Introduction to Mobile Robotics

Wheeled Locomotion

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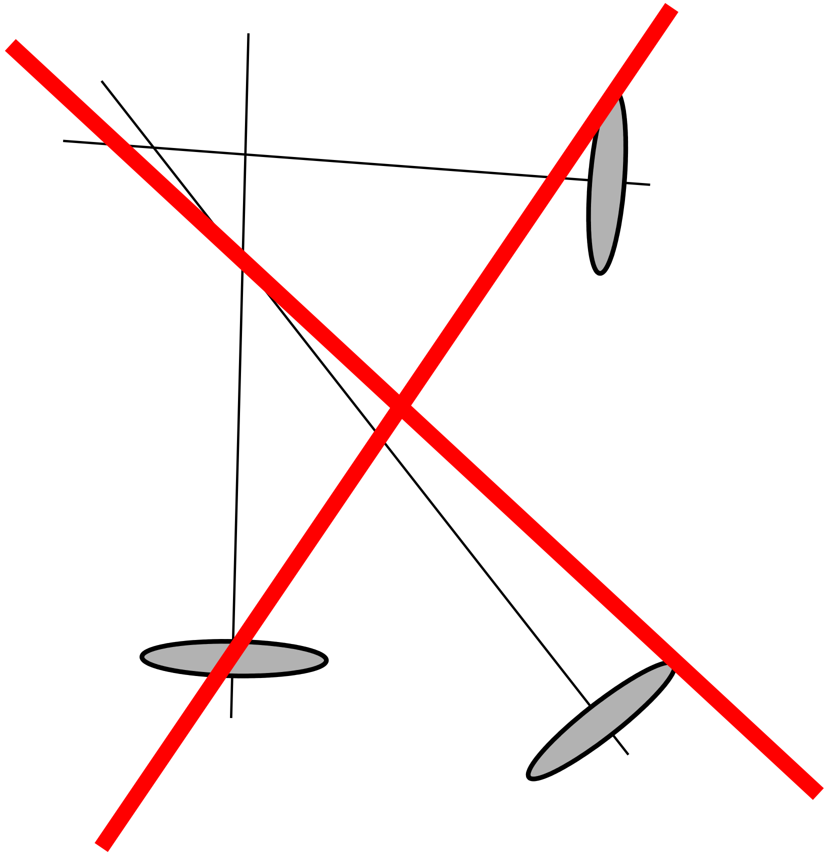
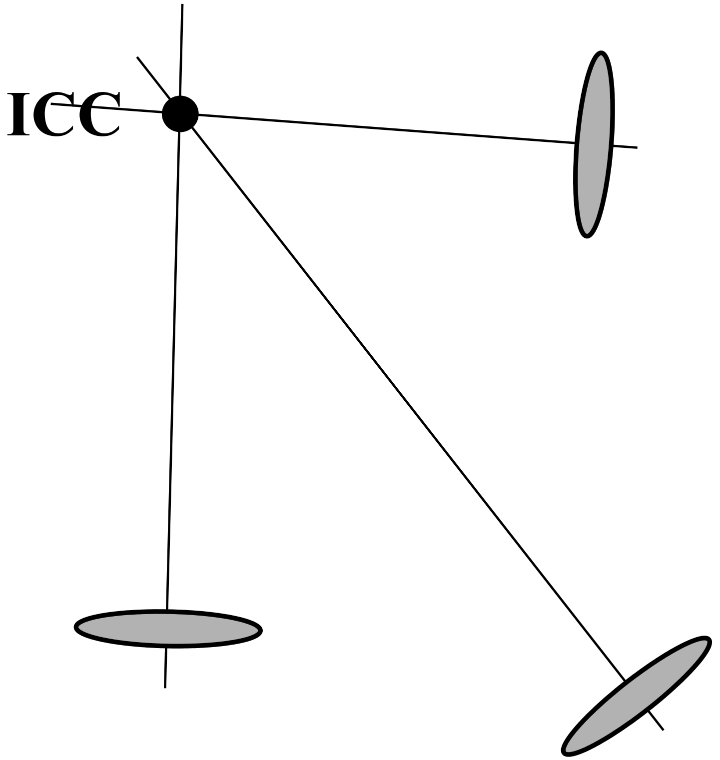
UTN

