Mahidol University
Wisdom of the Land

Logic and Computer Design Fundamentals

Chapter 2 – Combinational Logic Circuits

Part 1 – Gate Circuits and Boolean Equations

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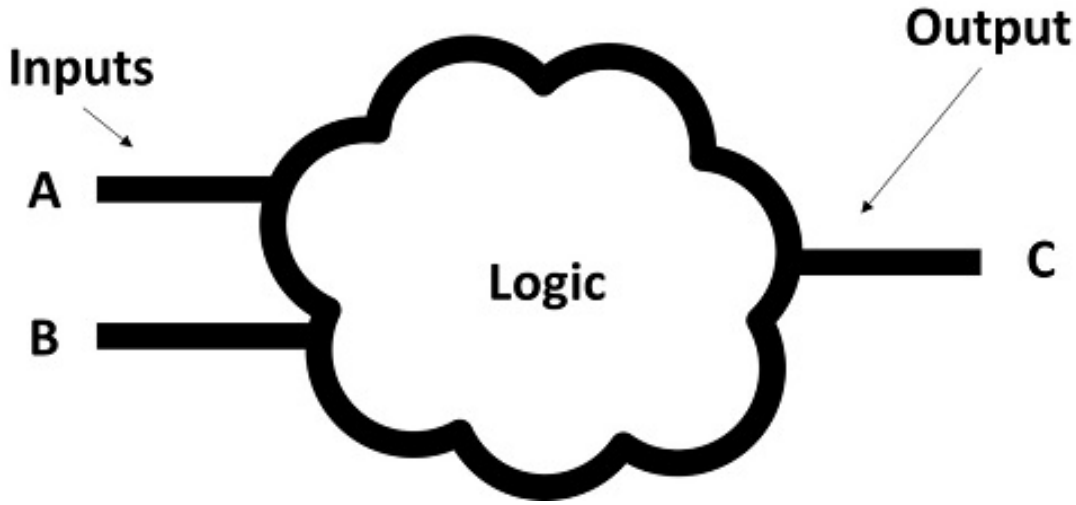
Overview

- **Part 1 – Gate Circuits and Boolean Equations**
  - **Binary Logic**
  - **Logic Gates**
  - **Boolean Algebra**

I. Binary Logic

Logic



Binary Logic and Gates

- Binary variables take on one of two values.
- Logical operators operate on binary values and binary variables.
- Basic logical operators are the logic functions AND, OR and NOT.
- Logic gates implement logic functions.
- Boolean Algebra: a useful mathematical system for specifying and transforming logic functions.
- We study Boolean algebra as a foundation for designing and analyzing digital systems!

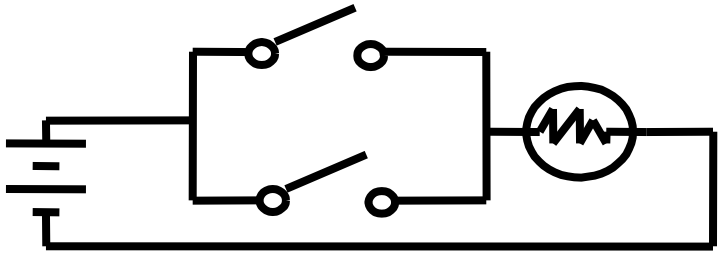
Binary Variables

- Recall that the two binary values have different names:
  - True/False
  - On/Off
  - Yes/No
  - 1/0
- We use 1 and 0 to denote the two values.
- Variable identifier examples:
  - A, B, x, y, or X_1 for now
  - RESET, START_IT, or ADD1 later

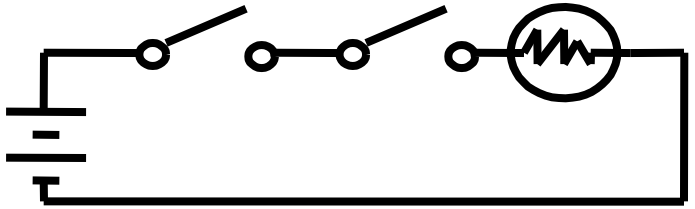
Logic Function Implementation

- Using Switches
  - For inputs:
    - logic 1 is switch closed
    - logic 0 is switch open
  - For outputs:
    - logic 1 is light on
    - logic 0 is light off.

