Simulation, Visualization and Experimental analysis for Population Protocols and Network Constructor in 2-Dimensional Case

Author: Haoxuan Wang
Supervisor: Dr. Othon Michail
Dr. Vitaliy Kurlin

May 7, 2018

Abstract

The abstract[1] [2] [3]

Contents

1 Introduction

1.1 Aims and Objectives

The project aimed to study general population protocols [1] and its two derived model, network constructor [2] and terminating grid network constructor [3] (specifically for grid network construction). It also attempted to experimentally simulate, visualise and compare these protocols via building the simulator and visualizer.

1.2 The challenges in the project

1.2.1 Heterogeneous for different types of Models

The theoretical models involved in three main different models initially originated in population protocols. These three models share inherently common points but there are also some conceptual differences in between them. For instance, the network constructor [2] and terminating grid network constructor [3] involve state of connections in between two nodes while the original population protocol does not. The node of terminating grid network constructor has its complexity structurally compared with the other two types of model.

1.2.2 Heterogeneous for different types of Protocols

The protocols discussed in the related papers [1, 2, 3] involve many different protocols. The protocols are totally different on many characteristics, such as their different computational ability, different ending in either or termination, computation target. These differences between protocol to protocol may lead the simulator and visualizer hard to develop and test.

1.2.3 Human factor: Lacking Experience for model visualization

Prior to this project, the author has totally no experiences on model simulation and also no knowledge on what kind of related library will be involved. Learning may take more time than its expected.

1.2.4 Brief introduction to the programme

The final programme contains an UI with an fix-sized area to illustrate the interaction process of a particular protocol and shows the states of elements[*] in the population. In addition, it contains a information panel containing some related information with regard to the population itself, including:

- Number of nodes
- Number of nodes distinguished in different status
- Number of selections for pairs of nodes[†] that scheduler had took
- Number of effective interactions the population executed

[*]"Elements" refers nodes in general population protocol, but also includes edge if the protocol involves edge states.

[†]may also include pair of ports for terminating grid network constructor

Additionally, it provides a set of parameters' settings regarding the initial configuration of a protocol and a population to be simulated, which includes:

- The number of nodes included in the simulation
- The initial state for each node[‡]
- The protocol type (and also different sets of transition rules for the protocol)
- Option on whether to use fast-forward simulation method for initially n-times selection, and the value of n if the option is enabled. A fast-forward simulation executed in the model but does not present that process in the viewer so it normally faster than the case that does not enable this option.

1.2.5 Evoluation of the project

In general, the simulator successfully implemented a series of different protocols for the three model mentioned above.

UI The UI functions of simulator is verified through a large number of different population simulations. This ensures the UI functions work as they expected in design stage. These experiments on theoretical model may also be asserted the correctness of model through the output configuration of these simulations.

Model The model partition of the simulator developed through Testing driven development method. According to the specification, the testing suits had been written before the written of any model code . The model code has to pass all testing suits after their implementation.

2 Background

2.1 Existing solution

There is an existing simulator called NETCS [5] for network constructor, which is the model covered in this paper [2], the simulator was written in Java, provided an web user interface and assisted related research.

This work will explore the possibility to simulate a further variance of network constructor, grid network constructor, as well as attempting to provide a uniform simulation to different models and protocols, which is easily extensible.

2.2 Brief Introduction to Population Protocols [1, 4]

Population protocols are theoretical models for distributed computation [1, 4]. The model contains a collection of indistinguishable agents. They (i.e. the agents) carry out computation tasks through directly pair-wised interactions. The interaction pattern of agents is unpredictable from perspective of agents themselves and is controlled through an adversary scheduler with fairness constraints [1, 4].

[‡]The state of edge for network constructors (i.e. network constructor and terminating grid network constructor) should be always "0" (i.e. inactivated) at initial, so it is omitted here.

Formal Definition [1] A protocol can be formally defined as:

- Q, a finite set of passible states for an agent,
- Σ, a finite set of input alphabet,
- Y, a finite set of output range,
- $\iota = \Sigma \to Q$, is an input map from $\sum$ to Q, hence $\iota(\sigma)$ represents the initial state whose input is $\sigma \in \Sigma$
- $\omega = Q \to Y$, is an output map from Q to Y, and
- $\delta \subseteq Q^4$, a transition relation describes how pairs of agents can interact.

A *configuration* for a population is a vector of all the agents' states. Because agents with a same state are indistinguishable with each other, each configuration could also an unordered multiset of states. It can be represented as C [1, 4].

At any point of the discrete time, the interaction is unpredictable from the perspective of the agents in the population and the population itself. The interactions at any time are decided by a adversary scheduler necessarily enforced with *fairness* condition [1, 4]. The fairness condition means that the scheduler cannot avoid a possible step forever. Formally, it means if a infinitely often configuration C and $C \to C^{'}$, the $C^{'}$ must also appear infinitely of in the execution [1].

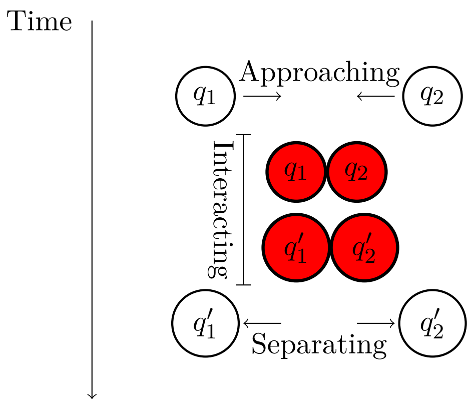


Figure 1: A typical (simple) interaction in population protocol

The current implementation of this work assumes a more strict scheduler called random scheduler, which resulting uniform random interactions (i.e. at each step, it presents equally possibility for every pair of agents to interact.) Essentially, a random scheduler is a "fair" scheduler, but a "fair" scheduler is not necessarily a random scheduler. The assumption actually increases the power of the model because it allows a leader agent detects the absence of agents in a particular state after a long enough waiting [1]. Note this is only for simplifying the current implementation and possibly extends to other fair schedulers that is not a random scheduler in the future.

