Optimal Learning When Forgetting

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Abstract

If people know that they may forget information over time, do they strategically respond to their memory decay? I develop a theoretical model of imperfect recall in which a decision-maker optimally shapes memory retention through learning effort. When the decision-maker attempts to recall previously learned information during the learning process, the success or failure of recall provides a signal about their own forgetting rate, leading to updated beliefs about memory strength and effort adjustment. This mechanism endogenously generates the spacing effect, a key property of human memory. I test the model's behavioral predictions with a novel laboratory experiment. The results show that participants are aware of their forgetting and choose their costly learning effort accordingly. Moreover, after observing negative feedback about their actual memory strength, participants adjust their behavior by choosing a higher effort. These findings suggest that individuals can deliberately manage their memory through effort, making imperfect recall an endogenous component of decision-making.

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1 Introduction

Forgetting is an integral and important feature of human learning. People's imperfect recall, defined as their inability to retrieve information they previously learned, has crucial implications for economic decision-making through shaping belief formation, and therefore choices and behavior (Mullainathan, 2002; Afrouzi et al., 2023; Fudenberg et al., 2024; Enke et al., 2024; Wachter and Kahana, 2024). However, the economics literature has paid very limited attention to investigate how imperfect recall arises. Most economic models that involve imperfect recall treat memory as exogenous, or as a fixed technology that restricts the agent's information set. In reality, people know that they may forget information over time. Their awareness raises a fundamental question: If agents anticipate forgetting the information they have learned, how do they strategically respond to their decaying memory?

I develop a theoretical and experimental framework of memory in which a decision-maker (DM) strategically makes choices to optimize an objective that trades off the costs and benefits of memory, and adjusts their behavior in response to signals about their memory strength. The model accommodates key stylized facts about memory from psychological research: the law of recency, the law of repetition, and the spacing effect (Kahana et al., 2024).[1] The law of recency indicates that people recall information that is more recently learned with a higher likelihood. The law of repetition describes the fact that recalling information becomes easier with repeated instances of learning. This fact is related to the role of effort on memory, as suggested by the findings of a stronger memory for items associated with a high-reward (Adcock et al., 2006), which has been shown to be driven by greater encoding effort during learning (da Silva Castanheira et al., 2022). Lastly, the spacing effect describes superior

[1] Kahana et al. (2024) list recency, contiguity, similarity, primacy, and repetition as potential laws of memory. Contiguity, similarity, and primacy effects involve how the relationship between different items affects their recall. Since I only consider a DM who is learning and reviewing the same material, I cannot evaluate how these facts can be accommodated within the framework of my model. Kahana et al. (2024) categorize the spacing effect under the law of repetition.

recall when repetitions of the information are spaced apart over time, a benefit that increases with the spacing interval up to an optimal point (Carpenter and Pan, 2025). This is a highly robust finding that has been documented for over a hundred years of psychological research (Cepeda et al., 2006).[2] While the recency and the repetition effects can be seen as naturally following from memory decay and practice effects, the spacing effect is striking because it contrasts with the law of recency: When instances of learning are farther apart, eventual memory is stronger, even though more total time has passed in which forgetting could occur.[3] These facts constitute foundational characteristics of memory in relation to learning and reviewing information, therefore it is important for any model that describes human memory to account for them.

To analyze the properties that describe the relationship between memory and the timing of the repeated instances of learning, the model includes three periods: the initial learning of some material, the review of the material, and the eventual recall. The DM initially learns some material, such as a fact, a concept, or an association. The likelihood of a successful eventual recall of the material declines over time due to forgetting, but can be increased through a review by exerting effort. The payoffs in the model consist of a reward that is received if the DM can successfully recall the material at the eventual recall stage, and the cost of effort exerted during the

[2]The spacing effect has been observed across diverse learning contexts, including facts (Cepeda et al., 2008), foreign languages (Karpicke and Bauernschmidt, 2011), motor skills (Shea et al., 2000), and various educational settings across different age groups (Carpenter et al., 2022), as well as in animal learning (Kramár et al., 2012; Menzel et al., 2001).

[3]While there is no single explanation that is agreed upon for the spacing effect, there exist four major categories of hypotheses as potential explanations: the deficient processing hypothesis, the encoding variability, the study-phase retrieval, and consolidation (Carpenter and Pan, 2025). The hypothesis of deficient processing argues that the lack of attention for information presented after a short amount of time is the driver of the spacing effect, which has been observed in experimental studies using eye-tracking (Koval, 2019). Encoding variability suggests that encoding information in different environments, which is more likely to happen with a longer spacing, creates richer associations in the brain, however there exists recent neuroscience evidence contradicting this mechanism (Xue et al., 2010). Another leading explanation is the study-phase retrieval, which claims that the repetition of the previously learned material leads the previous study to be retrieved from memory, and the difficulty of the successful recall is beneficial for strengthening memory (Gerbier and Toppino, 2015). Lastly, consolidation hypothesis proposes time-dependent neural processes that help stabilize memories, such as the effect of sleep on memory consolidation (Carpenter and Pan, 2025).

review stage. The DM is aware that the material can be forgotten over time, but is uncertain about the difficulty of retaining it. Therefore, the DM's decision is to choose how much effort to exert during the review stage to maximize their retention at the eventual recall stage according to their beliefs about their own memory strength. The resulting probability of recall is therefore the endogenous outcome of their effort choice. The analysis focuses on the *intensive margin* of learning. That is, how much effort to devote to reviewing the material in order to secure a high probability of recall in the future. This aspect of human learning has been largely overlooked in the economics literature relative to the extensive margin of learning, where binary decisions about whether to encode different types of information are made, the focus of most standard models of costly information acquisition and rational inattention that study "what to learn" (Zhong, 2022).

A key feature of the model is that the DM is uncertain about their true memory strength, the underlying state that determines how easily the learned material is retained, and consequently about the exact effect of their effort choice on the future probability of recall. The DM can, however, update their beliefs about their memory strength by attempting to recall the initially learned material before the review. The outcome of this recall attempt, referred to as the signal, indicates whether the recall was successful or not, and provides feedback about the DM's true retention rate since the likelihood of successful recall depends on how quickly the probability of recall has been decaying since the initial learning of the material. For instance, failing to recall the material some time after the initial learning may lead the DM to infer that the material is difficult to retain in the absence of additional effort, leading the DM to choose a higher effort level during review. Thus, the recall attempt leads the DM to adjust their effort choice depending on the outcome of the recall, and consequently affects the resulting retention rate. This signal mechanism comes through an intuitive process of self-monitoring, which can be illustrated by a student becoming aware of how much they have forgotten while restudying.

I consider two versions of the model: one without signals and one with the signal from attempting to recall the previously learned material. I show that, under natural assumptions about the recall function, which determines the probability of recall with respect to the chosen effort level, the DM's optimal choice always results in the recency effect, regardless of whether a signal is present. However, without the signals from recall attempts, the analogous assumptions do not accommodate the spacing effect. When the DM decides the effort level based only on prior beliefs, they may optimally exert less effort after a shorter spacing, which can be interpreted as paying less attention when the material is closely repeated. Yet the resulting probability of recall remains higher when the spacing between the initial learning and the review stage is shorter. This result indicates that, unless the recall function is assumed to exhibit implausibly high marginal returns to effort after long delays, a higher optimal effort choice for longer spacing is insufficient to generate the spacing effect.

On the other hand, when the DM receives a signal about the unobserved state of their memory strength by making a recall attempt before the review, the signal generated after a longer spacing becomes more informative regarding how quickly the recall probability decays. Fast and slow forgetting rates produce a larger difference between recall outcomes over longer intervals, allowing the DM to infer more precisely their true retention ability. The updated beliefs following a more informative signal about one's cognitive constraints, and the corresponding optimal effort adjustment is shown to be the key drivers that endogenously generate the spacing effect. A longer spacing leads to more informative signals which can lead the DM to choose a much higher effort level after a failed recall attempt. The exertion of lower encoding effort after a short spacing–the deficient processing hypothesis–has been suggested as one of the main explanations of the spacing effect in the psychology literature (Carpenter and Pan, 2025). My model demonstrates that if the DM is not learning about their own retention rate through recall attempts at the review stage, the higher effort that is optimally chosen is insufficient to offset the loss in memory due to a longer spacing. Crucially, the model provides a theoretical explanation for the spacing effect by showing how longer spacing makes the DM better informed about their memory limitations, a channel that can occur naturally but is typically uncontrolled for in spaced-learning experiments.[4]

To test the model's behavioral predictions, I conducted a pre-registered laboratory experiment with a novel experimental design featuring an incentivized task to measure participants' choices of costly learning effort. In this experiment, participants chose their learning effort to memorize a list of word-pairs, where the learning effort was measured by the time allocated to studying the word-pairs during the review, for the purpose of maximizing their probability of successful recall at the final test that is incentivized by a monetary reward. Exerting more effort was costly, so that they needed to pay a monetary cost for studying longer. A practice quiz that is conducted before the review served as a signal mechanism to provide participants with feedback on their memory decay. Four randomly assigned treatments determined whether the participants were assigned to a short or long spacing, and whether they received feedback or not. Using random-incentive scheme, I measured the ex-ante effort choices that are selected before the spacing for short and long spacing scenarios, in addition to the ex-post effort choices that are selected after spacing. The elicitation of these choices allowed for a within-subject comparison of effort over different learning schedules, and availability of signals. In addition to the effort choices, I

[4] Several computational models of memory in psychology, which provide mathematical formulations for the probability of recall, have been suggested to additionally accommodate for the spacing effect such as Raaijmakers (2003)'s generalized Search of Associative Memory (SAM), Pavlik Jr and Anderson (2005)'s version of Adaptive Control of Thought-Rational (ACT-R), and Walsh et al. (2018)'s Predictive Performance Equation (PPE). SAM uses an encoding variability explanation where longer delays causes the contextual state of the memory to drift; this results in spaced repetitions being encoded in more varied contexts, increasing the probability of a match during a future retrieval attempt. ACT-R describes that the decay rate for a new memory trace is dependent on how accessible the memory is at the moment of review. PPE provides a functional specification where the learning schedule changes the decay rate directly. While the mathematical formulations in SAM and ACT-R are chosen to represent specific psychological processes, in all three models the spacing effect is a direct mathematical consequence of these chosen functional forms. Additionally, these models do not have an agent who is decision-making, nor an effort choice.

elicited the participants' beliefs regarding their expected practice quiz performance using an incentive-compatible mechanism that deters the participants to underperform on purpose at the practice quiz. The difference between their expected quiz scores and their actual performance was then used as a signal about their memory strength after spacing. The randomly implemented order of the practice quiz and the ex-post effort choice determined whether this signal was available to the participant while making their ex-post effort choice. Finally, the participants' probability of recall was measured by their final test performance.

With this experiment, I tested 9 pre-registered hypotheses that are derived from the behavioral predictions and assumptions of the model, mainly focusing on how learning effort varies over different learning schedules, and in response to feedback about memory. The results of the experiment are consistent with the theoretical predictions of the model. The participants are found to be aware of their own forgetting, and to incorporate this into their learning effort choices: participants expect to forget more from their initial learning until the review following a longer spacing, and choose a higher ex-ante effort level for this long spacing scenario. Furthermore, participants adjust their effort levels depending on the feedback that they receive about their memory strength: if they receive a signal that indicates remembering worse at the practice quiz compared to their initial prediction, they choose a higher ex-post effort. According to the final test outcomes, I do not find evidence for the spacing effect in the presence of signals. However, I show that this outcome can be reconciled with the model by accounting for the observed difference in participants' ex-ante effort choices.

The primary contribution of this study is that it is the first economic model of imperfect recall where the probability of recall is an endogenous outcome of an optimal learning effort, strategically chosen in response to the agent's own forgetting over time.[5] This strategic response to forgetting highlights the importance of modeling

[5]A closely related model of optimal learning with endogenous memory decay is by Neligh (2024), where a DM chooses costly effort to determine the accuracy of encoding, while the information in

memory as an endogenous process in dynamic choice models with imperfect recall, since the existence of different incentives for remembering different types of information can generate different rates of probability of recall. Another contribution of this model is its consideration of metamemory control, defined as the ability to monitor and control one's own memory, whereby the DM learns about their memory limitations by recall. This natural feature of learning from the observation of own recall outcomes is shown to produce a new explanation for the spacing effect. Moreover, as the signal about forgetting is found to be effective on the learning effort choice and future recall, the use of external feedback about memory can be suggested as a possible economic tool to design the level of imperfect recall for the DMs, which can then impact their choices and behavior. Finally, this paper presents the first experiment on memory where the costly learning effort is measured with incentives, and contributes to the experimental literature on memory in economics by providing evidence that decision-makers choose a higher effort level when they anticipate to forget more, and increase their effort choices after receiving negative feedback about their memory strength.

Although the framework of the model is somewhat specific in terms of the periods of learning and the signal mechanism, it can be modified and expanded to discuss important economic applications. I present two such examples. The first demonstrates how the interaction of beliefs and the memory resulted by endogenous learning effort can affect consumer choice. In this example, following surprising news, depending on the current strength of their memory, the DM may or may not choose to relearn the decayed information in memory to form new beliefs. This endogeneity for the rehearsal of previously learned information leads to contrasting predictions about consumer

memory becomes noisier over time. The recency effect is generated endogenously in this model by the higher precision of more recent memories being given a larger weight in the posterior beliefs about the state of the world. The decay in memory in the framework of Neligh (2024) is the decline of the information quality, while the memory decay in my framework is affecting whether a piece of information can be successfully recalled. Another difference of my model is the uncertainty that the DM is facing about his memory decay, and the learning mechanism about memory through recall attempts.

choice. The second application shows how metamemory control can improve the efficiency of program design, using job retraining for the unemployed as an example. By delaying the intervention, agents are given time to monitor their own skill decay, which facilitates efficient self-selection and prevents costly, unnecessary enrollment. These examples illustrate the potential importance of the analysis of imperfect recall through endogenous learning decisions under the awareness of forgetting.

This paper is structured as follows. Section 2 reviews the related literature, by discussing the theoretical work on optimal learning and endogenous memory, and the experimental studies about memory in economics. Section 3 presents the core theoretical framework, introducing the formal model and then illustrating its mechanics with a simple example. Section 4 details the experimental design and outlines the pre-registered hypotheses derived from the theoretical model, followed by the experimental results presented in Section 5. Finally, Section 6 illustrates potential economic applications of the model, and Section 7 concludes.

2 Related Literature

Several works in the economics literature have studied the design of an optimal memory subject to various limitations. Wilson (2014) characterizes how a DM should optimally summarize a sequence of informative signals about the state of the world into a finite number of memory states, given that the DM cannot recall the full history of observed signals and is only able to observe their current memory state. Chen et al. (2010) studies the effect of consumers' optimal encoding of price information to a memory with a bounded capacity on price competition between firms. Afrouzi et al. (2023) document experimental evidence on the overreaction to most recent observations in a forecasting framework. They explain this result with a model where the DM, who can only use the information in the working memory, can freely access the most recent observation but must choose which other observations to retrieve from long term memory into the working memory. The cost of keeping a set of information in the working memory is determined by its informativeness. None of these papers feature unintentional memory decay.

Azeredo da Silveira et al. (2024) provides another explanation for the overreaction result in Afrouzi et al. (2023), using a model of memory with decay. In their framework, the DM flexibly decides on the structure of how to store or retrieve information, when the cost of memory depends on how precise it is in terms of its informativeness. The closest study to my framework is Neligh (2024)'s analysis of a rational memory in which a DM chooses costly effort to decide how accurately to encode information, as the information recorded in memory becomes noisier over time. Neligh (2024) shows that this model generates the recency effect, and discusses the implications of this memory structure in various economic settings. The type of memory decay caused by time in this model differs from the one in my framework, which is affecting the probability of recall. In addition, Neligh (2024) assumes common knowledge about the memory decay mechanism, whereas the DM in this paper is uncertain about the degree of the memory decay. Another stream of economic research involving an endogenous memory structure is models of motivated memory, such as Bénabou and Tirole (2002)'s model on memory and self-confidence, where a DM chooses whether to retain information about past successes and failures.

The analysis of memory from a rational perspective is also studied in the psychology literature. Anderson and Milson (1989)'s seminal work provides a rational perspective on human memory, where the probability of retrieving information from memory depends on how useful it is estimated to be. Building on this framework, a recent study by Callaway et al. (2024) constructs a computational model of memory recall, where the time allocated to the search for memories is optimally chosen based on the cost of searching and the potential utility of recall. Similar to my framework, their model includes a component of metacognitive monitoring: the agent who is not able to recall after a long search updates their "feeling of knowing" for the target. The main difference from my model is that the optimization occurs at the recall stage (choosing search duration), whereas my model focuses on optimal effort exertion during the learning stage. The idea of optimal learning and metacognition is mostly studied in educational contexts within the psychology literature. In an unincentivized experiment, Metcalfe and Finn (2008) shows that high self-reported beliefs regarding how well a participant knows some material affect their decision on how much more to study it. Another unincentivized experimental study by Bahrick and Hall (2005) analyzes how longer spacing between learning sessions can lead the students to select more efficient studying methods, such as verbal or visual elaboration over rereading and increase the final recall. They suggest this occurs because the long spacing interval makes students aware of the inefficiency of certain methods as they result in more retrieval failures during studying. This idea closely aligns with the theoretical mechanism I propose for the spacing effect.

The literature about memory limitations in economics demonstrates diverse mechanisms on how memory can affect beliefs and behavior. In a game-theoretic setting, Deck and Sarangi (2009) induce imperfect recall in participants by dividing their attention, but imperfect recall is not found to be affecting the rationality of participants' behavior. Several studies analyze the concept of motivated memory in experimental settings. Li (2013) finds that in social settings, individuals remember kinder acts better, and perceive their own unkind acts as less unkind over time, which suggest that people can be strategically manipulating their memory to maintain self-esteem. Conlon (2025) experimentally studies the effect of rehearsal of previous experiences on beliefs and recall, and finds that utility from revisiting specific type of experiences, such as enjoying thinking about one's own previous accomplishments, drive biased beliefs and distorted future recall as a result of naivete about rehearsal effects on memory. The choice of rehearsal acts in a similar way to the choice of learning effort in this paper in terms of increasing probability of recall. However, in Conlon (2025)'s framework, the cost of rehearsal is indirect through the biased beliefs, and there is no consideration of a memory decay over time. Other papers focus on selective recall mechanisms. Caballero and López-Pérez (2024) find that participants have better memory for information that was previously received as good news regarding a payoff. Enke et al. (2024) studies the effect of associative recall through the context of signals on the overreaction of beliefs. In a theoretical and experimental framework, Bordalo et al. (2023) show the impact of selective recall through similarity and interference on the estimation of probabilities. Lastly, Graeber et al. (2024) demonstrate that the qualitative context of the information, whether it is a story or statistic, affects selective recall. My experiment contributes to this growing literature with the analysis of strategic decisions about memory through the choice of learning effort.

3 Model

Consider a three period setting, where a decision-maker (DM) learns some material at the initial learning date, then studies the material at the review date by choosing how much learning effort to exert, and finally attempts to recall the material at the test date. If the DM successfully remembers the material at the test date, they will receive a reward, otherwise they will not receive a reward.[6] The utility of receiving the reward is normalized to 1, where the utility of no reward is 0.

