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附录 C　狄拉克场的量子化

接下来我们试图量子化一个狄拉克场, 也就是作用量: $S = \int \bar{\psi} \wedge (i\partial\!\!\!/ - m)\psi$ 所代表的场.

C.1　经典狄拉克场

首先经典的狄拉克场就与我们先前遇到的那些场不同, 我们需要将泛函 $\psi(x)$ 之间的乘法视作反对易的. 我们用 “$\wedge$” 标记这个乘法. 而像 $\psi(x) \wedge \psi(y)$ 这样的泛函需要作用在两个给定函数上才能得到结果:

$$\psi(x) \wedge \psi(y)(g, h) = g(x)h(y) - h(x)g(y) \tag{C.1}$$

接下来我们给出 ψ 在 $\mathrm{SL}(2, \mathbb{C}) \ltimes \mathbb{R}^4$ 元素 (A, a) 下的行为:

$$\psi(x) \mapsto S(A^{-1})\psi(\pi(A)x + a) \tag{C.2}$$

其中 $S(A)$ 定义为:

$$\begin{pmatrix} A & \\ & (A^{-1})^{\dagger} \end{pmatrix} \tag{C.3}$$

我们定义 γ 矩阵:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{C.4}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \tag{C.5}$$

$$\tag{C.6}$$

于是 $S(A)$ 的生成元是:

$$S(\omega_{\mu\nu}) = I + \frac{i}{2}\omega_{\mu\nu}S^{\mu\nu} + O(\omega_{\mu\nu}^2) \tag{C.7}$$

$$S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] \tag{C.8}$$

接下来我们考察经典对易关系, 为此先转到哈密顿形式, 计算动量为:

$$\pi(x) = \frac{\partial_r \mathcal{L}(x)}{\partial(\partial_0 \psi(x))} = i\psi(x)^\dagger \tag{C.9}$$

$$\bar{\pi}(x) = \frac{\partial \mathcal{L}(x)}{\partial(\partial_0 \bar{\psi}(x))} = 0 \tag{C.10}$$

这样就有:

$$H = -\int d\mathbf{x} \left(\mathcal{L} - \pi(x) \wedge \partial_0\psi(x) - \partial_0\bar{\psi}(x) \wedge \bar{\pi}(x)\right) \tag{C.11}$$

$$= \int d\mathbf{x} \left(\bar{\psi} \wedge (-i\partial_i\gamma^i + m)\psi\right) \tag{C.12}$$

$$\frac{\delta H}{\delta \pi(x)} = \partial_0\psi(x) \tag{C.13}$$

$$\frac{\delta_r H}{\delta \psi(x)} = -\partial_0\pi(x) \tag{C.14}$$

泊松括号定义为:

$$\{A, B\} = \int d\mathbf{x} A \left(\frac{\delta_r}{\delta\psi(x)} \wedge \frac{\delta}{\delta\pi(x)} + \frac{\delta_r}{\delta\pi(x)} \wedge \frac{\delta}{\delta\psi(x)}\right) B \tag{C.15}$$

于是我们熟悉的坐标动量关系为:

$$\{\psi(t,\mathbf{x}), \pi(t,\mathbf{y})\} = \{\pi(t,\mathbf{y}), \psi(t,\mathbf{x})\} = \delta(\mathbf{x} - \mathbf{y}) \tag{C.16}$$

扩展到任意时空点就有:

$$\{\psi(x), i\psi(y)^\dagger\} = \left(S^+(x-y) + S^-(x-y)\right)\gamma^0 \tag{C.17}$$

其中:

$$S^+(x) = (i\partial\!\!\!/_x + m)\Delta_m^+(x) \tag{C.18}$$
$$= \frac{1}{(2\pi)^3}\int dp(p\!\!\!/ + m)\theta(p^0)\delta(p^2-m^2)e^{-ipx} \tag{C.19}$$
$$S^-(x) = -(i\partial\!\!\!/_x + m)\Delta_m^+(-x) \tag{C.20}$$
$$= \frac{1}{(2\pi)^3}\int dp(p\!\!\!/ - m)\theta(p^0)\delta(p^2-m^2)e^{ipx} \tag{C.21}$$