2.3 From Population Protocols to Network Constructor [2]

A variance of population protocol is called "network constructor" [2]. It involves the extra elements do not existing in general population protocol called "edge", which is the connection in between any two agents (or ”processes”) [2]. In general, the connection is similar to the agents, on the characteristic that both of them have a finite number of states. For instance, if a connection have $k + 1$ states, state 0 may represent the connection does not exist and for state $i \in \{1, 2, 3, ..., k\}$ shows the strength for a particular connection.

In the following paragraph and the currently implemented simulator, we solely consider the simplest case containing only *on* and off two states for edges [2]. A connection is said to be in *on* or *active* state, if at any discrete time, the connection exists in between two particular nodes; otherwise if the connection is stated as in off or *inactive* state [2]. At initial, all edge are in off state, which means that there is no connection in a given population at discrete time 0 [2]. The state for an edge may or may not change through an interaction in between the two nodes it connected or disconnected to.

The network constructor also follows an adversary scheduler with ”fairness” constraint. This is exactly same as the paragraphs mentioned in the previous section 2.2.

Formal Definition [2] Formally, a network constructor can be defined as:

- Q, a finite set of passible states for an agent,
- Q_{out}, a finite set of output range,
- $q_0 \in Q$, is initial state of node,
- $\delta : Q \times Q \times \{0, 1\} \to Q \times Q \times \{0, 1\}$, a transition function, where the $\{0, 1\}$ is the states for edges with initial value 0.

The main target of this model concentrates on network construction rather than specific function computation, which is the task for general population protocol [2]. Notice that a transition can be either *effective* or *ineffective*. Define $\delta(a, b, c) = (a^{'}, b^{'}, c^{'})$ as a transition function (Here, a, b are ”node-state” whereas c is state for edge.) as it is in the formal definition, $\delta_1(a, b, c) = a^{'}$, $\delta_2(a, b, c) = b^{'}$ (representing two nodes states' change) and $\delta_3(a, b, c) = c^{'}$ (representing edge state change), the transition $(a, b, c) \to (a^{'}, b^{'}, c^{'})$ is called effective if at least one $x \in \{a, b, c\} \neq x^{'}$; otherwise it is called ineffective [2].

Name the set of nodes (or ”distributed processes” under this context) V_I and the set of pairs of nodes as E_I. A configuration C is a mapping $V_I \cup E_I \to Q \cup \{0, 1\}$ determines the states of nodes and edges in the population at any discrete time. The output of configuration C is defined as the graph $G(C) = (V, E)$ where $V = \{u \in V_I : C(u) \in Q_{out}\}$ and $E = \{uv : u, v \in V, u \neq v$, and $C(uv) = 1\}$ [2].

2.4 Grid Terminating Network Constructor [3]

Another paper [3] presents a different automata but very similar to network constructor. Each node in this model has a fixed number of ports with it. In the 2-dimensional case, it will have 4 distinguished ports p_y, p_x, p_{-y} and p_{-x}, may simply donated as u, r, d and l, respectively [3] . The port, which neighbour with each other, are also perpendicular to each other, forming local axes. Hence, $u \perp r$, $r \perp d$, $d \perp l$, and $l \perp u$ [3] . The coordinates are only for local purposes and do not necessarily represent the actual orientation of a node. A connection (or edge) can only be built through pairs of ports, which is different from the previous model (network constructor).

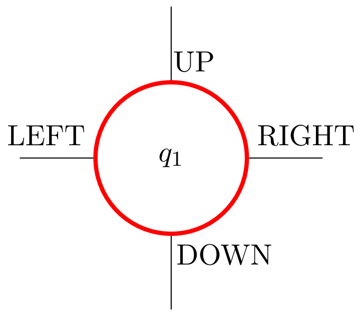


Figure 2: Structure of a node in Grid Terminating Network Constructor model

Formal Definition [3] Formally, a terminating grid network constructor in 2-D can be defined as:

- Q, a finite set of passible states for an agent,
- Q_{out}, a finite set of output range,
- $q_0 \in Q$, is initial state of node
- $\delta : (Q \times P) \times (Q \times P) \times \{0, 1\} \to Q \times Q \times \{0, 1\}$, a transition function, where the $\{0, 1\}$ is the states for edges with initial value 0.
- When required, there will be also a special initial leader-state $L_0 \in Q$ defined.

A transition can be either *effective* or *ineffective*. Define $\delta((a, p_1), (b, p_2), c) = (a^{'}, b^{'}, c^{'})$ as a transition function as it is in the formal definition, it is called effective if at least one $x \in \{a, b, c\} \neq x^{'}$; otherwise it is called ineffective [3].

Every configuration C forms a set of shapes $G[A(C)]$, where a shape means that the network induced by active edges of C [3]. Given the geometrical restrictions (i.e. the connection are made at unit distance and are perpendicular whenever they correspond to consecutive ports of a node), not all possible $A(C)$ are valid [3]. $A(C)$ is valid if any connected component defined by it is a subnetwork of 2D grid network with unit distances. A valid $A(C)$ at time $t - 1$ also restricts the possible valid $A(C)$ at time $t + k$, where $k \in Integer$ and $k \geq 0$ [3].

The paper [3] also defines a set of shapes $G_{out}(C) = \{V_s, E_s\}$ as output of a configuration where $V_s = \{u \in V : C_v(u) \in Q_{out}\}$ and $E_s = A(C) \cap \{(v_1, p_1) : v_1 \neq v_2 \in V_s; p_1, p_2 \in P\}$ [3]. Less formally, the output shapes of a configuration contains all nodes in output states and the active edges in between them. This model focuses on terminating protocols so it assumes $Q_{out} \subseteq Q_{halt} \subseteq Q$. All rules containing $q_{halt} \in Q_{halt}$ is ineffective [3].

