B+ Tree's: Primary method for managing data on disks (in RDBMS)

Objective: Understand the implications disk access on the design of data structures.

- fat fanout
- split/promote

Reading:

- Text Ch. 14
- For very helpful interactive demo, see
  https://www.cs.usfca.edu/~galles/visualization/BPlusTree.html

Slide thanks: many slides, Garcia-Molina, (but you may want to see his original set), http://infolab.stanford.edu/~hector/cs245/notes.htm

Last Time: Declarative and Asssociative Access

Invention	
iid	name
1	structure
2	sequencing_ma
3	expression_chip

Inventor		
iid	first	last
1	Francis	Crick
1	James	Watson
1	Rosaly	Franklin
1	Mauric	Wilkens
2	Lee	Hood
3	David	Botstein

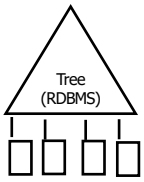
- Declarative Programming

  select inventor
  from inventions, inventor
  where …

- We say we want from sets

  no details how to do it.

  (seems like,
  iterate over sets, O(n)
  Cartesian product of sets, O(n^2))



First part of *the fix*

- Introduce indexes, (data structures for set membership)

  For tree structured indexes O(n) operations become "O(log n) "

Second part of the fix

- query optimizer decides when to use them

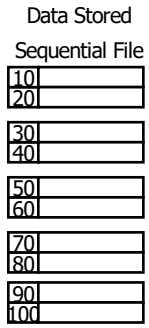
First there were B-trees

- Bayer & McCreight 1972

Then they were promptly improved, B+ Trees

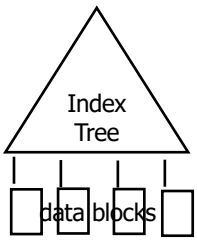
B+ trees, are so much better

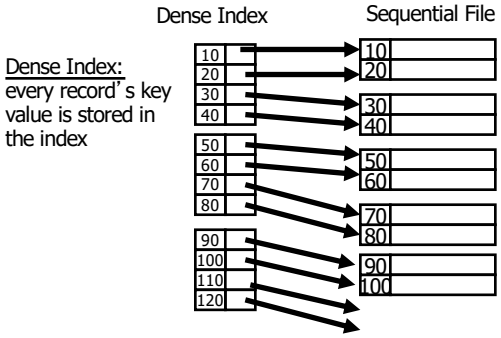
- We've forgotten about B-trees,
- Common usage B-tree = B+ Tree

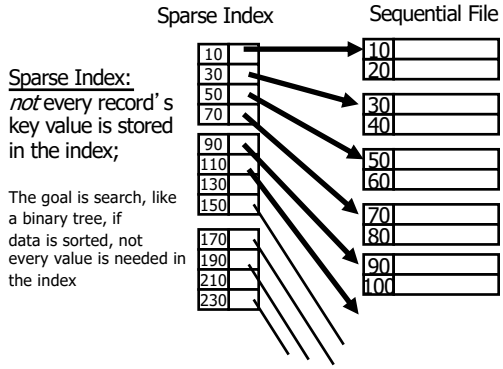


Data records (rows) stored in one file

Index records stored separately.







Sparse vs. Dense Tradeoff

- Sparse: Less index space per record
  can keep more of index in memory
- Dense: Can tell if any record exists
  without accessing file

(Later:
- sparse better for insertions
- dense needed for secondary indexes)

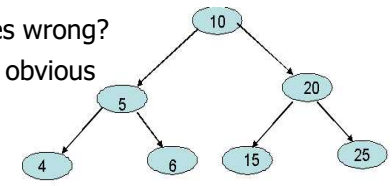
The B+ Tree

- Main memory binary trees evolved to disk.

Naïve Use of Binary Tree on Disk

One node per disk block

- What goes wrong?
- What are obvious fixes?