C.2 狄拉克场的量子化

量子化时我们就有条件:

$$U(\pi(A),a)\psi(x)U(\pi(A),a)^{-1} = S(A^{-1})\psi(\pi(A)x+a) \tag{C.22}$$

我们希望在某个希尔伯特空间上表出我们的场算符, 于是我们做傅里叶变换, 并且由于我们希望场算符至少满足 KG 方程, 傅里叶变换的系数被限制在了能壳上:

$$\psi(x) = \int d\mu(p)\left(u^\sigma(p)a_\sigma(p)e^{-ipx} + v^\sigma(p)b_\sigma^\dagger(p)e^{ipx}\right) \tag{C.23}$$

带入条件得到:

$$\begin{aligned} U(\pi(A),a)\psi(x)U(\pi(A),a)^{-1} = \\ \int d\mu(p)u^\sigma(p)U(\pi(A),a)a_\sigma(p)U(\pi(A),a)^{-1}e^{-ipx} \\ + \int d\mu(p)v^\sigma(p)U(\pi(A),a)b_\sigma^\dagger(p)U(\pi(A),a)^{-1}e^{ipx} \end{aligned} \tag{C.24}$$

由于我们已经知道在 Fock 空间上表示的具体形式了, 带入得到:

$$U(\pi(A),a)\psi(x)U(\pi(A),a)^{-1} = \int d\mu(p)u^{\sigma}(p)U^*(R)^{\sigma'}{}_{\sigma}a_{\sigma'}(\pi(A)p)e^{-ipx-i\pi(A)pa} + \int d\mu(p)v^{\sigma}(p)U(R)^{\sigma'}{}_{\sigma}b^{\dagger}_{\sigma'}(\pi(A)p)e^{ipx+i\pi(A)pa} \tag{C.25}$$

其中 $L(\pi(A)p)R = AL(p)$. 另一方面:

$$S(A^{-1})\psi(\pi(A)x+a) = \int d\mu(p)S(A^{-1})u^{\sigma}(p)a_{\sigma}(p)e^{-ip(\pi(A)x+a)} + \int d\mu(p)S(A^{-1})v^{\sigma}(p)b^{\dagger}_{\sigma}(p)e^{ip(\pi(A)x+a)} \tag{C.26}$$

做积分变量代换得到:

$$S(A^{-1})\psi(\pi(A)x+a) = \int d\mu(p)S(A^{-1})u^{\sigma}(\pi(A)p)a_{\sigma}(\pi(A)p)e^{-ipx-i\pi(A)pa} + \int d\mu(p)S(A^{-1})v^{\sigma}(\pi(A)p)b^{\dagger}_{\sigma}(\pi(A)p)e^{ipx+i\pi(A)pa} \tag{C.27}$$

也就是说:

$$u^{\sigma}(p)U^*(L(\pi(A)p)^{-1}AL(p))^{\sigma'}{}_{\sigma} = S(A^{-1})u^{\sigma'}(\pi(A)p) \tag{C.28}$$

$$v^{\sigma}(p)U(L(\pi(A)p)^{-1}AL(p))^{\sigma'}{}_{\sigma} = S(A^{-1})v^{\sigma'}(\pi(A)p) \tag{C.29}$$

我们把 u 和 v 写成矩阵 $u^{\sigma a}$ 的形式, 也就有:

$$U^*(L(\pi(A)p)^{-1}AL(p))^{\sigma'}{}_{\sigma}u^{\sigma a}(p) = u^{\sigma b}(\pi(A)p)S(A^{-1})^{a}{}_{b} \tag{C.30}$$

$$U(L(\pi(A)p)^{-1}AL(p))^{\sigma'}{}_{\sigma}v^{\sigma a}(p) = v^{\sigma b}(\pi(A)p)S(A^{-1})^{a}{}_{b} \tag{C.31}$$

我们先考虑 p_0 处的取值, 为此先看 $A \in \mathrm{SU}(2)$ 的情况, 于是要求:

$$U^*(R)u(p_0) = u(p_0)S(R^{-1})^T \tag{C.32}$$

$$U(R)v(p_0) = v(p_0)S(R^{-1})^T \tag{C.33}$$

由于我们假定了 U 是不可约表示, 也就是说 S 中需要存在 U 和 U^* 这两个表示. 如果这两个表示同构, 那也可以只有一个. 而在我们的旋量表示中, 正好是有两个互为复共轭的表示. 具体来看:

$$R^* \begin{pmatrix} u_L(p_0) & u_R(p_0) \end{pmatrix} = \begin{pmatrix} u_L(p_0) & u_R(p_0) \end{pmatrix} \begin{pmatrix} R^* & \\ & R^* \end{pmatrix} \tag{C.34}$$

$$R \begin{pmatrix} v_L(p_0) & v_R(p_0) \end{pmatrix} = \begin{pmatrix} v_L(p_0) & v_R(p_0) \end{pmatrix} \begin{pmatrix} R^* & \\ & R^* \end{pmatrix} \tag{C.35}$$