Switches in parallel => OR



Switches in series => AND



Logical Operations

- The three basic logical operations are:
  - AND
  - OR
  - NOT
- AND is denoted by a dot (·).
- OR is denoted by a plus (+).
- NOT is denoted by an overbar ($\bar{}$), a single quote mark (') after, or dash (~) before the variable.

Notation Examples

- Examples:
  - $\mathbf{Y = A \cdot B}$ is read "Y is equal to A AND B"
  - $\mathbf{a = x + y}$ is read "a is equal to x OR y"
  - $\mathbf{X = \overline{A}}$ is read "X is equal to NOT A"

the reduced form of AND is Y = AB

- Note: The statement:

  1 + 1 = 2 (read "one plus one equals two")

  is not the same as

  1 + 1 = 1 (read "1 or 1 equals 1")

Operator Definitions

Operations are defined on the values "0" and "1" for each operator:

AND	OR	NOT
$0 \cdot 0 = 0$	$0 + 0 = 0$	$\bar{0} = 1$
$0 \cdot 1 = 0$	$0 + 1 = 1$	$\bar{1} = 0$
$1 \cdot 0 = 0$	$1 + 0 = 1$	
$1 \cdot 1 = 1$	$1 + 1 = 1$	

Boolean Operator Precedence

- The order of evaluation in a Boolean expression is:
  1. Parentheses
  2. NOT
  3. AND
  4. OR
- Consequence: Parentheses appear around OR expressions
- Example: $F = A(B + C)(C + \overline{D})$

Truth Tables

- *Truth table* – a tabular listing of the values of a function for all possible combinations of values on its arguments
- Example: Truth tables for the basic logic operations:

AND		
X	Y	Z = X·Y
0	0	0
0	1	0
1	0	0
1	1	1

OR		
X	Y	Z = X+Y
0	0	0
0	1	1
1	0	1
1	1	1

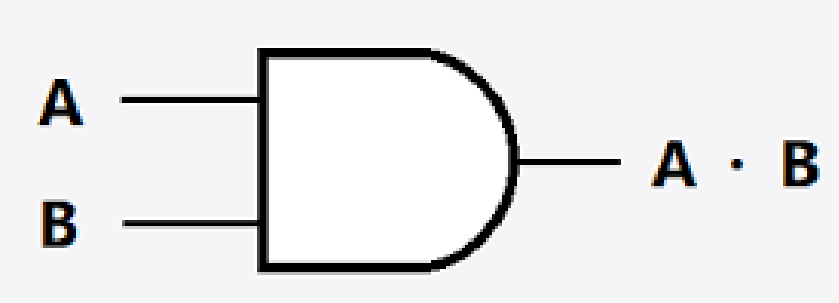
NOT	
X	$Z = \overline{X}$
0	1
1	0

Practice: Q1

Complete the following truth table:

A	B	C	$A \cdot B \cdot C$	$\overline{A}+(B \cdot C)$	$(A+B) \cdot \overline{C}$
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Logic Gates

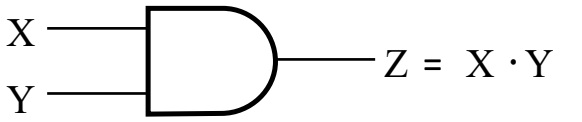


Logic Gates

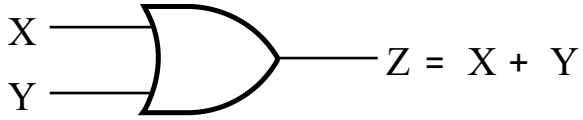
- A logic gate is an idealized model of computation or physical electronic device implementing the Boolean function, a logical operation performed on one or more binary inputs that produces a single binary output.

