EKF Algorithm

$$H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t}$$

$$G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$$

1. **Extended_Kalman_filter**(μ_{t-1}, Σ_{t-1}, u_t, z_t):

2. Prediction:

3. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ $\longleftarrow$ $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$

4. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + Q_t$ $\longleftarrow$ $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + Q_t$

5. Correction:

6. $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + R_t)^{-1}$ $\longleftarrow$ $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + R_t)^{-1}$

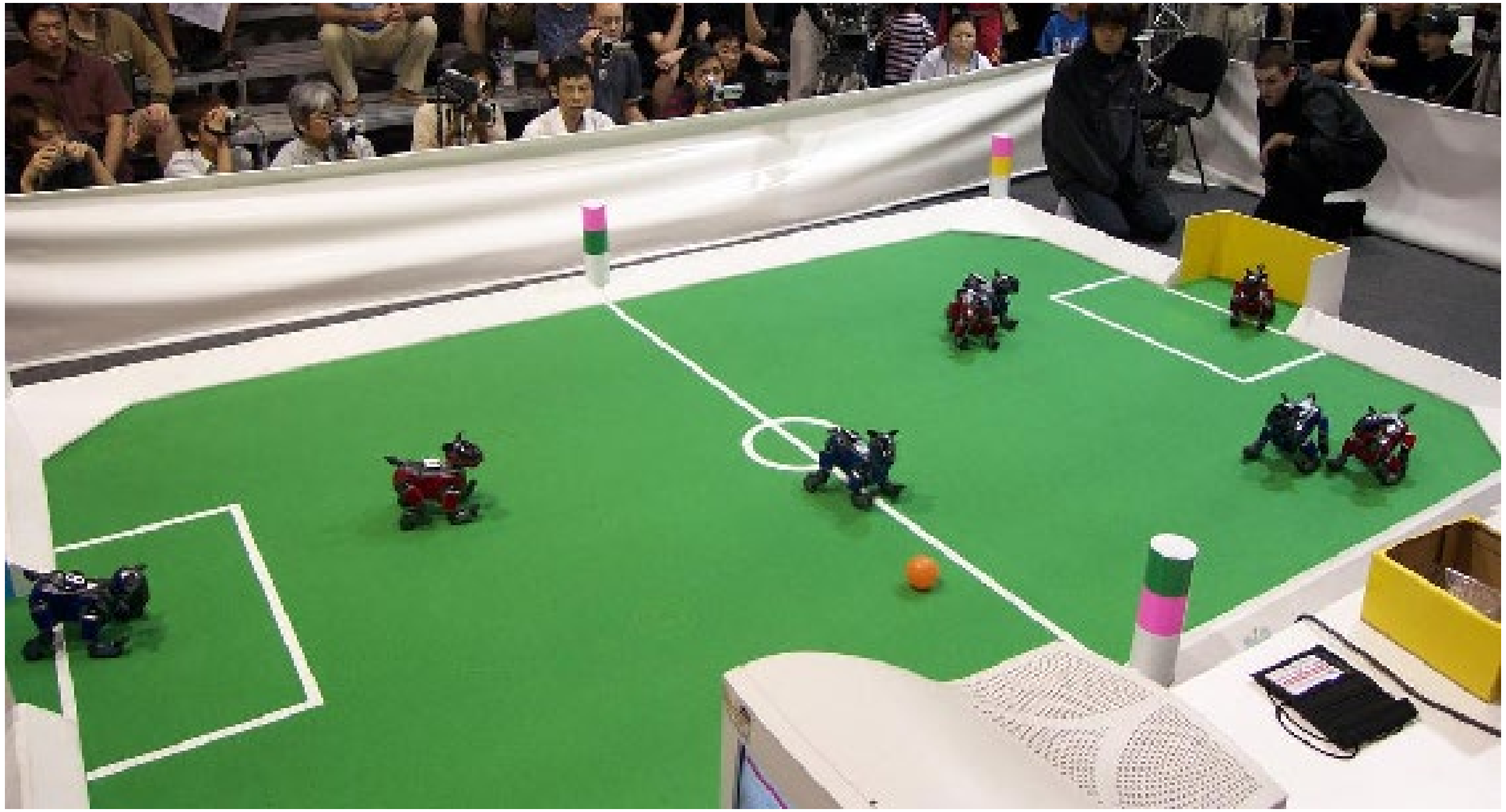
7. $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$ $\longleftarrow$ $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$

8. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$ $\longleftarrow$ $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

Example: EKF Localization

- EKF localization with landmarks (point features)



EKF_localization(μ_{t-1}, Σ_{t-1}, u_t, z_t, m):

Prediction:

$$G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial \mu_{t-1}} = \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix}$$

Jacobian of g w.r.t location

$$V_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial u_t} = \begin{pmatrix} \frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\ \frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\ \frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t} \end{pmatrix}$$

Jacobian of g w.r.t control

$$Q_t = \begin{pmatrix} (\alpha_1 |v_t| + \alpha_2 |\omega_t|)^2 & 0 \\ 0 & (\alpha_3 |v_t| + \alpha_4 |\omega_t|)^2 \end{pmatrix}$$

Motion noise

$$\bar{\mu}_t = g(u_t, \mu_{t-1})$$

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + V_t Q_t V_t^T$$

Predicted mean

Predicted covariance (V maps Q into state space)

EKF_localization($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, m$):

Correction:

$$\hat{z}_t = \begin{pmatrix} \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2} \\ \operatorname{atan2}(m_y - \bar{\mu}_{t,y}, m_x - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix}$$

Predicted measurement mean (depends on observation type)

$$H_t = \frac{\partial h(\bar{\mu}_t, m)}{\partial x_t} = \begin{pmatrix} \frac{\partial r_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \phi_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial \phi_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial \phi_t}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix}$$