2.5 Vector transformation: handle coordinate rotations in 2-dimension

As mentioned in the previous section 2.4, Grid Terminating Network Constructor enforces perpendicular ports and geometrical restrictions. This means that handling rotation in some cases becomes necessary. This section will cover some basic but related vector mathematics. The detailed algorithms will be discussed later on, in the section (4).

2.5.1 2D centred coordinate rotation, with origin point as the centre

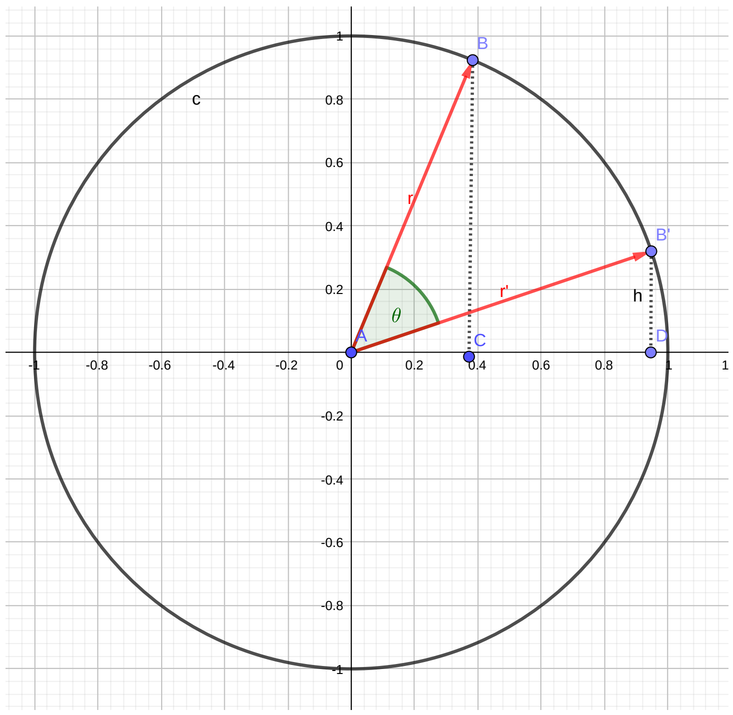


Figure 3: Vector $\vec{r}$ and $\vec{r\prime}$ in unit circle

The figure 3 has two same-length vectors, $\vec{r}$ (or $\vec{AB}$) and $\vec{r\prime}$ (or $\vec{AB\prime}$) in its Cartesian coordinate and their destination point located at the same unit circle. The point A is the origin point with coordinate $(0,0)$. Suppose it has already known for the coordinate of B is (x,y), and the θ angle. The question is that what the coordinate of $B^{'}$ (representing in $(x^{'},y^{'})$) is, after the first vector $\vec{r}$ rotates the given angle θ and becomes $\vec{r\prime}$. Note that, the "positive" rotation direction in current context and the implementation of the simulator is defined as "clock-wised".

To solve this, the following equation can be concluded:

$$\begin{bmatrix} x^{'} \\ y^{'} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{1}$$

A brief derivation can be found in appendix partition below and it also can be found in [6] with a more precise and general proof.

2.5.2 2D centred coordinate rotation, with any points as the centre

Given the equation (1), it can be easily deduced that the rotation equation for any centred points c with coordinate (c_x, c_y) can be achieved through panning c to origin point with coordinate $(0,0)$, carry out the rotation and panning back to its original coordinate (c_x, c_y). Suppose the target point that is required to be rotated is called p (with coordinate (x,y)), it then has :

$$\begin{bmatrix} x^{'} \\ y^{'} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \end{bmatrix} \right) + \begin{bmatrix} c_x \\ c_y \end{bmatrix} \tag{2}$$

$p\prime$ (with coordinate $(x\prime, y\prime)$) will be point after p rotating centred on c.

2.5.3 Affine transformation form

After a series of transformation, the equation 2 becomes the following form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c_x(1-\cos\theta) - c_y\sin\theta \\ c_y(1-\cos\theta) + c_x\sin\theta \end{bmatrix} \tag{3}$$

Let $\mathbf{M} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} c_x(1-\cos\theta) - c_y\sin\theta \\ c_y(1-\cos\theta) + c_x\sin\theta \end{bmatrix}$, then $\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix} + \mathbf{B}$, where it forms an *affine transformation* [7]. **M** and **B** can be calculated directly after knowing the rotation centre (c_x, c_y) and the rotation agle θ, and therefore the calculated **M** and **B** can be applied for multiple target coordinates (those coordinates required to be rotated). This simplifies the calculation for a large batch of centred rotation with multiple target coordinates.

3 Data Required

4 Design

4.1 Anticipated components of the system

The main partition of the system was a software that enables to simulate and visualise each interaction before convergence (or terminate). The project would also attempt to explore one or more protocol to check their internal status after finishing the software.

The simulator would allow users to choose from a set of well-defined protocol and specify the parameters for these protocol (e.g. number of agents participating). The simulator would allow users to easily extend their own protocol though writing structured few lines of code, an the extension should be as easy as possible.

The system expected to have a ability to simulate some variations (which at least includes the network constructor [2] and terminating grid network constructor in 2D [3]) of the original protocol according the protocol and dynamically visualise how process (nodes) transfer for each interaction.

The interface provided would be a desktop application running on Java Virtual Machine Platform. A web interface was desirable but would not be guaranteed to deliver.