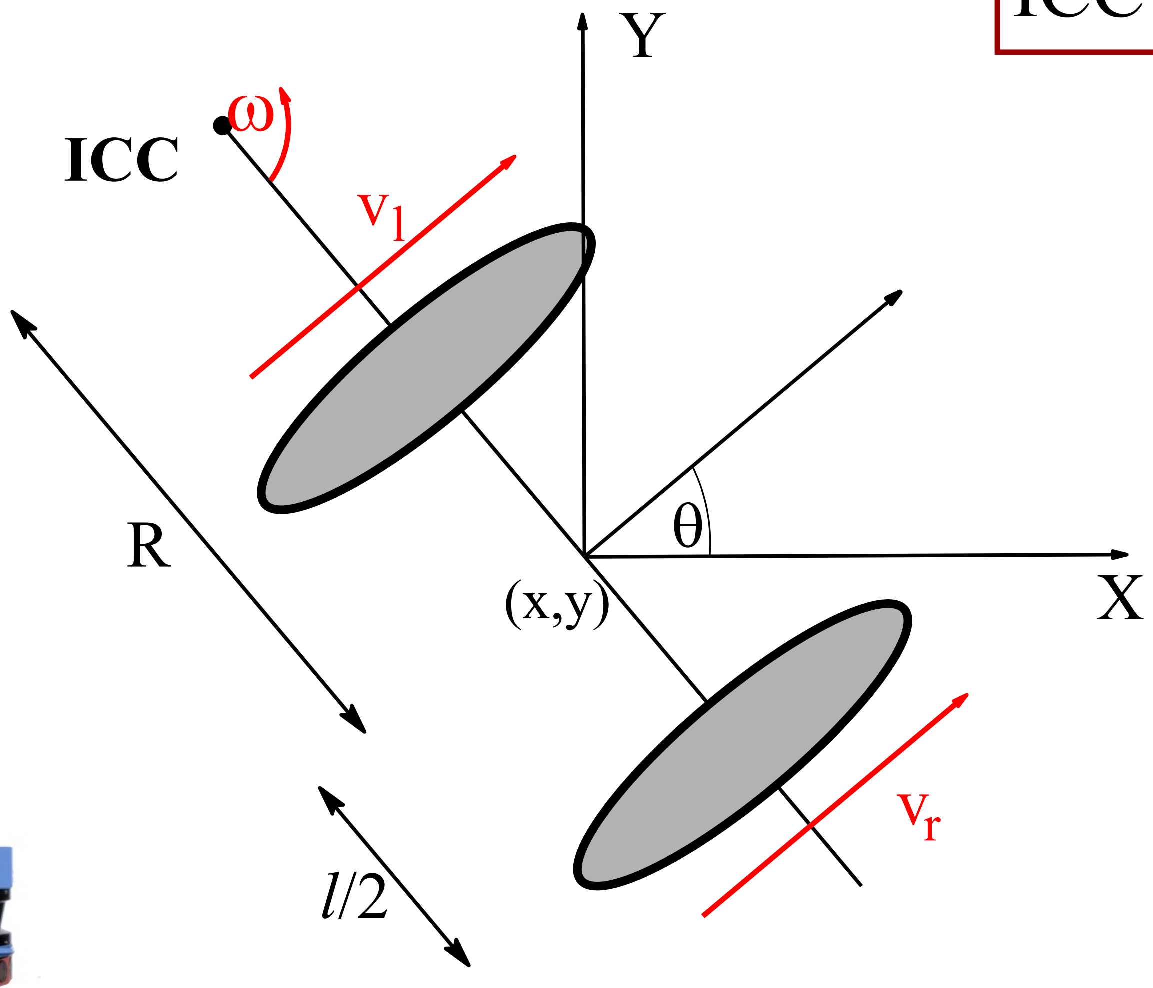
Instantaneous Center of Curvature





- For rolling motion to occur, each wheel has to move along its y-axis

Differential Drive



$$ICC = [x - R\sin\theta,\ y + R\cos\theta]$$

$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

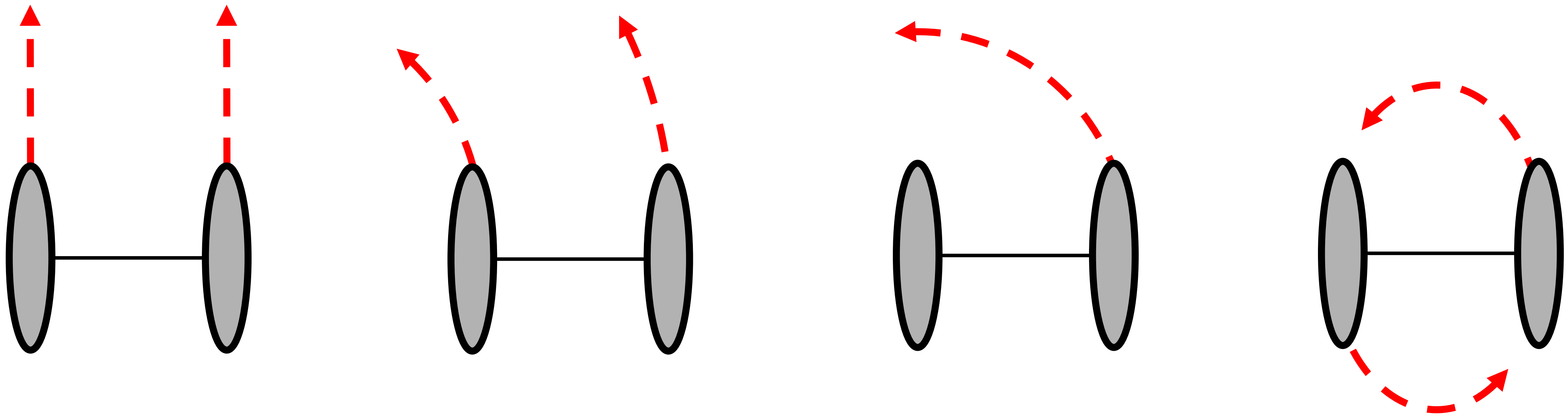
$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

$$v = \frac{v_r + v_l}{2}$$

Differential Drive Motion Patterns

$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}, \qquad \omega = \frac{v_r - v_l}{l}$$