The DM initially learns the material at $t = 0$. From the initial learning until the review date, which is at $t = \tau \in \mathbb{R}_+$, the DM's probability of recall for this material declines over time due to forgetting. The time interval τ between the initial learning

[6]To relate this setting to the standard models of learning in economics with memory limitations, the material that is learned can be thought as the true state of the world. The DM initially learns this state, and encodes this information with complete accuracy in their memory as a memory state. We can think the unintentional forgetting over time in my model corresponding to a stochastic transition from the memory state with the correctly encoded information to another memory state with no information. The DM's purpose to correctly remember the material in my setting could then be reframed as choosing an action to match the state of the world that pays the reward only if they can match the state correctly. As there is a single type of material that the DM is trying to memorize in my model, and because there is no analysis on partially forgetting the material in terms of its informational content, I omit using the notion of memory states in my framework.

and the review date is defined as *spacing*. The DM's probability of recall depends on how difficult the material is to retain in the memory. Let the state $\omega \in [0, 1]$ represent the difficulty of the material. Notice that the state in this model represents the strength of the DM's memory that the DM does not know. Therefore, the state of the nature in this model represents the DM's unobserved level of skill on memory retention. Suppose the DM has a prior belief that the difficulty of the material, ω, lies in the set $\{\omega_1, \ldots, \omega_n\}$ where $0 = \omega_1 < \omega_2 < \cdots < \omega_n = 1$. Let $\mu_i \in [0, 1]$ be the DM's prior belief that $\omega = \omega_i$, $i \in \{1, 2, \ldots, n\}$, with $\sum_{i=1}^{n} \mu_i = 1$, $\mu = (\mu_1, \ldots, \mu_n)$.

Denote the probability of recall before studying the material at the review date $t = \tau$ as $\pi(\tau; \omega)$.[7] π is assumed to be decreasing in the spacing τ and the difficulty ω, as the DM forgets over time, and the forgetting rate is higher when the material is more difficult. If the DM attempts to recall the material at the review date before exerting effort, they will successfully remember the material with probability $\pi(\tau; \omega)$. This recall attempt can act as a mechanism that generates a binary signal $s \in \{0, 1\}$ about the actual difficulty of the material: the DM successfully recalls the material and receives the signal $s = 1$ with probability $\pi_1(\tau; \omega) := \pi(\tau; \omega)$, or fails to recall the material and receives $s = 0$ with probability $\pi_0(\tau; \omega) := 1 - \pi(\tau; \omega)$. Let $\mu_{0,i}(\tau)$ and $\mu_{1,i}(\tau)$ be the posterior beliefs that $\omega = \omega_i$ after receiving $s = 0$ or 1, respectively.

At the test date, which is $T \in \mathbb{R}_+$ periods after the review date, the DM attempts to recall the material, and receives the reward if succeeds. To increase the probability of receiving the reward, the DM must exert learning effort at the review date $t = \tau$. The DM studies the material by choosing the amount of effort $e \in \mathbb{R}_+$ to exert. Exerting effort is costly. The cost of effort is denoted by $c(e)$, and is assumed to be increasing and convex in e.[8] Studying the material with effort e causes an instantaneous increase in the probability of recall at $t = \tau$, which then continues to decline until the test date $t = \tau + T$ due to forgetting.

[7] $\pi : \mathbb{R}_+ \times [0, 1] \to [0, 1]$

[8] $c : \mathbb{R}_+ \to \mathbb{R}_+$. c is assumed to be continuous.

Remember that the DM receives the reward 1 if they can successfully recall the material at the test date, and no reward otherwise. Assume that the preferences of the DM over the reward and the effort e admit an additively separable utility function, which yields the payoff $1 - c(e)$ when the reward is received, and $0 - c(e)$ when no reward is received.

Let $R(e, \tau, T; \omega)$ denote the probability of recall at the test date $t = \tau + T$, when effort e has been exerted at the review date $t = \tau$, and when the difficulty of the material is ω.[9] The probability of recall at the test date is assumed to take the following form:

$$R(e, \tau, T; \omega) = 1 - \omega + \omega \tilde{R}(e, \tau, T).$$

$R(e, \tau, T; \omega)$ can be interpreted as a convex combination of perfect recall, which always generates a probability of recall equal to 1, and the worst-case probability of recall, $\tilde{R}(e, \tau, T)$. Accordingly, when the difficulty ω is smaller, the weight on perfect recall increases, hence the material becomes easier to remember.

The probability of recall at the review date $t = \tau$ before studying the material, $\pi(\tau; \omega)$, and the probability of recall at the test date $t = \tau + T$, $R(e, \tau, T; \omega)$, are naturally linked to each other. The probability of recall at the review date $t = \tau$ before studying is equal to the probability of recall function R with an effort choice of 0 and the test date immediately following the review date at τ: $\pi(\tau; \omega) = R(0, \tau, 0; \omega)$.[10] Since these two functions are linked to each other, observing a signal before studying that is generated by $\pi(\tau; \omega)$ informs the DM about his future recall, $R(e, \tau, T; \omega)$. I denote the probability of recall before studying at the review date for the worst case as $p_\tau = \pi(\tau; 1)$, so that $\pi(\tau; \omega) = 1 - \omega + \omega p_\tau$.

The worst-case probability of recall, $\tilde{R}(e, \tau, T)$, is assumed to be strictly increasing

[9] $R : \mathbb{R}^3_+ \times [0, 1] \to [0, 1]$. R is assumed to be continuous and differentiable.

[10] This value is also equivalent to the case where $e = 0$, the review date coincides with the initial learning at $t = 0$, and the test date is at $t = \tau$, $R(0, 0, \tau; \omega)$.

and strictly concave in effort e, decreasing in T, and strictly decreasing in the spacing τ. Since R is a positive affine transformation of $\tilde{R}$, these properties of $\tilde{R}$ also hold for R. The concavity of R in e implies that the probability of recall is assumed to have decreasing marginal returns to effort, which is a plausible assumption considering that probability of recall is bounded at 1 from above. Moreover, the assumption of R being decreasing in T indicates that the retention rate is decreasing over time after studying the material. Furthermore, the assumption of R being strictly decreasing in τ indicates that exerting the same effort level at the review date leads to a lower probability of recall when the probability of recall before studying is lower due to a longer spacing.

Before proceeding to the main analysis, I first formally define the *spacing effect* and the *recency effect* within the framework of my model. The spacing effect is defined as the observed probability of recall to be higher due to a longer spacing τ, up to an optimal level τ^*, when the retention interval T is fixed. This means that the recall probability is non-monotonic in τ if the DM displays spacing effect. In contrast, the recency effect is defined as the observed probability of recall to be lower due to a longer retention interval T, when the spacing τ is fixed. Notice that the observed probability of recall depends on the chosen level of effort. Denote the effort level chosen by the DM as $e(\tau, T)$ when the spacing is τ and the retention interval is T.

Definition 1. The DM displays *spacing effect* if there exists $\tau^* > 0$ such that the observed probability of recall at the test date $R(e(\tau, T), \tau, T; \omega)$ is increasing in τ for $0 < \tau < \tau^*$, and decreasing in τ for $\tau > \tau^*$, given T and ω.

Definition 2. The DM displays *recency effect* if the observed probability of recall at the test date $R(e(\tau, T), \tau, T; \omega)$ is decreasing in T, given τ and ω.

I now turn to the analysis of optimal learning effort and the resulting probability of recall, which proceeds in two parts. First, in Section 3.1, I examine the DM's decision based on prior beliefs. I show that while this framework always predicts a recency effect, it cannot generate a spacing effect. Second, in Section 3.2, I turn to the analysis of a DM who observes a signal from a recall attempt and updates their beliefs regarding their memory strength. I show that the DM increases (decreases) effort after a negative (positive) signal. Furthermore, I demonstrate that the signal's informativeness increases with spacing, providing a mechanism that can, in turn, generate the spacing effect. Lastly, in Section 3.3, I provide a simple example and discuss the implications of the model.

3.1 Optimal Learning Under Prior Beliefs

Consider the case where the DM maximizes their ex-ante expected payoff, without updating their beliefs on the difficulty of retaining the material in memory through recall attempts:

$$\max_{e\geq 0} \mathbb{E}[R(e,\tau,T;\omega)] - c(e) \iff \max_{e\geq 0} \sum_{i=1}^{n} \mu_i \left(1-\omega_i+\omega_i \tilde{R}(e,\tau,T)\right) - c(e)$$

Let $e^*(\tau,T) = \text{argmax}_{e\geq 0}\, \mathbb{E}[R(e,\tau,T;\omega)] - c(e)$.

The uncertainty regarding the state ω can be interpreted as the DM being uncertain about the marginal returns to effort on the probability of recall, $\omega\tilde{R}_e(e,\tau,T)$, which determines how much effort matters for memory retention. Since the marginal return to effort is always smaller when the material is easier to retain in memory, the DM will choose a smaller effort level if they believe that the material is easier to remember. Alternatively, this uncertainty can be viewed as uncertainty about the forgetting rate, which also depends on ω. When the material is easier to remember, the forgetting rate is slower, therefore, a lower effort is sufficient to sustain a high level of probability of recall. Accordingly, Proposition 1 shows how the beliefs of the DM about their own forgetting rate affect the optimal effort choice.

Proposition 1. *If μ first-order stochastically dominates μ', $e^*(\tau,T|\mu) \geq e^*(\tau,T|\mu')$.*

The proof Proposition 1 follows from the larger marginal returns to effort when the material is more difficult to remember. The complete proof for Proposition 1 can be found in Appendix A.2.

Let us now consider whether the DM's optimal effort choice under the prior beliefs can generate the recency and the spacing effects. To observe the spacing effect, when τ is smaller than the optimal lag τ^*, the observed probability of recall must increase, therefore, the optimal effort choice must be increasing in the spacing τ. Otherwise, if the optimal effort choice is smaller when spacing is longer, then the probability of recall for the shorter spacing will definitely be higher, as R is decreasing in τ. For this reason, optimal effort choice being increasing in τ up to the optimal lag τ^* is a necessary condition for observing the spacing effect. If the spacing enhances the effectiveness of effort on memory retention, then the DM will choose a higher effort level for the longer spacing:

Remark 1. If $\tilde{R}_{e\tau}(e,\tau,T) > 0$ for all $e > 0$, given τ, T, then $\frac{\partial e^*(\tau,T)}{\partial \tau} > 0$.

Remark 1 follows from the marginal returns to effort being larger for the longer spacing. The proof can be found in Appendix A.3.

While effort to be increasing in spacing is a necessary condition to observe the spacing effect, a lower effort choice for a longer spacing is a sufficient condition for not observing the spacing effect. Therefore, if there exist $\tau < \tau' < \tau^*$ such that the DM finds it optimal to exert a lower level of effort for τ', the DM cannot exhibit the spacing effect where τ^* is the optimal lag:

Remark 2. If there exist $\tau < \tau' < \tau^*$ such that $e^*(\tau,T) > e^*(\tau',T)$, given T, then the DM cannot exhibit spacing effect for any $\omega \in [0,1)$.

Remark 2 follows from the probability of recall function R being strictly decreasing in the spacing τ, where T and $\omega \neq 1$ are given.

Even though the optimal effort choice for the longer spacing can be larger as shown in Remark 1, Proposition 2 shows that, unless the marginal returns to effort on the probability of recall are unreasonably high for longer spacing, as described in Assumption 1, the DM never exhibits the spacing effect. Assumption 1 states that whenever the DM exerts a higher effort level with a longer spacing to reach the same probability of recall with a shorter spacing, the marginal returns to effort on the probability of recall is lower for the longer spacing case, where the DM has already exerted a large amount of effort, compared to the shorter spacing case.

Assumption 1. For each $\tau < \tau'$, and each $e > 0$, fix e' such that $\tilde{R}(e, \tau, T) = \tilde{R}(e', \tau', T)$ whenever such e' exists, given T. Then, $\frac{\partial \tilde{R}}{\partial e}(e, \tau, T) \geq \frac{\partial \tilde{R}}{\partial e}(e', \tau', T)$.[11]

As R is a positive affine transformation of $\tilde{R}$, R also satisfies this assumption whenever Assumption 1 holds.

If the probability of recall function satisfies Assumption 1, then the resulting probability of recall when the DM chooses the learning effort optimally is always decreasing in the time between the initial learning and the review date. This result indicates that, even when the DM finds it optimal to exert a higher level of effort in order to compensate for the loss in memory from a longer delay, the increase in the optimal level of effort will never be sufficient to offset the stronger effect of forgetting with the longer spacing.

Proposition 2. *If $\tilde{R}$ satisfies Assumption 1, then $R(e^*(\tau, T), \tau, T; \omega)$ is decreasing in τ for $\tau \geq 0$ for any state ω, hence cannot exhibit the spacing effect.*

Proposition 2 follows from the fact that when there is a longer spacing, the marginal return to effort at an effort choice that generates a higher probability of recall will be lower. This effort level cannot be optimal with convex costs, leading to a an optimal effort choice that generates a smaller probability of recall, even when the optimal choice of effort is increasing in spacing. The proof for Proposition 2 can be found in

[11] Assumption 1 is equivalent to $\tilde{R}_e(e, \tau, T)\tilde{R}_{e\tau}(e, \tau, T) \leq \tilde{R}_{ee}(e, \tau, T)\tilde{R}_\tau(e, \tau, T)$, $\forall e,\ \tau,\ T$. The equivalence of this condition with Assumption 1 is shown in Appendix A.1

Appendix A.4.

Similarly, Assumption 2 considers the case that whenever the DM exerts a higher level of effort to be able to remember the material at a distant test date with the same probability of recall that could be achieved with a smaller effort for a more recent test date, the marginal returns to effort will be lower for the case with the more distant test date, where the DM has already exerted a large amount of effort, compared to the case with the sooner test date.

Assumption 2. For each $T < T'$, and each $e > 0$, fix e' such that $\tilde{R}(e, \tau, T) = \tilde{R}(e', \tau, T')$ whenever such e' exists, given τ. Then, $\frac{\partial \tilde{R}}{\partial e}(e, \tau, T) \geq \frac{\partial \tilde{R}}{\partial e}(e', \tau, T')$.[12]

While the spacing effect will not be observed when the DM decides the learning effort according to their prior beliefs, Proposition 3 shows that, assuming Assumption 2, the DM will always exhibit the recency effect.

Proposition 3. *If $\tilde{R}$ satisfies Assumption 2, then $R(e^*(\tau, T), \tau, T; \omega)$ is decreasing in T, given τ and ω.*

Proposition 3 can be shown by following the same reasoning as Proposition 2.

In conclusion, when the effort choice is determined according to the DM's prior beliefs, under reasonable assumptions regarding the probability of recall function, the recency effect will always be observed, whereas the spacing effect can never be observed. In the following section, I discuss the signal mechanism that is generated by making a recall attempt at the review date, which can result in the emergence of the spacing effect. For the remainder of the discussion in Section 3.2, T is held fixed; therefore, the variable T is omitted from the notation for simplicity.

[12]Similarly, Assumption 2 is equivalent to $\tilde{R}_e(e, \tau, T)\tilde{R}_{eT}(e, \tau, T) \leq \tilde{R}_{ee}(e, \tau, T)\tilde{R}_T(e, \tau, T)$, $\forall e$, τ, T.

3.2 Learning Decisions After Signals About Forgetting

Now consider the case where the DM receives a binary signal $s \in \{0, 1\}$ by making an attempt to recall the material that is learned at $t = 0$, before choosing the effort level at the review date $t = \tau$. The difficulty of the material, ω, determines how fast the DM will forget the material between the initial learning date until the review date. Therefore, the DM can observe the signal s to update their beliefs about how difficult the material is to retain in the memory, which will consequently affect the expected return to their effort choice on the probability of recall at the test date. Remember that $\mu_{0,i}(\tau)$ and $\mu_{1,i}(\tau)$ are the posterior beliefs that $\omega = \omega_i$ after receiving $s = 0$ or 1, respectively.[13]

After receiving the signal s, the DM maximizes their expected payoff with respect to their posterior belief:

$$\max_{e_s \geq 0} \mathbb{E}[R(e_s, \tau; \omega)|s] - c(e_s) \iff \max_{e_s \geq 0} \sum_{i=1}^{n} \mu_{s,i}(\tau)(1 - \omega_i + \omega_i \tilde{R}(e_s, \tau)) - c(e_s)$$

Let $e^*_s(\tau)$ be the optimal effort choice after receiving s.

Then, for any state ω, the expected value for the probability of recall when effort is optimally chosen after observing the signal will be $\pi_1(\tau; \omega)[R(e^*_1(\tau), \tau; \omega)] + \pi_0(\tau; \omega)[R(e^*_0(\tau), \tau; \omega)]$.

Receiving the signal helps the DM to understand the effectiveness of the effort choice on the probability of recall. After receiving the signal $s = 0$, the DM updates their belief to assign greater likelihood to the material being more difficult than under their ex-ante belief. This implies that the DM places greater weight on the larger values of ω, thereby increasing the weight on the worst-case probability of recall, $\tilde{R}(e, \tau)$. As a result, the marginal returns to effort of the expected value of the recall function are

[13] Given $p_\tau < 1$, which is guaranteed if $\tau > 0$, $\mu_{0,i}(\tau) = \frac{\mu_i(1-p_\tau)\omega_i}{\sum_{j=1}^{n} \mu_j(1-p_\tau)\omega_j} = \frac{\mu_i \omega_i}{\sum_{j=1}^{n} \mu_j \omega_j}$, $\quad \mu_{1,i}(\tau) = \frac{\mu_i(1-\omega_i+\omega_i p_\tau)}{\sum_{j=1}^{n} \mu_j(1-\omega_j+\omega_j p_\tau)}$. If $\tau = 0$ and $p_0 = 1$, then $\mu_{0,i}(0) = \mu_{1,i}(0) = \mu_i$.

larger according to their posterior beliefs, leading to a larger effort choice. By the same reasoning, the optimal effort after receiving $s = 1$ will be smaller.

Proposition 4. *The DM chooses a higher effort level $e_0^*(\tau)$ after a failed recall attempt ($s = 0$), and a lower effort level $e_1^*(\tau)$ after a successful recall attempt ($s = 1$) compared to the effort choice without signals $e^*(\tau)$.*

The complete proof for Proposition 4 can be found in Appendix A.5

We can now analyze the implications of a longer spacing on the signal mechanism. after the DM receives the signal s. The key mechanism is that a longer spacing increases the amount of forgetting that occurs before the review date. The DM anticipates this, meaning they expect their baseline recall probability to be lower, as formalized below.

Remark 3. The expected probability of recall before studying at the review date, $\mathbb{E}[\pi(\tau;\omega)]$, is decreasing in τ.

Remark 3 follows directly from p_τ being decreasing in τ, which implies that $\mathbb{E}[\pi(\tau;\omega)] = \mathbb{E}[1 - \omega + \omega p_\tau]$ also being decreasing in τ.

While the DM expects to forget more on average over a longer spacing, the DM also expects to have a larger difference between the probability of recall levels when the difficulty is different. As the difference between the states increases with more spacing due to having more time to forget the initially learned material, the informativeness of the signal s increases in τ. As an example, consider the case where $p_0 = 1$. If the DM attempts to recall the material at $t = 0$, the probability of receiving $s = 1$ is going to be 1 for all states, which will not be helpful to understand which state is more likely. When spacing is larger, p_τ will be smaller, leading to a larger difference across states in the probability of making a successful recall attempt. Formally, the informativeness of s can be measured as the mutual information of the difficulty of the material and the signal.

Proposition 5. *The mutual information $I(\omega; s) = H(\omega) - H(\omega|s)$ of the difficulty of the material ω and the signal s is increasing in τ, where $H(.)$ and $H(.|.)$ are the entropy and the conditional entropy functions, respectively.*

The proof for Proposition 5 follows from the log sum inequality theorem (Cover and Thomas, 1991, Theorem 2.7.1). The complete proof can be found in Appendix A.6.

Receiving signals about their own memory changes how the spacing between the initial learning and the review sessions influences DM's recall at the test date. When the DM receives a signal, the spacing affects the probability of recall at the test date through three distinct channels. First, spacing directly lowers recall because the probability of recall function is decreasing in the spacing τ ($\tilde{R}_\tau(e^*_s(\tau), \tau) < 0$). Second, conditional on each type of signal, the DM will adjust their optimal effort choice, with larger spacing leading to an indirect effect on recall ($\tilde{R}_e(e^*_s(\tau), \tau)e^{*\prime}_s(\tau)$). These two forces are the same as the case with no updating, where the aggregate impact of spacing was shown to be always negative ($\tilde{R}_e(e^*_s(\tau), \tau)e^{*\prime}_s(\tau) < |\tilde{R}_\tau(e^*_s(\tau), \tau)|$) when Assumption 1 holds.

