Introduction to Mobile Robotics

Probabilistic Sensor Models

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UTN

Bayes Filters are Familiar!

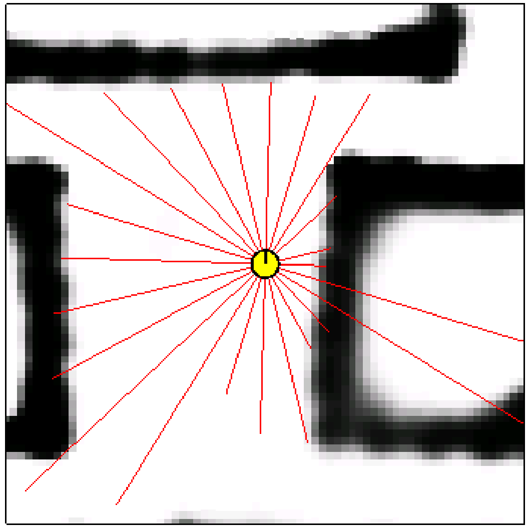
$$Bel(x_t) = \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

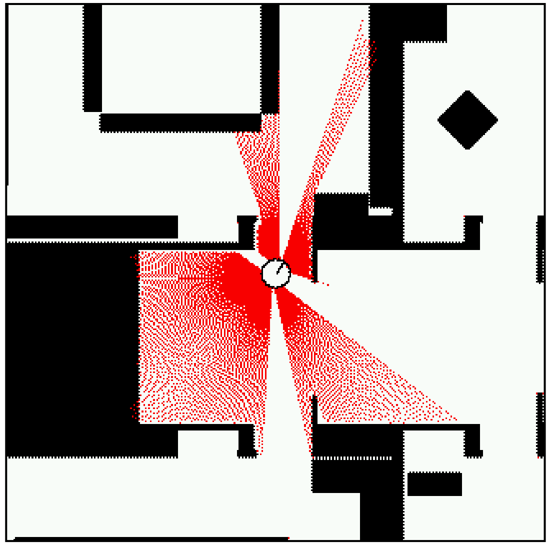
- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

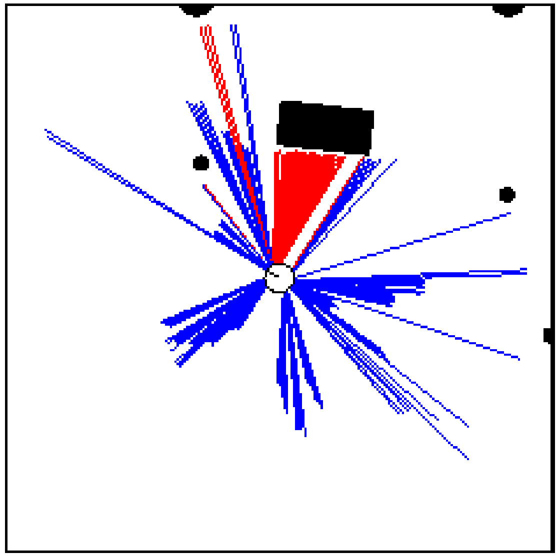
Sensors for Mobile Robots

- Contact sensors: Bumpers
- Proprioceptive sensors
  - Accelerometers (spring-mounted masses)
  - Gyroscopes (spinning mass, laser light)
  - Compasses, inclinometers (earth magnetic field, gravity)
- Proximity sensors
  - Sonar (time of flight)
  - Radar (phase and frequency)
  - Laser range-finders (triangulation, tof, phase)
  - Infrared (intensity)
- Visual sensors: Cameras
- Satellite-based sensors: GPS

Proximity Sensors







- The central task is to determine *P(z|x)*, i.e., the probability of a measurement *z* given that the robot is at position *x* (and given the map).
- **Question**: Where do the probabilities come from?
- **Approach**: Let's try to explain a measurement.

In this Course: Two types of Range Measurement Models

- **Beam-based model**
  - tries to explain the measurement
  - model parameters can be learned
  - requires ray-casting

- **Scan-based model**
  - tries to be fast
  - ignores the fact that the measurement is a ray

Both are approximations!

Independence Assumption of both Models

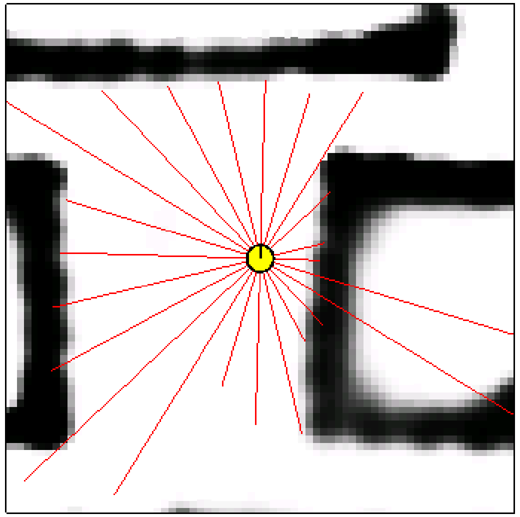
- Scan z consists of K measurements.

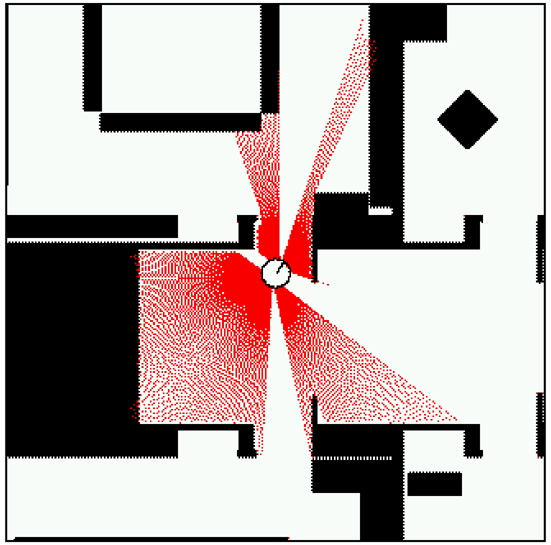
$$z = \{z_1, z_2, \ldots, z_K\}$$

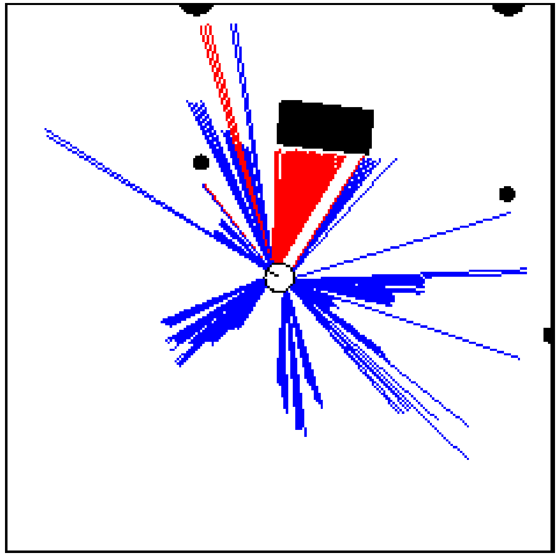
- Individual measurements are independent given the robot position.

$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

Typical Range Scans (Sonar and LiDAR)



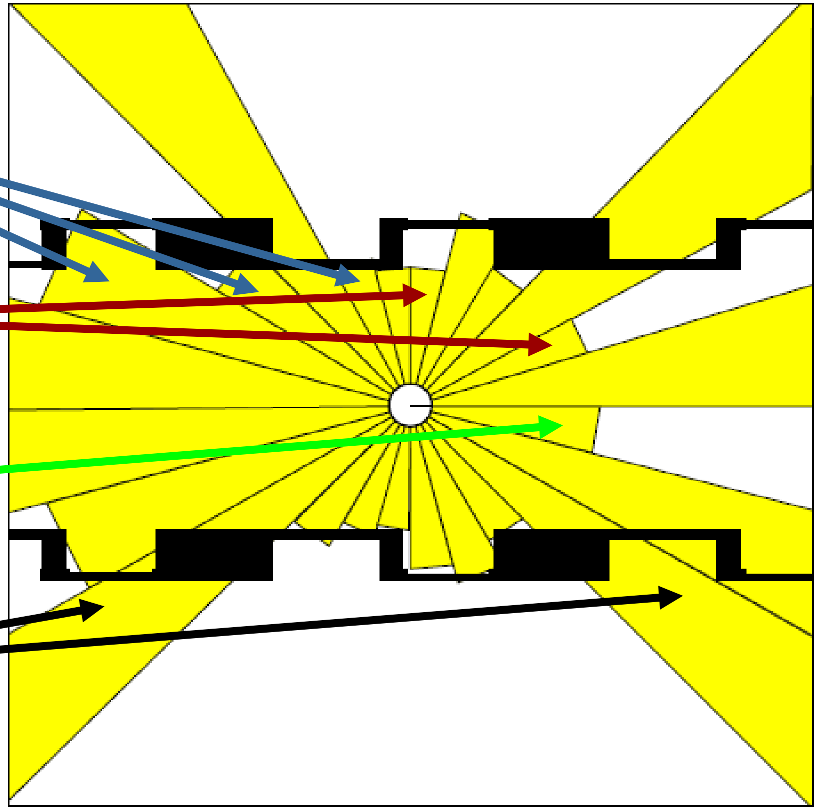




$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

Typical Errors of Range Measurements

1. Beams reflected by obstacles
2. Beams reflected by people / caused by crosstalk
3. Random measurements
4. Maximum range measurements



