UNIVERSITY OF WATERLOO

Department of Systems Design Engineering

SYDE 113 | Elementary Engineering Mathematics | Fall 2022

INSTRUCTOR: Prof. Chrystopher L. Nehaniv

Matlab Assignment 2

Put all of your code in .m files. You have been assigned to groups of 3 in the Learn system. In your submission state clearly the names of each group member, student id number, and their contribution. This lab is due by 11:59 pm Monday 2022 December 5.

1. Plotting

   You can plot a conic in matlab using the ezplot function. For example, entering at the command line:

   $ezplot('x^2 - y = 4');$

   will plot the parabola $x^2 - y = 4$

   For the parabola given above, find the vertex and the focal length (c).

   By holding the plot for the parabola given above (enter hold on at the command line) plot *on the same graph* two additional parabolas, one with a larger c value and one with a smaller c value. How does increasing/decreasing c change your parabola? A sentence is sufficient.

2. Matrix Transpose. Set MatLab's random number generator seed to your group's number. That is, replace *seed* in the command below by your group's number.
   rng(*seed*)
   Now generate two random matrices by typing in:
   A=round(10*rand(4))
   B=round(10*rand(4))
   Then compute the following matrices (using the *transpose* command, and $*$ to multiply matrices in Matlab):

   - $C = (AB)^T$
   - $D = (A^T)(B^T)$
   - $E = (B^T)(A^T)$
   - $F = (BA)^T$

   Compare C, D, E and F. Which of them are equal? Use this result to write down the general rule for the transpose of a matrix product.

3. (a) We will now generate a 2×2 matrix whose entries are random integers between $+10$ and -10. You can do this using the Matlab command given below. Note that your matrix should be different from that of your neighbours. First set your random number generator seed to your group's number. That is, replace *seed* by your group's number:
   rng(seed)
   A=randi([-10,10],[2,2]);
   Write down your matrix below.

   (b) Find the determinant of your matrix by hand.

   (c) Check your solution using the Matlab function det.

   (d) State the rank of your system from inspection, and check your answer using the Matlab function rank(A).

4. Matrix Inverse

   Create the following matrices in Matlab. (Use square brackets. One can separate matrix entries on the same row by spaces (or commas), and lines of the matrix by semicolons " **;** " .)

$$A = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & -1 & 0 & -1 & 0 \\ 1 & 0 & 3 & 0 & 2 \end{bmatrix}$$

   The inverse of a matrix A, can be obtained in matlab using $inv(A)$.

   (a) For the given A and B, find $inv(A)$ and $inv(B)$. Enter the result below.

   (b) For the given A and B, find $inv(AB)$. Enter the result below. Check that it is equal to $inv(B)inv(A)$.

   (c) Compute $C = inv(B) * M$ and BC. Notice B undoes what $inv(B)$ did to M.

5. Plotting powers of complex numbers

   Be sure to comment your Matlab code and give your variables intuitive names.

   Obtain a complex number $z = u + iv$ where the real and imaginary parts are chosen according to:

   rng(*seed*)

First set your random number generator seed to your group's number as above. That is, replace *seed* in the command above by your group's number.

u=round(10*rand(1),1)
v=round(10*rand(1),1)

Plot the successive powers of z, i.e. $z^1, z^2, \ldots, z^n$ where $n = 3$ on the complex plane. Verify the Matlab plot by hand calculation. Use De Moivre's theorem in your hand calculation.
Hand in: code, picture and hand calculation

6. Plotting roots of complex numbers

   Plot the three roots of $z^3 = -27i$ on the complex plane. Verify your solution using a hand calculation. How would your plot change if you were looking at $z^n = -27i$ where n is greater than 3 (e.g. 5, 10)?
   Hand in: code, picture and hand calculation

7. Plotting a conic

   Plot the hyperbola $x^2 - y^2/4 = 1$ in Matlab. To do this we will use the symbolic toolbox as follows:

```
syms x y
fimplicit(x^2-y^2/4==1)
h=fimplicit(x^2-y^2/4==1)  % store the  plot implicit function in h
X=h.XData  % row vector of x coordinates from plot
Y=h.YData  % row vector of y coordinates from plot
plot(X,Y)
```

   This produces two vectors X and Y, which contain the points on your hyperbola. Now define a rotation matrix

$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix},$$

   obtain the equation of your hyperbola in a rotated frame using matrix vector multiplication

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R \begin{bmatrix} x \\ y \end{bmatrix}$$

   Choose a value for θ and plot both the original and rotated hyperbola on the same graph.

   Create data matrix D with 2 rows, with each column an x,y pair in the hyperbola: D(1,:)=X
   D(2,:)=Y

   Multiply D by the rotation matrix by R to get that rotated hyperbola's coordinates and plot the result.

   Find the foci of the hyperbola on both the rotated and non-rotated frame.
   Hand in Code, picture, hand calculation for location of foci.

8. Eigenvectors, Eigenvalues, and Advanced Matrix Constructs (Singular Value Decomposition). *Use matrices A, B, and M from question 4.*

   (a) *e = eig(A)* returns a column vector containing the eigenvalues of A. *[V,D] = eig(A)* returns diagonal matrix D of eigenvalues and matrix V whose columns are the corresponding (right) eigenvectors, so that $A * V = V * D$. Try this on A and B and on AB. [Try this on $N = [3\ 1; 0\ 3]$, a 2×2 matrix that cannot be diagonalized. How are the two **eigenvectors** matlab finds related?]

   (b) Find the singular value decomposition of M, using $[U, S, V] = svd(M)$. **This returns 3 matrices decomposing M into 3 linear operations:** $M = USV^T$. U and V describe higher dimensional "orthogonal transformations" (U^T is U^{-1}, and V^T is V^{-1} so all the rows of U are orthogonal to each other, and the same is true for V.) Matrix S only has nonzeroes on the diagonal and describes stretching by M after these V^T is applied and before U is applied.

9. Quadratic equation

   Show (by hand) that the Matlab code below will yield the roots of a quadratic equation $x^2 - 2bx + c = 0$

```
if b > 0
       x1 = b + sqrt(b^2 -c)
       x2 = c/x1
  else
      x2 = b -sqrt(b^2 - c)
      x1 = c/x2
  end
```

   If your algorithm produces better roots than the standard quadratic formula for the coefficients given below, explain why. Are there any other problems that you think could come up when solving the quadratic equation? If so list one or two and give a plausible solution.

   $a = 1, b = -10^5, c = 1$
   Hand in: hand calculations and answer to questions.