ITCS 111 Chapter 1
Review of Limits

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Two Basic Problems of Calculus

- The concept of “l**imit**” is the fundamental building block on which all calculus concepts are based.
- We will study limits informally, with the goal of developing an intuitive feel for the basic ideas, then we will focus on computational methods and precise definitions.

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Two Basic Problems of Calculus

Many of the ideas of calculus originated with the following two geometric problems:

THE TANGENT LINE PROBLEM Given a function f and a point $P(x_0, y_0)$ on its graph, find an equation of the line that is tangent to the graph at P (Figure 1.1.1).

THE AREA PROBLEM Given a function f, find the area between the graph of f and an interval $[a, b]$ on the x-axis (Figure 1.1.2).

The solution to both of these problems requires the use of **limits.**

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Two Basic Problems of Calculus

Tangent Line

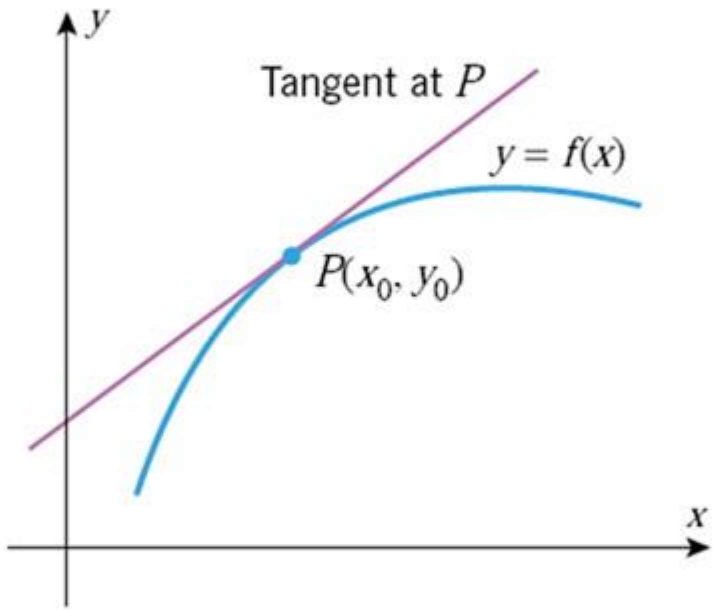

Figure 1.1.1

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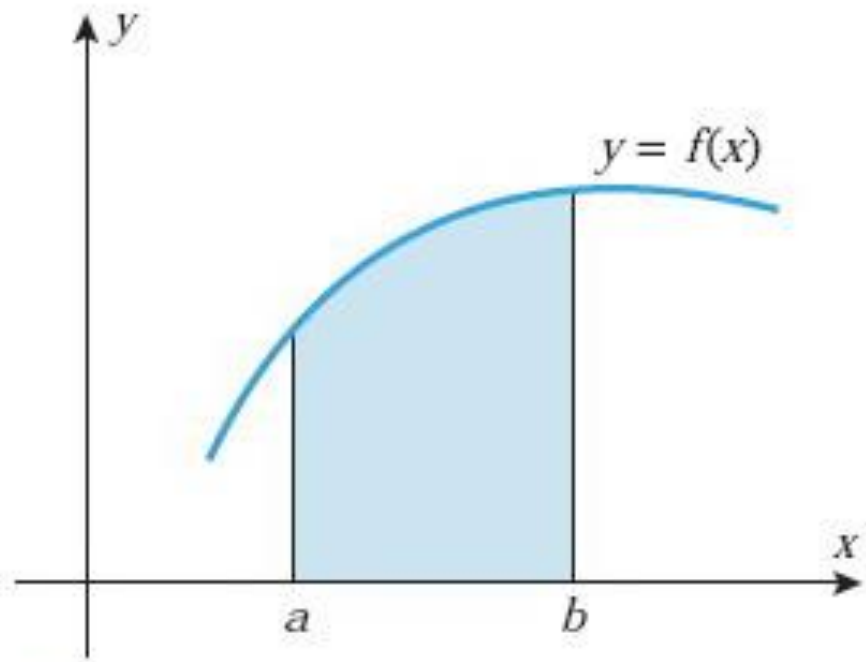
Two Basic Problems of Calculus

Area



▲ Figure 1.1.2

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1.1.1 LIMITS (AN INFORMAL VIEW) If the values of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a (but not equal to a), then we write

$$\lim_{x \to a} f(x) = L \tag{6}$$

which is read "the limit of $f(x)$ as x approaches a is L" or "$f(x)$ approaches L as x approaches a." The expression in (6) can also be written as

$$f(x) \to L \quad \text{as} \quad x \to a \tag{7}$$

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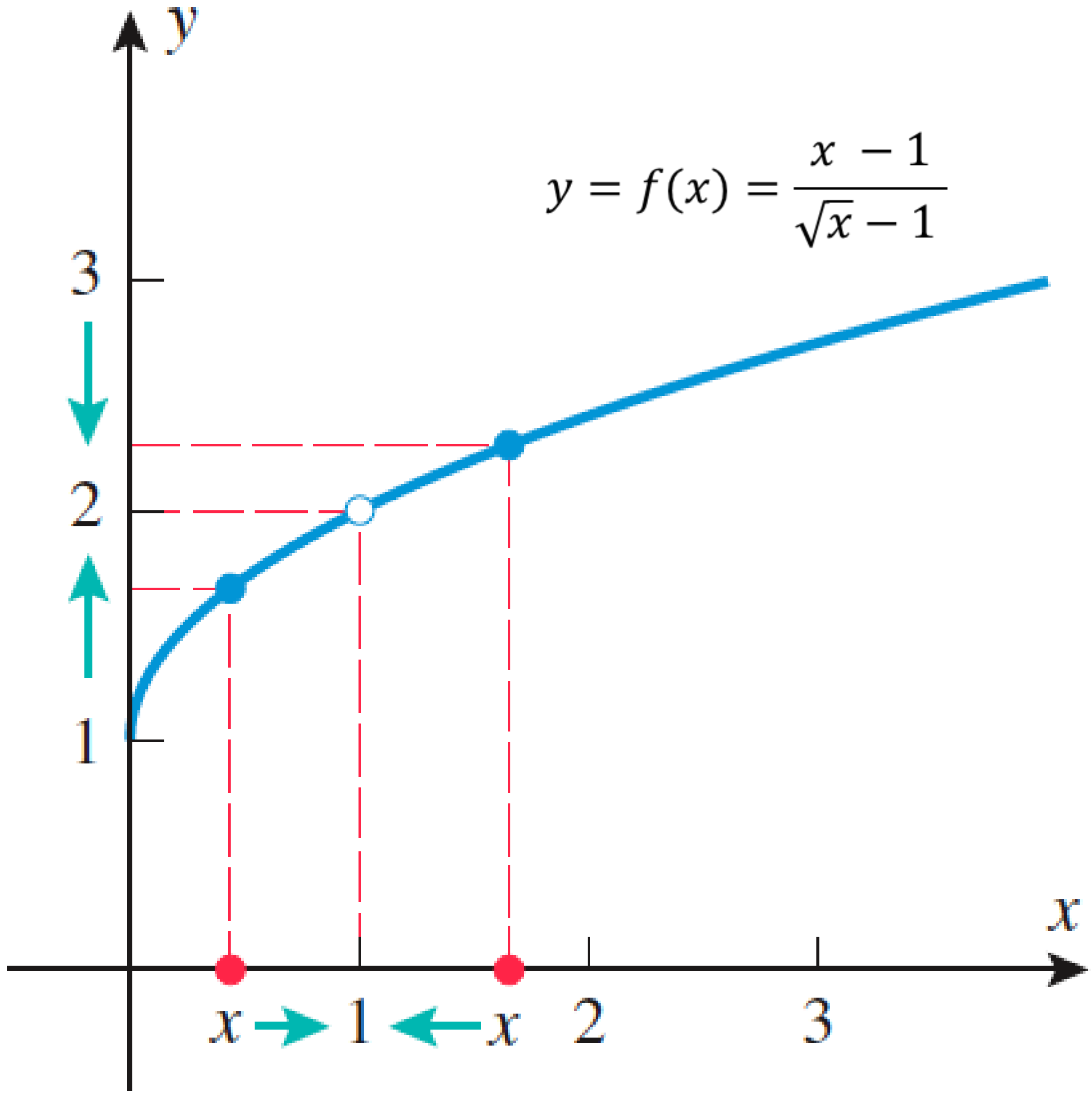