Jacobian of *h* w.r.t location

$$R_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{pmatrix}$$

$$S_t = H_t \bar{\Sigma}_t H_t^T + R_t$$

Innovation covariance

$$K_t = \bar{\Sigma}_t H_t^T S_t^{-1}$$

Kalman gain

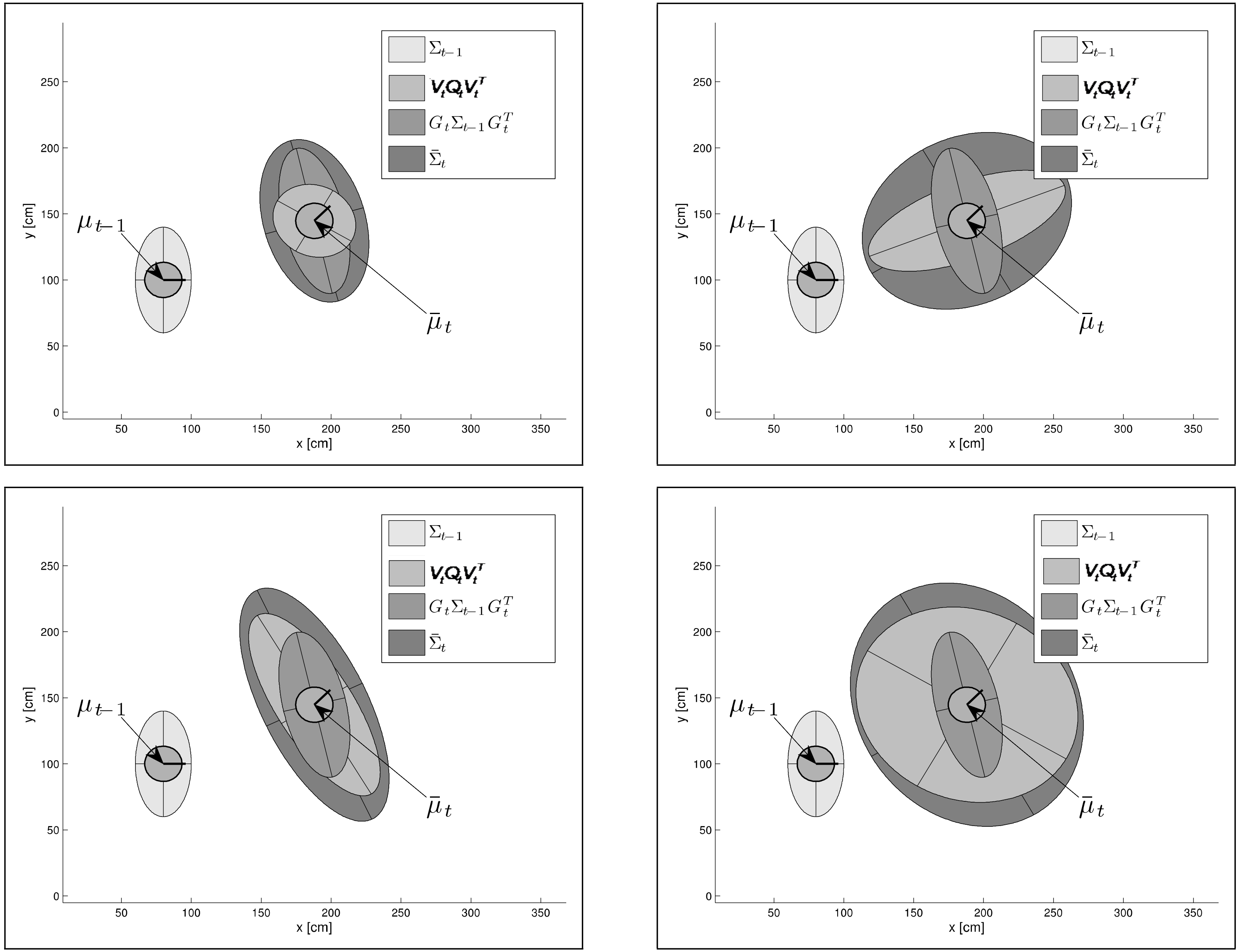
$$\mu_t = \bar{\mu}_t + K_t(z_t - \hat{z}_t)$$

