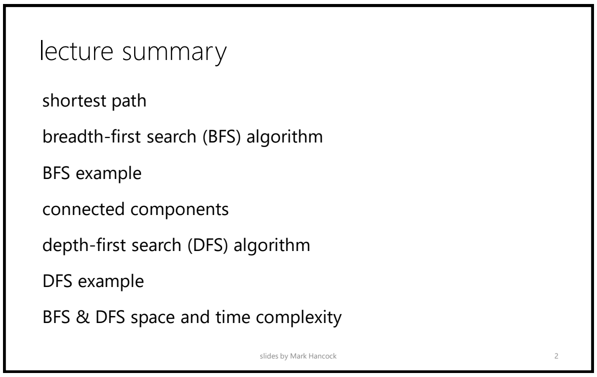
Breadth-First Search & Depth-First Search

MSCI 240: Algorithms & Data Structures



lecture summary

shortest path

breadth-first search (BFS) algorithm

BFS example

connected components

depth-first search (DFS) algorithm

DFS example

BFS & DFS space and time complexity

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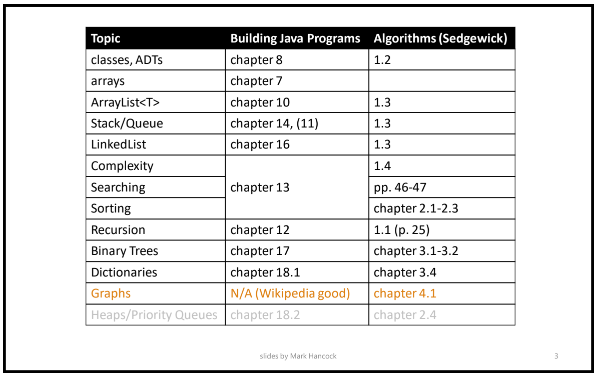

Topic	Building Java Programs	Algorithms (Sedgewick)
classes, ADTs	chapter 8	1.2
arrays	chapter 7	
ArrayList<T>	chapter 10	1.3
Stack/Queue	chapter 14, (11)	1.3
LinkedList	chapter 16	1.3
Complexity	chapter 13	1.4
Searching		pp. 46-47
Sorting		chapter 2.1-2.3
Recursion	chapter 12	1.1 (p. 25)
Binary Trees	chapter 17	chapter 3.1-3.2
Dictionaries	chapter 18.1	chapter 3.4
Graphs	N/A (Wikipedia good)	chapter 4.1
Heaps/Priority Queues	chapter 18.2	chapter 2.4

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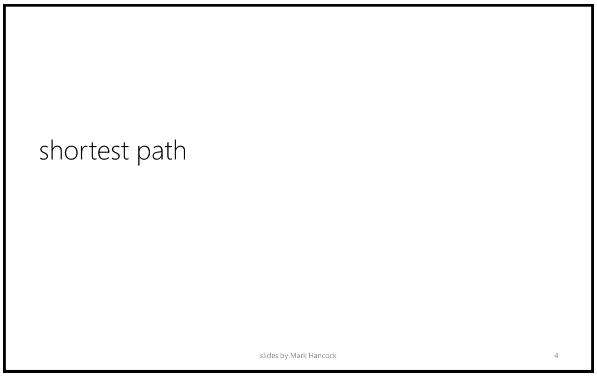

shortest path

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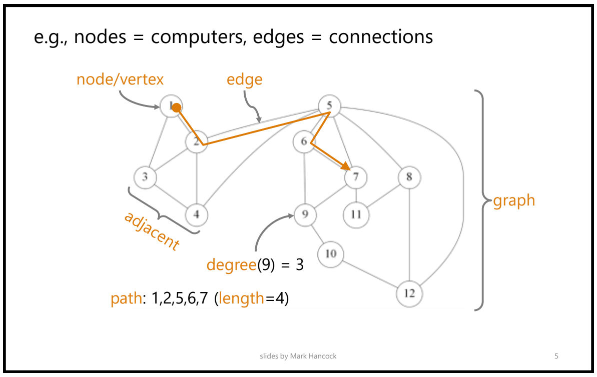

e.g., nodes = computers, edges = connections

node/vertex

edge

graph

adjacent

degree(9) = 3

path: 1,2,5,6,7 (length=4)