By contrast, with the existence of a signal, an additional force arises: as spacing increases, the signal becomes more informative, and receiving a negative signal becomes more likely, potentially leading to a much higher choice of effort. This informativeness channel provides a third pathway through which spacing influences recall. With this additional effect, the expected probability of recall when the optimal effort is chosen after observing the signal may increase with spacing, in contrast with the monotonically decreasing pattern without signals under Assumption 1.

Proposition 6. *The expected probability of recall conditional on receiving signal exhibits spacing effect if there exists $\tau^* > 0$ such that $-\frac{\partial \pi(\tau;\omega)}{\partial \tau}[\tilde{R}(e^*_0(\tau), \tau) - \tilde{R}(e^*_1(\tau), \tau)] - |\pi_1(\tau;\omega)[\tilde{R}_e(e^*_1(\tau), \tau)e^{*\prime}_1(\tau) + \tilde{R}_\tau(e^*_1(\tau), \tau)] + \pi_0(\tau;\omega)[\tilde{R}_e(e^*_0(\tau), \tau)e^{*\prime}_0(\tau) + \tilde{R}_\tau(e^*_0(\tau), \tau)]| \geq 0$ for all $\tau < \tau^*$, and < 0 for all $\tau \geq \tau^*$.*

The complete proof for Proposition 6 where the spacing effect is characterized in terms of the primitives of the model can be found in Appendix A.7. To prove Proposition 6, the change in the expected probability of recall is decomposed into three parts as explained above. The resulting probability of recall conditional on receiving $s = 0$ $(R(e_0^*(\tau), \tau; \omega))$, or $s = 1$ $(R(e_1^*(\tau), \tau; \omega))$, is decreasing in spacing in either of the cases due to the same reason in Proposition 2: even though $e_s^*(\tau)$ is increasing in τ, it will not be enough to compensate for the loss in memory due to a larger spacing. However, the spacing will cause the probability of receiving $s = 0$ to increase, leading $R(e_0^*(\tau), \tau; \omega)$ to happen more frequently than $R(e_1^*(\tau), \tau; \omega)$. Since $e_0^*(\tau) > e_1^*(\tau)$ as shown in Proposition 4, the effect of spacing on the signal can then lead to an increase in the expected probability of recall.

The presence of a signal does not change the conclusion regarding the recency effect. The probability of recall with the optimal effort choice after observing $s = 0$ or $s = 1$ will both be decreasing in T. When τ is fixed, the probability of observing $s = 1$ will remain constant, therefore, the average probability of recall with optimal the effort choice will continue to exhibit recency effect when signals are present.

3.3 A Simple Example

To illustrate the theoretical findings in a simpler setting, the following stylized case is considered as an example. Assume that the DM's probability of recall function is $R(e, \tau; \omega) = 1 - \omega + \omega \tilde{R}(e, \tau)$ where the material to be learned is either easy $(\omega = \gamma < 1)$ or difficult $(\omega = 1)$. If the material is difficult, $R(e, \tau; 1) = \tilde{R}(e, \tau)$. If the material is easy, $R(e, \tau; \gamma) = 1 - \gamma + \gamma \tilde{R}(e, \tau)$ which can be interpreted as the convex combination of perfect recall that always generates a probability of recall equal to 1, and the difficult case.

Let $\tilde{R}$ be as follows:

$$\tilde{R}(e, \tau) = 1 - (1 - \exp(-\lambda (\tau + T))) \exp\left(-\frac{ae}{1 + b\tau}\right)$$

To analyze the properties of $R(e, \tau; \omega)$, let us firstly focus on the difficult case where $R(e, \tau; 1) = \tilde{R}(e, \tau)$. If the DM chooses $e = 0$ at the review date ($t = \tau$), the probability of recall will be $p_\tau = \exp(-\lambda\tau)$ at the review date and $\exp(-\lambda(\tau + T))$ at the test date ($t = \tau + T$). This implies that when no effort is exerted, the DM forgets the material with a constant rate $\lambda > 0$ until the test date. If the DM chooses $e > 0$ instead, there will be an immediate gain in the probability of recall at the time of review $t = \tau$ by $(1 - p_\tau)(1 - \exp(-\frac{ae}{1+b\tau}))$, and the forgetting rate until the test date will be smaller than λ.[14] $a > 0$ describes the effectiveness of effort on increasing the probability of recall, and $b > 0$ describes the penalty on the effectiveness of effort on memory due to the initial forgetting until the review date $t = \tau$. $\tilde{R}$ is increasing and strictly concave in effort, decreasing in the spacing τ, and also decreasing in the time elapsed until the test T. In addition, the function satisfies the cross-derivative conditions of Assumption 1, and Assumption 2.[15]

Assume that $c(e) = ce$. Let $\mu \in (0, 1)$ be the DM's prior belief that the material is easy to remember, $\omega = \gamma$. According to this belief, the DM's ex-ante problem is as follows:

$$\max_{e \geq 0} \mathbb{E}[R(e, \tau; \omega)] - ce \iff \max_{e \geq 0} \mu(1 - \gamma) + (\mu\gamma + 1 - \mu)\tilde{R}(e, \tau) - ce$$

The DM's optimal effort choice according to the ex-ante problem $e^*(\tau)$ [16] results in the following probability of recall:

$$R(e^*(\tau), \tau; \omega) = 1 - \omega + \omega \max\left(1 - \frac{c(1 + b\tau)}{a(\mu\gamma + 1 - \mu)}, \exp(-\lambda(\tau + T))\right) \tag{1}$$

Here I summarize the comparative statics of the optimal study effort for the ex-

[14]Given $e > 0$, the forgetting rate between the review date and the test date is $\frac{\lambda \exp(-\lambda(\tau+T) - \frac{ae}{1+b\tau})}{1 - \exp(-\frac{ae}{1+b\tau}) + \exp(-\lambda(\tau+T) - \frac{ae}{1+b\tau})} < \lambda$.

[15]These properties are formally verified in Appendix A.8.

[16]$e^*(\tau) = \max\left(0, \frac{1+b\tau}{a} \log\left(\frac{a(1-\exp(-\lambda(\tau+T)))(\mu\gamma+1-\mu)}{c(1+b\tau)}\right)\right)$.

ante problem, $e^*(\tau)$, and the resulting recall probability, $R(e^*(\tau), \tau; \omega)$. When the marginal cost of effort, c, is higher, the DM chooses to exert less effort, which reduces the resulting probability of recall. Similarly, when the prior probability of the easy state, μ, increases, the DM anticipates a lower marginal benefit from studying and therefore chooses to reduce effort, causing a decrease in the probability of recall. When the effectiveness of effort on memory, a, rises, the DM chooses to increase effort if a is small,[17] but may reduce effort if a is already large since less effort is needed to achieve a comparable recall level. Nonetheless, the direct positive effect of a on the probability of recall function outweighs the indirect effect due to the adjustment in effort, so the probability of recall increases in a. Similarly, when the penalty b on the effectiveness of effort due to initial forgetting becomes more severe, the DM may respond by increasing effort if b is small, or by reducing effort if b is already large. However, the direct negative effect of b on recall dominates, so the probability of recall decreases. Moreover, when γ decreases, indicating that the material is easier to retain in the easy state, the DM reduces effort because the perceived marginal return to effort falls. If the true state is difficult, this choice lowers recall; but if the state is easy, the direct improvement in recall dominates, therefore the probability of recall increases despite the reduced effort. Finally, by Proposition 2, $R(e^*(\tau), \tau; \omega)$ is decreasing in τ for any $\omega \in \{\gamma, 1\}$. We can confirm this claim from equation (1) which is decreasing in τ. This indicates that the DM will not exhibit spacing effect according to the ex-ante problem in either the easy or the difficult case.

Now assume that before choosing the effort level, the DM tries to recall the material. The DM then receives a binary signal $s \in \{0, 1\}$ indicating either a successful ($s = 1$) or a failed ($s = 0$) recall attempt according to their probability of recall before studying. If the material is difficult, the DM receives $s = 1$ with probability p_τ. If the material is easy, the DM receives $s = 1$ with probability $1 - \gamma + \gamma p_\tau$. When τ

[17] I classify a as "large" and b as "small" if $\log\left(\frac{a\left(\gamma\mu+1-\mu\right)\left(1-\exp(-\lambda(\tau+T))\right)}{c(1+b\tau)}\right) > 1$.

is close to 0, $p_\tau = \exp(-\lambda\tau)$ is close to 1 as there is almost no time to forget the initially learned material, so the DM can make a successful recall attempt with a very high probability in both the easy and the difficult states. When the time between the initial learning and the review date τ increases, the difference in the probability of a successful recall attempt between the easy and the difficult states increases as well, leading to a more informative signal. Similarly, the informativeness of the signal is also increasing in the difference of the difficulty of cases, $1-\gamma$.

Let $\mu_0(\tau)$ and $\mu_1(\tau)$ be the posterior beliefs of the material being easy after receiving $s=0$ or 1, respectively.[18] Then, after receiving signal s, the DM's problem is

$$\max_{e_s\geq 0}\mathbb{E}[R(e_s,\tau;\omega)|s]-c(e_s) \iff \max_{e_s\geq 0}\mu_s(\tau)(1-\gamma)+(\mu_s(\tau)\gamma+1-\mu_s(\tau))\tilde{R}(e_s,\tau)-ce_s$$

Let $e_s^*(\tau)$ be the optimal effort choice after receiving s.[19] Without loss of generality, assume the case when the material is in fact difficult ($\omega=1$). Then, on average, we will observe the resulting probability of recall when effort is optimally chosen after observing the signal as $\mathbb{E}_s[\tilde{R}(e_s^*(\tau),\tau)] = p_\tau\tilde{R}(e_1^*(\tau),\tau)+(1-p_\tau)\tilde{R}(e_0^*(\tau),\tau)$.[20]

Figure 1 shows how the optimal effort choice and the resulting probability of recall before and after receiving the signal changes with the spacing τ when the DM believes that the material is more likely to be easy ($\omega=\gamma=0.1$) with a prior probability $\mu=0.85$.[21] Without the signal, the DM chooses $e^*(\tau)$ according to their prior belief. The optimal effort choice without the signal is increasing slightly in τ when τ is small,

[18] If $\tau>0$, $\mu_0(\tau)=\frac{\mu\gamma}{\mu\gamma+(1-\mu)}$, $\mu_1(\tau)=\frac{\mu(1-\gamma+\gamma\exp(-\lambda\tau))}{\mu(1-\gamma+\gamma\exp(-\lambda\tau))+(1-\mu)\exp(-\lambda\tau)}$. If $\tau=0$, $\mu_0(\tau)=\mu_1(\tau)=\mu$.

[19] $e_s^*(\tau)=\max\left(0,\frac{1+b\tau}{a}\log\left(\frac{a(1-\exp(-\lambda(\tau+T)))(\mu_s(\tau)\gamma+1-\mu_s(\tau))}{c(1+b\tau)}\right)\right)$.

[20] $\mathbb{E}_s[\tilde{R}(e_s^*(\tau),\tau)]=\exp(-\lambda\tau)\max\left(\exp(-\lambda(\tau+T)),\,1-\frac{c(b\tau+1)}{a\left(1-\frac{\mu(1-\gamma)(\gamma\exp(-\lambda\tau)+(1-\gamma))}{\mu(\gamma\exp(-\lambda\tau)+(1-\gamma))+(1-\mu)\exp(-\lambda\tau)}\right)}\right)$
$+(1-\exp(-\lambda\tau))\max\left(\exp(-\lambda(\tau+T)),\,1-\frac{c(b\tau+1)}{a\left(1-\frac{\mu(1-\gamma)\gamma}{\mu\gamma-\mu+1}\right)}\right)$.

[21] The example illustrated in Figure 1 uses the following values for the parameters: $a=1.25$, $b=0.1$, $\lambda=1$, $T=2$, $c=0.22$.

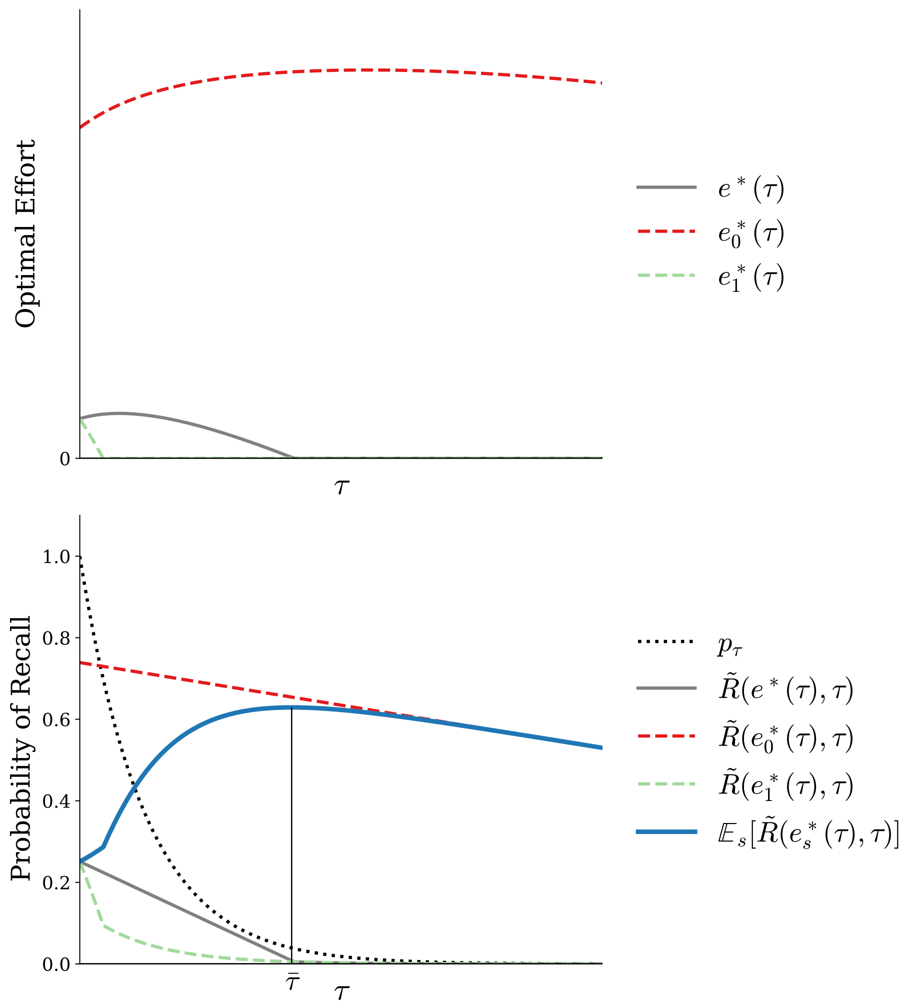

Figure 1: Optimal effort choice and the probability of recall

however, this increase in effort is not enough to offset the loss in memory due to a longer spacing, as can be seen from the decreasing probability of recall with $e^*(\tau)$, $\tilde{R}(e^*(\tau), \tau)$, in Figure 1. This is not the case when the DM receives a signal s by making a recall attempt prior to choosing the effort level. After receiving $s = 1$, the DM will be more confident that the material is indeed easy, and will choose a smaller effort level, $e_1^*(\tau)$, leading to a lower probability of recall $\tilde{R}(e_1^*(\tau), \tau)$ compared to the ex-ante problem. On the other hand, after receiving $s = 0$, the DM will update their belief and choose a significantly higher effort level, $e_0^*(\tau)$, as the returns to effort is much larger when the material is difficult. When $\tau = 0$, the probability of making a successful recall attempt will be 1 in either the difficult and the easy state, therefore, the signal will be completely uninformative. As more time passes between the initial learning and the review date and τ becomes larger, there will be more time to possibly forget the material, and the difference in the probability of receiving $s = 0$ will grow larger, making the signal much more informative. Therefore, when τ is larger, the DM will receive $s = 0$ with a higher probability and strongly update their beliefs towards the difficult state. In Figure 1, we can see that even though the probability of recall with $e_0^*(\tau)$ and $e_1^*(\tau)$ are both decreasing in τ, the average probability of recall that we will observe, $\mathbb{E}_s[\tilde{R}(e_s^*(\tau), \tau)]$, is increasing in τ until $\tau = \bar{\tau}$ as the probability of receiving $s = 0$ $(1 - p_\tau)$ increases. After $\bar{\tau}$, the increase in the optimal effort level $e_0^*(\tau)$ is not enough to offset the decrease in the recall level due to a larger spacing, hence the average probability of recall after the signal is decreasing in τ after this point. This non-monotonic pattern in the probability of recall is the spacing effect.

4 Experimental Design

I conduct a laboratory experiment to study the participants' decision-making about learning and their perception of their own memory. The experimental design mirrors the framework described in the theoretical model. There are two periods of learning as in the theoretical model, *initial learning* and *review*, followed by a rewarded test. Similar to the model, the initial learning period is identical for all participants. All of the decisions made by the participants are related to their learning during the review period. The participants also complete a practice quiz before the review period, which serves as a signal mechanism.

Treatments The experiment has 4 different treatments: {*Short spacing (S), Long spacing (L)*}× {*Feedback (F), No feedback (NF)*}×{*Before spacing (Pre), After spacing (Post)*}. Short and long spacing groups differ in the time interval between the initial learning and the review periods. Participants in the short spacing ($\tau = 0$) group start the review session immediately after making their effort choices following the initial learning session. In contrast, participants in the long spacing ($\tau = 20$) group complete a filler task of 20-minutes before the review session. To keep the overall duration of the experiment consistent across treatments, participants in the short spacing group complete the same filler task after the test. Feedback and No feedback groups differ in whether the participant completes the practice quiz before or after making their effort choices. While the feedback group completes the practice quiz and see their quiz score before choosing their learning effort, no feedback group completes the practice quiz after choosing their learning effort and does not see their quiz score. Before spacing and after spacing groups differ in which of their effort choice is randomly selected to be implemented for their actual review session. All of the choices in the experiment are made prior to this random selection. Therefore, whether the effort choice before or after spacing is implemented only affects the actual level of exerted effort and the observed test score, and has no effect on any of the observed choices in this experiment.

Measuring effort The participants are asked to memorize a list of 30 word pairs which need to be remembered to succeed at the rewarded test. In each learning session, the participants are shown the word pairs one by one on the screen. Each word pair is shown for 5 seconds in the initial learning session. For the review session, participants choose how many seconds per word pair that they would like to study, ranging from 0 to 30 seconds. During the review session, this duration applies uniformly across all word pairs, so the participants cannot choose different times for different pairs. The participants are given a \$9 endowment, and each additional second of studying per word pair has a constant marginal cost of \$0.01. I measure the choice of effort as the number of seconds chosen per word pair for the review session. Since any additional second of studying is costly, I assume that participants do not choose more time than they need and do not sit idle during any excess time.

Effort choices Participants make three effort choices during the experiment and know that any of these choices may be randomly selected for implementation. Immediately after the initial learning session, and before learning how much time they will wait until the review session, they are asked to choose how many seconds per word pair they would like to study in two possible scenarios: if the review session were to occur immediately, denoted by $e^{pre}(0)$, or after waiting 20 minutes, denoted by $e^{pre}(20)$. After making these two choices, the participants are informed of the actual waiting time until the review session, which depends on their randomly assigned treatment group. They then make a final effort choice for the review session, denoted by $e^{post}(0)$ for the short-spacing group and $e^{post}(20)$ for the long-spacing group. Depending on their treatment group for spacing $\tau \in \{0, 20\}$, either $e^{pre}(\tau)$ (for *before spacing* group) or $e^{post}(\tau)$ (for *after spacing* group) is randomly selected to be implemented, and the participants study according to this randomly selected effort choice during the review session. The cost of effort is determined by the implemented effort level.