Figure 1.1.9

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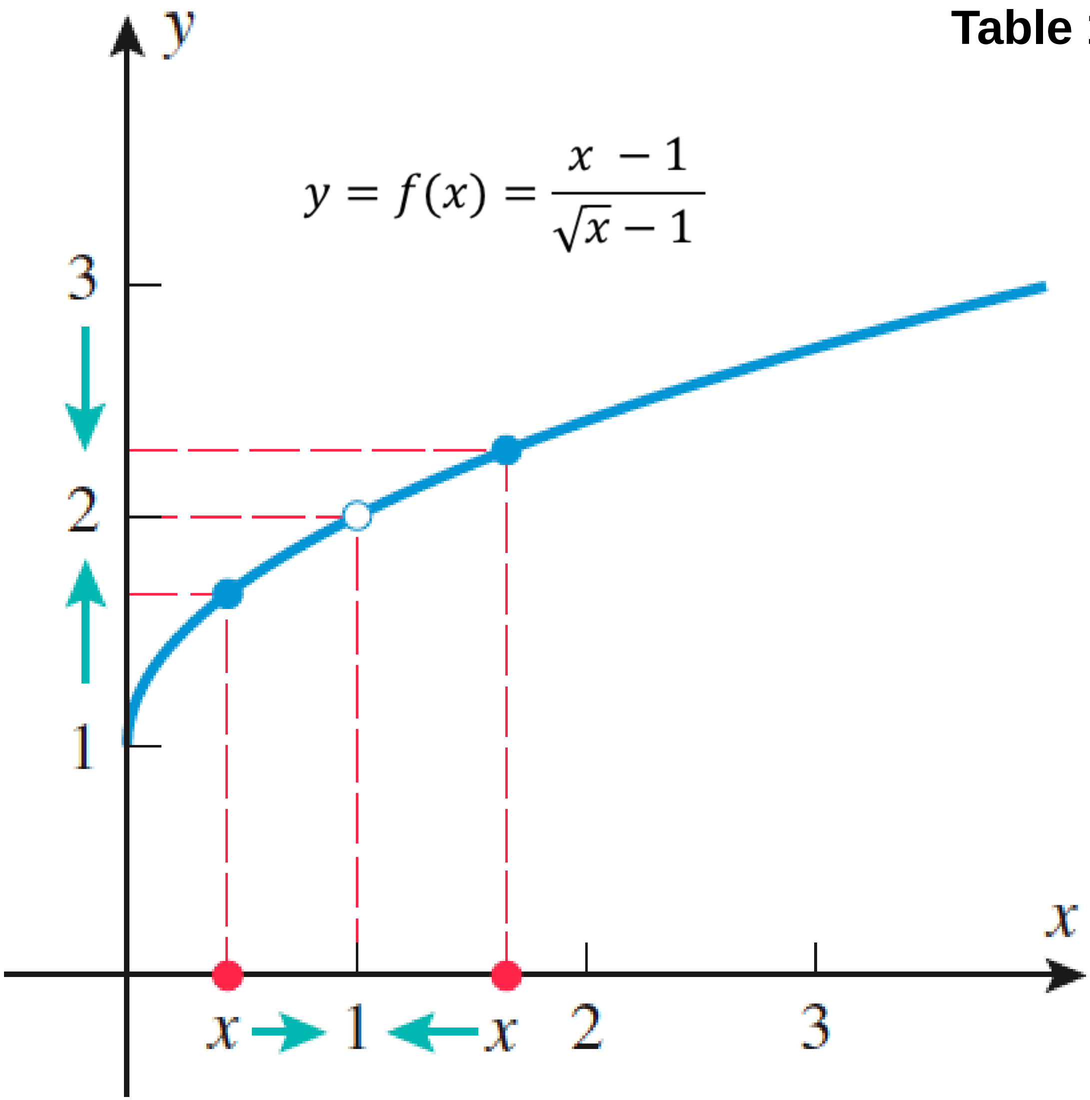

Figure 1.1.9

Table 1.1.1

x	f(x)
0.99	1.994987
0.999	1.999500
0.9999	1.999995
1.00001	2.000005
1.0001	2.000050
1.001	2.000500
1.01	2.004988

f(x) is undefined at *x*=1, this has no bearing on the limit. Table 1.1.1 shows sample *x*- values approaching 1 from the left side and from the right side. In both cases, the corresponding *f(x)* appear to get closer and closer to 2. **Hence we conjecture that** $\lim_{x\to 1} f(x)$ **= 2**

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1.1.2 ONE-SIDED LIMITS (AN INFORMAL VIEW) If the values of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a (but greater than a), then we write

$$\lim_{x \to a^+} f(x) = L \tag{14}$$

and if the values of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a (but less than a), then we write

$$\lim_{x \to a^-} f(x) = L \tag{15}$$

Expression (14) is read "the limit of $f(x)$ as x approaches a from the right is L" or "$f(x)$ approaches L as x approaches a from the right." Similarly, expression (15) is read "the limit of $f(x)$ as x approaches a from the left is L" or "$f(x)$ approaches L as x approaches a from the left."

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1.1.3 THE RELATIONSHIP BETWEEN ONE-SIDED AND TWO-SIDED LIMITS The two-sided limit of a function $f(x)$ exists at a if and only if both of the one-sided limits exist at a and have the same value; that is,

$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

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Find $\lim_{x \to 0} |x|/x$

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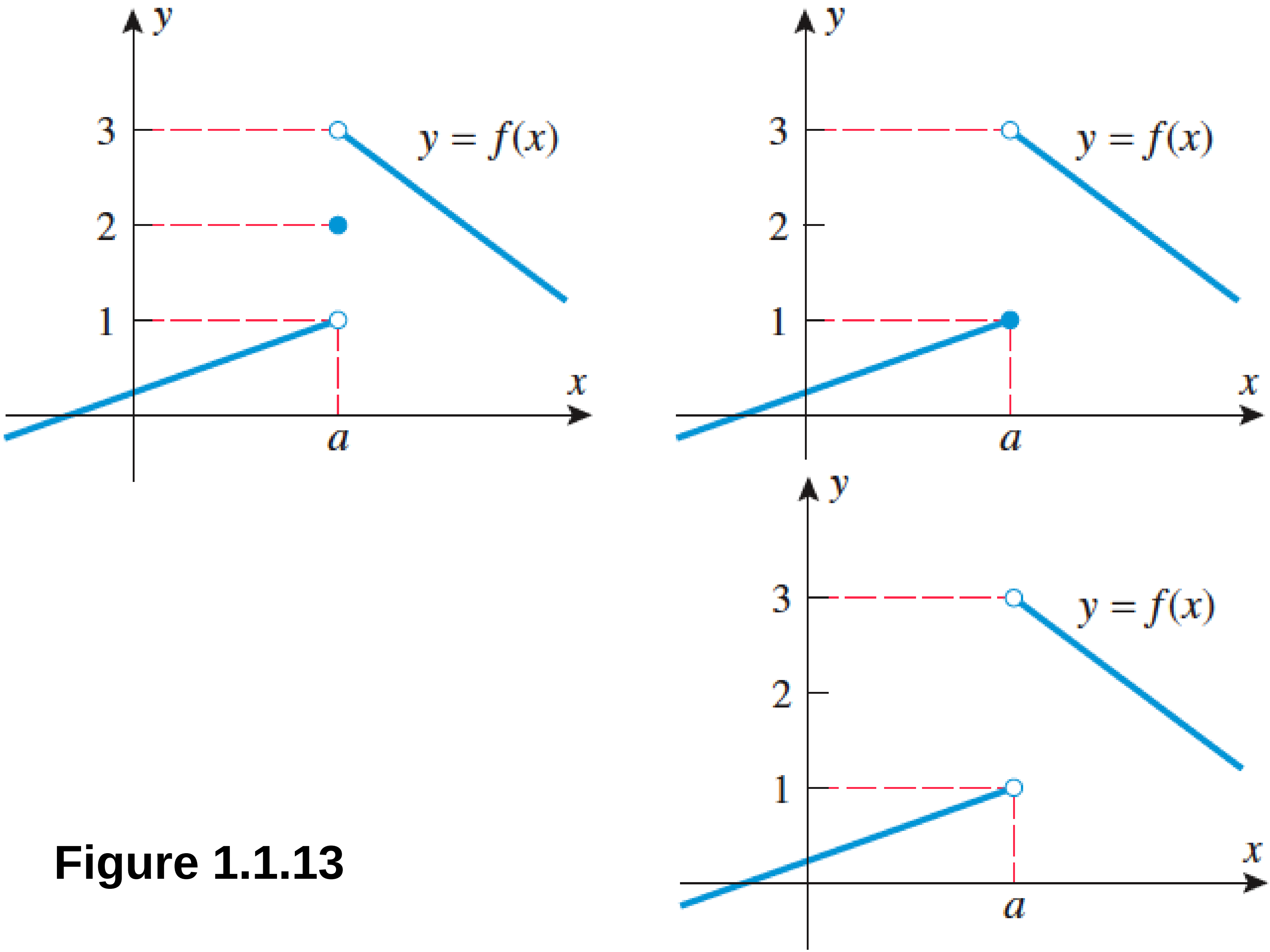

Figure 1.1.13

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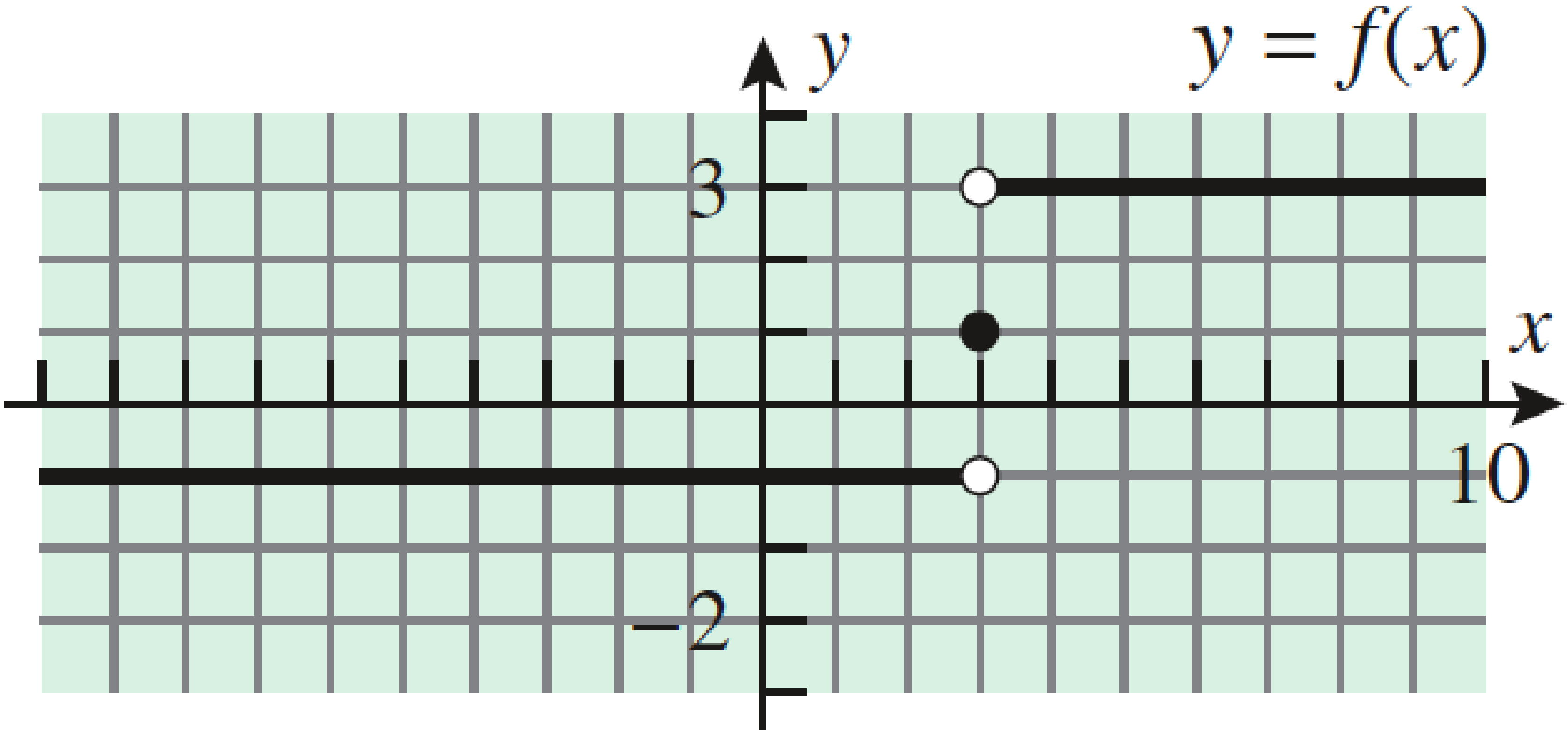

