Notes on the 1-loop power spectrum in scale-independent modified gravity

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1 General equations

We start discussing perturbation theory in modified gravity. We will introduce counter-terms later. We work in the Jordan frame, where matter is minimally coupled to the gravitational metric. Therefore, the continuity and Euler equations are standard and read

$$\dot{\delta} + a^{-1}\partial_i\left((1+\delta)v^i\right) = 0 \,, \tag{1.1}$$

$$\dot{v}^i + Hv^i + \frac{1}{a}v^j\partial_j v^i + \frac{1}{a}\partial_i\Phi = 0 \,. \tag{1.2}$$

Let us define the conformal Hubble rate as $\mathcal{H} \equiv Ha$ and use a prime to denote the derivative with respect to the scale factor a. In Fourier space, and in terms of the scale factor a, the equations of motion for the dark-matter overdensity δ and the rescaled velocity divergence,

$$\theta \equiv -\partial_i v^i/(f_+\mathcal{H}) \,, \tag{1.3}$$

are

$$\begin{aligned} a\,\delta'(\mathbf{k},a) - f_+\theta(\mathbf{k},a) = \int_{\mathbf{k}_1,\mathbf{k}_2} &(2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) \\ &\times f_+\alpha(\mathbf{k}_1,\mathbf{k}_2)\theta(\mathbf{k}_1,a)\delta(\mathbf{k}_2,a) \end{aligned} \tag{1.4}$$

$$\begin{aligned} a\,\theta'(\mathbf{k},a) - f_+\theta(\mathbf{k},a) + \frac{3}{2}\frac{\mu\Omega_\mathrm{m}}{f_+}\theta(\mathbf{k},a) + \frac{1}{f_+}\frac{k^2}{\mathcal{H}^2}\Phi(\mathbf{k},a) = \int_{\mathbf{k}_1,\mathbf{k}_2} &(2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) \\ &\times f_+\beta(\mathbf{k}_1,\mathbf{k}_2)\theta(\mathbf{k}_1,a)\theta(\mathbf{k}_2,a) \end{aligned} \tag{1.5}$$

where α and β are the standard dark matter interaction vertices,

$$\alpha(\mathbf{q}_1,\mathbf{q}_2) = 1 + \frac{\mathbf{q}_1\cdot\mathbf{q}_2}{q_1^2} \qquad \text{and} \qquad \beta(\mathbf{q}_1,\mathbf{q}_2) = \frac{|\mathbf{q}_1+\mathbf{q}_2|^2\mathbf{q}_1\cdot\mathbf{q}_2}{2q_1^2q_2^2} \,, \tag{1.6}$$

and we have used the notation $\int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} \equiv \int \frac{d^3k_1}{(2\pi)^3} \cdots \int \frac{d^3k_n}{(2\pi)^3}$.

In general relativity, one closes these two equations with the Poisson equation. In modified gravity, one uses the field equations to express the Laplacian of Φ in terms of the density contrast. This expression will contain linear terms in δ, as in the Poisson equation, but in general, in the presence of higher-derivative terms such as in models with Vainshtein screening, there will also be higher-order terms. Here we are interested in computing the spectra up to 1-loop calculation. Thus, we will need all terms up to third order in δ. Adopting the notation of [**?**], we can then write the generalized Poisson equation directly in Fourier space as

$$
\begin{aligned}
-\frac{k^2}{\mathcal{H}^2}\Phi(\mathbf{k},a) = {}& \mu(a)\frac{3\,\Omega_{\rm m}}{2}\,\delta(\mathbf{k},a) \\
&+ \mu_2(a)\left(\frac{3\,\Omega_{\rm m}}{2}\right)^2 \int_{\mathbf{k}_1,\mathbf{k}_2} (2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}_{12})\,\gamma_2(\mathbf{k}_1,\mathbf{k}_2)\delta(\mathbf{k}_1,a)\delta(\mathbf{k}_2,a) \\
&+ \mu_3(a)\left(\frac{3\,\Omega_{\rm m}}{2}\right)^3 \int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} (2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}_{123})\gamma_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta(\mathbf{k}_1,a)\delta(\mathbf{k}_2,a)\delta(\mathbf{k}_3,a) \\
&+ \mu_{22}(a)\left(\frac{3\,\Omega_{\rm m}}{2}\right)^3 \int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}_1,\mathbf{q}_2} (2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}_{12})(2\pi)^3\delta_D(\mathbf{k}_2-\mathbf{q}_{12}) \\
&\qquad \times \gamma_2(\mathbf{k}_1,\mathbf{k}_2)\gamma_2(\mathbf{q}_1,\mathbf{q}_2)\delta(\mathbf{k}_1,a)\delta(\mathbf{q}_1,a)\delta(\mathbf{q}_2,a)\ ,
\end{aligned}
\tag{1.7}
$$

