

## Introduction to Magic Sets a.k.a. Sideways Information Passing (SIP)

Objective:

- Learn about the key *logical* optimization concerning
  - Nested queries (so also views)
  - Recursive Datalog programs and SQL queries (SQL 99)
  - It follows, central to graph database query engines, (but almost none are there yet).

Slide thanks: (originals can be found under readings)

- Ramakrishnan and Gehrke text slides
- Not borrowed for lecture, but are very goo, Stefan Brass's course slides

## Scholarship:

With the identification of recursion in the relational model...

- In the beginning, invited paper, SIGMOD '86  
François Bancilhon, Raghu Ramakrishnan:  
**An Amateur's Introduction to Recursive Query Processing Strategies.** 16-52

## Scholarship:

With the identification of recursion in the relational model...

And then (>1986), annually

- SIGMOD, VLDB, ICDE & PODS, at least one full session:

### Recursion

- Isabel F. Cruz, Alberto O. Mendelzon, Peter T. Wood:  
**A Graphical Query Language Supporting Recursion.** 323-330
- H. V. Jagadish, Rakesh Agrawal, Linda Naser:  
**A Study of Transitive Closure As a Recursion Mechanism.** 331-344
- Weining Zhang, Clement T. Yu:  
**A Necessary Condition for a Doubly Recursive Rule to be Equivalent to**  
[SIGMOD '87]

## Critical Papers:

The beginning of the end:

- 1990
  - (PODS) "Magic Conditions", Mumick et.al.... *Ramakrishnan*
  - (SIGMOD) "Magic is Relevant", same authors

Then:

- 1996
  - (SIGMOD) "Cost-Based Optimization for Magic: Algebra and Implementation", Seshadri, Hellerstein,..., *Ramakrishnan*, ...

Commercial implementations followed, then standardized: SQL 99

- On going research – occasional paper, not >12 major papers a year

Two more things to know, upfront,  
... to get your attention

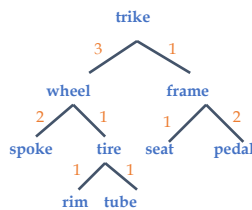
- Conceptual, logical, physical....
  - Our interest, relative to graph database queries: *logical*
- Our physical targets are very different, but:
 

**“Pushing  $\theta$ -Semijoin through Join:** We present below a transformation rule that describes how to push  $\theta$ -semijoins through joins.  
[Cost-Based Optimization for Magic: Algebra and Implementation]
- Bloom filters:
  - Central to cloud-native data stores
  - Enable, fast, approximate, implementation of semi-join

Today’s running example:

- From Ramakrishnan and Gehrke

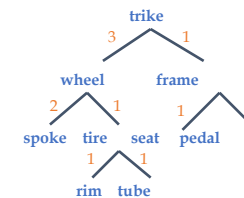
A Graph DB,  
- directed, (not illustrated)  
- edge labeled



	part	subpart	number
trike	wheel	3	
trike	frame	1	
frame	seat	1	
frame	pedal	2	
wheel	spoke	2	
wheel	tire	1	
tire	rim	1	
tire	tube	1	

Assembly instance

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	part	subpart	number
trike	wheel	3	
trike	frame	1	
frame	seat	1	
frame	pedal	1	
wheel	spoke	2	
wheel	tire	1	
tire	rim	1	
tire	tube	1	

Assembly instance

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- Find [all] components of “trike”.
- Can you write a relational algebra query to compute answer on the given instance of Assembly?

part	subpart	number
trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Assembly instance

- Can determine, **trike** is made of **wheels** and **frame**
  - 1 self-join of Assembly
- Union, **spoke, tire, seat** and **pedal**
  - Starting with trike – two self joins
  - Each such self-join, extends the path by 1
- Union,
  - ... need as many self-joins as height of the tree.
  - But, can't know tree height, without looking at the data
  - can't write the relational algebra before looking at the data.

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part	subpart	number
trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Assembly instance

- All components of trike  $\equiv$  transitive closure of trike
- Can't express that in pure relational algebra

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part	subpart	number
trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Assembly instance

- Recall: relational algebra  $\equiv$  nonrecursive Datalog
- Unspecified path length  $\rightarrow$  recursive Datalog

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part	subpart	number
trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Assembly instance

- Unspecified path length  $\rightarrow$  recursive Datalog

Rule 1:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- } \text{Assembly}(\text{Part}, \text{Subpt}, \text{Qty})$ .