于是我们发现, 只要满足:

$$u(p_0) = \begin{pmatrix} u_L(p_0)\sigma^0 & u_R(p_0)\sigma^0 \end{pmatrix} \tag{C.36}$$

$$v(p_0) = \begin{pmatrix} v_L(p_0)\sigma^2 & v_R(p_0)\sigma^2 \end{pmatrix} \tag{C.37}$$

接下来考虑不同 p 处的取值, 利用:

$$U^*(L(\pi(A)p_0)^{-1}A)u(p_0) = u(\pi(A)p_0)S(A^{-1})^T \tag{C.38}$$

$$U(L(\pi(A)p_0)^{-1}A)v(p_0) = v(\pi(A)p_0)S(A^{-1})^T \tag{C.39}$$

取 $A = L(\pi(A)p_0)$:

$$u(p_0)S(A)^T = u(p) \tag{C.40}$$

$$v(p_0)S(A)^T = v(p) \tag{C.41}$$

于是:

$$u(p) = \begin{pmatrix} u_L(p_0)L(p)^T & u_R(p_0)\left(L(p)^{-1}\right)^* \end{pmatrix} \tag{C.42}$$

$$v(p) = \begin{pmatrix} v_L(p_0)\sigma^2 L(p)^T & v_R(p_0)\sigma^2\left(L(p)^{-1}\right)^* \end{pmatrix} \tag{C.43}$$

设:

$$L(p) = e^{\frac{1}{2}\operatorname{arctanh}\frac{|\mathbf{p}|}{p^0}} \frac{1}{2}(1 + \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{|\mathbf{p}|}) + e^{-\frac{1}{2}\operatorname{arctanh}\frac{|\mathbf{p}|}{p^0}} \frac{1}{2}(1 - \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{|\mathbf{p}|}) \tag{C.44}$$

化简之后得到:

$$L(p) = \sqrt{\frac{p \cdot \sigma}{2m}} = \frac{p \cdot \sigma + m}{2\sqrt{m(p^0 + m)}} \tag{C.45}$$

于是最终重新定义 u 和 v 在 p_0 处值之后并作一个转置之后, 我们有:

$$u(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} u_L(p_0) \\ \sqrt{p \cdot \bar{\sigma}} u_R(p_0) \end{pmatrix} \tag{C.46}$$

$$v(p) = \begin{pmatrix} \sqrt{p \cdot \sigma}(-i\sigma^2) v_L(p_0) \\ \sqrt{p \cdot \bar{\sigma}}(-i\sigma^2) v_R(p_0) \end{pmatrix} \tag{C.47}$$

考虑对易关系:

$$[\psi(t, \mathbf{x}), \psi(t, \mathbf{y})^\dagger]_+ = i\hbar\{\psi(t, \mathbf{x}), -i\pi(t, \mathbf{y})\} = \hbar\delta(\mathbf{x} - \mathbf{y}) \tag{C.48}$$

$$[\psi(t, \mathbf{x}), \psi(t, \mathbf{y})]_+ = 0 \tag{C.49}$$

有:

$$|u_L(p_0)|^2 = |u_R(p_0)|^2 = |v_L(p_0)|^2 = |v_R(p_0)|^2 = 1 \tag{C.50}$$

由于狄拉克场实际上满足更强的方程:

$$(i\partial\!\!\!/ - m)\psi = 0 \tag{C.51}$$

于是:

$$\int d\mu(p) \left((p\!\!\!/ - m) u(p) a_\sigma(p) e^{-ipx} + (-p\!\!\!/ - m) v(p) b_\sigma^\dagger(p) e^{ipx} \right) = 0 \tag{C.52}$$

这给出了要求:

$$u_L(p_0) = u_R(p_0) \tag{C.53}$$

$$v_L(p_0) = -v_R(p_0) \tag{C.54}$$

最后我们把 u, v 写作:

$$u(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \\ \sqrt{p\cdot\bar{\sigma}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\end{pmatrix} = \frac{\not{p}+m}{\sqrt{2(p^0+m)}}\begin{pmatrix}1 & 0\\ 0 & 1\\ 1 & 0\\ 0 & 1\end{pmatrix} \tag{C.55}$$

