Introduction to R & R-Studio
Spring 2018

Simple Linear Regression

Start Your R Session

1. Preliminaries

Consider having a core set of preliminary commands that you always execute. These may vary depending on your preferences. The following are mine.

```
setwd("/Users/cbigelow/Desktop/")         # Set the working directory to desktop
rm(list=ls())                             # Clear current workspace
options(scipen=1000)                      # Turn off scientific notation
options(show.signif.stars=FALSE)          # Turn off display of significance stars
```

2. Install Packages (One time)

Often, in your R work you will want to use commands that are only available in packages which you must download from the internet.

Tip #1 – Always do your package installation at the console, **NEVER** within an R Markdown file.

Tip #2 – To execute any of the installations below, simply delete the leading "#".

```
# install.packages("ggplot2")
# install.packages("mosaic")
# install.packages("gridExtra")
# install.packages("car")
```

I – Simple Linear Regression

1. Introduction to Example and Load Data

Source:
Chatterjee, S; Handcock MS and Simonoff JS *A Casebook for a First Course in Statistics and Data Analysis.* New York, John Wiley, 1995, pp 145-152.

Setting:
Calls to the New York Auto Club are possibly related to the weather, with more calls occurring during bad weather. This example illustrates descriptive analyses and simple linear regression to explore this hypothesis in a data set containing information on calendar day, weather, and numbers of calls.

R Data Set:
ers.Rdata
In this illustration, the data set ***ers.Rdata*** is accessed from the PubHlth 640 website directly.
It is then saved to your current working directory.

Simple Linear Regression Variables:
Outcome Y = calls
Predictor X = low.

Launch R and load R data = ers.Rdata

```
#2.  Load data.  View structure
setwd("/Users/cbigelow/Desktop/")
load(file="ers.Rdata")
str(ersdata)

## 'data.frame':    28 obs. of  12 variables:
##  $ day    : int  12069 12070 12071 12072 12073 12074 12075 12076 12077 12078 ...
##  $ calls  : int  2298 1709 2395 2486 1849 1842 2100 1752 1776 1812 ...
##  $ fhigh  : int  38 41 33 29 40 44 46 47 53 38 ...
##  $ flow   : int  31 27 26 19 19 30 40 35 34 32 ...
##  $ high   : int  39 41 38 36 43 43 53 46 55 43 ...
##  $ low    : int  31 30 24 21 27 29 41 40 38 31 ...
##  $ rain   : int  0 0 0 0 0 0 1 0 1 0 ...
##  $ snow   : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ weekday: int  0 0 0 1 1 1 1 0 0 1 ...
##  $ year   : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ sunday : int  0 1 0 0 0 0 0 0 1 0 ...
##  $ subzero: int  0 0 0 0 0 0 0 0 0 0 ...
##  - attr(*, "datalabel")= chr ""
##  - attr(*, "time.stamp")= chr ""
##  - attr(*, "formats")= chr  "%8.0g" "%8.0g" "%8.0g" "%8.0g" ...
##  - attr(*, "types")= int  252 252 251 251 251 251 251 251 251 251 ...
##  - attr(*, "val.labels")= chr  "" "" "" "" ...
##  - attr(*, "var.labels")= chr  "" "" "" "" ...
##  - attr(*, "version")= int 8
```

We see that this data set has n=28 observations on several variables. For this illustration of simple linear regression, we will consider just two variables: calls and low. These are highlighted in red.

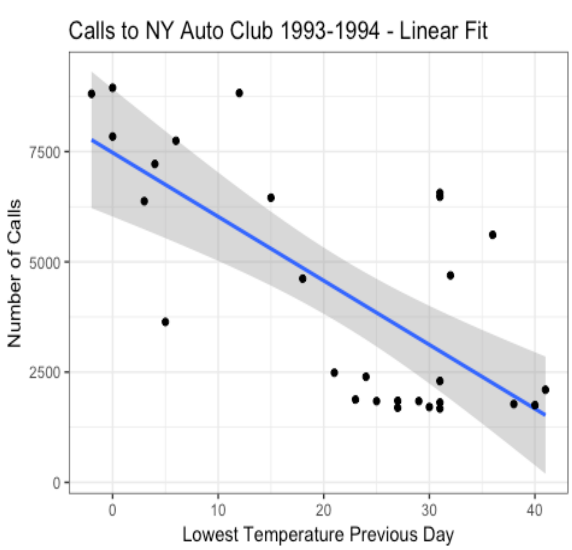
2.   Preliminaries – Descriptives

```
# summary(DATATFRAME$VARIABLE)
summary(ersdata$low)

##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   -2.00   10.50   26.00   21.75   31.00   41.00

summary(ersdata$calls)

##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    1674    1842    3062    4319    6498    8947
```

```
# To get summary statistics for EVERY variable in the dataframe
# summary(DATATFRAME)
```

```
summary(ersdata)

##       day             calls          fhigh            flow
##  Min.   :12069   Min.   :1674   Min.   :10.00   Min.   : 4.00
##  1st Qu.:12076   1st Qu.:1842   1st Qu.:29.75   1st Qu.:18.75
##  Median :12258   Median :3062   Median :35.00   Median :27.00
##  Mean   :12258   Mean   :4319   Mean   :34.96   Mean   :24.46
##  3rd Qu.:12440   3rd Qu.:6498   3rd Qu.:41.75   3rd Qu.:32.00
##  Max.   :12447   Max.   :8947   Max.   :53.00   Max.   :40.00
##       high            low              rain             snow
##  Min.   :10.00   Min.   :-2.00   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:32.00   1st Qu.:10.50   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :39.50   Median :26.00   Median :0.0000   Median :0.0000
##  Mean   :37.46   Mean   :21.75   Mean   :0.3214   Mean   :0.2143
##  3rd Qu.:43.25   3rd Qu.:31.00   3rd Qu.:1.0000   3rd Qu.:0.0000
##  Max.   :55.00   Max.   :41.00   Max.   :1.0000   Max.   :1.0000
##     weekday            year          sunday          subzero
##  Min.   :0.0000   Min.   :0.0   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.0   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :1.0000   Median :0.5   Median :0.0000   Median :0.0000
##  Mean   :0.6429   Mean   :0.5   Mean   :0.1429   Mean   :0.1786
##  3rd Qu.:1.0000   3rd Qu.:1.0   3rd Qu.:0.0000   3rd Qu.:0.0000
##  Max.   :1.0000   Max.   :1.0   Max.   :1.0000   Max.   :1.0000
```

```
Scatterplots
library(ggplot2)

SCATTERPLOT: Y=calls v X=low with least squares fit
# Tip - request line first, then overlay the points on top
gg <- ggplot(data=ersdata, aes(x=low, y=calls)) + geom_smooth(method="lm") + geom_point()
gg <- gg + xlab("Lowest Temperature Previous Day") + ylab("Number of Calls")
plotxy_linear <- gg + ggtitle("Calls to NY Auto Club 1993-1994 - Linear Fit") + theme_bw()
plotxy_linear
```

