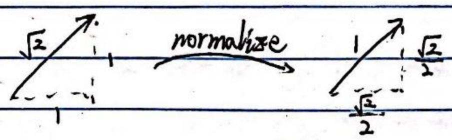
1.

① No, each $g_n$ may not all belongs to $Q^N$.
Because it may generate real number when doing the normalization.




where $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) \notin Q^2$

② a. Since $Q$ is a dense subset of $R$, $\forall x \in R$ and $\varepsilon > 0$, $\exists y \in Q$
s.t. :

$$\| x - y \| < \varepsilon$$

According to Gram-Schmidt, exist orthonormal basis $\{g_n\}_{n=1}^{N}$ for every non-zero vector space.
Suppose $\{g_n\}_{n=1}^{N}$ is basis of $Q^N$, then :

$$y = \sum_{i=1}^{N} b_n g_n, \quad \{b_n\}_{n=1}^{N} \in Q$$

Therefore,

$$\| x - \sum_{i=1}^{N} b_n g_n \| > \varepsilon$$

b. It is useful. Because we know that $R^n$ is an infinite vector space, but $Q$ is a finite vector space. It means we can use a set of basis in finite space to represent the vector in infinite space.

1.2

```
image = imread('hendrix_final.png');
image = double(image);
imageR=[image(:,:,1)];
[m,n]=size(imageR);
Q=zeros(m,n);
R=zeros(m,n);
for j=1:n
    v=imageR(:,j);
    for i=1:j-1
        R(i,j)=Q(:,i)'*imageR(:,j);
        v=v-R(i,j)*Q(:,i);
    end
    R(j,j)=norm(v);
    Q(:,j)=v/R(j,j);
end
```

2.

3. For $x$ :

Let $f(x) = \phi(y) - \phi(x) - (y-x)\phi'(x)$

$z \in [x, y]$ $\quad f'(z) = (z-y)\phi''(z)$

$f''(z) = \phi''(z) + (z-y)\phi'''(z)$

Since cannot make sure $f''(z) \geq 0$, cannot decide whether $f(x)$ is convex or not.

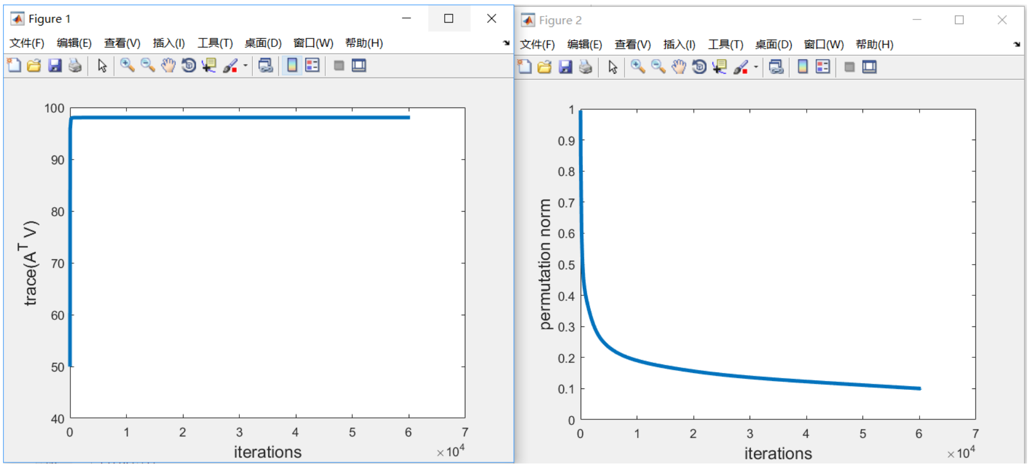
For $y$: $\quad f(y) = \phi(y) - \phi(x) - (y-x)\phi'(x)$