Measuring recall I measure the probability of recall using the practice quiz and the test scores. The test consists of all 30 word pairs. One word is missing from each pair, and the participants are asked to type the missing word. All participants take the test 20 minutes after the review session, so the duration from the last learning session until the test is constant ($T = 20$) for all treatments. At the end of the experiment, one of the word pairs is randomly selected and the participant earns the test reward if their answer to that question is correct. This payment system makes the potential gain from exerting effort equivalent to the probability of recall, consistent with the theoretical model. Depending on the participant's treatment group for spacing $\tau \in \{0, 20\}$, let $R(e^{post}(\tau), \tau)$ or $R(e^{pre}(\tau), \tau)$ denote the percentage test score, depending on which of their effort choices is randomly selected to be implemented. In addition to the test score, I use the practice quiz score to measure the probability of recall just before the review session, that is, after spacing delay has occurred but before any effort is exerted during the review session. The practice quiz follows the same format as the test but includes only the last 5 word pairs that the participants see in their initial learning session. [22] Depending on their treatment group for spacing $\tau \in \{0, 20\}$, let $Q(\tau)$ denote their practice quiz score.

Signal about forgetting The practice quiz serves as a mechanism to receive a signal about the participants' current probability of recall. After the waiting time following the initial learning session is completed according to their spacing treatment, the participants in the *Feedback* treatment group complete the practice quiz and learn their quiz score as a number out of 5. Then, they are asked to choose their effort level $e^{post}(\tau)$, knowing their quiz score. Therefore, the practice quiz score works as a signal which gives them information about the proportion of the word pairs that they still remember up to that point before making their effort decision. [23] The participants in the *No feedback* treatment group also complete the practice quiz, but only after making their effort choices. I do not show the participants in the *No feedback* treatment their practice quiz score. Even though they do not receive an explicit signal regarding their current recall, even solving the practice quiz itself can be informative and generate an unobserved signal. For instance, if a participant leaves one of the answers blank, they will know that they do not know the answer of that question for sure. However, receiving this type of unobservable signal through the practice quiz will not be an issue as the effort choices are completed before solving the quiz for the *No feedback* group. All participants are required to complete the practice

[22] The reason to use the last 5 word pairs of the initial learning session, rather than a random selection of 5 word pairs, is to ensure that the signal received for the short spacing treatment group is actually coming after a short spacing, and as close to 0 minutes as possible. As the initial learning list has random order for the word pairs, the five quiz questions will be different for each participant.

[23] In Section 3.2, a single recall attempt is used as a signal, as opposed to multiple recall attempts as in here. The extension of the theoretical model where the signal consists of 5 independent recall attempts is discussed in Appendix A.10.

quiz to avoid any practice effects on the test scores between different treatments.

Beliefs about forgetting I elicit participants' beliefs about their probability of recall after the spacing. After the initial learning session and choosing $e^{pre}(0)$ and $e^{pre}(20)$, participants are asked their expected quiz score out of 5 if they were to take the quiz immediately, or after waiting 20 minutes. I introduce an adaptation of the standard frequency method (Schlag and Tremewan, 2021) to elicit this belief, by paying them a reward if their quiz score guess exactly matches their actual quiz score, and an additional reward depending on their practice quiz success. Eliciting beliefs truthfully regarding their practice quiz performance is not a trivial exercise as the participants can always choose to underperform at the practice quiz to earn the additional reward. To avoid this issue and to incentivize the participants to solve as many practice questions as they can to reveal their actual probability of recall, the participants earn \$0.25 for each correct practice quiz question, and an additional \$0.25 if their quiz score guess is accurate. This incentivization method guarantees that the participants are always better off to answer as many practice quiz questions correctly as possible, as solving an extra question above their guess offsets the loss of the reward for a correct guess. For participants with risk-neutral preferences, this method reveals the mode of the participant's belief truthfully. For participants with risk-averse preferences, the stated guess is either equal or smaller than the mode of their belief. Intuitively, a risk-averse participant would like to insure themselves by stating a lower guess thereby compensating for the lower reward they would receive in states where they answer fewer questions. The proof for the conditions on the incentive-compatibility of this belief elicitation method is provided in Appendix A.9.

The reason for using this elicitation method as opposed to more complex methods that are incentive-compatible for any risk preferences such as probability matching is to ensure that it: (i) is guaranteed not to distort the quiz performance; (ii) is easy to understand and makes it transparent to the participants that they are always better off performing as well as possible on the quiz, and (iii) elicits a value (e.g. a score out of 5, rather than the indifference point for the probability in a choice list) that the participants can easily compare to the actual signal that they receive, which is the quiz score out of 5, $Q(\tau)$. [24] Since the effect of receiving a signal about recall is a primary objective in this experiment, it is crucial that participants demonstrate their actual recall in the practice quiz to generate a meaningful signal. The modified frequency method that I use satisfies this requirement. Let $q(0)$ and $q(20)$ be the stated expected quiz score for spacing $\tau = 0$ and $\tau = 20$, respectively. I interpret performing worse than the stated expected quiz score ($Q(\tau) - q(\tau) < 0$) as a negative signal, and performing as well as the expected quiz score ($Q(\tau) - q(\tau) \geq 0$) as positive signal in the analysis of the results. I discuss the correspondence of this signal structure to a single recall attempt as the signal in the theoretical model in Appendix A.10.

The exact order of all the decisions and tasks that are completed for each treatment is summarized in Figure 2 which shows the timeline of the experiment.

Implementation The experiment was conducted in person in September 2025 at the Toronto Experimental Economics Laboratory (TEEL). Participants were recruited from the TEEL participant pool via e-mail invitations sent through ORSEE

[24]For example, Möbius et al. (2022) elicit the probability equivalent of whether a participant's quiz score exceeds the median score among other participants in the experiment, for participants who have already completed a quiz, using the Becker-DeGroot-Marschak mechanism (Becker et al., 1964). Möbius et al. (2022) show that this method is incentive-compatible for any risk preferences and does not create hedging incentives to report a lower belief and deliberately perform poorly on the quiz. One difference between their setting and our experiment is that the beliefs are elicited before taking the practice quiz in our setting. Consider, for example, asking participants to choose between a lottery that pays a if their answer is correct for one randomly selected quiz question and another lottery that pays a with probability p, where p increases from 0 to 1 in a choice list. This would truthfully elicit the expected recall rate at the quiz. However, a participant who incorrectly believes their recall rate to be zero would always choose the second lottery except when $p = 0$, and thus have almost no incentive to attempt recalling during the quiz, as receiving the reward a would almost never depend on their performance, even when they might have been able to recall more word-pairs otherwise. Moreover, with this method, the indifference point stated by participants is not straightforward to compare with the signal that they receive which is a score out of 5.

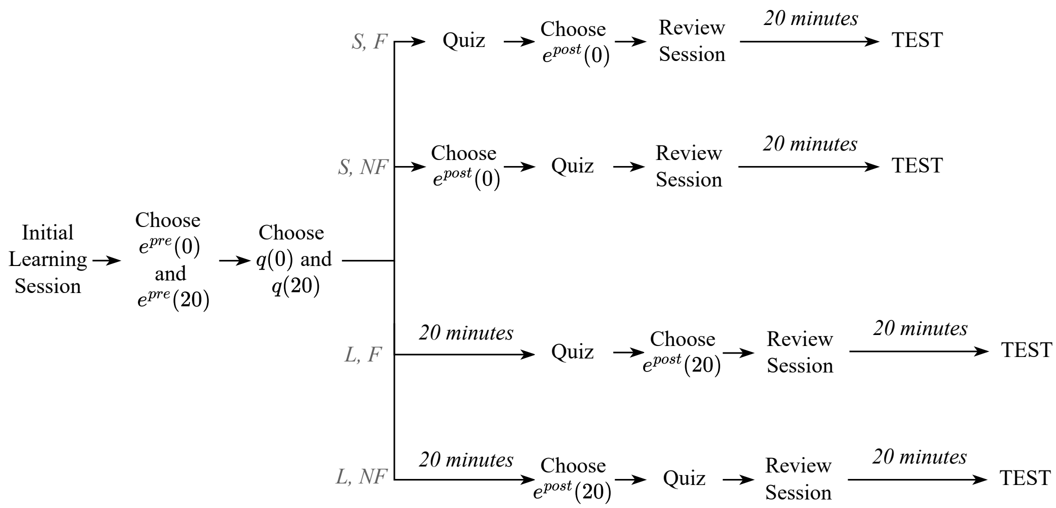

Figure 2: Timeline of the experiment for each treatment

(Greiner, 2015). To prevent participants from taking pictures of the learning material or writing down the words for use in the rewarded test, the experiment enforced a strict no-phone-use and no-writing rule. Participants were further informed that everyone will leave the laboratory at the end of the session together to avoid creating an additional incentive to choose a lower effort level for the purpose of leaving the laboratory earlier. After signing the consent forms, participants read the instructions and completed a five-question comprehension check. To increase attention to the instructions, participants who answered all comprehension check questions correctly on their first attempt earned an additional \$1 reward. The experiment was programmed with oTree (Chen et al., 2016). The order of the questions for the effort and quiz belief choices made before spacing for both the short and long spacing scenarios were randomized to avoid order effects.[25]

[25]No order effects are found for the effort choices. The participants are found to guess 0.2 less number of correct answers out of 5 on average for their practice quiz score for both the short and the long spacing scenarios when the guess for long spacing was asked first, however this effect is not significant at the 5% level. No order effects is found for the within-subject difference between the quiz score guesses for the long and short spacing scenarios. The results regarding the order effects are provided in Appendix B.1.

A total of 412 participants were recruited for this experiment. In line with the exclusion criteria specified in the pre-registration, 1 participant was excluded from the analysis for leaving the experiment before completion, and 13 were excluded for making more than 4 mistakes on the comprehension check questions. [26] The number of participants in each treatment group is summarized in Table 1. Treatments were assigned using balanced randomization within each laboratory session by the computer program. The distinction between the *before spacing* and *after spacing* group only affects the test score data, while the implementation of the after spacing effort choice is necessary to test the predictions of the theoretical model. For this reason, only 5 participants were assigned to treatments in which the before spacing effort choice was implemented, as outlined in the pre-registration. To avoid deception, the participants were informed that any of the scenarios that they make choices for could have been selected, but not all scenarios had an equal likelihood of being chosen.

Treatment	Frequency
Long, No feedback, Before spacing	5
Long, No feedback, After spacing	94
Long, Feedback, Before spacing	5
Long, Feedback, After spacing	94
Short, No feedback, Before spacing	5
Short, No feedback, After spacing	94
Short, Feedback, Before spacing	5
Short, Feedback, After spacing	96
Total	398

Table 1: Number of participants by treatment conditions

Payment The earnings of the participants consist of a combination of cash and Interac eTransfer payments. The cash payment is made at the end of the session and includes a show-up fee of $10, the remaining amount from the $9 endowment

[26] The participants had to answer each comprehension check question correctly to continue the experiment, so the participants who answered a question incorrectly on their initial attempt had to try again until selecting the correct answer.

depending on the cost of the effort level they choose, \$0.25 for each correct practice quiz answer, \$0.25 if their quiz score guess is accurate, and an additional \$1 if they answer all comprehension check questions correctly on their first attempt following the instructions of the experiment. For the test reward, one question is selected randomly from the test, and they earn \$30 the day after the session via Interac eTransfer if they answer the randomly selected question correctly. As previously described, participants make three effort choices throughout the experiment and Random Incentive Scheme is used to determine which of their effort choice is selected to be implemented. During the review session, participants learn the material for the number of seconds according to the implemented effort, and they pay the cost of the implemented effort as $\$0.01 \times 30 \times$ the number of seconds they chose. Note that while the cost of effort is paid on the same day as the session, the reward from a successful recall is paid the next day. Assuming participants have separable preferences over different days, the utility function of participants would be additively separable over the reward from the test and the cost of effort, following the functional form in the theoretical model. Moreover, for participants with risk-neutral or risk-averse preferences, this two-part payment system guarantees a convex cost of effort that is consistent with the theoretical model due to the evaluation of the remaining part of their endowment with a weakly concave utility function. The detailed explanation for the correspondence of this payment system to the theoretical model can be found in Appendix A.11.

Learning material The participants study the same list of 30 word pairs during both study sessions. The word pairs consist of randomly selected and randomly matched nouns from MRC Psycholinguistic Database (Wilson, 1988) that have between 3 and 7 letters, have concreteness rating larger than 400, with a maximum age of acquisition rating of 500. The order of word pairs in the study sessions and the tests are randomized to avoid participants using special studying strategies (e.g. study every other word for twice the chosen number of seconds). The list of word-pairs can be found in Appendix C.

Waiting tasks During the 20-minute waiting periods, participants played simple visual attention games serving as filler tasks that required continuous attention to the screen. The games were designed to prevent participants from focusing on anything other than the game, yet remained simple enough that any attentive participant could complete them successfully. In each game, participants were allowed only a limited number of mistakes, a limit that could be quickly reached unless they maintained continuous attention. Participants were informed that exceeding the maximum number of allowed errors in any game would result in the failure of the waiting task and dismissal from the experiment with only the show-up fee. This rule was intended to prevent participants from rehearsing the word list outside of the designated learning sessions. None of the participants failed the filler tasks, and the average number of errors were around 5% the allowed maximum number of errors for each task.[27] This suggests that, on average, participants paid a high level of attention to playing these games. Four different games were used in the experiment, each consisting of one minute of instructions followed by nine minutes of gameplay. Two of these games were assigned to the waiting period between the initial learning session and the review session for the long spacing group, and to the waiting period after the test for the short spacing group. The other two games were used between the review session and the test for all participants. This arrangement ensured that participants in the long spacing group did not gain an advantage in familiarity with the filler tasks over those in the short spacing group between the review session and the test, which could be the case if identical tasks had been used for all waiting periods. Detailed descriptions of each game can be found in the experimental instructions provided in Appendix D.

Hypotheses The following pre-registered hypotheses are tested with the results of the experiment:

[27] The summary statistics for the filler task performance is provided in Appendix B.2.

$\mathbf{H_1}$. $e_0(0) \leq e_0(20)$: *Participants choose more effort for the longer spacing scenario.*

This hypothesis implies that the participants take their forgetting into account when deciding on learning effort, and find it optimal to choose a higher level of effort to make up for the greater forgetting until the review session. The hypothesis follows from the probability of recall function R having larger marginal returns to effort for the longer spacing, as shown in Remark 1.

$\mathbf{H_2}$. $q(0) \geq q(20)$: *Participants expect to forget more between the learning sessions with long spacing.*

Hypothesis $\mathbf{H_2}$ follows from Remark 3.

$\mathbf{H_3}$. $Q(0) \geq Q(20)$: *Participants forget more until the review session with long spacing.*

Hypothesis $\mathbf{H_3}$ follows from the probability of recall function being decreasing in spacing τ.

Define a positive signal $s = 1$ as when $Q(\tau) - q(\tau) \geq 0$ so that the DM performs equal to or better than their quiz score guess, and a negative signal $s = 0$ when $Q(\tau) - q(\tau) < 0$.

$\mathbf{H_4}$. $(e^{post}(\tau) - e^{pre}(\tau)|s = 0)_F \geq (e^{post}(\tau) - e^{pre}(\tau)|s = 0)_{NF}$, $\tau \in \{0, 20\}$: *Participants choose a higher effort after a negative signal about their forgetting.*

$\mathbf{H'_4}$. $(e^{post}(\tau) - e^{pre}(\tau)|s = 1)_F \leq (e^{post}(\tau) - e^{pre}(\tau)|s = 1)_{NF}$, $\tau \in \{0, 20\}$: *Participants choose a lower effort after a positive signal about their forgetting.*

Hypotheses $\mathbf{H_4}$ and $\mathbf{H'_4}$ follow from Proposition 4.

$\mathbf{H_5}$. $(e^{post}(\tau) - e^{pre}(\tau)|s = 0)_{NF} = (e^{post}(\tau) - e^{pre}(\tau)|s = 1)_{NF} = 0$, $\tau \in \{0, 20\}$. *No change in effort choice if no feedback is received about forgetting.*

Hypothesis $\mathbf{H_5}$ follows from the assumption that the practice quiz is the only information source to generate a signal about forgetting.

$\mathbf{H_6}$. $(e^{post}(0) - e^{pre}(0))_F \leq (e^{post}(20) - e^{pre}(20))_F$: *The change in effort choice after receiving feedback is higher for the long spacing.*

Hypothesis $\mathbf{H_6}$ describes the effort as the underlying force behind the spacing effect on the probability of recall, which is tested in the following hypotheses.

$\mathbf{H_7}$. $R(e^{post}(0), 0)_F \leq R(e^{post}(20), 20)_F$: *Probability of recall at the test is higher for longer spacing if the participants have received feedback about their forgetting.*

$\mathbf{H_7'}$. $R(e^{post}(0), 0)_{NF} \geq R(e^{post}(20), 20)_{NF}$: *Probability of recall at the test is lower for longer spacing if the participants have received no feedback about their forgetting.*

Hypotheses $\mathbf{H_7}$ and $\mathbf{H_7'}$ follow from Proposition 6 and Proposition 2, respectively.

5 Experimental Results

The experimental results are organized into three parts, according to the pre-registered hypotheses. Firstly, in Section 5.1, I analyze the baseline effort choices for different spacing levels which are chosen before spacing, and discuss the beliefs about forgetting across different time intervals. Secondly, I study the effect of signals about forgetting on the choices of learning effort in Section 5.2. Lastly, in Section 5.3 I evaluate the effect of different spacing levels on the probability of recall through the effect of these signals.

Including the hypotheses that are evaluated separately for both spacing groups (H_4, H_4', H_5), a total of 12 hypotheses are tested. To address multiple hypothesis testing concerns, Holm-Bonferroni step-down procedure (Holm, 1979) is used to control for the family-wise error rate.

5.1 Baseline effort choices and beliefs on forgetting

$\mathbf{H_1}$. Participants choose more effort for the longer spacing scenario.

While the average effort choice for short spacing ($e^{pre}(0)$) is 9.15 seconds per word pair, the average effort choice for long spacing ($e^{pre}(20)$) is 9.70 seconds. The distribution of these effort choices is presented in Figure 3, which shows that the participants tend to choose higher effort levels for long spacing. To test whether participants choose a higher effort level for the longer spacing than the shorter spacing case, I compare the within-subject difference in effort choices between long and short spacing made before spacing. This test can be conducted using the full sample of participants as these choices are made under identical conditions across treatments at the beginning of the experiment and before any differences between treatments are implemented. The Wilcoxon signed-rank test indicates that the effort choice for long spacing is significantly higher (z=-3.95, p=0.0001) and remains significant after the Holm-Bonferroni correction at the 1% level. The summary statistics for these effort choices are reported in Table 2.

The small average difference of 0.54 seconds between these two effort choices is driven by 52% of participants who choose the same effort level for both short and long spacing. However, consistent with the hypothesis, 31% of the participants choose a higher effort level for long spacing, compared to only 17% of the participants who choose a higher effort for short spacing.