where

$$\Omega_{\rm m} \equiv \frac{\bar{\rho}_{\rm m}}{3M^2H^2}\ , \tag{1.8}$$

where M is the effective Planck mass, which can depend on time. Moreover, we denote $\mathbf{k}_{1\ldots n} = \mathbf{k}_1 + \ldots + \mathbf{k}_n$. The new kernels inside the integrals are given by

$$
\begin{aligned}
\gamma_2(\mathbf{k}_1,\mathbf{k}_2) &= \left[1-\left(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)^2\right] \\
\gamma_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) &= \left[1+2\left(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)\left(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_3\right)\left(\hat{\mathbf{k}}_2\cdot\hat{\mathbf{k}}_3\right)-\left(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_3\right)^2-\left(\hat{\mathbf{k}}_2\cdot\hat{\mathbf{k}}_3\right)^2-\left(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)^2\right]\ .
\end{aligned}
\tag{1.9}
$$

We anticipate that the cubic vertex proportional to μ_3 does not contribute to the power spectrum at one loop because it enters as $\gamma_3(\mathbf{k},\mathbf{q},-\mathbf{q}) = 0$ [**?**], but we include it here for completeness.

2 Models

2.1 nDGP

Let us discuss the nDGP model. In this case, the Friedmann equation is given by

$$-\frac{H}{r_c} = H^2\left(1-\Omega_m(a)\right)\ . \tag{2.1}$$

Then we have

$$\mu(a) = 1 + \frac{1}{3\beta(a)}\ , \qquad \beta(a) = 1 + \frac{H(a)}{H_0}\frac{1}{\sqrt{\Omega_{rc}}}\left(1+\frac{aH'(a)}{3H(a)}\right)\ , \tag{2.2}$$

where $\Omega_{rc} = 1/(4r_c^2H_0^2)$ parametrizes the cross-over scale, while

$$\mu_2(a) = -2H^2r_c^2\left(\frac{1}{3\beta}\right)^3\ , \qquad \mu_{22}(a) = 8H^4r_c^4\left(\frac{1}{3\beta}\right)^5\ . \tag{2.3}$$

As discussed above, we do not need to specify μ_3. General relativity is recovered for instance by sending $Hr_c \to \infty$.

3 Perturbative solution

The perturbation equations are

$$a\delta'_{\mathbf{k}} - f_+\theta_{\mathbf{k}} = (2\pi)^3 \int_{\mathbf{q}_1\mathbf{q}_2} \delta_D(\mathbf{k}-\mathbf{q}_{12}) f_+ \alpha(\mathbf{q}_1,\mathbf{q}_2)\theta_{\mathbf{q}_1}\delta_{\mathbf{q}_2}, \tag{3.1}$$

$$\begin{aligned} a\theta'_{\mathbf{k}} - f_+\theta_{\mathbf{k}} + \frac{3}{2}\mu\frac{\Omega_m}{f_+}(\theta_{\mathbf{k}}-\delta_{\mathbf{k}}) = (2\pi)^3 \int_{\mathbf{q}_1\mathbf{q}_2} & \delta_D(\mathbf{k}-\mathbf{q}_{12})\times \\ & \left[f_+\beta(\mathbf{q}_1,\mathbf{q}_2)\theta_{\mathbf{q}_1}\theta_{\mathbf{q}_2} + \frac{\mu_2}{f_+}\left(\frac{3\Omega_{\mathrm{m}}}{2}\right)^2 \gamma_2(\mathbf{q}_1,\mathbf{q}_2)\delta_{\mathbf{q}_1}\delta_{\mathbf{q}_2} \right] \\ & + (2\pi)^3 \int_{\mathbf{q}_1\mathbf{q}_2\mathbf{k}_1\mathbf{k}_2} \delta_D(\mathbf{k}_2-\mathbf{q}_{12})\delta_D(\mathbf{k}-\mathbf{k}_{12})\times \\ & \frac{\mu_{22}}{f_+}\left(\frac{3\Omega_{\mathrm{m}}}{2}\right)^3 \gamma_2(\mathbf{q}_1,\mathbf{q}_2)\gamma_2(\mathbf{k}_1,\mathbf{k}_2)\delta_{\mathbf{k}_1}\delta_{\mathbf{q}_1}\delta_{\mathbf{q}_2}, \end{aligned} \tag{3.2}$$

