



Research article

Federated learning and information sharing between competitors with different training effectiveness

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ABSTRACT

Federated Learning (FL) is an innovative technique that allows multiple firms to collaborate in training machine learning models while preserving data privacy. This is especially important in industries where data is sensitive or subject to regulations like the General Data Protection Regulation (GDPR). Despite its substantial benefits, the adoption of FL in competitive markets faces significant challenges, particularly due to concerns about training effectiveness and price competition. In practice, data from different firms may not be independently and identically distributed (non-IID) and heterogenous, which can lead to differences in model training effectiveness when aggregated through FL. This paper explores how initial product quality, data volume, and training effectiveness affect the formation of FL. We develop a theoretical model to analyze firms' decisions between adopting machine learning (ML) independently or collaborating through FL. Our results show that when the initial product quality is high, FL can never be formed. Moreover, when the initial product quality is low, and when data volume is low and firms' training effectiveness differences are small, FL is more likely to form. This is because the competition intensification effect is dominated by the market expansion effect of FL. However, when there is a significant difference in training effectiveness, firms are less likely to adopt FL due to concerns about competitive disadvantage (i.e., the market expansion effect is dominated by the competition intensification effect). This paper contributes to the literature on FL by addressing the strategic decisions firms face in competitive markets and providing insights into how FL designers and policymakers can encourage the formation of FL.

1. Introduction

Federated Learning (FL) is a decentralized machine learning technique that enables multiple parties to collaboratively train a shared model without sharing raw data. This innovative approach addresses two critical issues: data privacy and data silos. As firms and institutions increasingly rely on large volumes of sensitive data, concerns about data privacy, security, and compliance with regulations such as the General Data Protection Regulation (GDPR) have made traditional centralized data storage and analysis methods less viable. FL allows firms to retain their private data locally while still benefiting from the insights and advancements that

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come from pooling information and training models collectively. By enabling this kind of collaboration, FL offers a potential solution to data silos and the challenges posed by data privacy, which are particularly significant in industries like finance, healthcare, and autonomous driving (Kairouz et al., 2021).

Essentially, FL may exhibit different adoption patterns compared to other innovations, particularly those related to machine learning, due to its unique characteristics. Unlike traditional machine learning models, FL allows data to remain local, thus addressing privacy concerns and facilitating collaborative learning without centralizing data. This decentralized nature may appeal to organizations that are resource-constrained or face regulatory challenges around data privacy. Additionally, the collaborative aspect of FL can foster inter-firm cooperation, which may be distinct from the competitive dynamics typically associated with other machine learning innovations. These differences could influence the pace and extent of FL adoption across various industries.

Despite its substantial benefits, the adoption of FL in competitive markets faces significant challenges. One of the major concerns is the issue of training effectiveness (Wang et al., 2024a; Wen et al., 2023; Lai et al., 2022; Wei et al., 2020; Nilsson et al., 2018). Training effectiveness in the context of FL refers to how well the model trained using federated data (data distributed across multiple firms or devices) performs in terms of both accuracy and generalization. In FL, training effectiveness can be low due to several factors inherent in its decentralized nature. One of the main reasons is data heterogeneity (Mora et al., 2024; Yang et al., 2024; Vahidian et al., 2023; Ye et al., 2023; Qu et al., 2022; Li et al., 2020b), which means that the data distributed across different firms is often non-identical and non-independent (non-IID). Since each firm may collect data from different sources, regions, or market segments, the data can significantly differ in terms of quality, distribution, and representation of consumer behavior. This variation can hinder the model's ability to learn effectively from the combined data, as the training process assumes a certain level of uniformity in the data. As a result, when the data is highly heterogeneous, the global model aggregated from different firms' local models may fail to generalize well for all participants, leading to poor training effectiveness for some firms. Additionally, the communication and synchronization challenges in FL, where model updates are shared iteratively between firms and a central server, can introduce delays or inconsistencies in the learning process, further lowering the overall training effectiveness. Thus, these factors collectively contribute to the reduced training effectiveness in FL, making it harder to achieve optimal model performance across all participants.

