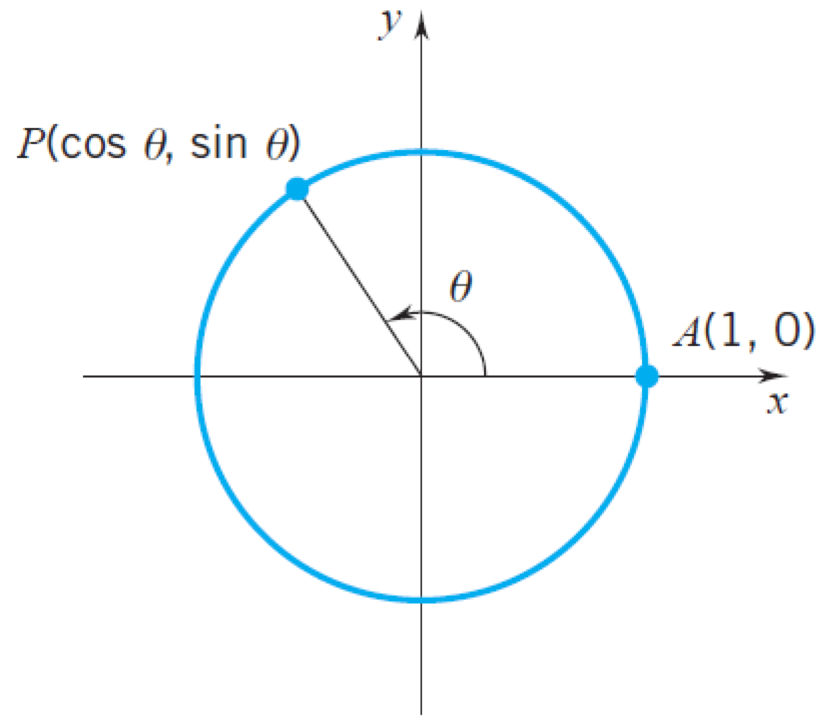
Review

Trig, Exp, Log functions

Cosine & Sine

- Unit circle (radius = 1)
- x-coordinate = cos θ
- y-coordinate = sin θ



Other trigonometric

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}, \qquad \sec\theta = \frac{1}{\cos\theta}, \qquad \csc\theta = \frac{1}{\sin\theta}.$$

They are defined from basic trig functions:

$\boldsymbol{\sin\theta}$, and $\boldsymbol{\cos\theta}$.

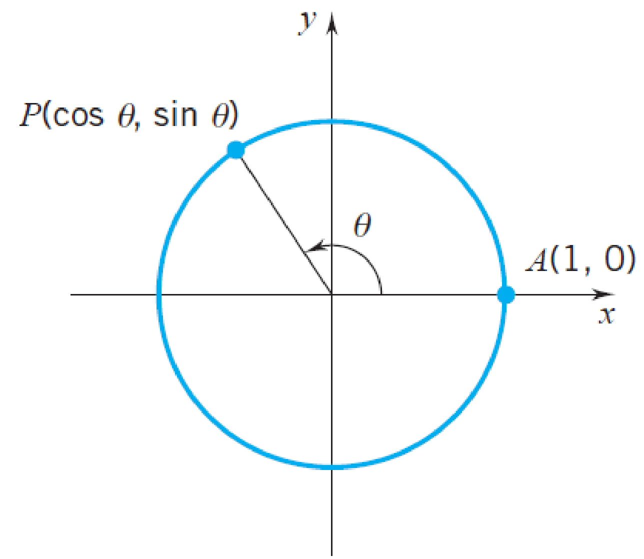
Inverse trigonometric

$\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$

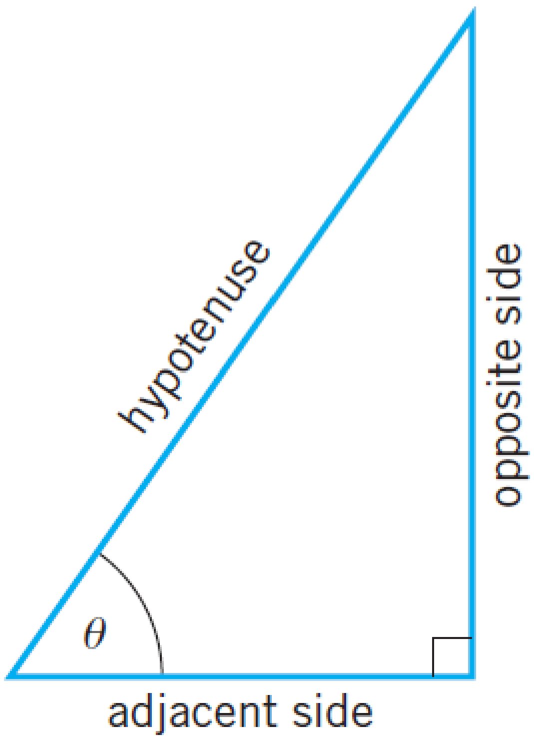
$\csc^{-1}(x)$, $\sec^{-1}(x)$, $\cot^{-1}(x)$