(C.56)

$$v(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} \\ -\sqrt{p\cdot\bar{\sigma}}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\end{pmatrix} = \frac{\not{p}+m}{\sqrt{2(p^0+m)}}\begin{pmatrix}0 & 1\\ -1 & 0\\ 0 & -1\\ 1 & 0\end{pmatrix} \tag{C.57}$$

(C.58)

C.3　对称性的生成元

我们现在来尝试计算几个对称性生成元用产生湮灭算符表示的形式.

$$J^{\mu\nu} = \int d\mathbf{x}\pi(t,\mathbf{x})\left(x^\mu\partial^\nu - x^\nu\partial^\mu - iS^{\mu\nu}\right)\psi(t,\mathbf{x}) \tag{C.59}$$

带入 ψ 表达式得到:

$$\begin{aligned} J^{\mu\nu} = i\int d\mathbf{x} \\ &\int d\mu(p)\left(a(p)^\dagger u(p)^\dagger e^{ipx} + b(p)v(p)^\dagger e^{-ipx}\right) \\ &\qquad\left(x^\mu\partial^\nu - x^\nu\partial^\mu - iS^{\mu\nu}\right) \\ &\qquad\quad\int d\mu(q)\left(u(q)a(q)e^{-iqx} + v(q)b(q)^\dagger e^{iqx}\right) \end{aligned} \tag{C.60}$$

简单起见只考虑角动量, 调整积分变量 p 得到

$$
\begin{aligned}
J^{ij} = \int \frac{d\mathbf{p}}{(2\pi)^3\sqrt{2\omega_{\mathbf{p}}}} \int \frac{d\mathbf{q}}{(2\pi)^3\sqrt{2\omega_{\mathbf{q}}}} \left(a(\mathbf{p})^\dagger u(\mathbf{p})^\dagger + b(-\mathbf{p})v(-\mathbf{p})^\dagger\right) \\
i\int d\mathbf{x}\left(-i\mathbf{x}^i\mathbf{q}^j + i\mathbf{x}^j\mathbf{q}^i - iS^{ij}\right)e^{-i(\mathbf{p}-\mathbf{q})\mathbf{x}} \\
\left(u(\mathbf{q})a(\mathbf{q}) + v(-\mathbf{q})b(-\mathbf{q})^\dagger\right)
\end{aligned}
\tag{C.61}
$$

把空间部分积掉之后:

$$
\begin{aligned}
J^{ij} = \int \frac{d\mathbf{p}}{(2\pi)^3\sqrt{2\omega_{\mathbf{p}}}} \int \frac{d\mathbf{q}}{(2\pi)^3\sqrt{2\omega_{\mathbf{q}}}} \left(a(\mathbf{p})^\dagger u(\mathbf{p})^\dagger + b(-\mathbf{p})v(-\mathbf{p})^\dagger\right) \\
\left(\int d\mathbf{x}\left(-i\boldsymbol{\partial}^i_{\mathbf{q}}\mathbf{q}^j + i\boldsymbol{\partial}^j_{\mathbf{q}}\mathbf{q}^i + S^{ij}\right)e^{-i(\mathbf{p}-\mathbf{q})\mathbf{x}}\right) \\
\left(u(\mathbf{q})a(\mathbf{q}) + v(-\mathbf{q})b(-\mathbf{q})^\dagger\right)
\end{aligned}
\tag{C.62}
$$

$$
\begin{aligned}
J^{ij} = \int \frac{d\mathbf{p}}{(2\pi)^3\sqrt{2\omega_{\mathbf{p}}}} \int \frac{d\mathbf{q}}{(2\pi)^3\sqrt{2\omega_{\mathbf{q}}}} \left(a(\mathbf{p})^\dagger u(\mathbf{p})^\dagger + b(-\mathbf{p})v(-\mathbf{p})^\dagger\right) \\
\left(\left(-i\boldsymbol{\partial}^i_{\mathbf{q}}\mathbf{q}^j + i\boldsymbol{\partial}^j_{\mathbf{q}}\mathbf{q}^i + S^{ij}\right)(2\pi)^3\delta(\mathbf{p}-\mathbf{q})\right) \\
\left(u(\mathbf{q})a(\mathbf{q}) + v(-\mathbf{q})b(-\mathbf{q})^\dagger\right)
\end{aligned}
\tag{C.63}
$$