Modification against original design The original design mentioned it could include "a configuration parser to enable user define a protocol from configuration files" finally replaced by a set of well-defined set of protocol and a programmable interface. This is because the original design requires a design and implementation of Domain Specific Language (DSL), and a parser of this kind of "new language", which is a expected large work load and might be infeasible to finish all of them during the project period. This might be done in the following development plan after finishing the project itself.

4.2 Data structure, algorithms and pseudo-code for key method

4.2.1 Graph and adjacency list

For some specific kinds of model, specifically ,the network constructor, a data structure to maintain a undirected graph G formed from current configuration C is required. In other words, it requires to maintain a group of status of *active* edges. It aims to construct "shapes" and there are some status for the edge uv in between any nodes (processors) u and v in the population.

To achieve this, an adjacency list would be used to record the connections associated with each nodes. Initially, it would be an map with all nodes in the Population as keys and an empty list associated as the value. If the connection in between u and v builds up, the node u will be added to the list which the node v corresponds to and vice versa (i.e. the v will be added to the list which the key u corresponds to). When an *active* connection, say a connection between $u^{'}$ and $v^{'}$ "cancelled" (i.e. running from *active* state to *inactive* state). The node reference $v^{'}$ in the list that $u^{'}$ as the key would be deleted and so the node reference $u^{'}$ in the list that $v^{'}$ as the key.

Modification against original design The original design used an adjacency matrix rather than a adjacency list to represent the undirected graph possibly appeared in some model to be simulated. The modification was made based on the consideration on space cost. A adjacency matrix requires a fixed $S(N^2)$ space allocated overhead for N nodes in the population while the proposed adjacency list implementation would consume $S(E)$ where E is the number of "*active*" edges, which at most has a value of $\frac{N*(N-1)}{2}$.

For every model defined, the initial configuration starts with no *active* edge so it would be a zero matrix if the adjacency matrix is used in the program and maintain a sparse state for most of time during simulation process. A adjacency list might be a better choice since the list would save spaces required than adjacency matrix.

4.2.2 Quad-direction linked structure

In 2-dimensional case, the agents in grid terminating grid network constructor model would have exactly 4 ports associated with them and any edges have to be connected through ports. As mentioned before, the port themselves are perpendicular with each other. To implement port concept and connection attached to ports, the system would use a quad-direction linked structure to construct this. Each nodes would have 4 null-able references to other nodes, one of which represents a port associated to this node. A connection (or edge) e between node n and node $n^{'}$ is treaded as "active" on a port p of n if the reference that p associated pointing to an another non-null node $n^{'}$ and so similar for the symmetric reference in $n^{'}$. In contrast, the connection (or "edge") on a port is deemed as "inactive" if the port reference pointing to null.

Listing 1: Pseudo-code demonstration of quad-direction linked structure

```
class LocallyCoordinatedModelNode(...){
  var up: LocallyCoordinatedModelNode? = null
  var down: LocallyCoordinatedModelNode? = null
  var left: LocallyCoordinatedModelNode? = null
  var right: LocallyCoordinatedModelNode? = null
  ...
}
```

Relative node coordinate and rotation For convenience of implementation, the relative node coordinate would be defined here. Given a node n, with it coordinate (x_n, y_n). The nodes that connected to its port p_x, p_{-x}, p_y, p_{-y} will have coordinate $(x_n+1, y_n), (x_n-1, y_n), (x_n, y_n+1), (x_n, y_n-1)$ respectively. Note that the coordinate is localized and from different node, they will have a different perspective of coordinate. It is also required to define a parameter φ_i representing the rotated degree from normal Cartesian coordinates. Here, the "positive" direction can be defined as clockwise.

Relative port degree In terminating grid network construction model, the 4 ports associated a node is called port p_y, p_x, p_{-y} and p_{-x}, may simply donated as u(UP), r(RIGHT), d(DOWN) and l(LEFT), respectively. They are perpendicular to each other. To model the perpendicularity of node numerically, it proposed to notate a degree for each port. Specifically, u(UP) is with a degree value of 0°, r(RIGHT) is with a degree value of 90°, d(DOWN) is with a degree value of 180° and d is with a degree value of 270°.

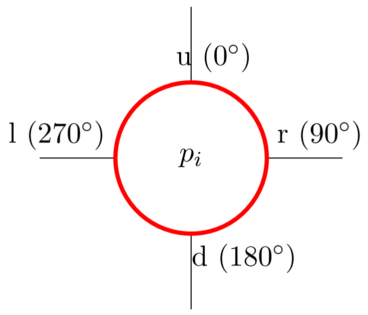


Figure 4: Defined degree for each port

The difference θ of degree between two ports p_1, p_2 can be defined as

$$\theta = ||\theta_1 - \theta_2|| = ||Angle(p_1) - Angle(p_2)||$$

where $Angle$ is a mapping from a port $p_i \in \{p_y, p_x, p_{-y}, p_{-x}\}$ to its degree $\theta_i \in \{0°, 90°, 180°, 270°\}$. Suppose there two nodes a, b and two ports where p_a is one port of a and p_b is one port of b, then the **"relative difference"** Θ could be:

$$\Theta_{p_{ab}} \equiv ||\theta_{p_a} - \varphi_a - (\theta_{p_b} - \varphi_b)|| \bmod 360 \tag{4}$$

which is the port difference after involving the rotation of nodes.

Relative docking port Also for convenience of implementation, here two pairs of "relative docking port" would be defined. A pair of relative docking port is the pair of ports of which relative difference is 180°. Certainly, this limits only two possible pairs of relative docking port: port $\{p_x, p_{-x}\}$ as the first pair of docking port, and port $\{p_y, p_{-y}\}$ as the second pair of docking port. Suppose there are two nodes shown as exactly in diagram 4, that is, two nodes with 0° rotations and if assuming there is no rotation allowed in the configuration, the only legal connection will be connections in between the two pairs of relative docking port, where the two ports belonging to different nodes and the two nodes can be interchangeable. For instance, suppose the port of the first node is selected as p_x, then the only legal choice for the port of the second node is p_{-x}, which is the relative docking port of p_x.