$$\mathcal{G}_1^\lambda(a) = \int_0^1 \left[G_1^\lambda(a,\tilde{a}) f_+(\tilde{a}) + G_2^\lambda(a,\tilde{a}) \frac{\mu_2(\tilde{a})}{f_+(\tilde{a})}\left(\frac{3\Omega_{\mathrm{m}}(\tilde{a})}{2}\right)^2 \right] \frac{D_+^2(\tilde{a})}{D_+^2(a)} d\tilde{a} \tag{3.3}$$

$$\mathcal{G}_2^\lambda(a) = \int_0^1 G_2^\lambda(a,\tilde{a}) \left[f_+(\tilde{a}) - \frac{\mu_2(\tilde{a})}{f_+(\tilde{a})}\left(\frac{3\Omega_{\mathrm{m}}(\tilde{a})}{2}\right)^2 \right] \frac{D_+^2(\tilde{a})}{D_+^2(a)} d\tilde{a} \tag{3.4}$$

$$\mathcal{U}_1^\lambda(a) = \int_0^1 \left\{ G_1^\lambda(a,\tilde{a}) f_+(\tilde{a}) \mathcal{G}_1^\delta(\tilde{a}) + \frac{G_2^\lambda(a,\tilde{a})}{f_+(\tilde{a})} \left(\frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right)^2 \left[\mu_2(\tilde{a}) \mathcal{G}_1^\delta(\tilde{a}) + \frac{\mu_{22}(\tilde{a})}{2} \frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right] \right\} \frac{D_+^3(\tilde{a})}{D_+^3(a)} d\tilde{a} \tag{3.5}$$

$$\mathcal{U}_2^\lambda(a) = \int_0^1 \left\{ G_1^\lambda(a,\tilde{a}) f_+(\tilde{a}) \mathcal{G}_2^\delta(\tilde{a}) + \frac{G_2^\lambda(a,\tilde{a})}{f_+(\tilde{a})} \left(\frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right)^2 \left[\mu_2(\tilde{a}) \mathcal{G}_2^\delta(\tilde{a}) - \frac{\mu_{22}(\tilde{a})}{2} \frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right] \right\} \frac{D_+^3(\tilde{a})}{D_+^3(a)} d\tilde{a} \tag{3.6}$$

$$\mathcal{V}_{11}^\lambda(a) = \int_0^1 \left\{ G_1^\lambda(a,\tilde{a}) f_+(\tilde{a}) \mathcal{G}_1^\theta(\tilde{a}) + \frac{G_2^\lambda(a,\tilde{a})}{f_+(\tilde{a})} \left(\frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right)^2 \left[\mu_2(\tilde{a}) \mathcal{G}_1^\delta(\tilde{a}) + \frac{\mu_{22}(\tilde{a})}{2} \frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right] \right\} \frac{D_+^3(\tilde{a})}{D_+^3(a)} d\tilde{a} \tag{3.7}$$

$$\mathcal{V}_{21}^\lambda(a) = \int_0^1 \left\{ G_1^\lambda(a,\tilde{a}) f_+(\tilde{a}) \mathcal{G}_2^\theta(\tilde{a}) + \frac{G_2^\lambda(a,\tilde{a})}{f_+(\tilde{a})} \left(\frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right)^2 \left[\mu_2(\tilde{a}) \mathcal{G}_2^\delta(\tilde{a}) - \frac{\mu_{22}(\tilde{a})}{2} \frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right] \right\} \frac{D_+^3(\tilde{a})}{D_+^3(a)} d\tilde{a} \tag{3.8}$$

$$\mathcal{V}_{12}^\lambda(a) = \int_0^1 G_2^\lambda(a,\tilde{a}) \left\{ f_+(\tilde{a}) \mathcal{G}_1^\theta(\tilde{a}) - \frac{1}{f_+(\tilde{a})} \left(\frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right)^2 \left[\mu_2(\tilde{a}) \mathcal{G}_1^\delta(\tilde{a}) + \frac{\mu_{22}(\tilde{a})}{2} \frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right] \right\} \frac{D_+^3(\tilde{a})}{D_+^3(a)} d\tilde{a} \tag{3.9}$$

$$\mathcal{V}_{22}^\lambda(a) = \int_0^1 G_2^\lambda(a,\tilde{a}) \left\{ f_+(\tilde{a}) \mathcal{G}_2^\theta(\tilde{a}) - \frac{1}{f_+(\tilde{a})} \left(\frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right)^2 \left[\mu_2(\tilde{a}) \mathcal{G}_2^\delta(\tilde{a}) - \frac{\mu_{22}(\tilde{a})}{2} \frac{3\Omega_\mathrm{m}(\tilde{a})}{2}\right] \right\} \frac{D_+^3(\tilde{a})}{D_+^3(a)} d\tilde{a} \tag{3.10}$$

References