NUMBER BASES

Properties of Integers

Learning objectives! Know what you will learn today

Self-Reflection! Rate levels of your understanding

❍ Checklist of key topics. Keep catching up with the course.

- ☐ Base conversion and finding the base the conversion is in
- ☐ Addition and subtraction in any bases
- ☐ Multiplication and division in any bases
- ☐ Finding the base an arithmetic calculation is in
- ☐ Application: Bacon's Code encryption and decryption

Confident	Got it	Okay	Fuzzy	Not a clue

To represent any number, no matter how large or how small

Numbering Systems	Base	Digits
Binary	2	0, 1
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

These are common bases in digital technology; in general, base $b \in \mathbb{Z}^+$

- Know how to **count** and how to **convert** from one base to another
- Know how to **add**, **subtract**, **multiply**, and **divide** in different bases
- Know how to encode & decode messages using base 26 **Bacon's code**

Conversion between any bases

Power expansion **Repeated division**

Step1: convert base 13 to base 10

$$AB58_{13} = 10 \times 13^3 + 11 \times 13^2 + 5 \times 13^1 + 8 \times 13^0$$
$$= 21970 + 1859 + 65 + 8$$
$$= 23902_{10}$$

Step2: convert base 10 to base 18

$$18 \mid 23902$$
$$18 \mid 1327 \quad 16$$
$$18 \mid 73 \quad 13$$
$$4 \quad 1$$

$$AB58_{13} = 41DG_{18}$$

Conversion by grouping of digits

Conversion by grouping of digits. This works when the larger base is some power of a smaller base.

Convert from binary to base 8
combining a group of 3 bits

$1\ 010\ 100\ 011\ 110_2$

⇨ 12436_8

Convert from base 8 to binary
expanding by a group of 3 bits

27504_8

⇨ $10\ 111\ 101\ 000\ 100_2$

RECAP

Conversion from 29BD5 base 14 to base 9

Step1: convert base 14 to base 10

Step2: convert base 10 to base 9

Conversion between base 4 and base 16

$2\ 23\ 11\ 22\ 33_4$ ⇨

0xEB625 ⇨

Finding bases of the conversion

- Find the value of the base b, given $(AB)_b$ = 0x83

WORKED EXAMPLES

- Find the value of the base b, given $(142)_b + (112)_{b-2} = (75)_8$

PRACTICE PROBLEMS

- Find the value of the base b, given $(152)_b = 0x6A$

- Find the value of the base b, given $(1204)_b = (292)_{10}$

PRACTICE PROBLEMS

- Find the value of the base b, given $(100)_b = (16)_{10}$

- When the number n is written in base b its representation is the two-digit number AB where $A = b - 2$ and $B = 2$. What is the representation of n in base $(b - 1)$?

Base Arithmetic

No matter which base is used to represent integers, elementary rules of **addition**, **subtraction**, **multiplication**, and **division** are still valid

- **Carrying** concept → same as base 10
- **Borrowing** concept → same as base 10

Goal: be able to do it in any bases (binary and others)

Addition in binary

- $(111)_2 + (110)_2$

$$\begin{array}{r} 1\ 1\ 1 \\ 1\ 1\ 0\ + \\ \hline \\ \hline \end{array}$$

Addition in any bases

- $(A1B5)_{20} + (E7)_{20}$

$$\begin{array}{r} A\ 1\ B\ 5 \\ E\ 7\ + \\ \hline \\ \hline \end{array}$$

Subtraction in binary

- $(1011)_2$ – $(101)_2$

```
  1  0  1  1
     1  0  1 -
  ___________
  ___________
```