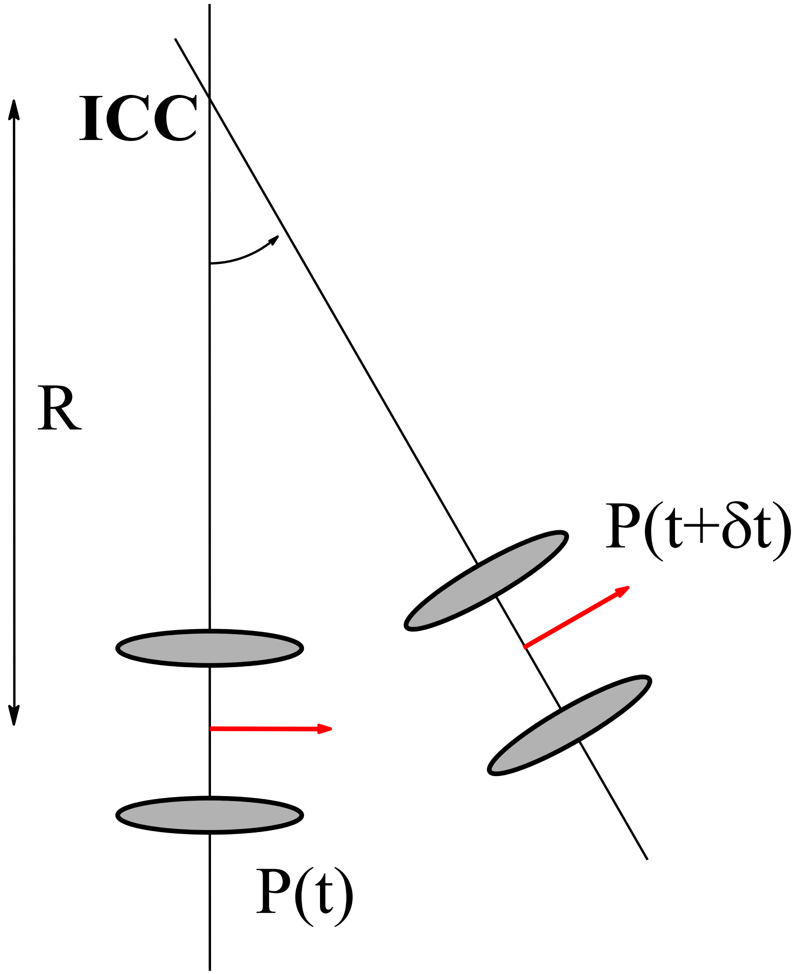
$v_l = v_r$

$v_l < v_r$
$v_l > 0$

$v_l = 0$
$v_r > 0$

$v_l = -v_r$

Differential Drive: Forward Kinematics



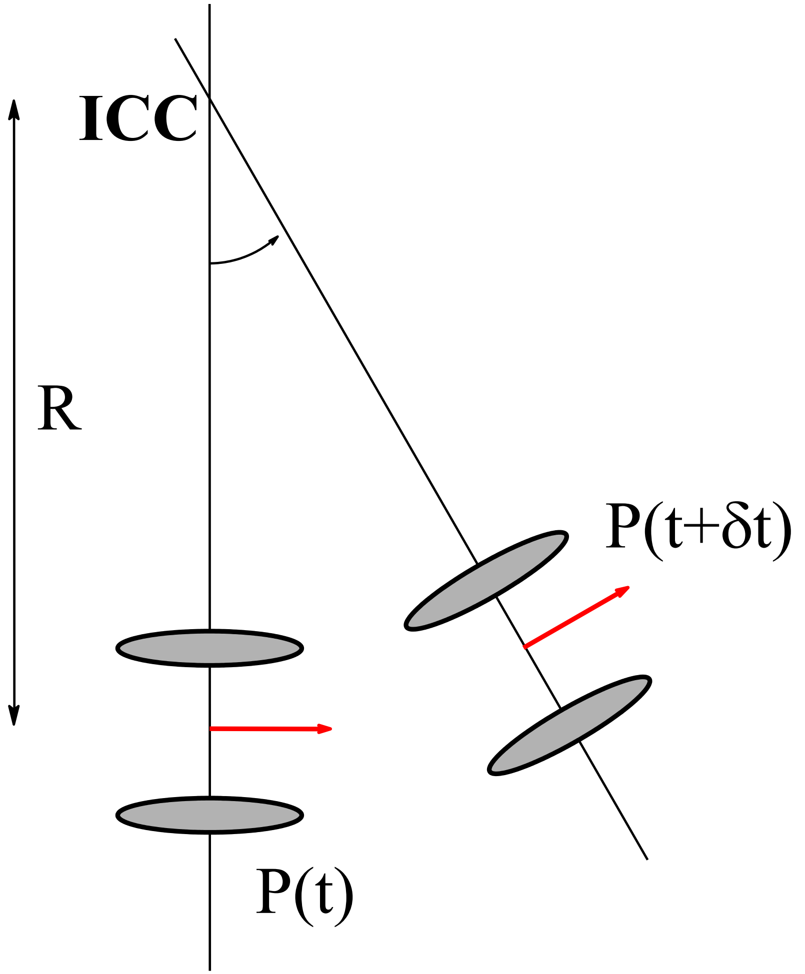
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \mathrm{ICC}_x \\ y - \mathrm{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \mathrm{ICC}_x \\ \mathrm{ICC}_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t v(t') \ \cos[\theta(t')]dt'$$