A simple binary tree

A Word About

- Constants
- Big O()

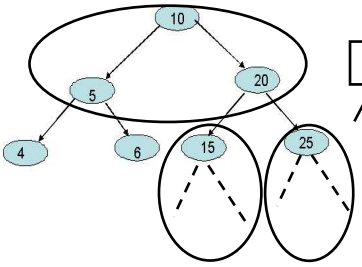
&

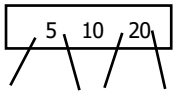
- Databases

- n, size of input, is BIG,
- c, so are the constants // how big?

Fatten the Nodes

(*e.g. not a B+ node yet*)





Disk access?

But these are integers what about records?

Do better?

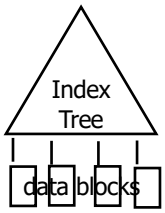
In databases

- We like algorithmic complexity (why?)

- But we don't really get to throw away the constants, etc.

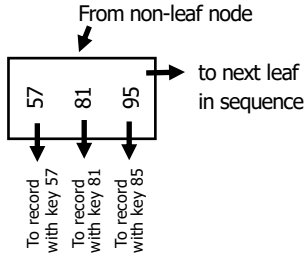
In the index, store only search keys, not the records

- Fatter fanout - more space for *split* keys
- All the data is in the data file.
- (if dense index, all keys are in the index)

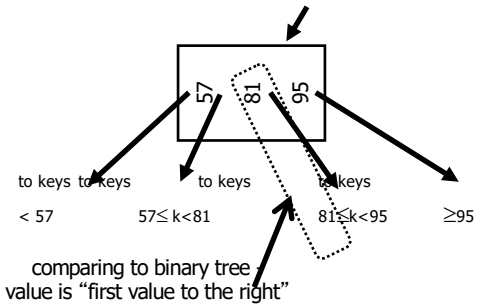


This is the primary difference between, B and B+ trees

Sample (dense) leaf node:

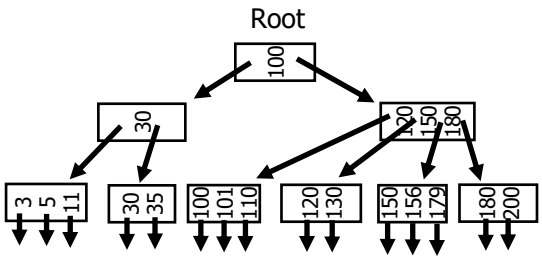


Sample non-leaf



comparing to binary tree value is "first value to the right"

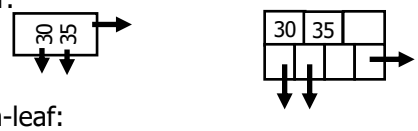
B+Tree Example n=3



Size of nodes: { n+1 pointers, n keys } (fixed)

In textbook's notation n=3

Leaf:



Non-leaf:

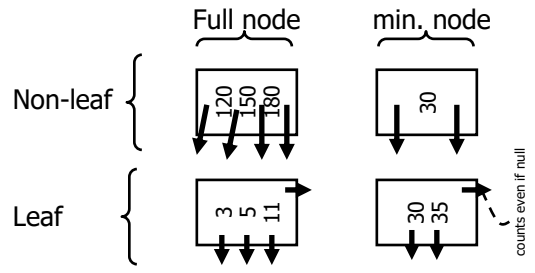
Don't want nodes to be too empty

- Use at least

  Non-leaf: $\lceil (n+1)/2 \rceil$ pointers

  Leaf: $\lfloor (n+1)/2 \rfloor$ pointers to data

n=3



B+tree rules

(1) All leaves at same lowest level (balanced tree)

(2) Pointers in leaves point to records except for "sequence pointer"

(3) Number of pointers/keys for B+tree

	Max ptrs	Max keys	Min ptrs→data	Min keys
Non-leaf (non-root)	n+1	n	$\lceil (n+1)/2 \rceil$	$\lceil (n+1)/2 \rceil - 1$
Leaf (non-root)	n+1	n	$\lfloor (n+1)/2 \rfloor$	$\lfloor (n+1)/2 \rfloor$
Root	n+1	n	1	1