$$
\begin{aligned}
J^{ij} = \int \frac{d\mathbf{p}}{(2\pi)^3\sqrt{2\omega_{\mathbf{p}}}} \int \frac{d\mathbf{q}}{(2\pi)^3\sqrt{2\omega_{\mathbf{q}}}} \left(a(\mathbf{p})^\dagger u(\mathbf{p})^\dagger + b(-\mathbf{p})v(-\mathbf{p})^\dagger\right) \\
(2\pi)^3\delta(\mathbf{p}-\mathbf{q})\left(i\boldsymbol{\partial}^i_{\mathbf{q}}\mathbf{q}^j - i\boldsymbol{\partial}^j_{\mathbf{q}}\mathbf{q}^i + S^{ij}\right) \\
\left(u(\mathbf{q})a(\mathbf{q}) + v(-\mathbf{q})b(-\mathbf{q})^\dagger\right)
\end{aligned}
\tag{C.64}
$$

$$
\begin{aligned}
J^{ij} = \int \frac{d\mathbf{p}}{(2\pi)^3 2\omega_{\mathbf{p}}} \left(a(\mathbf{p})^\dagger u(\mathbf{p})^\dagger + b(-\mathbf{p})v(-\mathbf{p})^\dagger\right) \\
\left(i\boldsymbol{\partial}^i_{\mathbf{p}}\mathbf{p}^j - i\boldsymbol{\partial}^j_{\mathbf{p}}\mathbf{p}^i + S^{ij}\right) \\
\left(u(\mathbf{p})a(\mathbf{p}) + v(-\mathbf{p})b(-\mathbf{p})^\dagger\right)
\end{aligned}
\tag{C.65}
$$

$$\left(i\boldsymbol{\partial}_{\mathbf{p}}^{i}\mathbf{p}^{j}-i\boldsymbol{\partial}_{\mathbf{p}}^{j}\mathbf{p}^{i}+S^{ij}\right)u(\mathbf{p})=\left(i\boldsymbol{\partial}_{\mathbf{p}}^{i}\mathbf{p}^{j}-i\boldsymbol{\partial}_{\mathbf{p}}^{j}\mathbf{p}^{i}+S^{ij}\right)\frac{\not{p}+m}{\sqrt{2(p^{0}+m)}}\begin{pmatrix}1&0\\0&1\\1&0\\0&1\end{pmatrix} \tag{C.66}$$

$$\left(i\boldsymbol{\partial}_{\mathbf{p}}^{i}\mathbf{p}^{j}-i\boldsymbol{\partial}_{\mathbf{p}}^{j}\mathbf{p}^{i}+S^{ij}\right)u(\mathbf{p})=\left(i\boldsymbol{\partial}_{\mathbf{p}}^{i}\mathbf{p}^{j}-i\boldsymbol{\partial}_{\mathbf{p}}^{j}\mathbf{p}^{i}+S^{ij}\right)\frac{p^{0}\gamma^{0}-\mathbf{p}^{i}\boldsymbol{\gamma}^{i}+m}{\sqrt{2(p^{0}+m)}}\begin{pmatrix}1&0\\0&1\\1&0\\0&1\end{pmatrix} \tag{C.67}$$

$$\begin{aligned}&\left(i\boldsymbol{\partial}_{\mathbf{p}}^{i}\mathbf{p}^{j}-i\boldsymbol{\partial}_{\mathbf{p}}^{j}\mathbf{p}^{i}+S^{ij}\right)u(\mathbf{p})=\\&\qquad\frac{i}{2}\boldsymbol{\gamma}^{i}\boldsymbol{\gamma}^{j}\frac{p^{0}\gamma^{0}-\mathbf{p}^{i}\boldsymbol{\gamma}^{i}+m}{\sqrt{2(p^{0}+m)}}\begin{pmatrix}1&0\\0&1\\1&0\\0&1\end{pmatrix}+\frac{-i\boldsymbol{\gamma}^{i}\mathbf{p}^{j}+i\boldsymbol{\gamma}^{j}\mathbf{p}^{i}}{\sqrt{2(p^{0}+m)}}\begin{pmatrix}1&0\\0&1\\1&0\\0&1\end{pmatrix}\end{aligned} \tag{C.68}$$

