



# An incentive mechanism design for federated learning with multiple task publishers by contract theory approach

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## ARTICLE INFO

### Keywords:

Incentive mechanism  
Federated learning  
Contract theory

## ABSTRACT

In the process of model training of the federated learning system, how to design an incentive mechanism to attract more high-quality worker nodes to join is a key issue. The existing researches on federated learning incentive mechanism only consider the scenario of single task publisher with multiple worker nodes. In the scenario of multi task publishers and multiple worker nodes, the competition between different task publishers makes the entire research process more complicated, and the contract design in the single task publisher scenario cannot be directly applied. To solve this problem, this paper proposes an incentive mechanism based on contract theory in the multi task publisher scenario and studies its application in federated learning. Simulation experiments show that this mechanism is effective for federated learning and can achieve the purpose of encouraging worker nodes to join and improve the efficiency of federated learning.

## 1. Introduction

In recent years, federated learning as an emerging machine learning technology has received extensive attention [1]. In the traditional machine learning process, the central server needs to obtain the local data of different devices and store these data. During this period, there is a risk of user privacy leakage [2]. Through federated learning technology, edge devices can participate in a large-scale machine learning model training process without exposing local data sets [3], which not only protects the data privacy of edge devices, but also helps improve the efficiency of model training [4]. A federation learning platform usually consists of a global server that coordinates the training process and a group of edge devices that each have their own local dataset [5]. A complete process of Federated learning usually includes multiple rounds of training [3]. During each round of training, the edge device downloads the latest machine learning training model from the global server and iteratively trains a new local model with its own data set locally, and then uploads the new local model to the global server. The server updates the global model according to the received model parameters and feeds the new model parameters back to the edge devices for them to continue training in the next round. Servers and edge devices continue to repeat the above process until the accuracy of the global model obtained by the server reaches their expectations [6]. Federated learning allows users to keep data on their own devices, which protects users' data privacy.

Although federated learning technology has many benefits mentioned above, it still faces many challenges. Most of the existing researches focus on how to optimize the model training algorithm to improve the efficiency of federated learning, but these researches

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<https://doi.org/10.1016/j.ins.2024.120330>

Received 19 June 2022; Received in revised form 14 June 2023; Accepted 14 February 2024

Available online 20 February 2024

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optimistically assume that worker nodes are willing to participate in the federated learning training process [3,7,8]. In fact, there are many risks in the federated learning system that may reduce the willingness of worker nodes to participate [9]. For example, although federated learning can effectively protect the user's data privacy, the user's local data is still at risk of being stolen by malicious attackers [10]. The attacker may be an untrusted global model owner in the federated learning system. It can infer whether the target data sample is in the user's data set through the local model uploaded by the user, and can even infer the feature distribution of the user's data set [11]. The attacker may also be a malicious worker node participating in the model training in the federated learning system. Based on the above risks, worker nodes may be unwilling to participate in federated learning due to concerns about local data leakage [9,12].

In addition to the data privacy problem of the data owner, worker nodes also need to consume a lot of costs in the process of model training and model communication [12]. Therefore, if there is no appropriate incentive mechanism, the willingness of worker nodes to participate in federated learning will not be very high, which will lead to a decrease in the efficiency of federated learning. To solve this dilemma, task publishers need to evaluate a large amount of private information of worker nodes (such as training cost and communication delay, etc.), and then design different incentive strategies for different types of worker nodes, to attract more high-quality worker nodes to join the federated learning system at the lowest possible cost, which can improve training efficiency [13].

Most of the existing studies only consider the scenario of one task publisher and multiple worker nodes, which is unrealistic in real life. In practice, task publishers are not unique. Worker nodes that require large amounts of data for training and model generation can also have the status of task publisher. When there are multiple task publishers, the federated learning system is always in a time slot where it has a set of federated learning tasks, and worker nodes are no longer limited by the situation of a unique task and have the right to actively choose and compare the benefits. Since the resources of worker nodes within the federated learning system are limited, and worker nodes obviously prefer tasks with high rewards, the task publisher needs to offer more attractive rewards than other publishers in order to recruit worker nodes to train models for himself. But for the task publisher, there is always a need to consider the rate of return to be gained and paid from the model, and blindly increasing the reward is not feasible. How to ensure that task publishers have a stake in the competition is a huge challenge in the design of incentives. It is also important to consider how to ensure that the incentive designed by the task publisher for the participant is effective in a competitive scenario, i.e. the participant will choose to participate in this current publisher's training task and reject other publishers' training tasks. Therefore, based on the scenario of multi task publishers and multi worker nodes, this paper proposes a federated learning incentive mechanism based on contract theory.

The main structure of this article is as follows: The second part introduces the concept of contract theory and federated learning incentive mechanism, and summarizes the work of other researchers in integrating contract theory into the scenario of federated learning incentive mechanism; the third part introduces an optimal contract design in a multi-task publisher scenario based on contract theory in detail; numerical results are presented in the fourth part followed by the conclusions in the fifth part.

## 2. Related works

### 2.1. Contract theory

Contract theory is an application of game theory. When the goals of different roles contradict each other, contract theory is a powerful tool to solve optimization problems [14]. In most cases, this problem is usually expressed as maximizing the objective function that represents the employer's compensation [15].

#### 2.1.1. Problem classification

Due to the information asymmetry between different roles, the following problems may arise in the process of designing the contract [16].

**Adverse selection:** In the problem of adverse selection, many characteristics of employees are hidden from employers. The contract is often designed by the employer who lacks information. Employers usually respond to adverse selection with the principle of revelation, which forces employees to choose a contract that conforms to their true identity.

