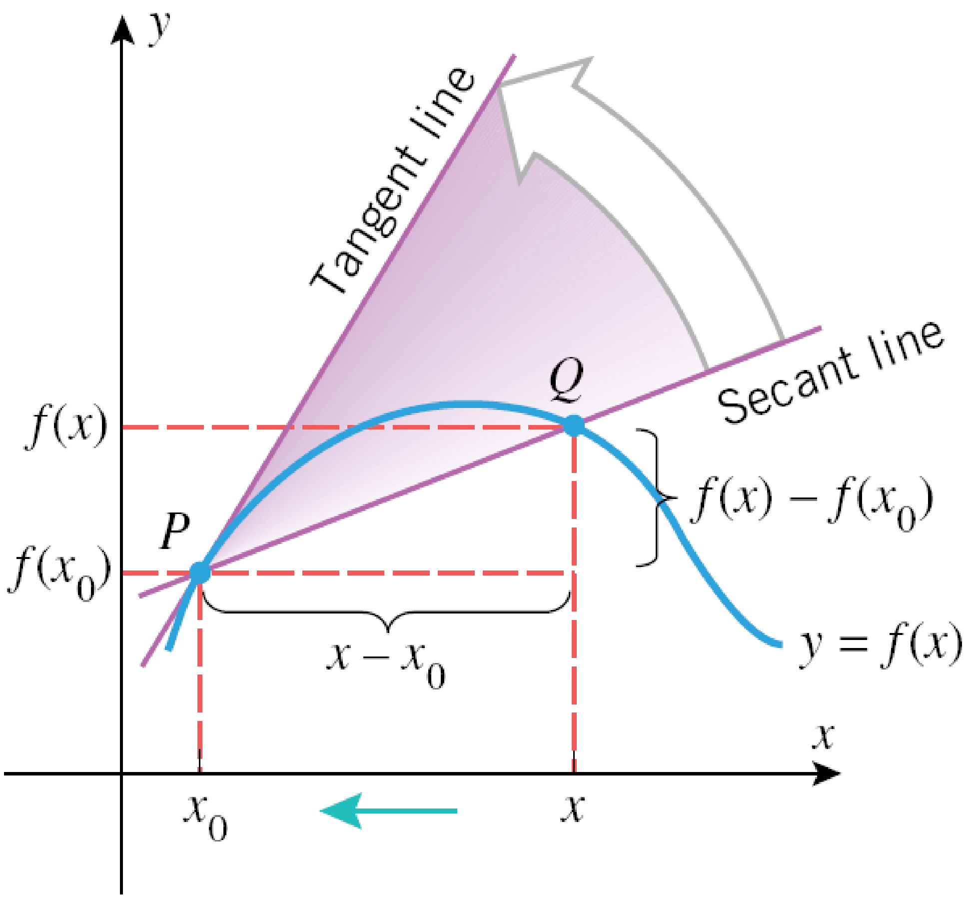
ITCS 111
Chapter 2: *Derivatives*

Some of the material in these slides is from *Calculus* 10/E by Howard Anton, Irl Bivens, and Stephen Davis

2.1 Tangent Line
Derivative = Slope of Tangent Line



Tangent line intersects only one point on a curve.

Secant line intersects two or more point on a curve.

Derivative = Slope of Tangent Line

2.1.1 DEFINITION Suppose that x_0 is in the domain of the function f. The ***tangent line*** to the curve $y = f(x)$ at the point $P(x_0, f(x_0))$ is the line with equation