$\arcsin(x)$, $\arccos(x)$, $\arctan(x)$

$\text{arccsc}(x)$, $\text{arcsec}(x)$, $\text{arccot}(x)$



In terms of a <u>right</u> triangle



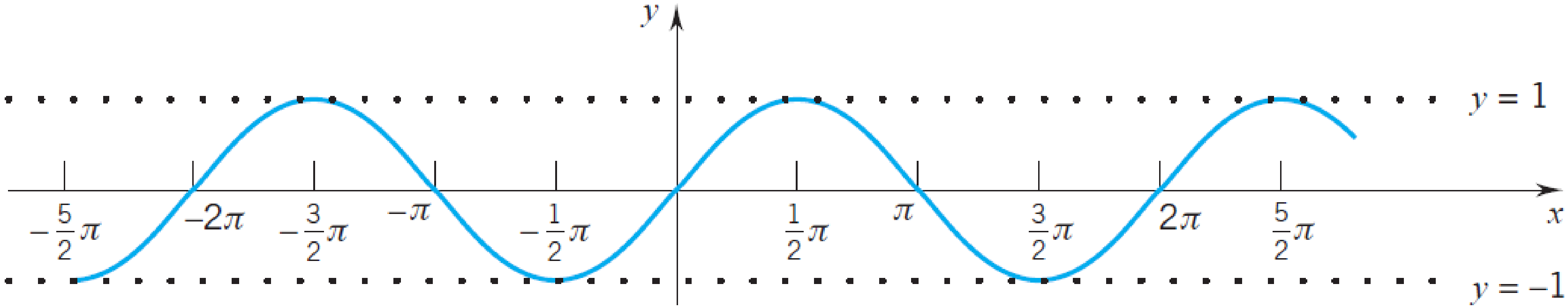
$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}, \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite side}},$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}, \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}},$$

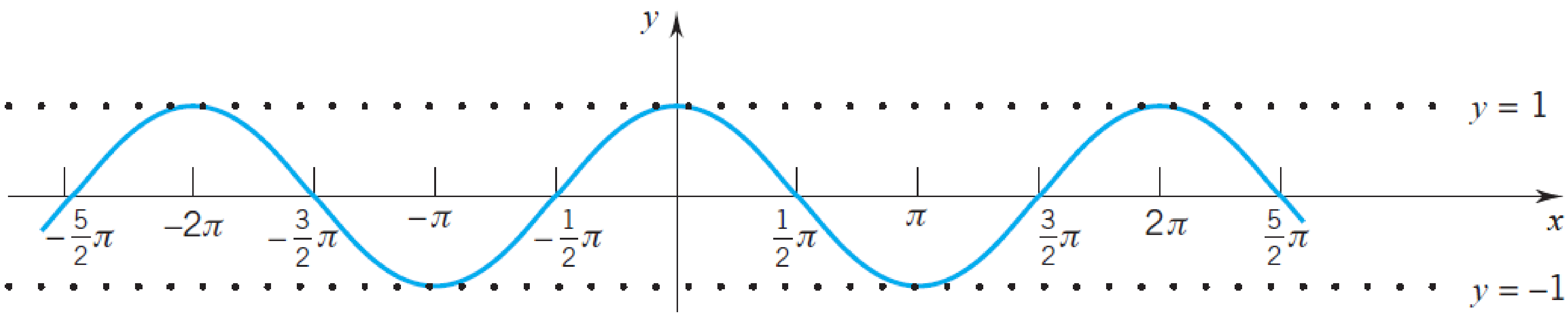
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}, \qquad \cot\theta = \frac{\text{adjacent side}}{\text{opposite side}}.$$

Graphs of Sine and Cosine

- The graphs of sine and cosine are waves that repeat themselves on every interval of length 2π.



