ITCS 111

Integration

Indefinite Integral & Integration by Substitution

Some of the material in these slides is from *Calculus* 9/E by
Howard Anton, Irl Bivens, and Stephen Davis

Antiderivatives

5.2.1 DEFINITION A function F is called an ***antiderivative*** of a function f on a given open interval if $F'(x) = f(x)$ for all x in the interval.

Example: $F(x) = (1/3)x^3$ is an ***antiderivative*** of x^2 on the interval $(-\infty, +\infty)$ because $F'(x) = \frac{d}{dx}[(1/3)x^3] = x^2 = f(x)$

The function $G(x) = (1/3)x^3 + c$ is also an ***antiderivative*** of f since $G'(x) = x^2$.

Indefinite Integral

The process of finding antiderivatives is called **antidifferentiation** or **integration**.

$$\frac{d}{dx}[F(x)] = f(x)$$

The **integrating** (or **antidifferentiating**) the function $f(x)$ produces an antiderivative of form $F(x) + C$

$$\int f(x)dx = F(x) + C$$

Integral Notation

Indefinite Integral

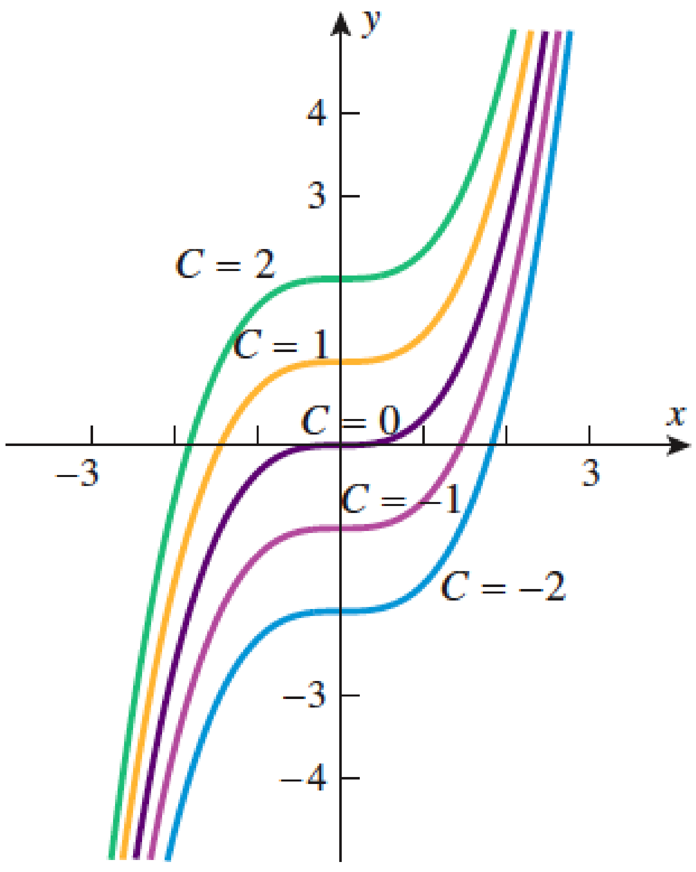
Example:

$$\int x^2 dx = \frac{1}{3}x^3 + C$$

is equivalent to $$\frac{d}{dx}[\frac{1}{3}x^3] = x^2$$

Antiderivatives = Family of Functions

$$\int x^2 dx = \frac{1}{3}x^3 + C$$



$y = \frac{1}{3}x^3 + C$

Calculus, 9/E by Howard Anton, Irl Bivens, and Stephen Davis

Properties of the indefinite integral

5.2.3 THEOREM *Suppose that $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ and $g(x)$, respectively, and that c is a constant. Then:*

(*a*) *A constant factor can be moved through an integral sign; that is,*

$$\int cf(x)\,dx = cF(x) + C$$

(*b*) *An antiderivative of a sum is the sum of the antiderivatives; that is,*

$$\int [f(x) + g(x)]\,dx = F(x) + G(x) + C$$

(*c*) *An antiderivative of a difference is the difference of the antiderivatives; that is,*

$$\int [f(x) - g(x)]\,dx = F(x) - G(x) + C$$

Basic integration formulas can be obtained directly from their companion differentiation formulas.