Rule 2:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- } \text{Assembly}(\text{Part}, \text{Part2}, \text{Qty}), \text{Comp}(\text{Part2}, \text{Subpt})$ .

Dependency graph:

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## If not Relational Algebra, what about SQL?

In SQL

- Keyword, WITH
- A statement forms a *temporary, intermediate table*,
  - Statement is called a <Common Table Expression>, (CTE)
- As needed, keyword, RECURSIVE

## If not Relational Algebra, what about SQL?

In SQL

- Keyword, WITH
- A statement forms a *temporary, intermediate table*,
  - Statement is called a <Common Table Expression>, (CTE)
- As needed, keyword, RECURSIVE

Documentation:

- <https://www.postgresql.org/docs/11/queries-with.html>

Tutorials:

- <https://www.sqlservertutorial.net/sql-server-basics/sql-server-cte/>
- <https://www.sqlservertutorial.net/sql-server-basics/sql-server-recursive-cte/>

## Recursive SQL Query Example

```
WITH RECURSIVE Comp(Part, Subpt) AS /* Define Comp */
```

```
(SELECT A1.Part, A1.Subpt FROM Assembly A1)
```

```
UNION
```

```
(SELECT A2.Part, C1.Subpt
FROM Assembly A2, Comp C1
WHERE A2.Subpt=C1.Part)
```

```
SELECT * FROM Comp C2 /* Returns all parts of a trike */
```

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## Recursive SQL Query Example

```
WITH RECURSIVE Comp(Part, Subpt) AS /* Define Comp */
```

*Think of Comp is a View name*

```
(SELECT A1.Part, A1.Subpt FROM Assembly A1)
```

```
UNION
```

```
(SELECT A2.Part, C1.Subpt
FROM Assembly A2, Comp C1
WHERE A2.Subpt=C1.Part)
```

```
SELECT * FROM Comp C2 /* Returns all parts of a trike */
```

1) Initialize:  $Comp(Part, Subpt) :- Assembly(Part, Subpt, Qty).$

2) Add through repeated, recursive, execution  
 $Comp(Part, Subpt) :- Assembly(Part, Part2, Qty), Comp(Part2, Subpt)$

3) Get a result as is the net result, Comp, is the name of a view

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## Review: Revisit Naïve Evaluation

### Beginning Cycle 1

Comp

Assembly

trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Comp, is an empty EDB table

Rule 2 is not satisfied

Eval rule 1, and get

Rule 1:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- } \text{Assembly}(\text{Part}, \text{Subpt}, \text{Qty})$ .  
 Rule 2:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- } \text{Assembly}(\text{Part}, \text{Part2}, \text{Qty}), \text{Comp}(\text{Part2}, \text{Subpt})$ .

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### End Cycle 1

Comp (1)

Assembly

trike	wheel	
trike	frame	
frame	seat	
frame	pedal	
wheel	spoke	
wheel	tire	
tire	rim	
tire	tube	

trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Rule 1:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- } \text{Assembly}(\text{Part}, \text{Subpt}, \text{Qty})$ .  
 Rule 2:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- } \text{Assembly}(\text{Part}, \text{Part2}, \text{Qty}), \text{Comp}(\text{Part2}, \text{Subpt})$ .

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Assembly

Comp (1)

trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

trike	wheel	
trike	frame	
frame	seat	
frame	pedal	
wheel	spoke	
wheel	tire	
tire	rim	
tire	tube	

Rule 1, again:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- } \text{Assembly}(\text{Part}, \text{Subpt}, \text{Qty})$ .

Set theory – new rows  
don't appear twice

Rule 2:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- } \text{Assembly}(\text{Part}, \text{Part2}, \text{Qty}), \text{Comp}(\text{Part2}, \text{Subpt})$ .

