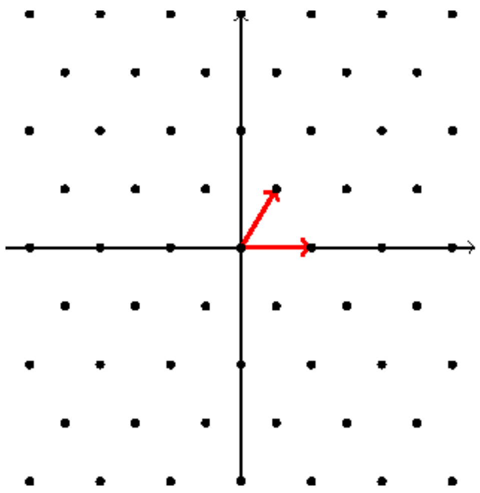
Summative Assignment

Module code and title	COMP2221 Programming Paradigms
Academic year	2023-24
Coursework title	System Programming - Summative
Coursework credits	10 credits
% of module's final mark	50%
Lecturer	Amir, Stuart
Submission date*	Tuesday, January 16, 2024 14:00
Estimated hours of work	20 hours
Submission method	Gradescope (code)

Additional coursework files	*None*
Required submission items and formats	*All source and header files needed to compile and run the program, Makefile that can generate the executable and report in pdf format.*

* This is the deadline for all submissions except where an approved extension is in place.

Managing Memory and Time in Future Cryptography: the Shortest Vector Problem



1. Background

Many real-world applications require significant computational resources to run at scale. As a result, it is very important to be able to manage the memory requirements and run-time at implementation time to ensure the applications can be efficiently and effectively deployed.

In this assignment, we are dealing with The Shortest Vector Problem (SVP). It represents a challenging computational task with far-reaching implications in the domains of lattice-based cryptography and computational mathematics. Within cryptography, SVP can serve as the foundation for secure lattice-based cryptographic systems and post-quantum cryptography. Beyond cryptography, SVP is utilised in error-correcting codes, network design, signal processing, and combinatorial optimisation problems. Its relevance extends to quantum computing, coding theory, integer programming, and diverse mathematical and computational domains, underpinning a wide range of real-world applications and cryptographic security.

The core objective of SVP is to find the shortest nonzero vector within a given lattice, where a lattice is a discrete set of points in n-dimensional space formed by linear combinations of a set of basis vectors. In mathematical terms, SVP seeks to determine a vector within the lattice such that its Euclidean norm (length) is minimised.

2. Problem Definition

The Shortest Vector Problem is the most famous and widely studied computational problem on lattices. A *Lattice* is a discrete and periodic grid-like structure formed by linear combinations of a set of *basis vectors* in n-dimensional Euclidean space.

The key characteristic of a lattice is that it extends infinitely in all directions. In the context of computational mathematics and computer science, lattice problems often refer to tasks related to these lattice structures. Common lattice problems include finding short vectors within a lattice, which is the Shortest Vector Problem (SVP).

The SVP problem that you need to solve through your implementation as part of this assignment is as defined as follows:

Given a lattice structure L embedded in n-dimensional Euclidean space $\mathbb{R}^n$, where L is generated by a set of linearly independent basis vectors $\{b_1, b_2, ..., b_n\}$, the objective is to identify a nonzero lattice vector $v \in L$ such that the Euclidean norm (L^2) of v is minimised. In short:

Find $v \in L,\ v \neq 0$, such that $||v||_2$ is minimised,

where $||v||_2$ is the Euclidean norm of vector v, defined as: $||v||_2 = \sqrt{\sum_1^n (v_i)^2}$, where v_i represents the components of vector v in the n-dimensional space.

Your goal is to implement a solution to this problem at an arbitrary number of dimensions n with considerations for runtime and memory requirements.

3. Hints

Here are a few hints to help get you started:

- Please read the submission instructions carefully. Since this is a programming assignment, deviations from the instructions might lead to you losing marks as a result of simple submission mistakes that can lead to your submission not running.

- Your implementation should be as efficient in its performance as possible, both in terms of memory and run-time.

- It is important that your solution can generalise to an arbitrary number of dimensions. You can start with a fixed small number of dimensions, like two, and then try to adjust your implementation to perform within an n-dimensional space.

- Creating the right data structures to hold your vector components and functions that perform the required operations on your vectors can make your implementation easier.