Breadth-First Search (BFS) Algorithm [8] on Quad-direction linked structure Breadth-First Search (BFS) algorithm is a famous and broadly-used graph traversing algorithm and is also foundation for some other algorithm [8]. Here, it would not be repeated for the details of BFS algorithm but there is a modification when applying the algorithm the Quad-direction linked structure. Because there are four nodes reference (edge) attached to one node, it treat all the 4 references as possible source of ”son nodes” and if any of them are not null and also had not been visited, they will be marked and be added to the tail of a queue for further traversal.

Consider a *clique* or a connected (sub -)graph in a configuration for the terminating grid network constructor model, the BFS can be used to aggregate this kind of clique from any node inside that clique as a set. This set of nodes S can be used as an input for a function applying on a batch of nodes, for instance, rotating all nodes according to a centred node.

4.2.3 Terminating grid network constructor: Check acceptable configuration & Handle rotation

Problem: Detecting legal configuration As mentioned in background section, not all possible configurations are acceptable, or say, belong to $A(C)$ given the geometrical restriction[3]. Hence, this will requires some inspections to ensure that only the acceptable configurations form after a node interacts with each other during simulation process. There are two ways to finish this kind of inspections:

1. List all possible interactions at each discrete step, filter out all illegal ones to get $A(C)$, and then select one configuration from $A(C)$.
2. Select one possible interaction from all possible interactions, reject if the interaction forms a configuration $c_w \notin A(C)$ and repeat the process until a acceptable configuration is found. The entire process would be happened in one discrete step.

The second approach is more feasible since it supposed to cost a impracticable long time to enumerate all possible elements in $A(C)$ at a particular discrete step. Hence, it would be discussed for the cases that should be rejected in an interaction.

Avoid self-connection Recall the definition of transition function of grid network constructor, two nodes and two edges associated them respectively are selected. This not excludes the case that one same identity select twice, which leading to a unsound connection on the selected node itself. This should be avoid through prohibiting the second nodes being selected if it is the same entity with the first one.

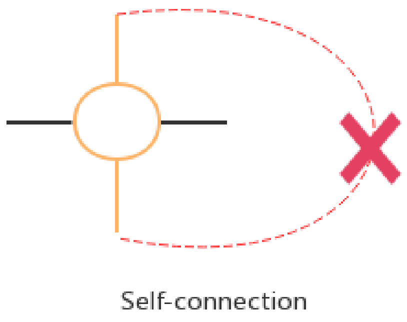


Figure 5: Illegal configuration: Self-connection

Avoid unit-distance violation A connection in terminating grid network constructor has to follow the unit distance restriction.

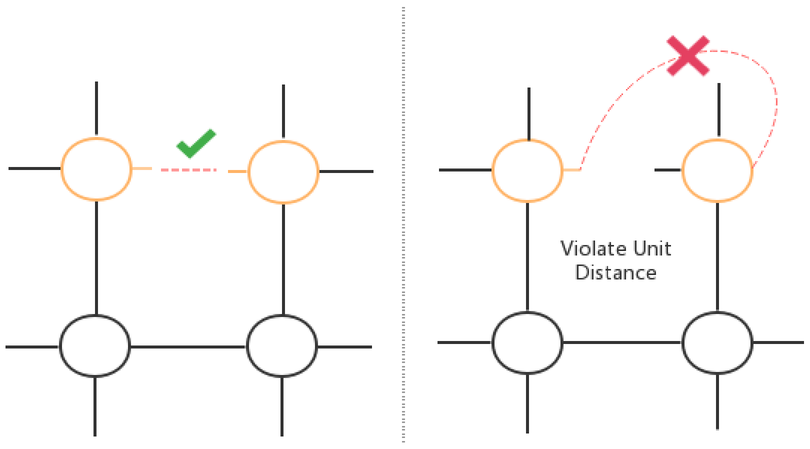


Figure 6: Illegal configuration: Unit distance violation

As illustrated in diagram 6, the left-hand side configuration is acceptable but the right-hand side configuration is not because it violate the unit distance. After the connection in the right-hand side being active, the configuration will not be sub-graph of 2-D grid network with unit distance restrictions.

It can be noticed that unit distance violation always happens when two interacting nodes are belonging to one same clique since a rotation will helps if the two nodes are in different clique. The condition that two nodes are in the same clique simplifies the judgement process. First, it requires to check the whether selected two port are docking port according to equation 4 (so this ensures after activating connection the direction of two ports will conform the grid network restriction). If the two ports selected are a pair of docking port, the initializer node then will mark itself with coordinate $(0, 0)$ and populate the coordinate using BFS algorithm. Finally, it will check the Euclidean distance of the two interacting nodes using propagated coordinates, which is asserted to be 1 otherwise it means that unit-distance restriction will be broken after interaction.

Algorithm 1 Algorithm for detecting unit distance violation

```
1:  procedure DETECT
2:      $((nodeA, portA), (nodeA, portA) \leftarrow scheduler.select()$
3:      $isViolation \leftarrow false$
4:      if $nodeA.belongToGroup \neq nodeB.belongToGroup$ then return $isViolation$
5:      if $isDockingPort(portA, portB)$ then
6:          $nodeA.coordinate \leftarrow (0, 0)$
7:          $connectedSet \leftarrow bfsForPopulatingCoordinate(nodeA)$
8:          $distance \leftarrow euclideanDistance(nodeA \in connectedSet, nodeB \in connectedSet)$
9:          if $distance \neq 1$ then
10:             $isViolation \leftarrow true$
11:     else
12:         $isViolation \leftarrow true$
        return $isViolation$
```

Avoid overlapping Overlapping is another situation that persuades a configuration become an acceptable configuration. This normally happens when two nodes that belongs to different cliques interacting with each other. This is not allowed for some cases when more than one nodes appears in the same position.

