RENEGOTIABLE CONTRACT-THEORETIC INCENTIVE MECHANISM IN FEDERATED LEARNING

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ABSTRACT

Federated learning (FL) has risen in importance due to heightened concerns over data privacy, yet effective incentive mechanisms remains underdeveloped. These mechanisms are crucial for motivating data owners (DOs) to contribute towards FL tasks. The challenge is intensified by privacy restrictions, which limit task owner's ability to accurately evaluate the capabilities and efforts of DOs, leading to moral hazard and information asymmetry issues. This study employs contract theory to devise optimal incentive mechanisms that promotes honest participation while mitigating risks, fostering a competitive and sustainable FL environment. Unlike previous contract-based approaches, our framework addresses potential changes in DOs' behavior and budget constraints by enabling renegotiation for `Re-Contract`, which allows for more flexible and adaptive incentives. Our extensive simulations on real-world datasets demonstrated that our approach delivers superior performance, with an average utility yield improvement of up to 27% on average, compared to other state-of-the-art methods.

***Index Terms*—** Federated learning, Contract theory, Incentive Mechanism.

1. INTRODUCTION

FL research has recently garnered attention due to data privacy and security concerns. However, a gap remains in developing effective incentive mechanisms to encourage honest contributions from DOs, whose participation is critical for global model convergence but may divert resources from their primary tasks [1]. However, due to privacy requirements, FL servers cannot directly observe DOs' capabilities or effort, leading to a moral hazard problem and information asymmetry, that can result in inefficient outcomes [2]. To overcome this challenge, we propose using contract theoretic approach. Contract theory [2] explores how entities could reach optimal agreements amid conflicting interests and varying levels of information. The FL servers will present a set of contracts that specify the level of contribution and corresponding rewards, allowing DOs to autonomously select an option tailored to their type and engage in local training. The contract mechanism has a self-revealing property, enabling the elicitation of optimal provisions even under information asymmetry and moral hazard. Traditionally, designing optimal contracts require certain assumptions to be held, which may not be practical in real-world scenarios. Firstly, optimal contracts necessitate full commitment from both parties, which may not always be feasible. DOs may unintentionally drop out due to issues like network failures or weak battery capacity, or they may do so intentionally, exhibiting semi-honest behavior. Secondly, designing an optimal contract requires the FL task owner to accurately estimate the necessary resources—such as budget, time, data, computational power, and communication energy—needed to complete the task, which is often challenging. In practical competitive environments, it is common for large projects to exceed their budget or schedule, leading to contract renegotiation or renewals [3].

Furthermore, due to the nature of FL tasks, which involve training the model over multiple rounds, there is a concern regarding the sustained commitment [4] of the DOs involved. Moreover, the discrepancy between expected and actual contributions generates uncertainty regarding rewards [5]. Such uncertainty can result in sub-optimal contributions and overbudgeting. It would therefore be in both parties interest to readjust their cost-reward expectations. Consequently, it is neither feasible, fair, nor efficient for FL task owners and DOs to commit to a single contract for the entire duration of the task. Most prior works in contract theory [6, 7, 8], have not adequately addressed these issues. To address these uncertainties, we propose a two-stage renegotiable contract framework. Specifically, we investigate how FL task owners can create an initial optimal contract based on preliminary assumptions and subsequently, after several rounds of observation, draft a new optimal contract where applicable. Thereby, realigning their interests throughout the task's completion.

2. RELATED WORKS

Incentive mechanisms are essential for maintaining a sustainable and efficient FL system, but substantial challenges remains [9]. Due to the privacy preserving nature of FL, information asymmetry exists. Information asymmetry refers to a situation where one agent has more information than the other agent. This imbalance can lead to inefficiencies and market failures, as the agent with less information may make decisions that are not in their best interest. In the case of FL, the FL task owner wants to minimise cost of hiring while the DOs wants to maximise their reward. Contract theory studies how arrangements can be structured to align incentives and encourage cooperative behavior among parties. Jiawen Kang et al. [6] addressed the challenge of selecting reliable DOs and incentivizing their participation in FL by using contract theory and reputation. The contract design involves specifying the resources DOs should contribute (i.e., data, computation) and the corresponding rewards they will receive. Similarly, in the studies [7] and [8], they proposed the use of contract theory to design an incentive mechanism that motivates their participants while ensuring sustainable and privacy-preserving collaboration among users. However, most existing research lacks flexibility in contracts, despite the possibility that some initial assumptions may be incorrect.

