ITCS 111
Chapter 2: *Derivatives of Trig., Inverse Trig., Exp, Log Functions*

Some of the material in these slides is from *Calculus* 10/E by Howard Anton, Irl Bivens, and Stephen Davis

Derivatives of *Trigonometric Functions*

Formula (1)

$$\frac{d}{dx}[\sin x] = \cos x$$

Formula (2)

$$\frac{d}{dx}[\cos x] = -\sin x$$

Derivatives of *Trig Functions*

Formula (3)

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

Formula (4)

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

Formula (5)

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

Formula (6)

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

Formula 3-6 can be derived based on 1-2

Derivatives of *Trig Functions*

Example: Find $f''(\pi/4)$ if $f(x) = \sec x$.

Derivatives of *Trig Functions*

Exercise 9

Derivatives of Inverse Trig Functions (p 467)

$$\frac{d}{dx}[\sin^{-1} u] = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\cos^{-1} u] = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

Derivatives of Inverse Trig Functions (p 467)

$$\frac{d}{dx}[\tan^{-1} u] = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}[\cot^{-1} u] = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}[\sec^{-1} u] = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}[\csc^{-1} u] = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

Derivatives of Inverse Trig Functions

Example: Find dy/dx when $y = \csc x + \tan^{-1} x$

Derivatives of Inverse Trig Functions

Example: Find $\frac{dy}{dx}$ if

$$y = \sin^{-1}(x^3)$$

Derivatives of Inverse Trig Functions

Exercise

In Exercises 1-7, find the derivatives.

1) $y = \sin^{-1}(3x)$

2) $y = \sin^{-1}(\sqrt{x})$

3) $y = \sqrt{\tan^{-1} x}$

4) $y = x\tan^{-1}(x^2)$

5) $y = \sec^{-1}(x^4)$

6) $y = 6\sec^{-1}\sqrt{x} + 1$

7) $y = \sin^{-1}(x\cos^{-1} x)$

Derivatives of Inverse Trig *and exp* Functions

Example: Find $\frac{dy}{dx}$ if

$$y = \sec^{-1}(e^x)$$

Derivatives of Inverse Trig and exp Functions

Example: Find $\frac{dy}{dx}$ if

$$y = \sec^{-1}(e^x)$$

Solution:

$$\frac{dy}{dx} = \frac{1}{e^x\sqrt{(e^x)^2-1}}\frac{d(e^x)}{dx}$$

$$= \frac{e^x}{e^x\sqrt{(e^{2x})-1}}$$

$$= \frac{1}{\sqrt{e^{2x}-1}}$$

Derivatives of trig, log, exp Functions

Exercise

Find the derivatives

1) $y = e^{3x+1}$
2) $y = e^{\sin x} + \sin(e^x)$
3) $y = xe^{\sqrt{x}}$
4) $y = x^{2x}$

Derivatives of trig, log, exp Functions

Exercise

Find the derivatives

1) $y = \ln(x^4) + \ln^5 x$

2) $y = x \ln x$

3) $y = \ln(\tan x)$

4) $y = \dfrac{\log x}{x}$

Derivatives of trig, log, exp Functions

Exercise

Find derivatives

1) $y = x^{2x}$

2) $y = \frac{(2x-1)(3x+1)(x+1)^{10}}{\sqrt{x^2+1}}$

3) $y = (\cot x)^{\ln x}$

Basic differentiation formulas

DIFFERENTIATION FORMULA	DIFFERENTIATION FORMULA
1. $\frac{d}{dx}[x] = 1$	8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$
2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \quad (r \neq -1)$	9. $\frac{d}{dx}[e^x] = e^x$
3. $\frac{d}{dx}[\sin x] = \cos x$	10. $\frac{d}{dx}\left[\frac{b^x}{\ln b}\right] = b^x \quad (0 < b, b \neq 1)$
4. $\frac{d}{dx}[-\cos x] = \sin x$	11. $\frac{d}{dx}[\ln \lvert x \rvert] = \frac{1}{x}$
5. $\frac{d}{dx}[\tan x] = \sec^2 x$	12. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$
6. $\frac{d}{dx}[-\cot x] = \csc^2 x$	13. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$
7. $\frac{d}{dx}[\sec x] = \sec x \tan x$	14. $\frac{d}{dx}[\sec^{-1} \lvert x \rvert] = \frac{1}{x\sqrt{x^2-1}}$