$$\left(i\boldsymbol{\partial}_{\mathbf{p}}^{i}\mathbf{p}^{j}-i\boldsymbol{\partial}_{\mathbf{p}}^{j}\mathbf{p}^{i}+S^{ij}\right)u(\mathbf{p})\quad=\quad\frac{i}{2}\frac{p^{0}\gamma^{0}-\mathbf{p}^{i}\boldsymbol{\gamma}^{i}+m}{\sqrt{2(p^{0}+m)}}\boldsymbol{\gamma}^{i}\boldsymbol{\gamma}^{j}\begin{pmatrix}1&0\\0&1\\1&0\\0&1\end{pmatrix} \tag{C.69}$$

对 v 类似有:

$$\left(i\boldsymbol{\partial}_{\mathbf{p}}^{i}\mathbf{p}^{j}-i\boldsymbol{\partial}_{\mathbf{p}}^{j}\mathbf{p}^{i}+S^{ij}\right)v(\mathbf{p})\;=\;\frac{i}{2}\frac{p^{0}\gamma^{0}-\mathbf{p}^{i}\boldsymbol{\gamma}^{i}+m}{\sqrt{2(p^{0}+m)}}\boldsymbol{\gamma}^{i}\boldsymbol{\gamma}^{j}\begin{pmatrix}0&1\\-1&0\\0&-1\\1&0\end{pmatrix} \tag{C.70}$$

所以:

$$J^{ij}=\int\frac{d\mathbf{p}}{(2\pi)^3 2\omega_{\mathbf{p}}}\left(a(\mathbf{p})^\dagger u(\mathbf{p})^\dagger+b(-\mathbf{p})v(-\mathbf{p})^\dagger\right)\\ \left(i\boldsymbol{\partial}_{\mathbf{p}}^i\mathbf{p}^j-i\boldsymbol{\partial}_{\mathbf{p}}^j\mathbf{p}^i+S^{ij}\right)\\ \left(u(\mathbf{p})a(\mathbf{p})+v(-\mathbf{p})b(-\mathbf{p})^\dagger\right) \tag{C.71}$$

$$J^{ij}=\int\frac{d\mathbf{p}}{(2\pi)^3}\\ \left(a(\mathbf{p})^\dagger\left(i\boldsymbol{\partial}_{\mathbf{p}}^i\mathbf{p}^j-i\boldsymbol{\partial}_{\mathbf{p}}^j\mathbf{p}^i+\frac{1}{2}\epsilon^{ijk}\sigma^k\right)a(\mathbf{p})+b(-\mathbf{p})\left(i\boldsymbol{\partial}_{\mathbf{p}}^i\mathbf{p}^j-i\boldsymbol{\partial}_{\mathbf{p}}^j\mathbf{p}^i-\frac{1}{2}\epsilon^{ijk}\sigma^k\right)b(-\mathbf{p})^\dagger\right) \tag{C.72}$$

$$J^{ij}=\int\frac{d\mathbf{p}}{(2\pi)^3}\\ \left(a(\mathbf{p})^\dagger\left(i\boldsymbol{\partial}_{\mathbf{p}}^i\mathbf{p}^j-i\boldsymbol{\partial}_{\mathbf{p}}^j\mathbf{p}^i+\frac{1}{2}\epsilon^{ijk}\sigma^k\right)a(\mathbf{p})+b(\mathbf{p})^\dagger\left(i\boldsymbol{\partial}_{\mathbf{p}}^i\mathbf{p}^j-i\boldsymbol{\partial}_{\mathbf{p}}^j\mathbf{p}^i+\frac{1}{2}\epsilon^{ijk}\sigma^k\right)b(\mathbf{p})\right) \tag{C.73}$$

这是我们早就能预料的结果, 当然计算看起来非常繁琐. 事实上这个关系式已经蕴含在我们对场算符和产生湮灭算符在洛伦兹变化下的行为的要求中了.

C.4 编时传播子

$$\langle\Omega|T\psi(x)\bar{\psi}(y)|\Omega\rangle=\hbar S_F(x-y) \tag{C.74}$$

$$S_F(x-y)=\frac{i}{(2\pi)^4}\int dp\frac{\not{p}+m}{p^2-m^2+i0}e^{-ip(x-y)} \tag{C.75}$$