3. FRAMEWORK AND PROCESS OVERVIEW

We consider a scenario where there are $\mathcal{N} = \{1, \ldots, n, \ldots N\}$ DOs available to participate under $\mathcal{F} = \{1, \ldots, f, \ldots F\}$ FL servers. For simplicity, we assume a one to one correspondent between task and FL server. The FL tasks would entail the server to train a global model $\omega_f(t)$ for a duration of $t \in [0, \mathcal{T}]$ global communication rounds or until the model attains a specific threshold level of performance for its generalization accuracy, whichever comes first. The summary of the flow is shown in Figure 1. We explain the process in detail below: **Stage 1: Task Initialisation and contract design.** In this stage, each FL server initiates an FL task. For consistency, we standardize these tasks to involve training a global model, $\omega_f(t)$, by minimizing the loss function. The global model performance is represented as $\xi(\omega_f(t))$, where t denotes the current global communication round. **Stage 2: Selection process.** Each FL task owner would divide available DOs into Θ with K levels $\Theta = \{\theta_1, \ldots, \theta_K\}$ based on their types, which is sorted in ascending order: $\theta_1 < \ldots < \theta_k < \ldots \theta_K \forall k \in K$. This first attempt in profiling DO types is described as the state of the world, and may change as more information is revealed through repeated interaction. In situations of information asymmetry, the FL task owner does not have direct access to the DO's type information and can only infer the likelihood of the DO belonging to a specific type based on observations, which satisfies $\sum_k^K \rho_k = 1$. The specific reward from the menu of contract written by the FL server f will be denoted as $\Upsilon_f^0 = (R_k^f(e_k)) \forall k \in K$. e_k denotes the effort that DO of type k will need to contribute, τ_k is the number of local epochs times the amount of quantity sample,d_k, DO of type k will train with $e_k = \tau_k \cdot d_k$. **Stage 3: Model initialisation** Upon choosing the contract, the FL servers will send the current global model, $\omega_f(t)$, to the participating DOs. **Stage 4: Local Model training and update.** The optimization of the global model's accuracy, $\xi(\omega_f(t))$, involves minimizing the weighted mean of participants' local loss functions [10]. Each participating DO will train its local model, $\omega_n(t)$, for some number of local epochs, τ_k, using its individual dataset and computational resources to maximise the local model's accuracy rate. **Stage 5: Model updating.** After τ_k local training rounds, the DO submits their model updates to the FL server. At round t, the FL server checks if the training and budget expenditure progresses as planned. Coupled with observations of the DOs' behavior and the current conditions, the FL task owner may aim to resolve prior uncertainties and design a new contract, Υ_f^1. Both parties would have to agree to the new contract Υ_f^1 for it to be held and previous contract will be void, otherwise Υ_f^0 will still be outstanding. **Stage 6: Model Aggregation and contribution assessment.** Upon receiving all the local updates, the FL servers will perform model aggregation following algorithms such as FedAvg [10] to update the global model $\omega_f(t)$. **Stage 7: Reward payment** After the DO fulfills the contract, the FL server rewards them accordingly. The FL training process then repeats from step 1 (excluding task initialization) to step 7 until either $t = T$ or the task is completed.

4. SYSTEM MODEL

4.1. Data owners' energy cost

The communicational cost per global communicational round is defined as:

$$E_n^{comm}(t) = T_n^{comm} p_n^{trans} = \frac{s_n(t) \cdot p_n^{trans}}{z_n}, \qquad (1)$$

where p_n^{trans} represents the transmission power of DO n. We use s to represent the size of local model, $\omega_n(t)$ and it is assumed to be constant for all participants under the same FL task (i.e., training the same model). The transmission rate, z_n of DO n given the transmission bandwidth β can be represented as:

$$z_n = \beta \ln \left(1 + \frac{\kappa_{n,f} p_n^{trans}}{\mathcal{H}_0} \right), \qquad (2)$$

where $\kappa_{n,f}$ denotes the channel gain of the link between DO n and FL server f and $\mathcal{H}_0$ denotes the background noise. Finally we can represent $E_n^{comm}(t) = \frac{s_n(t) \cdot p_n^{trans}}{S \ln(1 + \frac{G_{n,f} p_n^{trans}}{\mathcal{H}_0})}$.