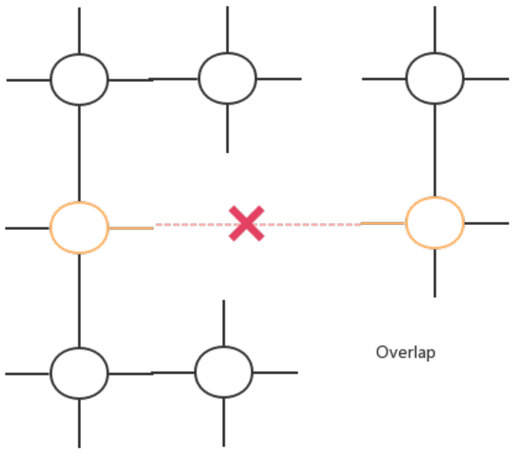


Figure 7: Illegal configuration: Unit distance violation

The figure 7 presents a configuration for highlighted nodes and edges. This kind of interaction should be rejected. To resolve this, it can be assumed that the interaction had already finished and see whether there is at least one pair of nodes owning a same coordinate. If it does, the interaction should be rejected because it causes an overlap otherwise it should accept the connection.

Note that after populating the coordinate, the coordinates of nodes in clique that the receiver belonging to is based on the receiver nodes' perspective. Hence, these coordinates of nodes require to do a centralized rotation with receiver as the center to confirm to the coordinates from initializer's perspective. The rotation degree is given by

$$180 - ||Angle(port_{receiver}) - Angle(port_{initializer}))||$$

which is the "port difference without node rotation involves", because the internal shape of two cliques is no related to their rotation degree).

Some cases might be more complicated. The two cliques may overlap with each other but their axisymmetric one will not. An example could be:

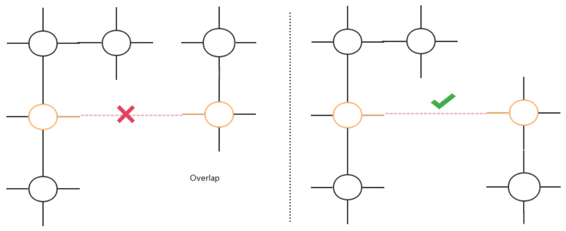


Figure 8: Axisymmetric shape eliminate overlap

The two cliques will be overlapped with each other after interaction in the left-hand of the figure 8 but if one of them flip about the "x-axis" of another clique the overlap will be eliminated. For interaction happens with p_x(RIGHT) and p_{-x}(LEFT) of initializer node, it may need to check the clique that the receiver node belongs to and its x-axisymmetric shape. For interaction happens with p_y(UP) and p_{-y}(DOWN) of initializer node, it may need to check the clique that the receiver node belongs to and its y-axisymmetric shape.

Modification against original design This section is the solution towards the legal configuration checking problem in original design document. The original version proposed such a problem without answers. For now, the solution have been finished.

Algorithm 2 Algorithm for detecting overlapping

```
1: procedure DETECT
2:     $((nodeA, portA), (nodeA, portA) \leftarrow scheduler.select()$
3:     $isOverlap \leftarrow false$
4:     if $nodeA.belongToGroup = nodeB.belongToGroup$ then return $isOverlap$
5:     else
6:         $nodeA.coordinate \leftarrow (0, 0)$
7:         $connectedSetA \leftarrow bfsForPopulatingCoordinate(nodeA)$
8:         $nodeB.coordinate \leftarrow getCoordinateReceiver(nodeA, portA)$
9:         $connectedSetB \leftarrow bfsForPopulatingCoordinate(nodeB)$
10:        $connectedSetB \leftarrow bfsForRotation(connectedSetB, 180 - ||portB.angle - portA.angle||)$
11:        $symmetricSetB \leftarrow symmetric(connectedSetB, portA)$
12:        if $(connectedSetA \cap connectedSetB \neq \emptyset) \wedge (connectedSetA \cap symmetricSetB \neq \emptyset)$ then
13:            if $(connectedSetA \cap connectedSetB \neq \emptyset)$ then
14:                $changeToSymmtricStructure(nodeB, portA)$
15:            $isOverlap \leftarrow true$
    return $isOverlap$
```