Exercise 1.1.3

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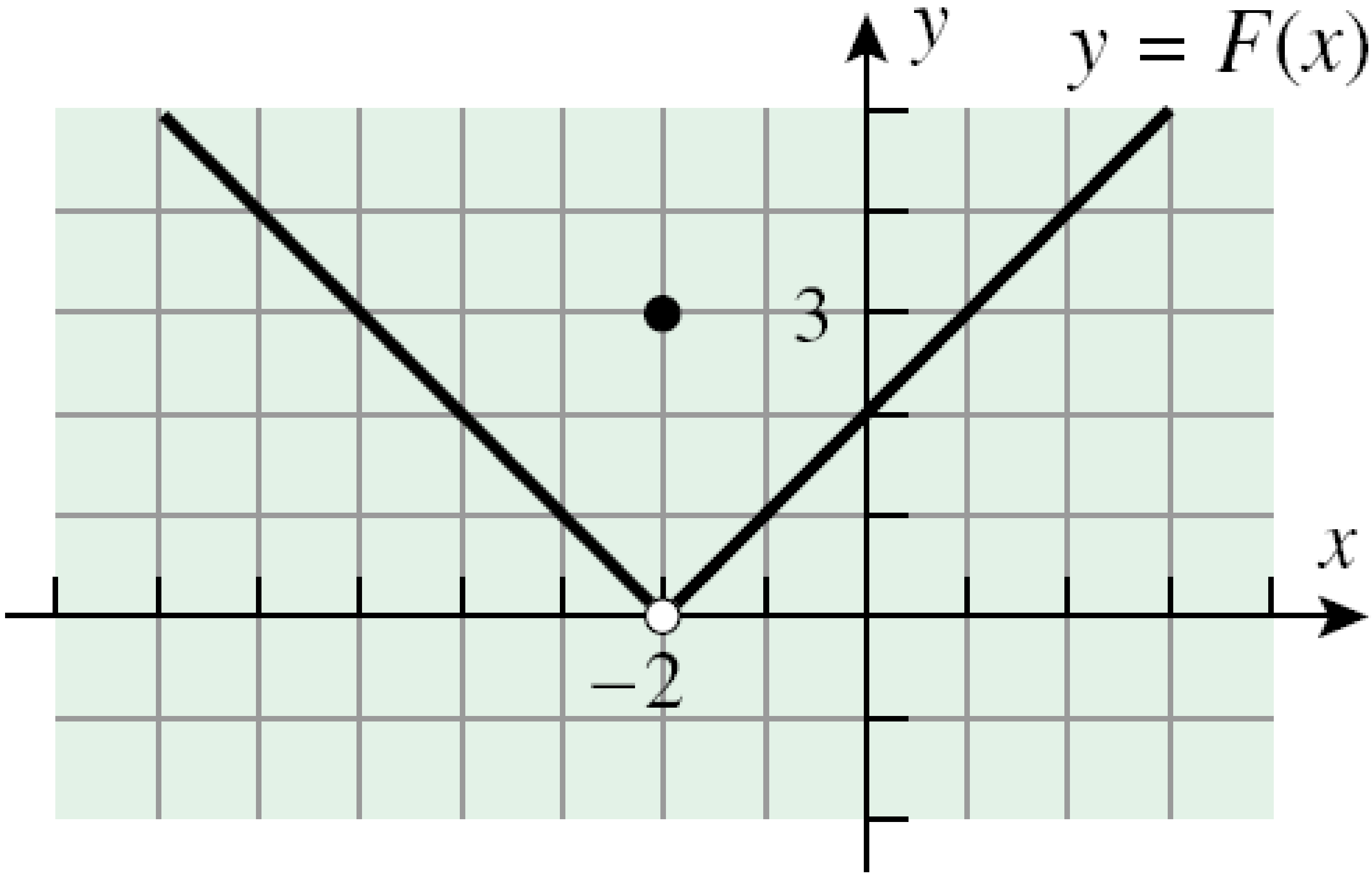

Exercise 1.1.5

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1.1.4 INFINITE LIMITS (AN INFORMAL VIEW)

The expressions

$$\lim_{x \to a^-} f(x) = +\infty \quad \text{and} \quad \lim_{x \to a^+} f(x) = +\infty$$

denote that $f(x)$ increases without bound as x approaches a from the left and from the right, respectively. If both are true, then we write

$$\lim_{x \to a} f(x) = +\infty$$

Similarly, the expressions

$$\lim_{x \to a^-} f(x) = -\infty \quad \text{and} \quad \lim_{x \to a^+} f(x) = -\infty$$

denote that $f(x)$ decreases without bound as x approaches a from the left and from the right, respectively. If both are true, then we write

$$\lim_{x \to a} f(x) = -\infty$$

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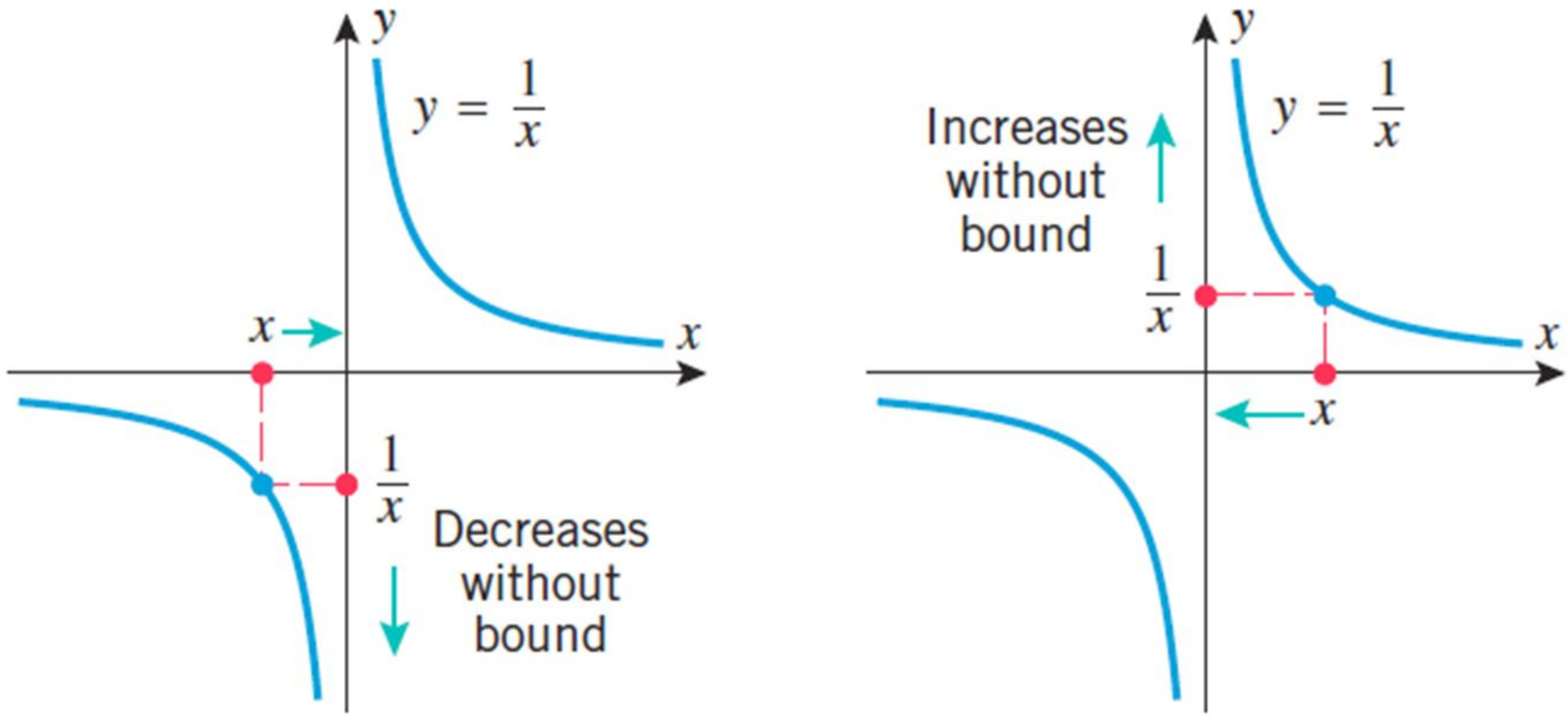

x	−1	−0.1	−0.01	−0.001	−0.0001	0	0.0001	0.001	0.01	0.1	1
$\frac{1}{x}$	−1	−10	−100	−1000	−10,000		10,000	1000	100	10	1