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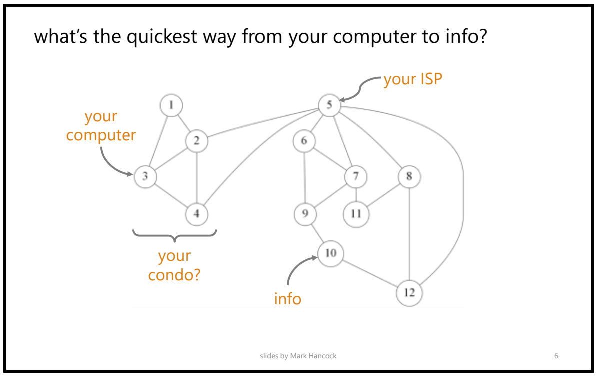

what's the quickest way from your computer to info?

your computer

your ISP

your condo?

info

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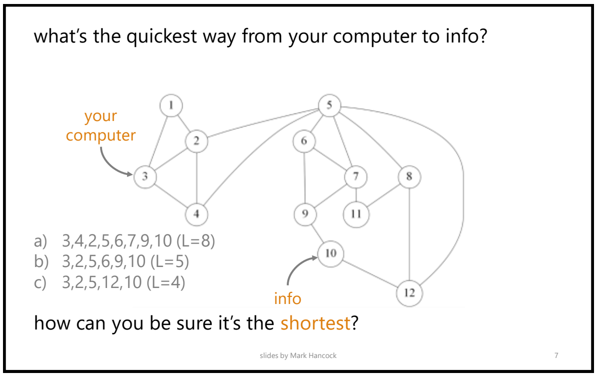
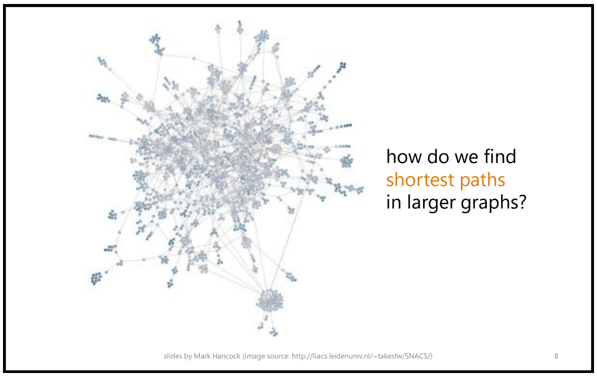


what's the quickest way from your computer to info?

a) 3,4,2,5,6,7,9,10 (L=8)
b) 3,2,5,6,9,10 (L=5)
c) 3,2,5,12,10 (L=4)

how can you be sure it's the shortest?



how do we find
shortest paths
in larger graphs?

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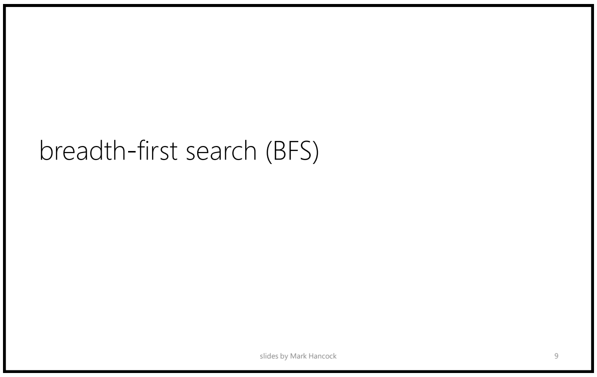
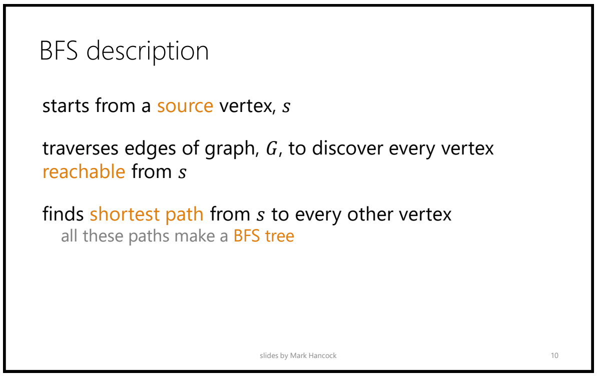


breadth-first search (BFS)