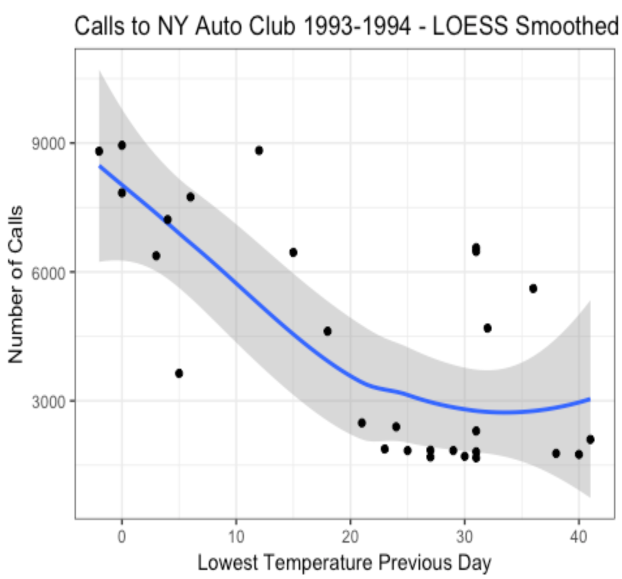


The scatterplot on the previous page suggests, as we might expect, that lower temperatures are associated with more calls to the NY Auto Club. We also see that the data are a bit messy.

Below, for illustration, is a scatterplot with an overlay lowess smoother.

Unfamiliar with LOWESS regression? LOWESS regression stands for "locally weighted scatterplot smoother". It is a technique for drawing a smooth line through the scatter plot to obtain a sense for the nature of the functional form that relates X to Y, not necessarily linear. The method involves the following: At each observation (x,y), the observed data point is fit to a line using some "adjacent" points. It's handy for seeing where in the data linearity holds and where it no longer holds. Handy!

```
SCATTERPLOT Y=calls v X=low with lowess smoother
# Tip - request loess smoother first, then overlay the points on top
# Key - span=0 is very wiggly while span=1 is less wiggly
gg <- ggplot(data=ersdata, aes(x=low, y=calls)) + geom_smooth(method="loess", span=1) + geom_point()
gg <- gg + xlab("Lowest Temperature Previous Day") + ylab("Number of Calls")
plotxy_loess <- gg + ggtitle("Calls to NY Auto Club 1993-1994 - LOESS Smoothed") + theme_bw()
plotxy_loess
```



The lowess smoothed fit suggests that perhaps the linear relationship stops being linear as the temperature increases above 20-25 degrees.

For now, we're going to just do linear regression.

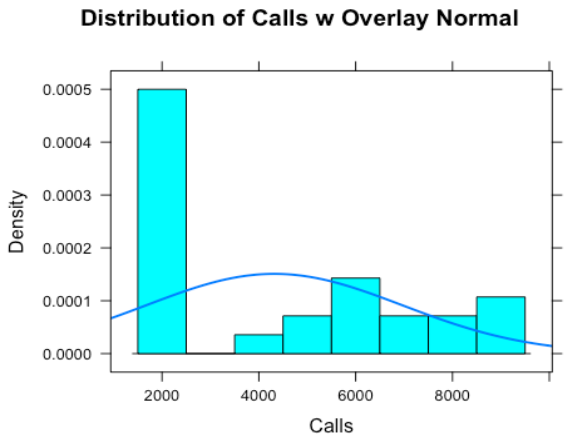
3. Assess Normality of Y

Recall. In normal theory regression, we assume that the outcome variable (in this case, Y=calls) can reasonably be assumed to be distributed normal (more on violations of this later…) So a preliminary is often to check this assumption before doing any model fits. If gross violations are apparent then, possibly, Y will be replaced by some transformation of Y that is better behaved.

Recall. It's okay for the predictor X (in this case X=low) to be NOT distributed normal. In fact, it is regarded as fixed (not random at all!)

Here is a lengthy bit of R code for you, so that you can pick and choose between the basic and the more fancy!