Insert into B+tree

(a) simple case
- space available in leaf

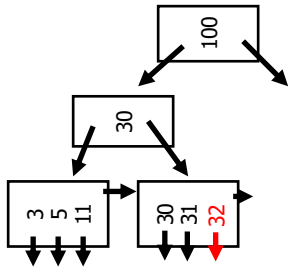
(b) leaf overflow

(c) non-leaf overflow

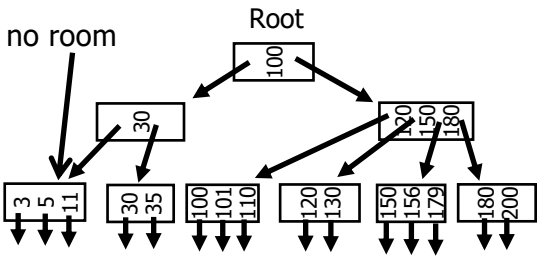
(d) new root

(a) Insert key = 32

n=3



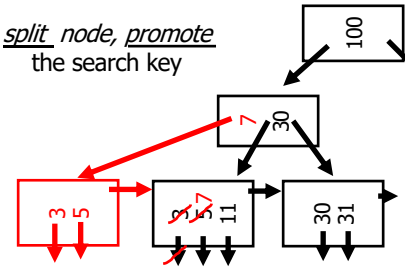
(b) Insert key = 7



(b) Insert key = 7

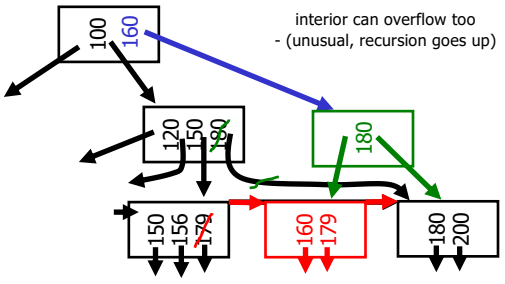
n=3

split node, promote the search key



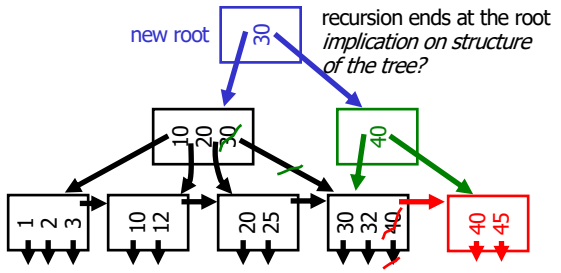
(c) Insert key = 160

n=3



(d) New root, insert 45

n=3

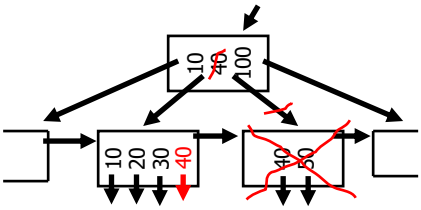


Deletion from B+tree

(a) Simple case - no example
(b) Coalesce with neighbor (sibling)
(c) Re-distribute keys
(d) Cases (b) or (c) at non-leaf

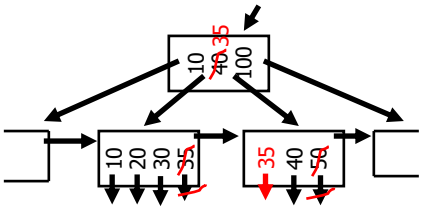
(b) Coalesce with sibling
– Delete 50

n=4



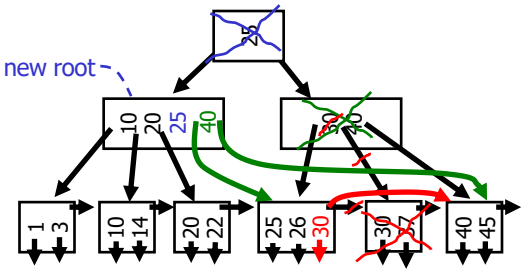
(c) Redistribute keys
– Delete 50

n=4



(d) Non-leaf coalesce
– Delete 37

n=4



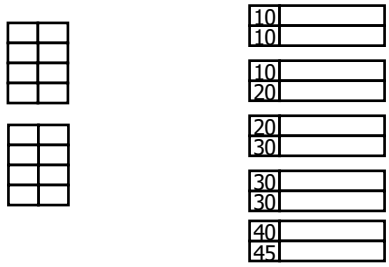
B+tree deletions in practice

– Often, coalescing is not implemented
  – Overhead not worth it! (why?)