DIFFERENTIATION FORMULA	DIFFERENTIATION FORMULA
1. $\frac{d}{dx}[x] = 1$	8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$
2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \quad (r \neq -1)$	9. $\frac{d}{dx}[e^x] = e^x$
3. $\frac{d}{dx}[\sin x] = \cos x$	10. $\frac{d}{dx}\left[\frac{b^x}{\ln b}\right] = b^x \quad (0 < b,\ b \neq 1)$
4. $\frac{d}{dx}[-\cos x] = \sin x$	11. $\frac{d}{dx}[\ln\lvert x\rvert] = \frac{1}{x}$
5. $\frac{d}{dx}[\tan x] = \sec^2 x$	12. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$
6. $\frac{d}{dx}[-\cot x] = \csc^2 x$	13. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$
7. $\frac{d}{dx}[\sec x] = \sec x \tan x$	14. $\frac{d}{dx}[\sec^{-1}\lvert x\rvert] = \frac{1}{x\sqrt{x^2-1}}$

Basic integration formulas can be obtained directly from their companion differentiation formulas.

Table 5.2.1
INTEGRATION FORMULAS

DIFFERENTIATION FORMULA	INTEGRATION FORMULA	DIFFERENTIATION FORMULA	INTEGRATION FORMULA
1. $\frac{d}{dx}[x] = 1$	$\int dx = x + C$	8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$	$\int \csc x \cot x \, dx = -\csc x + C$
2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \quad (r \neq -1)$	$\int x^r \, dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$	9. $\frac{d}{dx}[e^x] = e^x$	$\int e^x \, dx = e^x + C$
3. $\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x dx = \sin x + C$	10. $\frac{d}{dx}\left[\frac{b^x}{\ln b}\right] = b^x \quad (0 < b, b \neq 1)$	$\int b^x \, dx = \frac{b^x}{\ln b} + C \quad (0 < b, b \neq 1)$
4. $\frac{d}{dx}[-\cos x] = \sin x$	$\int \sin x \, dx = -\cos x + C$	11. $\frac{d}{dx}[\ln \lvert x \rvert] = \frac{1}{x}$	$\int \frac{1}{x} \, dx = \ln\lvert x \rvert + C$
5. $\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$	12. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$
6. $\frac{d}{dx}[-\cot x] = \csc^2 x$	$\int \csc^2 x \, dx = -\cot x + C$	13. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$
7. $\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + C$	14. $\frac{d}{dx}[\sec^{-1}\lvert x \rvert] = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1}\lvert x \rvert + C$

Indefinite Integral

Example: Calculate

$$\int x^8 dx$$

$$\int x(1 + x^3) dx$$

$$\int (2 + y^2)^2 dy$$

Indefinite Integral

Example:

Calculate $\int \left[5x^{3/2} - 2\csc^2 x\right] dx$

Indefinite Integral

Example:

Calculate $\int [5x^{3/2} - 2\csc^2 x]\,dx$

Solution

$$\int [5x^{3/2} - 2\csc^2 x]\,dx$$

$$= 5\int x^{3/2}dx - 2\int \csc^2 x\,dx$$

$$= 5\left(\frac{2}{5}\right)x^{5/2} + C_1 - 2(-\cot x) + C_2$$

$$= 2x^{5/2} + 2\cot x + C$$

Indefinite Integral

Example: Evaluate the following

$$\int \frac{dx}{\sqrt{4-x^2}}$$

$$\int \frac{dx}{x^2+36}$$

Indefinite Integral

Exercise #12

Integration by Substitution

Substitution can often be used to transform complicated integration problems into simple ones.

U-substitution

Integration by Substitution

Guidelines for u-Substitution

Step 1. Look for some composition $f(g(x))$ within the integrand for which the substitution

$$u = g(x), \quad du = g'(x)\,dx$$

produces an integral that is expressed entirely in terms of u and its differential du. This may or may not be possible.

Step 2. If you are successful in Step 1, then try to evaluate the resulting integral in terms of u. Again, this may or may not be possible.

Step 3. If you are successful in Step 2, then replace u by $g(x)$ to express your final answer in terms of x.

Integration by Substitution

Example: Evaluate $\int (x^2 - 1)^4 x \, dx$

Integration by Substitution

Example: Evaluate $\int (x^2 - 1)^4 x \, dx$

We know that $\quad \dfrac{d}{dx}[(x^2 - 1)^5] = 10(x^2 - 1)^4 x$

We just work back

$$\int (x^2 - 1)^4 x \, dx = \tfrac{1}{10} \int 10(x^2 - 1)^4 x \, dx = \tfrac{1}{10}(x^2 - 1)^5 + C$$

Integration by Substitution

Example: Evaluate $\displaystyle\int \frac{1}{(3+5x)^2}\,dx.$

Integration by Substitution

Example: Evaluate $\int x^2\sqrt{4+x^3}\,dx$

Integration by Substitution

Example: Evaluate $\int_0^2 (x^2 - 1)(x^3 - 3x + 2)^3 \, dx.$

Integration by Substitution

Exercise #13