

# Nonlinear Dynamic Systems

- Most realistic robotic problems involve nonlinear functions

$$\cancel{x_t = Ax_{t-1} + Bu_t + \varepsilon_t}$$



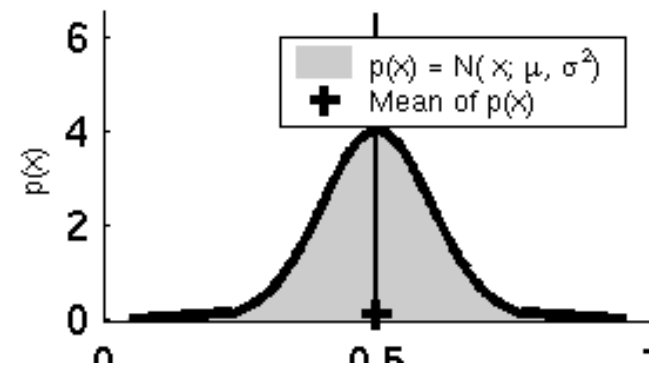
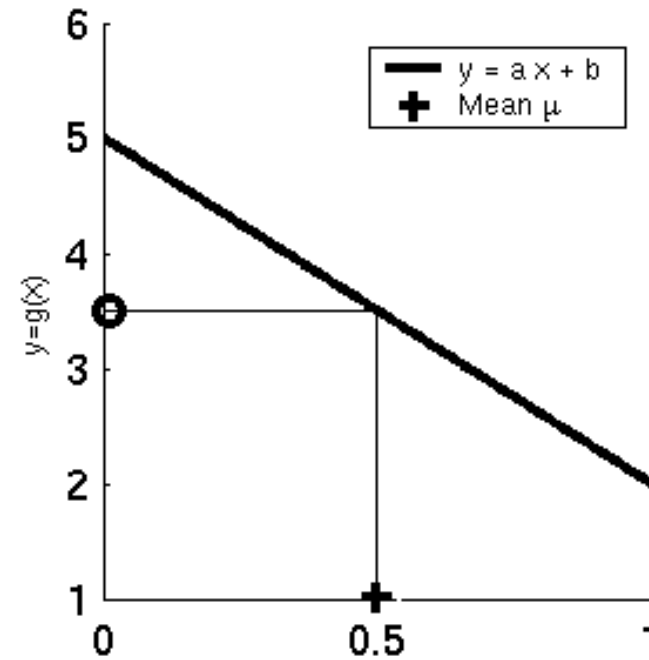
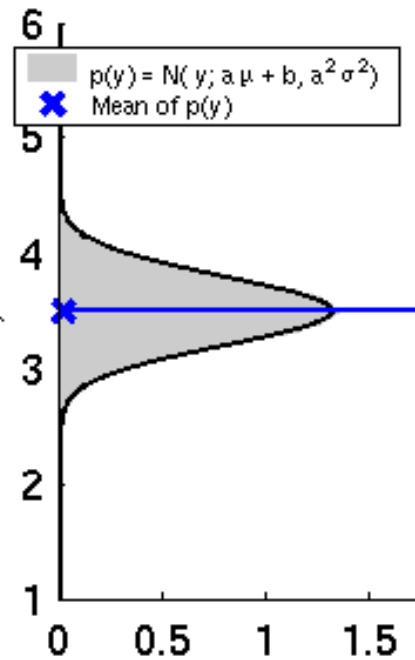
$$x_t = g(u_t, x_{t-1})$$