$$y(t) = \int_0^t v(t') \ \sin[\theta(t')]dt'$$

$$\theta(t) = \int_0^t \omega(t')\, dt'$$

Differential Drive: Forward Kinematics



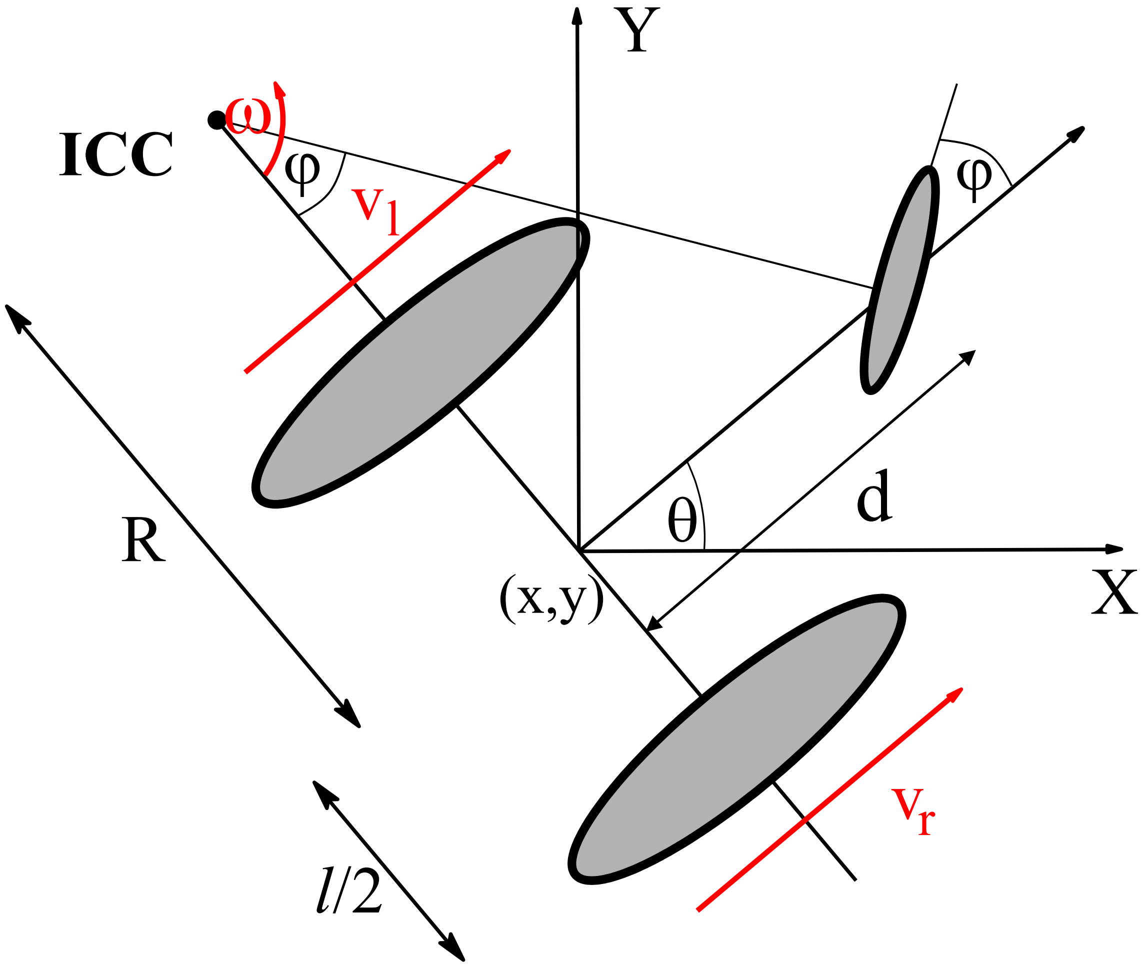
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \mathrm{ICC}_x \\ y - \mathrm{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \mathrm{ICC}_x \\ \mathrm{ICC}_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \frac{1}{2}\int_0^t [v_r(t') + v_l(t')] \ \cos[\theta(t')]dt'$$

$$y(t) = \frac{1}{2}\int_0^t [v_r(t') + v_l(t')] \ \sin[\theta(t')]dt'$$

$$\theta(t) = \frac{1}{l}\int_0^t [v_r(t') - v_l(t')] \, dt'$$

Ackermann Drive



$$ICC = [x - R\sin\theta, y + R\cos\theta]$$

$$R = \frac{d}{\tan\varphi}$$

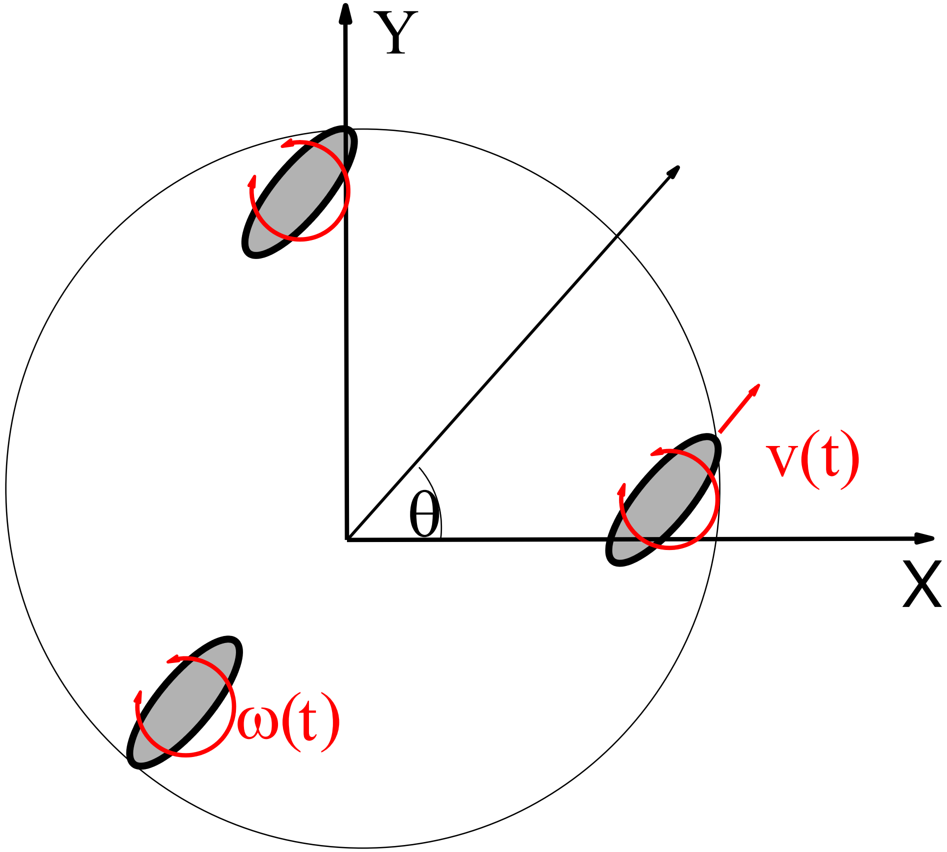
$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