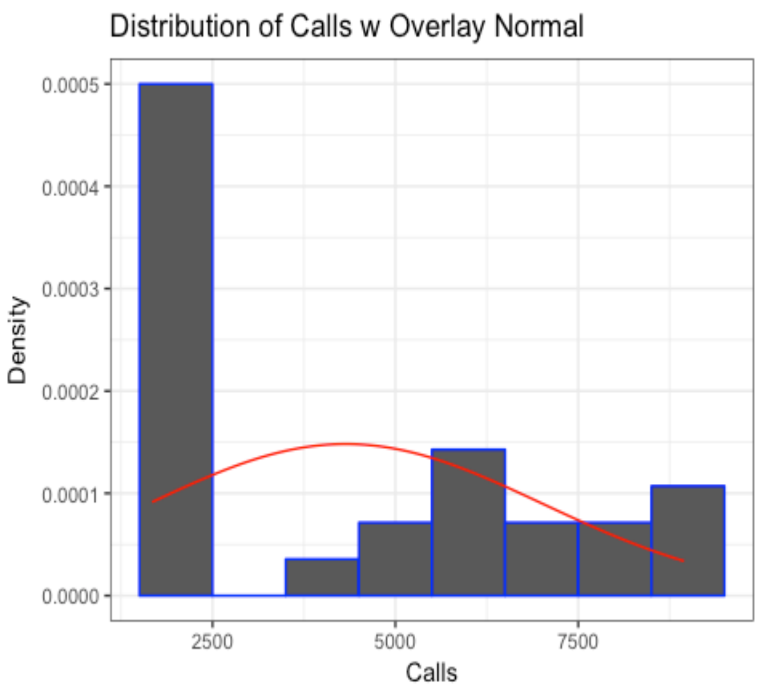
```
# Shapiro Wilk Test (Null:  distribution is normal)
shapiro.test(ersdata$calls)

##
##  Shapiro-Wilk normality test
##
## data:  ersdata$calls
## W = 0.82902, p-value = 0.0003628

# Histogram w Overlay Normal - Basic
# Command is histogram in package=mosaic
# Tip - Might want to tweak width=1000

library(mosaic)
histogram(ersdata$calls, width=1000, main="Distribution of Calls w Overlay Normal", xlab="C
alls", fit="normal")
```



The null hypothesis of normality of Y=calls is rejected (p-value = .00036). Tip- sometimes the cure is worse than the original violation. For now, we'll charge on.

For ggplot2 fans

```
library(ggplot2)
Histogram w Overlay Normal - w Aesthetics
# Tip- Might want to tweak binwidth=1000
# ggplot(DATAFRAME, aes(x=VARIABLENAME)) + stuff below
gg <- ggplot(ersdata, aes(x=calls))
gg <- gg + geom_histogram(binwidth=1000, colour="blue",
                          aes(y=..density..))
gg <- gg + stat_function(fun=dnorm,
                         color="red",
                         args=list(mean=mean(ersdata$calls),
                                   sd=sd(ersdata$calls)))
gg <- gg + ggtitle("Distribution of Calls w Overlay Normal")
gg <- gg + xlab("Calls") + ylab("Density")
plot_histogramcalls <- gg + theme_bw()
plot_histogramcalls
```



A bit fancier. The conclusion is the same. The null hypothesis of normality of Y=calls is rejected (p-value = .00036). But for now, we'll charge on.

4. Fit Model

```
Simple Linear Regression- Fit, Coefficients Table, ANOVA Table and R-squared
library(mosaic)

# FIT
# MODELNAME <- lm(YVARIABLE ~ XVARIABLE, data=DATAFRAME)
model_simple <- lm(calls ~ low, data=ersdata)


# Basic report of fit
# summary(MODELNAME)
summary(model_simple)

##
## Call:
## lm(formula = calls ~ low, data = ersdata)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -3112  -1468   -214   1144   3588
##
## Coefficients:
##             Estimate Std. Error t value       Pr(>|t|)
## (Intercept)  7475.85     704.63  10.610 0.000000000061
## low          -145.15      27.79  -5.223 0.000018649091
##
## Residual standard error: 1917 on 26 degrees of freedom
## Multiple R-squared:  0.5121, Adjusted R-squared:  0.4933
## F-statistic: 27.28 on 1 and 26 DF,  p-value: 0.00001865



# 95% CI for the regression coefficients (betas)
# confit(MODELNAME)
confint(model_simple)

##                 2.5 %     97.5 %
## (Intercept) 6027.4605 8924.23745
## low         -202.2744  -88.03352
```

```
# Analysis of Variance Table
# anova(MODELNAME)
anova(model_simple)

## Analysis of Variance Table
##
## Response: calls
##           Df    Sum Sq   Mean Sq F value     Pr(>F)
## low        1 100233719 100233719  27.285 0.00001865
## Residuals 26  95513596   3673600


# Obtain R-squared = % Variance Explained by Model
# rsquared (MODELNAME)
r2_simple <- rsquared(model_simple)
r2_simple <- 100*round(r2_simple,2)
paste("Percent Variance Explained, R-squared =",r2_simple,"%")

## [1] "Percent Variance Explained, R-squared = 51 %"
```

Note – We didn't really need to do this. You could have seen that R-squared = 51% from the initial report. There, you'll see it as MULTIPLE R-squared = .5121.

Putting this all together
Remarks

- *The fitted line is* $\widehat{calls} = 7{,}475.85 - 145.15*[low]$
- R^2 *= .51 indicates that 51% of the variability in calls is explained.*
- *The overall F test significance level "PROB > F" < .0001 suggests that the straight line fit*

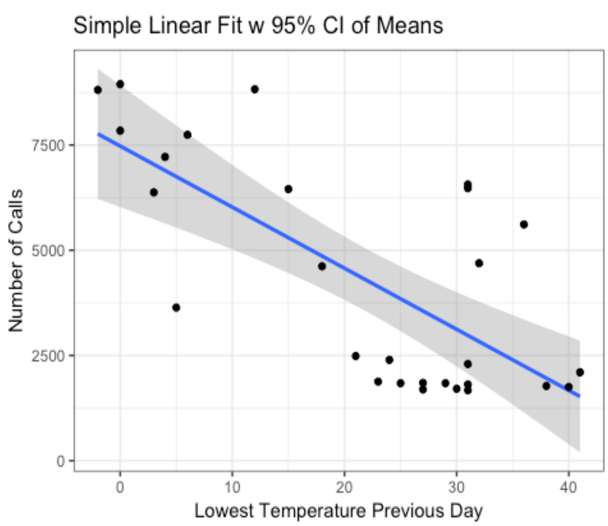
  performs better in explaining variability in calls than does $\bar{Y}$ *= average # calls*
- *From this output, the analysis of variance is the following:*