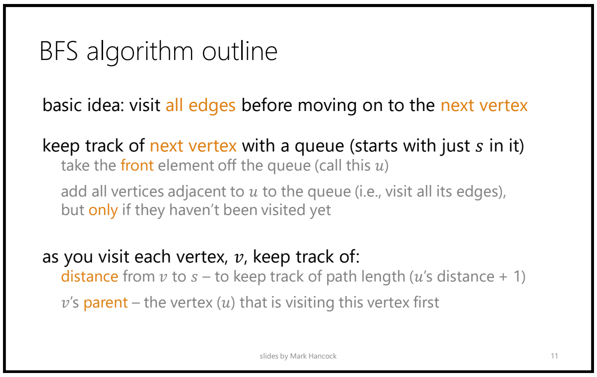
BFS description

starts from a source vertex, s

traverses edges of graph, G, to discover every vertex reachable from s

finds shortest path from s to every other vertex

- all these paths make a BFS tree



BFS algorithm outline

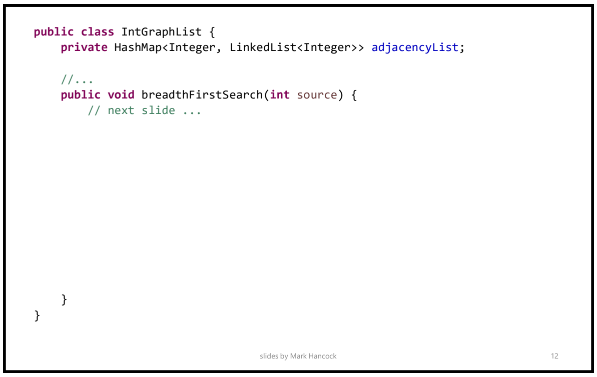
basic idea: visit all edges before moving on to the next vertex

keep track of next vertex with a queue (starts with just s in it)

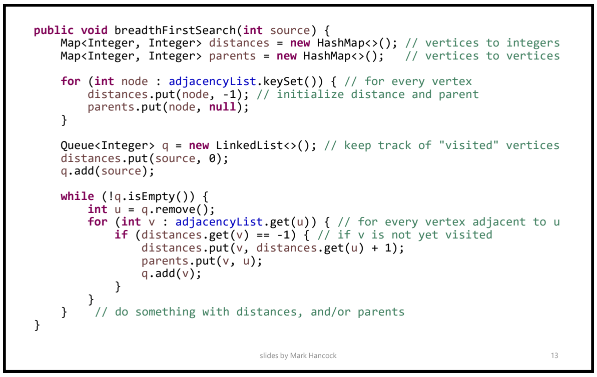
- take the front element off the queue (call this u)
- add all vertices adjacent to u to the queue (i.e., visit all its edges), but only if they haven't been visited yet

as you visit each vertex, v, keep track of:

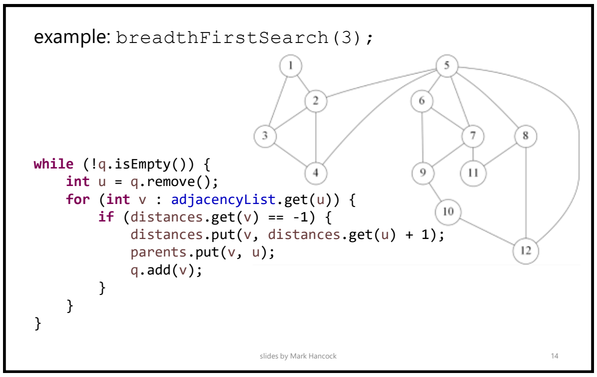
- distance from v to s – to keep track of path length (u's distance + 1)
- v's parent – the vertex (u) that is visiting this vertex first



```
public class IntGraphList {
    private HashMap<Integer, LinkedList<Integer>> adjacencyList;

    //...
    public void breadthFirstSearch(int source) {
        // next slide ...








    }
}
```

