Vectors, Lines, Planes

Parametric Equations of Lines, Planes

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Parametric Equations of Lines

Parametric Equations

11.5.1 THEOREM

(*a*) *The line in 2-space that passes through the point* $P_0(x_0, y_0)$ *and is parallel to the nonzero vector* $\mathbf{v} = \langle a, b\rangle = a\mathbf{i} + b\mathbf{j}$ *has parametric equations*

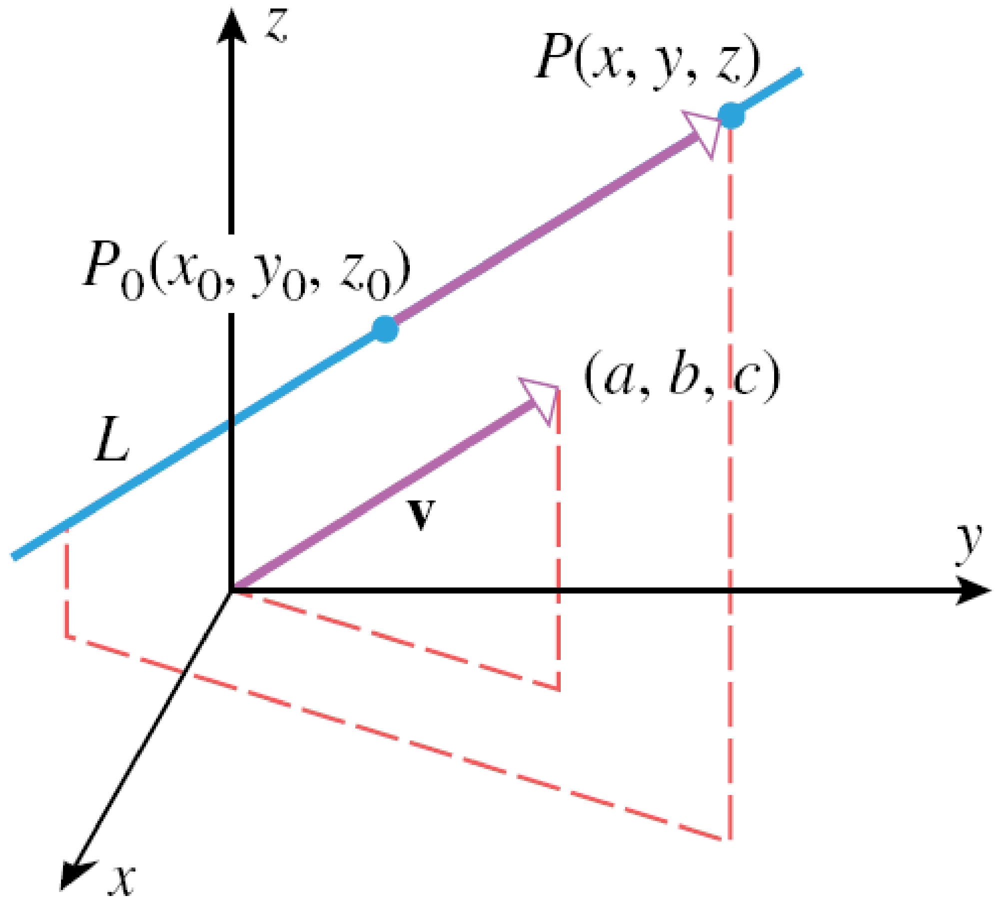
$$x = x_0 + at, \quad y = y_0 + bt \tag{1}$$

(*b*) *The line in 3-space that passes through the point* $P_0(x_0, y_0, z_0)$ *and is parallel to the nonzero vector* $\mathbf{v} = \langle a, b, c\rangle = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ *has parametric equations*

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct \tag{2}$$

Straight-Line Equations (2D)