4.3 Interaction (Sequence) Design

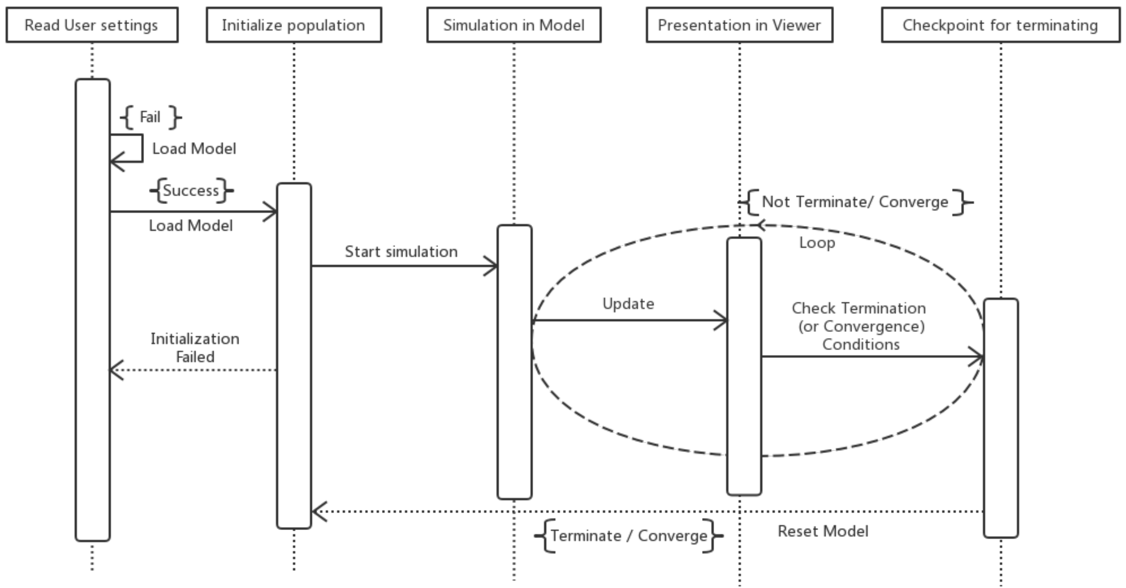


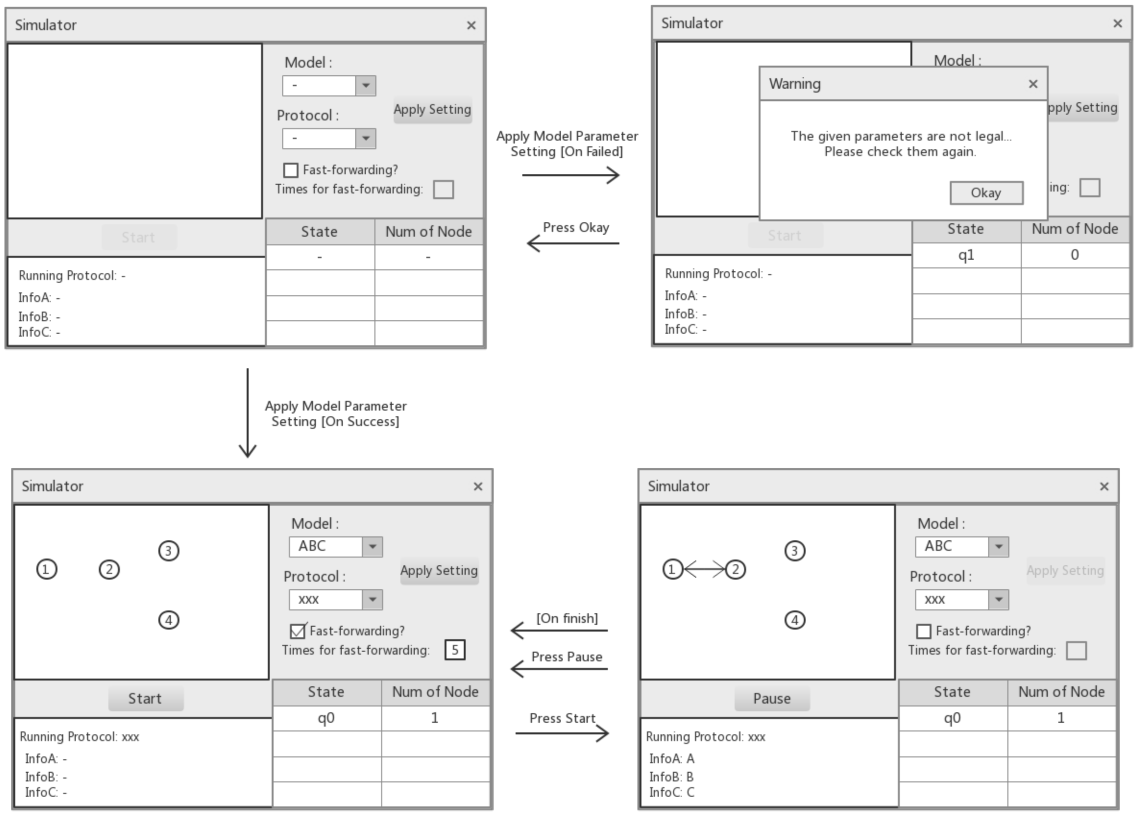
Figure 9: Interaction Design Diagram

The figure 9 illustrates the entire interaction process of the simulator. Initially, the user select one protocol from a set of well-defined protocols and then select a set of parameters required such as how many nodes to be included in the simulation, the initial status distribution for nodes and so forth. Then the user click the ”Apply settings button” to apply the model settings. before applying the model, the program itself would check whether all parameters are well-defined, i.e. locating in the value range they should be. If they are, the software would load the model otherwise it will reject this particular request from user and allow the users to attempt their (possibly) another setting.

After the program successfully loaded a model, the user can press the ”Start simulation” button to start a simulation. A simulation contains multiple rounds. For each round, the model executes interaction simulation under the control of scheduler specified in one of parameters mentioned before and then the model would update the states of elements in viewer to show the what happened in the model itself. Following that, the model would check whether it reaches a configuration that should stop the simulation. A stop can be caused by user’s temporarily interruption (pause) or the fact that the number of consistent accumulation of inefficient interactions overwhelms a pre-configured parameters defined in the model (which indicates a terminating or convergence with a high possibility). If the model does not detect a stop, it will continue the ”interact - update - checking stop” process as a loop. Once it detect a stop, the software will return to state of waiting for starting of a simulation process (while the model would keep its internal states and user can build an another new model under this situation).

Modification against original design The current design removed a step of sequence in original sequence design called ”Initialize scheduler”, which located after ”Initialize population” step. the ”Initialize scheduler” process had been integrated into ”Initialize population” because the scheduler partition does not necessarily separated from the population and can be treated as a partition of a ”population” in concept.

4.4 UI Interface Design



Model : -
Protocol : -
Fast-forwarding?
Times for fast-forwarding:

State	Num of Node
-	-

Running Protocol: -
InfoA: -
InfoB: -
InfoC: -

The given parameters are not legal...
Please check them again.

