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Let $P^{(0)}$ be an upper triangular parent matrix, with $P_{ij}^{(0)} = 1$ if $\pi(i) = j$, i.e. j is the parent of vertex i. This matrix will have a single nonzero entry per row and represents a forest. Given a set of undirected edges E, define upper triangular adjacency matrix A, such that $A_{ij} = 1$ if $(i, j) \in E$ with $i < j$. We seek to find a parent matrix which corresponds to a forest in which each tree is a connected component in $G = (V, E)$. Let $A^{(0)} = A$. Initialize, $\pi(i) = \max(i, \max_j A_{ij} j)$ and $P^{(0)}$ accordingly. Compute

$$B_{ij}^{(k)} = \max_l P_{li}^{(m)} A_{lj}^{(k)}$$

$$A_{ij}^{(k)} = \begin{cases} 0 & : i > j \\ \max(B_{ij}^{(k)}, B_{ji}^{(k)}) & : i \leq j \end{cases}.$$

So that if $A_{ij}^{(k)} = 1$, then $B_{\pi(i),j}^{(k)} = 1$, and $A_{\pi(i),j}^{(k)} = 1$ if $\pi(i) < j$ or $A_{j,\pi(i)}^{(k)} = 1$ if $\pi(i) \geq j$. Execute until convergence for some k. Then, set $\pi(i) = \max(i, \max_j A_{ij}^{(k)} j)$ and $P^{(m+1)}$ accordingly.