Updated mean

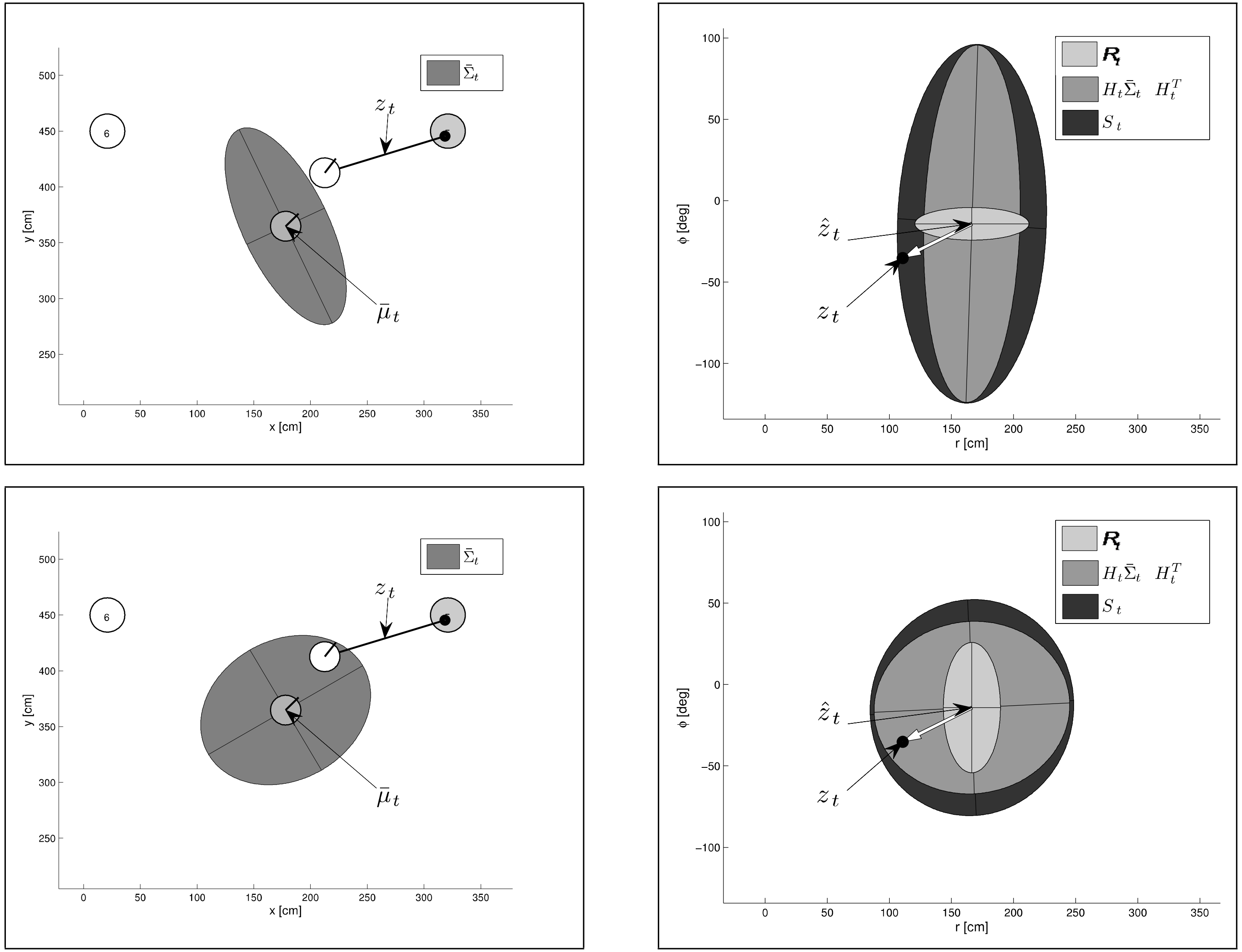
$$\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t$$