**Moral hazard:** The issue of moral hazard refers to the employee's behavior that is not assessable to the employer. Unlike adverse selection, the information asymmetry in moral hazard appears after the contract is signed. In the issue of moral hazard, a contract is a series of behavior reward packages.

**Combination:** In practice, it is often difficult to decide which of these two issues is more important, that is, whether it is a moral hazard issue or an adverse selection issue. In fact, most incentive problems are a combination of moral hazard and adverse selection.

#### 2.1.2. Model classification

The design of the contract changes as the model changes.

**One-dimensional or multi-dimensional:** One-dimensional model only considers one feature or one task. On the contrary, in the multidimensional model, the employer evaluates the abilities of employees from multiple dimensions or assigns multiple tasks to employees. As an extension of the one-dimensional model, the multi-dimensional model can also be analyzed in a similar way to the one-dimensional model.

**Bilateral or multilateral:** Bilateral contract is the basic one-to-one contract mode, that is, an employer and an employee trade with each other. However, in a multilateral situation, there is usually a transaction between one employer and multiple employees.

Static or repeated: A static contract refers to a one-time transaction between an employer and an employee. In a repeated contract, the employee's transaction history will directly affect the next contract. The repeated iterations between the two parties of the contract bring new incentive problems and therefore increase the complexity compared with the static contract [17].

## 2.2. Federated learning incentive mechanism

In the process of participating in federated learning tasks, worker nodes will inevitably consume a lot of costs [18]. In addition, the research of Hu et al. [19] shows that training models are vulnerable to privacy attacks such as attribute inference attacks, and that these server nodes that launch malicious attacks can infer sensitive attributes of target data records based on model output or non-sensitive attribute information, making these worker nodes at risk of private information leakage. Because of the above reasons, the task publisher in the federated learning system needs to design an appropriate incentive mechanism to attract worker nodes participating in the federated learning task. Positive incentives motivate others by promising rewards, while negative incentives aim to avoid personal malicious behavior through punishment [3]. In the federated learning system, the basis for the design of the worker node incentive mechanism is mainly based on the following two aspects:

### 2.2.1. Incentive mechanism design driven by worker nodes' data value

In the federated learning system, task publishers often design incentive mechanisms by evaluating the value of worker nodes' data. Evaluation indicators can be summarized into two categories, which are data quality and data quantity. In terms of evaluating data quality, the Shapley value [20,21] has been widely used. Shim et al. [22] proposed an adversarial scoring method based on Shapley values. In terms of evaluating data quantity, Weng et al. [23] used blockchain technology to record the size of the data set for each worker node to train the local model and reward the worker nodes according to the size of the data set they used and their degree of honesty.

### 2.2.2. Incentive mechanism design driven by worker nodes' reputation

In the federated learning system, reputation is an important indicator for task publishers to choose worker nodes. Rehman et al. [24] include the three roles of edge device, fog node, and cloud server into the evaluation of the reputation system, and these three roles can be rated with each other. The system uses smart contracts to calculate and record the reputation score of each participant in the federated learning system, thereby improving the efficiency of the federated learning system.

## 2.3. The application of contract theory in the federated learning incentive mechanism

In the federated learning system, there is a problem of information asymmetry between task publishers and worker nodes. The task publisher does not know the key information of each worker node such as data quality or computing power, but the task publisher can calculate the quality distribution of worker nodes based on experience, and then infer the probability that each worker node belongs to a certain type [25]. Therefore, in the existing research on federated learning incentive mechanism, contract theory is often used to solve the problem of information asymmetry between task publishers and worker nodes.

Based on the Differential Private Federated Learning Framework (DPFL), which can prevent data leakage of worker nodes, Wu et al. [10] model the computational cost, communication cost, and privacy cost of worker nodes and their contribution to the federated learning system, and design an incentive mechanism based on contract theory. Because the three types of costs of worker nodes are unknown to the task publisher. To maximize the profit under this asymmetry of three-dimensional information, the author uses a three-dimensional contract method to design an incentive mechanism. Specifically, he calculates the optimal rewards for different types of worker nodes. The simulation experiment verifies the effectiveness of the proposed incentive mechanism.

Ding et al. [6] analyze and study the optimal incentive mechanism design of task publishers based on the multidimensional private information of worker nodes. The author's main contribution is to transform the user's multidimensional private information into a one-dimensional standard, and through the performance of the incentive mechanism in three scenarios, it reveals the influence of different degrees of information asymmetry on federated learning. When the training data of the worker nodes are independent and identically distributed, weak and incomplete information will not increase the cost of the task publisher; but when the training data does not satisfy the independent and identical distribution, it usually increases the cost of the task publisher. In addition, under the condition of strong incomplete information, the design of the optimal incentive mechanism will be more challenging. The incentive mechanism proposed by the author cannot always motivate users to join federated learning at the lowest cost.

To solve the problem of information asymmetry and attract more high-quality worker nodes to join the federated learning system, Kang et al. [4] design an effective contract based on the contract theory by mapping the resources contributed by the worker nodes into appropriate rewards. Incentives. Specifically, the author defines the data accuracy and other related parameters of the worker node as the type of worker node in the contract theory model. Worker nodes with higher precision can get higher rewards. The experimental results show that the incentive mechanism proposed by the author is better than the existing methods.

Li et al. [26] proposed an incentive mechanism based on contract theory (IMCT) using federation learning to train high-quality global models in a health crowdsensing scenario. The fact that worker nodes come from different hospitals and community treatment centres makes the quality of each node's locally trained model varies significantly. The authors therefore match the training reward with the contribution to the global model, transforming the incentive problem for worker nodes into a utility optimization problem to attract nodes that can provide high-quality local models to participate in this training.