Since each type of DO can vary the amount resources and directly influence the number of local epochs. The computational energy $E_n^{comp}(x_n) = P_n^{cmp} \cdot T_n^{cmp}(x_n)$ per global training round, is mathematically defined as: $P_n^{cmp} = \zeta_n \nu_n^2 \digamma_n$ and $T_n^{cmp}(x_n) = \frac{\mu_n d_n x_n}{\digamma_n}$. Where d_n is the amount of data sample DO n will supply to train the FL model. ν_n is the supply voltage required of DO n's processor. μ_n represent the total number of CPU cycles required for DO to train a unit size of data. $\digamma_n$ denotes n's CPU operating frequency, and ζ_n represents the effective load capacitance of n's computation chip-set. x_n represents the number of local round DO n will train for that particular global communication round.

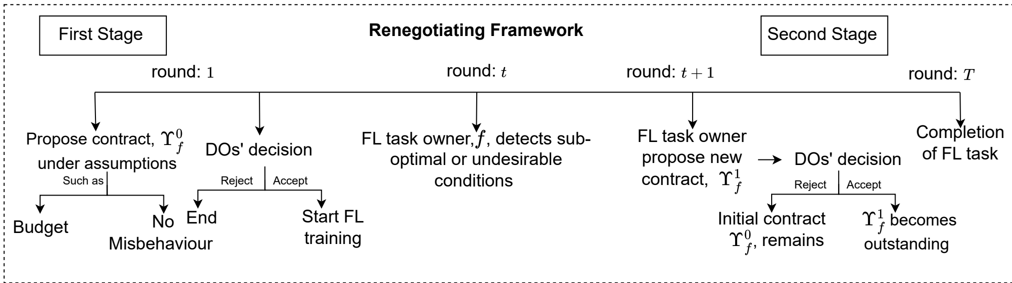

Fig. 1: Summary of the flow for `Re-Contract` process

The total cost for DO to participate in FL for a single global communication round consists of computational and communicational costs, $C_n^{total}(x_n, t) = \gamma_n \cdot (E_n^{cmp}(x_n) + E_n^{comm}(t))$. Where γ_n is the cost conversion parameter for DOs' energy consumption.

4.2. Utility functions

In our context, the FL servers play the role of *principal* while the participating DOs plays the role of *agents*. We denote the effort exerted by a DO of type k as e_k, where $e_k = d_k x_k$. Here, effort represents the amount of data quantity and local training rounds that the DO can contribute. The utility function for DOs of type k who is under server f's contract its own preferences to contribute e_k level of effort for a global communication round will yield:

$$U_n^f(e_k) = R_k^f(e_k) - C_k^{total}(e_k) \tag{3}$$

Considering that there might be several contract offers for a single DO coupled with the assumption of rationality, the DOs might demonstrate selfish behavior by prioritizing the maximization of their own utility, by choosing contracts that offer the highest reward, at the lowest cost. In other words, DOs will be more inclined to stick with federated learning (FL) servers where they have already built a strong reputation, and would be penalised if they misbehave or exhibit semi-honest behaviours. Thus DO will try to maximise its utility by choosing the best contract set available:

$$\underset{(R_k^f, e_k)}{max}\ U_n(e_k) = \rho_k b_k^f \Big(R_n^k(e_k) - C_k^{total}(e_k) \Big) \tag{4}$$

ρ_k is the probability of DO m belonging to k^{th} type and that $\sum_{k=1}^{K} \rho_k = 1$. b_k^f is a binary variable, if n of type k is selected to participate in f's FL task, $b_n^f = 1$ otherwise 0. To clarify, we assume that each DO can only participate in one task at a time. The utility yield for each FL task owner for all the contract created can be defined as,

$$U_f(e_k, R_k^f) = \sum_k^K \rho_k Q_f \Bigg(\xi(\omega_k) + \ln\Big[T_{max} - \frac{d_k \mu_k x_k}{F_k} - T_k^{comm} \Big] - \theta_k R_k^f \Bigg) \tag{5}$$

Q_f is the revenue conversion function. The DO will be rewarded with R_k^f, this is also known as the the accounting cost for executing the task.