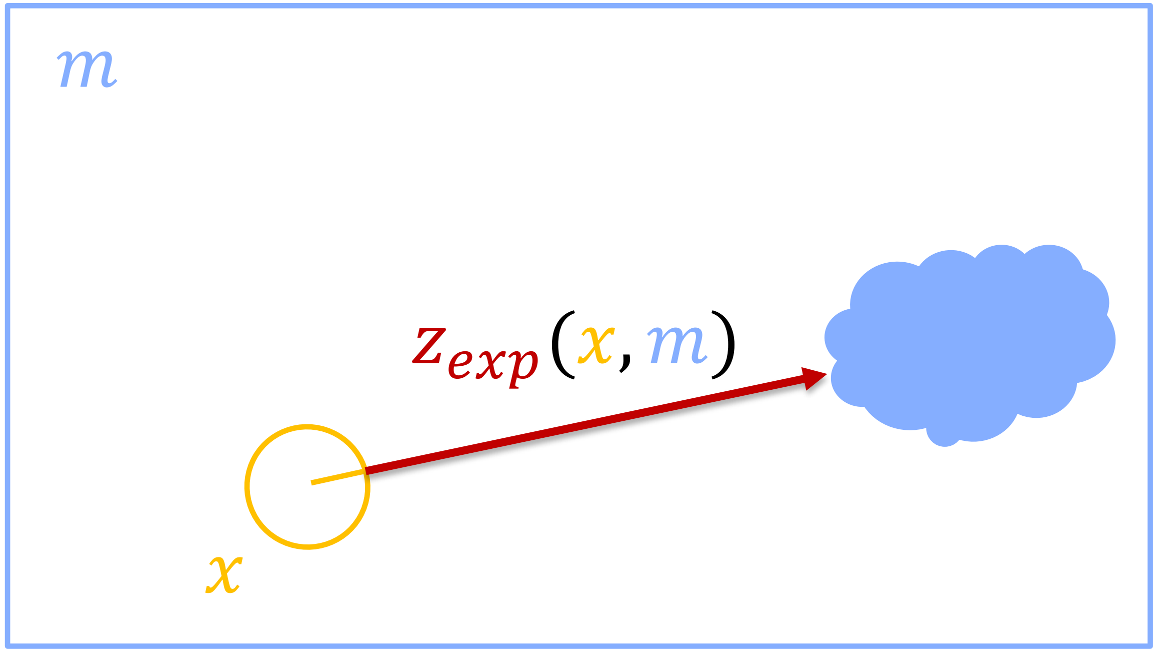
Proximity Measurement

- Measurement can be caused by …
  - a known obstacle,
  - cross-talk,
  - an unexpected obstacle (people, furniture, …), or
  - missing all obstacles (total reflection, glass, …).
- Noise is due to uncertainty …
  - in measuring distance to known obstacle,
  - in position of known obstacles,
  - in position of additional obstacles, or
  - whether obstacle is missed.

Key Idea of the Beam-based Model

- Considers beams individually
- Uses the following approximation:

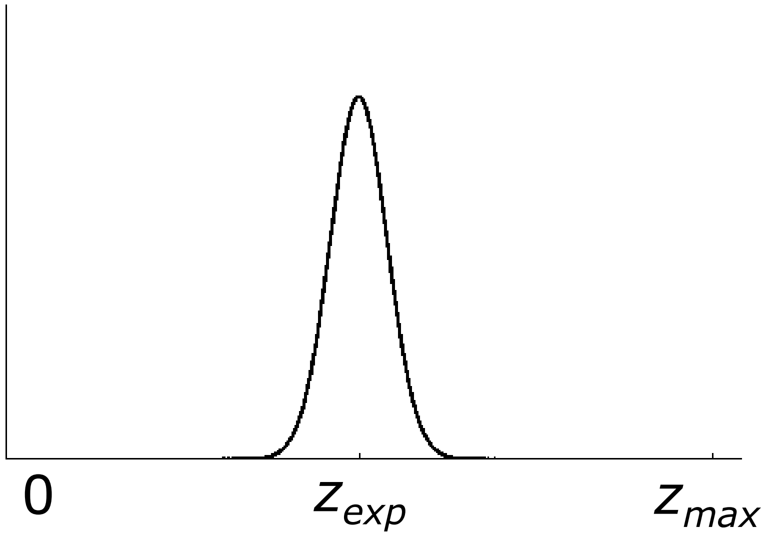
$$P(z \mid x) = P(z \mid x, m) \approx P(z \mid z_{exp}(x, m)) = P(z \mid z_{exp})$$



$z_{exp}(x, m)$ equals distance to closest obstacle in direction of measurement (obtained by ray casting)

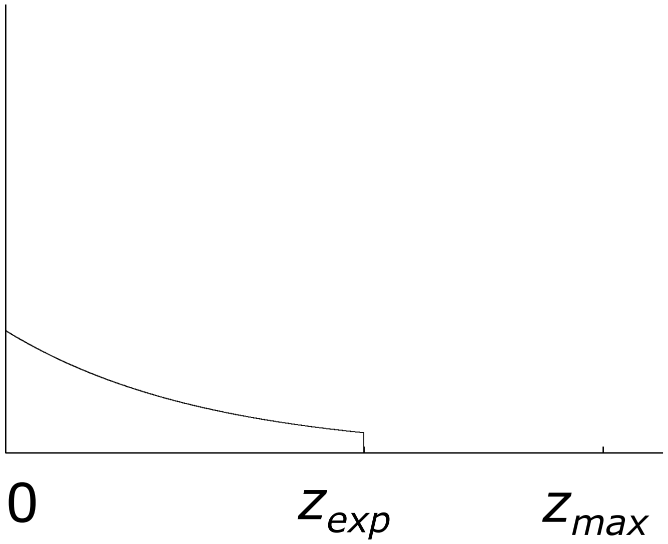
Beam-based Proximity Model

Measurement noise



$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\frac{(z - z_{\exp})^2}{b}}$$

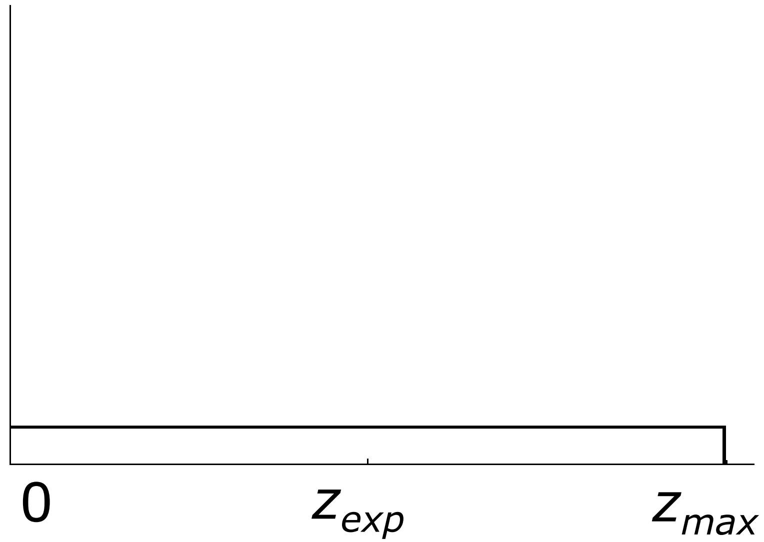
Unexpected obstacles



$$P_{\text{unexp}}(z \mid x, m) = \begin{cases} \eta\, \lambda\, e^{-\lambda z} & z < z_{\exp} \\ 0 & \textit{otherwise} \end{cases}$$

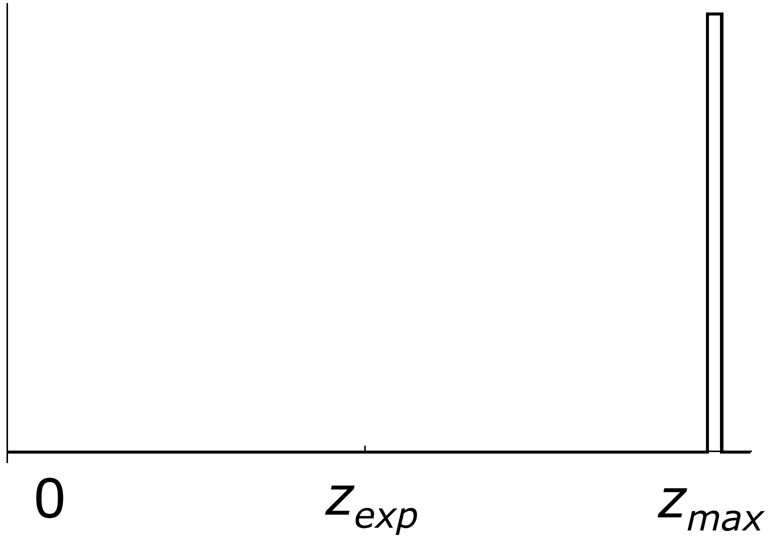
Beam-based Proximity Model

Random measurement



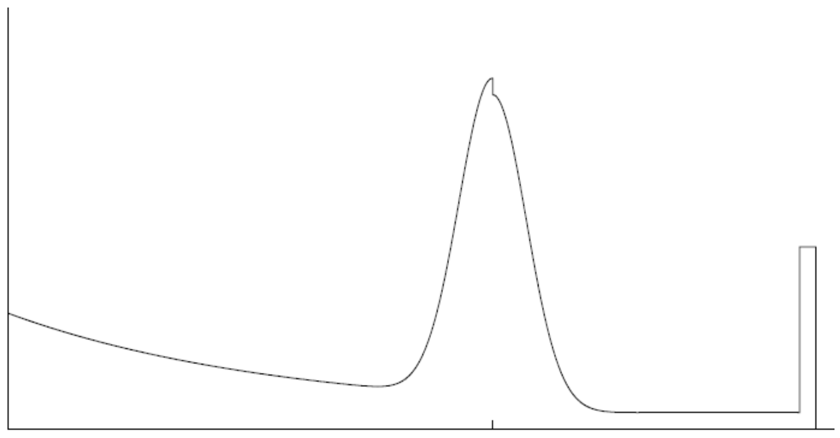
$$P_{rand}(z \mid x, m) = \eta \frac{1}{z_{\max}}$$

Max range



$$P_{\max}(z|x,m) = \begin{cases} 1 & z = z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Resulting Mixture Density

