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附录 B　诺特定理

我们再次回顾一下诺特定理, 这次我们给出一个非常不动脑子的暴力证明, 如果拉格朗日量在坐标变换以及场变化:

$$x \mapsto \tilde{x} \tag{B.1}$$

$$\phi^a(x) \mapsto \tilde{\phi^a}(\tilde{x}) \tag{B.2}$$

之后仍然保持原先的形式:

$$\mathcal{L}(\phi^a(x), \partial\phi^a(x), x) = \mathcal{L}(\tilde{\phi^a}(\tilde{x}), \tilde{\partial}\tilde{\phi^a}(\tilde{x}), \tilde{x}) \tag{B.3}$$

因为仅仅是替换变量所以:

$$\int_{x\in O} dx \mathcal{L}(\phi^a, \partial\phi^a, x) = \int_{\tilde{x}\in O} d\tilde{x} \mathcal{L}(\phi^a(\tilde{x}), \tilde{\partial}\phi^a(\tilde{x}), \tilde{x}) \tag{B.4}$$

进而就有:

$$\int_{x\in O} dx \mathcal{L}(\tilde{\phi^a}(\tilde{x}), \tilde{\partial}\tilde{\phi^a}(\tilde{x}), \tilde{x}) = \int_{\tilde{x}\in O} d\tilde{x} \mathcal{L}(\phi^a(\tilde{x}), \tilde{\partial}\phi^a(\tilde{x}), \tilde{x}) \tag{B.5}$$

$$= \int_{x\in \tilde{x}^{-1}(O)} dx \left|\frac{d\tilde{x}}{dx}\right| \mathcal{L}(\phi^a(\tilde{x}), \tilde{\partial}\phi^a(\tilde{x}), \tilde{x}) \tag{B.6}$$

$$\tag{B.7}$$

如果有 $\left|\frac{d\tilde{x}}{dx}\right| = 1$ 的话, 那么:

$$\int_{x\in O} dx \mathcal{L}(\tilde{\phi^a}(\tilde{x}), \tilde{\partial}\tilde{\phi^a}(\tilde{x}), \tilde{x}) = \int_{x\in \tilde{x}^{-1}(O)} dx \mathcal{L}(\phi^a(\tilde{x}), \tilde{\partial}\phi^a(\tilde{x}), \tilde{x}) \tag{B.8}$$

$$\approx \int_{x\in O} dx \mathcal{L}(\phi^a(\tilde{x}), \tilde{\partial}\phi^a(\tilde{x}), \tilde{x}) - \int_{x\in\partial O} dS \cdot \delta x \mathcal{L}(\tilde{\phi^a}(\tilde{x}), \tilde{\partial}\tilde{\phi^a}(\tilde{x}), \tilde{x}) \tag{B.9}$$

然后:

$$\int_{x\in O} dx\mathcal{L}(\tilde{\phi^a}(\tilde{x}), \tilde{\partial}\tilde{\phi^a}(\tilde{x}), \tilde{x}) - \int_{x\in O} dx\mathcal{L}(\phi^a(\tilde{x}), \tilde{\partial}\phi^a(\tilde{x}), \tilde{x}) \approx -\int_{x\in\partial O} dS\cdot\delta x\mathcal{L}(\tilde{\phi^a}(\tilde{x}), \tilde{\partial}\tilde{\phi^a}(\tilde{x}), \tilde{x}) \tag{B.10}$$

$$\int_{x\in O} dx\left(\frac{\partial\mathcal{L}}{\partial\phi^a}(\tilde{\phi^a}(\tilde{x}) - \phi^a(\tilde{x})) + \frac{\partial\mathcal{L}}{\partial(\partial\phi^a)}\tilde{\partial}(\tilde{\phi^a}(\tilde{x}) - \phi^a(\tilde{x}))\right) \approx -\int_{x\in\partial O} dS\cdot\delta x\mathcal{L}(\tilde{\phi^a}(\tilde{x}), \tilde{\partial}\tilde{\phi^a}(\tilde{x}), \tilde{x}) \tag{B.11}$$

定义:

$$\tilde{\phi^a}(\tilde{x}) - \phi^a(\tilde{x}) = \phi^a(x) - \phi^a(\tilde{x}) + \delta\phi^a(x) \tag{B.12}$$

$$\approx -\partial\phi^a(x)\cdot\delta x + \delta\phi^a(x) \tag{B.13}$$

最后:

$$\int_{x\in O} dx\left(\frac{\partial\mathcal{L}}{\partial\phi^a}(-\partial\phi^a(x)\cdot\delta x + \delta\phi^a(x)) + \frac{\partial\mathcal{L}}{\partial(\partial\phi^a)}\partial(-\partial\phi^a(x)\cdot\delta x + \delta\phi^a(x))\right) \approx -\int_{x\in\partial O} dS\cdot\delta x\mathcal{L}(\phi^a(x), \partial\phi^a(x), x) \tag{B.14}$$

$$\int_{x\in\partial O} dS\cdot\left(\delta x\mathcal{L}(\phi^a(x), \partial\phi^a(x), x) + \frac{\partial\mathcal{L}}{\partial(\partial\phi^a)}(-\partial\phi^a(x)\cdot\delta x + \delta\phi^a(x))\right) \approx 0 \tag{B.15}$$

注. 所有约等号的意思都是在最多相差一阶小量下成立.