5. CONTRACT DESIGN AND `Re-Contract`

For DO n to prefer FL server f's contract over others, the contract must meet two key constraints: incentive compatibility (IC) and individual rationality (IR). **Definition 1.** IC ensures that participants are incentivized to truthfully disclose their private information, leading each DO to select the contract that maximizes their expected utility. That is,

$$\theta_k R_k^f(e_k) - C_k^{total}(e_k) \geqslant \theta_j R_j^f(e_j) - C_k^{total}(e_j),$$
$$\forall k, j \in \{1, \dots, K\}, k \neq j. \tag{6}$$

Definition 2. IR ensures that participants are not worse off by participating, meaning each DO will only contribute if their expected utility is at least zero. This could be represented as,

$$U_k^f(e_k) = R_k^f(e_k) - C_k^{total}(e_k) \geqslant 0. \tag{7}$$

Definition 3. Budget feasibility: Total amount paid per global communication round cannot exceed the maximum budget, B_f^{max}, which is determined prior. This would be represented as $\sum_k^K R_k^f \leq B_f^{max}$.

To maximize utility, the server must balance total payments to DOs with global model performance. However, maximizing Eq (5) doesn't meet convex optimization conditions, making it challenging to find an optimal solution. To address this, we first relax the IR and IC constraints and then iteratively verify if any solutions satisfy the Local Downward Incentive Compatibility (LDIC) and Local Upward Incentive Compatibility (LUIC) constraints [11]. The mathematical transformation and proofs are in the appendix.

Under the assumption, that all DOs have similar communication environment for all communication rounds, that is for any t, $E_1^{comm} = E_2^{comm} = \ldots = E_k^{comm} \forall k \in K$ and that γ_k is stays constant. Thus, we have the following 'reduced' objective function to maximise:

$$\underset{(R_k^f, e_k)}{max}\, U_f(x_k, R_k^f) = \sum_k^K \rho_k Q_f\Bigg(\xi(\omega_k) + \ln\Bigg[T_{max} - \frac{\mu_k e_k}{f_k} - T_k^{comm}\Bigg] - \theta_k R_k^f\Bigg). \tag{8}$$

Subjected to:

$$\theta_1 R_1^f - C_1^{total}(e_1) = 0, \tag{9}$$

$$\theta_k R_k^f - (\mu_k \zeta_k \nu_k^2 e_k) \geqslant \tag{10}$$

$$\theta_k R_{k-1}^f - (\mu_{k-1}\zeta_{k-1}\nu_{k-1}^2 e_{k-1}), k \in \{2, \ldots, K\},$$

$$\sum_{k=1}^K \theta_k R_k^f \leqslant B_f^{max}, \forall k \in K \tag{11}$$

By systematically incorporating the constraints from the relaxed problem, we can derive R_k^f as follows:

$$R_k^f = \sum_{k=2}^K \frac{1}{\theta_k}\mu_k \zeta_k \nu_k^2 (e_k - e_{k-1}) + \frac{1}{\theta_1}\Bigg(C_1^{total}(e_1)\Bigg) \tag{12}$$

In Eq. (12), the optimal reward R_k^f is now dependent on the DO's effort e_k. As such we would be able to solve Eq. (8) with only one variable. In other words, we are able to obtain the optimal contract reward $R_k^f(e_k)$ given the set of feasible effort each DO can contribute and that the set satisfies the monotonicity constraint, $e_1 \geqslant e_k \ldots \geqslant e_K$. Substituting R_k^f into the function $\sum_k^K \rho_k \theta_k R_k^f$ we can derive the total rewards required for DOs of types k across the probability distribution ρ_k:

$$\sum_k^K \rho_k \theta_k R_k^f = \sum_k^K X_k^f + \frac{C_1^{comm}}{\theta_1}\sum_k^K \theta_k \rho_k, \tag{13}$$

when $k < K$, X_k^f would be equivalent to:

$$\mu_k \zeta_k \nu_k^2 e_k + \mu_k \zeta_k \nu_k^2 e_k (\frac{1}{\theta_k} - \frac{1}{\theta_{k+1}}) \sum_{i=k+1}^K \theta_i \rho_i, \tag{14}$$

otherwise, when $k = K$, X_k^f would equal to $\mu_K \zeta_K \nu_K^2 e_K$. Using the closed-form solution we would be able to reduce the objective function into a single variable problem. Using convex optimization tools, we would be able to derive the optimal effort, $\hat{e}_k$ and reward value $\hat{R}_k^f$. Initially, we assumed a uniform distribution for the types. The number of local training epochs can be determined according to, $\tau_k = \frac{e_k}{d_k}$. Subsequently, after t communication rounds and FL task owners will have new observations ψ on participating DOs' behaviours. FL task owner can therefore utilise the Bayes theorem, $\Pr(\theta = k|\psi) \propto \Pr(\psi)\Pr(\psi|\theta = k)$, to update their prior beliefs based on the observations.