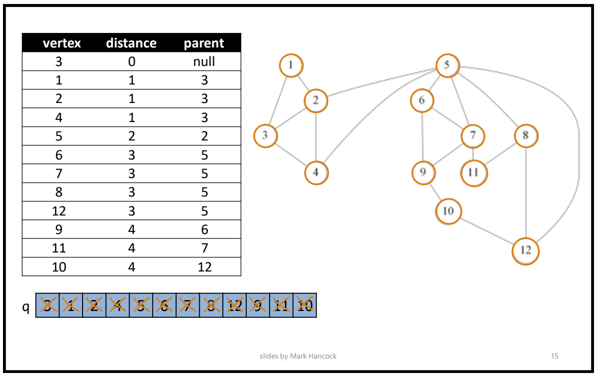


```
public void breadthFirstSearch(int source) {
    Map<Integer, Integer> distances = new HashMap<>(); // vertices to integers
    Map<Integer, Integer> parents = new HashMap<>();   // vertices to vertices

    for (int node : adjacencyList.keySet()) { // for every vertex
        distances.put(node, -1); // initialize distance and parent
        parents.put(node, null);
    }

    Queue<Integer> q = new LinkedList<>(); // keep track of "visited" vertices
    distances.put(source, 0);
    q.add(source);

    while (!q.isEmpty()) {
        int u = q.remove();
        for (int v : adjacencyList.get(u)) { // for every vertex adjacent to u
            if (distances.get(v) == -1) { // if v is not yet visited
                distances.put(v, distances.get(u) + 1);
                parents.put(v, u);
                q.add(v);
            }
        }
    }    // do something with distances, and/or parents
}
```



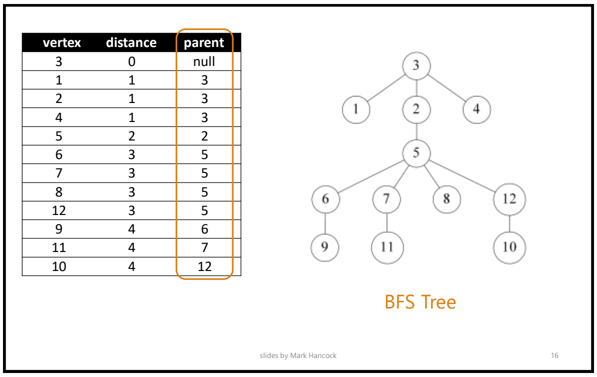
example: `breadthFirstSearch(3);`

```
while (!q.isEmpty()) {
    int u = q.remove();
    for (int v : adjacencyList.get(u)) {
        if (distances.get(v) == -1) {
            distances.put(v, distances.get(u) + 1);
            parents.put(v, u);
            q.add(v);
        }
    }
}
```



vertex	distance	parent
3	0	null
1	1	3
2	1	3
4	1	3
5	2	2
6	3	5
7	3	5
8	3	5
12	3	5
9	4	6
11	4	7
10	4	12

q ~~3~~ ~~1~~ ~~2~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~12~~ ~~9~~ ~~11~~ ~~10~~



vertex	distance	parent
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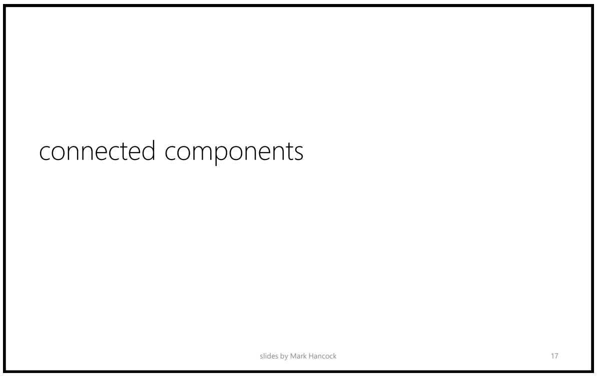
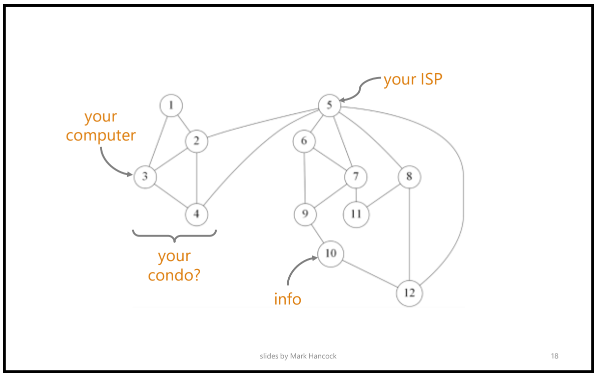
8	3	5
12	3	5
9	4	6
11	4	7
10	4	12