Source	Df	Sum of Squares	Mean Square
Model "Regression"	**1**	**MSS =** $\sum_{i=1}^{n}\left(\hat{Y}_i - \bar{Y}\right)^2$ **= 100,233,719**	**MSS/1 = 100,233,719**
Residual "Error"	**(n-2) = 26**	**RSS =** $\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2$ **= 95,513,596**	**RSS/(n-2) = 3,673,600**
Total, corrected	**(n-1) = 27**	**TSS =** $\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2$ **= 195,747,315**	

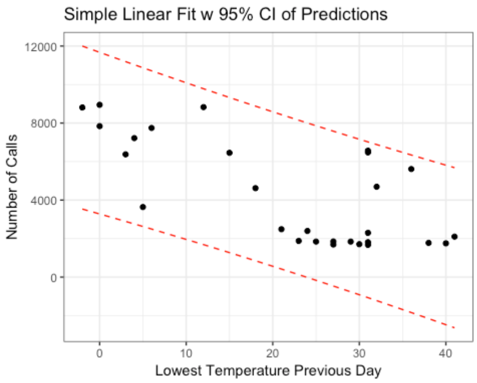
5. Post Fit Model Examination

Three plots in ggplot2 are shown: a) plot of 95% CI of the mean; b) plot of 95% CI of the individual predictions and c) combined plot showing both 95% CI of mean and 95% CI of individual predictions.

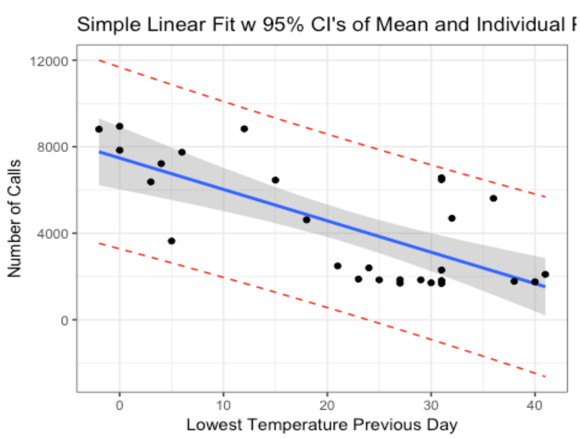
```
library(ggplot2)

#####  a)  95% CI of mean w overlay scatter
gg <- ggplot(ersdata, aes(x=low, y=calls))+ geom_smooth(method=lm, level=.95, se=TRUE)
gg <- gg + geom_point()
gg <- gg + xlab("Lowest Temperature Previous Day") + ylab("Number of Calls")
gg <- gg + ggtitle("Simple Linear Fit w 95% CI of Means")
plot_CImean <- gg + theme_bw()
plot_CImean
```




```
#####  b)  95% CI of individual prediction w overlay scatter
yhat <- predict(model_simple, interval="prediction")

## Warning in predict.lm(model_simple, interval = "prediction"): predictions on current dat
a refer to _future_ responses

temp_df <- cbind(ersdata, yhat)
gg <- ggplot(temp_df, aes(x=low, y=calls))
gg <- gg + geom_line(aes(y=lwr), color = "red", linetype = "dashed")
gg <- gg + geom_line(aes(y=upr), color = "red", linetype = "dashed")
gg <- gg + geom_point()
gg <- gg + xlab("Lowest Temperature Previous Day") + ylab("Number of Calls")
gg <- gg + ggtitle("Simple Linear Fit w 95% CI of Predictions")
plot_CIpredict <- gg + theme_bw()
plot_CIpredict
```




```
#####  c)  COMBINED:  95% CI of mean, 95% CI of prediction + overlay scatter
yhat <- predict(model_simple, interval="prediction")

## Warning in predict.lm(model_simple, interval = "prediction"): predictions on current dat
a refer to _future_ responses

temp_df <- cbind(ersdata, yhat)
gg <- ggplot(temp_df, aes(x=low, y=calls))
gg <- gg + geom_line(aes(y=lwr), color = "red", linetype = "dashed")
gg <- gg + geom_line(aes(y=upr), color = "red", linetype = "dashed")
gg <- gg + geom_smooth(method=lm, level=.95, se=TRUE)
gg <- gg + geom_point()
gg <- gg + xlab("Lowest Temperature Previous Day") + ylab("Number of Calls")
gg <- gg + ggtitle("Simple Linear Fit w 95% CI's of Mean and Individual Predictions")
plot_CIboth <- gg + theme_bw()
plot_CIboth
```