Next:

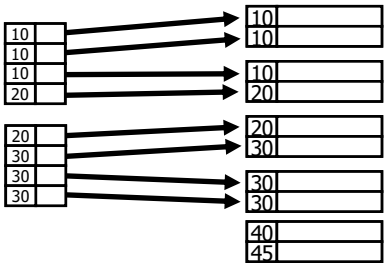
- Duplicate keys
- Deletion/Insertion
- Secondary indexes

Duplicate keys



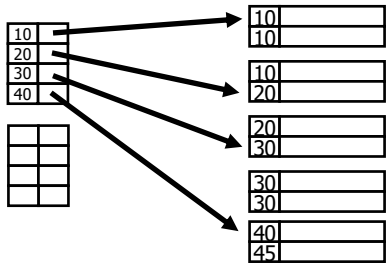
Duplicate keys

Dense index, one way to implement?



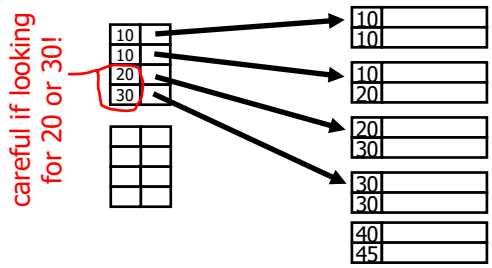
Duplicate keys

Dense index, better way?



Duplicate keys

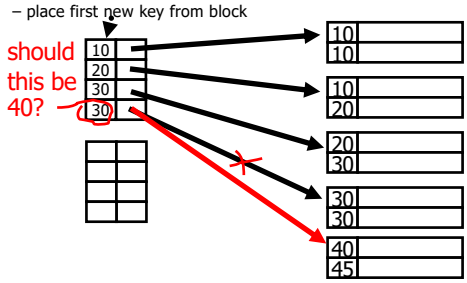
Sparse index, one way?



Duplicate keys

Sparse index, another way?

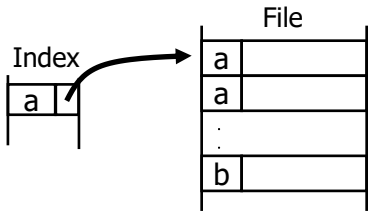
– place first new key from block



Summary

Duplicate values, primary index

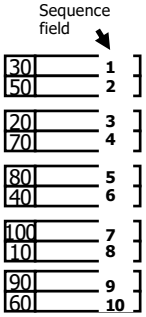
- Index may point to first instance of each value only



Primary vs. Secondary Index

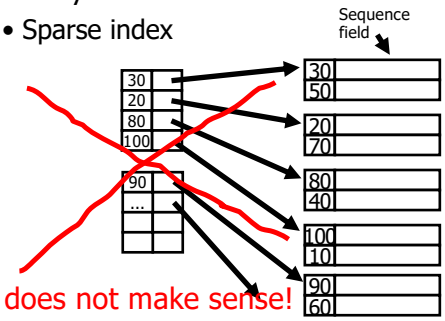
- Primary is sorted on the index key *and placed on disk that way*
- Secondary index,
  - does not "cluster"
  - consider (x, y)