Left side → ← Right side

Figure 1.1.15 : Find $\lim_{x\to 0-}\frac{1}{x}$, $\lim_{x\to 0+}\frac{1}{x}$

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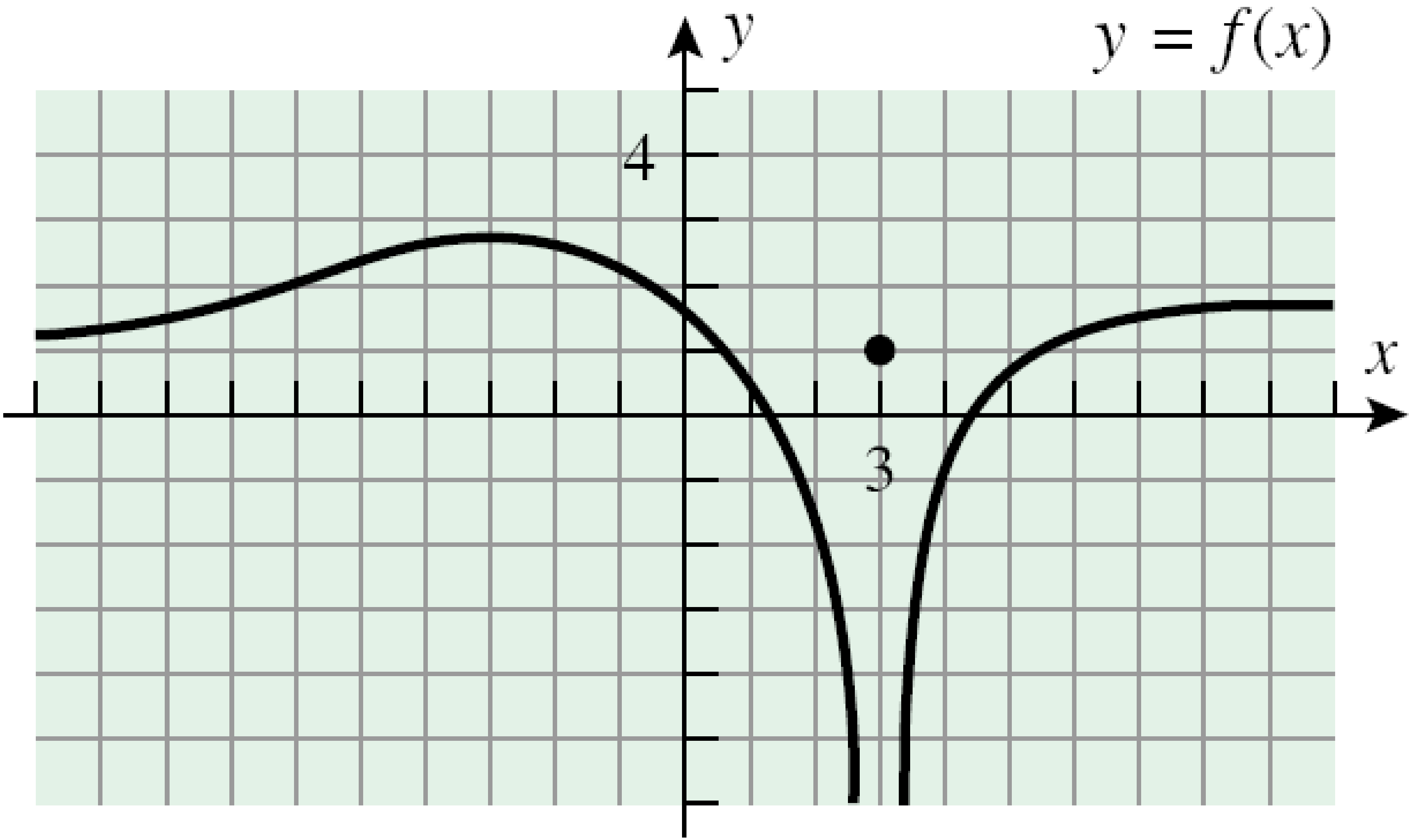

Exercise 1.1.7

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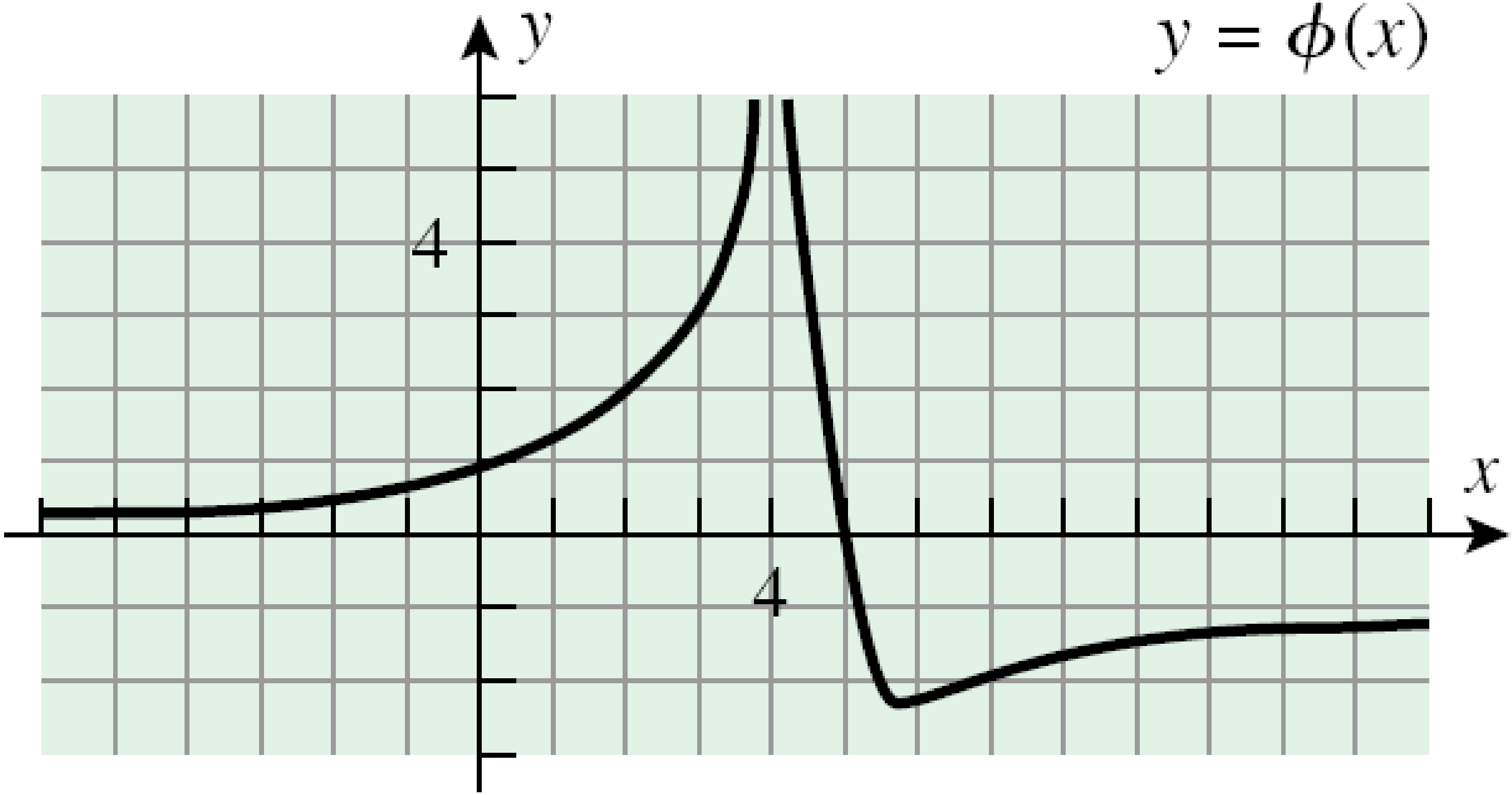

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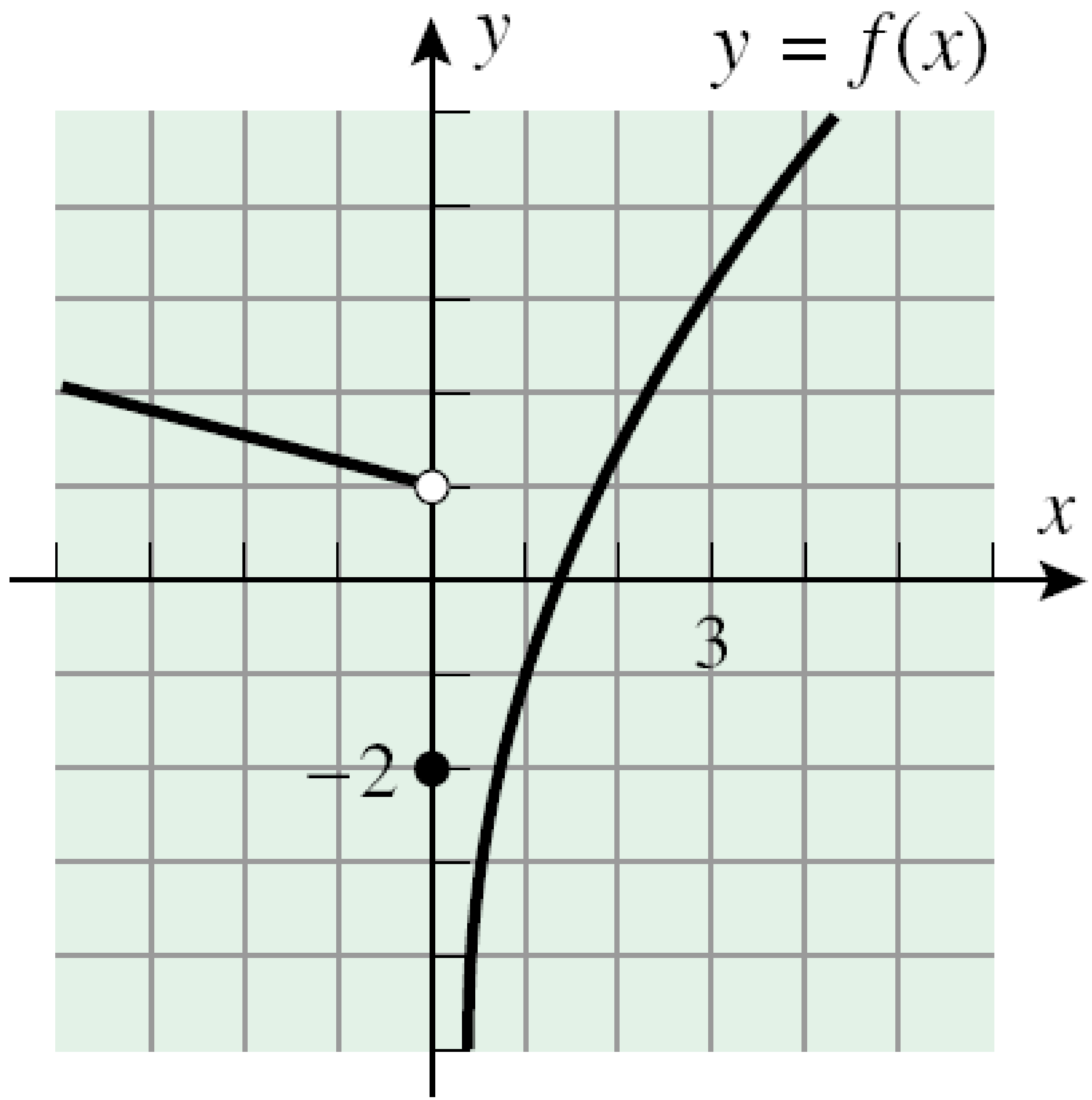