In addition, there are existing studies in which researchers have used other forms of motivation to provide solutions for motivation in multi-task competition scenarios. For example, Yu et al. [27] formulated the incentive framework for a multi-task cross-device federated learning and analysis system as a multi-leader-follower game. Saadatmand et al. [28] proposed a reverse auction based on multi-task location dependence in mobile crowd sensing. However, these forms of incentives do not have the self-disclosure property of contract theory and cannot address the problem of information heterogeneity.

As a conclusion, many researchers have used contract theory to study the federated learning incentive mechanism. However, most of them only consider a scenario where there is one task publisher and multiple worker nodes, which is unrealistic in real life. In the scenario of multi-task publishers and multiple worker nodes, the competition between different task publishers makes the entire research process more complicated. Therefore, the contract design in the single-task publisher scenario cannot be directly applied. It is necessary to design a contract in the scenario of multi-task publishers and multi-worker nodes. On the one hand, in order to compete with each other, task publishers need to increase the remuneration to attract more worker nodes; on the other hand, task publishers need to reduce the remuneration they pay to worker nodes in order to make themselves more profitable in federated learning tasks. For worker nodes, they can choose the best contract from different task publishers to get the most utility.

### 3. Models

In this paper, we consider the contract design problem among multiple task publishers and multiple worker nodes, and study its application in the federated learning incentive mechanism. When designing contracts, the goal of each task publisher is to maximize its own profit while satisfying the two constraints, which are Individual Rationality (IR) and Incentive Compatibility (IC). The premise for worker nodes to participate in federated learning tasks is the non negative benefit of worker nodes in terms of willingness to participate, which is guaranteed by IR; IC ensures that worker nodes can obtain optimal utility only when they choose contracts corresponding to their own types [29].

#### 3.1. System structure

The system structure diagram of this article is shown in Fig. 1. There are multiple task publishers and a set of heterogeneous worker node sets  $N = \{1, \dots, N\}$  in the federated learning system. They maintain relevant information through the blockchain. Take task publishers A and B as examples. The two task publishers design specific contracts according to the quality of worker nodes, and save the content of each contract in the blockchain. Each worker node can sign a contract provided by the two task publishers to maximize his profit. After that, the heterogeneous worker nodes use their own unique data set to train the model locally, and save the updated model parameters to the blockchain. The task publisher obtains the updated model parameters on the blockchain and locally aggregates the models uploaded by all worker nodes. After the federated learning task is completed, the task publisher needs to pay the corresponding reward to the worker node according to the contract signed with the worker node, and save the payment record to the blockchain.

To solve the problem of information asymmetry, task publishers need to calculate the cost and utility function when designing contracts for different quality types of worker nodes. The following sections will give the specific derivation process of the optimal contract

#### 3.2. The cost of worker nodes

##### 3.2.1. Training cost of worker nodes

Each worker node  $n \in N$  has its own local data set. We use  $s_n$  to represent the size of the local dataset of worker node  $n$ , and use  $f_n$  to represent the CPU cycle frequency of worker node  $n$ ; The number of CPU cycles consumed by worker node  $n$  to train a unit size of data is  $c_n$ . According to [4], the calculation time  $T_n^{cmp}$  and the corresponding CPU resource cost  $E_n^{cmp}$  required to train a local model for the worker node  $n$  in each iteration are denoted as follows:

$$T_n^{cmp}(f_n) = \frac{c_n s_n}{f_n} \quad (1)$$

$$E_n^{cmp}(f_n) = \zeta c_n s_n f_n^2 \quad (2)$$

##### 3.2.2. Transmission cost of worker nodes

In the federated learning system, the worker node transmits each updated local model to the task publisher through wireless communication. Each time worker node  $n$  is trained locally, the accuracy of the model must reach  $\epsilon_n$ . To make the accuracy of the local model meet expectation, the expected number of local iterations for each training is  $\log\left(\frac{1}{\epsilon_n}\right)$ . According to Shannon's theorem, the rate at which the worker node transmits the model parameters to the task publisher during each iteration is denoted as

$$r_n = B \ln \left( 1 + \frac{\rho_n h_n}{N_0} \right) \quad (3)$$

In formula (3),  $\rho_n$  represents transmission power,  $h_n$  represents channel gain, and  $N_0$  represents background noise.