Comp tuples by  
applying Rule 2  
once, (in cycle 2)

trike	wheel
trike	frame
frame	seat
frame	pedal
wheel	spoke
wheel	tire
tire	rim
tire	tube

trike	spoke
trike	tire
trike	seat
trike	pedal
wheel	rim
wheel	tube

### Cycle 2

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Cycle 3

trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Assembly instance

trike	spoke
trike	tire
trike	seat
trike	pedal
wheel	rim
wheel	tube

Comp tuples by applying Rule 2 again in cycle 3

trike	spoke
trike	tire
trike	seat
trike	pedal
wheel	rim
wheel	tube

Union, New, Comp tuples by applying Rule 2, again in cycle 3

Comp(Part, Subpt) :-  
 Assembly(Part, Part2, Qty),  
 Comp(Part2, Subpt).

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Cycle 4

trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Assembly instance

trike	spoke
trike	tire
trike	seat
trike	pedal
wheel	rim
wheel	tube

Comp tuples by applying Rule 2 again in cycle 3

trike	spoke
trike	tire
trike	seat
trike	pedal
wheel	rim
wheel	tube
trike	rim
trike	tube

Union, New, Comp tuples by applying Rule 2, again in cycle 4

Comp(Part, Subpt) :-  
 Assembly(Part, Part2, Qty),  
 Comp(Part2, Subpt).

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In cycle 4 we finished computing the transitive closure

- Cycle 5 – no new rows → achieved fixed point.
- But, we have, on cycle 5, recomputed the entire contents of the EDB

Independent of Efficiency...  
 Have we forgotten something?

Yes, we've only spoken to computing the transitive closure.

What if we don't need the transitive closure, but have a more specific query.

e.g. What are the parts of the wheel?

In SQL, actually kind of nice:

```
WITH RECURSIVE Comp(Part, Subpt) AS
(SELECT A1.Part, A1.Subpt FROM Assembly A1)
UNION
(SELECT A2.Part, C1.Subpt
FROM Assembly A2, Comp C1
WHERE A2.Subpt=C1.Part)
```

General construction

```
SELECT * FROM Comp C2
Where C2.part = wheel.
```

Specific query

The optimizer worries the rest

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What are the optimizations?

- Avoid Repeated Inferences:
- Avoid Unnecessary Inferences:

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What are the optimizations?

- Avoid Repeated Inferences:
  - Inference  $\equiv$  Query Evaluation
  - Semi-naïve evaluation
- Avoid Unnecessary Inferences:
  - Magic-set transformation

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Intuition From SQL Version

If the following were actually a materialized view:

```
WITH RECURSIVE Comp(Part, Subpt) AS
(SELECT A1.Part, A1.Subpt FROM Assembly A1)
UNION
(SELECT A2.Part, C1.Subpt
FROM Assembly A2, Comp C1
WHERE A2.Subpt=C1.Part)
```

Given query:

```
SELECT * FROM Comp C2
WHERE C2.part = trike;
```

The answer is the entire transitive closure of trike

Computer everything – no waste

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## Avoiding Unnecessary Inferences

If the following were a materialized view:

```
WITH RECURSIVE Comp(Part, Subpt) AS
(SELECT A1.Part, A1.Subpt FROM Assembly A1)
UNION
(SELECT A2.Part, C1.Subpt
FROM Assembly A2, Comp C1
WHERE A2.Subpt=C1.Part)
```

**Given query:**

```
SELECT * FROM Comp C2
WHERE C2.part = wheel.
```

From the entire materialized view,  
Answer = {spoke, tire, rim, tube}

So we did not need to compute the entire  
transitive closure.

→ Much of initial query execution was not  
necessary to determine the answer.

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## Same explanation: strict Datalog terminology

- Unnecessary inferences:

- If we just want to find components of a particular part, say **wheel**, then first computing general fixpoint of Comp program and then at end selecting tuples with **wheel** in the first column is wasteful.
- This would compute many irrelevant facts.

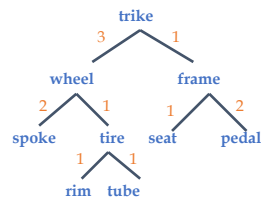
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## Magic Sets [a different] Example: Avoiding Unnecessary Inferences

```
SameLev(S1,S2) :- Assembly(P1,S1,Q1),
                  Assembly(P1,S2,Q2).
SameLev(S1,S2) :- Assembly(P1,S1,Q1),
                  SameLev(P1,P2),
                  Assembly(P2,S2,Q2).
```

Observe: Much like the  
cousin example

• Semantics?