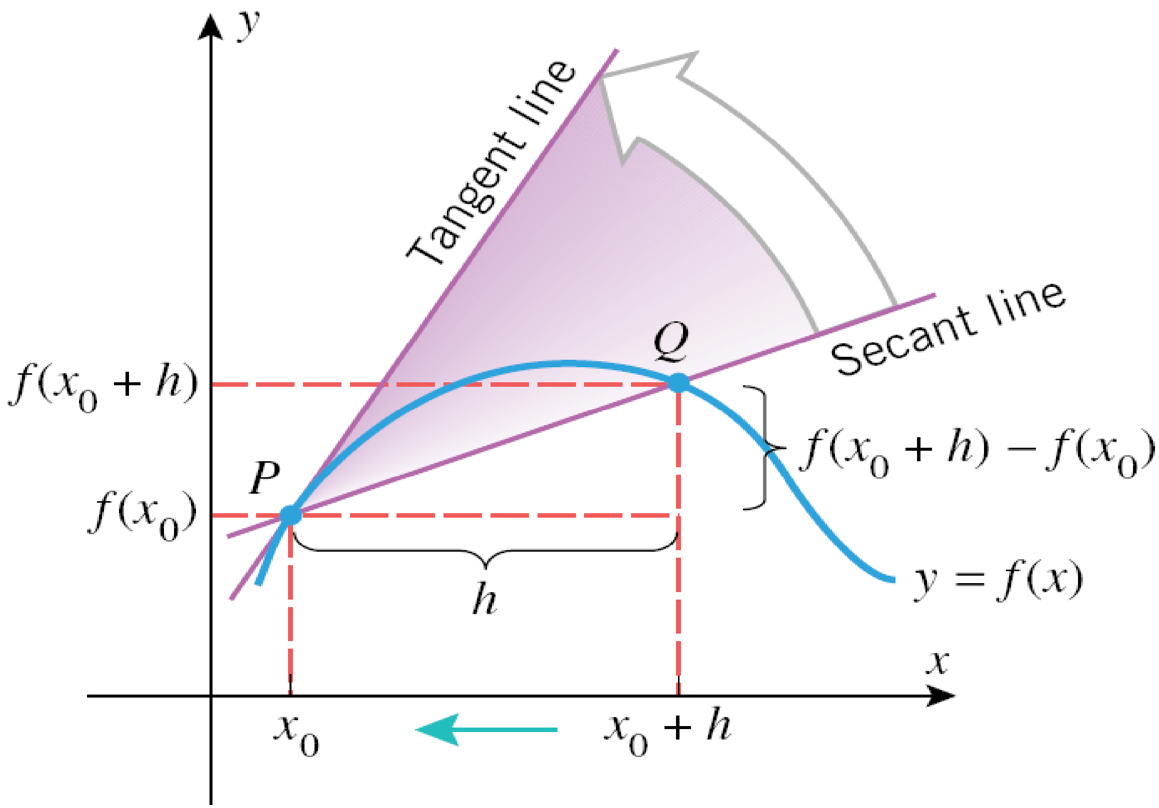
$$y - f(x_0) = m_{\tan}(x - x_0)$$

where

$$m_{\tan} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \tag{1}$$

provided the limit exists. For simplicity, we will also call this the tangent line to $y = f(x)$ at x_0.

Alternate Derivative Formulations



Example (p111, 112)

2.2 Derivative Function

Derivative *Function* Definition

2.2.1 DEFINITION The function f' defined by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{2}$$

is called the ***derivative of f with respect to x***. The domain of f' consists of all x in the domain of f for which the limit exists.

Differentiability

2.2.2 DEFINITION A function f is said to be ***differentiable at x_0*** if the limit

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} \tag{5}$$

exists. If f is differentiable at each point of the open interval (a, b), then we say that it is ***differentiable on (a, b)***, and similarly for open intervals of the form $(a, +\infty)$, $(-\infty, b)$, and $(-\infty, +\infty)$. In the last case we say that f is ***differentiable everywhere***.

Compute derivatives by using the derivative function definition

Example: Calculate $f'(-2)$, where $f(x) = 1 - x^2$

- First, find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[1 - (x+h)^2] - [1 - x^2]}{h}$$

$$= \lim_{h \to 0} (-2x - h)$$

$$= -2x$$

- Substitute -2 for x

$$f'(-2) = -2(-2) = 4$$

Differentiability

2.2.2 DEFINITION A function f is said to be ***differentiable at*** x_0 if the limit

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exists. If f is differentiable at each point of the open interval (a, b), then we say that it is ***differentiable on*** (a, b), and similarly for open intervals of the form $(a, +\infty)$, $(-\infty, b)$, and $(-\infty, +\infty)$. In the last case we say that f is ***differentiable everywhere***.

2.2.3 THEOREM

If a function f is differentiable at x_0, then f is continuous at x_0.

Derivative Notations

A derivative can be indicated by "double-d" notation

Example:

$\frac{dy}{dx}$	if	y	is a function of x
$\frac{dy}{dt}$	if	y	is a function of t,
$\frac{dy}{dz}$	if	y	is a function of z

$\frac{dy}{dx} = \frac{df(x)}{dx} = f'(x)$: $\frac{dy}{dx}$ read "the derivative of y with respect to x",
$f'(x)$ is called "the derivative of f with respect to x".

The value of derivative at a specific value x=x_0 is written as $\frac{dy}{dx}|_{x\ =\ x0} = f'(x_0)$

Derivative Function

Remarks:

1) The process of finding a derivative is called **differentiation**.
2) If x_0 is not in the domain of f or if the limit does not exist, then we say that f is **not differentiable at x_0**.
3) If f is differentiable at every value of x in an open interval (a, b), then we say that f is **differentiable on (a, b).**

Exercise

EXERCISE# 5: The Derivative Function