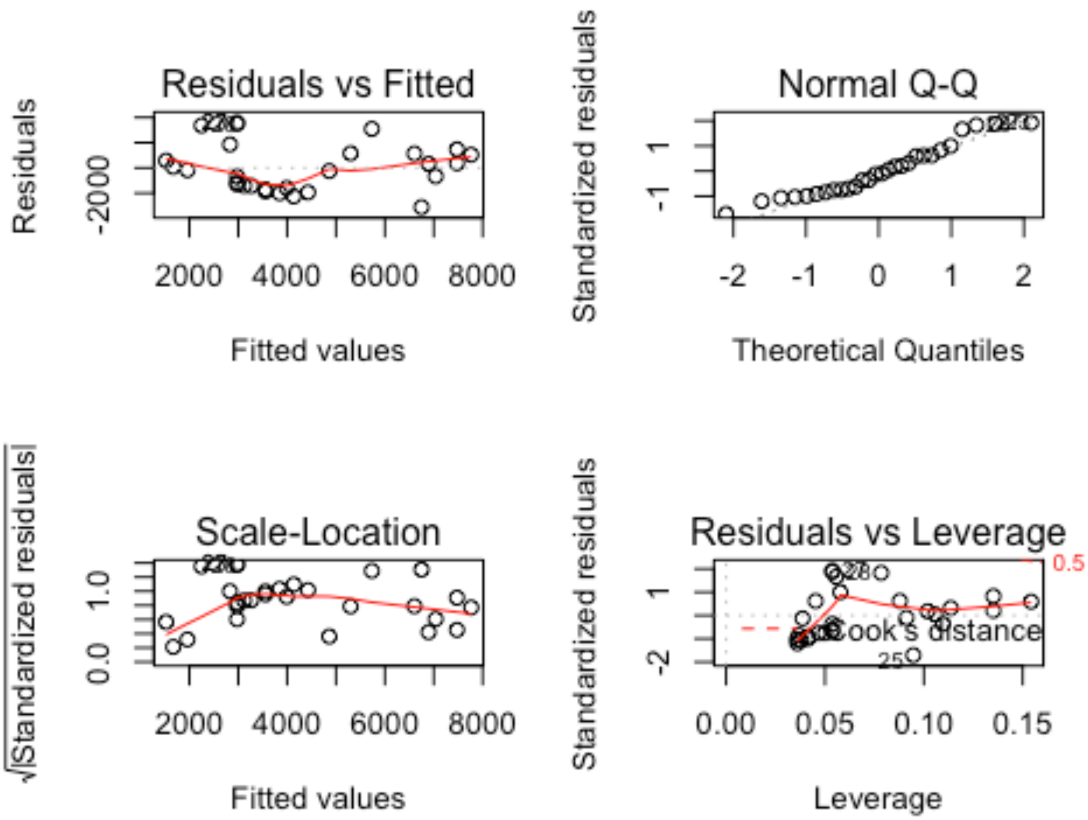


Remarks

- *The overlay of the straight line fit is reasonable but substantial variability is seen, too.*
- *There is a lot we still don't know, including but not limited to the following ---*
- *Case influence, omitted variables, variance heterogeneity, incorrect functional form, etc.*

6. Some Graphical Diagnostics

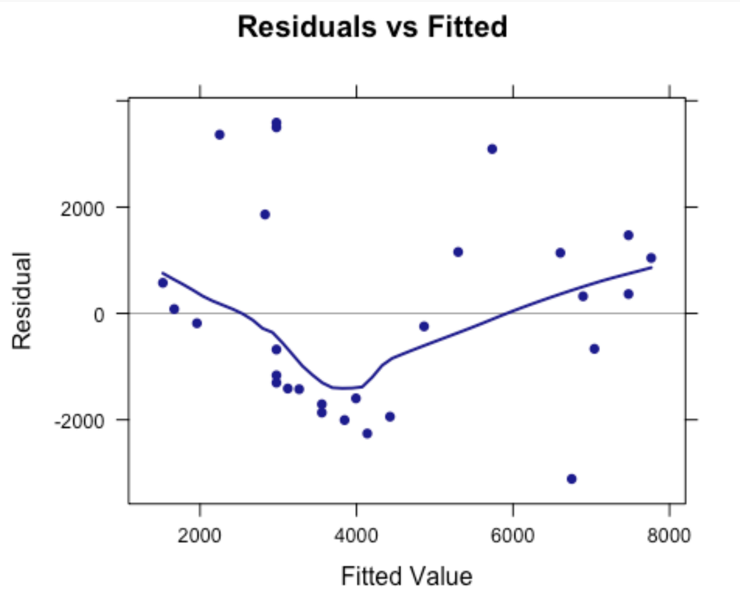
```
library(mosaic)
library(ggplot2)
library(gridExtra)

# BASIC - Produces 4 plots in a single panel
# Key:  par(mfrow=c(2,2)) says arrange the 4 plots in a 2x2 array
par(mfrow=c(2,2))
plot(model_simple)
```



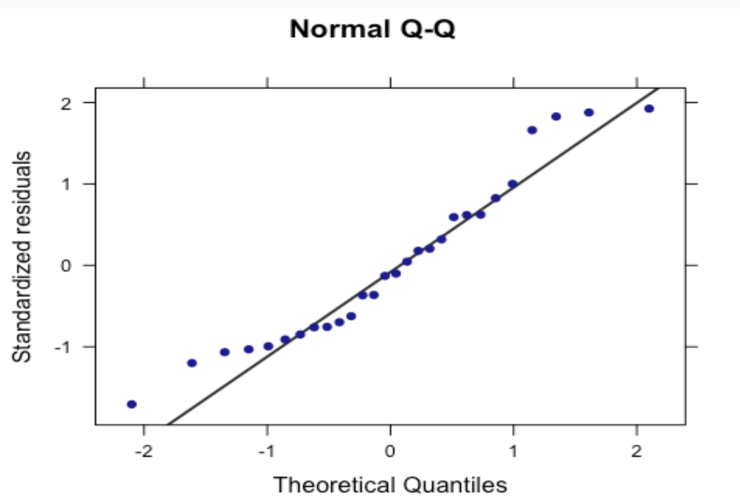
A little hard to see what's going on here. I think I'll look at these plots one at a time.