Before continuing with the remaining hypotheses, it is important to report an anomaly regarding the distribution of the difference in effort for long and short spacing ($e^{pre}(20) - e^{pre}(0)$) across treatments. As $e^{pre}(0)$ and $e^{pre}(20)$ are measured before any of the randomly assigned treatment groups are treated, we should expect no difference across the groups. However, participants in the *(Long spacing, Feedback)* group have an average of -0.04 for this difference. This indicates that, on average, they choose a slightly higher effort level for the shorter spacing case. The average difference in

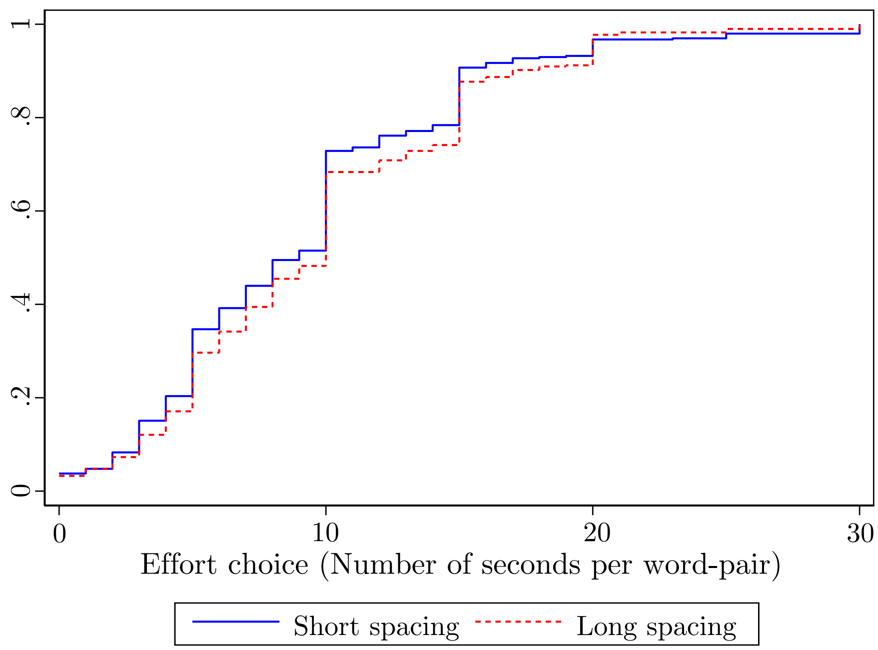

Figure 3: Empirical CDF of effort choices made before spacing

effort choice is 0.73 for all other treatment groups, which is significantly different than the *(Long spacing, Feedback)* group using the Mann-Whitney U test (z=1.99, p=0.047). [28] This indicates that the participants that are randomly assigned to the *(Long spacing, Feedback)* group have a different perception about how their memory will interact with waiting. As $-0.04 < 0$, I cannot claim that $R_{e\tau} > 0$ for this group on average.

$\mathbf{H_2}$. Participants expect to forget more between the learning sessions with long spacing.

To understand how participants expect to forget from their initial learning of the word-pairs until the review session, I compare the stated quiz score guesses for the

[28] Drawing 10,000 random samples of size 100 with replacement from the full sample (N=398) generates 9% of these samples that have at least this much difference in the mean of $e^{pre}(20) - e^{pre}(0)$ compared to the mean of the full sample, which indicates that this anomaly is unlikely but not impossible. Due to the higher incidence of observations with $e^{pre}(20) - e^{pre}(0) = 0$, the average of this variable is sensitive to over-sampling from the left tail of this distribution.

short spacing ($q(0)$) and the long spacing ($q(20)$) scenario within subjects. As the belief elicitation of quiz scores takes place at the beginning of the experiment, and before any of the treatment groups are actually treated, the full sample can be used for the analysis of this hypothesis. On average, participants expect to have 2.79 correct answers out of 5 questions for the short spacing scenario, which corresponds to taking the practice quiz immediately after making their choices. For the long spacing scenario, which includes an additional 20 minutes of waiting until the quiz, the average quiz score guess decreases to 2.19. The distribution of quiz score is shown in Figure 4(a), which clearly indicates that participants expect to forget more over time. In particular, 63% of the participants state a lower quiz score guess for the long spacing scenario, and 24% of the participants guess an equal score for both. Using the Wilcoxon signed rank test, the quiz score guesses for the long spacing are found to be significantly lower than the short spacing scenario ($z = 10.404, p < 0.0001$). The result remains significant after the Holm-Bonferroni correction at the 1% level. The summary statistics for the quiz score guesses are reported in Table 2.

H_3. Participants forget more until the review session with long spacing.

Practice quiz scores of participants in the *Short spacing* and *Long spacing* groups are compared to assess differences in forgetting across different time intervals. The average quiz score in the short spacing group is 1.86 out of 5 question, which is higher than the average score in the long spacing group (1.26). The difference in forgetting across short and long spacing is significant ($z = 3.887, p = 0.0001$) according to the Mann-Whitney U test, and remains significant at the 1% level after the Holm-Bonferroni correction. The distribution of quiz scores for both groups is shown in Figure 4(b). A two-sample Kolmogorov-Smirnov test rejects the equality of distributions ($p = 0.006$), significant at the 10% level but not at the 5% level after the Holm-Bonferroni correction. Overall, these results indicate that participants forget

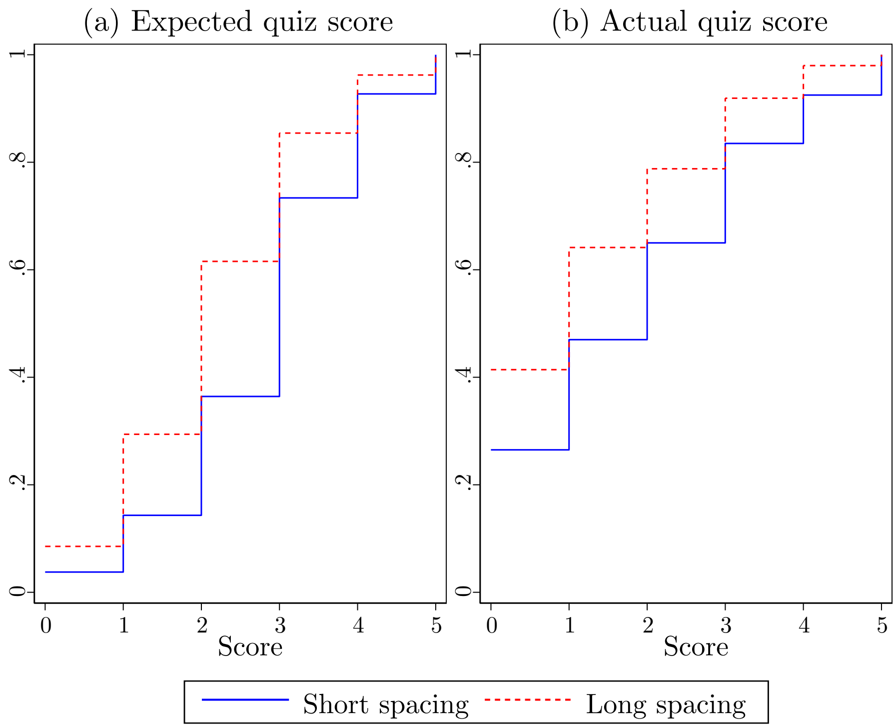

Figure 4: Empirical CDF of quiz score guesses and the actual quiz scores

significantly more over longer intervals, in consistent with their expectations.[29]

5.2 Effect of signals on effort

Before evaluating the remaining hypotheses to test the predictions of the theoretical model regarding the effect of signals on effort choice and memory, I first report the findings on the signals that participants received (for the *Feedback* group) or would have received (for the *No feedback* group). As shown in in Figure 4, comparing the distribution of expected quiz scores and the actual scores, participants tend to overestimate their probability of recall for both the short and long intervals of spacing.

[29]This result also shows that the waiting tasks were effective in preventing participants to rehearse the word-pairs in their minds while playing the games which avoids a situation where the participants in the long spacing group have an advantage of having an extra 20 minutes to mentally practice the word-pairs.

Table 2: Summary Statistics

Variable Name	Obs.	Mean	S.D.	Min.	Max.
Effort choice (seconds per word pair)					
$e^{pre}(0)$	398	9.15	5.95	0	30
$e^{pre}(20)$	398	9.70	5.78	0	30
$e^{pre}(20) - e^{pre}(0)$	398	0.54	3.42	-15	13
$e^{post}(0) \mid$ *Feedback*	101	11.32	6.64	0	30
$e^{post}(0) \mid$ *No Feedback*	99	10.06	6.87	0	30
$e^{post}(20) \mid$ *Feedback*	99	10.26	5.86	0	30
$e^{post}(20) \mid$ *No Feedback*	99	9.20	6.03	2	30
$e^{post}(0) - e^{pre}(0) \mid$ *Feedback*	101	2.10	3.81	-5	20
$e^{post}(0) - e^{pre}(0) \mid$ *No Feedback*	99	0.61	2.18	-5	10
$e^{post}(20) - e^{pre}(20) \mid$ *Feedback*	99	0.72	3.68	-14	15
$e^{post}(20) - e^{pre}(20) \mid$ *No Feedback*	99	0.31	3.01	-7	15
Expected quiz score (out of 5)					
$q(0)$	398	2.79	1.20	0	5
$q(20)$	398	2.19	1.24	0	5
$q(0) - q(20)$	398	0.61	1.08	-3	3
Quiz score (out of 5)					
$Q(0)$	200	1.86	1.57	0	5
$Q(20)$	198	1.26	1.38	0	5
Signal					
$Q(0) - q(0)$	200	-0.96	1.69	-5	3
$Q(20) - q(20)$	198	-0.88	1.56	-5	4

The summary statistics for the difference between the actual quiz scores and the quiz score guesses ($Q(\tau) - q(\tau)$) are presented in Table 2. On average, the participants in the short spacing group overestimate their quiz score by 0.96 on a 5-question scale, compared to 0.88 in the long spacing group. This tendency to overestimate quiz performance suggests that the potential underreporting of the quiz score guess due to risk-averse preferences, as discussed in Section 4, is unlikely to be a concern.

To illustrate the relationship between the signal received and the corresponding change in effort choice, Figure 5 presents a scatterplot of the difference in effort choice before and after spacing ($e^{post}(\tau) - e^{pre}(\tau)$) against the signal from the practice quiz ($Q(\tau) - q(\tau)$) for each treatment. Figure 5 shows that when participants receive no feedback, their quiz performance has no correlation in the short spacing group or a slightly negative correlation for the long spacing group. However, when participants observe their quiz score before making their effort choices, the increase in the effort choice is negatively correlated with how well they performed on the practice quiz relative to their initial guess.

For the following hypotheses, I interpret a negative signal as occuring when the actual quiz score is below the participant's guess, and a positive signal as occuring when the score is greater than or equal to their guess. With a longer spacing, we observe less accurate guesses, and a higher frequency of negative signals about participants' forgetting. In the short spacing group, 24.5% of the participants made correct guesses of their quiz scores, compared to 20.2% in the long spacing group. The majority of participants (54.5%) in the short spacing group scored below their stated guess, and this share is even larger in the long spacing group (60%). The prevalence of lower accuracy and greater likelihood of negative signals with longer spacing is consistent with the signal structure of the theoretical model.

H_4. Participants choose a higher effort after a negative signal about their forgetting.

The theoretical model predicts that participants choose a higher effort level after observing a negative signal, compared to their effort choice based on prior beliefs. In this experiment, however, we cannot simply test whether participants who receive a negative signal choose a different effort level than before spacing and attribute this change to the signal, since receiving either a positive or negative signal reflects participant characteristics, such as innate memory ability, that can also influence effort choices. To identify the causal effect of a negative signal, I only use the participants who receive or would receive a negative signal (those who perform worse than their guess in

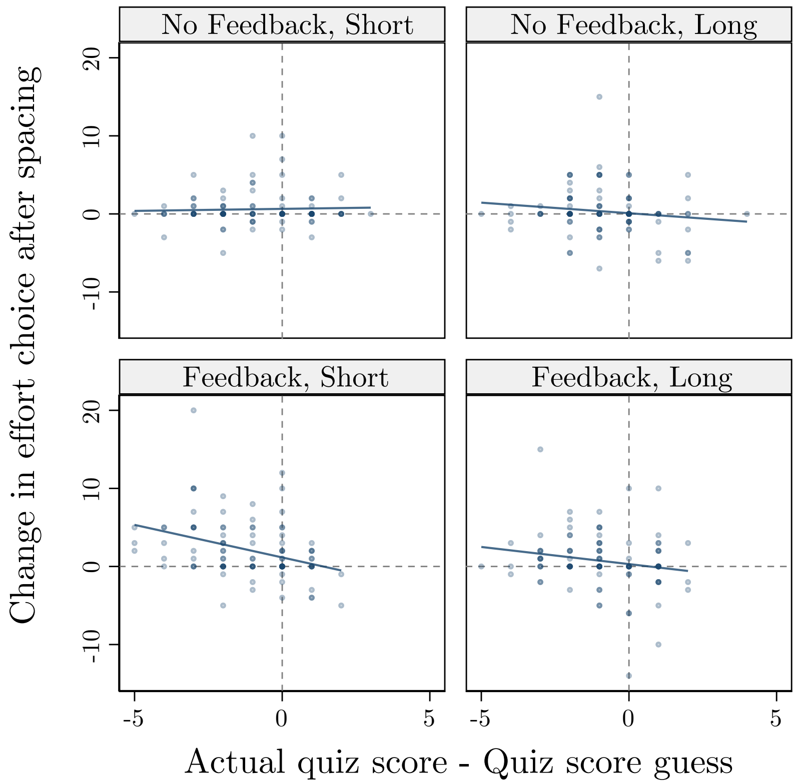

Figure 5: Change in effort choice after spacing vs. signal about forgetting

Difference between the effort choices before and after spacing with respect to the received signal from the quiz is plotted for each treatment group with an added linear fit line. Darker points indicate a higher number of observations that have the same values.

the practice quiz), and compare their change in effort after spacing ($e^{post}(\tau) - e^{pre}(\tau)$) between the *feedback* group who observe the signal before choosing $e^{post}(\tau)$, and the *no feedback* group, who solve the practice quiz after choosing $e^{post}(\tau)$, separately for short and long spacing groups. Since signal observation is randomly determined by the treatment group, this comparison allows us to understand the effect of observing a negative signal on the choice of effort.

After observing a negative signal, participants in the short spacing group increase their effort choice by an average of 3 seconds per word-pair relative to their effort choice before spacing. In contrast, participants in the no feedback group who would otherwise receive a negative signal increase their effort choice by only 0.6 seconds.

The Mann-Whitney U test indicates that receiving a negative signal about forgetting until the review session significantly increases the effort choice (z=-3.94, p=0.0001), which remains significant at the 1% level after the Holm-Bonferroni correction.

Conducting the same analysis on participants in the long spacing group who perform worse at the practice quiz compared to their guess, participants who observe the negative signal increase their effort choice by 1.37 seconds per word-pair, compared to an increase of 0.77 for the participants who do not observe the signal. Even though participants who observe the negative signal increase their effort choice more, this difference is not statistically significant using the Mann-Whitney U test ($z = -1.41, p = 0.158$).

As an additional check to test $\mathbf{H_4}$, I conduct the same analysis after excluding the participants who expect to remember more if they were to wait longer ($q(0) < q(20)$). [30] As expecting an inverse forgetting does not satisfy the fundamental assumptions of the model about the recall function and the DM's beliefs over forgetting, the theoretical model does not predict these participants to update their beliefs and choose a higher effort after a negative signal. After the exclusion of these participants with non-standard beliefs over forgetting, I find that receiving a negative signal significantly increases the effort choice for both the short spacing ($z = -4.16, p < 0.001$), and the long spacing ($z = -2.39, p = 0.0169$) groups using the Mann-Whitney U test.

$\mathbf{H'_4}$. Participants choose a lower effort after a positive signal about their forgetting.

To see the effect of a positive signal about forgetting on the effort choice, I compare the change in effort between the *feedback* and *no feedback* groups. In the short spacing group, participants increase their effort choice slightly, by an average of 0.93 seconds for the feedback group and by 0.62 seconds for the no feedback group, but

[30] This analysis is not pre-registered.

this difference is not statistically significant ($z = -0.93, p = 0.35$) according to the Mann-Whitney U test. In the long spacing group, consistent with the hypothesis, the participants who perform equal to or better than their guess in the practice quiz decrease their effort choice by an average of 0.34 seconds, regardless of whether they observe the signal, resulting in no significant difference across these groups ($z = -0.21, p = 0.84$). These conclusions remain unchanged if participants who expect to remember more in the practice quiz following a longer wait are excluded. Although the null hypothesis that a positive signal has no effect on effort cannot be rejected, the observed tendency to reduce the effort choice after waiting 20 minutes in the long spacing group, contrasting with the short spacing group, may suggest a more informative internal signal of forgetting when participants wait longer.

H_5. No change in effort choice if no feedback is received about forgetting.

To assess whether the effort choice remains unchanged without a signal, I compare the effort choices within-subject for participants in the no feedback group for each spacing level and signal type with Wilcoxon signed-rank test, and then compare the change between different signal types, separately for each spacing group with Mann-Whitney U test. For short spacing, participants who would receive a negative signal show an increase in effort after spacing ($z = -1.964$, $p = 0.049$), while those who would receive a positive signal also exhibit a small increase in effort that is not statistically significant ($z = -1.44, p = 0.151$). The change does not differ between the negative and positive signals ($z = 0.573$, $p = 0.567$). For long spacing, participants who would receive a negative signal choose a slightly higher effort that is marginally significant ($z = -1.72, p = 0.085$), and participants who would receive a positive signal slightly decrease their effort choices that is not statistically significant ($z = 1.01, p = 0.316$). Unlike the short spacing group, participants in the long spacing group who receive a positive or negative signal have a slightly different reaction in terms of how they adjust their effort choices after spacing ($z = 1.851$, $p = 0.064$). This difference can suggest that unlike the short spacing group where participants slightly increase their effort irrespective of whether they remembered better or worse than their guess, participants in the long spacing group adjust their effort according to the type of signal they would have received. This result may indicate that waiting longer creates a stronger internal signal about own forgetting. Overall, these results reject $\mathbf{H_5}$ for short spacing due to a small increase in effort choice for both types of signals, while providing no evidence against $\mathbf{H_5}$ for long spacing.

5.3 Effect of spacing on recall

In this section, I discuss the effects of spacing on effort choice and the resulting probability of recall. To test whether feedback generates the spacing effect through the effort choice, I compare the effort choices and the test outcomes between short and long spacing among participants who receive feedback about their forgetting from the practice quiz. As documented in Section 5.1, according to their ex-ante choices, participants in the *Long spacing, Feedback* group prefer to exert less effort for longer spacing on average, unlike the participants in the other treatments. According to the theoretical model, having these type of preferences is a sufficient condition for the absence of spacing effect in the setting without signals, because the probability of recall function is decreasing in spacing. Similarly, with a signal mechanism, the reduction in effort induced by longer spacing further offsets the recall gains from more informative signals. Accordingly, the lack of spacing effect in the experimental results is a prediction of the theoretical model.

$\mathbf{H_6}$. The change in effort choice after receiving feedback is higher for the long spacing.

Participants in the short spacing group increased their effort choice by an average of 2.1 seconds after receiving feedback, compared to 0.72 seconds in the long spacing. This difference is significant at the 5% level using the Mann-Whitney U test

($z = 2.35, p = 0.019$). Additionally, comparing the resulting effort choices after spacing, participants in the short spacing chose a higher effort level on average (11.32 seconds) than those in the long spacing group (10.27 seconds), but this difference is not statistically significant ($z = 0.9, p = 0.37$). Thus, the evidence does not support the hypothesis that the change in effort choice is higher for the long spacing group.

$\mathbf{H_7}$. Probability of recall at the test is higher for longer spacing if the participants have received feedback about their forgetting.

Since the average effort choice is higher for the short spacing group, the theoretical model predicts that the resulting probability of recall will be lower for the long spacing group. Table 3 reports the summary statistics for the test scores. Consistent with this prediction, the short spacing group achieved a higher mean test score (20.13 correct answers) compared to the long spacing group (19.45). The difference is not significant according to either the Mann-Whitney U test ($z = 0.54, p = 0.59$) or the Kolmogorov-Smirnov test ($p = 0.92$).

$\mathbf{H_7'}$ Probability of recall at the test is lower for longer spacing if the participants have received no feedback about their forgetting.

In the absence of signals, the theoretical model predicts no spacing effect. Consistent with this, the participants in the short spacing group have a slightly higher average probability of recall (18.39 correct answers) than the long spacing group (18.11 correct answers), but the difference is not statistically significant with either the Mann-Whitney U test ($z = 0.31, p = 0.76$), or the Kolmogorov-Smirnov test ($p = 0.96$). Therefore, I cannot reject the null hypothesis that there is no difference in recall probabilities across different spacing conditions when the signals are absent.