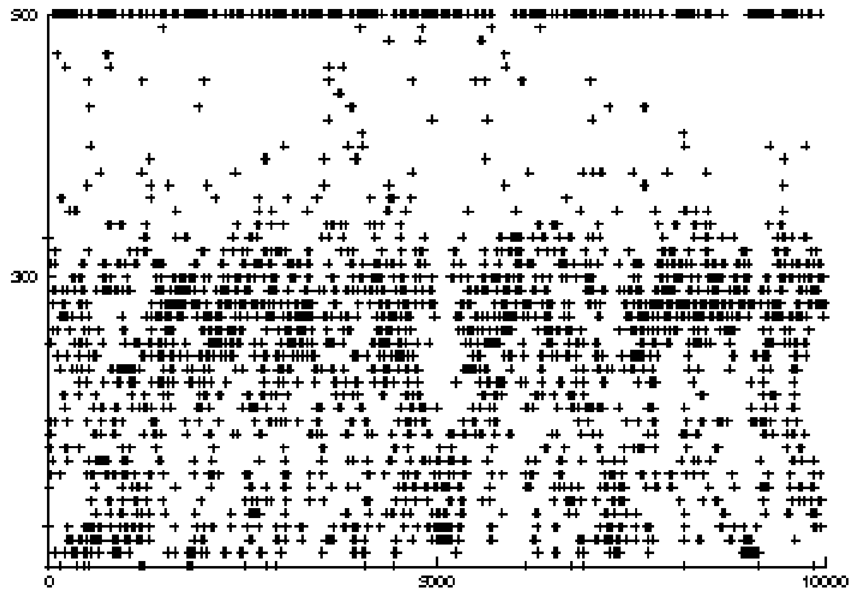


$$P(z \mid x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\max} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\text{hit}}(z \mid x, m) \\ P_{\text{unexp}}(z \mid x, m) \\ P_{\max}(z \mid x, m) \\ P_{\text{rand}}(z \mid x, m) \end{pmatrix}$$

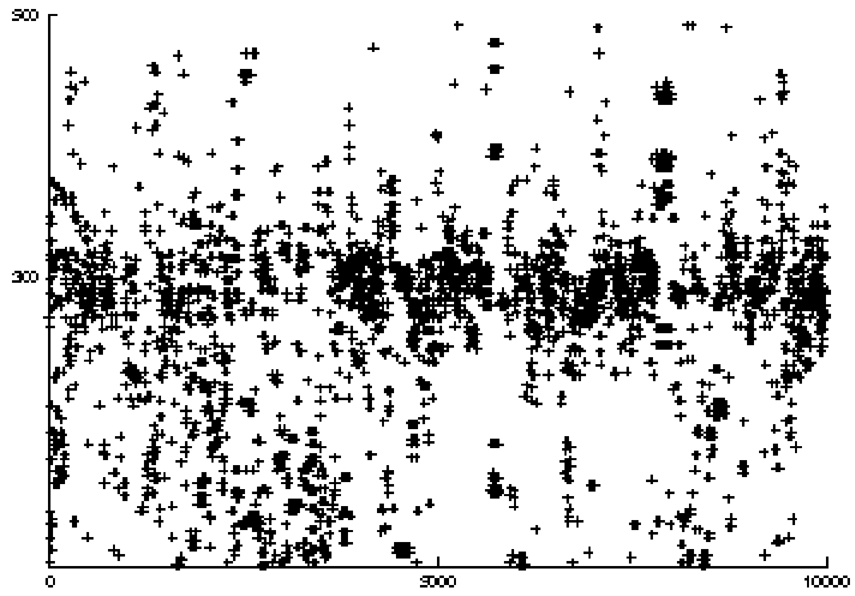
How can we determine the model parameters?

Raw Sensor Data

Measured distances for expected distance of 300 cm.



Sonar



Laser

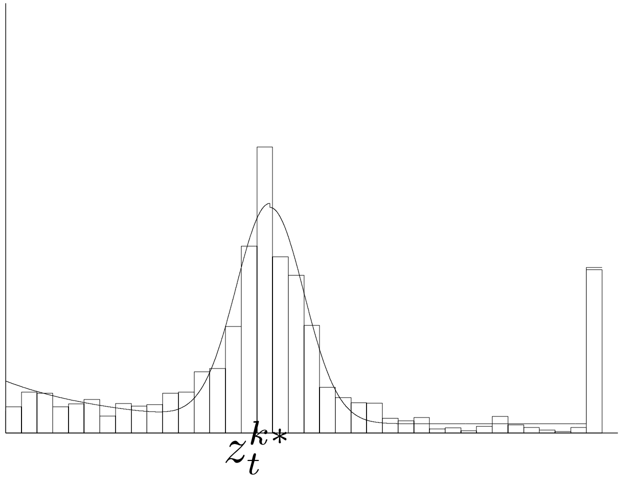
Approximation

- Maximize log likelihood of the data

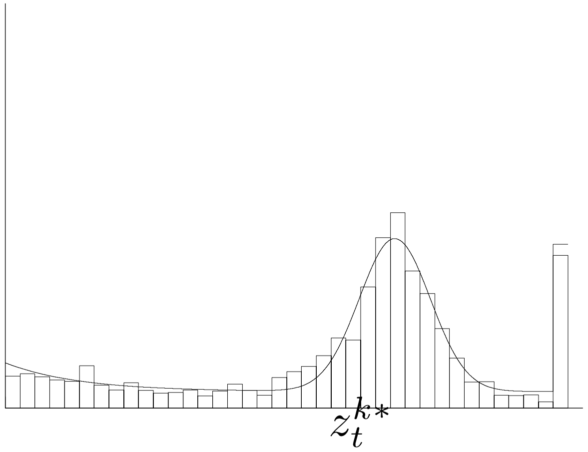
$$P(z \mid z_{\exp})$$

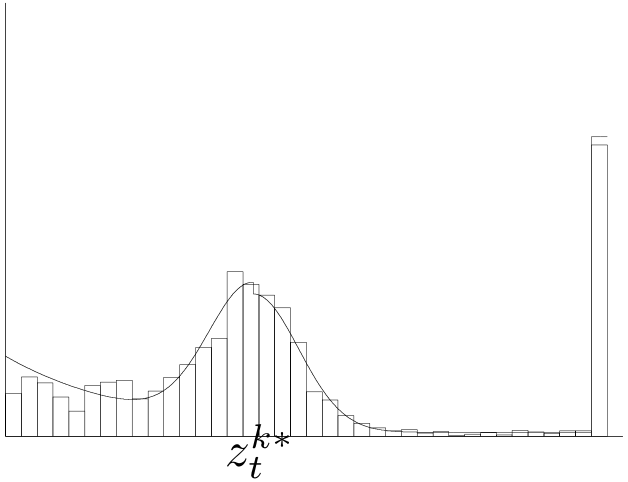
- Search space of n-1 parameters and deterministically compute the n-th parameter to satisfy normalization constraint.
  - Hill climbing
  - Gradient descent
  - Genetic algorithms
  - …

