PRIME FACTORIZATION
GCD AND LCM

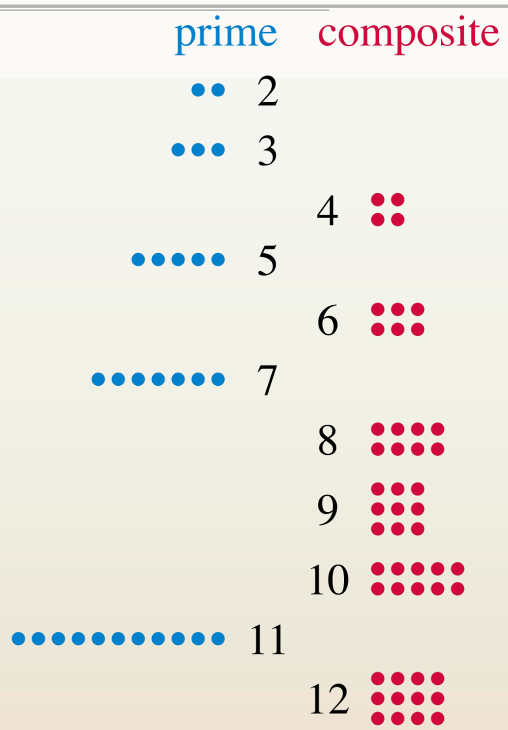
Properties of Integers

Prime Numbers

Prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself

The only positive integers that divide a prime number p are 1 and p

A natural number greater than one that is not a prime number is called a **composite number**



https://en.wikipedia.org/wiki/Prime_number

Factoring a number into its primes

Divide by primes: 2, 3, 5, 7, …

Continue until you get to 1

Write down the products of all prime divisors

1386 =

	1386

Every positive integer n > 1 can be broken into multiples of primes

$n = p_1^{k1} p_2^{k2} p_3^{k3} \ldots p_s^{ks}$ where $p_1 < p_2 < p_3 < \ldots < p_s$ are prime numbers

Factoring a number into its primes

Keep dividing the number by a prime

Stop when you get to 1

140 =

Every positive integer n > 1 can be broken into multiples of primes

$n = p_1^{k1} p_2^{k2} p_3^{k3} \ldots p_s^{ks}$ where $p_1 < p_2 < p_3 < \ldots < p_s$ are prime numbers

PRACTICE PROBLEMS

Write each integer as a product of powers of primes

- 75 =
- 512 =
- 3038 =
- 3401 =

WORKED EXAMPLES

- Find the largest prime divisor of 5! + 6!

- Find the prime factorization of 10!

PRACTICE PROBLEMS

- Find the largest prime divisor of 49! + 50! + 51! + 52!
- Find the largest prime divisor of $2^{16} - 1$

GCD Greatest Common Divisor

HCF – Highest Common Factor

The **largest** int that is a common divisor of a given set of numbers. It divides all integers in the set.

Factors of 15: 1, 3, 5, and 15

Factors of 20: 1, 2, 4, 5, 10, and 20

The GCD of 15 and 20 is 5

LCM Least Common Multiple

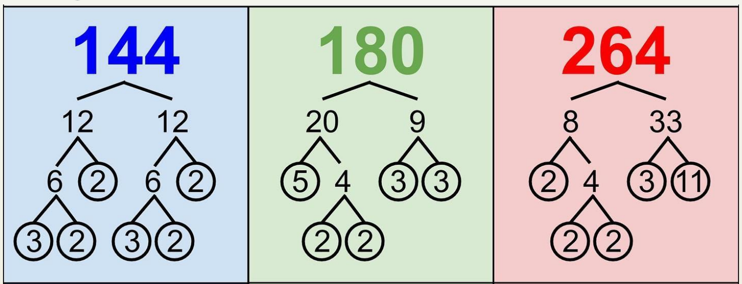
The **smallest** multiple that two or more numbers have in common. It is a multiple of all int in the set.

Multiples of 15: 15, 30, 45, 60, …

Multiples of 20: 20, 40, 60, 80, …

The LCM of 15 and 20 is 60

Recall & Review Your GCD & LCM!