Mosaic has 6 nice diagnostic plots. Here I obtain each of them. Note - the plotting requires ggplot2

```
#  FANCY - Produces 6 plots, separately or in one combined panel
#  Following uses commands in packages = mosaic, ggplot2, gridExtra


#####  a)  Y=residual v X =predicted  (Good if:  Even band at Y=0)
mplot (model_simple, which=1)
```

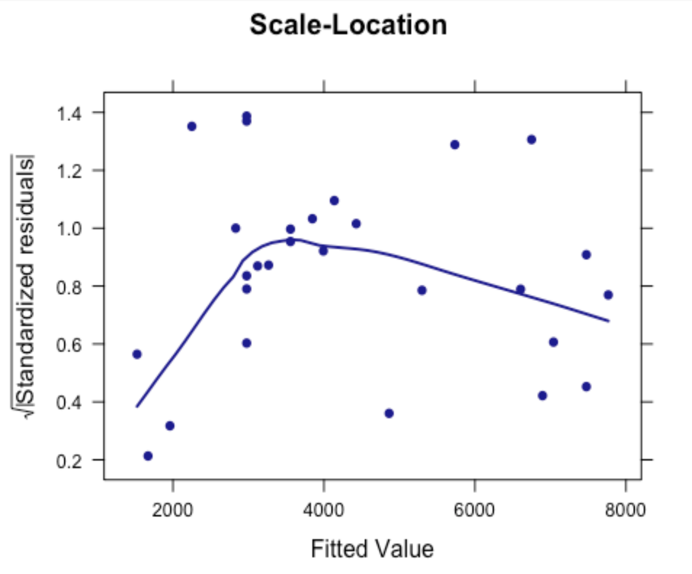



```
#####  b)  Normal Quantile Plot  (Good if:  X=Y 45 degree line)
mplot (model_simple, which=2)
```



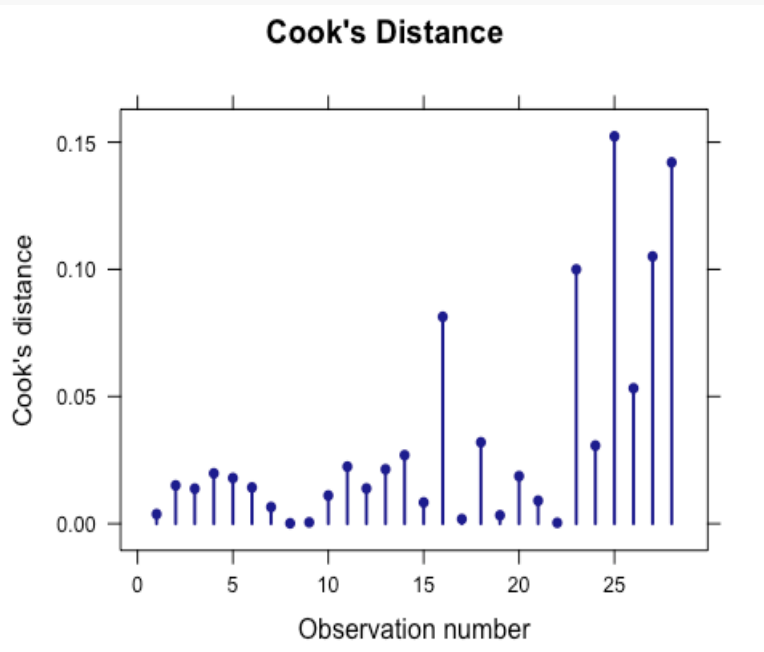
Not bad!

```
##### c) Y=standardized residual v X=predicted (Good if: Constant variance)
mplot (model_simple, which=3)
```



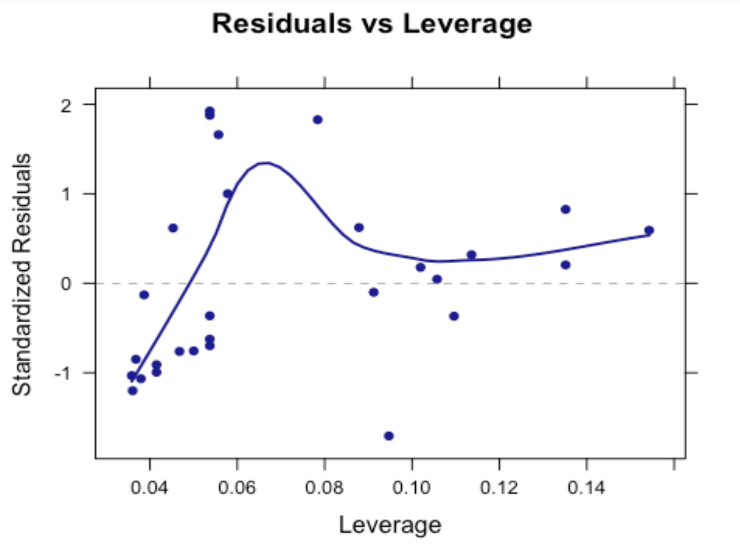
Also not bad. Note that the square root of the standardized residuals are the absolute values.

```
##### d) Y=Cook's Distance v X=Observation number (Good if: all are below .5)
mplot (model_simple, which=4)
```



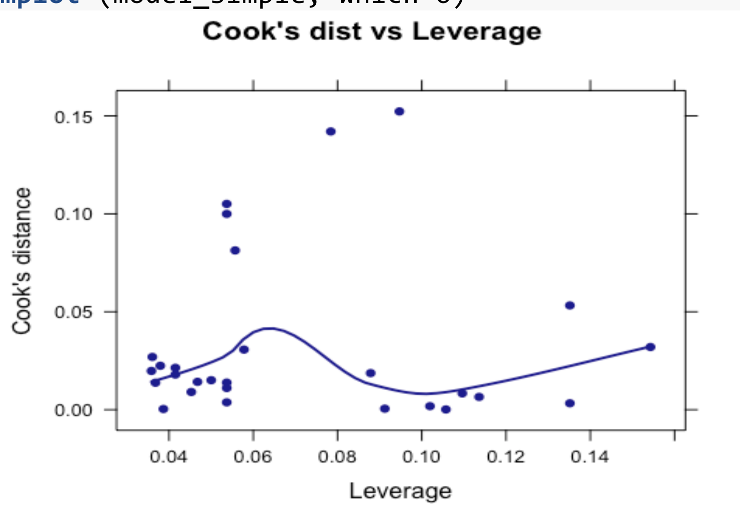
In simple linear regression, the rule of thumb is to notice a Cook's distance > 1. Clearly we have no problem here. The largest Cook distance is less than 0.15!

```
#####  e)  Y=residuals v X=leverage  (Good if: Nice even band centered at Y=0)
mplot (model_simple, which=5)
```