Secondary indexes



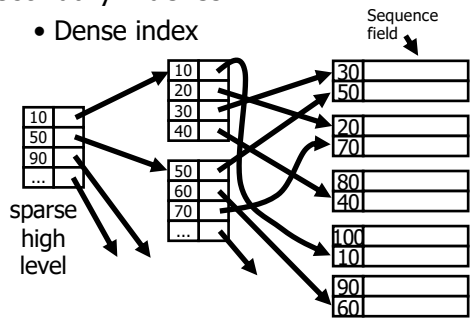
Secondary indexes

- Sparse index



Secondary indexes -->

- Dense index



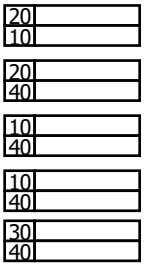
With secondary indexes:

- Lowest level is dense
- Other levels are sparse

Also: Pointers are record pointers

(not block pointers; not computed)

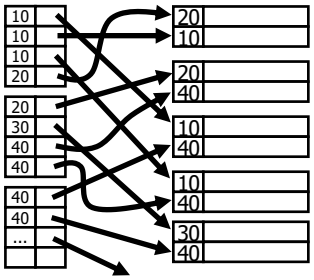
Duplicate values & secondary indexes



Duplicate values & secondary indexes

one option...

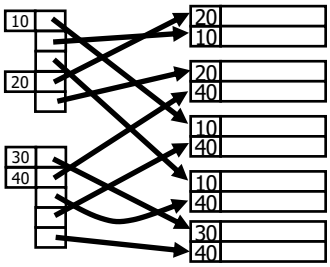
Problem:
excess overhead!
- disk space
- search time



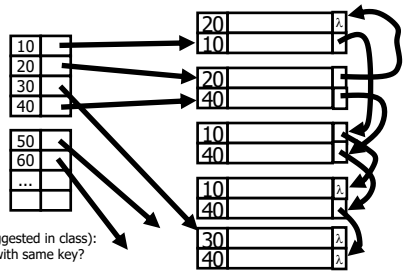
Duplicate values & secondary indexes

another option...

Problem:
variable size
records in
index!



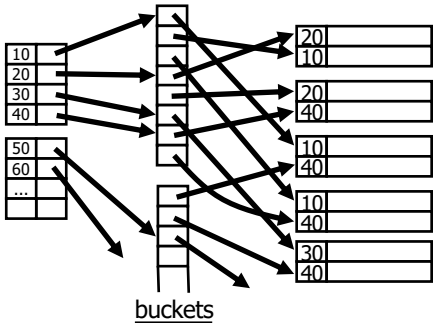
Duplicate values & secondary indexes



Another idea (suggested in class):
Chain records with same key?

Problems:
- Need to add fields to records
- Need to follow chain to know records

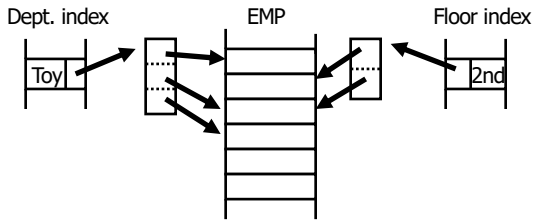
Duplicate values & secondary indexes



Why "bucket" idea is useful

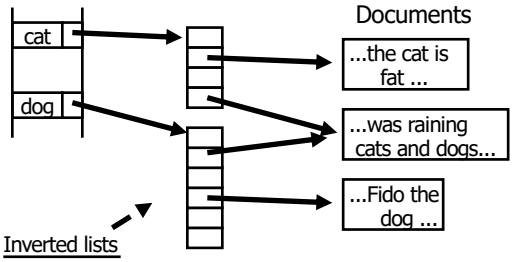
Indexes	Records
Name: primary	EMP (name,dept,floor,...)
Dept: secondary	
Floor: secondary	

Query: Get employees in
(Toy Dept) ∧ (2nd floor)



→ Intersect toy bucket and 2nd Floor bucket to get set of matching EMP's

This idea used in
text information retrieval



Review

- [Index] sequential file
- Search key (≠ primary key)
- Primary index (on Sequencing field)
- Secondary index
- Dense index (all Search Key values in)
- Sparse index
- Multi-level index