BFS Tree



connected components

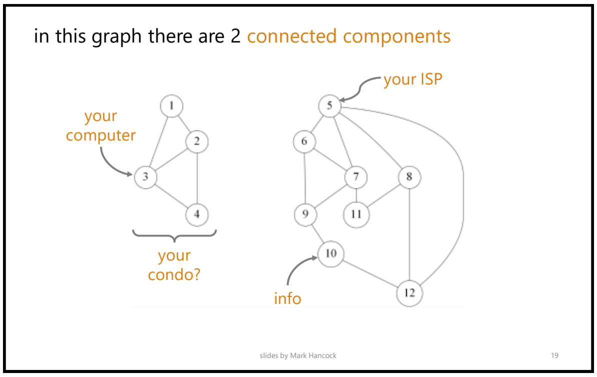


your computer

your condo?

your ISP

info



in this graph there are 2 connected components

your ISP

your computer

your condo?

info

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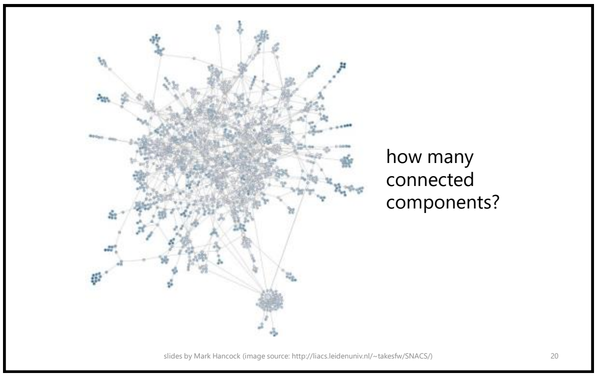

how many connected components?

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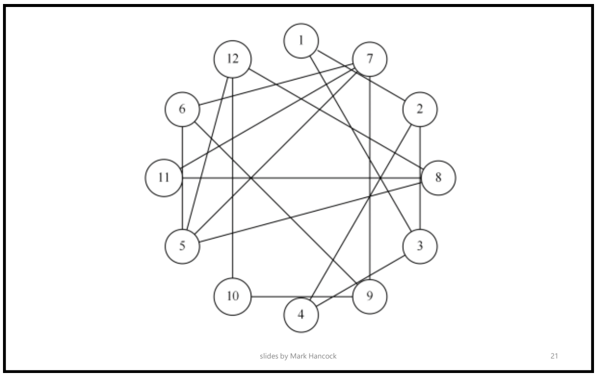

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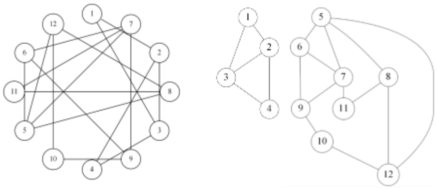
same graph!

∴ 2 connected components each



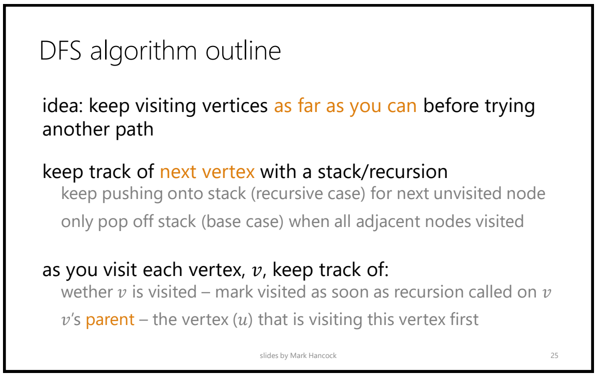
depth-first search (DFS)

DFS description

does not require a source vertex
but can have a starting point

traverses edges of graph, G, depth-first to discover connected components
every vertex reachable from every other vertex

all these paths make a DFS tree



DFS algorithm outline

idea: keep visiting vertices as far as you can before trying another path

keep track of next vertex with a stack/recursion

- keep pushing onto stack (recursive case) for next unvisited node
- only pop off stack (base case) when all adjacent nodes visited

as you visit each vertex, v, keep track of:

- wether v is visited – mark visited as soon as recursion called on v
- v's parent – the vertex (u) that is visiting this vertex first