$z \in [x, y]$ $\quad f'(z) = \phi'(y) - \phi'(x)$

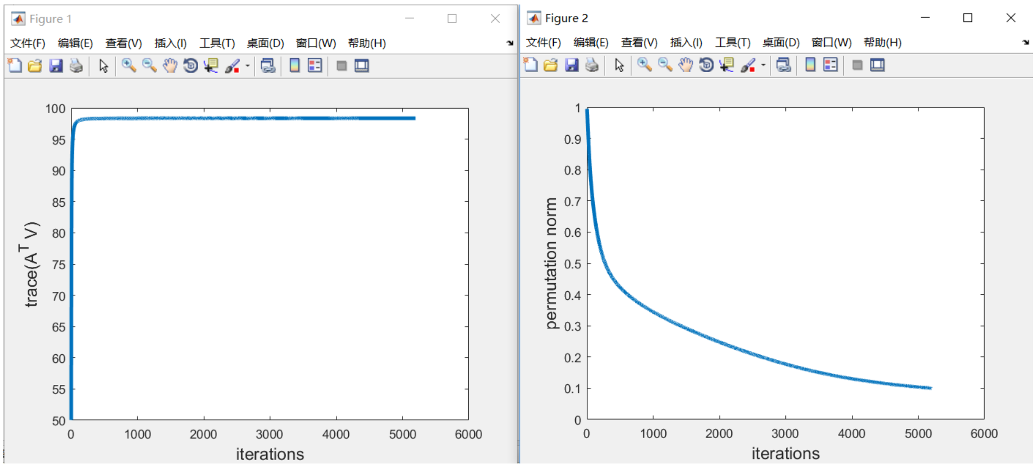
$f''(z) = \phi''(y)$

If $\phi(x)$ is convex, which means $\phi''(x)$ is greater or equal to zero, then $f(y)$ is convex. Otherwise, $f(y)$ is not convex assBreg:

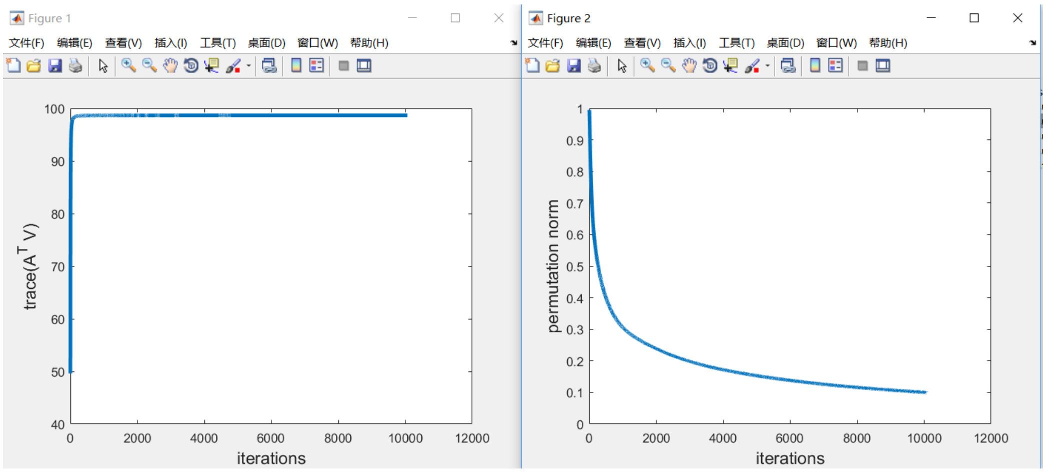
Alpha=1 l2thr=0.1 permnorm=0.1




Time: 214.763398s (assBreg) 0.017122s (lapjv)




Time: 9.916508s (assBreg) 0.015597 (lapjv)




Time: 27.131764s (assBreg) 0.015595 (lapjv)

The result is in data.mat in “hw7”. (I didn’t have enough time to run 10 matrices, so I just did 3 instead) (Github: https://github.com/billy607/math_homework.git)