ITCS 111
Chapter 2: *Derivatives (cont.)*

Some of the material in these slides is from *Calculus* 10/E by
Howard Anton, Irl Bivens, and Stephen Davis

2.3 Techniques of Differentiation
Derivative Rules

2.3.1 THEOREM **The derivative of a constant function is 0; that is, if c is any number, then $\frac{dy}{dx}[c] = 0$.**

Example

$$\frac{d}{dx}(-3) = 0 \qquad \frac{d}{dx}(1) = 0 \qquad \frac{d}{dx}(\pi) = 0$$

Derivative ***Rules***

2.3.2 THEOREM (*The Power Rule*) *If n is a positive integer, then*

$$\frac{d}{dx}[x^n] = nx^{n-1} \tag{5}$$

In words, *to differentiate a power function, decrease the constant exponent by one* and *multiply the resulting power function by the original exponent.*

Example 2 (p 135): $\frac{d}{dx}[x^4]$, $\frac{d}{dt}[t^{12}]$

Derivative ***Rules***

2.3.3 THEOREM (*Extended Power Rule*) *If r is any real number, then*

$$\frac{d}{dx}[x^r] = rx^{r-1} \tag{7}$$

To differentiate a power function, *decrease the constant exponent by one* and *multiply the resulting power function by the original exponent*.

Example 3 (p 136): $\frac{d}{dx}[x^{\pi}]$, $\frac{d}{dt}[t^{4/5}]$, $\frac{d}{dw}\left[\frac{1}{w^{100}}\right]$

Derivative ***Rules***

2.3.4 THEOREM (*Constant Multiple Rule*) *If f is differentiable at x and c is any real number, then cf is also differentiable at x and*

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)] \tag{8}$$

In words, *a constant factor can be moved through a derivative sign.*

Example 4 (p137): $\frac{d}{dx}[4x^8]$, $\frac{d}{dx}\left[\frac{\pi}{x}\right]$

Derivative ***Rules***

2.3.5 THEOREM ***(Sum and Difference Rules)*** *If f and g are differentiable at x, then so are $f+g$ and $f-g$ and*

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)] \tag{9}$$

$$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)] \tag{10}$$

In words, *the derivative of a sum equals the sum of the derivatives*, and *the derivative of a difference equals the difference of the derivatives.*

Examples 5 (p 137): $\frac{d}{dx}[2x^6 + x^{-9}]$

2.4 The Product and Quotient Rules
The Product *Rule*

2.4.1 THEOREM (*The Product Rule*) *If f and g are differentiable at x, then so is the product $f \cdot g$, and*

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)] \tag{1}$$

In words, *the derivative of a product of two functions is the first function times the derivative of the second plus the second function times the derivative of the first.*

Examples 1 (p 143 – 144): Find $\frac{dy}{dx}, y = (4x^2 - 1)(7x^3 + x)$

The Quotient *Rule*

2.4.2 THEOREM ***(The Quotient Rule)*** *If f and g are both differentiable at x and if $g(x) \neq 0$, then f/g is differentiable at x and*

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2} \tag{2}$$

In words, *the derivative of a quotient of two functions is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the denominator squared.*

Examples 3 (p 145) : Find $y'(x), y = \frac{x^3+2x^2-1}{x+5}$,

Examples 5 (p 137) Find $\frac{d}{dx}\left[\frac{\sqrt{x}-2x}{\sqrt{x}}\right]$

Summary of Derivative *Rules*

Table 2.4.1

RULES FOR DIFFERENTIATION			
$\frac{d}{dx}[c]=0$	$(f+g)'=f'+g'$	$(f\cdot g)'=f\cdot g'+g\cdot f'$	$\left(\frac{1}{g}\right)'=-\frac{g'}{g^2}$
$(cf)'=cf'$	$(f-g)'=f'-g'$	$\left(\frac{f}{g}\right)'=\frac{g\cdot f'-f\cdot g'}{g^2}$	$\frac{d}{dx}[x^r]=rx^{r-1}$

Derivatives of higher order

- Second order derivative

$$f'' = (f')' \qquad \text{or} \qquad \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

- Third order derivative

$$f''' = (f'')' \qquad \text{or} \qquad \frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$$

- n$^{\text{th}}$ order derivative

$$f^{(n)} = (f^{(n-1)})' \qquad \text{or} \qquad \frac{d^ny}{dx^n} = \frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right)$$

Example

Example 9 (p139): Let $f(x) = 3x^4 - 2x^3 + x^2 - 4x + 2$.

Find $f'(x), f''(x), f'''(x), f^{(4)}(x), f^{(5)}(x), f^{(6)}(x)$.

Exercises

EXERCISE# 6: Techniques of Differentiation

EXERCISE# 7: The Product and Quotient Rules