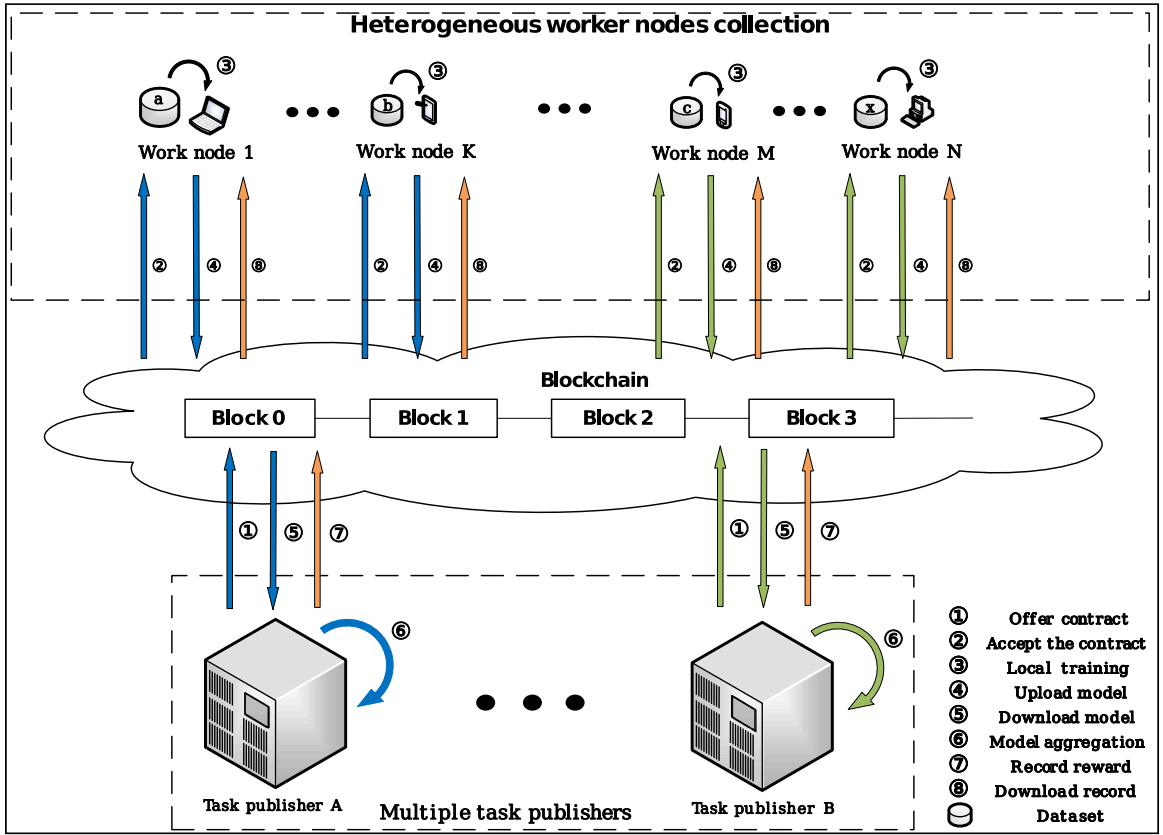


Fig. 1. System structure diagram.

Assume that the size of the model parameters passed by all worker nodes after each iteration is a fixed constant  $\sigma$ . According to [4], the communication time  $T_n^{com}$  and communication resource  $E_n^{com}$  that need to be consumed during each iteration of the worker node  $n$  are denoted as follows:

$$T_n^{com} = \frac{\sigma}{B \ln \left( 1 + \frac{\rho_n h_n}{N_0} \right)} \quad (4)$$

$$E_n^{com} = T_n^{com} \cdot \rho_n = \frac{\sigma \rho_n}{B \ln \left( 1 + \frac{\rho_n h_n}{N_0} \right)} \quad (5)$$

### 3.2.3. Total cost of worker nodes

According to the formula in the previous two subsections, the total time  $T_n^t$  and the total energy  $E_n^t$  that consumed by the worker node  $n$  participating in a global iterative process are denoted as follows:

$$T_n^t = \log \left( \frac{1}{\epsilon_n} \right) T_n^{cmp} + T_n^{com} \quad (6)$$

$$E_n^t = \log \left( \frac{1}{\epsilon_n} \right) E_n^{cmp} + E_n^{com} \quad (7)$$

## 3.3. Utility function

### 3.3.1. Utility function of task publisher

To attract different types of worker nodes to join the federated learning system, we define parameter  $\theta_n = \frac{\lambda}{\log \left( \frac{1}{\epsilon_n} \right)}$  to represent the quality of the worker nodes. Here,  $\lambda$  is a pre-defined parameter about the number of training iterations of the local model. We divide the quality of worker nodes into  $M$  categories in ascending order, namely

$$\theta_1 < \dots < \theta_m < \dots < \theta_M, m \in \{1, \dots, M\} \quad (8)$$

Although the task publisher cannot know the exact quality of each worker node, but he can infer the probability that each worker node belongs to a certain type based on experience [25]. Here, we use  $p_m$  to represent the probability that each worker node belongs to a certain type.  $\theta_M$ , and  $\sum_{m=1}^M p_m = 1$ .

When there are two or more task publishers in the federated learning system, the worker node can choose from the contracts provided by different task publishers to obtain the highest reward. Therefore, the parameter  $pc_i$  is defined in this paper to represent the probability that the worker node chooses task publisher  $i$ . In the scenario of a dual-task publisher,  $\sum_{i=1}^2 pc_i = 1$ .

Due to the problem of information asymmetry between the task publisher and the worker node, the task publisher  $i$  offers specific contracts for different quality types of worker nodes, which is denoted as

$$(R_n^i(f_n), f_n) \quad (9)$$

Formula (9) uses contract theory to map the quality types of worker nodes into corresponding rewards. Among them,  $(f_n)$  represents the CPU cycle frequency of worker node  $n$ , and  $R_n^i(f_n)$  is the corresponding reward of this type of worker node. The utility function  $U_p$  of the task publisher is denoted as

$$U_p = \ln(T_{max} - T_n^t) - R_n^i \quad (10)$$

Among them,  $T_{max}$  represents the longest tolerable time of the task publisher;  $T_n^t$  represents the total time consumed by the worker node  $n$  participating in a global iterative process;  $R_n^i$  represents the reward paid by the task publisher  $i$  to the worker node  $n$ .

The goal of task publishers is to maximize their own profits in the federated learning system. Due to the competition between different task publishers, it is also necessary to use the parameter  $pc_i$  based on formula (10) to calculate task publisher  $i$ 's profit when cooperating with the worker nodes. The total utility function of the task publisher  $i$  can be denoted by accumulating all profits from the contracts, which is

$$\begin{aligned} \max_{(R_n, f_n)} U_p &= \sum_{n=1}^N pc_i N p_n \ln \left[ T_{max} - (T_n^{com} + \frac{\lambda}{\theta_n} \cdot \frac{c_n s_n}{f_n}) \right] \\ &\quad - \sum_{n=1}^N pc_i N p_n R_n^i \end{aligned} \quad (11)$$

### 3.3.2. Utility function of worker nodes

For a worker node  $n$  who signs the contract (9) with the task publisher  $i$ , the utility function is denoted as

$$U_a = R_n^i - \mu E_n^t \quad (12)$$

Among them,  $R_n^i$  represents the worker node  $n$ 's reward paid by the task publisher  $i$ ;  $\mu$  is a pre-defined weight coefficient;  $E_n^t$  represents the total energy consumed by the worker node  $n$  participating in a global iteration process.