Another significant issue in FL is the market competition among firms. Specifically, the presence of competitive incentives in markets can lead to the phenomenon of free-riding, where firms might benefit from FL without fully contributing their data. This is particularly problematic in competitive settings, where firms may be reluctant to share their data, fearing that doing so could inadvertently improve their competitors' models, leading to a loss of competitive advantage (Meng et al., 2024; Chen et al., 2024; Bi et al., 2024; Zhang et al., 2023; Karimireddy et al., 2022). These issues have hindered the widespread adoption of FL, especially in industries where competitive dynamics are intense.

The application of FL spans a wide range of industries. In the context of electric vehicles (EVs), for example, firms can collaborate to improve autonomous driving and smart charging technologies by pooling data related to vehicle behavior, traffic patterns, and geographical information. Similarly, in the financial sector, FL can enable banks and other financial institutions to share data for fraud detection, credit scoring, and risk analysis, while preserving customer privacy. However, the challenge remains: *how can firms be incentivized to participate in FL when they fear that, in a competitive environment, different training effectiveness could lead to unequal improvements in model-based products, potentially undermining their competitive position?*

This paper aims to address these challenges by exploring firms' decisions to improve model-based product quality through either Machine Learning (ML) or FL. Specifically, we investigate how initial product quality, data volume, and training effectiveness affect the formation of FL. We will begin by examining the case of an initially high-quality product, and then analyze the case of an initially low-quality product. Through this analysis, we aim to uncover the conditions under which FL is likely to be formed, and how FL affects firms' profits and consumer welfare.

Our paper's key findings provide insights into the dynamics of FL formation in competitive markets. First, we show that if the initial product quality is high, ML is always formed, while FL cannot be formed. This is because in such a market, firms are facing intense price competition, and improvements in product quality are crucial for gaining a market advantage. Since one firm may benefit more from FL while the other may not due to different training effectiveness, the latter firm that does not benefit from FL will prefer to use ML independently rather than participate in FL.

Second, our results show that if the initial product quality is low, firms are more likely to form FL when data volume is low, and their training effectiveness difference is small. This is because, with lower data volume, firms cannot achieve high-quality products through their own ML alone. In such cases, FL becomes more attractive as it allows both firms to pool their data, leading to improved product quality and more expanded market demand. However, we also find that FL could also introduce two opposing effects: a positive market expansion effect and a negative competition intensification effect. If the difference in training effectiveness between the firms is small, the market expansion effect dominates, and FL is formed. On the other hand, when there is a large difference in training effectiveness, the firm with better training effectiveness benefits more from FL, capturing more market share at the expense of the other firm. This intensifies competition and makes FL unprofitable for the firm with weaker training effectiveness, leading it to prefer ML instead. As a result, FL formation becomes less likely when there is a large difference in training effectiveness.

Third, our results also suggest that to facilitate the formation of FL, it is crucial to improve FL training effectiveness, ensuring that FL leads to significant product quality improvements. If FL does not enhance product quality enough, firms will have little incentive to participate. Furthermore, FL designers should prioritize selecting firms with more homogeneous data for collaboration, as this would also help ensure that the training effectiveness of FL is not too low (because data heterogeneity lowers FL's training effectiveness). Furthermore, FL designers should ensure the fairness of FL, ensuring that the training effectiveness differences between participants are not too large.

The structure of this paper is as follows: In Section 2, we provide a literature review, discussing relevant studies and theories. Section 3 presents the model description, detailing the assumptions and structure of the analysis. In Section 4, we conduct the

equilibrium analysis, exploring the conditions under which FL can form and its implications. Finally, Section 5 concludes the paper, summarizing the key findings and offering insights and policy recommendations for FL practitioners and policymakers.