Synchronous Drive

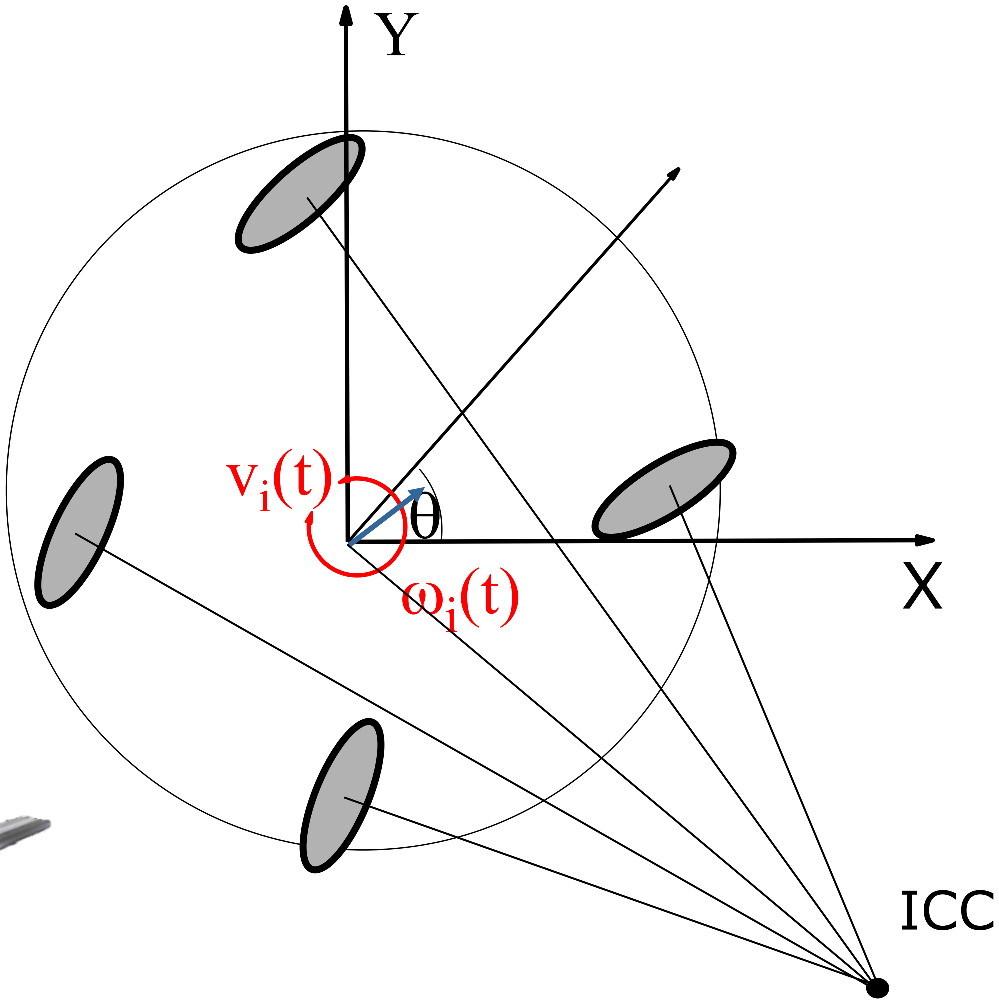


$$x(t) = \int_0^t v(t') \cos[\theta(t')] dt'$$

$$y(t) = \int_0^t v(t') \sin[\theta(t')] dt'$$

$$\theta(t) = \int_0^t \omega(t') \, dt'$$

XR4000 Drive

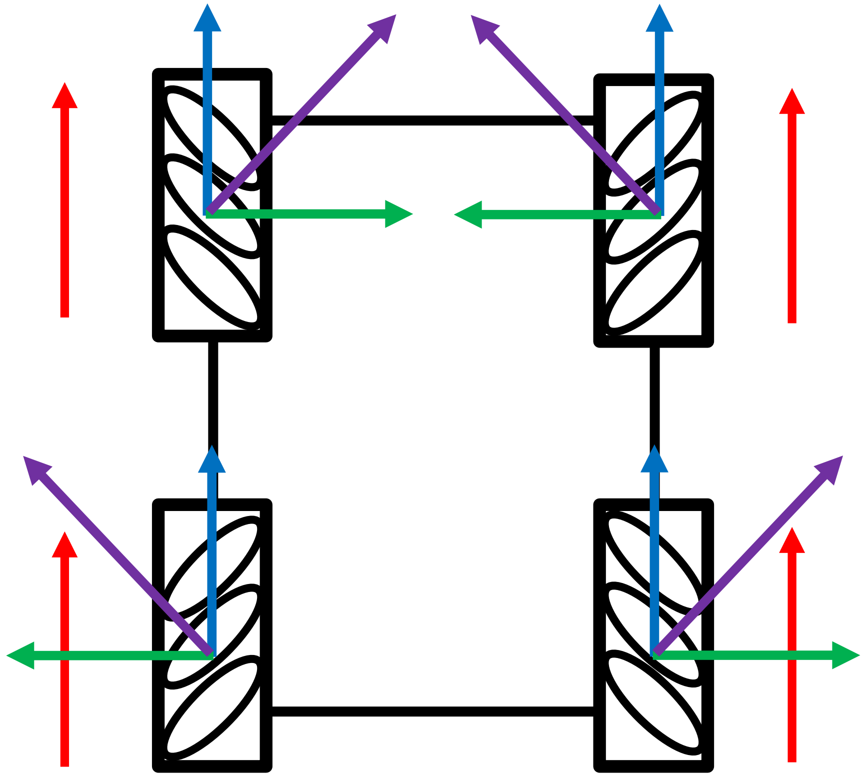


$$x(t) = \int_0^t v(t') \ \cos[\theta(t')] dt'$$

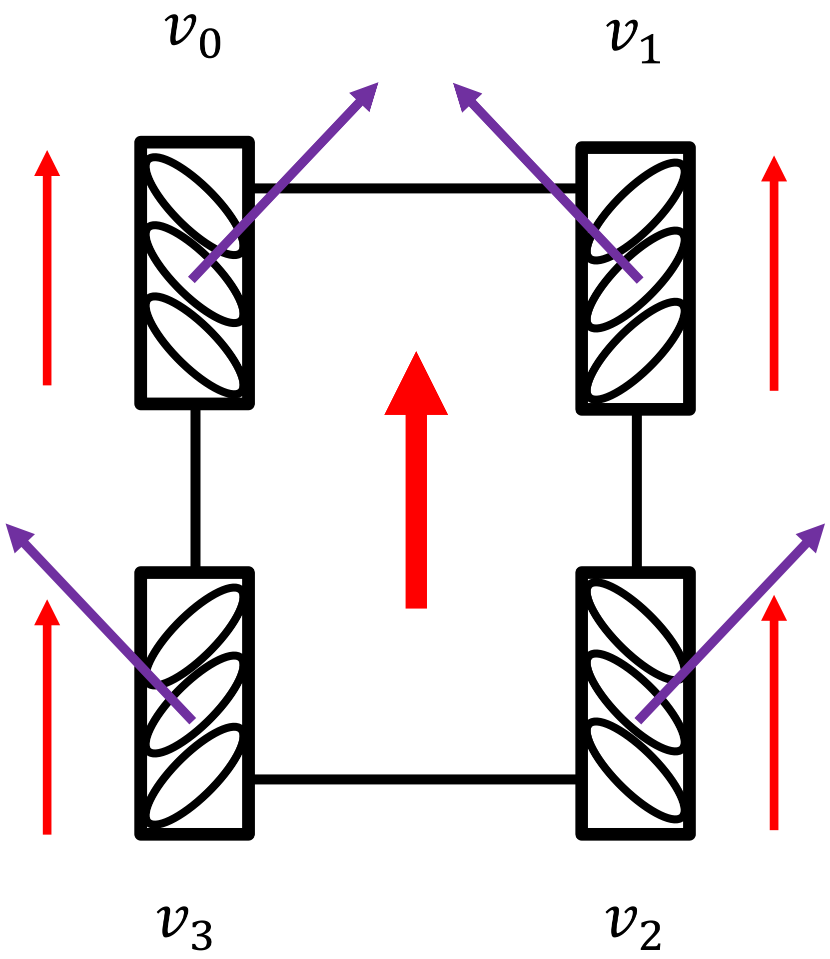
$$y(t) = \int_0^t v(t') \ \sin[\theta(t')] dt'$$