Intuitively, higher-quality worker nodes will get more rewards when they complete the federated learning task, because they contribute more to the federated learning system. Assume that all worker nodes are selfish and they hope to minimize their own consumption when performing federated learning tasks, thereby maximizing their own utility. Since there are multiple task publishers in the federated learning system, it is also necessary to use the parameter  $pc_i$  based on formula (10) to calculate worker nodes' profit. The total utility function of the worker node  $n$  can be denoted as follows

$$\max_{(R_n, f_n)} U_a = \sum_{i=1}^2 pc_i \left[ R_n^i - \mu \left( \frac{\lambda}{\theta_n} \zeta c_n s_n f_n^2 + E_n^{com} \right) \right] \quad (13)$$

### 3.4. Optimal contract design

To make contracts feasible, each contract must satisfy two constraints, which are Individual Rationality (IR) and Incentive Compatibility (IC).

IR constraint ensures that each worker node will participate in the federated learning task only when the income is nonnegative, which is denoted as

$$R_n^i - \mu \left[ \frac{\lambda}{\theta_n} \zeta c_n s_n f_n^2 + \frac{\sigma \rho_n}{B \ln \left( 1 + \frac{\rho_n h_n}{N_0} \right)} \right] \geq 0, \forall n \in N \quad (14)$$

IC constraint ensures that each worker node will only get the maximum profit when choosing a contract specifically designed for its own quality type, which is denoted as

$$R_n^i - \mu \frac{\lambda}{\theta_n} E_n^{cmp} \geq R_m^i - \mu \frac{\lambda}{\theta_m} E_n^{cmp}, \forall m, n \in N, n \neq m \quad (15)$$

According to the research of [4], we assume that  $\mu = 1$  and all worker nodes are in a similar wireless communication environment, which means  $E_1^{com} = \dots = E_n^{com} = \frac{\sigma \rho_n}{B \ln(1 + \frac{\rho_n h_n}{N_0})}$ ,  $n \in N$ . After the above derivation, the utility function of task publisher  $i$  can be expressed as an optimization problem as shown in formula (16),

$$\begin{aligned}
 \max_{(R_n, f_n)} U_p^i &= \sum_{n=1}^N p c_i N p_n \ln \left[ T_{max} - (T_n^{com} + \frac{\lambda}{\theta_n} \cdot \frac{c_n s_n}{f_n}) - R_n^i \right], \\
 s.t. \\
 R_n^i - (\frac{\lambda}{\theta_n} \zeta c_n s_n f_n^2 + \frac{\sigma \rho_n}{B \ln(1 + \frac{\rho_n h_n}{N_0})}) &\geq 0, \forall n \in N, \\
 R_n^i - \left[ \frac{\lambda}{\theta_n} \zeta c_n s_n f_n^2 + E_n^{com} \right] &\geq R_m^i - \left[ \frac{\lambda}{\theta_m} \zeta c_m s_m f_m^2 + E_m^{com} \right], \forall m, n \in N, n \neq m, \\
 \frac{c_n s_n}{f_n} &\leq T_{max}, \forall n \in N, \\
 \sum_{n=1}^{N_i} p c_i N_i p_n R_n^i &\leq R_{max}^i, \forall n \in N
 \end{aligned} \tag{16}$$

Among them,  $R_{max}^i$  represents the task publisher's reward budget. The above problem does not satisfy the form of the convex optimization problem, its solution can be found by performing the following transformation. To calculate the optimal contract, we must first solve the slack problem in formula (16) without monotonic constraints, and then check whether the obtained solution satisfies the monotonic condition [4]. Through the iterative method, we can obtain

$$\begin{aligned}
 R_n^i &= E_n^{com} + \frac{\lambda E_1^{cmp}}{\theta_n} + \sum_{k=1}^n r \Delta k, \\
 \Delta k &= \frac{\lambda E_k^{cmp}}{\theta_k} - \frac{\lambda E_{k-1}^{cmp}}{\theta_{k-1}}, \Delta 1 = 0
 \end{aligned} \tag{17}$$

To maximize its own profit, the worker node should use the optimal  $p^* c_i$ , which can be written in the form of formula (18),

$$p^* c_i = \operatorname{argmax} \left[ \sum_{i=1}^2 p c_i R_n^i - \sum_{i=1}^2 p c_i \left( \frac{\lambda}{\theta_n} E_n^{cmp} + E_n^{com} \right) \right] \tag{18}$$

According to the Local Downward Incentive Constraint (LDIC) [4], the constraint conditions in formula (16) can be simplified. And by substituting (18) into the problem (16), the simplified optimal contract formula can be denoted as (19),

$$\begin{aligned}
 \max_{(R_n, f_n)} U_p^i &= \sum_{n=1}^N p^* c_i N p_n \ln(T_{max} - T_n^{com} - \frac{\lambda}{\theta_n} T_n^{cmp} - R_n^i), \\
 s.t. \\
 R_n^i - (\frac{\lambda}{\theta_n} \zeta c_n s_n f_n^2 + \frac{\sigma \rho_n}{B \ln(1 + \frac{\rho_n h_n}{N_0})}) &= 0, \forall n \in N, \\
 R_n^i - (\frac{\lambda}{\theta_n} \zeta c_n s_n f_n^2 + E_n^{com}) &\geq R_{n-1}^i - \frac{\lambda}{\theta_n} \zeta c_{n-1} s_{n-1} f_{n-1}^2 - E_{n-1}^{com}, \forall n \in N, n \neq 1, \\
 \frac{c_n s_n}{f_n} &\leq T_{max}, \forall n \in N, \\
 \sum_{n=1}^{N_i} p c_i N_i p_n R_n^i &\leq R_{max}^i, \forall n \in N
 \end{aligned} \tag{19}$$