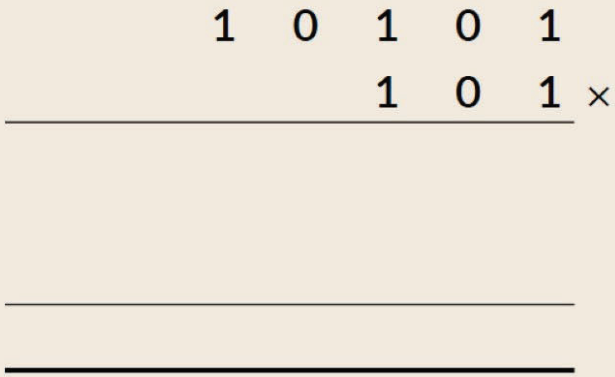

Subtraction in any bases

- $(10C48)_{13}$ – $(3963)_{13}$

```
  1  0  C  4  8
     3  9  6  3 -
  ________________
  ________________
```

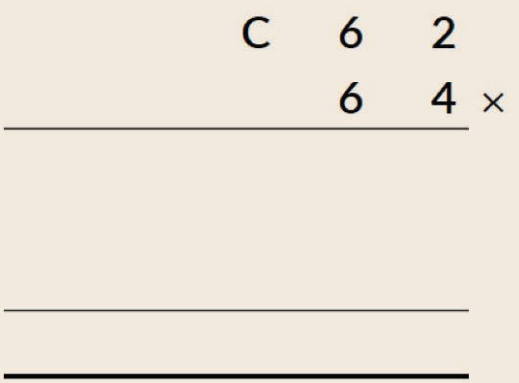
Multiplication in binary

- $(10101)_2 \times (101)_2$



Multiplication in any bases

- 0xC62 × 0x64



Division in binary

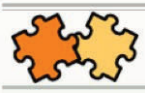
- $(11011)_2 \div (101)_2$

$$101 \overline{)\ 1\ 1\ 0\ 1\ 1}$$

Division in any bases

- $(21254)_6 \div (51)_6$

$$51 \overline{)\ 2\ 1\ 2\ 5\ 4}$$



PRACTICE PROBLEMS

Arithmetic in binary (base 2)

- $110 + 11 =$
- $101 - 11 =$
- $100 \times 11 =$
- $111 \div 11 =$

PRACTICE PROBLEMS

Arithmetic in other bases (non-binary)

Base 4: $(333)_4 + (222)_4 =$

Base 5: $(141)_5 - (104)_5 =$

Base 6: $(512)_6 \times (24)_6 =$

Base 7: $(2252)_7 \div (23)_7 =$

PRACTICE PROBLEMS

Arithmetic in other (higher) bases

Base 22: $(CAL1)_{22}$ + $(10L5)_{22}$ =

Base 19: $(H5A24)_{19}$ – $(9091)_{19}$ =

Base 15: $(BDC)_{15}$ × $(E3)_{15}$ =

Base 11: $(1A46A)_{11}$ ÷ $(AA)_{11}$ =

PRACTICE PROBLEMS

Find the base the calculation is in

In which base was each of the following calculations carried out?

- 7 + 5 = 13
- 5115 – 343 = 4442
- 846 – 55 = 7A1

PRACTICE PROBLEMS

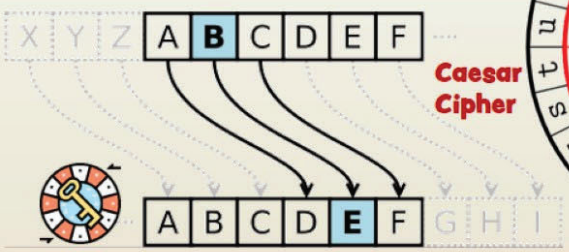
Find the base the calculation is in

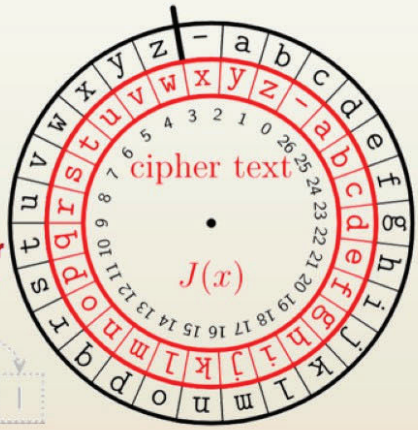
In which base was each of the following calculations carried out?

- $4 \times 5 = 32$
- $171 \times 12 = 2272$
- $432 \times 21 = 20122$

Applications

- The bases 2, 8 and 16 are frequently used in computer science
- The **base 26** is sometimes used in **cryptology**, the science of producing and deciphering secret codes
- Before, we learned Caesar (shift) cipher
- Next, code due to **Sir Francis Bacon**





https://www.rocketmatter.com/technology/evernote-lawyers-client-confidentiality-reasonable-care/

Number Bases and Cryptography

NEXT STEPS

How messages are encrypted and decrypted in Bacon's code based on base 26 and binary.

Base 26

Use 26 letters A, B, ..., Z of English alphabet to represent digits 0, 1, ... , 25

- What is the expansion in base 10 of $(TWO)_{26}$?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

PRACTICE PROBLEMS

Use 26 letters A, B, ..., Z of the English alphabet to represent digits 0, 1, ... , 25

- what is the base 26 representation of ZOO + FUN and ZOO – FUN?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

PRACTICE PROBLEMS

Use 26 letters A, B, ..., Z of the English alphabet to represent digits 0, 1, ... , 25

- what is the base 26 representation of ZOO × FUN and ZOO ÷ FUN?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Bacon Encryption Step 1 and 2 of 4

- Encode the message "FLEE NOW"
- Map each alphabet to its corresponding base-26 decimal value
- Convert the each decimal value to 5-digit base 2 representation

F L E E N O W

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Bacon Encryption Step 3 of 4

- Choose unrelated **dummy message exactly five times as long** (padded with a few extra letters if necessary)

Length of the given message **"FLEE NOW" is 7 letters**

Hence, the length of a dummy message is **7 × 5 = 35 letters**

Here, we choose a dummy message (length=35):