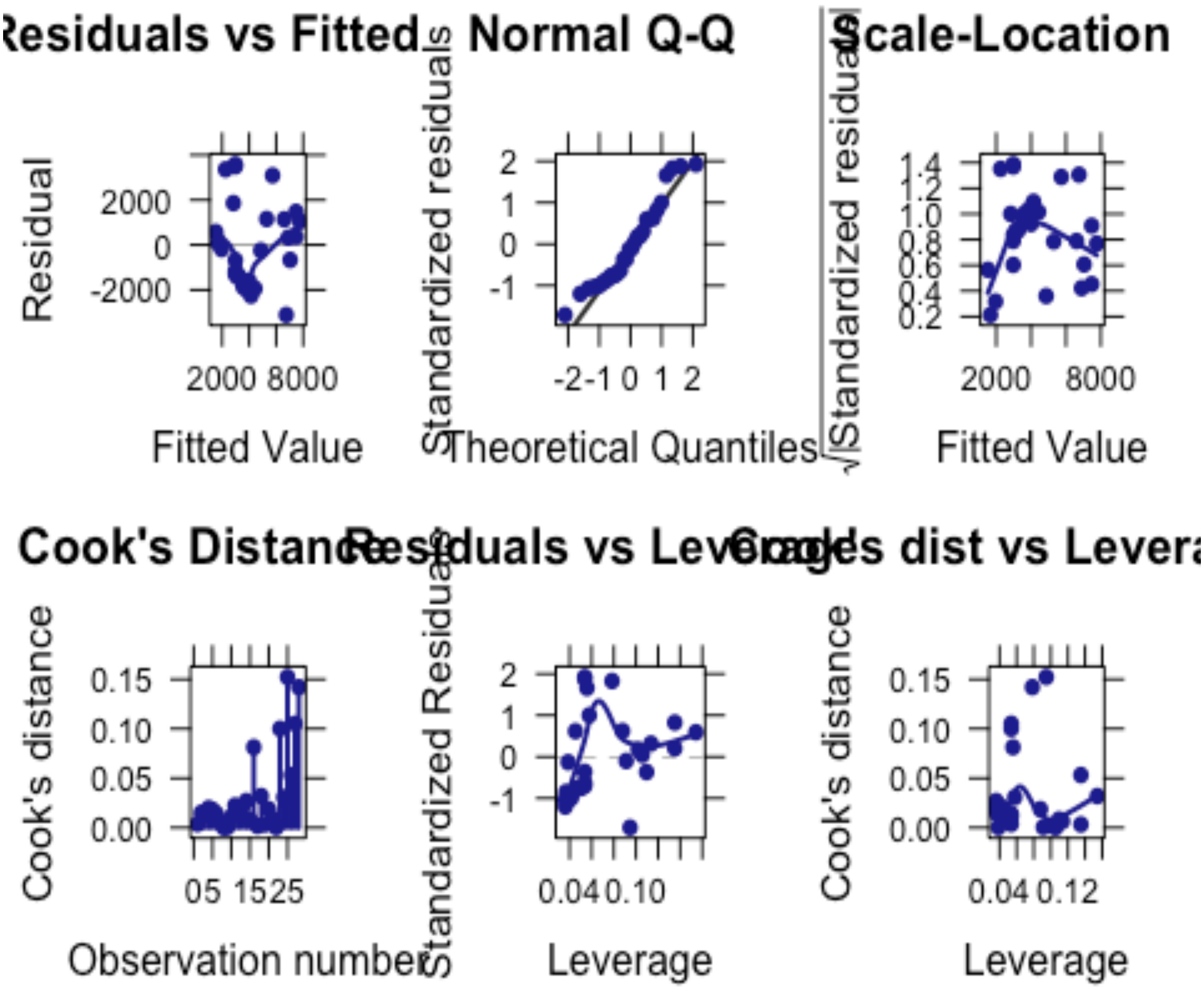
Looks okay

```
#####  f)  Y=Cook Distance v X=leverage (Good if:  no trend of any sort)
mplot (model_simple, which=6)
```



Also looks okay.

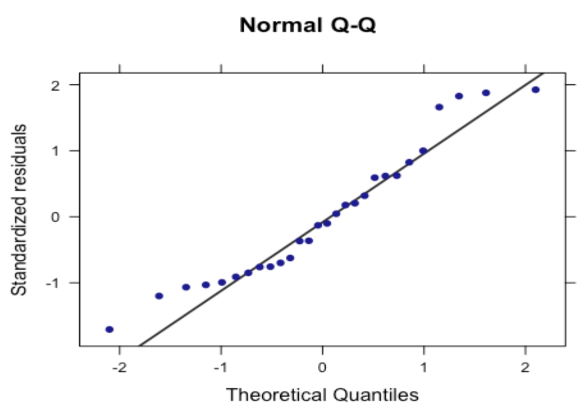
```
#####   The 6 mplots( )  above in a single panel
p1 <- mplot(model_simple, which=1)
p2 <- mplot(model_simple, which=2)
p3 <- mplot(model_simple, which=3)
p4 <- mplot(model_simple, which=4)
p5 <- mplot(model_simple, which=5)
p6 <- mplot(model_simple, which=6)
grid.arrange(p1, p2, p3, p4, p5, p6, ncol=3)
```