By substituting (17) into problem (19), problem (19) can be transformed into  $U_p^i$ 's optimization problem with respect to  $f_n$ . By differentiating  $U_p^i$  with respect to  $f_n$ , we can obtain  $\frac{d^2 U_p^i}{d f_n^2} \leq 0$ , so problem (19) is a convex optimization problem. With the help of the convex optimization tool CVX, the optimal contract  $R_n^i$  designed by the task publisher  $i$  under the condition of a given  $f_n$  can be calculated. With the help of the iterative method in [30], we can first fix the contract of task publisher  $A$ , and then derive the optimal contract of task publisher  $B$  under this condition. Next, we can fix the contract of task publisher  $B$ , and then derive the optimal contract of task publisher  $A$ . The above procedures are repeated until convergence where the optimal contracts for both task publishers are derived. Under this equilibrium condition, the IR constraint and IC constraint of each contract are satisfied, which means each worker node can guarantee the maximum profit when choosing a contract that matches its own type, and different task publishers can also obtain nonnegative profits under competitive conditions.



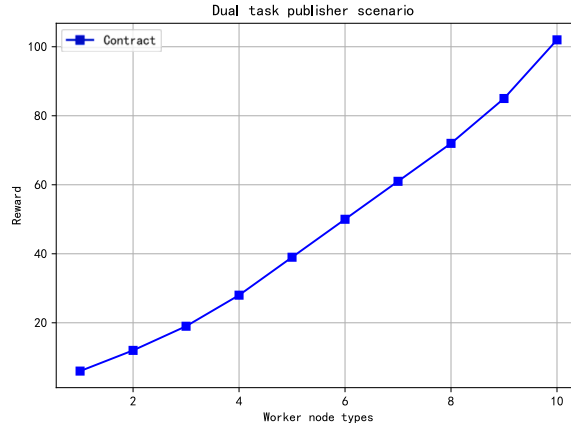


Fig. 2. Rewards for each type of worker node.

## 4. Results and analysis

### 4.1. Experimental design

In this part, we design Experiment 1 to verify the feasibility of the optimal contract in the scenario of two task publishers in the federated learning system. Specifically, we validate whether the IR and IC constraints are satisfied in the proposed method. We design Experiment 2 to show the effectiveness of the contract design in this paper in terms of the overall utility of task publishers and worker nodes. First, we calculate the profits that the task publisher and worker nodes can obtain in each type of contract in the multi task publisher scenario, and compare the profits of the task publisher in the dual task publishers scenario with the experimental results in [4]. Then we calculate the profits for the task publisher when the number of worker nodes changes from 10 to 100 in the dual task publishers scenario and compare it with an incentive scheme designed for a multi-task scenario using a game in [27], demonstrating the superiority of an incentive mechanism based on contract theory. At the end we compare the profits of each type of worker node in the scenarios of dual task publishers, three task publishers, and four task publishers with the experimental results in [4], and compare the changes in the overall profits of task publishers and worker nodes in these four scenarios. Experiment 3 aims to compare the changes in the overall profits of the task publisher when the quality distribution of the worker nodes is different.

In the simulation experiment, we assume that there are 100 members in the worker node set  $N$ , the 100 worker nodes are divided into 10 categories according to their quality. *Type1* has the lowest quality and *type10* has the highest quality. The probability of each type is 0.1; The number of CPU cycles consumed by worker node  $n$  to train a unit of data  $c_n = 5$  and the local data set size  $s_n = 20$ . The communication time consumed by worker node  $n$  during each iteration  $T_n^{com} = 10$  and the communication resources that need to be consumed  $E_n^{com} = 20$ . The longest tolerance time of the task publisher  $T_{max} = 600$  and the task publisher's reward budget  $R_{max}^i = 10000$ . With the help of the scipy tool in Python, we can solve the optimal contract design problem in the scenario of multi task publishers and multi worker nodes. By substituting the optimal contract into the utility function of the worker nodes and the task publisher, we can calculate the maximum profits that the task publisher and worker nodes can obtain in the federated learning task.

### 4.2. Experimental verification

#### 4.2.1. Contract feasibility in the dual task publisher scenario

In this part of the simulation experiment, we first give Fig. 2 to show each type of worker node and its corresponding reward. It can be seen that the higher the quality of the worker node, the higher the reward obtained in the corresponding contract. After that, the rewards of each type of contract are used to calculate the profits of the task publisher and worker nodes, and then it is verified that the optimal contract design satisfies the IC constraint and the IR constraint. Specifically, Fig. 3a is given to show that the optimal contract design in the dual task publishers scenario satisfies IC constraint, which means each type of worker node can only guarantee the maximum profit when choosing its own type of contract, and any deviation will lead to a decline in the profit. The figure lists the profits obtained by the four types of worker nodes when they choose different types of contracts, verifying that the contracts meet the IC constraint. Fig. 3b is used to show the optimal contract design in the dual task publishers scenario satisfies IR constraints, which means the profits of worker nodes and task publishers are nonnegative under optimal contract conditions. The figure shows that after each type of worker node signs and fulfills its own type of contract, both the worker nodes and the task publisher can obtain nonnegative benefits, which verifies that the optimal contract design satisfies the IR constraints.