Approximation Results

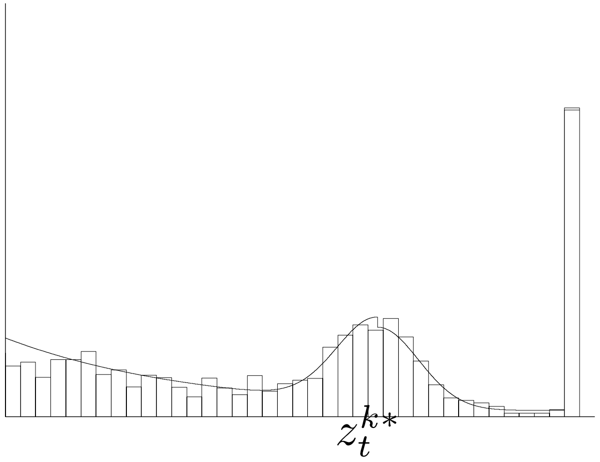


Laser





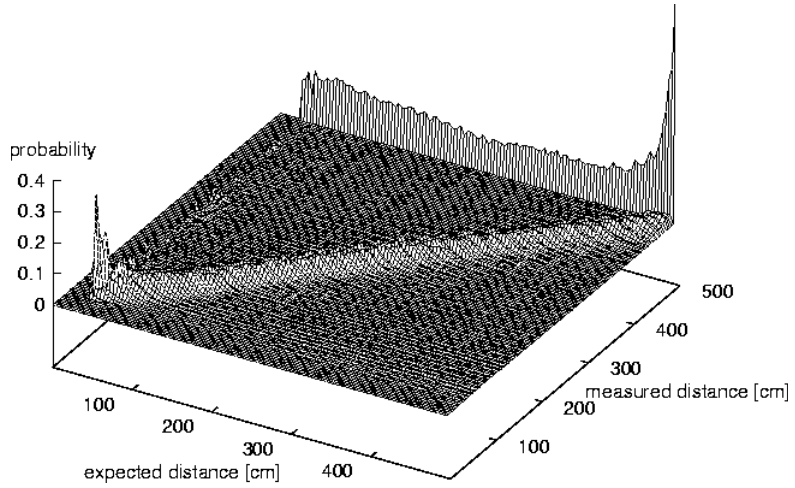
Sonar



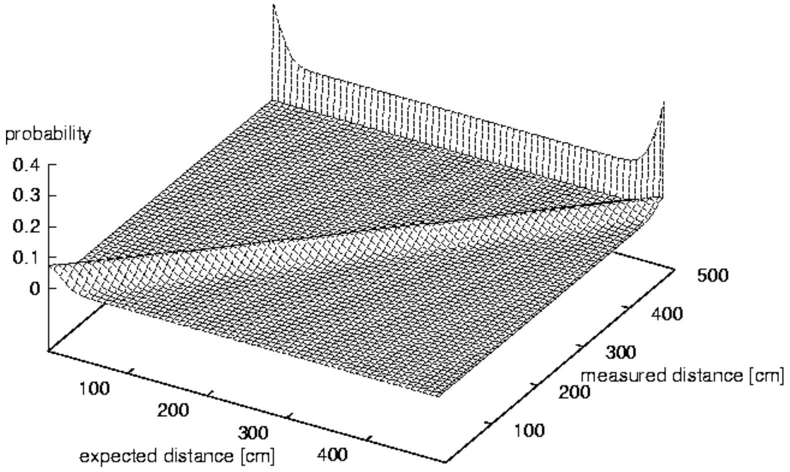
300cm

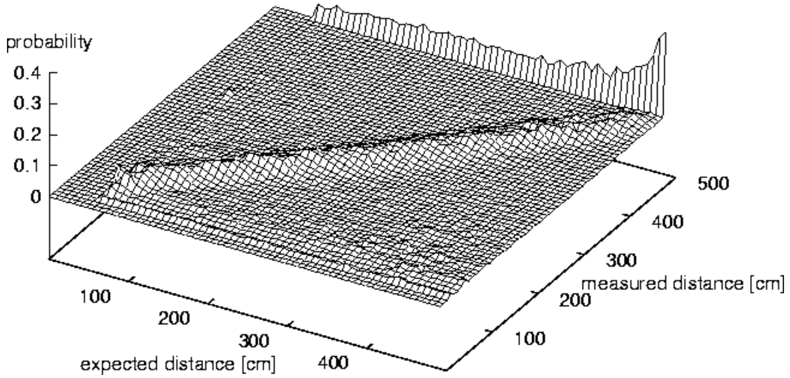
400cm

Approximation Results

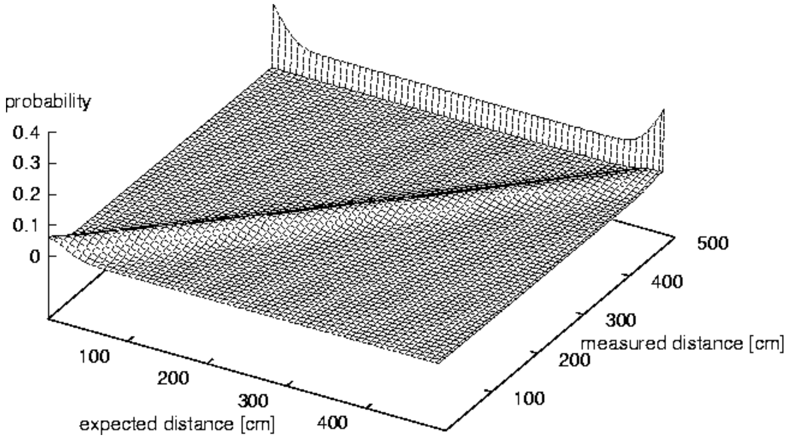


Laser



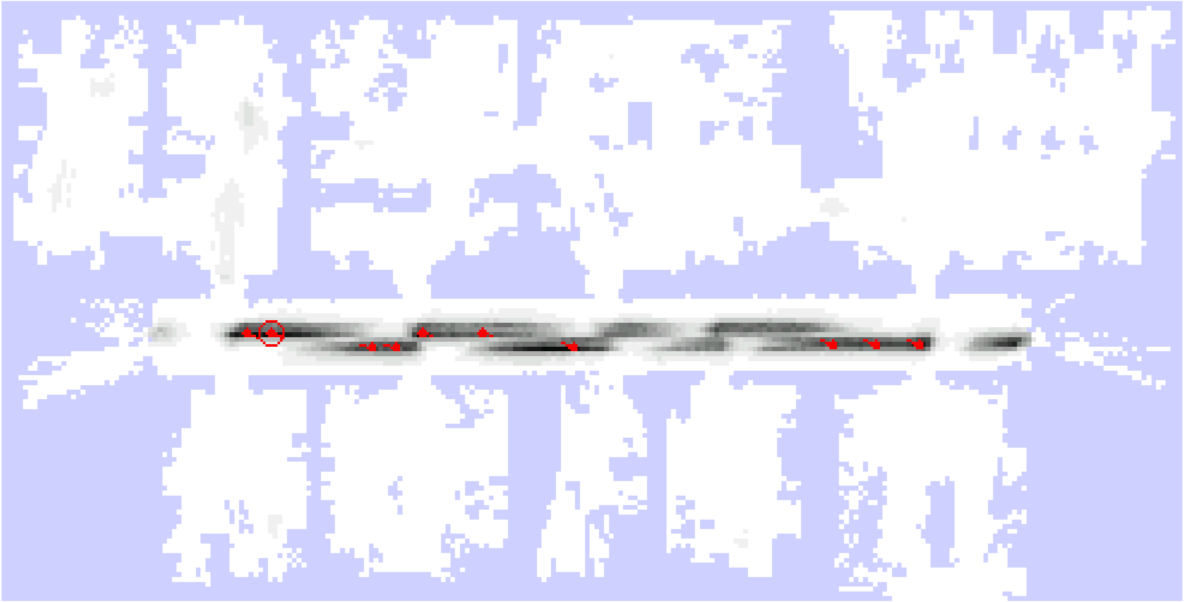


Sonar



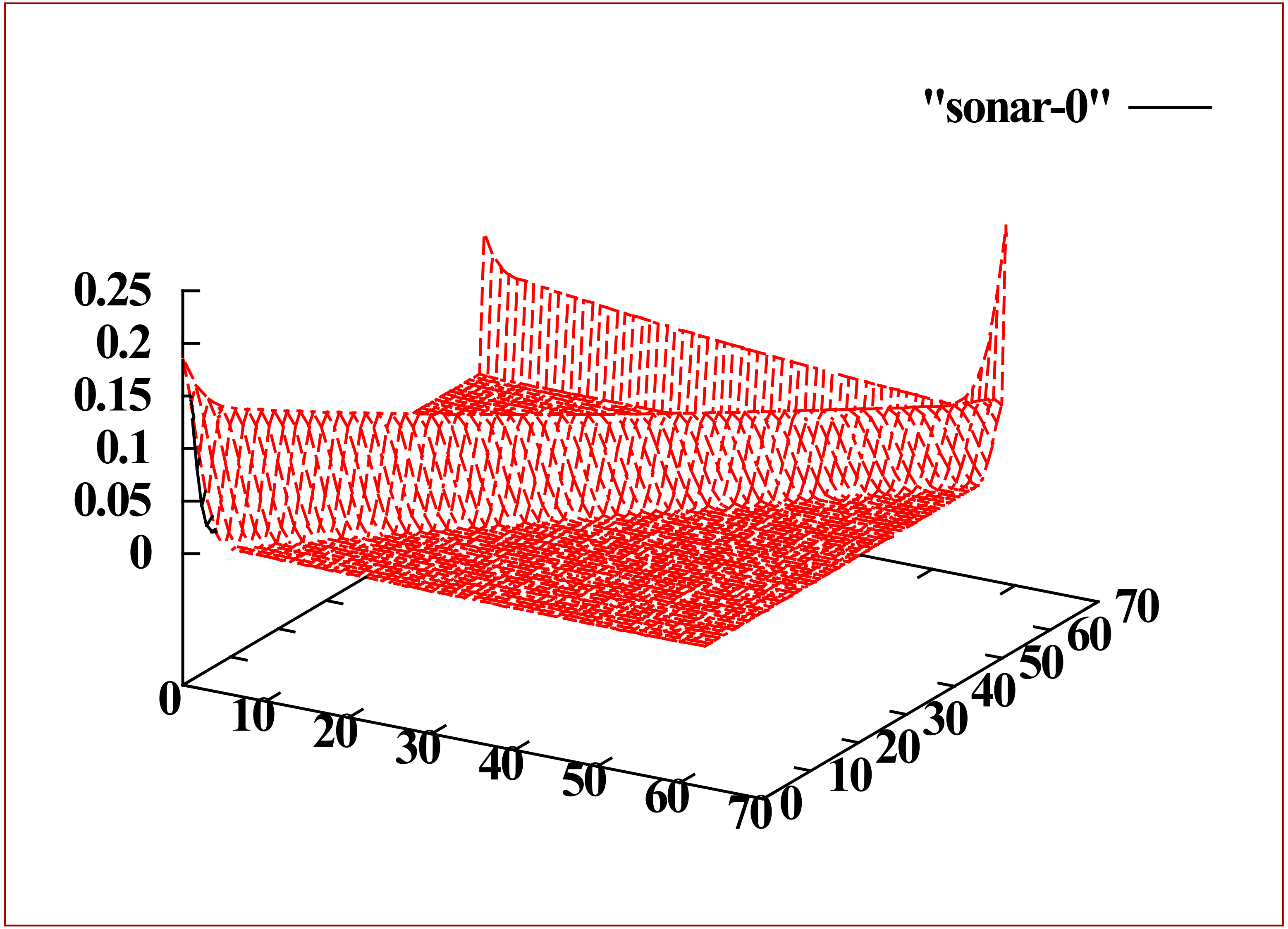
Example

z

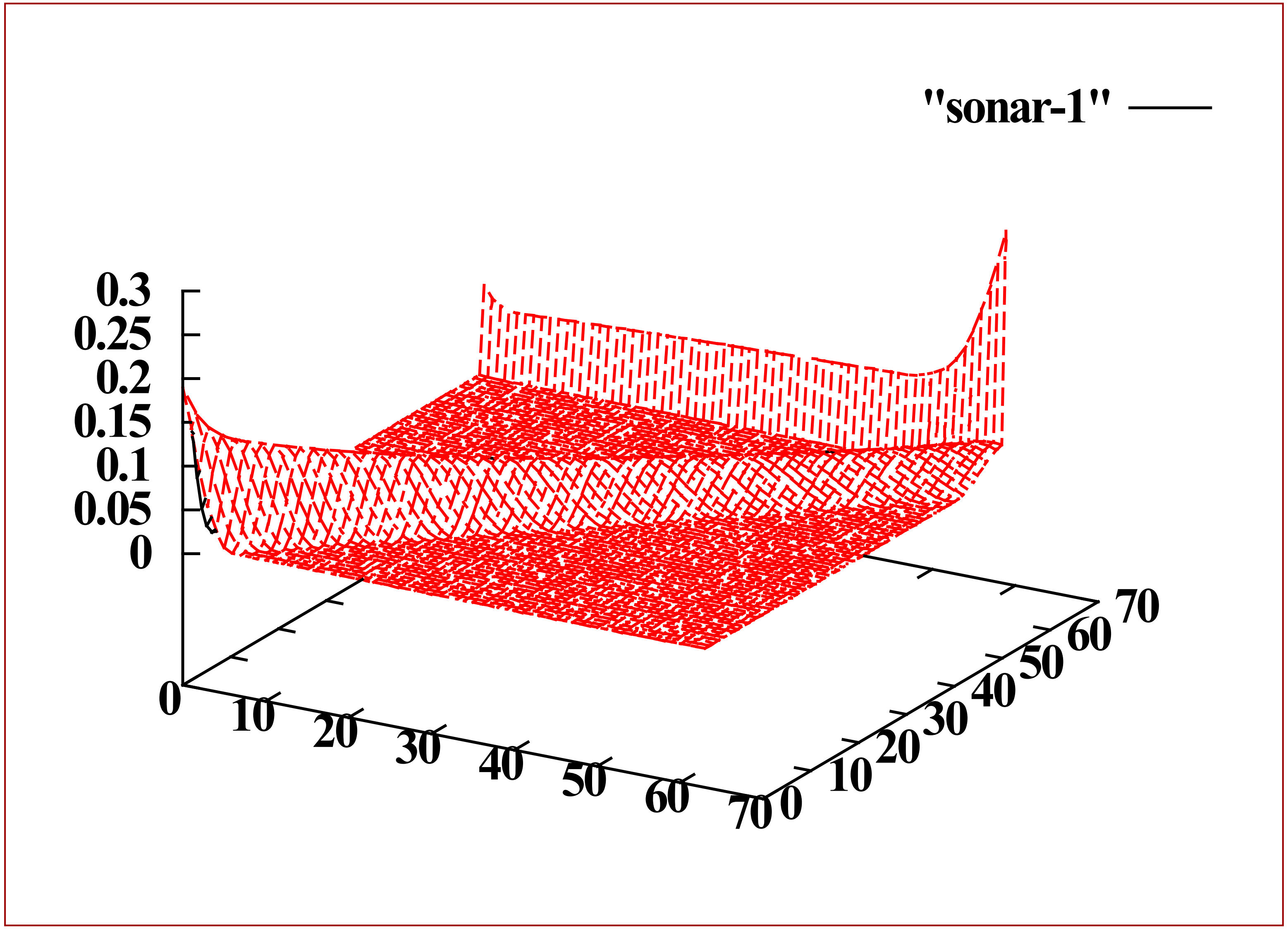


$P(z|x,m)$

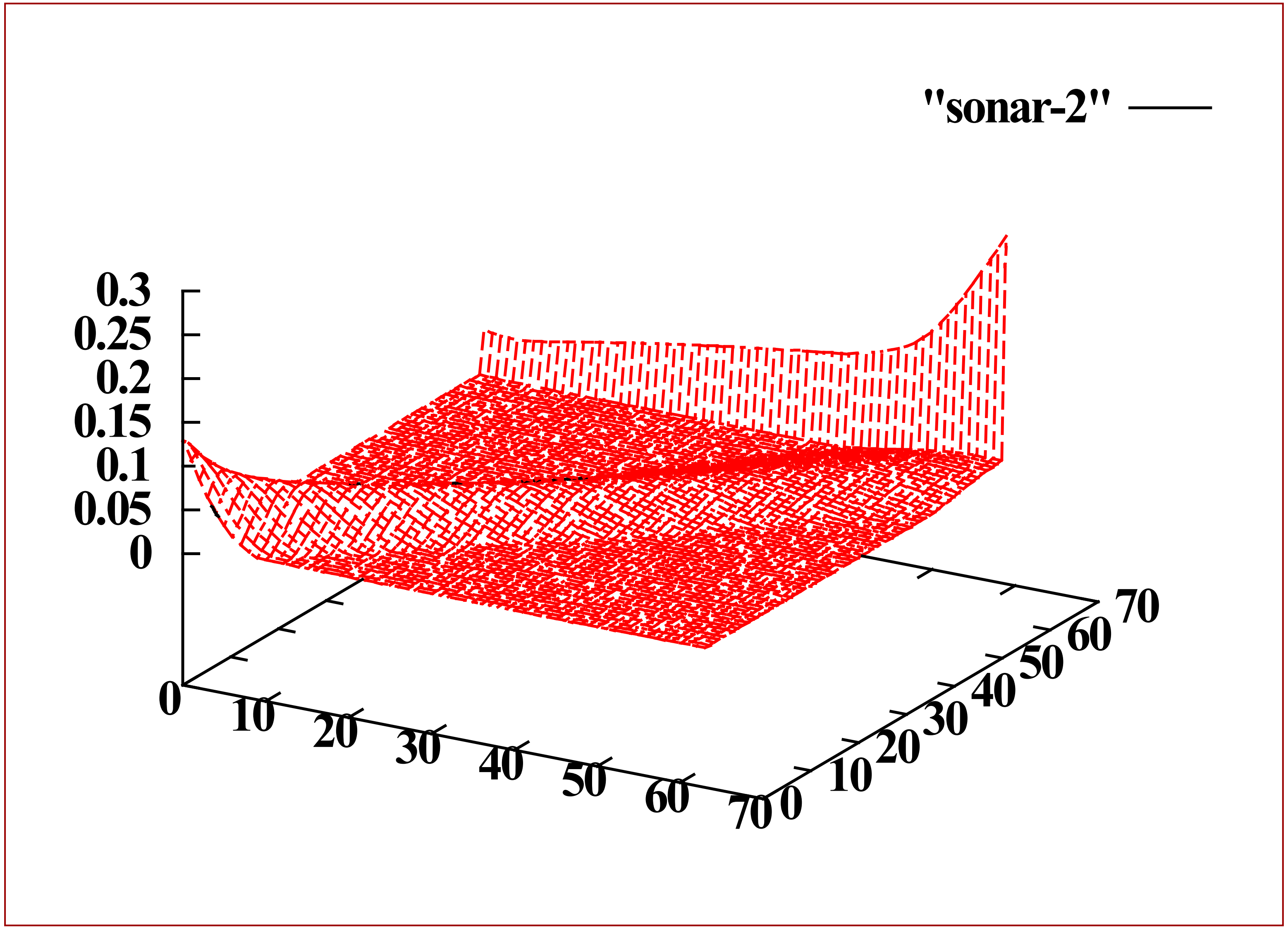
Influence of Angle to Obstacle



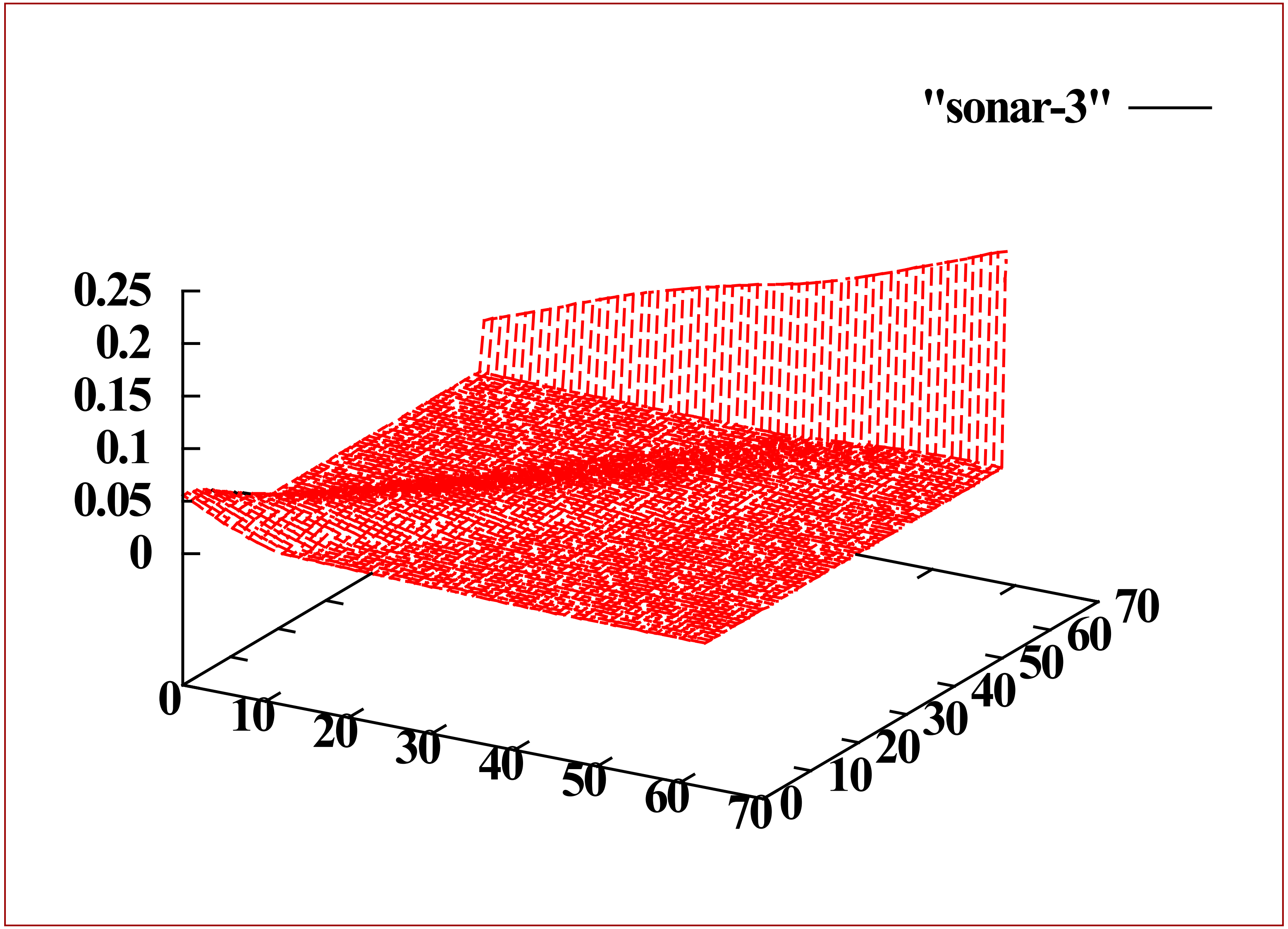
Influence of Angle to Obstacle



Influence of Angle to Obstacle



Influence of Angle to Obstacle



Summary Beam-based Model

- Assumes independence between beams.
  - Justification?
  - Overconfident!
- Models physical causes for measurements.
  - Mixture of densities for these causes.
  - Assumes independence between causes. Problem?
- Implementation
  - Learn parameters based on real data.
  - Different models should be learned for different angles at which the sensor beam hits the obstacle.
  - Determine expected distances by ray-casting.
  - Expected distances can be pre-processed.