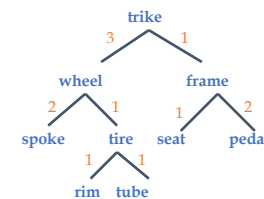
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## Example for considering Avoiding Unnecessary Inferences

```
SameLev(S1,S2) :- Assembly(P1,S1,Q1),
                  Assembly(P1,S2,Q2).
SameLev(S1,S2) :- Assembly(P1,S1,Q1),
                  SameLev(P1,P2),
                  Assembly(P2,S2,Q2).
```

Observe: Much like the  
cousin example  
Sibling(S1, S2) :- Par(P1, S1, \_),  
Par(P1, S2, \_)

• Semantics?



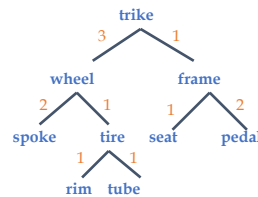
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## Avoiding Unnecessary Inferences

```
SameLev(S1,S2) :- Assembly(P1,S1,Q1),
                  Assembly(P1,S2,Q2).
SameLev(S1,S2) :- Assembly(P1,S1,Q1),
                  SameLev(P1,P2),
                  Assembly(P2,S2,Q2).
```

- Tuple (S1,S2) in SameLev if there is path up from S1 to some node and down to S2 with same number of up and down edges.



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## What if materialize the program Apply Rule 1

SameLev(S1,S2) :-		Assembly(P1,S1,Q1).		Assembly(P1,S2,Q2)			
Wheel	Frame	trike	wheel	3	trike	wheel	3
Spoke	Tire	trike	frame	1	trike	frame	1
Seat	Pedal	frame	seat	1	frame	seat	1
Rim	Tube	frame	pedal	1	frame	pedal	1
		wheel	spoke	2	wheel	spoke	2
		wheel	tire	1	wheel	tire	1
		tire	rim	1	tire	rim	1
		tire	tube	1	tire	tube	1

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## Apply Rule 2, first time

SameLev(S1,S2) :- Assembly(P1,S1,Q1), SameLev(P1,P2), Assembly(P2,S2,Q2).

Assembly(P1,S1,Q1)		SameLev(P1,P2)		Assembly(P2,S2,Q2)	
trike	wheel	3	Wheel	Frame	1
trike	frame	1	Spoke	Tire	1
frame	seat	1	Seat	Pedal	1
frame	pedal	1	Rim	Tube	1
wheel	spoke	2			
wheel	tire	1			
tire	rim	1			
tire	tube	1			

Column Break

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## Result

SameLev(S1,S2) :-

Rule 1

Rule 2, first application.. reaches fixed point

Wheel	Frame
Spoke	Tire
Seat	Pedal
Rim	Tube
Spoke	Seat
Spoke	Pedal
Tire	Seat
Tire	Pedal

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## Example Query

- Want all SameLev tuples with **spoke** in first column.

In Datalog,

SameLev(spoke,s):=.



Wheel	Frame
Spoke	Tire
Seat	Pedal
Rim	Tube
Spoke	Seat
Spoke	Pedal
Tire	Seat
Tire	Pedal

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## Approach:

- Recall, ordinary queries, pushing selects down (usually a constant) → big improvements
- We will augment a Datalog program with sets of constants, *Magic Sets*
  - Defined by new predicates, that yield the *Magic Sets* of constants
  - Substitute them into the Datalog program



Wheel	Frame
Spoke	Tire
Seat	Pedal
Rim	Tube
Spoke	Seat
Spoke	Pedal
Tire	Seat
Tire	Pedal

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## Avoiding Unnecessary Inferences

- Want all SameLev tuples with **spoke** in first column.
- Intuition:** "Push" this selection into fixpoint computation.

SameLev(S1,S2) :-  
Assembly(P1,S1,Q1),  
SameLev(P1,P2),  
Assembly(P2,S2,Q2).

SameLev(spoke,S2) :-  
Assembly(P1,spoke,Q1),  
SameLev(P1,P2),  
Assembly(P2,S2,Q2).

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## Avoiding Unnecessary Inferences

- Intuition:** "Push" this selection with **spoke** into fixpoint computation.