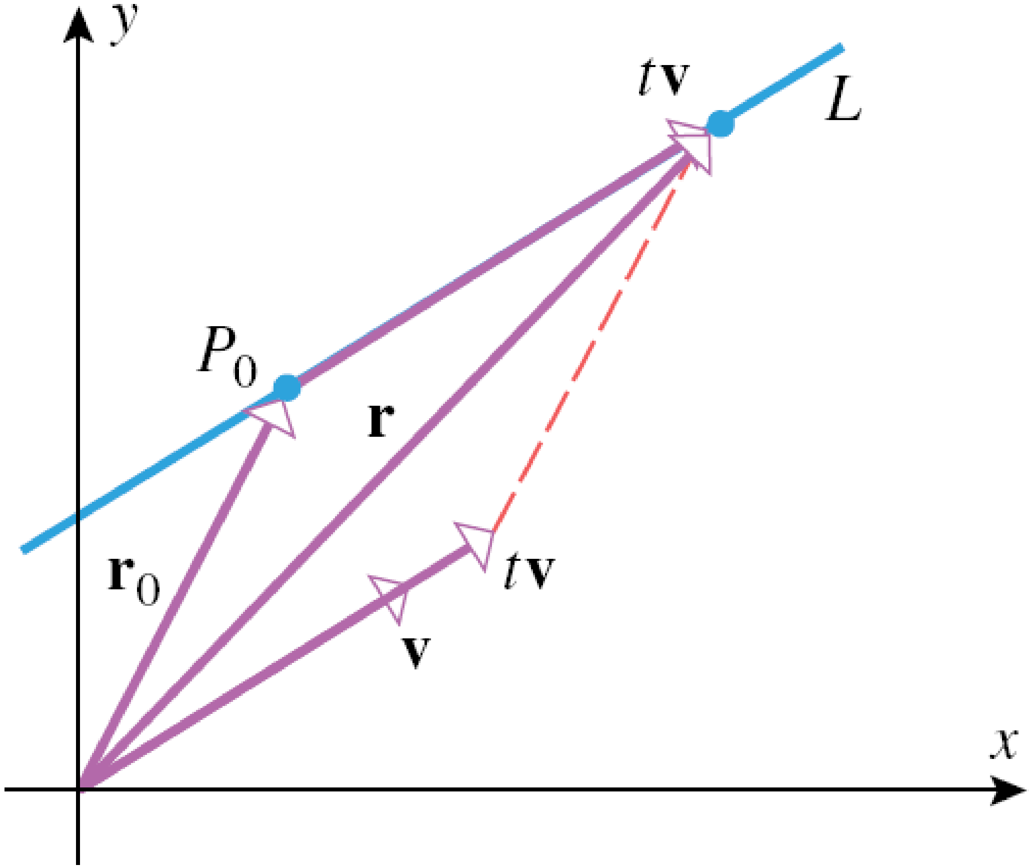
vertical line	$x = a.$	
horizontal line	$y = b.$	
point-slope form	$y - y_0 = m(x - x_0).$	
slope-intercept form	$y = mx + b.$	($y = b$ at $x = 0$)
two-intercept form	$\dfrac{x}{a} + \dfrac{y}{b} = 1.$	(x-intercept a; y-intercept b)
general form	$Ax + By + C = 0.$	(A and B not both 0)



Parametric equations of the line passing points P_0 and P :

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct$$

Vector Equation of a line



$$\boxed{\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}}$$

Example 1: Find parametric equations of the line L passing through the points (4, 2) and parallel to **v** = $\langle -1, 5 \rangle$.

Example 2: Find parametric equations of the line L passing through the points (1, 2, -3) and parallel to $\mathbf{v} = 4\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$.

Example 3: Let L_1 and L_2 be the lines

$$L_1: x = 1 + 4t,\ y = 5 - 4t,\ z = -1 + 5t$$

$$L_2: x = 2 + 8t,\ y = 4 - 3t,\ z = 5 + t$$

(a) Are the lines parallel?

(b) Do the lines intersect?