Exercise 1.1.9

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Exercise 1: Limits (Intuitive Approach)

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1.2 Calculating Limits – Basic Cases

Some Basic Limits

The strategy for finding limits algebraically has two parts:

- First we will obtain the limits of some simple functions.
- Then, we will develop a repertoire of theorems that will enable us to use the limits of those simple functions as building blocks for finding limits of more complicated functions.

1.2.1 THEOREM *Let a and k be real numbers.*

(a) $\lim_{x \to a} k = k$ (b) $\lim_{x \to a} x = a$ (c) $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ (d) $\lim_{x \to 0^+} \frac{1}{x} = +\infty$

Theorem 1.2.1

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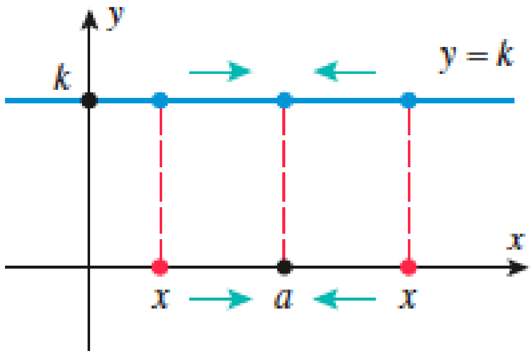
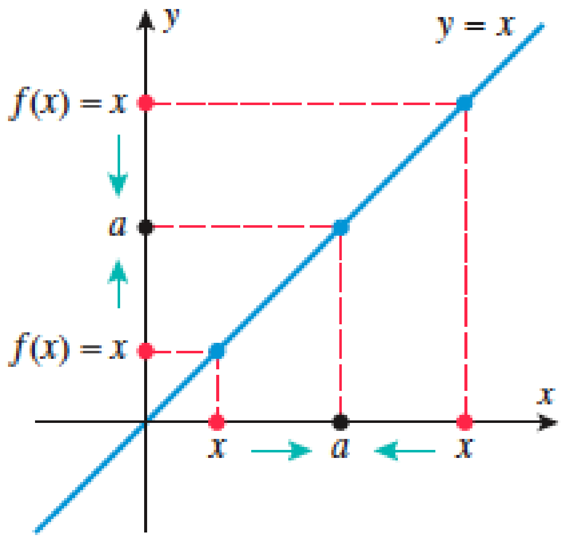
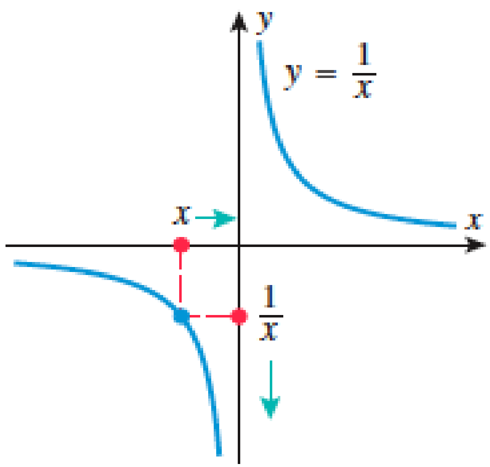
1.2 Calculating Limits – Basic Cases



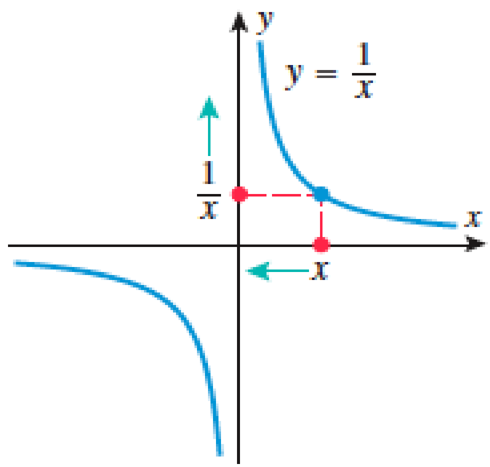
$$\lim_{x \to a} k = k$$



$$\lim_{x \to a} x = a$$



$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$



$$\lim_{x \to 0^+} \frac{1}{x} = +\infty$$

Figure 1.2.1

1.2 Calculating Limits – Algebra Rules

1.2.2 THEOREM *Let a be a real number, and suppose that*

$$\lim_{x\to a} f(x) = L_1 \quad \textit{and} \quad \lim_{x\to a} g(x) = L_2$$

That is, the limits exist and have values L_1 and L_2, respectively. Then:

(a) $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = L_1 + L_2$

(b) $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = L_1 - L_2$

(c) $\lim_{x\to a} [f(x)g(x)] = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right) = L_1 L_2$

(d) $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \dfrac{L_1}{L_2}$, *provided* $L_2 \neq 0$

(e) $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} = \sqrt[n]{L_1}$, *provided* $L_1 > 0$ *if* n *is even.*

Moreover, these statements are also true for the one-sided limits as $x \to a^-$ or as $x \to a^+$.

1.2 Calculating Limits – Algebra Rules

1.2.3 Theorem For any polynomial

$$p(x) = c_0 + c_1x + c_2x^2 + \ldots + c_nx^n$$

and any real number a,

$$\lim_{x\to a} p(x) = c_0 + c_1a + \ldots + c_na^n = p(a)$$

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1.2 Calculating Limits – Algebra Rules

Example: (1) Find $\lim\limits_{x \to 2} \dfrac{5x^3 + 4}{x - 3}$

(2) Find $\lim\limits_{x \to 4} \dfrac{2 - x}{(x - 4)(x + 2)}$

(3) Find $\lim\limits_{x \to 3} \dfrac{x^2 - 6x + 9}{(x - 3)}$

(4) Find $\lim\limits_{x \to -4} \dfrac{2x + 8}{(x^2 + x - 12)}$

(5) Find $\lim\limits_{x \to 5} \dfrac{x^2 - 3x - 10}{(x^2 - 10x + 25)}$

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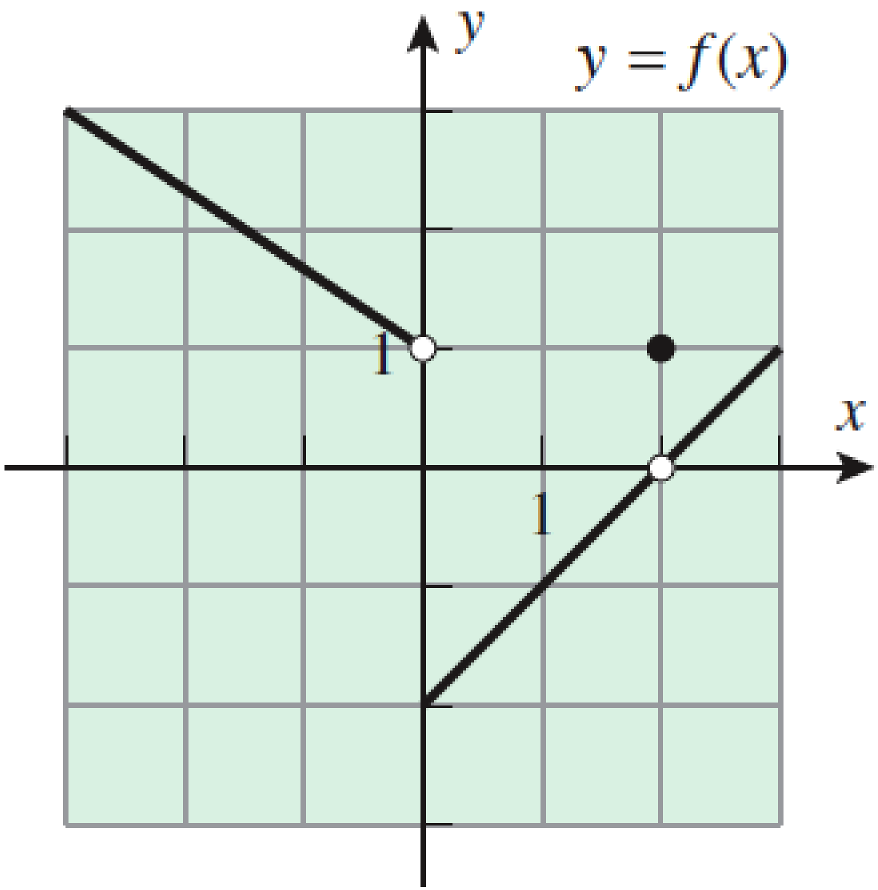