$144 = 2^4 \cdot 3^2$ | $180 = 2^2 \cdot 3^2 \cdot 5$ | $264 = 2^3 \cdot 3 \cdot 11$

SCALAR LEARNING www.ScalarLearning.com

$$GCF = 2^2 \cdot 3 = 12$$

$$LCM = 2^4 \cdot 3^2 \cdot 5 \cdot 11 = 7920$$

2	48 , 72 , 108
2	24 , 36 , 54
3	12 , 18 , 27
3	4 , 6 , 9
2	4 , 2 , 3
	2 , 1 , 3

$GCD = 2 \times 2 \times 3 = 12$

$LCM = 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 1 \times 3 = 432$

Using prime factors to find GCD and LCM

GCD – Greatest Common Divisor

$540 = 2^2 \times 3^3 \times 5$

$504 = 2^3 \times 3^2 \times 7$

Product of common min-power

LCM – Least Common Multiple

$540 = 2^2 \times 3^3 \times 5$

$504 = 2^3 \times 3^2 \times 7$

Product of every max-power

$$GCD(a,b) \times LCM(a,b) = ab$$

WORKED EXAMPLES

- Find the GCD and LCM of 27, 90, and 84

WORKED EXAMPLES

- The GCD of 70 and some $n \in N$ is 10. Their LCM is 210. Find n.

PRACTICE PROBLEMS

- Find the GCD of 11! and 6!
- Find the GCD of 11! and 5607

PRACTICE PROBLEMS

- Find the GCD of 11! and $(8!)^2$
- The GCD of 6! and some $n \in N$ is 144. Their LCM is 2880. Find n.

PRACTICE PROBLEMS

- The LCM of two numbers is six times their GCD. The sum of the LCM and the GCD is 210. If one number is 60, then what is the other?

EUCLIDEAN ALGORITHM

Properties of Integers

GCD (47376,78255) = ?

Factoring ?

Make the numbers smaller and find GCD of smaller pairs …

- a = kb + r — Given a & b we can find k & r
- GCD (a, b) = GCD (b, r) — Integers b & r are smaller than a & b

Euclidean algorithm – To compute the GCD of a and b, recursively find k and r such that a = kb + r and compute the GCD of b and r.

Use Euclidean Algorithm to Compute GCD(273,98)

Iteration	a	b	r	a = kb + r

GCD(273,98) = ?

Use Euclidean Algorithm to Compute GCD(273,98)

a = kb + r

From running the Euclid's algorithm, we got the GCD of **not one but multiple pairs** of integers

Extended Euclidean Algorithm

Used to find two integers s and t (not necessarily positive) such that

$$GCD(a, b) = d = sa + tb$$

You are not responsible for a proof of this theorem but ***you must be able to compute the GCD and integers s and t.***

Use Extended Euclid to find s,t in 7 = s × 273 + t × 98

a = kb + r

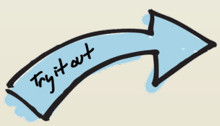
Extended Euclidean Algorithm

GCD (a, b) = d = sa + tb

Find integers x and y such that $z = xa + yb$

If $z = \text{GCD}(a,b)$, then x and y are s and t, by the extended Euclid's alg.

If z is not equal to the GCD of a and b, then we need one extra step



- ❑ Find x and y such that $(x)(273) + (y)(98) = 70$

http://mathforum.org/library/drmath/view/51595.html

PRACTICE PROBLEMS

Let a=108 and b=60, find d=GCD(a,b) and find s,t such that d=sa+tb

Determine whether there exist $(x, y) \in \mathbb{Z}$ such that $108x + 60y = 40$

PRACTICE PROBLEMS

Given integers: a = 7854 and b = 4746,

- determine the least common multiple of a and b
- find x and y such that xa + yb = GCD(a,b)
- find x and y such that xa + yb = 1134
- determine if the equation 1470x + 168y = 168 has a solution such that both x and y are integers. If it has, then find x and y.