Scan-based Model

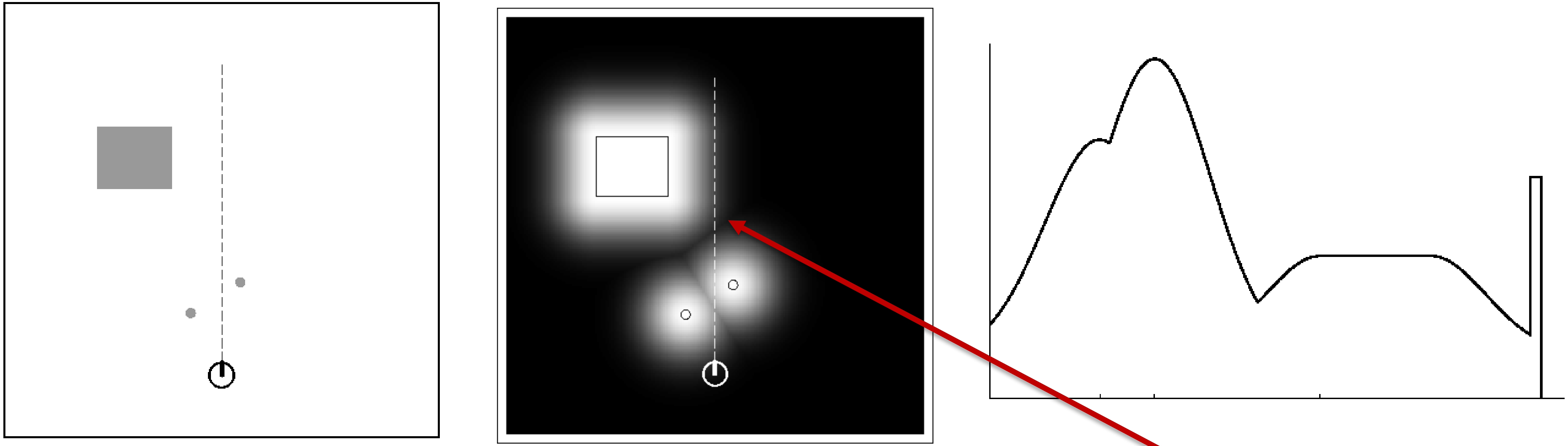
- Beam-based model is …
  - not smooth for small obstacles and at edges.
  - not very efficient (due to ray-casting).

- Idea: Instead of following along the beam, just check the end point.

Scan-based Model

- Probability is a mixture of …
  - a Gaussian distribution with mean at distance to closest obstacle,
  - a uniform distribution for random measurements, and
  - a small uniform distribution for max range measurements.
- It can be efficiently stored in a two-dimensional “likelihood field”
- Again, independence between different beams is assumed.

Example



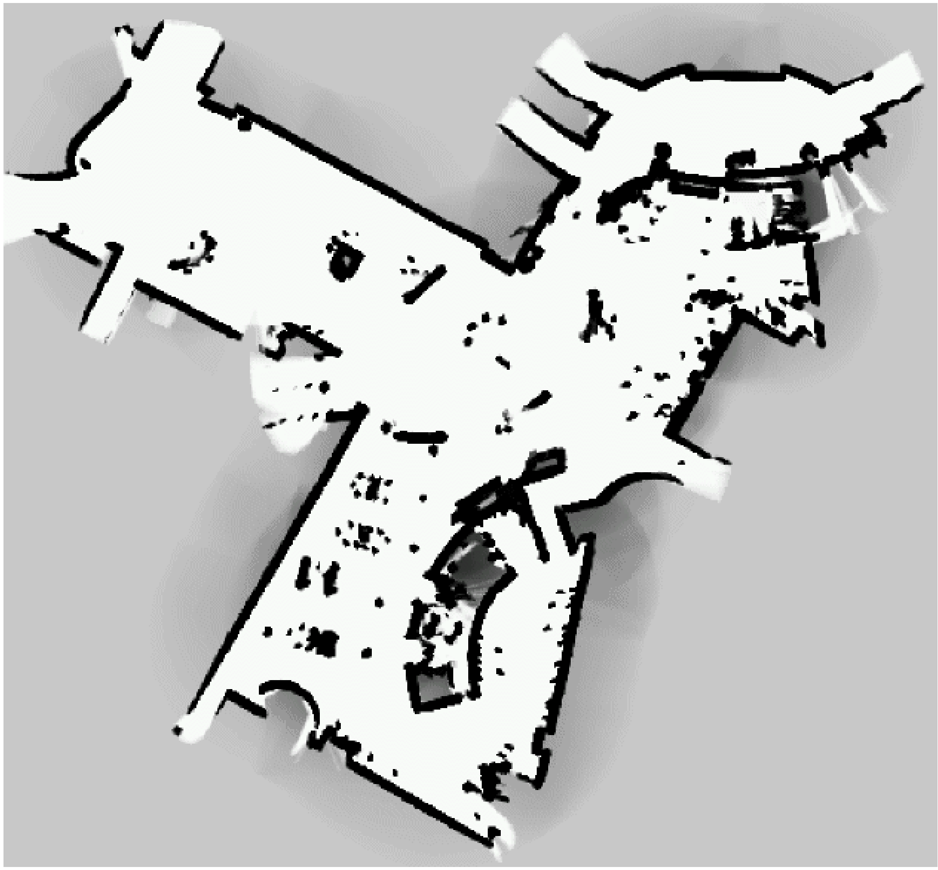
Map *m* and pose *x*

Likelihood field for *m*

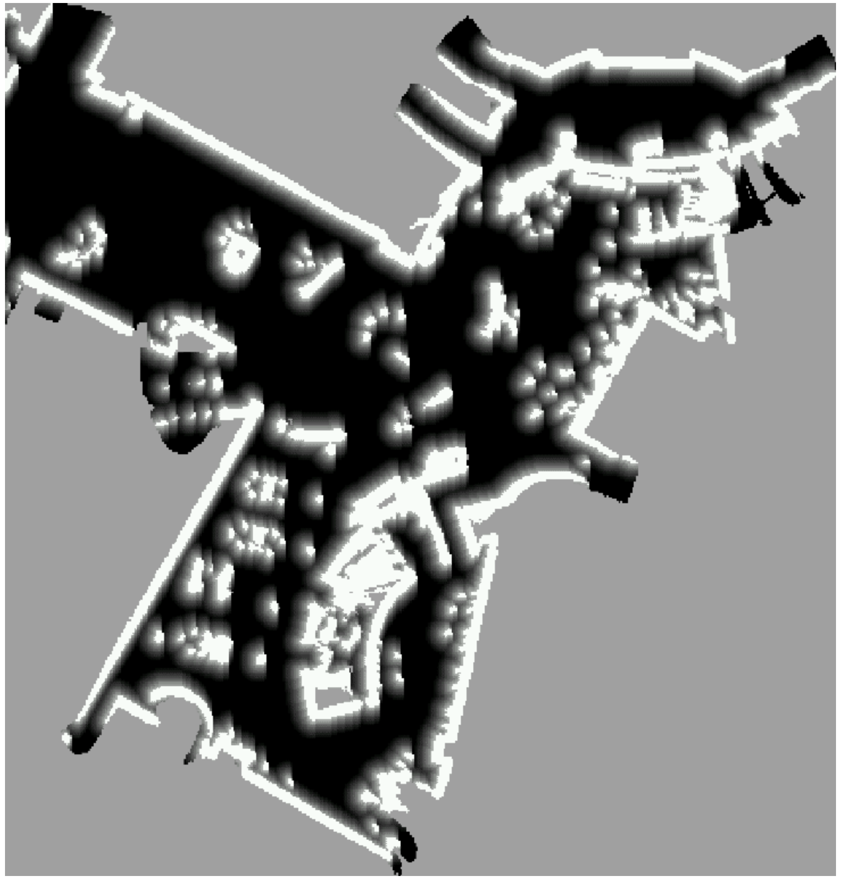
P(z|x,m)

For all *z* along this ray

San Jose Tech Museum



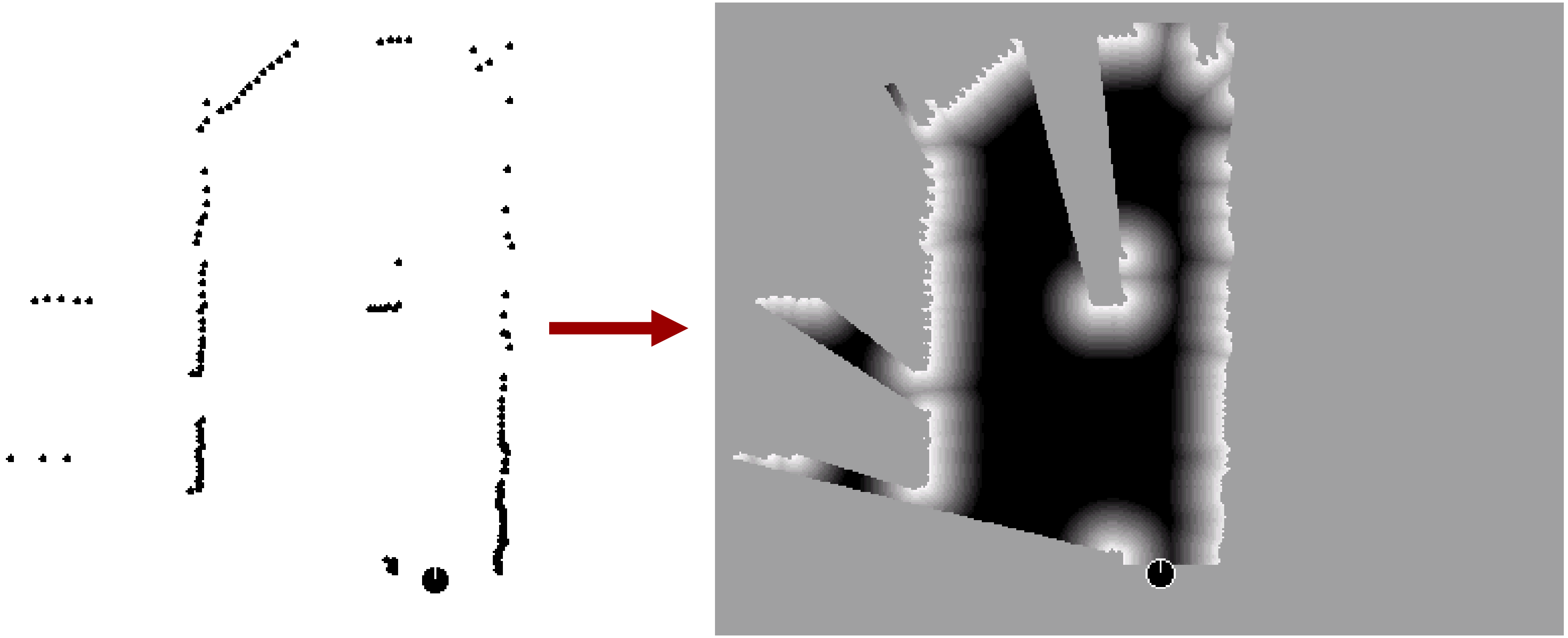
Occupancy grid map



Likelihood field

Scan Matching

- Extract likelihood field from scan and use it to match different scan.