$$\cancel{z_t = Cx_t + \delta_t}$$

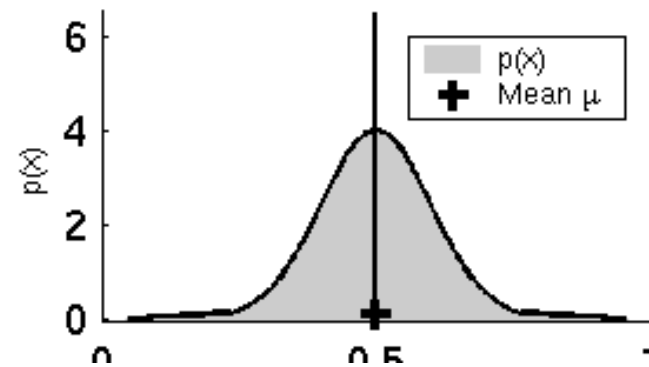
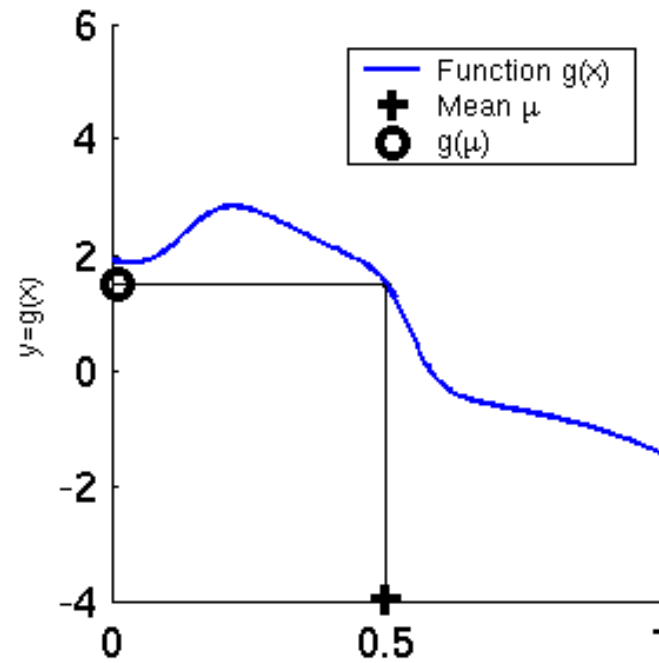
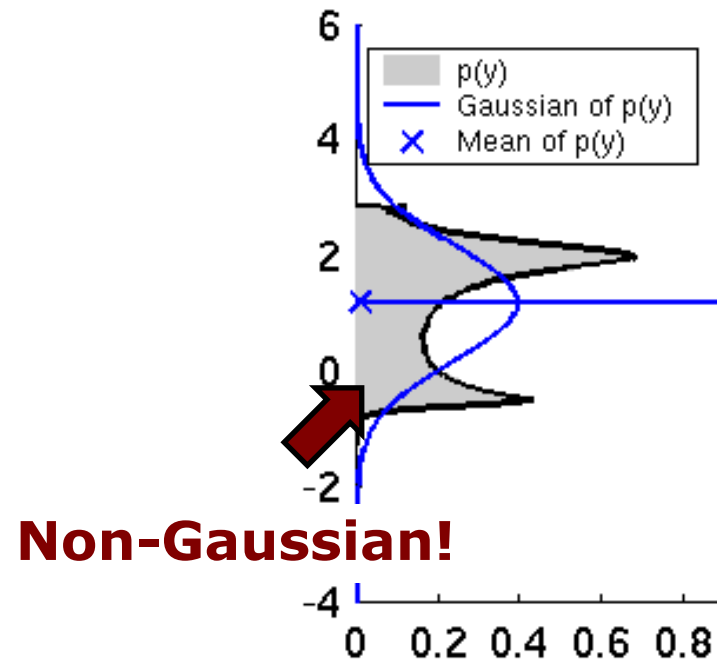


$$z_t = h(x_t)$$

# Linearity Assumption Revisited



# Non-Linear Function



# Non-Gaussian Distributions

- The non-linear functions lead to non-Gaussian distributions
- Kalman filter is not applicable anymore!

**What can be done to resolve this?**

# Non-Gaussian Distributions

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- Kalman filter is not applicable anymore!