Updated covariance

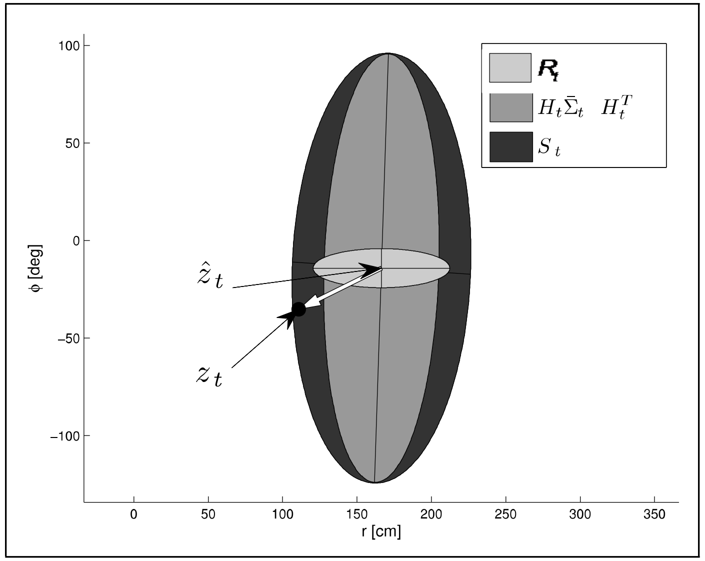
EKF Prediction Step Examples

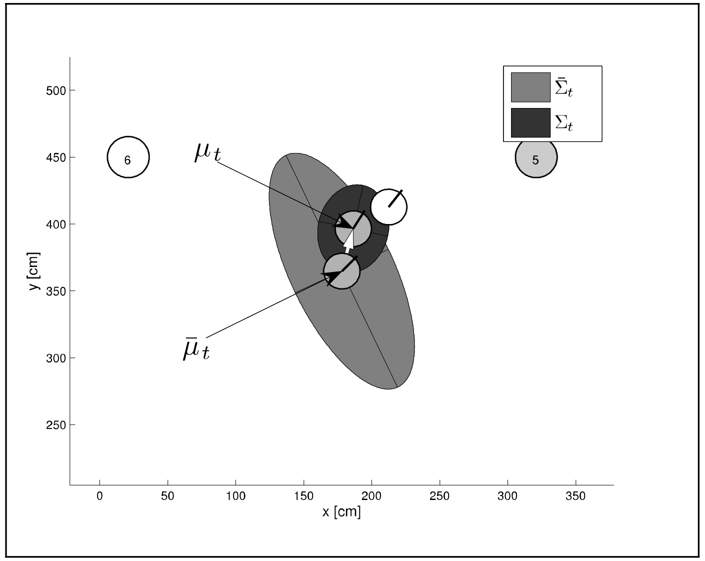


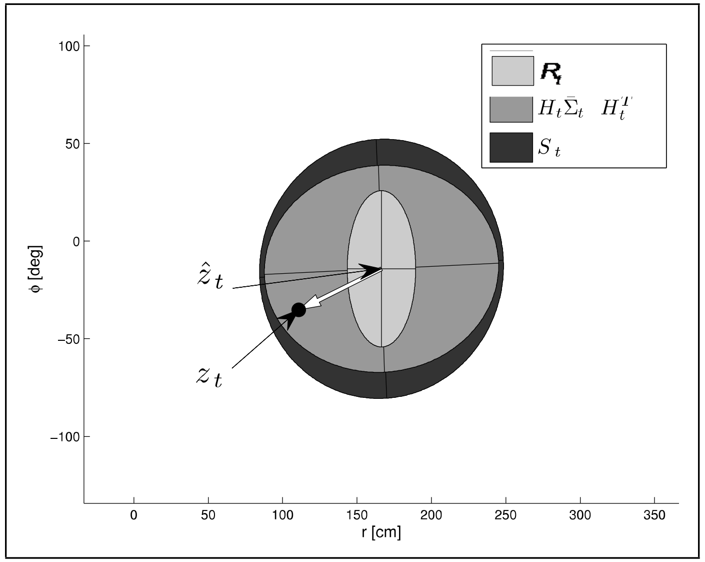
EKF Observation Prediction Step

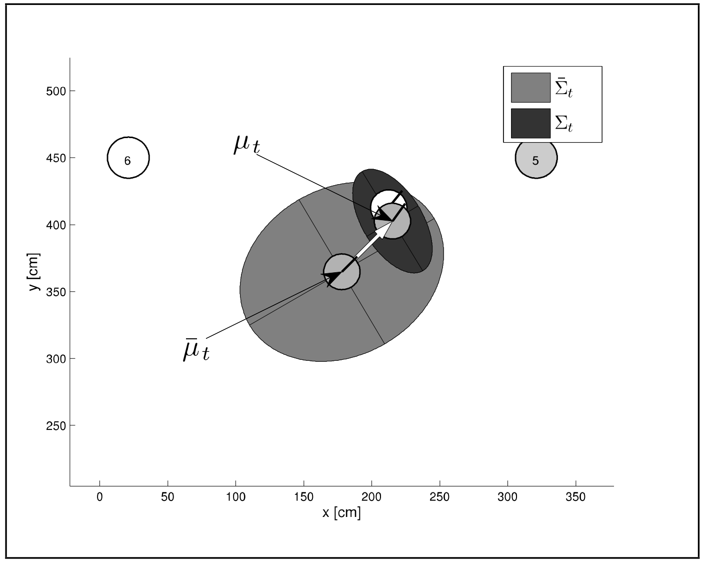


EKF Correction Step

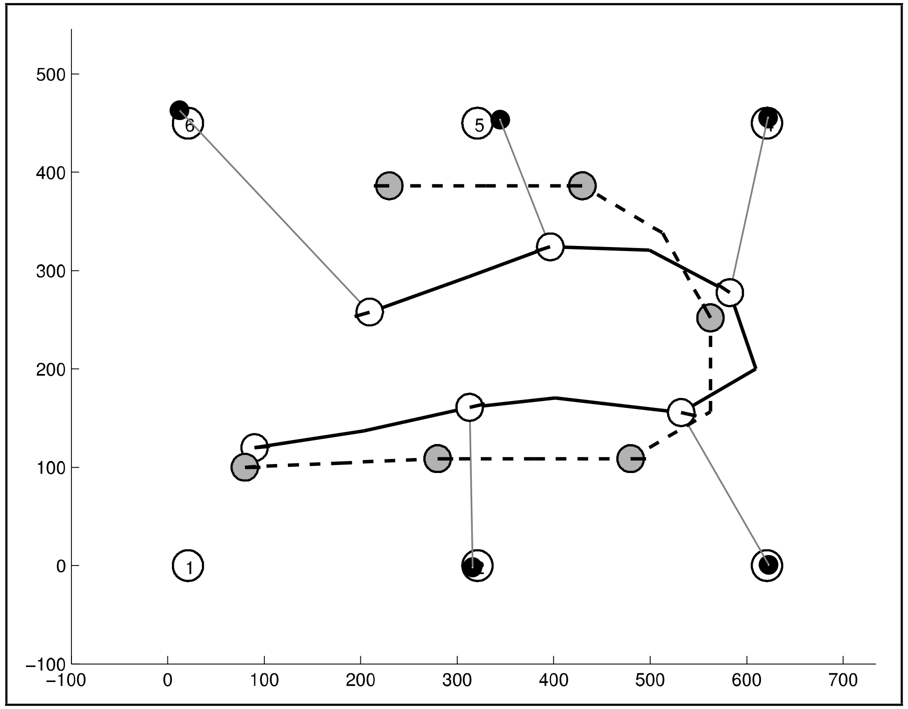