- You can implement **a brute-force search** to solve this problem. Enumerate all possible combinations of lattice vectors to find the shortest one. Keep track of the shortest vector found so far. Keep in mind that this brute-force approach can be highly inefficient and impractical for high-dimensional lattices, and of course, your implementation might be tested on a higher number of dimensions. As a result, **optimising the process is very important since part of the marks is given based on performance and memory requirements.**

- With the right optimisation, you can potentially get the full marks by implementing a brute-search approach. However, you can opt for more advanced algorithms.

- **You are not expected to develop novel algorithms**, but there is a class of [lattice reduction]() methods that can provide solutions in a much faster and more efficient manner than a brute-force search. Additional credit may be given for the use of more advanced algorithms. Consider the trade-off between memory and speed, when implementing advanced methods. A simple enumeration technique for an exhaustive search might use much less memory than a faster and more complex algorithm and can be easier to optimise.

- Implement error handling and bounds checking to ensure that your program handles invalid input, doesn't go into an infinite loop or other such issues. Not only will this make your implementation easier to test for yourself but is also part of the marking scheme and can affect your marks.

- Optimise your code for performance, especially if you are dealing with high-dimensional lattices. This might include algorithmic optimisations and careful memory allocation. Make sure your optimisation methods are explained in your report. Advanced optimisation methods will receive extra credit. Be mindful of memory usage, which is especially important in high-dimensional problems. Allocate and deallocate memory efficiently to avoid memory leaks.

- Test your implementation with known lattice problems where you already know the solution to verify its correctness. see https://www.latticechallenge.org/svp-challenge/ You can also try your solution on a "Lattice D", often denoted as D_n. It is one of the simplest and most fundamental lattices used for benchmarking and testing SVP algorithms. It serves as an excellent starting point for understanding the basic concepts of lattice problems and their solutions. Lattice D_n is an n-dimensional lattice formed by the integer combinations of the standard basis vectors in n-dimensional Euclidean space. In other words, it consists of all vectors whose components are integers. The basis vectors for Lattice D_n are the standard unit vectors. In two dimensions (D_2), you have basis vectors [1, 0] and [0, 1], while in D_3, you have basis vectors [1, 0, 0], [0, 1, 0], and [0, 0, 1], and so on. The shortest vector is always the shortest basis vector, which has a length of 1. This can be used as a benchmark to evaluate the correctness and efficiency of SVP-solving algorithms. Since the solution is known, you can verify if your algorithm correctly identifies the shortest vector.

- Understand the time and space complexity of your implementation to estimate its performance for different problem sizes. Cover this in your report.

- Make sure you submit your files well before the deadline to make sure that system or network issues do not affect your submission. Every year, we receive dozens of emails right after the submission deadline from students with their submissions marked as "late" due to a variety of reasons. Do not leave your submission to the final minutes and leave yourself plenty of time to deal with any issues that might affect your submission. Note that lecturers do not have the power to grant extensions and late submissions are handled by the office centrally.

4. Code and Submission Specifications

- Your program Your code should run on Linux. A Docker image of Ubuntu 22.04 with `` `apt-get install -y gcc` ``
will give you an identical environment to the one used for marking. Alternatively, you can use the University Linux system Mira. Failure to do so may result in all marks being lost regardless of whether it compiles and runs on your own windows machine. Note the Mira might occasionally experience downtime due to maintenance or other potential issues. **The deadline will not be extended due to issues with Mira** or other university provided systems. You are able to complete the implementation using the Docker image as explained above.

- You are allowed to use either C or C++ for your implementation.

- You are not allowed to use any external libraries but you can use all standard headers. Only use the headers that are needed. Do include what is not necessary as that will affect your marks.

- You may have as many source files and headers files as you need - considering that your code should be organised well and easy to follow - but your program should be run via an executable called `` `runme` ``, which should be produced by your program. This executable should only be called `` `runme` `` and should not have any extensions.

- You will need to include a Makefile and your Makefile should include an `` `all` `` option, which compiles all the necessary source files and headers that you submit and creates an executable `` `runme` ``. Our automated marking script will try to compile your program to get the executable by running the command `` `make all` ``.

- Your Makefile will be marked for functionality, clarity and correctness. Your Makefile should also contain `` `test` `` and `` `clean` `` options that respectively test the program with a known lattice and clean the compile environment.