In summary, the results show participants are aware that their probability of recall decreases over time. Accordingly, for the long spacing scenario, they anticipate more forgetting until the review session and choose a higher level of effort to review the

Table 3: Summary Statistics for Test Scores (out of 30)

Treatment	**Obs.**	**Mean**	**S.D.**	**Min.**	**Max.**
Long, Feedback, After spacing	94	19.45	8.31	1	30
Short, Feedback, After spacing	96	20.13	8.04	2	30
Long, No Feedback, After spacing	94	18.11	8.80	0	30
Short, No Feedback, After spacing	94	18.39	9.06	0	30

word-pairs. I also find that the participants significantly increase their effort choice after receiving negative feedback about their memory, however, no evidence for a lower effort choice following positive feedback is found. For the interaction effect of spacing and feedback, I do not find that the participants receiving feedback to choose a higher level of effort for long spacing, which then results in the inconclusive effect of spacing on memory. I find that the ex-ante effort choices of the participants in the *Long spacing, Feedback* group are significantly different than the other randomly assigned treatment groups, which can be a driver of this result.

6 Economic Applications of Metamemory Control

Strategic decision-making about learning and memory when agents are forgetting information has significant economic implications. This section provides two examples to illustrate the economic importance of endogenous learning decisions regarding memory and metamemory control. In the first application, I show an example of how a contrasting prediction for consumer choice emerges when imperfect recall is endogenously determined versus exogenously given. In the second application, I discuss how considering metamemory control can improve the efficiency of program design within the framework of job retraining programs for the unemployed.

6.1 Endogenous learning and consumer choice with surprising news

The economic decision-making models with exogenous imperfect recall study the effect of a limited set of information on the formation of beliefs, which consequently determines the behavior of decision-makers. In these models, the new information affects the behavior only through its impact on the beliefs. However, the inclusion of the awareness of forgetting and decision-making about memory can capture behavior which cannot be be predicted by solely modeling the effect of news on beliefs. To illustrate, consider a consumer who plans to buy a new computer after their next payday, and must choose between Brand A and Brand B. After an extensive analysis of technical differences and customer reviews, the consumer concludes that Brand A's technical features are superior for their needs and forms the belief that Brand A has higher value.

Now contrast two scenarios. In the first, the purchasing date is in two weeks and, one week before the purchase, the consumer learns of a small discount in price for Brand B. The consumer remembers initially preferring Brand A, but cannot recall the technical reasons in order to make a new comparison with the changed price; this failed recall attempt acts as a signal, leading the consumer to update their beliefs about the difficulty to retain information for this complex product category. In response, the consumer repeats the research, exerts a high learning effort to encode the technical details to form a new belief about the higher value of Brand A despite the discount for Brand B. When a slightly larger discount for Brand B appears a week later at the store, the consumer still prefers to purchase Brand A by using the clearly remembered technical information about the brands. In the second scenario, the purchasing date is in one week after the initial research, and the small discount for Brand B arrives the next day. The information about the technical aspects of the computers remains fresh in the memory, additional learning effort is not optimal, and the consumer decides that Brand A is still preferable. By the time of purchase, only the preference for Brand A is vaguely remembered, but the technical information cannot be recalled due to the lack of previous learning effort. In this case, the second, slightly larger discount that the consumer sees at the store can tip the choice toward Brand B. Notice that without the endogenous memory formation and awareness of forgetting, the DM with imperfect recall could overreact to the recent price cuts and select Brand B in both scenarios.

6.2 Metamemory control and the efficiency of job retraining programs

One straightforward implication of metamemory control is its consequences regarding training and learning in educational settings. As an example, consider the job retraining programs for the unemployed, which addresses skill decay that happens during unemployment over time. The efficiency of these programs are widely debated, as they are highly costly compared to other policies such as job search assistance, and can impose additional costs due to “lock-in effects" that deter active job searching while participating the program (Lechner and Wunsch, 2009). Due to these costs, these training programs are suggested to be inefficient for professional skills that decay slowly over time (Osikominu, 2021).

My model with optimal learning effort choices and signals about forgetting provides a new perspective to enhance program efficiency through the accurate self-selection of participants according to their skill decay rate, using the timing of the training. If a program is offered too early after becoming unemployed, participants will not be able to deduce their rate of forgetting since little time has passed to actually forget the occupational information and skills, even if the skill decay rate is high. This can lead to inefficient outcomes, such as unnecessary enrollment by those with slow skill decay as participation is costly in terms of creating lock-in effects, or low effort exertion during training from participants who underestimate their skill decay. Conversely, delaying the intervention allows individuals to first monitor their own skill loss. This monitoring period facilitates efficient self-selection: only those who observe a high rate of forgetting will opt into the program.

7 Conclusion

This paper proposes an economic model of imperfect recall where agents, aware of their forgetting, choose their learning effort to optimize memory retention. In this framework, imperfect recall becomes a strategically managed outcome rather than a static constraint. The model demonstrates how agents learn about their own memory strength from recall outcomes, which in turn affects their optimal effort choice. This dynamic endogenously generates stylized facts about memory, such as the spacing effect. The model's key predictions are supported by an incentivized lab experiment. These results imply that memory performance, and its subsequent effect on belief formation, can respond directly to economic incentives and the information provided about memory. This finding has broad implications for models of dynamic choice.

Furthermore, the model highlights a critical point: forgetting is not merely a cognitive limitation but can be an integral feature of efficient learning. By generating signals about memory efficacy between learning periods, forgetting can lead to more efficient learning outcomes. These results underscore the importance of considering strategic recall and forgetting in the study of human decision-making.

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Appendix

A Proofs and Supplementary Results

A.1 Equivalence to Assumption 1

Proof. I omit T from the representation of $\tilde{R}$ since T is fixed. Define $e_r(\tau)$ such that $\tilde{R}(e_r(\tau),\tau) = r$ for each $r \in [0,1]$. Assume Assumption 1 holds. Fix r. Using the implicit function theorem, we get $e_r'(\tau) = -\frac{\tilde{R}_\tau(e_r(\tau),\tau)}{\tilde{R}_e(e_r(\tau),\tau)}$. Assumption 1 implies that $\tilde{R}_e(e_r(\tau),\tau)$ is decreasing in τ, which is equivalent to $\tilde{R}_{ee}(e_r(\tau),\tau)e_r'(\tau) + \tilde{R}_{e\tau}(e_r(\tau),\tau) \leq 0 \iff \tilde{R}_{ee}(e_r(\tau),\tau)\left(-\frac{\tilde{R}_\tau(e_r(\tau),\tau)}{\tilde{R}_e(e_r(\tau),\tau)}\right) + \tilde{R}_{e\tau}(e_r(\tau),\tau) \leq 0 \iff \tilde{R}_e(e_r(\tau),\tau)\tilde{R}_{e\tau}(e_r(\tau),\tau) \leq \tilde{R}_{ee}(e_r(\tau)\tau)\tilde{R}_\tau(e_r(\tau),\tau)$. Since this is correct for any r, $\tilde{R}_e(e,\tau)\tilde{R}_{e\tau}(e,\tau) \leq \tilde{R}_{ee}(e,\tau)\tilde{R}_\tau(e,\tau)$, $\forall e,\tau$. Now assume that $\tilde{R}_e(e,\tau)\tilde{R}_{e\tau}(e,\tau) \leq \tilde{R}_{ee}(e,\tau)\tilde{R}_\tau(e,\tau)$, $\forall e,\tau$. Fix $r, \underline{\tau} < \overline{\tau}$ such that $\underline{e} = e_r(\underline{\tau}), \overline{e} = e_r(\overline{\tau})$. By the assumption, $\tilde{R}_{e\tau}(e_r(\tau),\tau) - \frac{\tilde{R}_{ee}(e_r(\tau),\tau)\tilde{R}_\tau(e_r(\tau),\tau)}{\tilde{R}_e(e_r(\tau),\tau)} \leq 0$ on $[\underline{\tau},\overline{\tau}]$. Then,

$$
\begin{aligned}
&\int_{\underline{\tau}}^{\overline{\tau}} \tilde{R}_{e\tau}(e_r(\tau),\tau) - \frac{\tilde{R}_{ee}(e_r(\tau),\tau)\tilde{R}_\tau(e_r(\tau),\tau)}{\tilde{R}_e(e_r(\tau),\tau)}\ d\tau \\
&= \int_{\underline{\tau}}^{\overline{\tau}} \tilde{R}_{e\tau}(e_r(\tau),\tau) + \tilde{R}_{ee}(e_r(\tau),\tau)e_r'(\tau)\ d\tau \\
&= \tilde{R}_e(\overline{e},\overline{\tau}) - \tilde{R}_e(\overline{e},\overline{\tau}) \leq 0
\end{aligned}
$$

□

A.2 Proposition 1

Let μ first order stochastically dominate μ'. Then $\mathbb{E}_\mu(\omega) \geq \mathbb{E}'_\mu(\omega)$ which is equivalent to $\sum_{i=1}^n \mu_i\omega_i \geq \sum_{i=1}^n \mu_i'\omega_i$. Using this inequality, we can see that, given τ and T, the expected marginal returns to effort will be larger under μ compared to μ':

$$\sum_{i=1}^{n} \mu_i \omega_i \tilde{R}_e(e, \tau, T) \geq \sum_{i=1}^{n} \mu_i' \omega_i \tilde{R}_e(e, \tau, T).$$

The larger marginal returns to effort for μ then implies that $e^*(\tau, T|\mu) \geq e^*(\tau, T|\mu')$, since $\tilde{R}$ is strictly increasing and concave in e, and c strictly increasing and convex in e.

A.3 Remark 1

The first order condition for the optimal effort choice is

$$\sum_{i=1}^{n} \mu_i \omega_i \tilde{R}_e(e^*(\tau, T), \tau, T) = c'(e^*(\tau, T)).$$

Then, taking the derivative with respect to τ, we can see that

$$\frac{\partial e^*(\tau, T)}{\partial \tau} \left[\frac{c''(e^*(\tau, T))}{\sum_{i=1}^{n} \mu_i \omega_i} - \tilde{R}_{ee}(e^*(\tau, T), \tau, T) \right] = \tilde{R}_{e\tau}(e^*(\tau, T), \tau, T)$$

If $\tilde{R}_{e\tau}(e, \tau, T) > 0$ for all e given τ, T, then $\frac{\partial e^*(\tau, T)}{\partial \tau} > 0$ follows from the concavity of $\tilde{R}$ in e and the convexity of c.

A.4 Proposition 2

Proof. Since T is fixed, I omit T from the representation of R when showing the following result.

First, consider the case when $\omega = \omega_n = 1$, and the DM is accurate and certain about his memory, such that $\mu_n = 1$. In this case, $R(e, \tau) = \tilde{R}(e, \tau)$. By the concavity of R and the convexity of c, $\text{argmax}_e R(e, \tau) - c(e) = e^*(\tau)$ exists and is unique. If $e^*(\tau) > 0$, then, $\frac{\partial R}{\partial e}(e^*(\tau), \tau) = c'(e^*(\tau))$. Let $\tau' > \tau$, $e' > e^*(\tau)$ be such that $R(e^*(\tau),\tau) = R(e',\tau')$.[31] $\frac{\partial R}{\partial e}(e^*(\tau),\tau) \geq \frac{\partial R}{\partial e}(e',\tau')$ by Assumption 1, so, $c'(e^*(\tau)) \geq \frac{\partial R}{\partial e}(e',\tau')$. Assume for a contradiction that $e^*(\tau') \geq e'$. Since c is convex, $\frac{\partial R}{\partial e}(e^*(\tau'),\tau') = c'(e^*(\tau')) \geq c'(e') \geq c'(e^*(\tau)) \geq \frac{\partial R}{\partial e}(e',\tau')$ which is not possible since R is strictly concave in e given τ'. If $e^*(\tau) = 0$, then $\lim_{e\to 0^+} c'(e) \geq \lim_{e\to 0^+} \frac{\partial R}{\partial e}(e,\tau)$. Let $h : e \mapsto h(e)$ be such that $R(e,\tau) = R(h(e),\tau')$, for each $e \in [0,\bar{e}]$ where $R(\bar{e},\tau) = \lim_{e\to\infty} R(e,\tau')$. By Assumption 1, $\frac{\partial R}{\partial e}(e,\tau) \geq \frac{\partial R}{\partial e}(h(e),\tau')$ for all $e \in [0,\bar{e}]$, then $\lim_{e\to 0^+} c'(e) \geq \lim_{e\to 0^+} \frac{\partial R}{\partial e}(e,\tau) \geq \lim_{e\to 0^+} \frac{\partial R}{\partial e}(h(e),\tau') = \frac{\partial R}{\partial e}(h(0),\tau')$. Assume for a contradiction that $e^*(\tau') \geq h(0)$. Since c is convex and $h(0) > 0$, $\frac{\partial R}{\partial e}(e^*(\tau'),\tau') = c'(e^*(\tau')) \geq c'(h(0)) > \lim_{e\to 0^+} c'(e) \geq \lim_{e\to 0^+} \frac{\partial R}{\partial e}(e,\tau) \geq \frac{\partial R}{\partial e}(h(0),\tau')$ which is not possible since R is strictly concave in e given τ'. This concludes that $R(e^*(\tau'),\tau') < R(h(0),\tau') = R(e^*(\tau),\tau)$.

Now consider the general case, where $\omega \in [0,1]$, and the DM has a non-degenerate prior belief μ over the possible states ω_i, $i \in \{1,\ldots,n\}$. If $\tilde{R}(e,\tau)$ satisfies Assumption 1, then $\mathbb{E}[R(e,\tau;\omega)]$ also does as it is an affine transformation of $\tilde{R}(e,\tau)$:

If $\tilde{R}_{ee}(e,\tau)\tilde{R}_{\tau}(e,\tau) - \tilde{R}_{e}(e,\tau)\tilde{R}_{e\tau}(e,\tau) \geq 0, \forall e,\tau$, then $\frac{\partial^2\mathbb{E}[R(e,\tau;\omega)]}{(\partial e)^2}\frac{\partial\mathbb{E}[R(e,\tau;\omega)]}{\partial\tau} - \frac{\partial^2\mathbb{E}[R(e,\tau;\omega)]}{(\partial e)(\partial\tau)}\frac{\partial\mathbb{E}[R(e,\tau;\omega)]}{\partial e} = (\sum_{i=1}^{n}\mu_i\omega_i)^2(\tilde{R}_{ee}(e,\tau)\tilde{R}_{\tau}(e,\tau) - \tilde{R}_{e}(e,\tau)\tilde{R}_{e\tau}(e,\tau)) \geq 0$.

Hence, the first part of the proof which derives the monotonicity of $\tilde{R}(e^*(\tau),\tau)$ from Assumption 1 implies that $\mathbb{E}[R(e^*(\tau),\tau;\omega)]$ is also decreasing in τ. If $\mathbb{E}[R(e^*(\tau),\tau;\omega)]$ is decreasing in τ, then $1-\omega+\omega\tilde{R}(e^*(\tau),\tau)$ is decreasing in τ for any $\omega \in \{\omega_1,\ldots;\omega_n\}$ as it is a monotonic transformation of $\mathbb{E}[R(e^*(\tau),\tau;\omega)]$. $\square$

[31] Notice that if no such e' exists due to $R(e^*(\tau),\tau) > R(e,\tau')$ for all $e \geq 0$, then $R(e^*(\tau),\tau) > R(e^*(\tau'),\tau')$.

A.5 Proposition 4

Proof. Let $e_s^*(\tau) = \text{argmax}_{e \geq 0} \sum_{i=1}^n \mu_{s,i}\omega_i \tilde{R}(e,\tau) - c(e)$ for $s \in \{0,1\}$, and $e^*(\tau) = \text{argmax}_{e \geq 0} \sum_{i=1}^n \mu_i\omega_i \tilde{R}(e,\tau) - c(e)$.

Firstly, note that if $\sum_{i=1}^n \mu_{0,i}\omega_i > \sum_{i=1}^n \mu_i\omega_i > \sum_{i=1}^n \mu_{1,i}\omega_i$, then $e_0^*(\tau) \geq e^*(\tau) \geq e_1^*(\tau)$ as the expected marginal return to effort $\sum_{i=1}^n \mu_{s,i}\omega_i \tilde{R}_e(e,\tau)$ will be larger for $s = 0$ and smaller for $s = 1$ for any $e \geq 0$ compared to receiving no signal.

To see that $\sum_{i=1}^n \mu_{0,i}\omega_i > \sum_{i=1}^n \mu_i\omega_i$, we can use Jensen's inequality as follows:

$$
\begin{aligned}
& \sum_{i=1}^n \mu_i(\omega_i)^2 > \left(\sum_{i=1}^n \mu_i(\omega_i)\right)^2 \\
\iff & (1-p_\tau)\sum_{i=1}^n \mu_i(\omega_i)^2 > (1-p_\tau)\left(\sum_{i=1}^n \mu_i(\omega_i)\right)^2 \\
\iff & \frac{\sum_{i=1}^n \mu_i(1-p_\tau)(\omega_i)^2}{\sum_{i=1}^n \mu_i(1-p_\tau)\omega_i} > \sum_{i=1}^n \mu_i\omega_i \\
\iff & \sum_{i=1}^n \mu_{0,i}\omega_i > \sum_{i=1}^n \mu_i\omega_i
\end{aligned}
$$

To see that $\sum_{i=1}^n \mu_i\omega_i > \sum_{i=1}^n \mu_{1,i}\omega_i$, we can once again use Jensen's inequality as follows:

$$
\begin{aligned}
& \left(\sum_{i=1}^n \mu_i\omega_i\right)^2 < \sum_{i=1}^n \mu_i(\omega_i)^2 \\
\iff & \sum_{i=1}^n \mu_i\omega_i - (1-p_\tau)\left(\sum_{i=1}^n \mu_i\omega_i\right)^2 > \sum_{i=1}^n \mu_i\omega_i - (1-p_\tau)\sum_{i=1}^n \mu_i(\omega_i)^2 \\
\iff & \sum_{i=1}^n \mu_i\omega_i\left(1 - \sum_{i=1}^n \mu_i\omega_i(1-p_\tau)\right) > \sum_{i=1}^n \mu_i\omega_i - \sum_{i=1}^n \mu_i(1-p_\tau)(\omega_i)^2
\end{aligned}
$$

$$\iff \sum_{i=1}^{n} \mu_i\omega_i > \frac{\sum_{i=1}^{n} \mu_i\omega_i(1-(1-p_\tau)\omega_i)}{\sum_{i=1}^{n} \mu_i(1-(1-p_\tau)\omega_i)}$$
$$\iff \sum_{i=1}^{n} \mu_i\omega_i > \frac{\sum_{i=1}^{n} \mu_i(1-\omega_i+\omega_i p_\tau)\omega_i}{\sum_{i=1}^{n} \mu_i(1-\omega_i+\omega_i p_\tau)}$$
$$\iff \sum_{i=1}^{n} \mu_i\omega_i > \sum_{i=1}^{n} \mu_{1,i}\omega_i$$

□

A.6 Proposition 5

Proof.

$$\begin{aligned}
I(\omega;s) &= H(\omega) - H(\omega|s) \\
&= \sum_{i=1}^{n} \mu_i\omega_i(1-p_\tau)\log\left(\frac{\mu_i\omega_i(1-p_\tau)}{\left[\sum_{j=1}^{n}\mu_j\omega_j(1-p_\tau)\right]\mu_i}\right) \\
&\quad + \sum_{i=1}^{n} \mu_i(1-\omega_i+\omega_i p_\tau)\log\left(\frac{\mu_i(1-\omega_i+\omega_i p_\tau)}{\left[\sum_{j=1}^{n}\mu_j(1-\omega_j+\omega_j p_\tau)\right]\mu_i}\right) \\
\frac{\partial I(\omega;s)}{\partial p_\tau} &= -\sum_{i=1}^{n}\mu_i\omega_i\log\left(\frac{(1-p_\tau)\omega_i}{1-\omega_i+\omega_i p_\tau}\right) \\
&+ \left(\sum_{i=1}^{n}\mu_i\omega_i\right)\log\left(\frac{\sum_{i=1}^{n}\mu_i\omega_i(1-p_\tau)}{\sum_{i=1}^{n}\mu_i(1-\omega_i+\omega_i p_\tau)}\right) \leq 0
\end{aligned}$$