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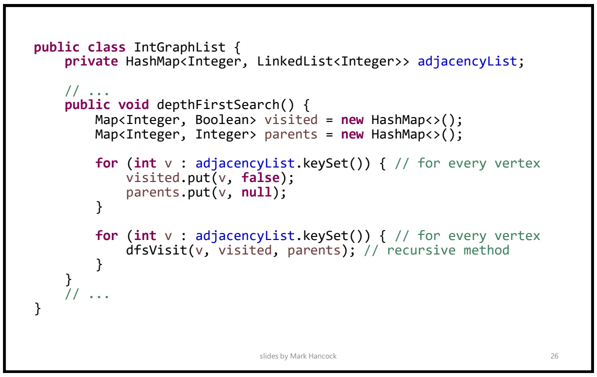

```
public class IntGraphList {
    private HashMap<Integer, LinkedList<Integer>> adjacencyList;

    // ...
    public void depthFirstSearch() {
        Map<Integer, Boolean> visited = new HashMap<>();
        Map<Integer, Integer> parents = new HashMap<>();

        for (int v : adjacencyList.keySet()) { // for every vertex
            visited.put(v, false);
            parents.put(v, null);
        }

        for (int v : adjacencyList.keySet()) { // for every vertex
            dfsVisit(v, visited, parents); // recursive method
        }
    }
    // ...
}
```

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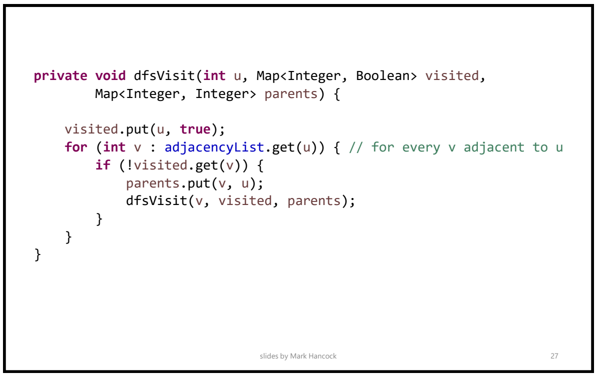

```
private void dfsVisit(int u, Map<Integer, Boolean> visited,
        Map<Integer, Integer> parents) {

    visited.put(u, true);
    for (int v : adjacencyList.get(u)) { // for every v adjacent to u
        if (!visited.get(v)) {
            parents.put(v, u);
            dfsVisit(v, visited, parents);
        }
    }
}
```

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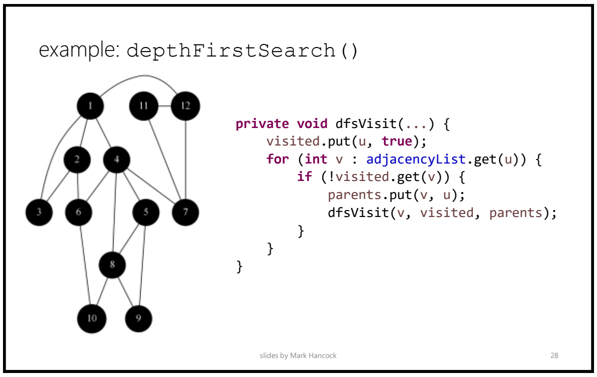
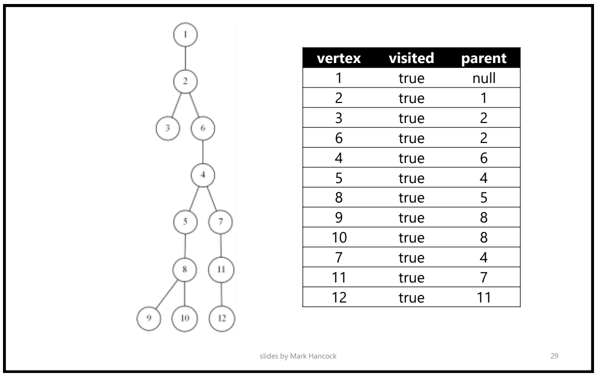


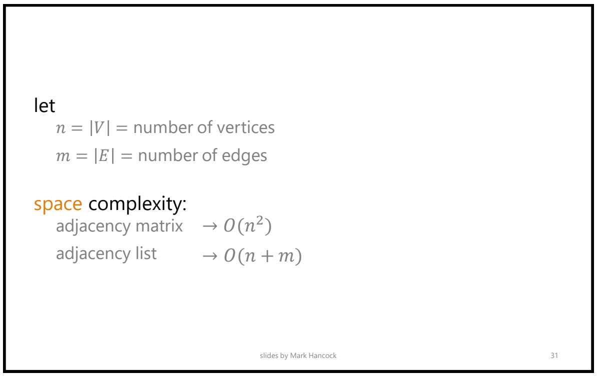
example: `depthFirstSearch()`

```
private void dfsVisit(...) {
    visited.put(u, true);
    for (int v : adjacencyList.get(u)) {
        if (!visited.get(v)) {
            parents.put(v, u);
            dfsVisit(v, visited, parents);
        }
    }
}
```



vertex	visited	parent
1	true	null
2	true	1
3	true	2
6	true	2
4	true	6
5	true	4
8	true	5
9	true	8
10	true	8
7	true	4
11	true	7
12	true	11