Properties of Scan-based Model

- Highly efficient, uses 2D tables only.
- Likelihood field is smooth w.r.t. to small changes in robot position.
- Allows gradient descent, scan matching.
- Ignores physical properties of beams.

Additional Models of Proximity Sensors

- Map matching (sonar, laser): generate small, local maps from sensor data and match local maps against global model.
- Scan matching (laser): map is represented by scan endpoints, match scan into this map.
- Features (sonar, laser, vision): Extract features such as doors, hallways from sensor data.

Landmarks

- Active beacons (*e.g.*, radio, GPS)
- Passive (*e.g.*, visual, retro-reflective)
- Standard approach is triangulation

- Sensor provides
  - distance, or
  - bearing, or
  - distance and bearing.

Distance and Bearing

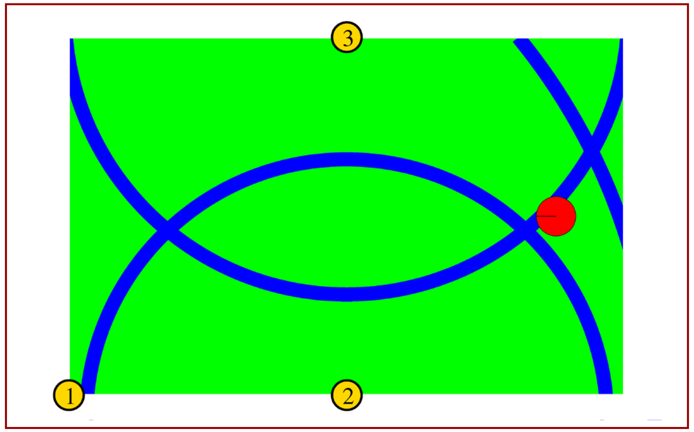
Probabilistic Model

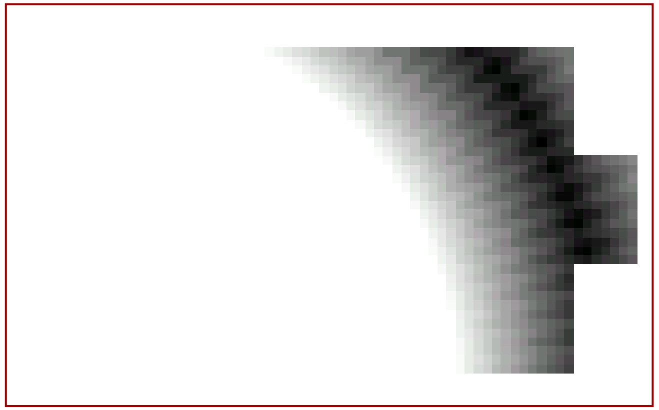
1. Algorithm **landmark_detection_model**(z,x,m):

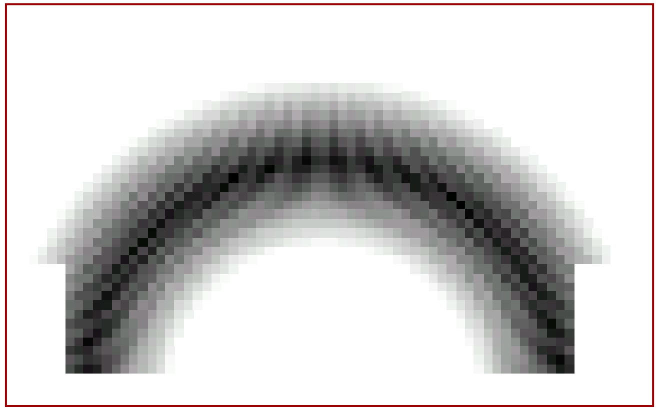
   $z = \langle i, d, \alpha \rangle, x = \langle x, y, \theta \rangle$

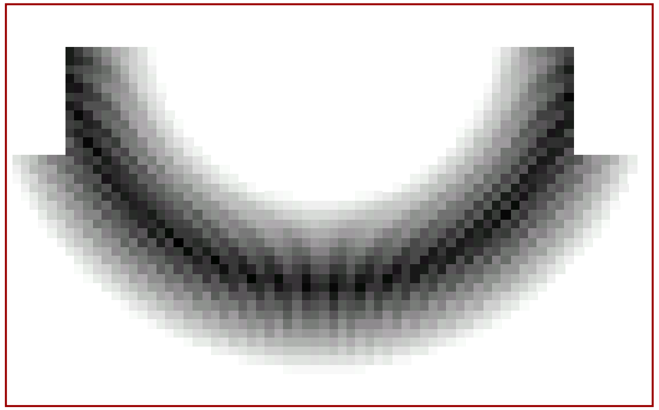
2. $\hat{d} = \sqrt{(m_x(i) - x)^2 + (m_y(i) - y)^2}$
3. $\hat{\alpha} = \operatorname{atan2}(m_y(i) - y, m_x(i) - x) - \theta$
4. $p_{\det} = \operatorname{prob}(\hat{d} - d, \varepsilon_d) \cdot \operatorname{prob}(\hat{\alpha} - \alpha, \varepsilon_\alpha)$
5. Return $p_{\det}$

