Advanced Analysis of Algorithms - Homework III

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1 Problems

1. ```
   int countInv(int A[], int start, int end) {
     if(start == end || start + 1 == end) {
       return 0;
     }
     int a, b;
     a = countInv(A, start, (start+end)/2);
     b = countInv(A, start+(start+end)/2, end);
     int i = start, j = start+(start+end)/2;
     while(i < (start+end)/2 && j < end) {
       if(
   }
   ```

2. a) The optimal substructure property does not hold when there are negative weight cycles.

   b) ```
      int floyd(int n, const float W[][], float D[][]) {
        int i, j, k;
        D = W; //do a deep copy, not shallow
        for(k=0; k<n; k++) {
          for(i=0; i<n; i++) {
            for(j=0; j<n; j++) {
              D[i][j] = min(D[i][j], D[i][k]+D[k][j]);
            }
          }
        }
        for(i=0; i<n; i++) {
          if(D[i][i] < 0) {
            return FALSE; //if path from i to i is negative, there has to be a negative cycl
          }
        }
        return TRUE;
      }
      ```

3. a)

   $$X = \begin{pmatrix} 9 & 3 \\ 2 & -1 \end{pmatrix} Y = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

For Strassen's matrix multiplication algorithm, we have to calculate matrices m_1 through m_7.

$$
\begin{aligned}
\mathrm{m}_1 &= (a_{11} + a_{22})(b_{11} + b_{22}) \\
\mathrm{m}_2 &= (a_{21} + a_{22})b_{11} \\
\mathrm{m}_3 &= a_{11}(b_{12} - b_{22}) \\
\mathrm{m}_4 &= a_{22}(b_{21} - b_{11} \\
\mathrm{m}_5 &= (a_{11} + a_{12})b_{22} \\
\mathrm{m}_6 &= (a_{21} - a_{11})(b_{11} + b_{12}) \\
\mathrm{m}_7 &= (a_{12} - a22)(b_{21} + b_{22})
\end{aligned}
$$

$X \times Y = C$ where

$$C = \begin{pmatrix} \mathrm{m}_1 + m_4 - m_5 + m_7 & \mathrm{m}_3 + m_5 \\ \mathrm{m}_2 + m_4 & \mathrm{m}_1 + m_3 - m_2 + m_6 \end{pmatrix}$$

So

$$
\begin{aligned}
\mathrm{m}_1 &= (9 + -1)(1 + -1) = 0 \\
\mathrm{m}_2 &= (2 + -1)(1) = 1 \\
\mathrm{m}_3 &= 9(2 - -1) = 27 \\
\mathrm{m}_4 &= -1(2 - 1) = -1 \\
\mathrm{m}_5 &= (9 + 3)(-1) = -12 \\
\mathrm{m}_6 &= (2 - 9)(1 + 2) = -21 \\
\mathrm{m}_7 &= (3 - -1)(2 + -1) = 4
\end{aligned}
$$

and

$$\mathrm{C} = \begin{pmatrix} 0 + -1 - (-12) + 4 & 27 + (-12) \\ 1 + -1 & 0 + 27 - 1 + (-21) \end{pmatrix} = \begin{pmatrix} 15 & 15 \\ 0 & 5 \end{pmatrix}$$

b)

4.

5. a)

   b)