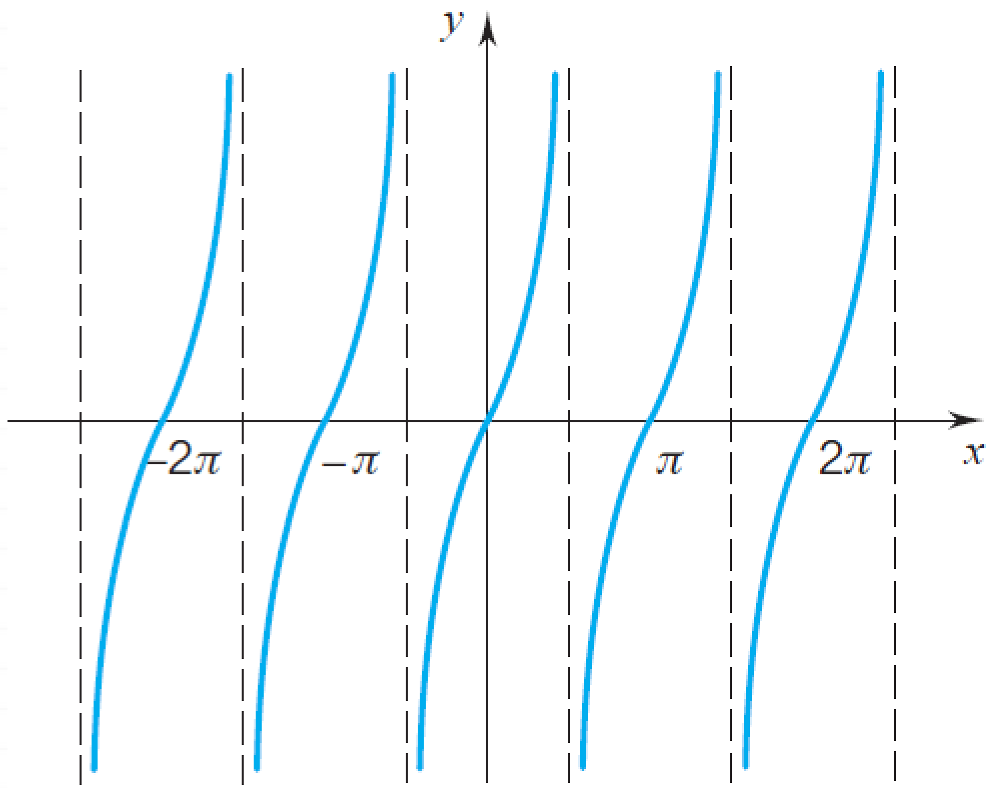
$y = \sin x$
period 2π



$y = \cos x$
period 2π

Graph of the tangent

- Consists of identical pieces separated every π units by asymptotes that mark the points x where cos x = 0



$y = \tan x$
period π

vertical asymptotes $x = (n + \frac{1}{2})\pi$, n an integer

Exponential Form
&
Logarithmic Form

Definition of the Logarithmic Function

For $x > 0$ and $b > 0, b \neq 1$,

$$y = \log_b x \text{ is equivalent to } b^y = x.$$

The function $f(x) = \log_b x$ is the **logarithmic function with base b.**

The equations

$$y = \log_b x \quad \text{and} \quad b^y = x$$

are different ways of expressing the same thing. The first equation is in **logarithmic form** and the second equivalent equation is in **exponential form.**

$$b^y = x \qquad \Longleftrightarrow \qquad y = \log_b x$$

Location of Base and Exponent in Exponential and Logarithmic Forms

Exponent

Logarithmic Form: $y = \log_b x$

Base

Exponent

Exponential Form: $b^y = x$

Base

Example

Evaluate.

a. $\log_3 81$ $\rightarrow 81 = 3^y \rightarrow 3^4 = 3^y \rightarrow y = 4$

b. $\log_{36} 6$ $\rightarrow 6 = 36^y \rightarrow 6^1 = (6^2)^y = 6^{2y} \rightarrow 2y = 1 \rightarrow y = \frac{1}{2}$

c. $\log_5 1$ $\rightarrow 1 = 5^y = 5^0 \rightarrow y = 0$

Basic Logarithmic Properties Involving One

1. $\log_b b = 1$ because 1 is the exponent to which b must be raised to obtain b. $(b^1 = b)$
2. $\log_b 1 = 0$ because 0 is the exponent to which b must be raised to obtain 1. $(b^0 = 1)$

Examples: $\log_8 8 = 1$

$$\log_6 1 = 0$$

Inverse Properties of Logarithms

For $b > 0$ and $b \neq 1$,

$$\log_b b^x = x$$

The logarithm with base b of b raised to a power equals that power.

$$b^{\log_b x} = x$$

b raised to the logarithm with base b of a number equals that number.

Examples: $\log_7 7^2 = 2$

$$5^{\log_5 8} = 8$$

Example

Use the properties of logarithms to find the answers.

a. $3^{\log_3 15}$

b. $\log_2 2^3$

c. $\log_9 9$

d. $\log_3 \frac{1}{3}$

15, 3, 1, -1