



# **Linear Systems and Cramer's Rule**

# Linear Systems and Cramer's Rule

## THEOREM 2.3.7 Cramer's Rule

*If  $A\mathbf{x} = \mathbf{b}$  is a system of  $n$  linear equations in  $n$  unknowns such that  $\det(A) \neq 0$ , then the system has a unique solution. This solution is*

$$x_1 = \frac{\det(A_1)}{\det(A)}, \quad x_2 = \frac{\det(A_2)}{\det(A)}, \quad \dots, \quad x_n = \frac{\det(A_n)}{\det(A)}$$

*where  $A_j$  is the matrix obtained by replacing the entries in the  $j$ th column of  $A$  by the entries in the matrix*

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

# Linear Systems and Cramer's Rule

## ► EXAMPLE 8 Using Cramer's Rule to Solve a Linear System

Use Cramer's rule to solve

$$\begin{aligned}x_1 + \quad + 2x_3 &= 6 \\-3x_1 + 4x_2 + 6x_3 &= 30 \\-x_1 - 2x_2 + 3x_3 &= 8\end{aligned}$$

**Solution**

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{bmatrix}$$

Therefore,

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{-40}{44} = \frac{-10}{11}, \quad x_2 = \frac{\det(A_2)}{\det(A)} = \frac{72}{44} = \frac{18}{11},$$
$$x_3 = \frac{\det(A_3)}{\det(A)} = \frac{152}{44} = \frac{38}{11} \quad \blacktriangleleft$$

## Exercises

1. Solve the linear system by Cramer's rule, if it can be applied.

$$3x_1 - x_2 + x_3 = 4$$

$$-x_1 + 7x_2 - 2x_3 = 1$$

$$2x_1 + 6x_2 - x_3 = 5$$

2. Solve for  $y$  of the linear system by Cramer's rule, if it can be applied.

$$4x + y + z + w = 6$$

$$3x + 7y - z + w = 1$$

$$7x + 3y - 5z + 8w = -3$$

$$x + y + z + 2w = 3$$

## Exercises

3. Solve the linear system by Cramer's rule, if it can be applied.

$$4x + 5y = 2$$

$$11x + y + 2z = 3$$

$$x + 5y + 2z = 1$$