**What can be done to resolve this?**

**Local linearization!**

# EKF Linearization: First Order Taylor Expansion

- Prediction:

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} (x_{t-1} - \mu_{t-1})$$

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t (x_{t-1} - \mu_{t-1})$$

- Correction:

$$h(x_t) \approx h(\bar{\mu}_t) + \frac{\partial h(\bar{\mu}_t)}{\partial x_t} (x_t - \bar{\mu}_t)$$

$$h(x_t) \approx h(\bar{\mu}_t) + H_t (x_t - \bar{\mu}_t)$$

Jacobian matrices



# Reminder: Jacobian Matrix

- It, in general, is a  $n \times m$  **non-square matrix**
- Given a vector-valued function

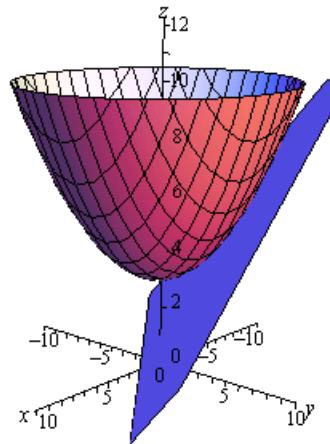
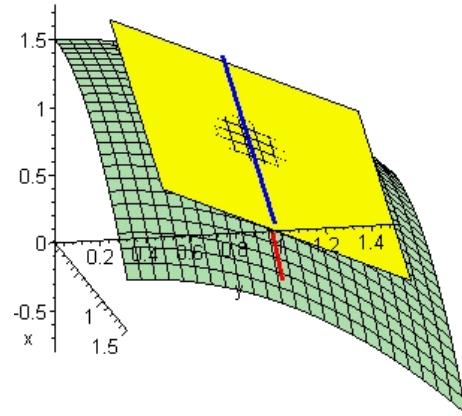
$$f(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_m(\mathbf{x}) \end{bmatrix}$$

- The **Jacobian matrix** is defined as

$$\mathbf{F}_{\mathbf{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

# Reminder: Jacobian Matrix

- It is the orientation of the tangent plane to the vector-valued function at a given point



- Generalizes the gradient of a scalar valued function



# EKF Linearization: First Order Taylor Expansion

- Prediction:

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} (x_{t-1} - \mu_{t-1})$$

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t (x_{t-1} - \mu_{t-1})$$

- Correction:

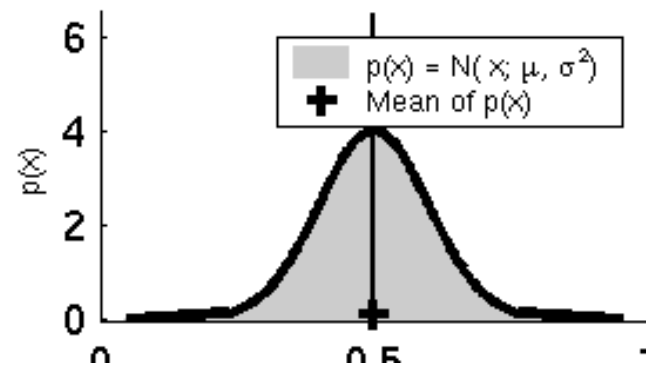
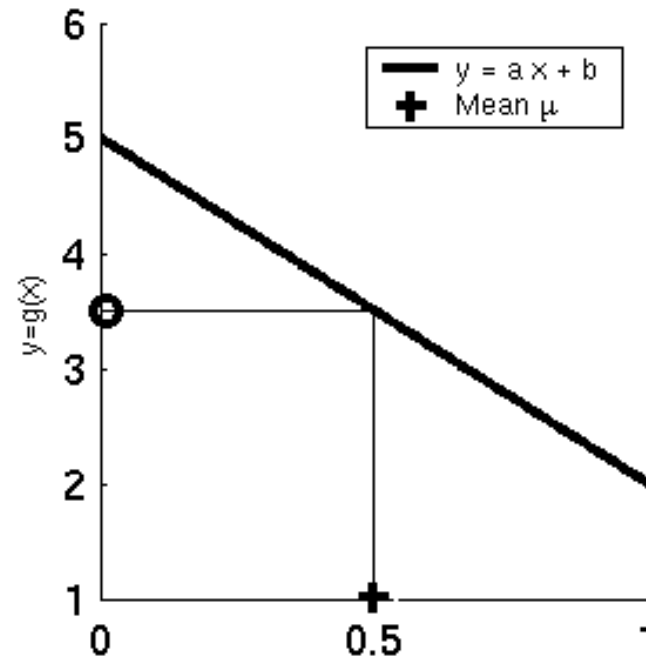
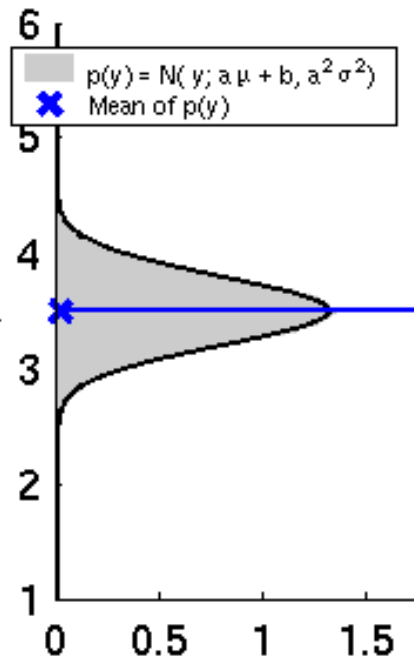
$$h(x_t) \approx h(\bar{\mu}_t) + \frac{\partial h(\bar{\mu}_t)}{\partial x_t} (x_t - \bar{\mu}_t)$$

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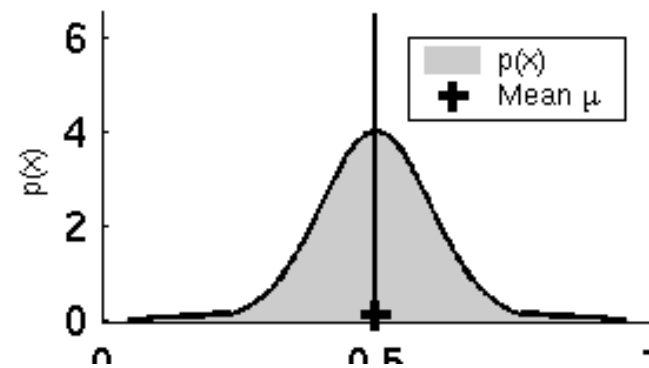
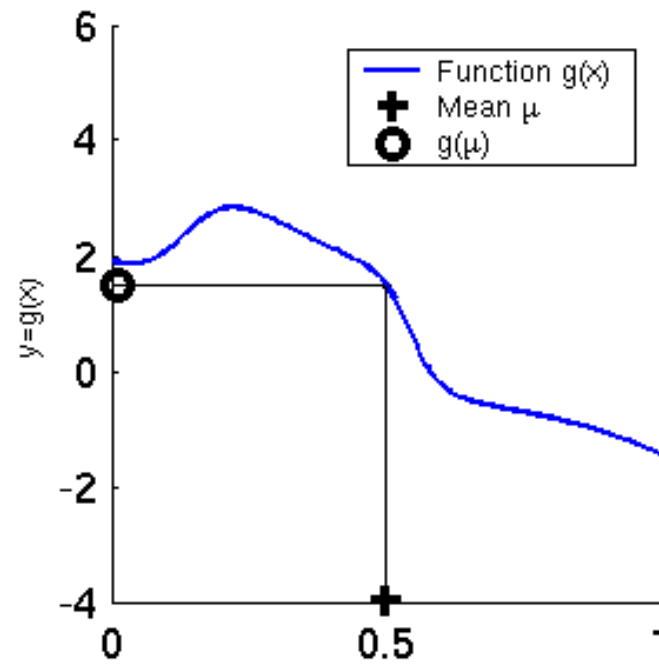
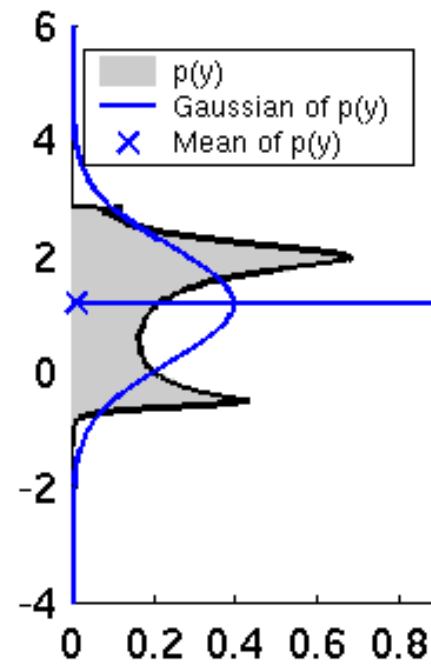
Linear function!