Algorithm 1 `Re-Contract`Algorithm

Initialize: $\omega_f(0), T$, training parameters.
Formulate the menu of contracts for DOs of types based on prior probability distribution, $\Upsilon_f^0 = (R_f^k(e_k)|\mathcal{P}_k)$;
Each DO n choose preferred contract and participate in FL task;
FL Task owner f, publishes $\omega_f(0)$ to all participating DOs;
while $t < T$ **do**
  When budget expenditure is available, perform FL training with FedAvg
  if $k = K$ **then**
    $\hat{R}_k^f = \mu_k \zeta_k \nu_k^2 e_k$;
  else
    $\hat{R}_k^f = \mu_k \zeta_k \nu_k^2 e_k + \mu_k \zeta_k \nu_k^2 e_k (\frac{1}{\theta_k} - \frac{1}{\theta_{k+1}})$;
  end if
  if $t = \frac{T}{a}$ & $\sum_t^{T/a}\sum_k^K R_k^f \leqslant \frac{B_f^{max}}{a}$ & $\omega_f(t) \leqslant \omega_f(t-1)$ **then**
    update ψ and probability distribution;
    $\Pr(\theta = k|\psi) = \frac{\Pr(\psi|\theta=k)\Pr(\theta=k)}{\Pr(\psi)}$;
    Reformulate contract, $\Upsilon_f^1 = [R_f^k(e_k)|(\mathcal{P}_k|\psi)]$;
    if n accepts new contract **then**
      Υ_f^1 takes effect, and f pays the new $\hat{R}_f^k$ to DO n of type k.
    else
      Υ_f^0 is still effective.
    end if
  end if
end while

The new type probability distribution becomes:

$$\Pr(\theta = k|\psi) = \frac{\Pr(\psi|\theta = k)\Pr(\theta = k)}{\Pr(\psi)} \tag{15}$$

where $\Pr(\psi)$ can be calculated using the law of total probability, that is $\Pr(\psi) = \sum_{k=1}^K \Pr(\psi|\theta = k)\Pr(\theta = k)$.

Afterward, the FL server f can propose the newly drafted contract, Υ_f^1, to participating DOs once the conditions are met. To ensure budget feasibility, the server verifies that the model is converging as expected and that sufficient budget remains at the point of re-contracting. The conditions $\sum_t^{T/a}\sum_k^K \theta_k R_k^f \leqslant \frac{B_f^{max}}{a}$ and $\omega_f(t) \leqslant \omega_f(t-1)$ are checked at round $t = \frac{T}{a}$, where a is a predetermined split. The new contract details will follow Eq (14), ensuring better alignment with the actual behaviors and capabilities observed, thus improving the agreement's effectiveness and fairness. The algorithm psuedo-code for `Re-Contract`will be outlined in Algorithm. 1.

6. EXPERIMENTAL EVALUATION

In this section, we experimentally evaluate our approach and compare it against other state-of-the-art approaches. The

Table 1: Experiment Settings

Parameters	Value	Parameters	Value
Budget	400	T_{max}	1500
Total Rounds	50	Total DOs	45
Batch Size	128	Learning rate	0.01
Q_f	$2(\cdot)$	γ_f	0.003
E_n^{cmp}	0.01	E_n^{comm}	0.1
Momentum	0.9	a	2

model for training on CIFAR-10 datatset[12] has $1,006,206$ parameters, with two CNN blocks followed by three fully connected layers, including dropouts before the third layer. Similarly, the model for EMNIST balanced datatset [13] has $907,491$ parameters, but with one input channel and no dropouts. The MNIST [14] model is the smallest, with $21,840$ parameters, consisting of two simple CNN and max pooling blocks and two smaller fully connected layers. We tested our approach alongside four other state-of-the-art methods in both IID and non-IID scenarios, demonstrating that our approach particularly outperforms the others in non-IID settings. For the `Re-Contract` approach, we assume that the DOs initially have a uniform prior distribution for their types. If, after 25 of 50 rounds, the FL server observes that a DO's type appears different than expected, the server can update its belief and propose a new contract if certain conditions are met.