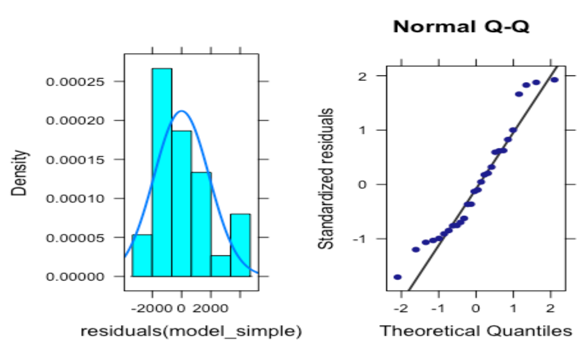
Hmmmm – I think I need to find a way to make the text in each of these 6 plots a lot SMALLER, so as to make more room for the plot itself!

7. Simple Linear Regression Diagnostics – A Suggested Approach

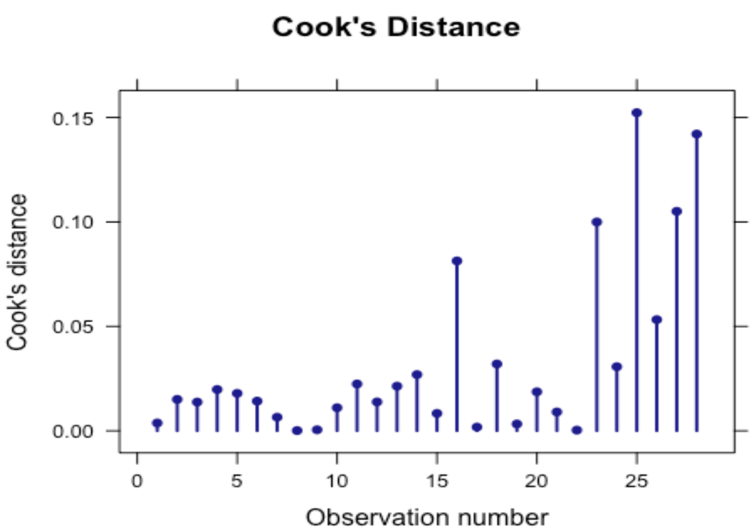
```
library(car)
library(mosaic)
library(ggplot2)
library(gridExtra)

# Retrieve some post estimation variables -
residual <- resid(model_simple)               # Simple residuals
sresidual <- rstudent(model_simple)           # Studentized residuals

#####  a) NORMALITY (Good if:  residuals are distributed normal)
# Test - - Shapiro Wilk Test of residuals (Null:  distribution is normal)
shapiro.test(residual)

##
##  Shapiro-Wilk normality test
##
## data:  residual
## W = 0.94073, p-value = 0.1154

# Plots - -
p1<- histogram(~residuals(model_simple), density=TRUE)
p2 <- mplot(model_simple, which=2)
```




```
grid.arrange(p1, p2, ncol=2)
```

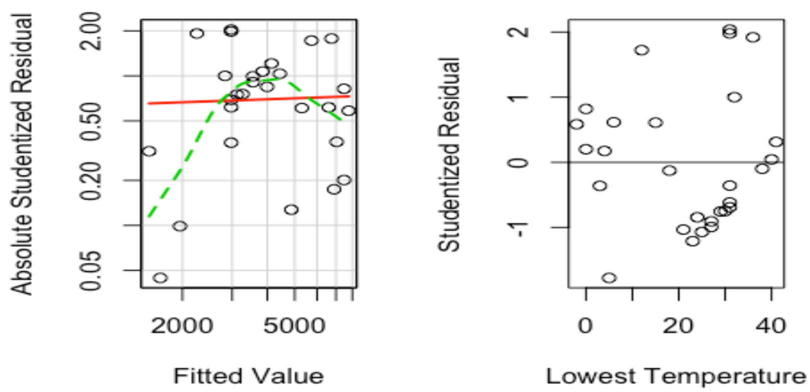


```
##### b)  Outlier Detection (Good if: no outliers)
# Test - - Test of Outliers (Null: no misspecification)
outlierTest(model_simple)

##
## No Studentized residuals with Bonferonni p < 0.05
## Largest |rstudent|:
##    rstudent unadjusted p-value Bonferonni p
## 27 2.037655           0.052299           NA

#####  c) Influential Observations Detection (Good if:  no observation is influential)
cook <- cooks.distance(model_simple)
# identify cook distance values > 4/(n-p-1)
cutoff <- 4/((nrow(ersdata)-length(model_simple$coefficients)-2))
# Plot
mplot(model_simple, which=4)
```




```
# List observations with cook distance values > cutoff  (Note: if all is well, you'll ge
t no output)
ersdata[cook>cutoff,]

##  [1] day     calls   fhigh   flow    high    low     rain    snow
##  [9] weekday year    sunday  subzero
## <0 rows> (or 0-length row.names)

####   d)  Non-Constant Variance (Good if: variance is constant)
# Test:  Homogeneity of variance (Null:  Variance is constant)
ncvTest(model_simple)

## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 0.3071705    Df = 1     p = 0.5794217
```

```
# Plots - -
par(mfrow = c(1, 2))                    # Set Plotting Arrangment to 1 row x 2 columns
spreadLevelPlot(model_simple,ylab="Absolute Studentized Residual", xlab="Fitted Value",
main="")

##
## Suggested power transformation:  0.9333337

plot(ersdata$low, sresidual,xlab="Lowest Temperature",ylab="Studentized Residual"); abli
ne(0,0)
```




```
# TIP!!!  Restore plotting arrangement to default setting of 1x1 single panel
par(mfrow = c(1, 1))
```