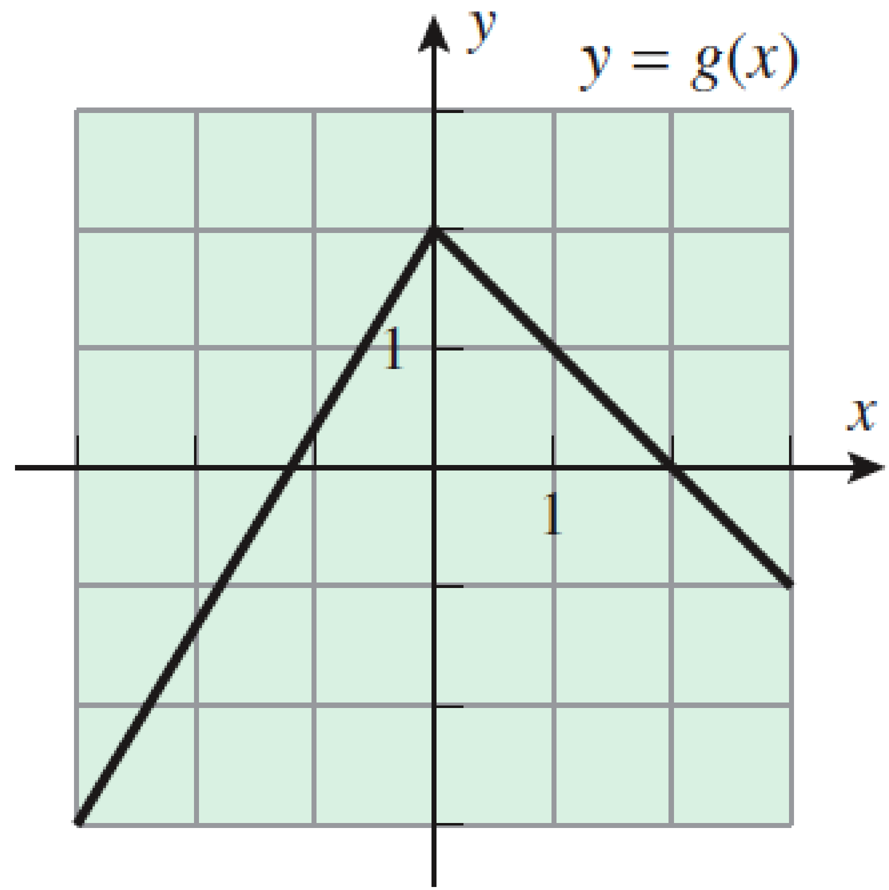

Exercise 1.2.2: Find $\lim_{x\to 2}(f(x)+g(x))$, $\lim\limits_{x\to 0-}(f(x)g(x))$, $\lim\limits_{x\to 0+}(f(x)/g(x))$, $\lim\limits_{x\to 0}(f(x)-g(x))$

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Calculating Limits – Rational Functions

1.2.4 THEOREM *Let*

$$f(x) = \frac{p(x)}{q(x)}$$

be a rational function, and let a be any real number.

(*a*) *If* $q(a) \neq 0$, *then* $\lim_{x \to a} f(x) = f(a)$.

(*b*) *If* $q(a) = 0$ *but* $p(a) \neq 0$, *then* $\lim_{x \to a} f(x)$ *does not exist.*

Theorem 1.2.4

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Exercise 2: Computing Limits

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1.3 Calculating Limits – As x goes to +,- Infinity

1.3.1 LIMITS AT INFINITY (AN INFORMAL VIEW) If the values of $f(x)$ eventually get as close as we like to a number L as x increases without bound, then we write

$$\lim_{x \to +\infty} f(x) = L \quad \text{or} \quad f(x) \to L \text{ as } x \to +\infty \tag{3}$$

Similarly, if the values of $f(x)$ eventually get as close as we like to a number L as x decreases without bound, then we write

$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad f(x) \to L \text{ as } x \to -\infty \tag{4}$$

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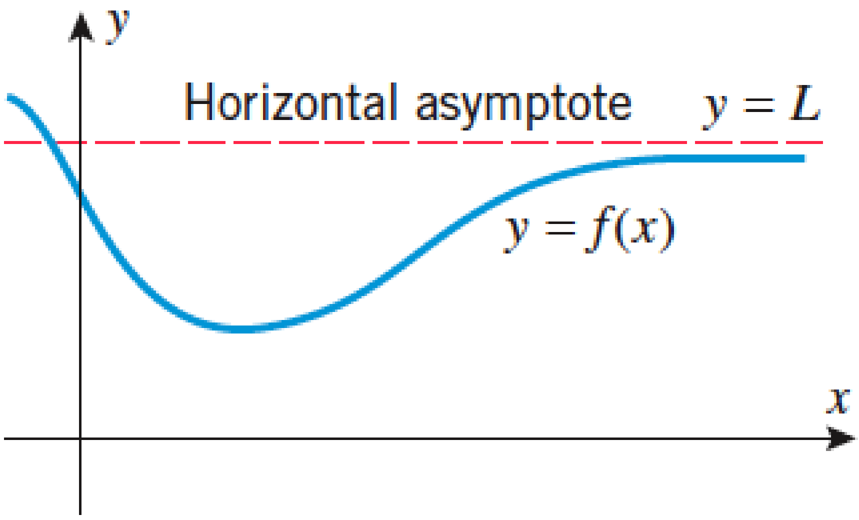
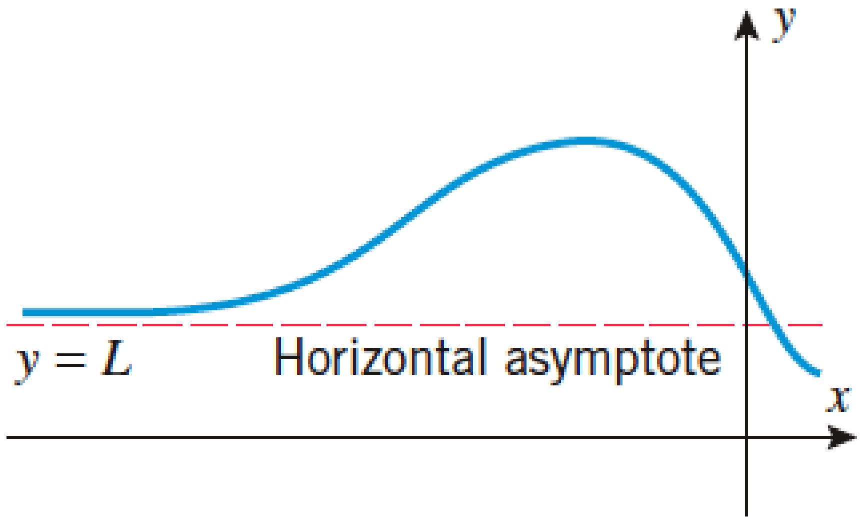


$$\lim_{x \to +\infty} f(x) = L$$



$$\lim_{x \to -\infty} f(x) = L$$

Figure 1.3.2

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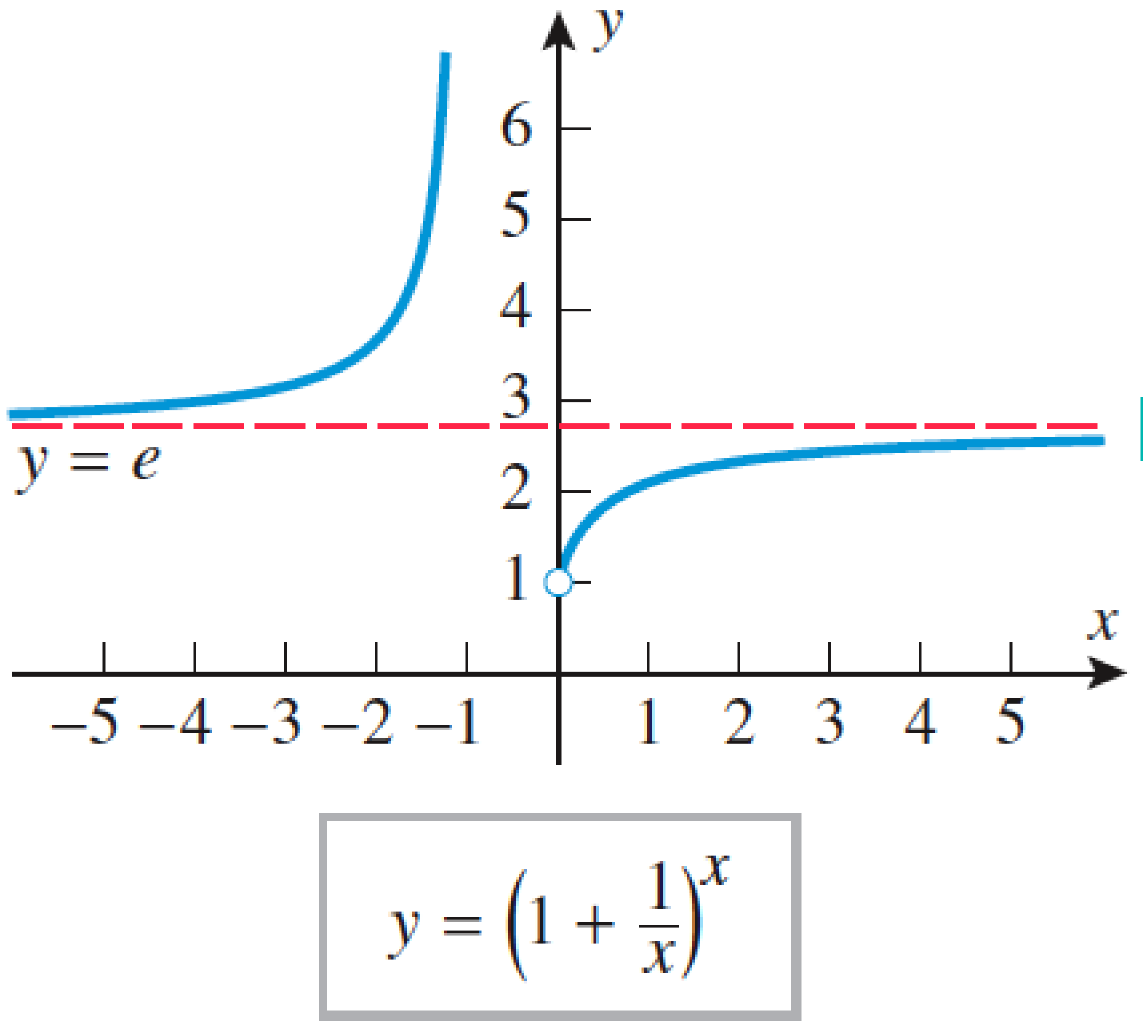

Figure 1.3.4

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1.3 Calculating Limits – As x goes to +,- Infinity

1.3.2 INFINITE LIMITS AT INFINITY (AN INFORMAL VIEW) If the values of $f(x)$ increase without bound as $x \to +\infty$ or as $x \to -\infty$, then we write

$$\lim_{x \to +\infty} f(x) = +\infty \quad \text{or} \quad \lim_{x \to -\infty} f(x) = +\infty$$

as appropriate; and if the values of $f(x)$ decrease without bound as $x \to +\infty$ or as $x \to -\infty$, then we write

$$\lim_{x \to +\infty} f(x) = -\infty \quad \text{or} \quad \lim_{x \to -\infty} f(x) = -\infty$$

as appropriate.