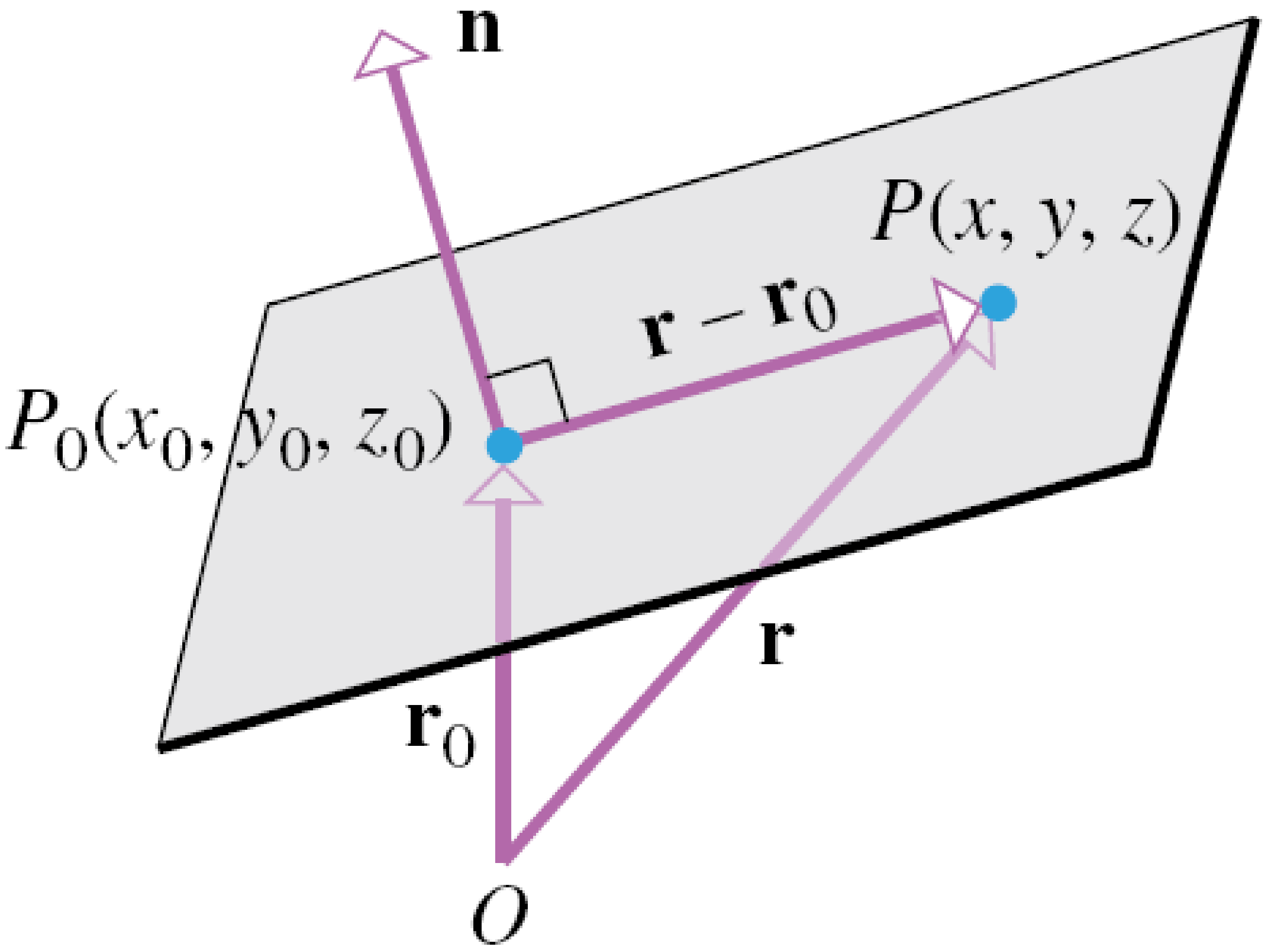
Planes determined by a point and a normal vector

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

$$\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Planes determined by a point and a normal vector



11.6.1 THEOREM *If a, b, c, and d are constants, and a, b, and c are not all zero, then the graph of the equation*

$$ax + by + cz + d = 0 \tag{6}$$

is a plane that has the vector $\mathbf{n} = \langle a, b, c \rangle$ *as a normal.*

General form of a plane

11.6.2 THEOREM *The distance D between a point $P_0(x_0, y_0, z_0)$ and the plane $ax + by + cz + d = 0$ is*

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} \qquad (10)$$

In 3D the **distance *d*** between the points $\mathbf{P_1}(x_1, y_1, z_1)$ and $\mathbf{P_2}(x_2, y_2, z_2)$ is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Distance between a point and a plane