ITCS 175
Definite Integral

Some of the material in these slides is from *Calculus* 9/E by
Howard Anton, Irl Bivens, and Stephen Davis

Definite Integral

5.5.1 DEFINITION A function f is said to be ***integrable*** on a finite closed interval $[a, b]$ if the limit

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

exists and does not depend on the choice of partitions or on the choice of the points x_k^* in the subintervals. When this is the case we denote the limit by the symbol

$$\int_a^b f(x)\,dx = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

which is called the ***definite integral*** of f from a to b. The numbers a and b are called the ***lower limit of integration*** and the ***upper limit of integration***, respectively, and $f(x)$ is called the ***integrand***.

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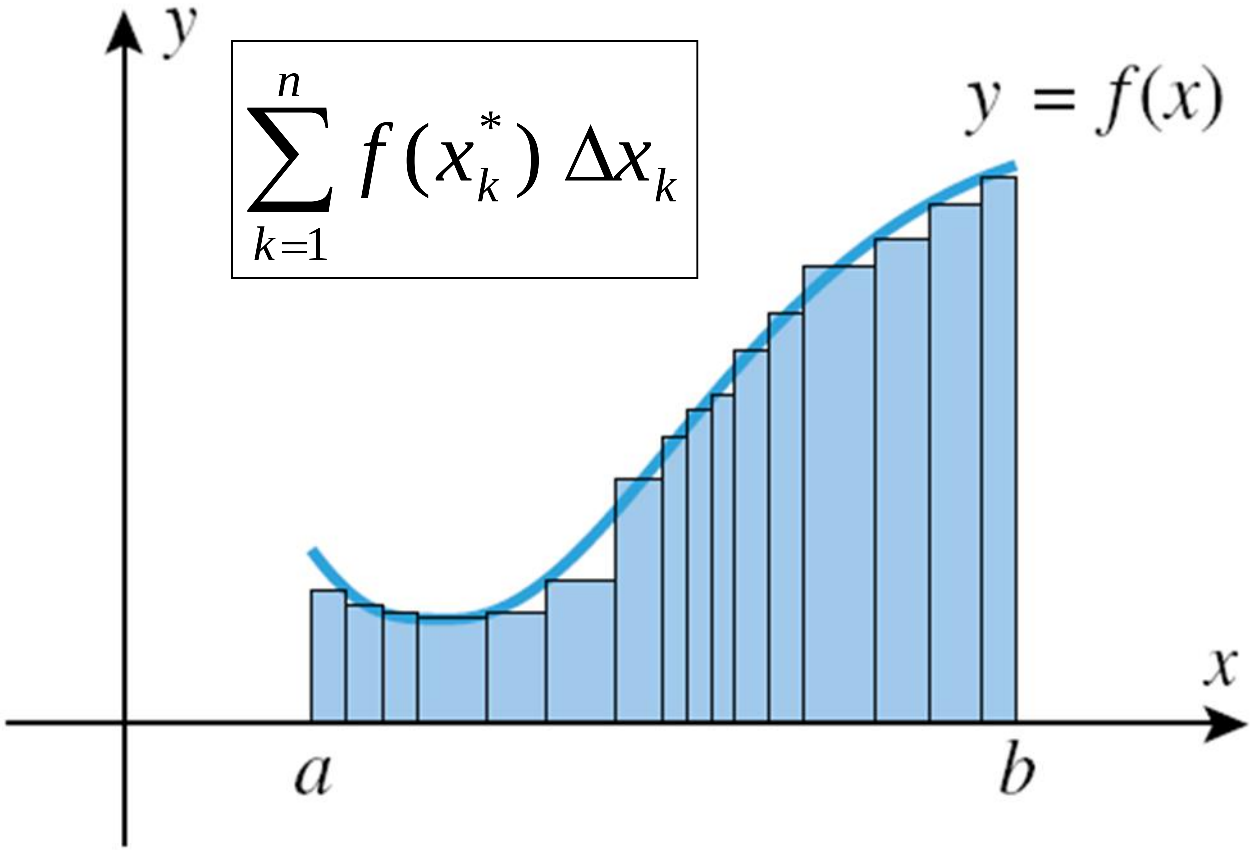
Definite Integral