Logic Gate Symbols and Behavior

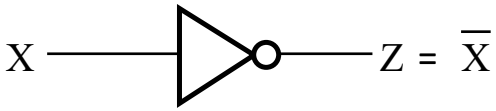
Three basic symbols for Logic gates:



AND gate



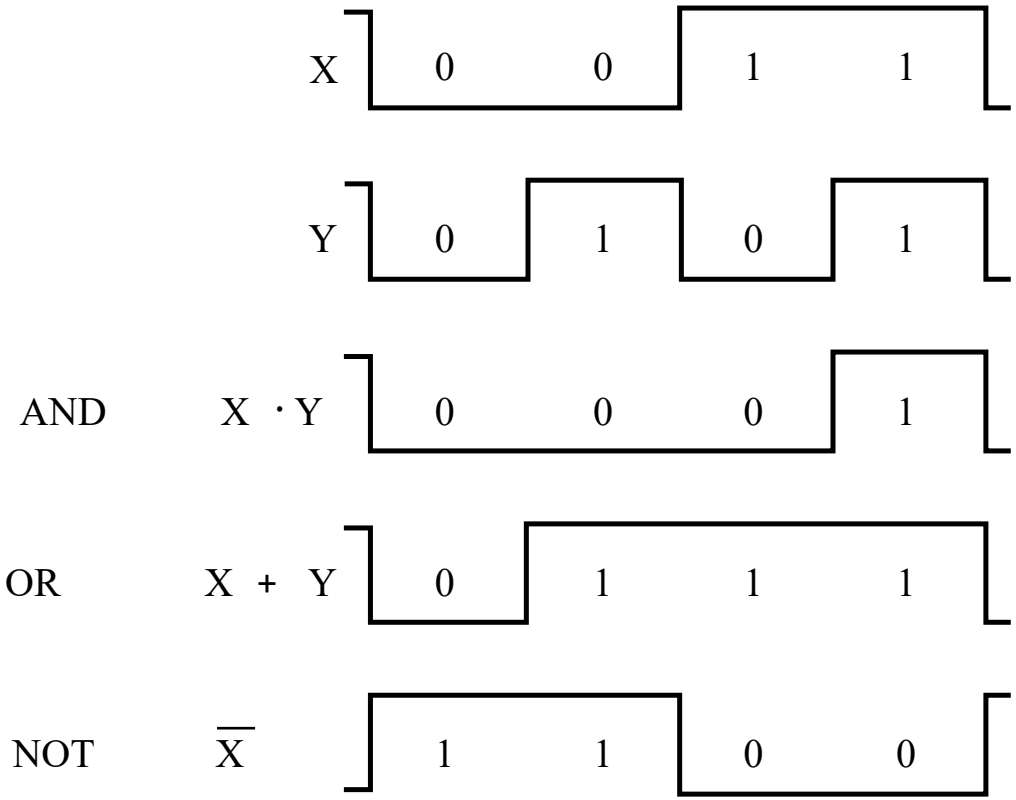
OR gate



NOT gate or
inverter

Timing Diagram

Timing Diagram represents the waveform behavior in time as follows



Logic Diagram

- Logic diagram is a diagram that displays graphically, by interconnection of logic symbols, the digital design of a logic circuit or system.

Logic Diagrams and Expressions

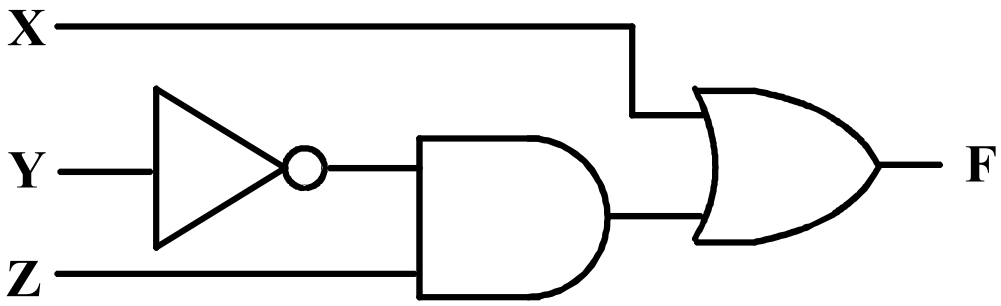
Truth Table

X Y Z	$F = X + \overline{Y} \cdot Z$
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

Equation

$$F = X + \overline{Y}\,Z$$

Logic Diagram



- Boolean equations, truth tables and logic diagrams describe the same function