6.1. Comparison Baselines

We compared our approach against four other state-of-the-art approaches that have demonstrated strong performance in the context of incentive-based client selection in FL. **GTG-SV** [15]: Shapley value(SV) is a method used to evaluate the contribution level of each participant, where rewards are distributed proportionately based on each participant's associated SV. To reduce SV's computational complexity, a guided estimation approach proposed in [15] was implemented. **OORT** [16]: This approach gradually and effectively selects participants, balancing the exploration-exploitation dilemma to identify high-performing contributors. textbfR-RAFL [17]: A reputation-aware incentive mechanism designed within a reverse auction framework, aiming to select the top k number of reputable participants while adhering to a given budget constraint. **Contract theory**: The baseline approach based purely on an optimal contract, with rewards corresponding to the equation from (12) and (14).

Figures 2, 4 and 3 presents the simulation results of the approaches under the MNIST, EMNIST and CIFAR10 dataset under IID scenarios, respectively. Figures 5, 7 and 6 presents the results in non-IID scenarios in the same order. All reported values are in the magnitude order of 10^2. We can observe from the charts that `Re-Contract`, performs very well when compared to other state-of-the-art approaches especially in non-IID scenarios. To illustrate, under MNIST non-iid dataset, `Re-Contract` scored a total utility of 4.93%, 30.45%, 34.38%, 38.58% higher than basic contract-based approach, 'GTG-SV', 'OORT', 'RRAFL', respectively. While under a more challenging CIFAR10 dataset under non-iid scenario, `Re-Contract` scored a total utility of 6.67%,11.12%,33.33%, 31.04% higher than the other approaches in the same order.

7. CONCLUSIONS AND FUTURE WORK

In this paper, we applied contract theory to create contracts that align with the capabilities and preferences of DOs. However, we acknowledge that designing an optimal contract relies on specific assumptions that may need further observations to ascertain. To address this, we developed a flexible framework that enables FL servers to initiate `Re-Contract`, allowing for adjustments. This dynamic approach is designed to encourage DOs to contribute their resources while ensuring they accurately disclose their true capabilities. In the future, we plan to enhance this work by exploring a more theoretical approach, such as using cross entropy, to determine the optimal timing for FL servers to initiate `Re-Contract`.

8. REFERENCES

[1] Xavier Tan, Wei Yang Bryan Lim, Dusit Niyato, and Han Yu, "Reputation-aware opportunistic budget optimization for auction-based federation learning," in *International Joint Conference on Neural Networks*. IEEE, 2023, pp. 1–8.

[2] Stephen A Smith, *Contract theory*, OUP Oxford, 2004.

[3] Jesper Kranker Larsen, Geoffrey Qiping Shen, Søren Munch Lindhard, and Thomas Ditlev Brunoe, "Factors affecting schedule delay, cost overrun, and quality level in public construction projects," *Journal of management in engineering*, vol. 32, no. 1, pp. 04015032, 2016.

[4] Patrick Bolton, "Renegotiation and the dynamics of contract design," *European Economic Review*, vol. 34, no. 2-3, pp. 303–310, 1990.

[5] Han Xu, Priyadarsi Nanda, and Jie Liang, "Reciprocal federated learning framework: Balancing incentives for model and data owners," *Future Generation Computer Systems*, 2024.

[6] Jiawen Kang, Zehui Xiong, Dusit Niyato, Shengli Xie, and Junshan Zhang, "Incentive mechanism for reliable federated learning: A joint optimization approach to combining reputation and contract theory," *IEEE Internet of Things Journal*, vol. 6, no. 6, pp. 10700–10714, 2019.

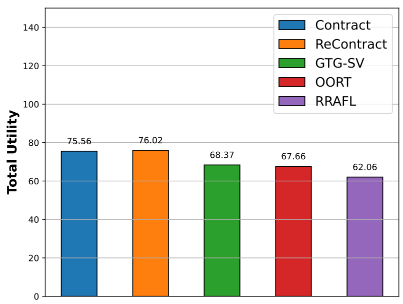

Fig. 2: Comparison of utility yield for MNIST dataset.

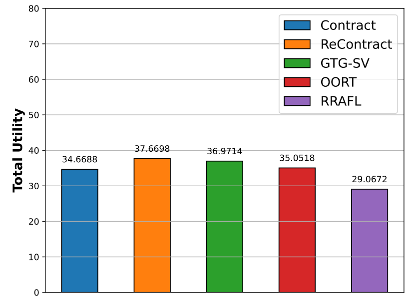

Fig. 3: Comparison of the utility yield for CIFAR10 dataset.