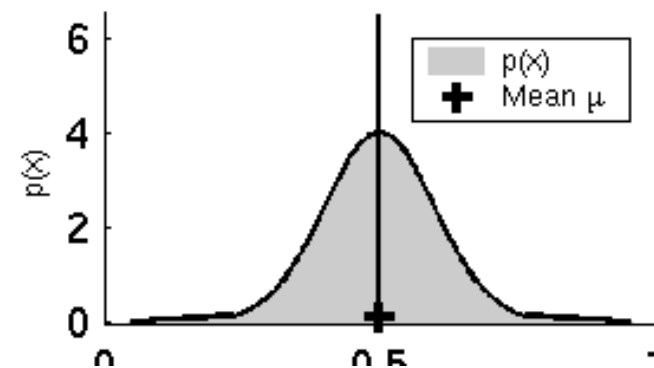
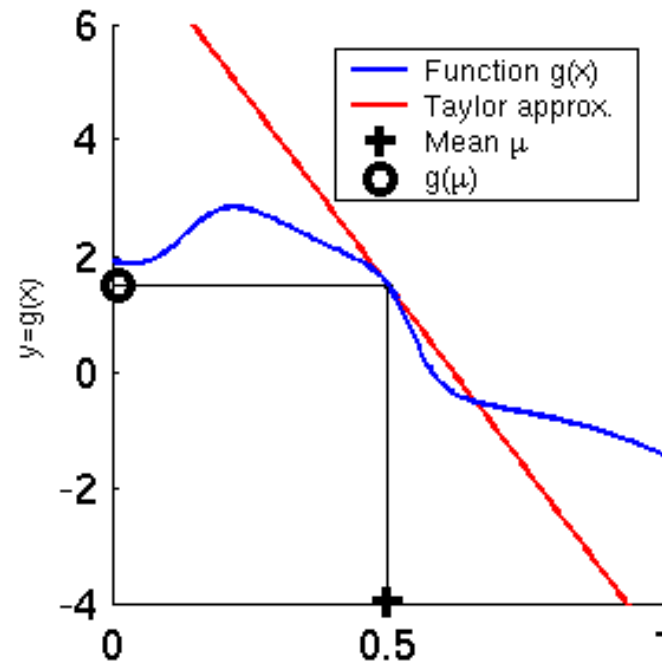
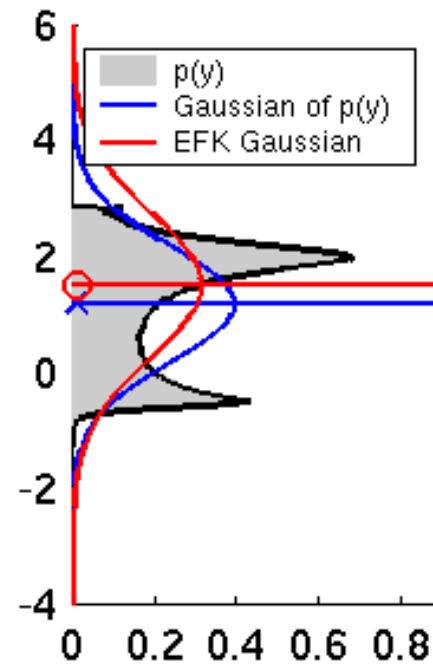
# Linearity Assumption Revisited