#### 4.2.2. The profits of different roles in the multi task publishers scenario

In this part of the experiment, we first calculate the profit of the task publisher in each type of contract in the dual task publishers scenario, and compare the profit of the task publisher in the single task publisher scenario in [4]. The results are shown in Fig. 4a.



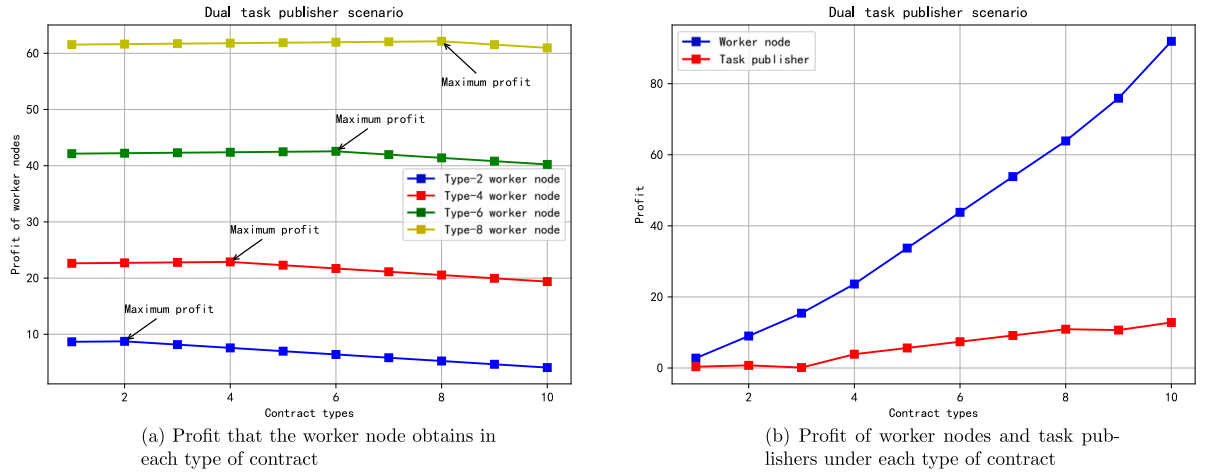


Fig. 3. The validation of the IR and IC constraints. (a) IR constraint. (b) IC constraint.

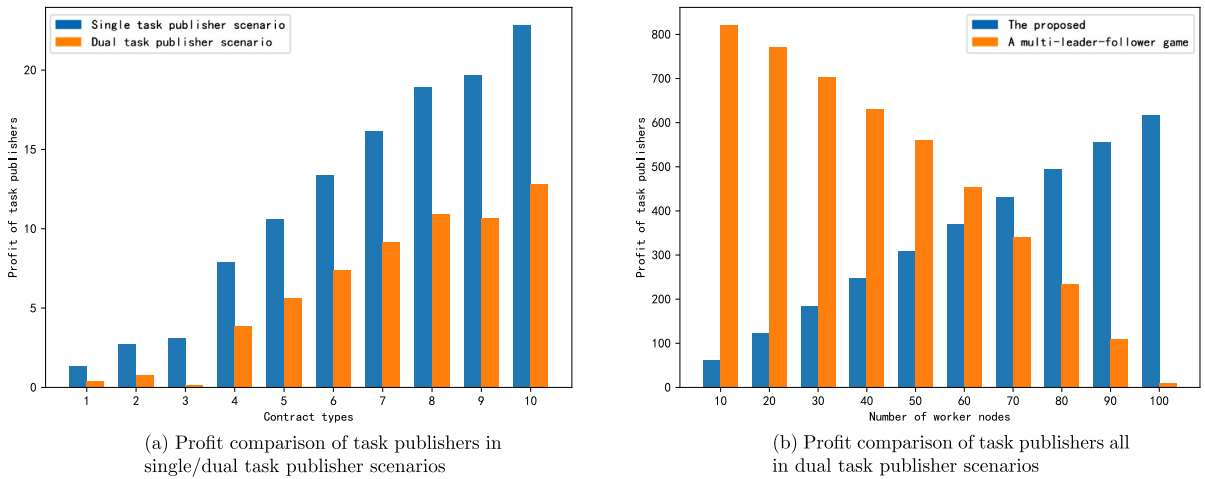


Fig. 4. Profit for task publishers with different incentive mechanisms. (a) in single/dual task publisher scenarios. (b) all in dual task publisher scenarios.

It can be seen that in the dual task publishers scenario, the profit of the task publisher will be reduced. This is because the task publisher needs to increase the reward it pays to the worker node to compete with the other one. However, in each type of contract, the profit of the task publisher is still nonnegative. Then we calculate the profits for the task publisher when the number of worker nodes changes from 10 to 100 in the dual task publishers scenario and compare it with an incentive scheme designed for a multi-task scenario using a game in [27]. The results are shown in Fig. 4b. It can be seen that in the dual task publishers scenario, our proposed incentive mechanism is able to provide higher profits to the task publisher as the total number of worker nodes increases, whereas the game is unable to guarantee the profits of the task publisher when the number of worker nodes increases. This is because our proposed contract theory-based incentive mechanism considers the benefits of both the task publisher and worker nodes, enabling both parties to maintain a win-win situation, which is beneficial for the long-term stability of the federated learning system. In addition, for federated learning model training tasks, it is necessary to recruit enough worker nodes to provide sufficient quality and quantity of data to ensure the accuracy of the model.