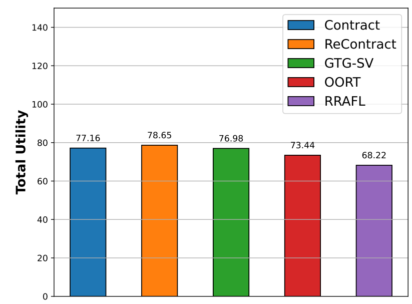

Fig. 4: Comparison of utility yield for EMNIST dataset.

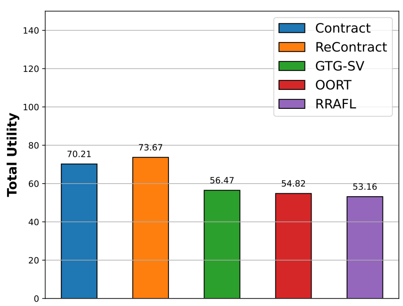

Fig. 5: Comparison of utility yield for MNIST dataset, under Non-IID setting.

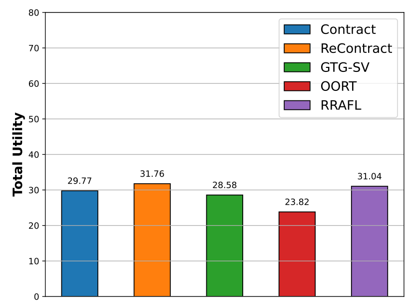

Fig. 6: Comparison of the utility yield for CIFAR10 dataset, under non-IID setting.

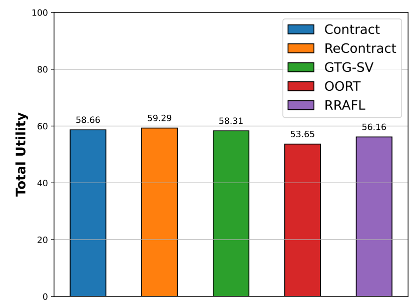

Fig. 7: Comparison of the utility yield for EMNIST dataset, under non-IID setting.

[7] Lim Wei Yang Bryan et al., "Dynamic contract design for federated learning in smart healthcare applications," *IEEE Internet of Things Journal*, vol. 8, no. 23, pp. 16853–16862, 2020.

[8] Li Li, Xi Yu, Xuliang Cai, Xin He, and Yanhong Liu, "Contract-theory-based incentive mechanism for federated learning in health crowdsensing," *IEEE Internet of Things Journal*, vol. 10, no. 5, pp. 4475–4489, 2022.

[9] Xuezhen Tu, Kun Zhu, Nguyen Cong Luong, Dusit Niyato, Yang Zhang, and Juan Li, "Incentive mechanisms for federated learning: From economic and game theoretic perspective," *IEEE transactions on cognitive communications and networking*, vol. 8, no. 3, pp. 1566–1593, 2022.

[10] Brendan McMahan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Aguera y Arcas, "Communication-efficient learning of deep networks from decentralized data," in *AISTATS*, 2017, pp. 1273–1282.

[11] Gorkem Celik, "Mechanism design with weaker incentive compatibility constraints," *Games and Economic Behavior*, vol. 56, no. 1, pp. 37–44, 2006.

[12] Alex Krizhevsky, Geoffrey Hinton, et al., "Learning multiple layers of features from tiny images," 2009.

[13] Gregory Cohen, Saeed Afshar, Jonathan Tapson, and Andre Van Schaik, "Emnist: Extending mnist to handwritten letters," in *International joint conference on neural networks*. IEEE, 2017, pp. 2921–2926.

[14] Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner, "Gradient-based learning applied to document recognition," *Proceedings of the IEEE*, vol. 86, no. 11, pp. 2278–2324, Nov 1998.

[15] Zelei Liu, Yuanyuan Chen, Han Yu, Yang Liu, and Lizhen Cui, "Gtg-shapley: Efficient and accurate participant contribution evaluation in federated learning," *ACM Transactions on intelligent Systems and Technology (TIST)*, vol. 13, no. 4, pp. 1–21, 2022.

[16] Fan Lai, Xiangfeng Zhu, Harsha V Madhyastha, and Mosharaf Chowdhury, "Oort: Efficient federated learning via guided participant selection," in *15th {USENIX} Symposium on Operating Systems Design and Implementation ({OSDI} 21)*, 2021, pp. 19–35.