2. Literature review

Federated learning (FL) is a decentralized machine learning approach that enables multiple participants to collaboratively train models without the need to share their raw data. In this framework, each participant trains a local model on its private dataset and then shares model updates with a central server for aggregation (McMahan et al., 2017; Konečný et al., 2017). This decentralized method ensures that sensitive data remains private and secure, as it is not exchanged between participants. Numerous studies have aimed to enhance FL algorithms by optimizing communication efficiency, improving model aggregation strategies, and reinforcing privacy-preserving mechanisms (McMahan et al., 2017; Geyer et al., 2018; Bonawitz et al., 2019; Li et al., 2020a; Tan et al., 2023; Zhang and Huang, 2024). For instance, the Federated Averaging (FedAvg) algorithm, proposed by McMahan et al. (2017), reduces the communication burden and strengthens privacy by averaging updates from local models.

FL addresses the challenge of regulatory compliance, particularly with strict data protection laws like the General Data Protection Regulation (GDPR) and the Health Insurance Portability and Accountability Act (HIPAA), by enabling collaborative model training without the need to centralize sensitive data (Peng et al., 2024; Mahon et al., 2024). This decentralized approach allows organizations to utilize the power of machine learning while keeping the data on local devices or servers, ensuring that personal information remains within the boundaries defined by regulatory frameworks. In industries such as healthcare and finance, where data privacy is paramount, FL offers a viable solution for adhering to compliance requirements while still benefiting from advanced data analytics and machine learning models.

Training effectiveness is a critical challenge in FL, as it directly impacts the ability of the global model to generalize and perform well across diverse clients (Mora et al., 2024; Yang et al., 2024; Vahidian et al., 2023; Ye et al., 2023; Qu et al., 2022; Li et al., 2020b). One of the primary factors undermining training effectiveness is data heterogeneity, which refers to the differences in data distributions across clients (Li and Lyu, 2024; Liu et al., 2024a; Zhang et al., 2024; Yang et al., 2019). When the data is non-IID (independent and identically distributed), the learning process becomes less effective. Misaligned updates during the model aggregation step, caused by varying data characteristics, slow down convergence or prevent the model from reaching an optimal solution. As a result, some clients may experience suboptimal training outcomes, leading to lower overall training effectiveness. Additionally, system heterogeneity, including differences in computational resources and network bandwidth, can further exacerbate these challenges, contributing to an uneven and unreliable training process. Together, these factors limit the global model's ability to generalize well, which diminishes the effectiveness of the entire federated learning process.

To address these challenges associated with low training effectiveness in FL, several solutions have been proposed (Wang et al., 2024c, 2024b; Liu et al., 2024b; Lu et al., 2024). Personalization techniques, such as fine-tuning the global model on each client's local data (e.g., Personalized FL) or employing meta-learning approaches like Model-Agnostic Meta-Learning (MAML), aim to adapt the model to the unique characteristics of each client's dataset. Additionally, clustering methods group clients with similar data distributions, allowing for more tailored model training within each cluster. To further address the impact of heterogeneous data, weighted aggregation and regularization techniques, like FedProx, adjust the influence of each client's update based on their data size and distribution (Li et al., 2020c). These methods help improve convergence, ensure fairness, and enhance the model's performance across all clients, regardless of their individual data characteristics.

However, existing research primarily focuses on FL in non-competitive environments, often overlooking the complex competitive behaviors between firms. In a competitive market, firms must balance the potential benefits of FL cooperation with the need to maintain their competitive advantage. This dynamic becomes particularly significant when the training effectiveness from FL differs across firms.

Our research attempts to address this gap by analyzing FL formation in a competitive market context, using an analytical model to capture the impact of training effectiveness differences on firms' strategic choices. Specifically, we examine conditions under which FL can be formed, focusing on factors like initial product quality, data volume, and training effectiveness. Our results reveal that when the initial product quality is high, FL is unlikely to form due to firms' competitive incentives. In contrast, when the initial product quality is low, FL formation is more feasible when data volume is low and the training effectiveness differences are small. However, as the training effectiveness differences increase, competitive disadvantages will discourage FL formation. This study provides novel insights into FL in competitive settings, offering guidance for FL design and policymakers to promote FL formation when market competition and training effectiveness differences exist.