$$\theta(t) = \int_0^t \omega(t') \, dt'$$

Mecanum Wheels



Mecanum Wheels Motion Patterns



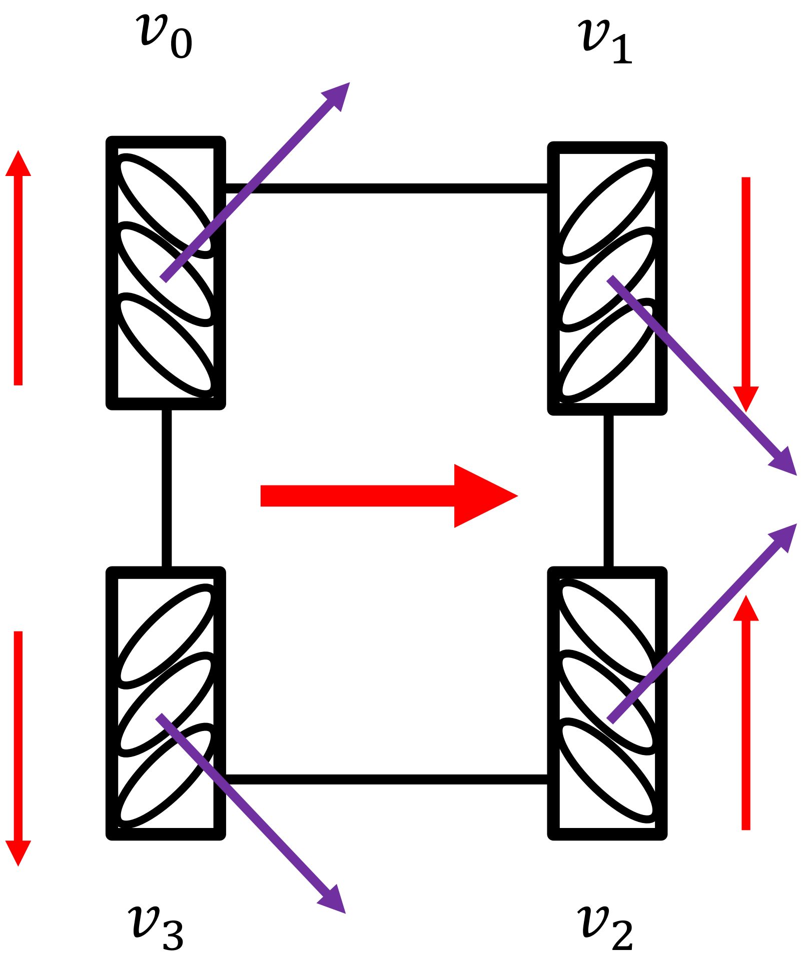
$$v_y = (v_0 + v_1 + v_2 + v_3)/4$$

$$v_x = (v_0 - v_1 + v_2 - v_3)/4$$

$$v_\theta = (v_0 - v_1 - v_2 + v_3)/4$$

$$v_{error} = (v_0 + v_1 - v_2 - v_3)/4$$

Mecanum Wheels Motion Patterns



$$v_y = (v_0 + v_1 + v_2 + v_3)/4$$

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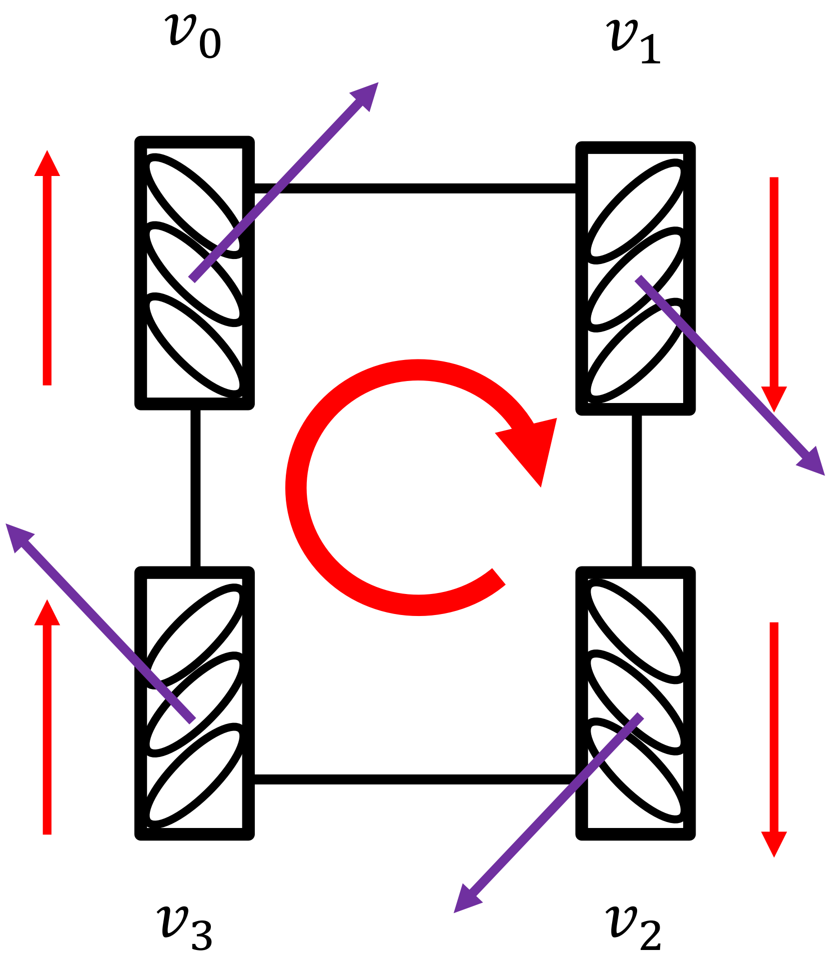
Mecanum Wheels Motion Patterns



$$v_y = (v_0 + v_1 + v_2 + v_3)/4$$

$$v_x = (v_0 - v_1 + v_2 - v_3)/4$$

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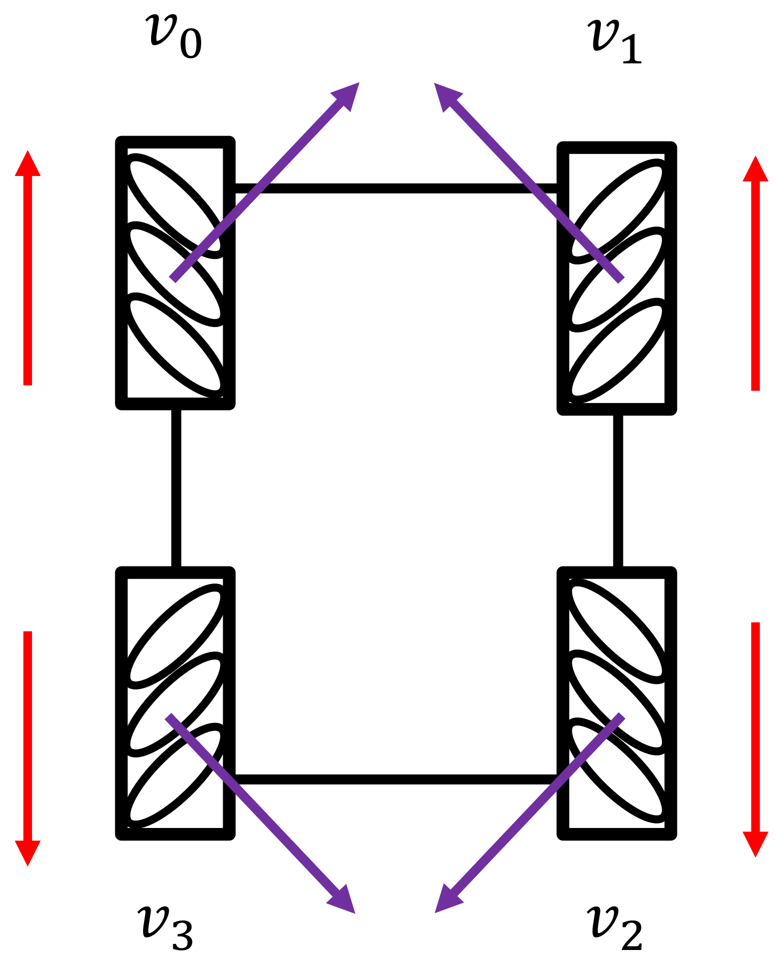
Mecanum Wheels Motion Patterns



$$v_y = (v_0 + v_1 + v_2 + v_3)/4$$

$$v_x = (v_0 - v_1 + v_2 - v_3)/4$$

$$v_\theta = (v_0 - v_1 - v_2 + v_3)/4$$

$$v_{error} = (v_0 + v_1 - v_2 - v_3)/4$$

Non-Holonomic Constraints

- Non-holonomic constraints limit the possible incremental movements within the configuration space of the robot.
- Robots with differential drive or synchro-drive move on a circular trajectory and cannot move sideways.
- Mecanum-wheeled robots can move sideways (they have no non-holonomic constraints).

Holonomic vs. Non-Holonomic

- Non-holonomic constraints reduce the control space with respect to the current configuration
  - E.g., moving sideways is impossible.