We can show this inequality by using the log sum inequality theorem (Cover and

Thomas, 1991, Theorem 2.7.1):

$$
\begin{aligned}
&\left(\sum_{i=1}^{n} \mu_i \omega_i (1-p_\tau)\right) \log\left(\frac{\sum_{i=1}^{n} \mu_i \omega_i (1-p_\tau)}{\sum_{i=1}^{n} \mu_i (1-\omega_i+\omega_i p_\tau)}\right) \\
&\quad - \sum_{i=1}^{n} \mu_i \omega_i (1-p_\tau) \log\left(\frac{\mu_i (1-p_\tau)\omega_i}{\mu_i (1-\omega_i+\omega_i p_\tau)}\right) \leq 0 \\
\iff (1-p_\tau) &\left(\sum_{i=1}^{n} \mu_i \omega_i\right) \log\left(\frac{\sum_{i=1}^{n} \mu_i \omega_i (1-p_\tau)}{\sum_{i=1}^{n} \mu_i (1-\omega_i+\omega_i p_\tau)}\right) \\
&\quad - (1-p_\tau) \sum_{i=1}^{n} \mu_i \omega_i \log\left(\frac{(1-p_\tau)\omega_i}{1-\omega_i+\omega_i p_\tau}\right) \leq 0
\end{aligned}
$$

As the informativeness of the signal is decreasing in p_τ, it is increasing in τ. □

A.7 Proposition 6

Proof. Let ω be the true state of the recall function. Hence, the DM will receive signal $s = 1$ with probability $\pi_1(\tau;\omega) = 1 - \omega + \omega p_\tau$ and choose $e_1^*(\tau)$, and receive $s = 0$ with probability $\pi_0(\tau;\omega) = \omega(1-p_\tau)$ and choose $e_0^*(\tau)$. Therefore, the expected probability of recall of the DM will be:

$$
(1-\omega+\omega p_\tau)\left(1-\omega+\omega \tilde{R}(e_1^*(\tau),\tau)\right) + \omega(1-p_\tau)\left(1-\omega+\omega \tilde{R}(e_0^*(\tau),\tau)\right).
$$

The derivative of the expected probability of recall with respect to τ will be:

$$
\begin{aligned}
\omega\Big(&- \frac{\partial p_\tau}{\partial \tau}\omega\big(\tilde{R}(e_0^*(\tau),\tau) - \tilde{R}(e_1^*(\tau),\tau)\big) \\
&+ (1-\omega+\omega p_\tau)\big(\tilde{R}_e(e_1^*(\tau),\tau)e_1^{*\prime}(\tau) + \tilde{R}_\tau(e_1^*(\tau),\tau)\big) \\
&+ \omega(1-p_\tau)\big(\tilde{R}_e(e_0^*(\tau),\tau)e_0^{*\prime}(\tau) + \tilde{R}_\tau(e_0^*(\tau),\tau)\big)\Big)
\end{aligned}
$$

where

$$
\begin{aligned}
e_s^*(\tau) &= \begin{cases} f_\tau^{-1}\left(\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tau)\omega_i}\right), & \text{if } \frac{1}{\sum_{i=1}^n \mu_{s,i}(\tau)\omega_i} < \lim_{e\to 0} f_\tau(e) \\ 0, & \text{if } \frac{1}{\sum_{i=1}^n \mu_{s,i}(\tau)\omega_i} \geq \lim_{e\to 0} f_\tau(e) \end{cases} \\
f_\tau(e) &= \frac{\tilde{R}_e(e,\tau)}{c'(e)} \\
\mu_{0,i}(\tau) &= \frac{\mu_i\omega_i}{\sum_{j=1}^n \mu_j\omega_j} \\
\mu_{1,i}(\tau) &= \frac{\mu_i(1-\omega_i+\omega_i p_\tau)}{\sum_{j=1}^n \mu_j(1-\omega_j+\omega_j p_\tau)} \\
{e^*}'_s(\tau) &= \begin{cases} -\frac{[c'(e_s^*(\tau))]^2 \frac{\sum_{i=1}^n {\mu'}_{s,i}(\tau)\omega_i}{\left[\sum_{i=1}^n \mu_{s,i}(\tau)\omega_i\right]^2} + c'(e_s^*(\tau))\tilde{R}_{e\tau}(e_s^*(\tau),\tau)}{\tilde{R}_{ee}(e_s^*(\tau),\tau)c'(e_s^*(\tau)) - \tilde{R}_e(e_s^*(\tau),\tau)c''(e_s^*(\tau))}, & \text{if } \frac{1}{\sum_{i=1}^n \mu_{s,i}(\tau)\omega_i} < \lim_{e\to 0} f_\tau(e) \\ 0, & \text{if } \frac{1}{\sum_{i=1}^n \mu_{s,i}(\tau)\omega_i} \geq \lim_{e\to 0} f_\tau(e) \end{cases} \\
\mu'_{0,i}(\tau) &= 0 \\
\mu'_{1,i}(\tau) &= \mu_i \frac{\partial p_\tau}{\partial \tau} \frac{\omega_i \sum_{j=1}^n \mu_j \big(1-\omega_j+\omega_j p_\tau\big) - \big(1-\omega_i+\omega_i p_\tau\big)\sum_{j=1}^n \mu_j\omega_j}{\left[\sum_{j=1}^n \mu_j\big(1-\omega_j+\omega_j p_\tau\big)\right]^2}
\end{aligned}
$$

for $s \in \{0,1\}$ and $\tau \in \{\tilde{\tau} | \forall s \in \{0,1\}, \frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau})\omega_i} \neq \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau})}{c'(e)}$, or $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau})\omega_i} = \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau})}{c'(e)}$ and $\exists \bar{\varepsilon} > 0$ such that $\forall \varepsilon > 0, \varepsilon < \bar{\varepsilon}$, either $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau}+\varepsilon)\omega_i} \geq \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau}+\varepsilon)}{c'(e)}$ and $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau}-\varepsilon)\omega_i} \geq \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau}-\varepsilon)}{c'(e)}$, or $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau}+\varepsilon)\omega_i} \leq \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau}+\varepsilon)}{c'(e)}$ and $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau}-\varepsilon)\omega_i} \leq \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau}-\varepsilon)}{c'(e)}\}$ so that the expected probability of recall is differentiable with respect to τ.

The complement of this set, $\{\tilde{\tau} | \exists s \in \{0,1\}$ such that $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau})\omega_i} = \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau})}{c'(e)}$ and $\forall \bar{\varepsilon} > 0, \exists \varepsilon > 0, \varepsilon < \bar{\varepsilon}$ such that $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau}+\varepsilon)\omega_i} > \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau}+\varepsilon)}{c'(e)}$ and $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau}-\varepsilon)\omega_i} < \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau}-\varepsilon)}{c'(e)}$, or $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau}+\varepsilon)\omega_i} < \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau}+\varepsilon)}{c'(e)}$ and $\frac{1}{\sum_{i=1}^n \mu_{s,i}(\tilde{\tau}-\varepsilon)\omega_i} > \lim_{e\to 0} \frac{\tilde{R}_e(e,\tilde{\tau}-\varepsilon)}{c'(e)}\}$, consists of the boundary points of the set of τ for which the optimal effort is a corner solution, and is countable when there exists $\delta > 0$ such that $[\sup_{0<h<\delta} \phi_s(\tilde{\tau}+h) \leq 0$ or $\inf_{0<h<\delta} \phi_s(\tilde{\tau}+h) \geq 0]$, and $[\sup_{0<h<\delta} \phi_s(\tilde{\tau}-h) \leq 0$ or $\inf_{0<h<\delta} \phi_s(\tilde{\tau}-h) \geq 0]$, $\phi_s(\tau) = \frac{1}{\sum_{i=1}^n \mu_{s,i}(\tau)\omega_i} - \lim_{e\to 0} \frac{\tilde{R}_e(e,\tau)}{c'(e)}$ to avoid oscillatory crossings across the interior and the corner solutions so that each of the points in this set is isolated. Assume that there exists such $\delta > 0$. In this case, the expected probability of recall is differentiable almost everywhere.

Then, the expected probability of recall will exhibit the spacing effect if there exists τ^* such that the derivative of the expected probability of recall as defined above is positive for any $\tau < \tau^*$ and negative for any $\tau \geq \tau^*$, whenever the expected probability of recall is differentiable at τ. □

A.8 Properties of the example function in Section 3.3

$$\tilde{R}_T(e,\tau,T) = -\lambda \mathrm{e}^{-\lambda(\tau+T)}\mathrm{e}^{-\frac{ae}{b\tau+1}} < 0$$

$$\tilde{R}_e(e,\tau,T) = \frac{a\left(1-\mathrm{e}^{-\lambda(\tau+T)}\right)\mathrm{e}^{-\frac{ae}{b\tau+1}}}{b\tau+1} > 0$$

$$\tilde{R}_{ee}(e,\tau,T) = -\frac{a^2\left(1-\mathrm{e}^{-\lambda(\tau+T)}\right)\mathrm{e}^{-\frac{ae}{b\tau+1}}}{(b\tau+1)^2} < 0$$

$$\tilde{R}_\tau(e,\tau,T) = -\lambda \mathrm{e}^{-\frac{ae}{b\tau+1}-\lambda(\tau+T)} - \frac{abe\left(1-\mathrm{e}^{-\lambda(\tau+T)}\right)\mathrm{e}^{-\frac{ae}{b\tau+1}}}{(b\tau+1)^2} < 0$$

$$\tilde{R}_{e\tau}(e,\tau,T) = \frac{a\,\mathrm{e}^{-\frac{ae}{b\tau+1}}}{(b\tau+1)^3}\left[\lambda\,\mathrm{e}^{-\lambda(\tau+T)}(b\tau+1)^2 + b\left(ae-(b\tau+1)\right)\left(1-\mathrm{e}^{-\lambda(\tau+T)}\right)\right]$$

$$\begin{aligned}&\tilde{R}_{ee}(e,\tau,T)\tilde{R}_\tau(e,\tau,T) - \tilde{R}_{e\tau}(e,\tau,T)\tilde{R}_e(e,\tau,T)\\ &\quad= \frac{a^2\,\mathrm{e}^{-\frac{2ae}{b\tau+1}}}{(b\tau+1)^4}\left(1-\mathrm{e}^{-\lambda(\tau+T)}\right)\left(2\lambda(b\tau+1)^2\,\mathrm{e}^{-\lambda(\tau+T)} \;+\; abe\left(1-\mathrm{e}^{-\lambda(\tau+T)}\right)\right) > 0\end{aligned}$$

$$\tilde{R}_{eT}(e,\tau,T) = \frac{a\lambda \mathrm{e}^{-\lambda(\tau+T)}\mathrm{e}^{-\frac{ae}{b\tau+1}}}{b\tau+1} > 0$$

$$\tilde{R}_{ee}(e,\tau,T)\tilde{R}_{T}(e,\tau,T) - \tilde{R}_{eT}(e,\tau,T)\tilde{R}_{e}(e,\tau,T) = 0$$

A.9 Incentive-compatibility of the belief elicitation method

Proof. Let $a > 0$ be the reward per correct quiz answer, and also the reward for the correct quiz score guess. Let $P(k)$, $k \in \{0, 1, \ldots, 5\}$, be the probability of having exactly k correct answers according to the DM's belief. Let $g \in \{0, 1, \ldots, 5\}$ be the DM's stated guess for the number of correct answers. Consider the case that the DM's actual number of correct answer is n, then the total reward that the DM receives will be $a \cdot n$ if $g \neq n$, and $a \cdot (n+1)$ if $g = n$. Let $U(g)$ be the expected utility of the DM for stating guess g, and $v(y)$ be the DM's utility over monetary payoffs $y \in \mathbb{R}$. Then,

$$U(g) = \left(\sum_{k=0}^{5} P(k)v(a\cdot k)\right) + P(g)\left[v(a\cdot(g+1)) - v(a\cdot g)\right].$$

Let $g^* = \operatorname{argmax}_g U(g)$ denote the DM's optimal stated guess given their belief. Note that $g^* = \operatorname{argmax}_g P(g)\left[v(a\cdot(g+1)) - v(a\cdot g)\right]$. If the DM is risk-neutral, then v is linear, implying that $g^* = \operatorname{argmax}_g P(g)$, so the DM should state the mode of their belief as their guess. If the DM is risk-averse, then $[v(a\cdot(g+1)) - v(a\cdot g)]$ is decreasing in g, which implies $g^* \leq \operatorname{argmax}_g P(g)$. $\square$

A.10 Multiple recall attempts as signal

Consider that prior to making effort choice, the DM makes 5 independent and identically distributed recall attempts. Depending on the state ω_i, the probability of having

k successful attempts out of 5 is

$$P(Q = k|\omega_i) = \binom{5}{k}(1 - \omega_i + \omega_i p_\tau)^k(\omega_i(1 - p_\tau))^{5-k}.$$

After observing k successful attempts, the DM will update their belief as follows:

$$\mu_{k,i} = P(\omega = \omega_i|Q = k) = \frac{\binom{5}{k}(1 - \omega_i + \omega_i p_\tau)^k(\omega_i(1 - p_\tau))^{5-k}\mu_i}{\sum_{j=1}^n \binom{5}{k}(1 - \omega_j + \omega_j p_\tau)^k(\omega_j(1 - p_\tau))^{5-k}\mu_j}.$$

According the this posterior belief, the DM chooses their effort choice as follows:

$$\max_{e\geq 0} \sum_{i=1}^n \mu_{k,i}(1 - \omega_i + \omega_i \tilde{R}(e, \tau)) - c(e).$$

The DM will choose a higher effort level e when $\alpha_k = \sum_{i=1}^n \mu_{k,i}\omega_i$ is larger. Let $r = 1 - \omega + \omega p_\tau$. Then, the DM will choose a higher effort level e when $\alpha_k = \frac{1}{1-p_\tau}\frac{\mathbb{E}[r^k(1-r)^{6-k}]}{\mathbb{E}[r^k(1-r)^{5-k}]}$ is larger. Without the signal, the DM would decide the effort choice according to $\alpha = \frac{\mathbb{E}[1-r]}{1-p_\tau} = \sum_{i=1}^n \mu_i\omega_i$. Hence, if $\alpha_k \leq \alpha$, the DM will decrease their effort choice after observing the signal. In contrast, if $\alpha_k \geq \alpha$, the DM will increase their effort choice after observing the signal.

Notice that $\alpha_k \lesseqgtr \alpha \iff \mathrm{Cov}(r^k(1 - r)^{5-k}, 1 - r) \lesseqgtr 0$. Consider the DM observes $k = 0$, so all of the recall attempts are unsuccessful. In this case we will have $\alpha_0 \geq \alpha$ so that the DM finds it optimal to increase their effort choice, as $\mathrm{Cov}((1-r)^{5-k}, 1-r) > 0$. Similarly, when the DM observes that $k = 5$, DM will choose to decrease their effort choice compared to their ex-ante effort choice, as $\mathrm{Cov}(r^5, 1 - r) < 0$. Another case where we can exactly determine whether α_k is smaller or larger than α is when the support for r according to the DM's prior belief is only on $[0, k/5]$, or $[k/5, 1]$. When $1 - \omega_i + \omega_i p_\tau \leq k/5$ for all $i \in \{1, 2, \ldots, n\}$, $r^k(1 - r)^{5-k}$ will be increasing in r for $r \in [0, k/5]$, where $1 - r$ is decreasing, hence $\mathrm{Cov}(r^5, 1 - r) < 0$ which indicates that $\alpha_k < \alpha$, leading a lower effort choice after observing the signal. Intuitively, the DM will be surprised to perform as well as he did, and choose to decrease the costly effort as his recall level is already high. We can use the same reasoning to show that the DM will increase his effort choice after observing k correct recall attempts if r has its support in $[k/5, 1]$.

Using these results, within the context of the experiment, we can claim that whenever a participant observes a quiz score of 0, it is a negative signal; and whenever a participant observes a quiz score of 5 out of 5, it is a positive signal. Moreover, if the participant has a strong belief about their potential quiz score, in the sense that there is high enough mass closely around the mode, observing a quiz score smaller or larger than the stated belief can then be interpreted directly as a negative or positive signal. Similarly, when there is more difference between the observed quiz score and the stated belief, it becomes more likely that the signal is interpreted as a negative signal if the difference is negative and vice versa. However, it is important to note that it could be possible for a participant to have a prior belief regarding their quiz score where the participant believes that any quiz score is equally likely or a prior belief with a bimodal structure such that the participant only believes that a very small or very large quiz score is possible. In these situations, it is not possible to cleanly interpret the difference between the quiz score and the stated expected quiz score as a negative or positive signal. As a robustness check, we test hypotheses $\mathbf{H_4}$ and $\mathbf{H'_4}$ that involve receiving a feedback by interpreting a negative signal as when the quiz score is 0, and a positive signal as when the quiz score is 5 in Appendix B.3.

A.11 Correspondance between the DM's value function in the theoretical model and the experiment

According to the experimental design, the participants pay the cost of their effort choice ($\$0.30 \times e$) out of their endowment of \$9, and receive the remaining part $\$(9 - 0.30e)$ at the end of the session in cash. The next day, they receive the test reward of \$30 via Interac eTransfer if their answer is correct for the randomly selected test question.

We assume that the participants' preferences can be represented by Discounted Expected Utility. Let $R(e, \tau)$ be the probability of answering the test question correctly, hence the probability of recall, when the exerted effort level is e and the spacing level is τ. Let $v(.)$ be the participant's utility function over monetary payoffs, and let δ be the discounting factor of a payment received the next day. After normalizing $v(0) = 0$, the value function that the participant will evaluate can be represented as:

$$\delta R(e, \tau) v(30) + v(9 - 0.30e)$$

If the participant is risk-neutral or risk averse, then v is linear or concave, hence the value function can be represented as

$$R(e, \tau)\hat{v}(30) - c(e)$$

where c is a convex cost function such that $c(e) = -v(9 - 0.30e)$ and $\hat{v}(30) = \delta v(30)$, which has exactly the same formulation as the theoretical model.

B Additional Experimental Results

B.1 Order Effects

Table B1: Order Effects

	$e^{pre}(0)$	$e^{pre}(20)$	$e^{pre}(20) - e^{pre}(0)$	$q(0)$	$q(20)$	$q(0) - q(20)$
Order(S–L)	0.163	0.054	-0.109	-0.203*	-0.214*	0.011
	(0.599)	(0.582)	(0.345)	(0.124)	(0.120)	(0.109)
Constant	9.079***	9.671***	0.593**	2.895***	2.295***	0.600***
	(0.405)	(0.394)	(0.233)	(0.084)	(0.087)	(0.077)
Observations	398	398	398	398	398	398

Each column reports an OLS regression of the listed variable on the order indicator (short spacing scenario first and long spacing scenario second). Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

B.2 Waiting task performance

Table B2: Summary Statistics for the Number of Errors

Variable	**Obs**	**Mean**	**Std. Dev.**	**Min**	**Max**
Fruit Catcher (before review)	198	1.42	3.95	0	26
Letter Z or M (before review)	198	1.56	3.85	0	28
Circle Game (before test)	398	0.73	3.25	0	27
Tetris (before test)	398	1.88	1.30	0	8
Fruit Catcher (after test)	200	1.95	4.53	0	29
Letter Z or M (after test)	200	2.55	5.37	0	29

B.3 Robustness check for $\mathbf{H_4}$ and $\mathbf{H_4'}$

In this section, I present the results for hypotheses $\mathbf{H_4}$ and $\mathbf{H_4'}$ using the definition of a positive signal as a quiz score (q) of 5, and a negative signal as a quiz score of 0 out of 5 questions, as discussed in Appendix A.10.

In the short spacing group, there exists 53 participants with $q = 0$, and 15 participants with $q = 5$. In the long spacing group, there exists 82 participants with $q = 0$

and 4 participants with $q = 5$. The number of participants who answer all quiz questions correctly are too low, which does not allow a meaningful statistical test to be conducted. For this reason, only the average values for the variables of interest will be reported for the hypotheses that include a positive signal without a statistical test.