State	Num of Node
q1	0

Model : ABC
Protocol : xxx
☑ Fast-forwarding?
Times for fast-forwarding: 5

State	Num of Node
q0	1

Running Protocol: xxx
InfoA: -
InfoB: -
InfoC: -

Model : ABC
Protocol : xxx
☐ Fast-forwarding?
Times for fast-forwarding:

State	Num of Node
q0	1

Running Protocol: xxx
InfoA: A
InfoB: B
InfoC: C

Figure 10: Interaction Design Diagram

Modification against original design The UI was redesigned entirely. This is because the original design built on the assumption that the users load their settings for some population from domain specification language (DSL) codes residing in some files. The DSL is infeasible to be finished due to the limited time of implementation. Hence, the design would allow user to set their population through the UI containing a set of well-defined protocols and would allow the users to write their own extensional protocol through few lines of codes. As the functionality changes, the UI removed the functionality entrance for loading files from disk and added some options to allow user setting the parameters of a protocol.

Bibliography

[1] J. Aspnes and E. Ruppert, "An introduction to population protocols," *Bulletin of the European Association for Theoretical Computer Science*, vol. 93, pp. 98–117, Oct. 2007.

[2] O. Michail and P. G. Spirakis, "Simple and efficient local codes for distributed stable network construction," *Distributed Computing*, vol. 29, no. 3, pp. 207–237, 2016. [Online]. Available: https://link.springer.com/article/10.1007/s00446-015-0257-4

[3] O. Michail, "Terminating distributed construction of shapes and patterns in a fair solution of automata," *Distributed Computing*, pp. 1–23, 2017. [Online]. Available: http://link.springer.com/article/10.1007/s00446-017-0309-z

[4] O. Michail, I. Chatzigiannakis, and P. G. Spirakis, *New Models for Population Protocols*, ser. N. A. Lynch (Ed), Synthesis Lectures on Distributed Computing Theory, N. A. Lynch, Ed. Morgan & Claypool, 2011.

[5] D. Amaxilatis, M. Logaras, O. Michail, and P. G. Spirakis, "NETCS: A new simulator of population protocols and network constructors," *CoRR*, vol. abs/1508.06731, 2015. [Online]. Available: http://arxiv.org/abs/1508.06731

[6] S. Widnall, "Lecture l3 - vectors, matrices and coordinate transformations," Fall 2009. [Online]. Available: https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-07-dynamics-fall-2009/lecture-notes/MIT16_07F09_Lec03.pdf Accessed: 2018-04-30.

[7] E. Weisstein, "Affine transformation." [Online]. Available: http://mathworld.wolfram.com/AffineTransformation.html Accessed: 2018-05-01.

[8] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms, Third Edition*, 3rd ed. The MIT Press, 2009.

A Brief Derivation for equation (1)

For convenience, here name $\angle CAB$ as α in figure 3. Then it has

$$|AD| = |\vec{r'}|\cos(\alpha - \theta)$$

$$|B^{'}D| = |\vec{r'}|\sin(\alpha - \theta)$$

By applying angle sum identities,

$$|AD| = |\vec{r'}|(\cos(\alpha)\cos(-\theta) - \sin(\alpha)\sin(-\theta))$$

$$|B^{'}D| = |\vec{r'}|(\sin(\alpha)\cos(-\theta) + \sin(-\theta)\cos(\alpha))$$

Because $|r^{'}| = |r|$, $\sin(\theta) = -\sin(-\theta)$ for $\theta \in [0, \pi/2)$ and $\cos(\theta) = \cos(-\theta)$ for $\theta \in [0, \pi/2)$,

$$|AD| = |\vec{r}|(\cos(\alpha)\cos(\theta) + \sin(\alpha)\sin(\theta))$$

$$|B^{'}D| = |\vec{r}|(\sin(\alpha)\cos(\theta) - \sin(\theta)\cos(\alpha))$$

Then,

$$|AD| = (|\vec{r}|\cos(\alpha))\cos(\theta) + (|\vec{r}|\sin(\alpha))\sin(\theta)$$
$$|B^{'}D| = (|\vec{r}|\sin(\alpha))\cos(\theta) - (|\vec{r}|\cos(\alpha))\sin(\theta)$$

Hence,

$$|AD| = |AC|\cos(\theta) + |BC|\sin(\theta)$$
$$|B^{'}D| = |BC|\cos(\theta) - |AC|\sin(\theta)$$

Finally,

$$|x^{'}| = |x|\cos(\theta) + |y|\sin(\theta)$$
$$|y^{'}| = |x|(-\sin(\theta)) + |y|\cos(\theta)$$

For $\theta \in [0, \pi/2)$, the equations can be written simply as

$$x^{'} = x\cos\theta + y\sin\theta$$

$$y^{'} = x(-\sin\theta) + y\cos\theta$$

which can be easily extended to $\theta \in [0, 2\pi + 2k\pi)$ via discussion under different cases, given that $k \in Integer$.

The two equations can be written in matrix form :

$$\begin{bmatrix} x^{'} \\ y^{'} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

B Brief Derivation for equation (3)

The equation 2 can be transformed to:

$$\begin{aligned}
\begin{bmatrix} x^{'} \\ y^{'} \end{bmatrix} &= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \end{bmatrix} \right) + \begin{bmatrix} c_x \\ c_y \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} c_x \\ c_y \end{bmatrix} + \begin{bmatrix} c_x \\ c_y \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} c_x \cos\theta + c_y \sin\theta \\ -c_x \sin\theta + c_y \cos\theta \end{bmatrix} + \begin{bmatrix} c_x \\ c_y \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c_x(1-\cos\theta) - c_y \sin\theta \\ c_y(1-\cos\theta) + c_x \sin\theta \end{bmatrix}
\end{aligned}$$