After that, the profits of each type of worker node in the scenarios of dual task publishers, three task publishers, and four task publishers are compared with the experimental results in [4], and the results are shown in Fig. 5a. It can be seen that as more task publishers join the competition of the federated learning system, each type of worker node can further increase its own profit after signing and fulfilling its own type of contract. This is mainly because different task publishers will compete for limited worker node resources by increasing the rewards paid to worker nodes, and worker nodes can choose from the contracts provided by multiple task publishers to get the highest reward. Fig. 5b shows when there are different numbers of task publishers in the federated learning system, the overall profits that the task publisher can obtain. It can be seen that when the number of task publishers is extended to 4, although the overall profits of the task publishers are further reduced, it is still nonnegative. That means in the scenario of multi task publishers, the optimal contract proposed in this article can still ensure that the profit of the task publisher is nonnegative while increasing the profits of the worker nodes, which makes the transaction in this scenario more efficient.

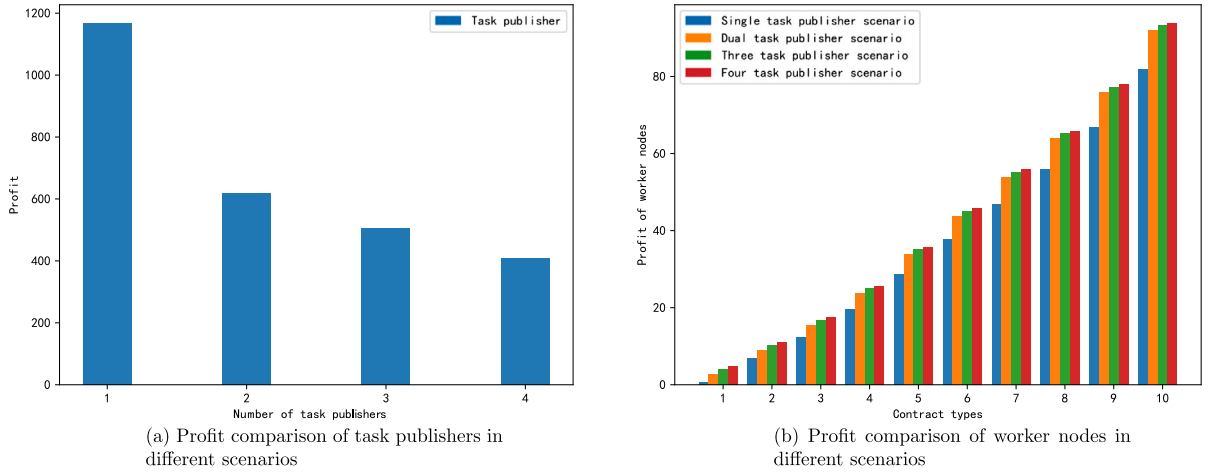


Fig. 5. Profit comparison. (a) Task publishers. (b) Worker nodes.

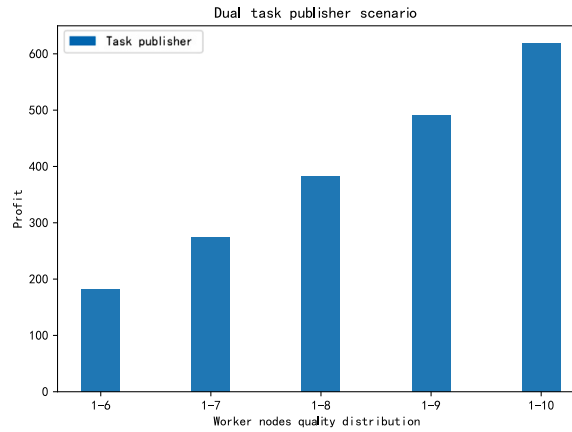


Fig. 6. The impact of worker nodes with different quality distributions on the profit of task publisher.

#### 4.2.3. The profits of different roles in the multi task publishers scenario

In this part of the experiment, we assume that the number of high-quality worker nodes participating in the federated learning system is limited, and calculate the changes in the overall profits of the task publisher when the types of the highest-quality worker nodes are different. In Fig. 6, the abscissa represents the quality distribution of the worker nodes (for example, 1-6 represents that the worker nodes of *type1* to *type6* are all participating in the federated learning task, and the other types of worker nodes do not participate), and the ordinate represents the overall profit of the publisher. As higher-quality worker nodes participate in the federated learning system, the overall profit of task publishers has also been significantly improved. Therefore, the task publisher should attract more high-quality worker nodes to join the federated learning system as much as possible, to obtain higher returns.

## 5. Conclusion

In this paper, we mainly study the incentive mechanism of worker nodes in the multi task publishers scenario in the federated learning system. By improving the optimal contract design in the single task publisher scenario, the optimal contract design in the multi task publishers scenario is solved. After experiments, the contract proposed in this paper can satisfy individual rational constraints and incentive compatibility constraints in the scenario of multi task publishers. In the meantime, the multi task publishers contract design can improve the utility of worker nodes compared with the single task publisher contract design, and achieve the purpose of incentivizing worker nodes to join the federated learning system.

### CRedit authorship contribution statement

**Shichang Xuan:** Conceptualization, Methodology, Writing – original draft. **Mengda Wang:** Methodology, Software, Writing – original draft. **Jingyi Zhang:** Formal analysis, Software, Writing – review & editing. **Wei Wang:** Data curation, Validation. **Dapeng Man:** Formal analysis, Resources, Validation. **Wu Yang:** Funding acquisition, Project administration, Resources, Supervision.

## Declaration of competing interest

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

## Data availability

The data that has been used is confidential.

## Acknowledgement

This work was supported by the National Natural Science Foundation of China (grant numbers U2003206 and U20B2048), the Defense Industrial Technology Development Program (grant number 2020604B004), and the Heilongjiang Provincial Natural Science Foundation of China (grant number LH2021F016).

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