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Technique for a rational function:

Divide each term in the **numerator** and **denominator** by the highest power of x that occurs in the denominator.

$$\lim_{x\to\infty}\frac{4x^2-5x}{1-3x^2}=\lim_{x\to\infty}\frac{4-\frac{5}{x}}{\frac{1}{x^2}-3}=-\frac{4}{3}$$

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Technique for a rational function:

L'Hospital's Rule

The rule tells us that if we have an indeterminate form 0/0 or *inf*/*inf* we need to do is differentiate the ***numerator*** and differentiate the ***denominator*** and then take the limit.

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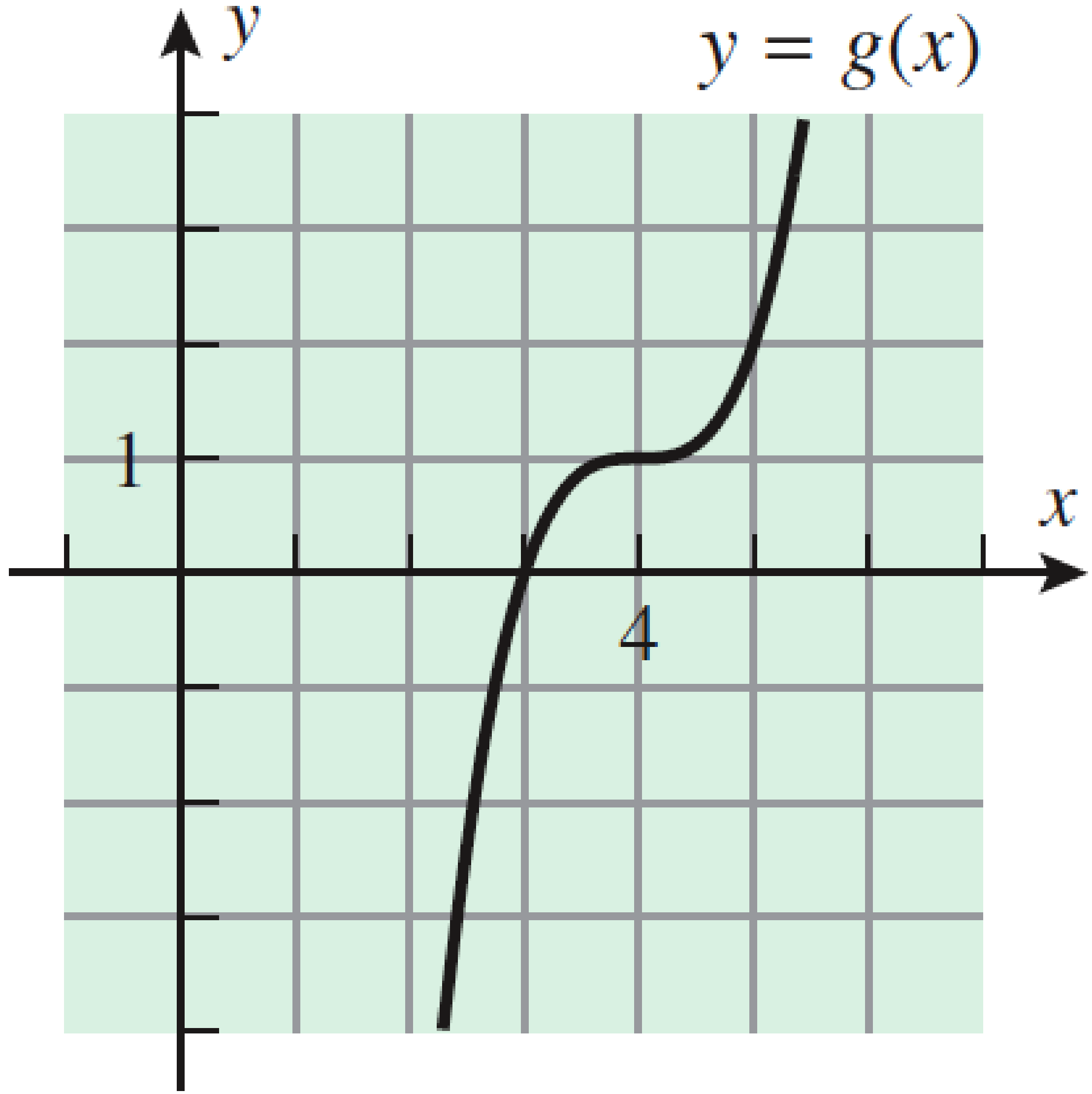

Exercise 1.3.1

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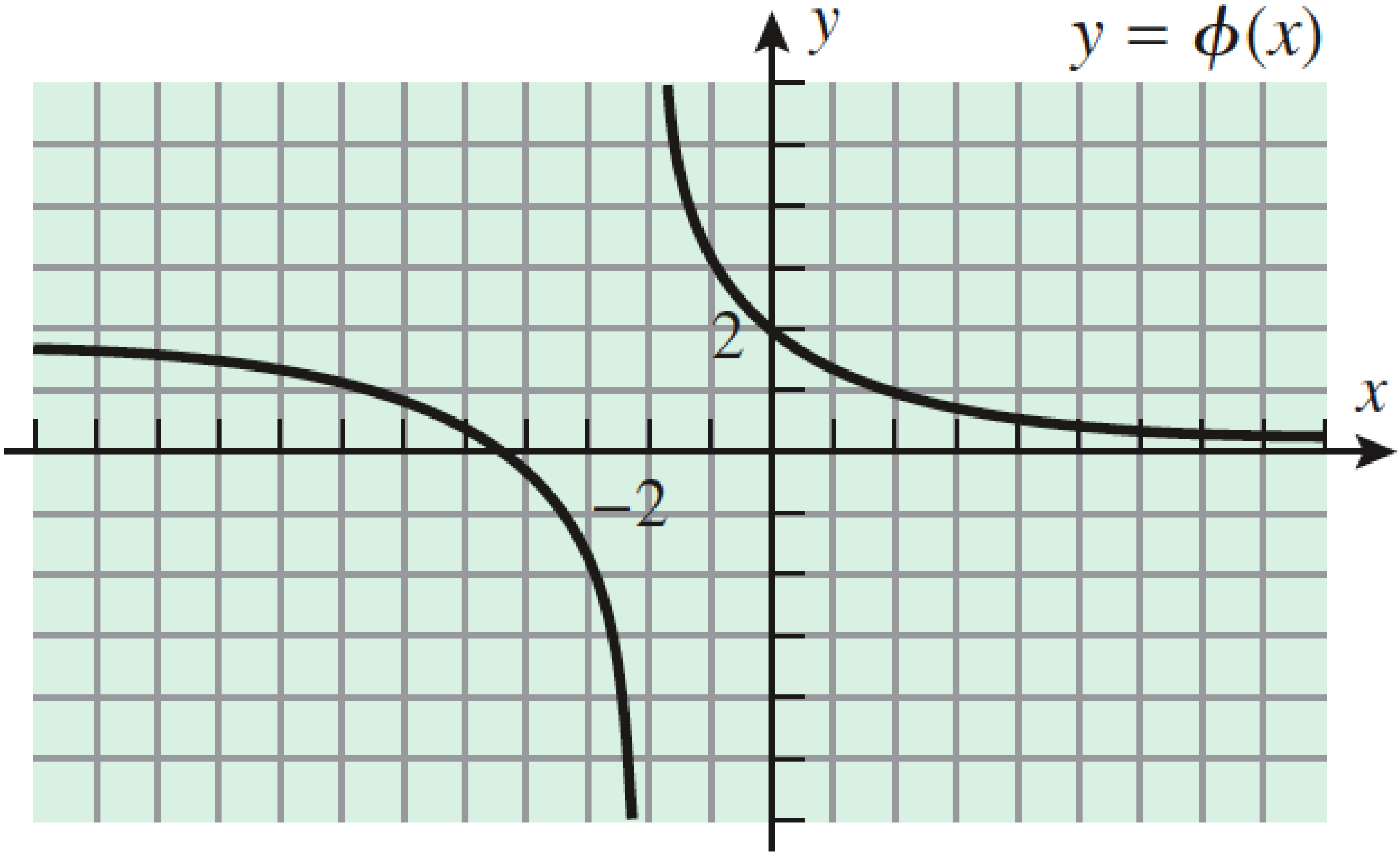