3. Model description

Consider a competitive market with two firms, denoted as firm 1 and firm 2, each offering a horizontally differentiated model-based product with zero production cost.

Consumers' strategy. Following the classical Hotelling model framework, consumers are uniformly distributed along a linear city represented by the interval $[0, 1]$, with unit density. Each consumer decides among three options—purchasing from firm 1, purchasing from firm 2, or not purchasing at all. Let each consumer be indexed by the parameter $x \in [0, 1]$, where x represents the consumer's location on the interval. Consumers incur a transportation cost proportional to the distance between their location and the firm from which they purchase. Given that firm 1 is fixed at the left side (location 0), and firm 2 is fixed at the right side (location 1), the utility for a consumer located at x purchasing from firm i (where $i \in \{1, 2\}$) is given by:

$$\begin{cases} u_1 = v_1 - p_1 - t(x - 0), \\ u_2 = v_2 - p_2 - t(1 - x), \end{cases} \quad (1)$$

where v_i represents the product quality, p_i denotes the price of product i , and t is a positive constant representing the transportation cost per unit of distance. We assume that the two firms are initially symmetric, with identical product quality, that is, $v_1 = v_2 = v$.

Firms' strategy. Assume each firm has a dataset with an information quantity of \mathcal{D} , where the data of the two firms could be non-IID and heterogeneous. The firms can enhance product quality through either individual machine learning or collaborative federated learning. Specifically, each firm has two strategies that can be used to improve the quality of its model-based product (v_i): (1) using ML training individually with its own data, or (2) forming FL with pooling information from both firms.

For ML, we use a linear function $f(x) = \alpha x$ to represent the training effectiveness on product improvement for both firms. Here, x denotes the amount of information, and α represents the training effectiveness of unit information. The more information available (x) or the higher the ML training effectiveness (α), the higher the improvement in product quality.

For FL, we use linear functions $g_1(x) = \beta_1 x$ and $g_2(x) = \beta_2 x$ to represent the training effectiveness on product improvement for firm 1 and firm 2 respectively. Here, β_1 and β_2 represent the training effectiveness of unit information for firm 1 and firm 2 respectively. It is reasonable to assume that the training effectiveness of the two firms is different ($\beta_1 \neq \beta_2$). It is because that, under FL, the two firms share a global model, but due to data heterogeneity, this global model will have different training effectiveness for each firm. In addition, since data cannot be centralized and are non-IID, the training effectiveness of FL is lower than ML (Yang et al., 2019). Thus, without loss of generality, we assume that $0 < \beta_2 < \beta_1 < \alpha$. We also ignore firms' free-riding behavior (where the firm only shares partial information) (Meng et al., 2024), and assume that in FL, each firm shares the total \mathcal{D} amount of information.

In the following text, we use superscript ml (fl) to represent the case with ML training (FL collaboration). Specifically, when firms use ML training, the consumer's utility in Equation (1) becomes:

$$\begin{cases} u_1 = v_1^{ml} - p_1 - t(x - 0), \\ u_2 = v_2^{ml} - p_2 - t(1 - x), \end{cases} \quad (2)$$

where v_i^{ml} is the updated quality of model-based product i under ML and is equal to $v_i + \alpha\mathcal{D}$ and the second term $\alpha\mathcal{D}$ represents the improved level of quality of product i under ML with firm i 's own data \mathcal{D} . Alternatively, when firms use FL collaboration, the consumer's utility in Equation (1) becomes:

$$\begin{cases} u_1 = v_1^{fl} - p_1 - t(x - 0), \\ u_2 = v_2^{fl} - p_2 - t(1 - x), \end{cases} \quad (3)$$

where v_i^{fl} is the updated quality of product i under FL and is equal to $v_i + 2\beta_i\mathcal{D}$ and the second term $2\beta_i\mathcal{D}$ represents the improved level of quality of product i under FL collaboration. Assume that each firm is rational and chooses the strategy that generates a higher (positive) profit. Notably, if at least one firm decides not to form FL, the situation degenerates to each firm conducting individual ML training on its own data. In other words, FL can only be formed when both firms find it profitable.