- Substitute S1 with spoke

SameLev(S1,S2) :-  
Assembly(P1,S1,Q1),  
SameLev(P1,P2),  
Assembly(P2,S2,Q2).

SameLev(spoke,S2) :-  
Assembly(P1,spoke,Q1),  
SameLev(P1,P2),  
Assembly(P2,S2,Q2).

SameLev(spoke,seat) :- Assembly(wheel,spoke,2),  
SameLev(wheel,frame), Assembly(frame,seat,1).

- Other SameLev tuples are needed to compute all such tuples with **spoke**,

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## “Magic Sets” Idea

- 1. Define “filter” table that computes all **relevant** values
- 2. Restrict computation of SameLev to infer only tuples with **relevant** value in first column.

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## Intuition

- Relevant** values: contains all tuples  $m$  for which we require to compute all same-level tuples with  $m$  in first column to answer query.
- Put differently, relevant** values are all Same-Level tuples whose first field contains value on path from spoke up to root.
- We call it Magic-SameLevel (Magic-SL)

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## “Magic Sets” in Example

- Idea:** Define “filter” table that computes all relevant values : Collect all parents of *spoke*.

```
Magic_SL(P1) :- Magic_SL(S1), Assembly(P1,S1,Q1).
Magic_SL(spoke) :- .
```

Make Magic table as Magic-SL

Notice:

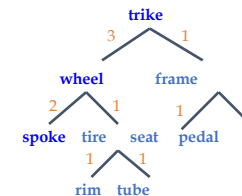
- rule with head, `Magic_SL(p1)`, has no constants
- `Magic_SL(spoke):-.` Is a separate rule for defining the filter table

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Define “filter” table that computes all relevant values :

- Idea:** Collect all parents of *spoke*.

```
Magic_SL(P1) :- Magic_SL(S1), Assembly(P1,S1,Q1).
Magic_SL(spoke) :- .
```



Make Magic table as Magic-SameLevel.

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## “Magic Sets” Idea

- **Idea:** Define a “filter” table (in the form a predicate) to restrict the computation of SameLev to only tuples with a relevant value in the first column

Define:  
`Magic_SL(P1) :- Magic_SL(S1), Assembly(P1,S1,Q1).`  
`Magic(spoke).`

Add predicate to same level rules (limiting the results to a subset of the original)  
`SameLev(S1,S2) :- Magic_SL(S1), Assembly(P1,S1,Q1),`  
`Assembly(P1,S2,Q2).`

`SameLev(S1,S2) :- Magic_SL(S1), Assembly(P1,S1,Q1),`  
`SameLev(P1,P2), Assembly(P2,S2,Q2).`

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## The Magic Sets Algorithm

1. Generate an “adorned” program
  - Program is rewritten to make pattern of **bound and free arguments** in query explicit
2. Add magic filters of form “Magic\_P”
  - for each rule in adorned program add a Magic condition to body that acts as filter on set of tuples generated (**predicate P to restrict** these rules)
3. Define new rules to define **filter tables**
  - Define new rules to define **filter tables** of form Magic\_P

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## Step 1: Generating Adorned Rules

- Adorned program for query pattern SameLev<sup>bf</sup>, assuming right-to-left order of rule evaluation :

```
SameLevbf (S1,S2) :- Assembly(P1,S1,Q1), Assembly(P1,S2,Q2).
SameLevbf (S1,S2) :- Assembly(P1,S1,Q1),
                        SameLevbf (P1,P2), Assembly(P2,S2,Q2).
```

- ❖ Argument of (a given body occurrence of) SameLev is:
  - ❖ **b** if it appears to the left in body,
  - ❖ or if it is a **b** argument of head of rule,
  - ❖ Otherwise it is *free*. (as in won't be bound)
- ❖ Assembly not adorned because explicitly stored table (EDB).