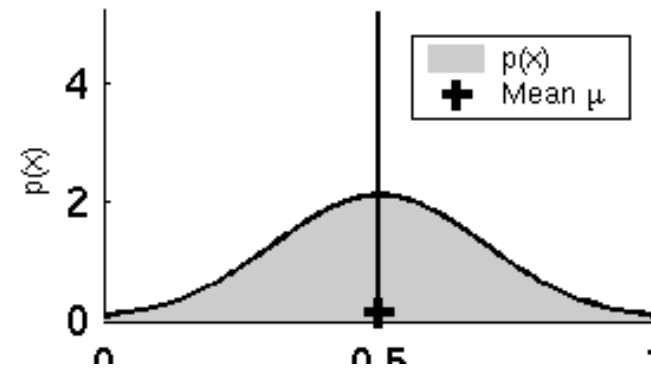
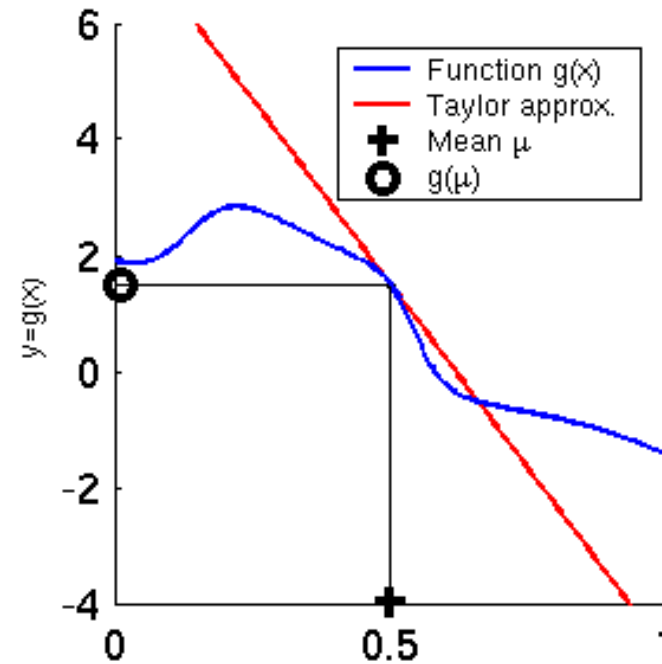
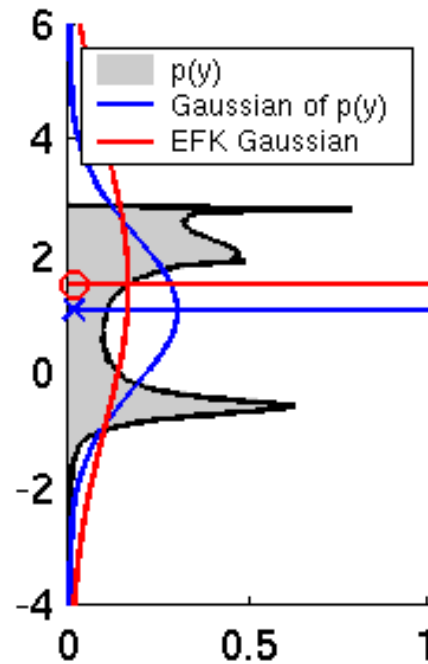
# Non-Linear Function



# EKF Linearization (1)



# EKF Linearization (2)



# EKF Linearization (3)