The timing of the game is as follows (see Figure 1): First, each firm decides whether to form FL: if both firms opt for FL, they share information with each other, and the total amount of information is $2\mathcal{D}$; otherwise, each firm uses ML training with its own data, with an information amount of \mathcal{D} . Second, each firm simultaneously decides its optimal price. Last, demands are realized and profits are obtained by each firm. We use backward induction to solve the game.

4. Equilibrium analysis

In this section, we will explore firms' decisions to improve product quality through either ML or FL. Specifically, we will investigate the impact of initial product quality (v), data volume (\mathcal{D}), and training effectiveness (β_1 and β_2) on the formation of FL. We will begin by examining the case where the initial product quality is relatively high, followed by an analysis of the case where the initial product quality is relatively low. Finally, we will discuss the issue of consumer welfare.

4.1. High initial product quality

In this subsection, we first explore the case in which the initial product quality is sufficiently high so that initially, there exists an indifferent consumer, with consumers to the left of this point purchasing from firm 1 and those to the right purchasing from firm 2,

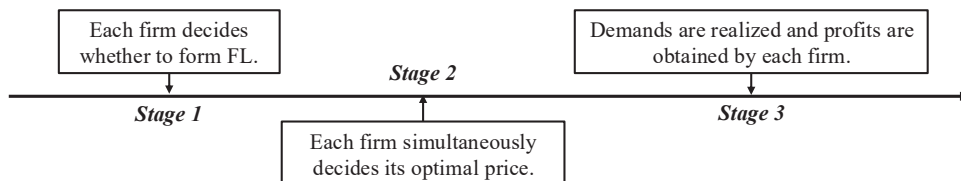


Fig. 1. Three-stage game.

with no consumers choosing not to purchase. That is, in this case, there is intense price competition between the two firms. When both firms improve their quality by the same amount, the product quality difference will remain unaltered, and thus their pricing will remain unchanged. The following proposition highlights the firms' choices between ML and FL.

Proposition 1. If the initial product quality is sufficiently high ($v \geq \frac{3}{2}t$), then FL cannot be formed.

Proof. Based on Equation (1), we know that the location of the indifferent consumer (\hat{x}) must satisfy $u_1(\hat{x}) = u_2(\hat{x})$. Therefore, given that $v_1 = v_2 = v$, we have

$$\hat{x} = \frac{1}{2} - \frac{p_1 - p_2}{2t}.$$

Thus, firm 1's profit is given by $\pi_1 = p_1\hat{x}$, and firm 2's profit is given by $\pi_2 = p_2(1 - \hat{x})$. Solving $\frac{\partial \pi_1}{\partial p_1} = 0$ and $\frac{\partial \pi_2}{\partial p_2} = 0$, and verifying $\frac{\partial^2 \pi_1}{\partial p_1^2} < 0$ and $\frac{\partial^2 \pi_2}{\partial p_2^2} < 0$, yields the equilibrium price

$$p_1^* = p_2^* = t.$$

Thus, it is easy to verify that $\hat{x} = \frac{1}{2}$. To ensure that the utility of the indifferent consumer is no less than zero, we have $u_1 = u_2 = v - t - \frac{1}{2}t \geq 0$. Thus, we obtain $v \geq \frac{3}{2}t$.