Approximations with variable width

Continuous Functions are Integrable

5.5.2 THEOREM *If a function f is continuous on an interval $[a, b]$, then f is integrable on $[a, b]$, and the net signed area A between the graph of f and the interval $[a, b]$ is*

$$A = \int_a^b f(x)\,dx \tag{1}$$

The <u>area under the graph</u> of ***f*** over the interval [***a***, ***b***] is represented by the <u>definite integral</u>.

The Definite Integral

Geometric Interpretation

5.5.3 DEFINITION

(a) If a is in the domain of f, we define

$$\int_a^a f(x)\,dx = 0$$

(b) If f is integrable on $[a, b]$, then we define

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

Definite Integral Rules

5.5.4 THEOREM *If f and g are integrable on $[a, b]$ and if c is a constant, then cf, $f + g$, and $f - g$ are integrable on $[a, b]$ and*

(*a*) $\displaystyle\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

(*b*) $\displaystyle\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

(*c*) $\displaystyle\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

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Definite Integral Rules

5.5.5 THEOREM *If f is integrable on a closed interval containing the three points a, b, and c, then*

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

no matter how the points are ordered.

Definite Integral

Example: Evaluate the integrals.

1) $\int_{-1}^{2} 3dx$
2) $\int_{1}^{2}(6x^2 + 2x - 1)dx$
3) $\int_{0}^{2} y(1 + y^3)dy$
4) $\int_{0}^{\pi/2}(\sin x + \cos x)dx$
5) $\int_{0}^{\pi/4} \sec^2 x\, dx$

Table 5.2.1

INTEGRATION FORMULAS

DIFFERENTIATION FORMULA	INTEGRATION FORMULA	DIFFERENTIATION FORMULA	INTEGRATION FORMULA
1. $\frac{d}{dx}[x] = 1$	$\int dx = x + C$	8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$	$\int \csc x \cot x \, dx = -\csc x + C$
2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \quad (r \neq -1)$	$\int x^r \, dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$	9. $\frac{d}{dx}[e^x] = e^x$	$\int e^x \, dx = e^x + C$
3. $\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x dx = \sin x + C$	10. $\frac{d}{dx}\left[\frac{b^x}{\ln b}\right] = b^x \quad (0 < b,\ b \neq 1)$	$\int b^x \, dx = \frac{b^x}{\ln b} + C \quad (0 < b,\ b \neq 1)$
4. $\frac{d}{dx}[-\cos x] = \sin x$	$\int \sin x \, dx = -\cos x + C$	11. $\frac{d}{dx}[\ln \lvert x \rvert] = \frac{1}{x}$	$\int \frac{1}{x} \, dx = \ln\lvert x \rvert + C$
5. $\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$	12. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$
6. $\frac{d}{dx}[-\cot x] = \csc^2 x$	$\int \csc^2 x \, dx = -\cot x + C$	13. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$
7. $\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + C$	14. $\frac{d}{dx}[\sec^{-1}\lvert x \rvert] = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1}\lvert x \rvert + C$

Definite Integral vs. Antiderivatives (Indefinite Integral)

We have seen two basic ideas so far:

Indefinite Integral: Computes a **family of functions**

$$\int f(x)dx$$

Definite Integral: Computes a **number** = **area**

$$\int_a^b f(x)dx$$

Fundamental Theorem of Calculus - I

5.6.1 THEOREM (*The Fundamental Theorem of Calculus, Part 1*) *If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then*

$$\int_a^b f(x)\,dx = F(b) - F(a) \tag{2}$$

Relationship between Definite and Indefinite Integrals

$$\int_a^b f(x)dx = \int f(x)dx \,\Big|_a^b$$

Fundamental Theorem of Calculus - I

Exercise #14-15