(Week 8)

# 21. Integration by Parts

Let *f* and *g* be *differentiable functions. By the product rule for derivatives,*

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

The integral of the above equality is

$$\begin{aligned}\int \frac{d}{dx}[f(x)g(x)]dx &= \int f(x)g'(x)dx + \int g(x)f'(x)dx \\ f(x)g(x) + c &= \int f(x)g'(x)dx + \int g(x)f'(x)dx \\ \int f(x)g'(x)dx &= f(x)g(x) - \int g(x)f'(x)dx + c\end{aligned}$$

Equivalently,

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx \qquad ***** \ (1)$$

Let $u = f(x)$ and $dv = g'(x)dx$, therefore
$du = f'(x)dx$ and $v = g(x)$.

Substitutions yeild

$$\int udv = uv - \int vdu \qquad ***** \ (2)$$

Both integrals in (1) and (2) are called integration by parts.

For definite integral the formula corresponding to (2) is

$$\int_a^b u\ dv = uv\Big]_a^b - \int_a^b v\ du$$

**Example:** Evaluate $\int xe^x dx$.

Solution: Let $u = x$ and $dv = e^x dx$,
so $du = dx$ and $v = \int e^x dx = e^x$

Thus,

$$\begin{aligned}\int xe^x dx &= \int udv \\ &= uv - \int vdu \\ &= xe^x - \int e^x dx \\ &= xe^x - e^x + C\end{aligned}$$