Next, we begin to prove why FL cannot be formed. Through a similar solution as above, we can derive that under ML, the equilibrium profits of both firms are $\pi_1^{ml} = \frac{(3t + v_1^{ml} - v_2^{ml})^2}{18t}$ and $\pi_2^{ml} = \frac{(3t + v_2^{ml} - v_1^{ml})^2}{18t}$; and under FL, the equilibrium profits of both firms are $\pi_1^f = \frac{(3t + v_1^f - v_2^f)^2}{18t}$ and $\pi_2^f = \frac{(3t + v_2^f - v_1^f)^2}{18t}$. Because $v_1^{ml} = v_2^{ml} = v + \alpha\mathcal{D}$, and $v_1^f = v + 2\beta_1\mathcal{D} > v_2^f = v + 2\beta_2\mathcal{D}$, one can readily verify that $\pi_1^f > \pi_1^{ml}$ and $\pi_2^f < \pi_2^{ml}$. Consequently, firm 2 always finds it unprofitable to form FL. As a result, FL cannot be formed. \square

Proposition 1 reflects the impact of firms' initial product quality on the formation of FL. Specifically, when the product quality of the firms is already sufficiently high, the firms have no incentive to form an FL. The underlying reason is that, at this point, their quality is already high enough to capture sufficient market demand, thus weakening their motivation to further improve the product through FL.

From **Proposition 1**, we can observe that when the initial product quality is high ($v \geq \frac{3}{2}t$), the market is fully covered. Therefore, **Proposition 1** can also be interpreted as follows: at this point, firms engage in intense price competition to capture market demand, and the firm with higher product quality will gain the advantage in this competition. In other words, in this case, a firm's profit depends not only on its own product quality (v_i) but also on the quality of its competitor's product (v_{-i}). As long as the product quality difference ($v_i - v_{-i}$) remains unaltered, the profits of both firms will remain unchanged. Therefore, under FL, because firm 1's training effectiveness (β_1) is better than that of firm 2 (β_2), firm 1 always finds forming FL profitable, while firm 2 is reluctant to form FL as it would reduce its profit. Ultimately, since Firm 2 is unwilling to form FL, ML is formed in equilibrium.

4.2. Low Initial Product Quality

In this subsection, we explore the case where the initial product quality is low so that initially, the left-hand consumers purchase from firm 1, the right-hand consumers purchase from firm 2, and there may exist some consumers in the middle who do not purchase anything.¹ That is, in this case, there is no intense price competition between the firms, and both firms aim to improve their product quality through ML or FL to increase their price or expand market demand. Unlike the previous subsection, in this case, when the two firms improve their quality by the same amount, their pricing can increase. Referring back to **Proposition 1**, this proposition indicates that if the initial product quality is sufficiently high, ML is always formed, regardless of data volume (\mathcal{D}) and training effectiveness (β_1 and β_2). However, this subsection will show that if the product quality is low initially, the data volume and training effectiveness will have an important impact on the firms' choices between ML and FL. The following proposition summarizes the results when the data volume is high.

Proposition 2. If the initial product quality is low ($0 < v < \frac{3}{2}t$) and the data volume is high ($\mathcal{D} \geq \frac{3t - 2v}{2\alpha}$), then FL cannot be formed.

Proof. If the initial product quality is low ($0 < v < \frac{3}{2}t$), we first derive the conditions under which the post-training quality through ML reaches sufficiently high. Based on Equation 2, we know that if the post-training quality is high enough after ML, the location of the indifferent consumer (\hat{x}) must satisfy $u_1(\hat{x}) = u_2(\hat{x})$. Therefore, given that $v_1^{ml} = v_2^{ml} = v + \alpha\mathcal{D}$, we have

$$\hat{x} = \frac{1}{2} - \frac{p_1 - p_2}{2t}.$$

Thus, firm 1's profit is given by $\pi_1 = p_1\hat{x}$, and firm 2's profit is given by $\pi_2 = p_2(1 - \hat{x})$. Solving $\frac{\partial \pi_1}{\partial p_1} = 0$ and $\frac{\partial \pi_2}{\partial p_2} = 0$, and verifying $\frac{\partial^2 \pi_1}{\partial p_1^2} < 0$ and $\frac{\partial^2 \pi_2}{\partial p_2^2} < 0$, yields the equilibrium price

$$p_1^* = p_2^* = t.$$

¹ Specifically, when there are consumers in the middle who choose not to purchase the product, the market is not fully covered. When there are no such consumers, the market will be in what Economides (1984) calls a "touching equilibrium".