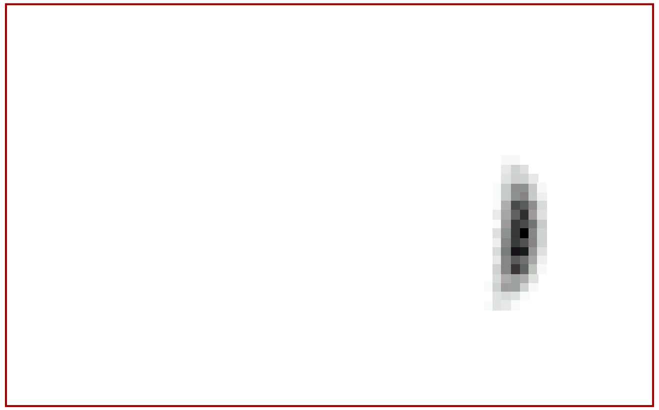
Distributions



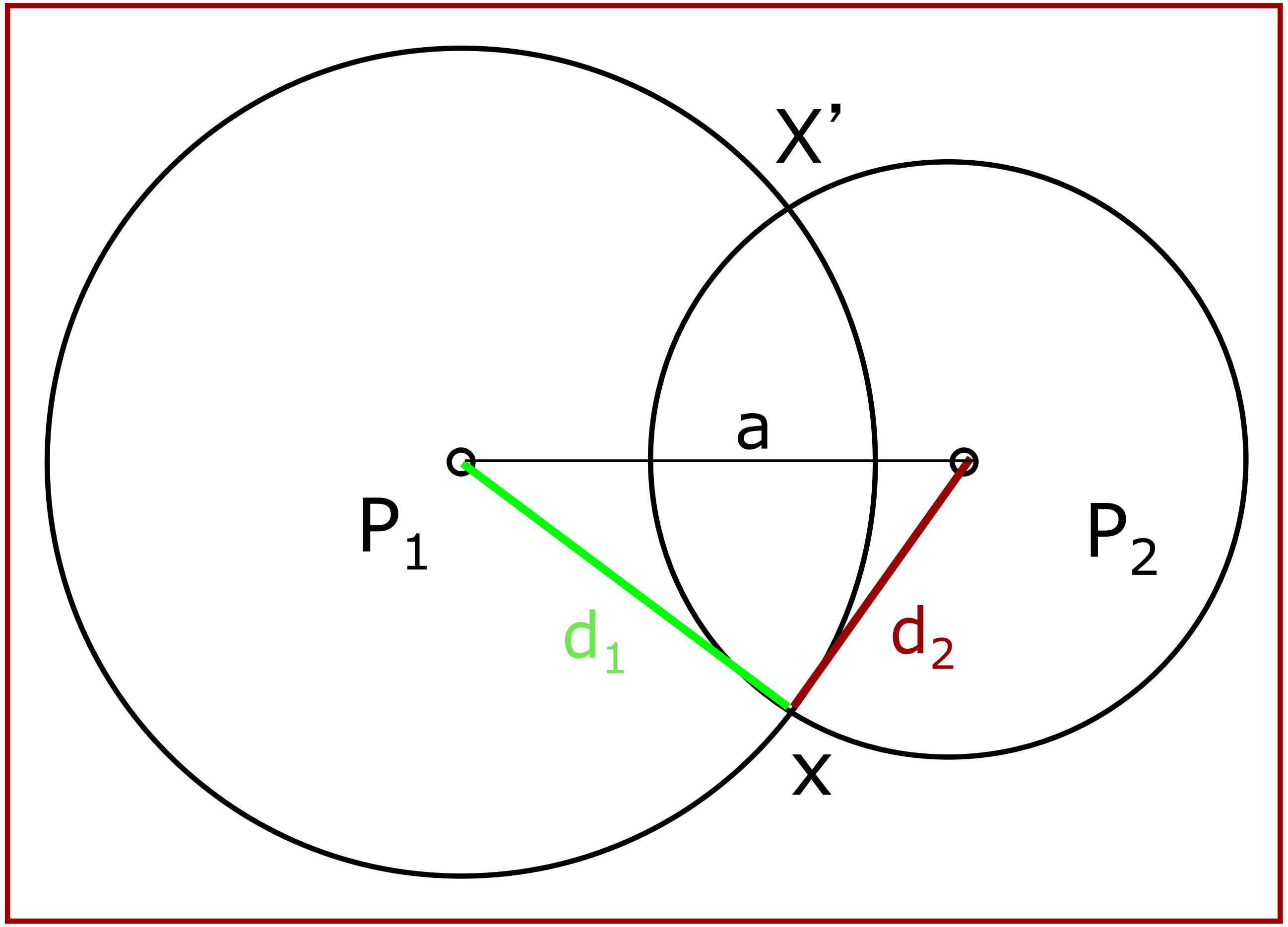








Distances Only, No Uncertainty



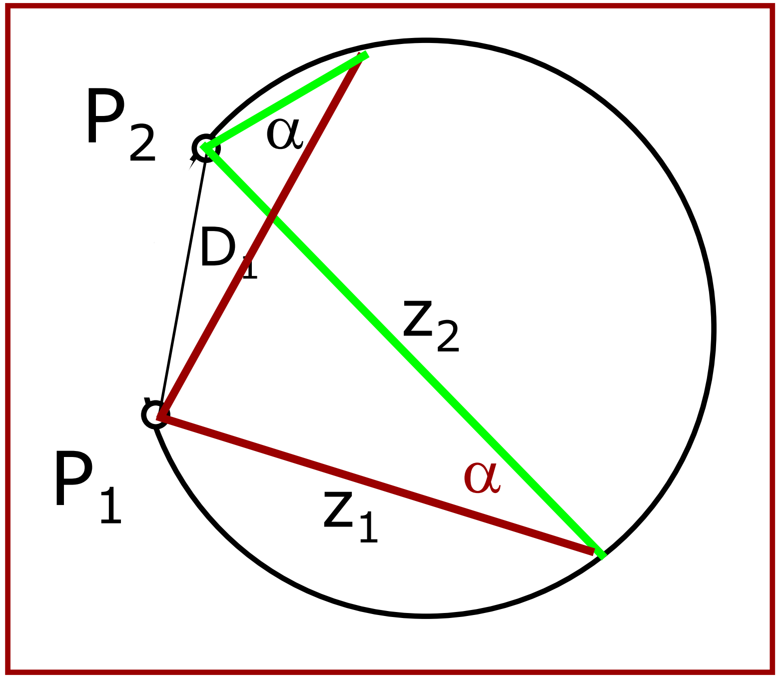
$$x = (a^2 + d_1^2 - d_2^2)/2a$$

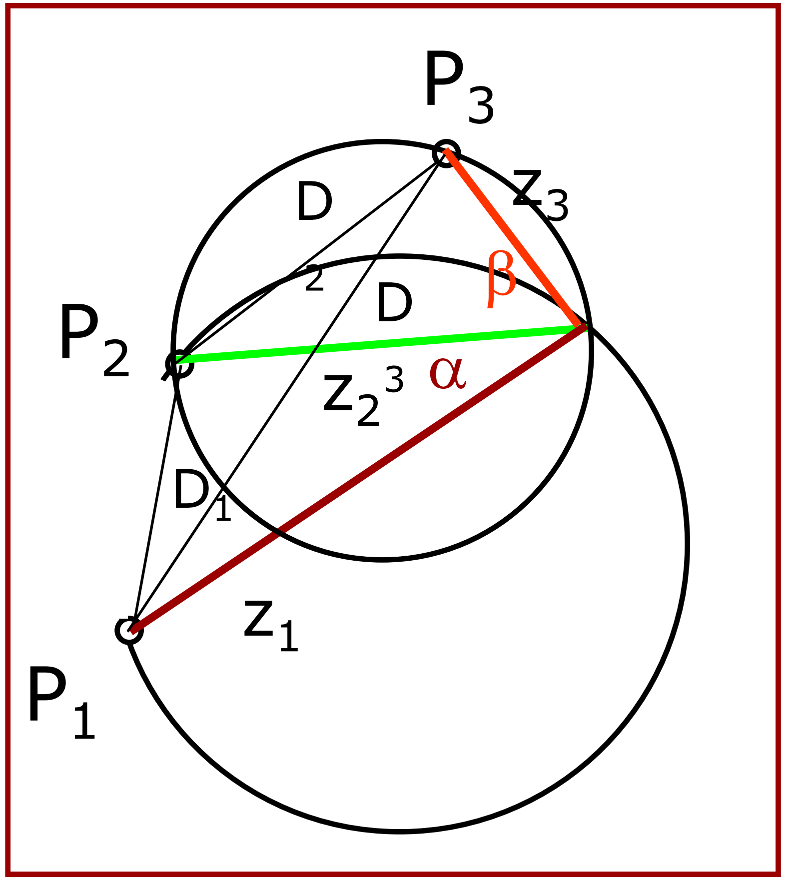
$$y = \pm\sqrt{(d_1^2 - x^2)}$$

$P_1=(0,0)$

$P_2=(a,0)$

Bearings Only, No Uncertainty





Law of cosine

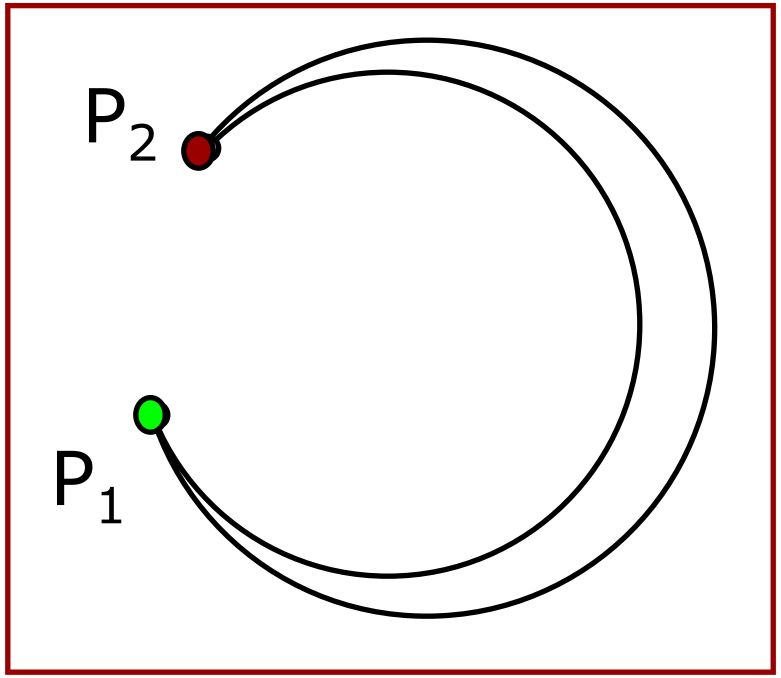
$$D_1^2 = z_1^2 + z_2^2 - 2\, z_1 z_2 \cos\alpha$$

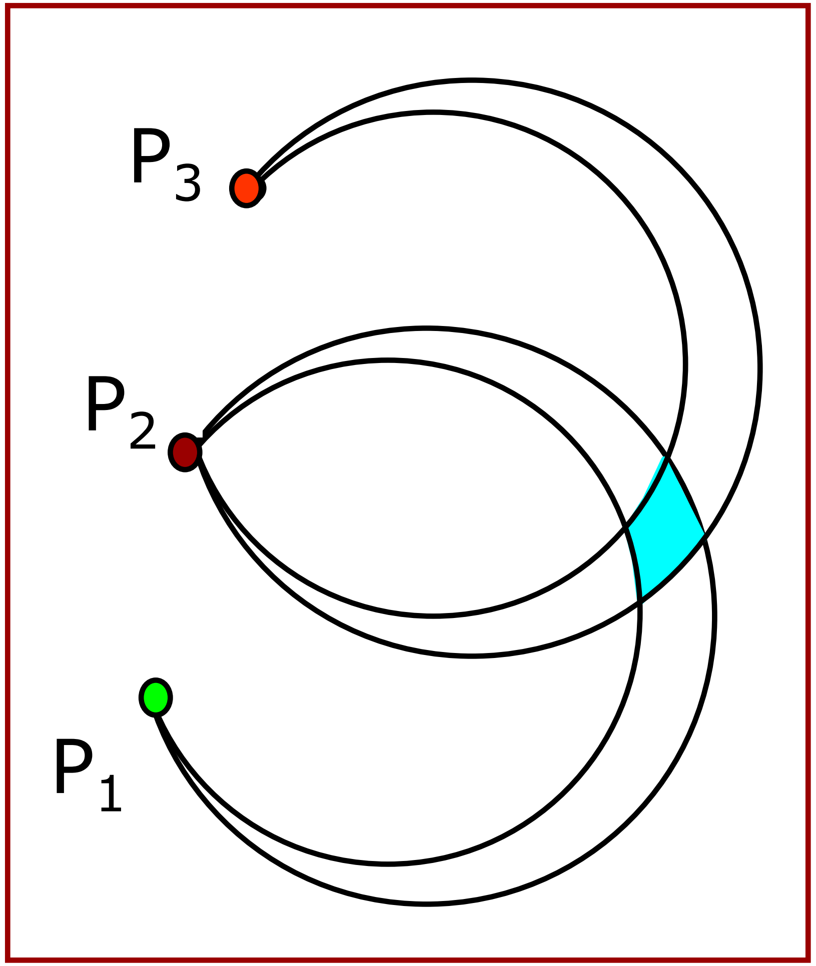
$$D_1^2 = z_1^2 + z_2^2 - 2\, z_1 z_2 \cos(\alpha)$$

$$D_2^2 = z_2^2 + z_3^2 - 2\, z_1 z_2 \cos(\beta)$$

$$D_3^2 = z_1^2 + z_3^2 - 2\, z_1 z_2 \cos(\alpha + \beta)$$

Bearings Only With Uncertainty





Most approaches attempt to find estimation mean.

Summary of Sensor Models

- Explicitly modeling uncertainty in sensing is key to robustness.
- In many cases, good models can be found by the following approach:
  1. Determine parametric model of noise free measurement.
  2. Analyze sources of noise.
  3. Add adequate noise to parameters (eventually mix in densities for noise).
  4. Learn (and verify) parameters by fitting model to data.
  5. Likelihood of measurement is given by "probabilistically comparing" the actual with the expected measurement.
- This holds for motion models as well.
- It is extremely important to be aware of the underlying assumptions!