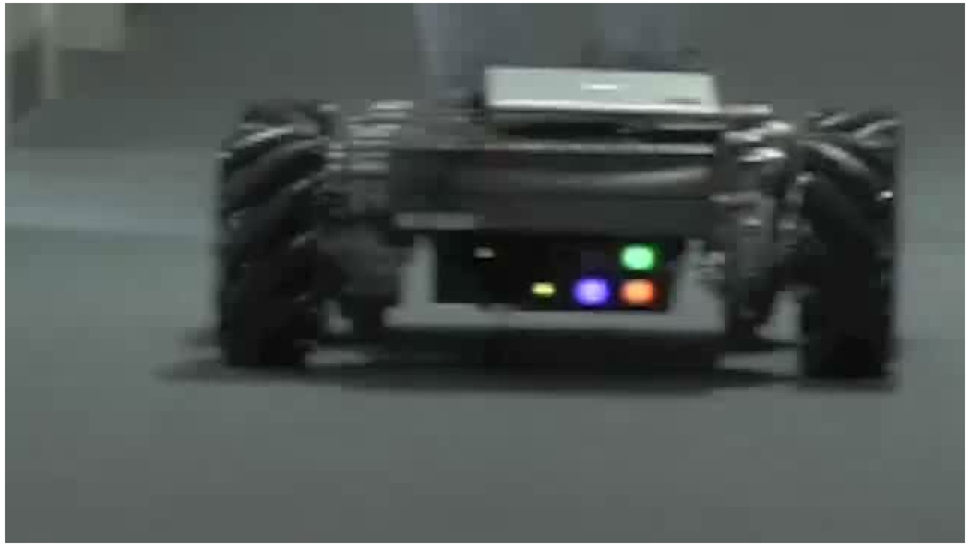
- Holonomic constraints reduce the configuration space.
  - E.g., a train on tracks (not all positions and orientations are possible)

Drives with Non-Holonomic Constraints

- Synchro-drive
- Differential drive
- Ackermann drive

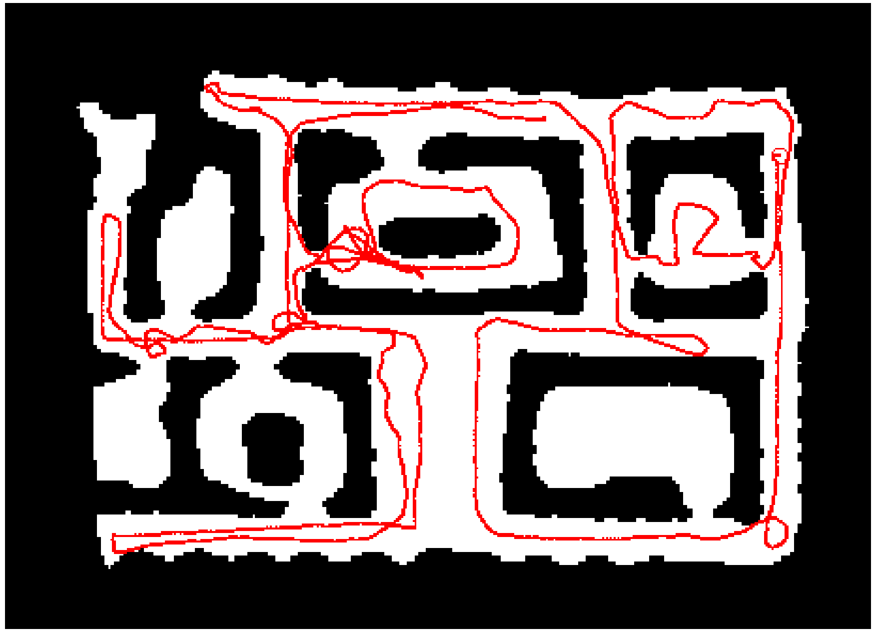
Drives without Non-Holonomic Constraints

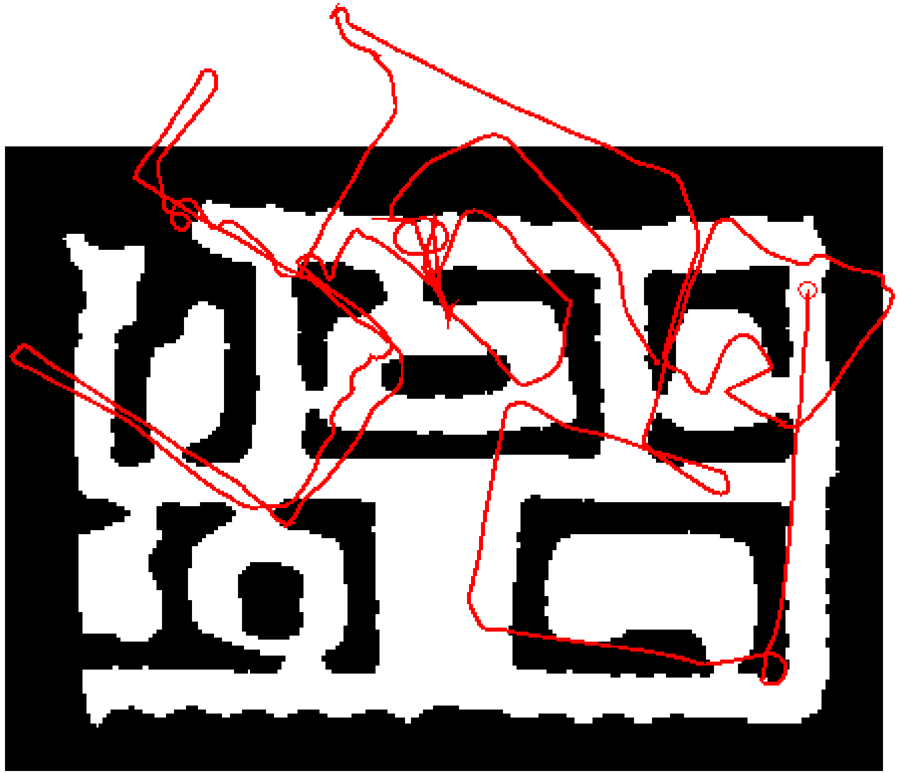
- Mecanum wheels



Dead Reckoning and Odometry

- Estimating the motion based on the issued controls/wheel encoder readings
- Integrated over time





Summary

- Introduced different types of drives for wheeled robots
- Math to describe the motion of the basic drives given the speed of the wheels
- Non-holonomic constraints
- Odometry and dead reckoning