"ONCE UPON A TIME IN THE WEST THERE WAS A TOWN"

Ch1.4, Ex.15

Bacon Encryption Step 4 of 4

- Place the dummy message in a third row
- Write each letter in one font if it corresponds to a 0 and another font if it corresponds to 1 (e.g., italic & bold, lower & upper case)

F L E E N O W

- Encoded message:

Ch1.4, Ex.15

Decode the Bacon's Code

- Decode the following message using the Bacon code
- Now Is The TIme for All gOOd mEN tO aiD tHE CountrY

NowIs TheTI mefor AllgO OdmEN tOaiD tHECo untrY

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

PRACTICE PROBLEMS

Use Bacon's code to encode the word COME BACK

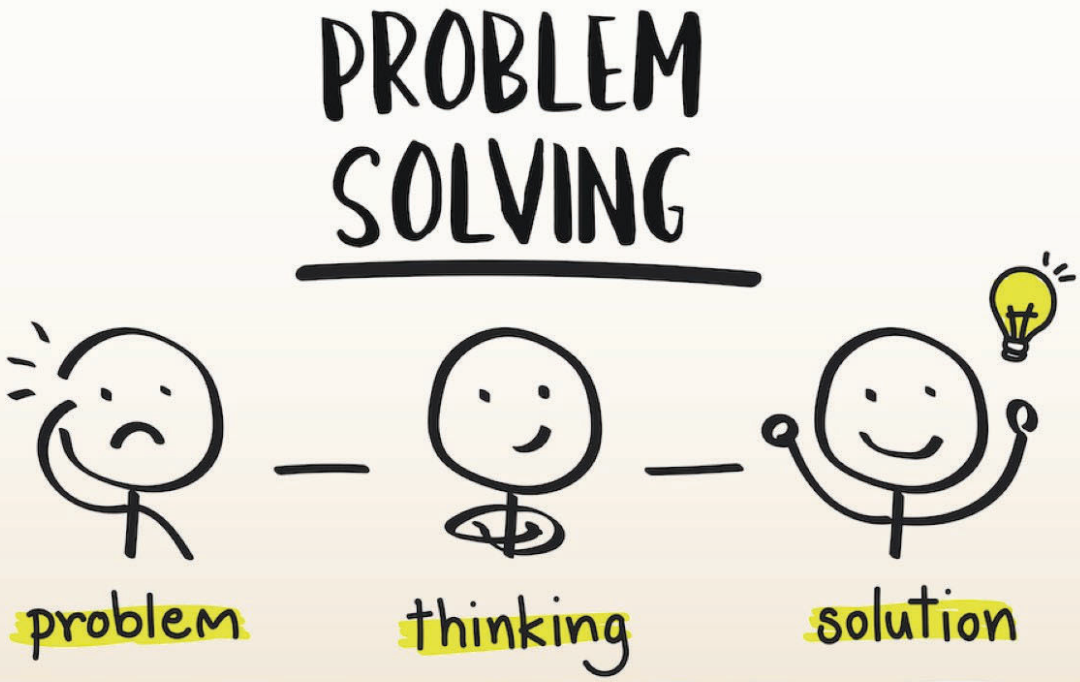
PRACTICE PROBLEMS

Use Bacon's code to decode
WheN tHe mOON cOmes ovER THe moUnTAin, tHe owLs FLy HiGH

PRACTICE PROBLEMS

- Modify Bacon's code so that it handles the spaces between words and the digits 0, 1, 2, . . . , 9.

Ch1.4, Q.46



Knowing how to count in any bases

PRACTICE PROBLEMS

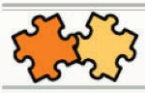
- What is $AAA_{11} + 1_{11}$ in decimal value?

No addition here! Just be able to count, knowing what the next number is.

PRACTICE PROBLEMS

- Given $9^6 = 531{,}441$, how would you represent 531,440 in base 9?

No base conversion here! Know how to count, what comes before or after.



PRACTICE PROBLEMS

- How many natural numbers require 3 digits when written in base 12, but require 4 digits when written in base 9?

PRACTICE PROBLEMS

- What is the smallest and the largest base 10 number that can be expressed as a 3-digit base 5 number?

Quiz08 (*Optional, though recommended*) Since the time is short, this quiz is half the length of the others and the scores will count towards extra credits. Use it as part of your review for the midterm. *If you missed earlier quizzes, this can help!*

WEEKS 09-15

AJ. Tee
Rawesak Tanawongsuwan

Microsoft Teams

Sec1: Mon Oct 9, 9am – noon
Sec4: Mon Oct 9, 1pm – 4pm

Weeks 9-10 Relations	Weeks 11-12 Prop. of Relations	Weeks 13-15 Functions
Product Sets, Partitions	Properties of Relations	Functions in CS
Relations and Digraphs	Equivalence Relations	Growth of Functions
Boolean Matrix of Rel.	Transitive Closure	Permutation Functions