BFS & DFS complexity



let

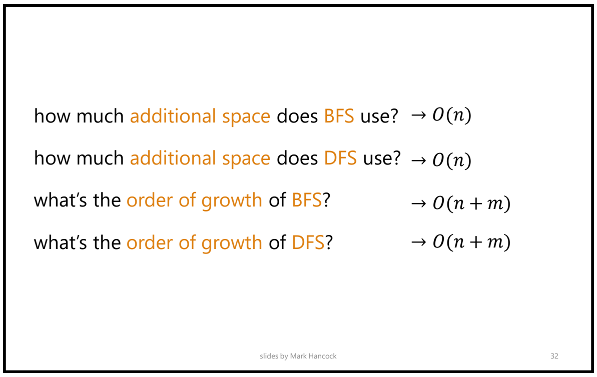
$n = |V| =$ number of vertices

$m = |E| =$ number of edges

space complexity:

adjacency matrix → $O(n^2)$

adjacency list → $O(n + m)$

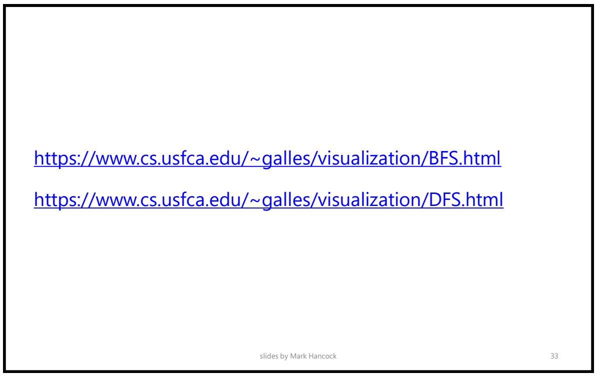


how much additional space does BFS use? → $O(n)$

how much additional space does DFS use? → $O(n)$

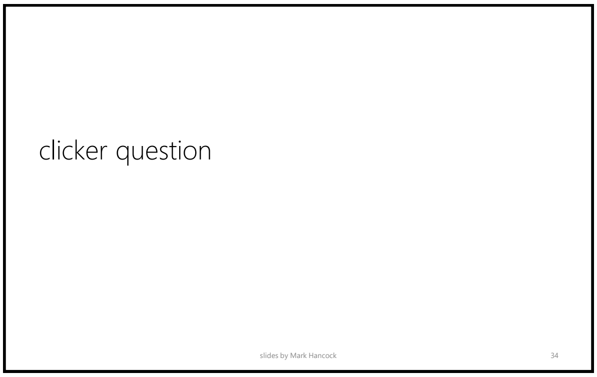
what's the order of growth of BFS? → $O(n + m)$

what's the order of growth of DFS? → $O(n + m)$



https://www.cs.usfca.edu/~galles/visualization/BFS.html

https://www.cs.usfca.edu/~galles/visualization/DFS.html



clicker question

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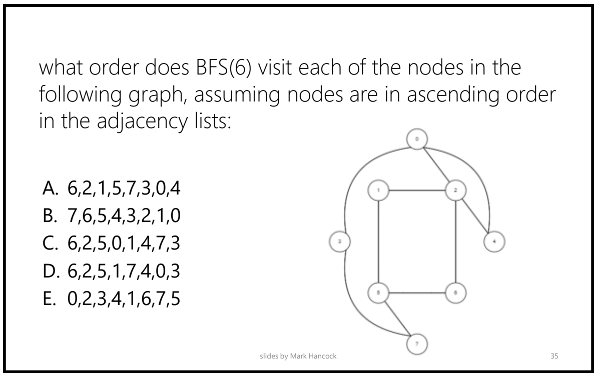

what order does BFS(6) visit each of the nodes in the following graph, assuming nodes are in ascending order in the adjacency lists:

A. 6,2,1,5,7,3,0,4
B. 7,6,5,4,3,2,1,0
C. 6,2,5,0,1,4,7,3
D. 6,2,5,1,7,4,0,3
E. 0,2,3,4,1,6,7,5

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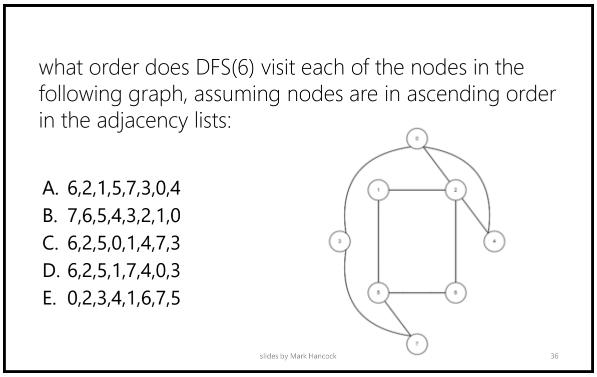

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E. 0,2,3,4,1,6,7,5

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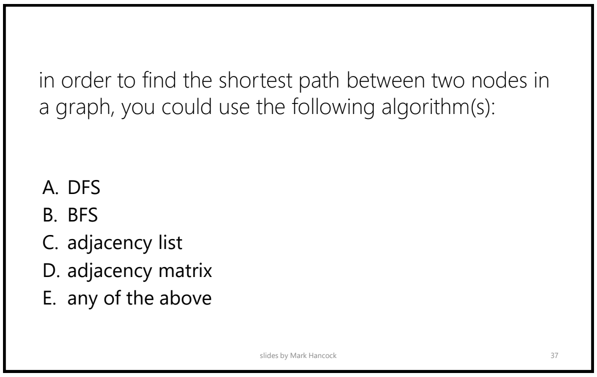
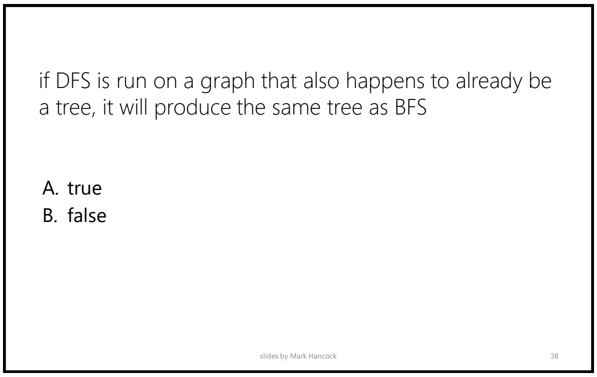


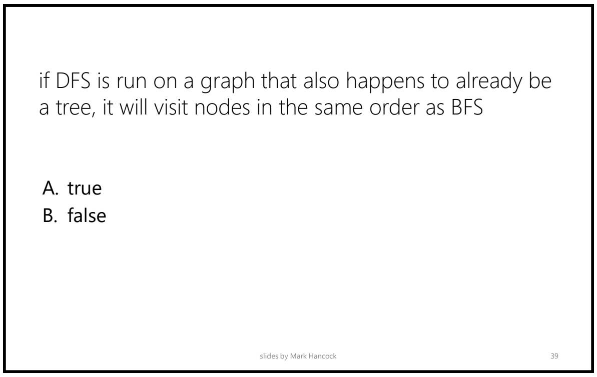
in order to find the shortest path between two nodes in a graph, you could use the following algorithm(s):

A. DFS
B. BFS
C. adjacency list
D. adjacency matrix
E. any of the above



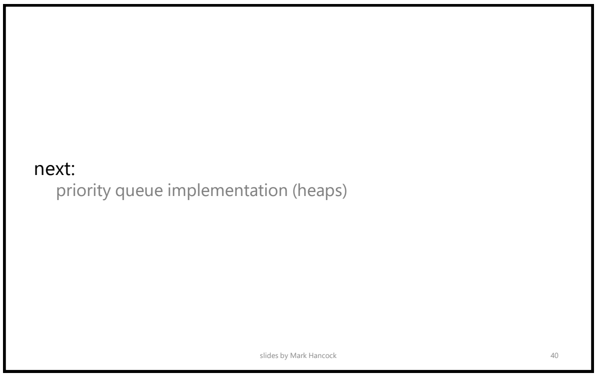
if DFS is run on a graph that also happens to already be a tree, it will produce the same tree as BFS

A. true
B. false



if DFS is run on a graph that also happens to already be a tree, it will visit nodes in the same order as BFS

A. true
B. false



next:

- priority queue implementation (heaps)

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