[17] Jingwen Zhang, Yuezhou Wu, and Rong Pan, "Incentive mechanism for horizontal federated learning based on reputation and reverse auction," in *Proceedings of the Web Conference 2021*, 2021, pp. 947–956.

A. PROOFS FOR LDIC AND LUIC

Lemma 1 *If θ_1's IR constraint is satisfied, all IR constraint for other higher types can be reduced.*

Proof 1 *It is given that,*

$$\theta_k R_k^f(e_k) - C_k^{total}(e_n) \geqslant \theta_k R_1^f(e_1) - C_1^{total}(e_1), \quad (16)$$

$$\theta_k R_1^f(e_k) - C_1^{total}(e_1) \geqslant \theta_1 R_1^f(e_1) - C_1^{total}(e_1), \quad (17)$$

we can reduce the IR constraints to the following,

$$\theta_1 R_1^f - C_1^{total}(e_1) = 0. \quad (18)$$

Lemma 2 *If $\theta_k \geqslant \theta_j$, then it must be true that $e_k \geqslant e_j$ such that inevitably $R_k \geqslant R_j$, where k is a higher type than j.*

Proof 2 *According to definition of IC and Eq. 6, we know for sure that*

$$\theta_k R_k^f(e_k) - C_k^{total}(e_k) \geqslant \theta_k R_j^f(e_j) - C_j^{total}(e_j), \quad (19)$$

$$\theta_j R_k^f(e_j) - C_j^{total}(e_j) \geqslant \theta_j R_j^f(e_k) - C_m^{total}(e_k), \quad (20)$$

Combining the above two equation will yield us $(\frac{1}{\theta_j} - \frac{1}{\theta_k})(e_k - e_j) \geqslant 0$ and $(R_k^f - R_j^f) \geqslant C_k^{total}(e_k) - C_j^{total}(e_j)$ which can be further simplified into $(R_k^f - R_j^f) \geqslant \mu_k \zeta_k \nu_k^2 (e_k - e_j)$.

$$(R_k^f - R_j^f) \geqslant \mu_k \zeta_k \nu_k^2 (e_k - e_j) \quad (21)$$

In other words, $(R_k \geqslant R_j)$ is true if and only if $e_k \geqslant e_j$, thus monotonicity must be held.

From lemma 2, the IC constraints can therefore be further reduced as a pair of LDIC and LUIC constraints, $\theta_k R_k^f - (\mu_k \zeta_k \nu_k e_k) \geqslant \theta_k R_{k-1}^f - (\mu_{k-1} \zeta_{k-1} \nu_{k-1}^2 e_{k-1}), k \in \{2, \ldots, K\}, k \in \{2, \ldots, K\}$ and $\theta_k R_k^f - (\mu_k \zeta_k \nu_k^2 e_k) \geqslant \theta_k R_{k+1}^f - (\mu_{k+1} \zeta_{k+1} \nu_{k+1}^2 e_{k+1}), k \in \{1, \ldots, K-1\}$ respectively. However, because of the monotonicity as aforementioned, the following can be deduced, $\theta_{k+1}(R_k^f - R_{k-1}^f) \geqslant \theta_k (R_k^f - R_{k-1}^f) \geqslant \mu_k \zeta_k \nu_k^2 (e_k - e_{k-1})$. Thereafter, we can combine and simplify the pair of LDIC and LUIC to be:

$$\theta_k R_k^f - (\mu_k \zeta_k \nu_k^2 e_k) \geqslant \theta_k R_{k-1}^f - (\mu_{k-1} \zeta_{k-1} \nu_{k-1}^2 e_{k-1}). \quad (22)$$

Proof 3 *From Eq. 18, it is in the interest of the FL servers to reduce R_1 as much as possible, such that they could maximise their utility yield (i.e., hiring DOs at cost price $\theta_1 R_1(e_1) - C_1^{total}(e_1) = 0$). This applies to LDIC as well, FL servers will want to reduce reward value until $\theta_k R_k(e_k) - C_1^{total} = \theta_k R_{k-1}(e_k) - C_1^{total}(e_{k-1})$. This can be reformatted as $\theta_k R_k(e_k) - \theta_k R_{k-1}(e_k) = (\mu_n \zeta_n \nu_k^2 e_k - e_{k-1})$ combining with Eq. (21) we can derive our reduced IC constraint Eq. (22).*