Exercise 1.3.2

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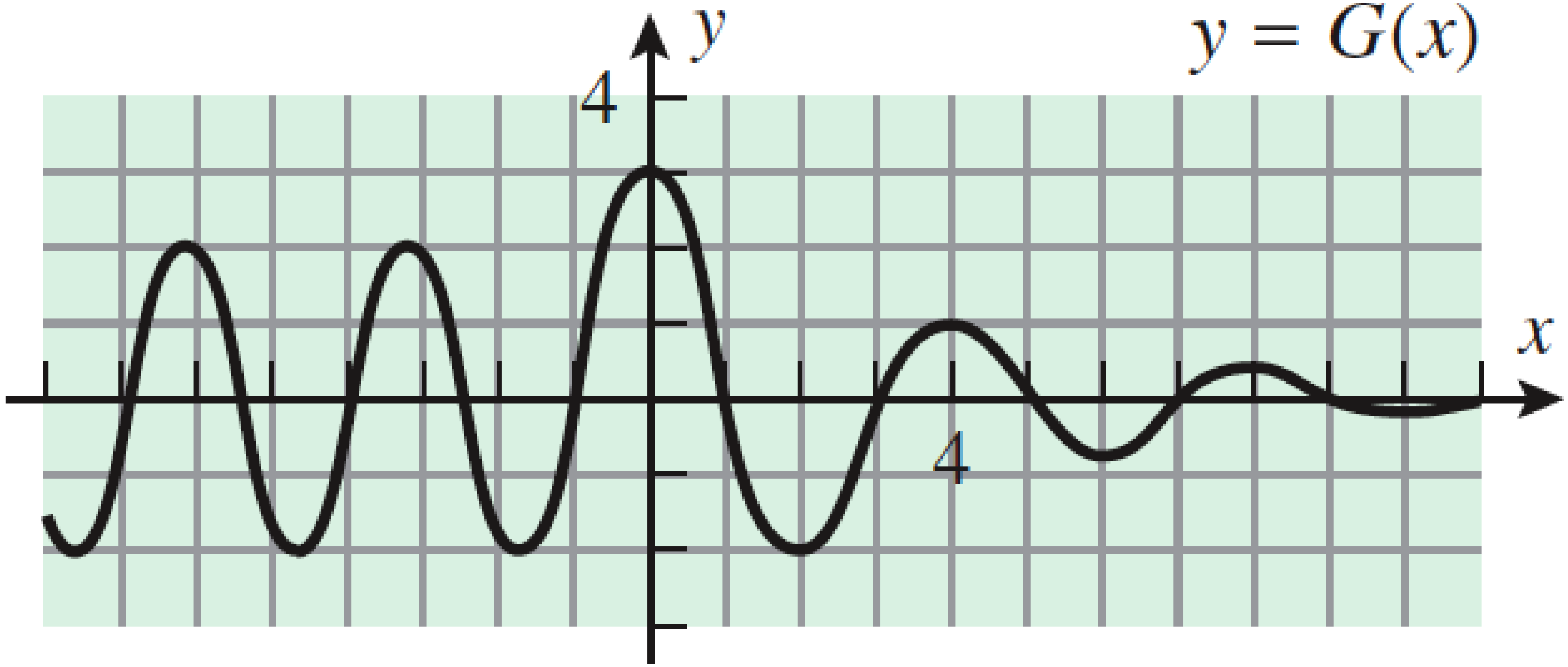

Exercise 1.3.4

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Exercise 3: Limit at Infinity,
End Behavior of a function

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1.5 Continuous Functions

1.5.1 DEFINITION A function f is said to be ***continuous at*** $x = c$ provided the following conditions are satisfied:

1. $f(c)$ is defined.

2. $\lim_{x \to c} f(x)$ exists.

3. $\lim_{x \to c} f(x) = f(c)$.

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1.5 Continuous Functions

1.5.2 DEFINITION A function f is said to be continuous on a closed interval $[a, b]$ if the following conditions are satisfied:

1. f is continuous on (a, b).
2. f is continuous from the right at a.
3. f is continuous from the left at b.

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1.5 Continuous Functions

1.5.3 THEOREM *If the functions f and g are continuous at c, then*

(*a*) $f + g$ *is continuous at* c.

(*b*) $f - g$ *is continuous at* c.

(*c*) fg *is continuous at* c.

(*d*) f/g *is continuous at* c *if* $g(c) \neq 0$ *and has a discontinuity at* c *if* $g(c) = 0$.

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1.5 Continuous Functions

1.5.4 THEOREM

(a) A polynomial is continuous everywhere.

(a) A rational function is continuous at every point where the denominator is nonzero, and has discontinuities at the points where the denominator is zero.

Continuity on $(-\infty, \infty)$:
If a function f is continuous at each number in $(-\infty, \infty)$, the we say that f is continuous on a $(-\infty, \infty)$, or f is continuous everywhere.

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1.5 Continuous Functions

1.5.5 THEOREM *If* $\lim_{x \to c} g(x) = L$ *and if the function* f *is continuous at* L, *then* $\lim_{x \to c} f(g(x)) = f(L)$. *That is,*

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right)$$

This equality remains valid if $\lim_{x \to c}$ *is replaced everywhere by one of* $\lim_{x \to c^+}$, $\lim_{x \to c^-}$, $\lim_{x \to +\infty}$, *or* $\lim_{x \to -\infty}$.

In the special case of this theorem where $f(x) = |x|$, the fact that $|x|$ is continuous everywhere allows us to write

$$\lim_{x \to c} |g(x)| = \left|\lim_{x \to c} g(x)\right| \tag{3}$$

Example:

$$\lim_{x \to 3} |5 - x^2| = \left|\lim_{x \to 3} (5 - x^2)\right| = |-4| = 4$$

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1.5 Continuous Functions

1.5.6 THEOREM

(*a*) *If the function* g *is continuous at* c, *and the function* f *is continuous at* $g(c)$, *then the composition* $f \circ g$ *is continuous at* c.

(*b*) *If the function* g *is continuous everywhere and the function* f *is continuous everywhere, then the composition* $f \circ g$ *is continuous everywhere.*

$$\lim_{x \to c} (f \circ g)(x) = \underbrace{\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right)}_{\text{Theorem 1.5.5}} \underbrace{=}_{g \text{ is continuous at } c.} f(g(c)) = (f \circ g)(c) \quad \blacksquare$$

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Exercise 4: Continuity

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1.6 Continuity of Trigonometric, Exponential, Inverse Functions

1.6.1 THEOREM *If c is any number in the natural domain of the stated trigonometric function, then*

$$\lim_{x \to c} \sin x = \sin c \qquad \lim_{x \to c} \cos x = \cos c \qquad \lim_{x \to c} \tan x = \tan c$$

$$\lim_{x \to c} \csc x = \csc c \qquad \lim_{x \to c} \sec x = \sec c \qquad \lim_{x \to c} \cot x = \cot c$$

Theorem 1.6.1 implies that the six basic trigonometric functions are continuous on their domains. In particular, $\sin x$ and $\cos x$ are continuous everywhere.

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1.6 Continuity of Trigonometric, Exponential, Inverse Functions

Example Find the limit

$$\lim_{x \to 1} \cos[(x^2 - 1)/(x - 1)]$$

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1.6 Continuity of Trigonometric, Exponential, Inverse Functions

1.6.2 THEOREM *If f is a one-to-one function that is continuous at each point of its domain, then f^{-1} is continuous at each point of its domain; that is, f^{-1} is continuous at each point in the range of f.*

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1.6 Continuity of Trigonometric, Exponential, Inverse Functions

1.6.3 THEOREM *Let* $b > 0$, $b \neq 1$.

(*a*) *The function* b^x *is continuous on* $(-\infty, +\infty)$.

(*b*) *The function* $\log_b x$ *is continuous on* $(0, +\infty)$.

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1.6 Two useful Trigonometric Limits

1.6.5 THEOREM

(a) $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$ (b) $$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

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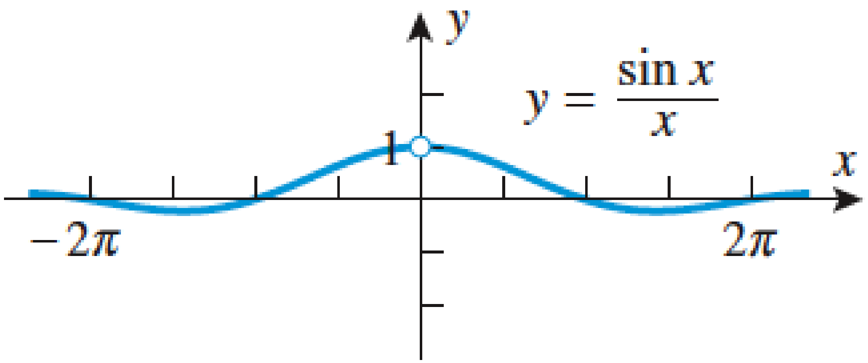
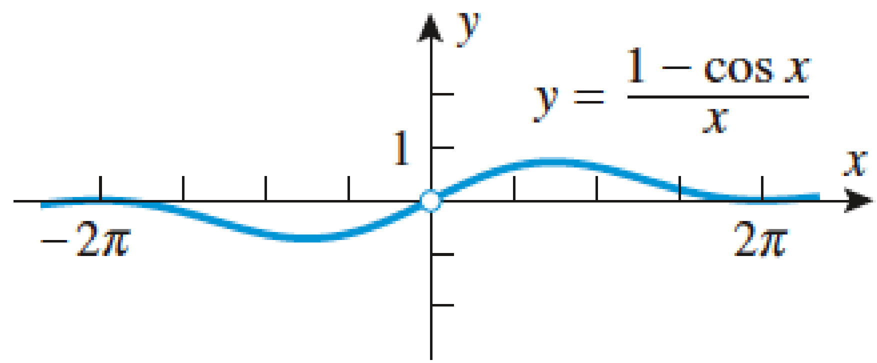
1.6 Two useful Trigonometric Limits



$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

Figure 1.6.3

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Exercises: Evaluate the following limits

1) $\lim_{x\to\pi}(\sin x + \cos 2x)$

2) $\lim_{x\to 0} \frac{\sin 5x}{x}$

3) $\lim_{x\to\infty} \tan(\frac{1}{x})$

4) $\lim_{t\to 0} \frac{t^2}{1-\cos^2 t}$

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