}{\partial t} \rho(t) + \rho(t) \text{utheta}(t) \frac{\partial}{\partial t} \theta(t)$$

```
a = subs(diff(v, t), ...  
        {diff(urho(t), t), diff(utheta(t), t)}, ...  
        {utheta(t) * diff(theta(t), t), -diff(theta(t), t) * urho(t)} ...  
        )
```

$$a(t) =$$
$$\text{urho}(t) \frac{\partial^2}{\partial t^2} \rho(t) - \rho(t) \text{urho}(t) \left( \frac{\partial}{\partial t} \theta(t) \right)^2 + \rho(t) \text{utheta}(t) \frac{\partial^2}{\partial t^2} \theta(t) + 2 \text{utheta}(t) \frac{\partial}{\partial t} \theta(t) \frac{\partial}{\partial t} \rho(t)$$

In stationary co-ordinate x-y system:

```
syms ux uy
```

```
urho_xy = rho(t) * cos(theta(t)) * ux + rho(t) * sin(theta(t)) * uy
```

$$\text{urho\_xy} = \text{ux} \cos(\theta(t)) \rho(t) + \text{uy} \sin(\theta(t)) \rho(t)$$

```
r_xy = rho(t) * urho_xy
```

$$r_{xy} = \rho(t) (\text{ux} \cos(\theta(t)) \rho(t) + \text{uy} \sin(\theta(t)) \rho(t))$$

```
v_xy = simplify(diff(r_xy, t))
```

$$v_{xy} =$$
$$\rho(t) \left( 2 \text{ux} \cos(\theta(t)) \frac{\partial}{\partial t} \rho(t) + 2 \text{uy} \sin(\theta(t)) \frac{\partial}{\partial t} \rho(t) + \text{uy} \cos(\theta(t)) \rho(t) \frac{\partial}{\partial t} \theta(t) - \text{ux} \sin(\theta(t)) \rho(t) \frac{\partial}{\partial t} \theta(t) \right)$$

```
a_xy = simplify(diff(v_xy, t))
```

$$\mathbf{a}_{xy} =$$

$$2 \, u_x \cos(\theta(t)) \, \sigma_2 + 2 \, u_y \sin(\theta(t)) \, \sigma_2 + 2 \, u_x \cos(\theta(t)) \, \rho(t) \, \sigma_4 + 2 \, u_y \sin(\theta(t)) \, \rho(t) \, \sigma_4 - u_x \cos(\theta(t)) \, \rho(t)^2 \, \sigma_1$$

where

$$\sigma_1 = \left( \frac{\partial}{\partial t} \theta(t) \right)^2$$

$$\sigma_2 = \left( \frac{\partial}{\partial t} \rho(t) \right)^2$$

$$\sigma_3 = \frac{\partial^2}{\partial t^2} \theta(t)$$

$$\sigma_4 = \frac{\partial^2}{\partial t^2} \rho(t)$$