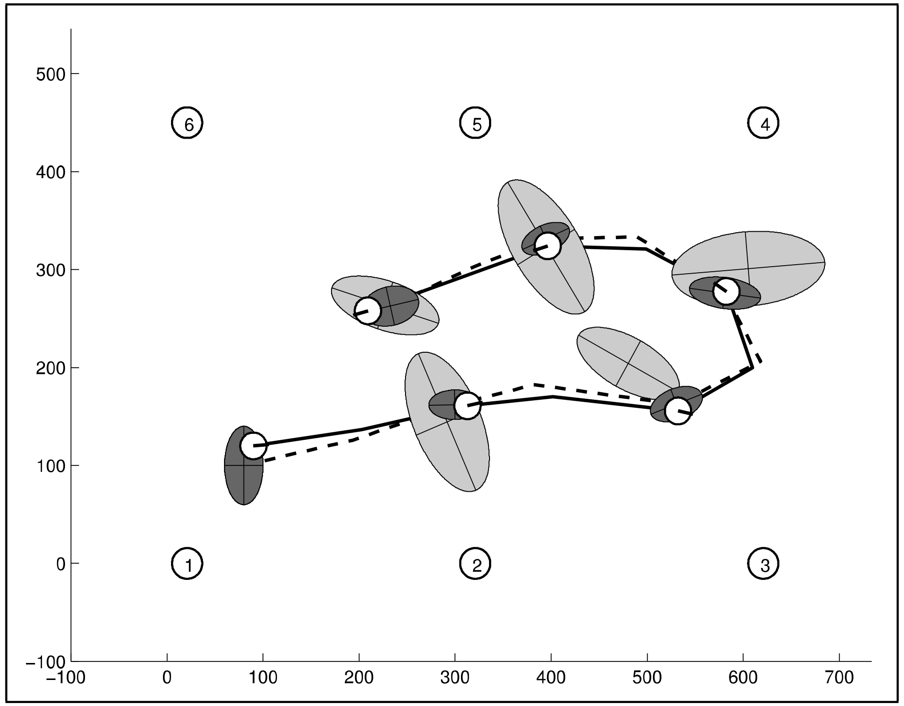




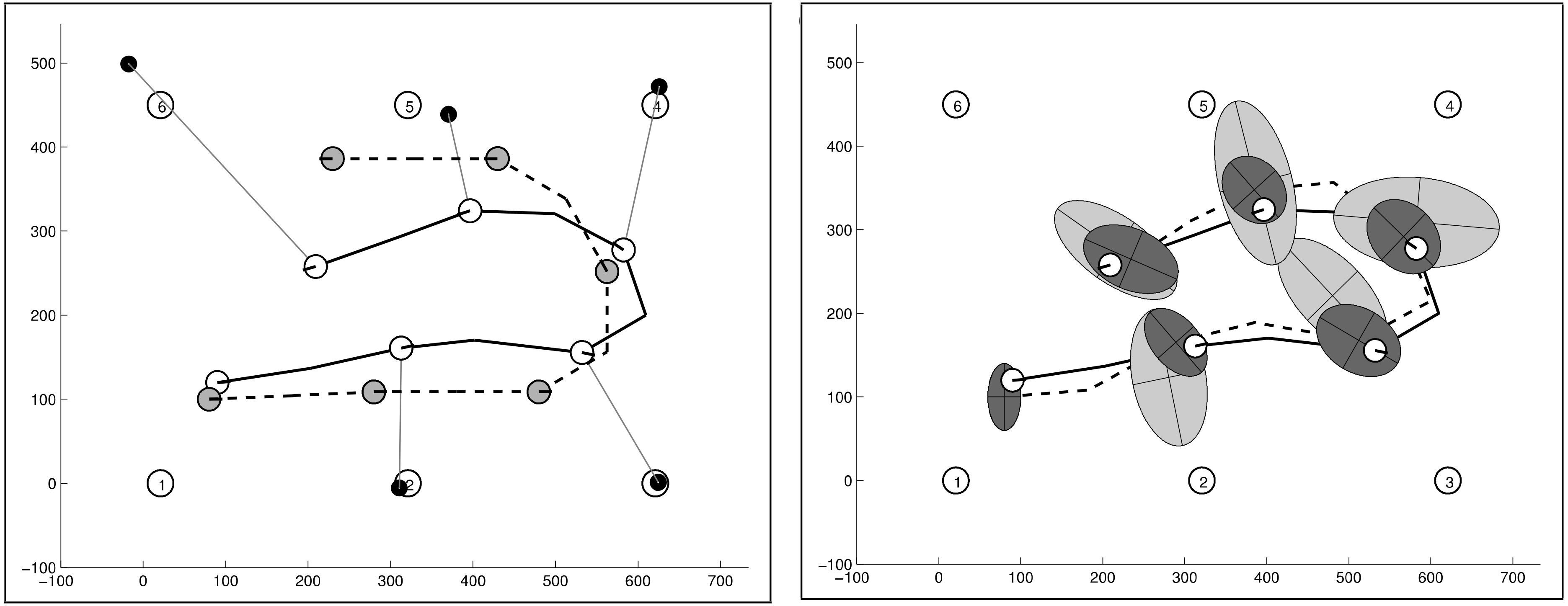


Estimation Sequence (1)





Estimation Sequence (2)



Extended Kalman Filter Summary

- The EKF is an ad-hoc solution to deal with non-linearities
- It performs local linearization in each step
- It works well in practice for moderate non-linearities (example: landmark localization)
- There exist better ways for dealing with non-linearities such as the unscented Kalman filter, called UKF
- Unlike the KF, the EKF in general is not an optimal estimator
- It is optimal if the measurement and the motion models are both linear, in which case the EKF reduces to the KF.