$\mathbf{H_4}$: For the short spacing group, among the participants who score $q = 0$, the participants who receive the feedback before their ex-post effort choice increase their effort by 2.89 seconds on average, compared to an increase of 0.81 seconds for the no feedback group. The difference is significant using the Mann-Whitney U-test (z=-2.43, p=0.015).

For the long spacing group, similarly to the results in Section 5, while the average increase in effort choice is larger for participants who receive the negative feedback (0.67 seconds) compared to the no feedback group (0.24 seconds), the difference is not statistically significant using the Mann-Whitney U-test (z=-0.87, p=0.39). However, as discussed in Section 5, excluding the participants who expect to remember more after waiting result in a statistically significant difference (z=-2.078, p=0.038).

$\mathbf{H_4'}$: Among the 15 participants who score $q = 5$ in the short spacing group, 5 participants who receive the feedback decrease their effort choice by 0.4 seconds on average compared to the 10 participants in the no feedback group who increase their effort choice by 0.6 seconds. Among the 4 participants with $q = 5$ in the long spacing group, 3 participants who received the feedback had an average decrease in their effort choice of -1.33 seconds, and the 1 participant in the no feedback group did not change their choice. While the direction of the results are in line with the hypothesis, we cannot conclude a result due to the very small sample size.

C Learning material

The word-pairs that were used in the experiment are as follows:

1. Head-Waste
2. Scene-Hide
3. Grape-War
4. Music-Pin
5. Paste-Bet
6. Coat-Route
7. Muscle-Home
8. Bowl-Thread
9. Scream-Blanket
10. Rent-Bath
11. Heap-Cider
12. Split-Cold
13. Tear-Tent
14. Supper-Metal
15. Post-Hobby
16. Wheat-Dog
17. Rubber-Lunch
18. Book-Rope
19. Liar-Pedal

20. Deposit-Rain
21. Button-School
22. Range-Penny
23. Item-Ocean
24. Pot-Knee
25. Shadow-Movie
26. Lift-Cherry
27. Deer-Flame
28. Trail-Bill
29. Cube-Autumn
30. Green-Hunger

D Experimental Instructions

Instructions

General Information

Welcome! This is an experiment in the economics of decision-making. You will now begin reading the instructions.

The following are strictly prohibited during the experiment:

- Communicating with other participants
- Using your phone or any other personal electronic device
- Writing or taking notes

If you need assistance of any kind during the experiment, please raise your hand.

After each section of the instructions, you will answer a short question to check your understanding. You must answer all of them correctly to continue to the experiment.

If you answer all the comprehension check questions correctly **on your first attempt**, you will earn an additional reward of **$1.00**.

After the instructions, you will make some choices. Your choices depend on your preferences and beliefs, so different participants will usually make different choices. You will be paid according to your choices, so read these instructions carefully and think before deciding.

The Basic Idea

In this experiment, you will make decisions about how much you would like to study in order to succeed on a test. If you succeed on the test, you will earn **a monetary reward**.

- First Reading: First, you will **read a list of word pairs** for a fixed amount of time.
- Make Decisions: You will then make decisions.
- Study Session: Then, you will have a single study session where you will study the same word pairs.
- Test: Finally, you will take the test on the same word pairs.

After the first reading, you will make decisions: You will decide how many seconds you want to study each word pair during the study session. **Each additional second of studying costs money**, which is deducted from the endowment allocated to you.

You will make these decisions for different scenarios. The computer has **randomly selected one scenario** to be implemented. Any of the scenarios you make choices for could have been selected. Not all scenarios have an equal likelihood of being chosen. During the study session, you will study all the word pairs for as many seconds as you chose for the randomly selected scenario.

Word Pairs and the Test

During the first reading and the study session, you will study **the same list of word pairs**. These word pairs will be shown to you on the screen one by one.

Word A - Word B

In the test, one word from each pair (either Word A or Word B) will be missing. You will be asked to type the corresponding missing word for every pair.

Word A Type here

The test consists of all word pairs. Only **one** of the questions in this test will be randomly selected by the computer to determine whether you win the reward. If you answer that question correctly, you will earn the **test reward**.

You should correctly answer as many questions as you can to earn the test reward. Having more correct answers in the test means a higher likelihood of earning the reward.

Scenarios and Decisions

During the experiment, you will make decisions regarding how many seconds you want to study per word pair during the study session. You will make these decisions **for different scenarios**.

The scenarios might differ in when your study session takes place.

One of the scenarios has been randomly selected by the computer. **Only your decision for the randomly selected scenario** will be implemented. During the study session, you will study the word pairs for the number of seconds you choose for that scenario.

Any of the scenarios you make choices for could have been selected. Not all scenarios have an equal likelihood of being chosen. This protocol suggests that you should choose in each scenario as if it were the only choice that determines the outcome of this experiment.

The number of seconds you choose to study determines how much you spend from your endowment: **studying longer costs more**.

You will pay the cost of studying out of your endowment according to your choice for the randomly selected scenario. You must pay this cost whether you earn the test reward or not.

Payment

Show-up fee:

You will earn a show-up fee of $10.00 at the end of this experiment for your participation. You will receive the show-up fee in cash.

Test Reward:

One of the questions in the test has been randomly selected by the computer. If you answer that question correctly, you will receive the test reward via **Interac e-Transfer tomorrow**.

Endowment:

You will also be given an endowment of $9.00. Your choices during the experiment will determine how you spend this endowment. One of the scenarios that you will face during the experiment has been randomly selected by the computer, and your choice in that scenario will determine how much is spent from the endowment. You will receive the remaining part of your endowment **in cash at the end of the experiment**.

This protocol of determining payments suggests that you should choose in each scenario as if it were the only scenario that determines your payment.

Additional Earnings:

During the experiment, you will have the opportunity to earn some additional earnings. The details will be explained during the experiment.

In summary:

- You will be paid your **show-up fee + (your endowment - your choice of spending)** in cash at the end of the experiment.
- Additionally, if you answer the randomly selected question correctly in the test, you will receive your **test reward** via Interac e-Transfer tomorrow.
- You can earn additional earnings in cash on top of the earnings above at the end of the experiment.

You will be informed of your payment at the end of the experiment.

Waiting Tasks

During the experiment, there will be designated waiting periods. While waiting, you will be asked to complete visual attention tasks for a fixed amount of time. The tasks will be easy to complete if you pay attention. You must complete these waiting tasks successfully. **If you fail to complete any of the waiting tasks, your experiment will end** and you will not be allowed to continue the experiment. **You will only receive your show-up fee** and will be asked to leave the experiment.

Frequently Asked Questions

• Is this some kind of psychology experiment with an agenda you haven't told us?
Answer: No, it is an economics experiment. If we do anything deceptive or don't pay you as described, then you can complain to the University of Toronto Research Ethics Board and we will be in serious trouble. These instructions are meant to clarify how you earn money and our interest is in seeing how people make decisions.

Comprehension Check

You must answer the question correctly to proceed. You will receive **$1.00** if you answer all the comprehension check questions correctly **on your first attempt**. You may revise the instructions anytime using the button at the end of this page.

Q1. Which of the following sentences is FALSE?

- ◯ There will be a single study session. During the study session, you will study all the word pairs as many seconds as you chose for the randomly selected scenario.
- ◯ You will make decisions for different scenarios.
- ◯ After the first reading, you will make decisions regarding how many seconds you want to study each word pair during the study session.
- ◯ There will be many study sessions.

Q2. Which of the following sentences is CORRECT about the test?

- ◯ The test consists of only one randomly selected word pair. You will win the test reward if you answer the question correctly.
- ◯ The test consists of all word pairs. You will win the test reward only if you answer all questions correctly.
- ◯ The test consists of all word pairs. One pair has been randomly selected by the computer. You will win the test reward if you answer that question correctly.

Q3. How is your spending out of your endowment determined?

- ◯ All of your choices for all scenarios jointly determine your spending.
- ◯ The number of seconds you choose to study for the randomly selected scenario determines your spending. You must pay this cost whether you earn the test reward or not.
- ◯ Your test score determines your spending.
- ◯ The number of seconds you choose to study for the randomly selected scenario and if you earn the test reward together determine your spending.

Q4. How will you receive your test reward if you succeed?

- ◯ Tomorrow via Interac e-Transfer
- ◯ Tomorrow in cash
- ◯ At the end of the experiment via Interac e-Transfer
- ◯ At the end of the experiment in cash

Q5. What happens if you fail to complete the waiting tasks?

- ◯ You will only receive your show-up fee and will be asked to leave the experiment.
- ◯ You will continue the experiment.

First Reading of the Word Pairs

You will now see 30 word pairs, each displayed for 5 seconds. Please note that the study session and the test will also include the same word pairs.

Please click "Next" when you are ready. The words will appear automatically.

Next

Item - Ocean

Time remaining: 3 seconds

Make Decisions

You will now make choices regarding how long you would like to study the word pairs during the study session for different scenarios.

The number of seconds that you choose determines how many seconds you will have to read each word pair during the study session. Each additional second of studying costs $0.01 per word pair, corresponding to $0.30 for all word pairs. You have $9.00 as endowment to pay for this cost.

Remember that one of the scenarios has been randomly selected by the computer to be implemented. Not all scenarios are chosen with an equal likelihood. This protocol suggests that you should choose in each scenario as if it were the only choice that determines the outcome of this experiment.

Make Decisions

Please choose how many seconds you would like to study each word-pair for the following scenarios. One of the scenarios in this experiment has been randomly selected by the computer to be implemented.

For your reference, you had 3 seconds for each of the 30 word-pairs during the first reading.

Remember that you will get the test reward if your answer is correct for the randomly selected question in the test. If you succeed, the reward will be paid tomorrow with Interac eTransfer.

You have $9.00 as endowment to pay for the cost of studying.

Scenario A:

- Your reward for the test will be **$30**.
- Your study session will start **20 MINUTES AFTER** you make your choices.

In Scenario A, how many seconds would you like to spend studying during the study session **for each word-pair**?

You chose to study **13 seconds** per word pair. This will cost $3.90.

Scenario B:

- Your reward for the test will be **$30**.
- Your study session will start **IMMEDIATELY** after you make your choices.

In Scenario B, how many seconds would you like to spend studying during the study session **for each word-pair**?

You chose to study **7 seconds** per word pair. This will cost $2.10.

Guess Practice Quiz Score

Before the study session, you will take a practice quiz. The practice quiz will include 5 of the word pairs. You will type the missing word for each word pair. Notice that you will take the practice quiz before studying the word pairs.

You will now guess your score for the practice quiz for different scenarios. You will earn a prize of **$0.25** if you guess your score correctly.

You will additionally earn **$0.25** for each correct answer in the practice quiz.

If you were to take the quiz **20 minutes later**, how many correct answers do you expect to give (out of 5)?

If you were to take the quiz **now**, how many correct answers do you expect to give (out of 5)?

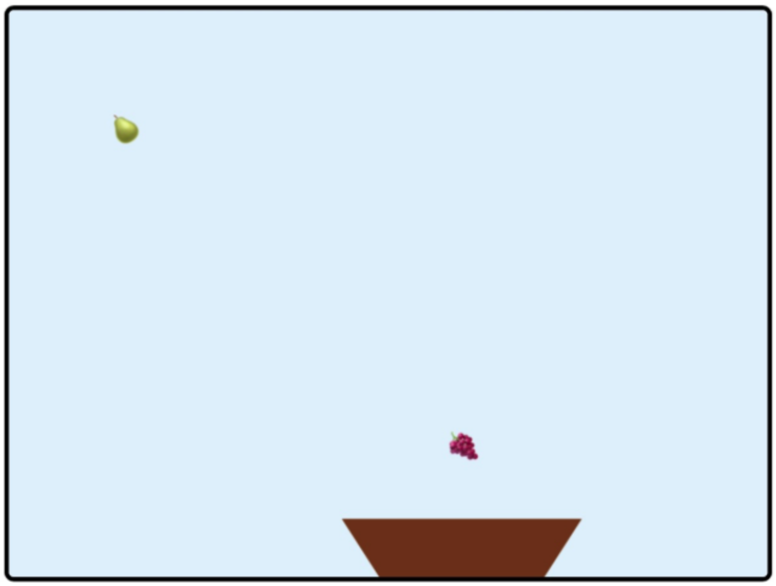
Submit

Waiting Task: Fruit Catcher

Now, you will do a waiting task for 9 minutes.

For this task, you will play the "Fruit Catcher" game.

In this game, you need to catch the fruits that are falling from above into the basket. You will use the left and right arrow keys (◀ ▶) on the keyboard to move the basket. Position the basket so that each fruit falls into the basket.



Each time you miss a fruit will be counted as 1 mistake. You are allowed to make **at most 29** mistakes.

If you make 30 mistakes:

You **fail** the waiting task and **the experiment will stop**. You will not be allowed to continue the experiment. You will only receive the show-up fee.

Waiting Task: Letter Z or Letter M

Now, you will do another waiting task for 9 minutes.

For this task, you will play the "Letter Z or Letter M" game.

At random times, either letter Z, or letter M will appear in a box on the screen.

- When the letter **Z** appears in the box, press **Z** on the keyboard immediately.
- When the letter **M** appears in the box, press **M** on the keyboard immediately.

Each letter will appear in the box for 5 seconds. When a letter is displayed in the box, pressing the wrong key or no key counts as a mistake.

When the box is empty, don't press any key. Pressing a key when no letter is displayed in the box also counts as a mistake.

You are allowed to make **at most 29** mistakes.

If you make 30 mistakes:

You **fail** the waiting task and **the experiment will stop**. You will not be allowed to continue the experiment. You will only receive the show-up fee.

Hint: Letter Z will always appear on the left side of the screen. Letter M will always appear on the right side of the screen. As you only need to use either key Z or M in this game, **USE YOUR LEFT HAND** to press **Z**, and **USE YOUR RIGHT HAND** to press **M**.

Practice Quiz

You will now take the practice quiz. Practice quiz has the **same format** as the final test.

You will be asked to type the missing word for 5 word pairs.

You will earn **$0.25** for each correct quiz answer.

Each word pair will appear one by one automatically. You will have **15 seconds** to type each missing word. You can directly start typing the answer for each question.

You don't need to click on the screen to write the answer, or to submit the answer. The answers submit automatically after 15 seconds for each question.

You don't need to capitalize the letters. Solutions are **not** case-sensitive.

Press "Next" when you are ready for the practice quiz, which will start immediately.

Next

Time left to complete this page: **0:14**

Type here **War**

Submit

Practice Quiz Feedback

You answered 0 questions correctly (out of 5).

Earlier, you guessed you would have 3 correct answers.

Next

Make Decisions

Please choose how many seconds you would like to study each word-pair for the following scenario.

For your reference, you had 3 seconds for each of the 30 word-pairs during the first reading.

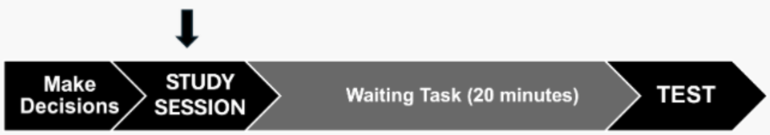
Remember that you will get the test reward if your answer is correct for the randomly selected question in the test. If you succeed, the reward will be paid tomorrow with Interac eTransfer.

You have $9.00 as endowment to pay for the cost of studying.

Practice Quiz Feedback : You had 0 correct answers out of 5 in the practice quiz. Earlier, your guess was 3 out of 5.

Scenario C:

- Your reward for the test will be **$30**.
- Your study session will start **IMMEDIATELY** after you make your choices.



In Scenario C, how many seconds would you like to spend studying during the study session **for each word pair** ?

You chose to study **17 seconds** per word pair. This will cost $5.10.

Submit

Random Scenario Result

Scenario C is selected randomly to be implemented:

- The reward level for the test is $30.
- Your choice for this scenario was to study each word pair for 17 seconds. This will cost $5.10.

Next

Study Session

You will now start the study session. According to your choice, you will see each word pair for 17 seconds.

Word pairs will appear after you press "Next". Press "Next" when you are ready.

Next

Waiting Task: Circle Game

Now, you will do a waiting task for 9 minutes.

In this task, you will play the Circle Game. A circle will appear on the screen at random times and locations for 5 seconds. You need to click **on** the circle with your mouse whenever it appears on the screen.

Every time you miss to click on the circle will be counted as 1 mistake. You are allowed to make **at most 29** mistakes.

If you do 30 mistakes:

You **fail** the waiting task and **the experiment will stop**. You will not be allowed to continue the experiment. You will only receive the show-up fee.

Press "Next" button when you are ready.

Next

You must stay on this page for at least 60 seconds before continuing.

Waiting Task: Tetris Game

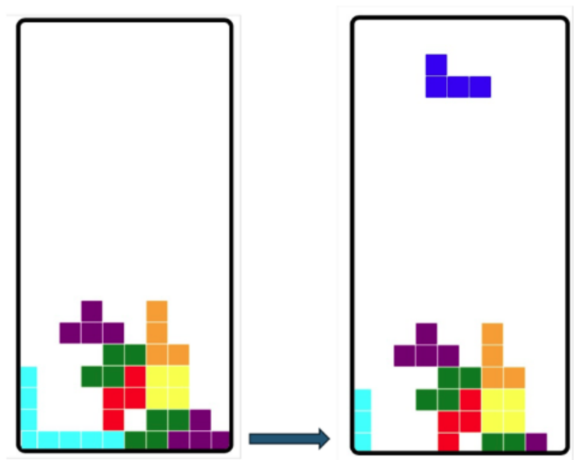
Now, you will do another waiting task for 9 minutes.

For this task, you will play the Tetris game.

You will use the left, right, down arrow keys (◀ ▼ ▶) to <u>move</u> the Tetris blocks.

Use the up (▲) arrow key to <u>rotate</u> the Tetris blocks.

Your goal is to fill complete rows horizontally with the blocks. When you fill a row, it disappears.



If the Tetris blocks **stack up and touch the top of the box, it will be counted as 1 mistake.**

You are allowed to make **at most 9** mistakes.

So your objective is **to fill as many rows horizontally as possible to make them disappear**, to avoid the blocks touching the top of the box.

If you make 10 mistakes:

You **fail** the waiting task and **the experiment will stop**. You will not be allowed to continue the experiment. You will only receive the show-up fee.

TEST

You will now do the test.

You will be asked to type the missing word for all word pairs.

Each word pair will appear one by one automatically. You will have **15 seconds** to type each missing word. You can directly start typing the answer for each question.

You don't need to click on the screen to write the answer, or to submit the answer. The answers submit automatically after 15 seconds for each question.

You don't need to capitalize the letters. Solutions are **not** case-sensitive.

Try to answer as many questions correctly as possible to earn the test reward.

Press "Next" when you are ready for the test, which will start immediately.

Next

Time left to complete this page: **0:15**

TEST

Heap Type here

Submit

End of the experiment

Thank you for your participation in this experiment.

Payment

Test Reward:

You have **5** correct answers out of 30 questions in the test.

The selected question is Button-School.

Your answer to this question was: bird. Unfortunately, you did not answer the question correctly. You did not win a reward for the test.

Spending:

You chose to study 17 seconds per word pair, which costs $5.10.

Your initial endowment was $9.00. After the costs, the remaining endowment will be **$3.90**.

Show-up Fee:

Your show-up fee is **$10.00**.

Additional Rewards:

You completed the comprehension check on your first try. You earned **$1.00**.

For the practice quiz, you had 0 correct answers. You earned **$0.00** for your practice quiz performance.

Your guess for the practice quiz was 3 correct answers. You guessed incorrectly.

Payment Summary:

Today: Your earning for today is **$14.90** in total. Please complete the Receipt for Payment now. On the Receipt for Payment, please write **$14.90** for "Amount of Payment" box, and also write your name, student number, and sign the Receipt.

Unfortunately, you will not receive a test reward tomorrow.