- Your executable should be able to receive an arbitrary number of input arguments. These input arguments will correspond to the basis vectors that define the lattice. For instance, your program will be tested by running the following command:

`./runme [1.0 0.0 0.0] [0.0 1.0 0.0] [0.0 0.0 1.0]`

In this example, I have passed three three-dimensional basis vectors as inputs to your program. Other more complex vectors will be used to test your submission.

- The arguments will be passed and should be read in the format of the example you see above. There will always be the same number of input vectors as their dimensions. Your program should parse the arguments, infer the number of dimensions, and solve the shortest vector problem.

- Your program should be able to handle incorrect argument formats and numbers.

- The basis vectors passed in as input arguments to define the lattice can have real numbers as their components.

- The output of your program should be the length (L^2 norm) of the shortest vector as a scalar. You should not output the vector components but just the length. Your program must save the output of your program in a text file called `` `result.txt` ``. Your program should create this text file and just save one number, the final answer that your program generates, in that file.

- Some of these code specifications will be checked automatically on Gradescope after you submit your files. If Gradescope is showing that you have submitted your files in the wrong format and they cannot be run, this means that **your assignment will not be marked, and you will receive a mark of zero**. Make sure you submit your files well before the deadline to make sure that they are in the right format, and you have time to re-adjust so they can be run correctly. Do not ignore the errors and warnings from Gradescope.

- If your program does not compile and generate the correct executable named `./runme` and is not run via the command (with this of different arguments passed in this format)
  `./runme [1.0 0.0 0.0] [0.0 1.0 0.0] [0.0 0.0 1.0]`
  or does not generate the `` `result.txt` `` file with the final solution inside, you will lose marks or might get a mark of zero.

- You are responsible for how your submission files are uploaded to Gradescope – e.g., by zipping and submitting everything or submitting files individually. No guidance is provided on this matter. Make sure you submit your files well before the deadline to make sure that they are handled by the Gradescope system correctly.

- Do not submit any extra files. Compressed files, git related files, files produced by the Mac operating system if that is what you are using (e.g., DS_Store or other files) or any other unnecessary files will result in your submission being marked down.

5. Submission

- Full program source code files for your final solution to the above task as a working C or C++ program, meeting the above *“**code and submission specifications**”* for testing.

- You must submit the following files:

- All source and header files that are needed to compile your program.
- A Makefile that has the `all`, `test` and `clean` options and will compile your program and generate the `runme` executable.
- Report (max. 750 words) detailing your approach to the problem and the success of your solution in the task specified. Provide any illustrative figures, diagrams and tables (as many as you feel necessary) of the performance of your solution. ***Any diagrams, images, titles, captions, tables, references, and graphs do not count towards the total word count of the report.*** Summarise the success of your system in performing the required task, speed during run-time and reducing memory requirements. Your report can follow any structure as long as it is clear and easy to follow. **Submit a PDF (not in any other format**). Your submission will be marked down if any file format other than a pdf is submitted as the report.

6. Marks

The marks will be awarded as follows:

● Correctness of the solution	15%
○ You **may** receive partial marks for certain parts of the solution but note that certain wrong solutions might lead to the loss of not only the entirety of this portion of the marks but other parts as well.	
● Makefile	5%
○ Your Makefile should have `all`, `test` and `clean` options and will be marked based on correctness, functionality and generalisability.	
● Performance of your program in terms of run-time (speed)	10%
● Performance of your program in terms of memory requirements	10%
● Quality of the implementation	
○ Clear, readable, well-documented (with sufficient comments) and well-presented program source code	10%
○ Error handling	5%
○ Extensibility	5%
● Report:	
○ Discussion/detail of solution design and choices made	10%
○ *Quantitative* evidence and analysis of performance	10%

- Additional credit for one or more of, but not limited to, the following:
    - design and use of an alternative or novel methods

- use of advanced optimisation methods to improve performance
- superior performance in terms of speed and memory use
- the capability to handle overly complex lattices
  (for any of the above, up to a maximum, dependent on quality) 20%

Total: 100%

Plagiarism: You must not plagiarise your work. Attempts to hide plagiarism by simply changing comments/variable names will be detected. You should have been made aware of the Durham University policy on plagiarism.

You should not make your submission publicly available or share it with others as that will be considered plagiarism.

Submissions should be made through the Gradescope system. Delayed submissions are handled centrally and are not under the control of the lecturers.