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## Step 2: Add Magic Filters

- For every rule in adorned program add a ‘magic filter’ predicate

`SameLevbf (S1,S2) :- Magic_SL (S1), Assembly(P1,S1,Q1),`  
`Assembly(P1,S2,Q2).`

`SameLevbf (S1,S2) :- Magic_SL (S1),`  
`Assembly(P1,S1,Q1),`  
`SameLevbf (P1,P2), Assembly(P2,S2,Q2).`

- Filter predicate: copy of head of rule, Magic prefix, and delete free variable

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### Step 3: Defining Magic Tables

- Rule for Magic\_P is generated from each occurrence of recursive P in body of rule:
  - Delete everything to right of P
  - Add prefix "Magic" and delete free columns of P
  - Move P, with these changes, into head of rule

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### Step 3: Defining Magic Table

- Rule for Magic\_P is generated from each occurrence of recursive P in body of rule:
  - Delete everything to right of P

$\text{SameLev}^{\text{bf}}(S1, S2) \text{ :- Magic\_SL}(S1), \text{Assembly}(P1, S1, Q1),$   
 $\text{SameLev}^{\text{bf}}(P1, P2), \text{Assembly}(P2, S2, Q2).$

- Add prefix "Magic" and delete free columns of P

$\text{Magic-SameLev}^{\text{bf}}(S1, S2) \text{ :- Magic\_SL}(S1), \text{Assembly}(P1, S1, Q1),$   
 $\text{Magic-SameLev}^{\text{bf}}(P1, \underline{\quad}).$

- Move P, with these changes, into head of rule

$\text{Magic\_SL}(P1) \text{ :- Magic\_SL}(S1), \text{Assembly}(P1, S1, Q1).$

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### Supplemental Slides: Semi-naïve Evaluation Example

#### Beginning Cycle 1

Comp

Assembly

trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Comp, is an empty EDB table

Rule 2 is not satisfied

Eval rule 1, and get

Rule 1:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- Assembly}(\text{Part}, \text{Subpt}, \text{Qty}).$   
 Rule 2:  $\text{Comp}(\text{Part}, \text{Subpt}) \text{ :- Assembly}(\text{Part}, \text{Part2}, \text{Qty}), \text{Comp}(\text{Part2}, \text{Subpt}).$

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## End Cycle 1

Comp (1)		Assembly		
trike	wheel	trike	wheel	3
trike	frame	trike	frame	1
frame	seat	frame	seat	1
frame	pedal	frame	pedal	1
wheel	spoke	wheel	spoke	2
wheel	tire	wheel	tire	1
tire	rim	tire	rim	1
tire	tube	tire	tube	1

Rule 1:  $Comp(Part, Subpt) :- Assembly(Part, Subpt, Qty).$   
 Rule 2:  $Comp(Part, Subpt) :- Assembly(Part, Part2, Qty), Comp(Part2, Subpt).$

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Assembly			Comp (1)	
trike	wheel	3	trike	wheel
trike	frame	1	trike	frame
frame	seat	1	frame	seat
frame	pedal	1	frame	pedal
wheel	spoke	2	wheel	spoke
wheel	tire	1	wheel	tire
tire	rim	1	tire	rim
tire	tube	1	tire	tube

Rule 1, no new assembly rows, do nothing, but Comp 1 stays:

Rule 2:  $Comp(Part, Subpt) :- Assembly(Part, Part2, Qty), Comp(Part2, Subpt).$

Compute Comp tuples by applying Rule 2 (in cycle 2)

## Cycle 2

trike	wheel
trike	frame
frame	seat
frame	pedal
wheel	spoke
wheel	tire
tire	rim
tire	tube

trike	spoke
trike	tire
trike	seat
trike	pedal
wheel	rim
wheel	tube

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trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Assembly instance

trike	spoke
trike	tire
trike	seat
trike	pedal
wheel	rim
wheel	tube

Consider only these, the new rows of Comp

trike	spoke
trike	tire
trike	seat
trike	pedal
wheel	rim
wheel	tube
trike	rim
trike	tube

Per incremental view maintenance, only these two rows get added

## Cycle 3

$Comp(Part, Subpt) :-$   
 $Assembly(Part, Part2, Qty),$   
 $Comp(Part2, Subpt).$

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trike	wheel	3
trike	frame	1
frame	seat	1
frame	pedal	1
wheel	spoke	2
wheel	tire	1
tire	rim	1
tire	tube	1

Assembly instance

Join new rows only.  
 Join result is empty.  
 No more work to do  
 → fixed point,  
 No cycle 5

## Cycle 4

$Comp(Part, Subpt) :-$   
 $Assembly(Part, Part2, Qty),$   
 $Comp(Part2, Subpt).$

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