Thus, it is easy to verify that $\hat{x} = \frac{1}{2}$. To ensure that the utility of the indifferent consumer is no less than zero, we have $u_1 = u_2 = v + \alpha\mathcal{D} - t - \frac{1}{2}t \geq 0$. Thus, we obtain $\mathcal{D} \geq \frac{3t-2v}{2\alpha}$. This implies that when the data volume is high ($\mathcal{D} \geq \frac{3t-2v}{2\alpha}$), the high post-training quality can be achieved through ML.

Next, we begin to prove why FL cannot be formed. Through a similar solution as above, we can derive that under ML, the equilibrium profits of both firms are $\pi_1^{ml} = \frac{(3t+v_1^{ml}-v_2^{ml})^2}{18t}$ and $\pi_2^{ml} = \frac{(3t+v_2^{ml}-v_1^{ml})^2}{18t}$. And under FL, (1) if the post-training quality is high enough (i.e., $v_1^f + v_2^f \geq 3t$), then the equilibrium profits of both firms are $\pi_1^f = \frac{(3t+v_1^f-v_2^f)^2}{18t}$ and $\pi_2^f = \frac{(3t+v_2^f-v_1^f)^2}{18t}$. Because $v_1^{ml} = v_2^{ml} = v + \alpha\mathcal{D}$, and $v_1^f = v + 2\beta_1\mathcal{D} > v_2^f = v + 2\beta_2\mathcal{D}$, one can readily verify that $\pi_1^f > \pi_1^{ml}$ and $\pi_2^f < \pi_2^{ml}$; (2) if the post-training quality is not high enough ($v_1^f + v_2^f < 3t$), one can readily verify that $\pi_1^f > \pi_1^{ml}$ and $\pi_2^f > \pi_2^{ml}$ cannot hold together. Consequently, at least one firm will find it unprofitable to form FL. As a result, FL cannot be formed. \square

Proposition 2 shows that if the initial product quality is low and the data volume is high, ML is always formed, whereas FL cannot be formed. This is primarily because, with a sufficiently large data volume, each firm can significantly improve its product through its own ML. Specifically, when the data volume is high enough ($\mathcal{D} \geq \frac{3t-2v}{2\alpha}$), both firms can train their models using their own data, which leads to higher demand or price. Furthermore, since both firms have equal initial product quality ($v_1 = v_2 = v$) and ML training effectiveness (α), the market will be split evenly between the two firms in equilibrium (i.e., each firm owns a market share of 1/2).

However, when the data volume is high enough, FL training may lead to two scenarios. First, if FL results in high post-training quality, firm 1 will gain a larger share of the market and pricing power due to its better training effectiveness ($\beta_1 > \beta_2$), while firm 2 will capture a smaller share and pricing power. In this case, firm 2 will find FL unprofitable. Therefore, FL cannot be formed in this situation. Second, if high post-training quality still cannot be achieved through FL (e.g., when β_1 and β_2 are very small), then at least one firm will have a market share smaller than 1/2, meaning that at least one firm will find FL unprofitable. Thus, in this case as well, FL cannot be formed.

Essentially, **Proposition 2** indicates that when a firm has enough data (i.e., the firm does not have a shortage of data), it can train a good model and improve its product quality through its own ML. Therefore, the firm has no incentive to improve product quality through FL. The above discussion shows that when data volume is high ($\mathcal{D} \geq \frac{3t-2v}{2\alpha}$), ML is always formed, while FL cannot be formed. Next, we will examine the strategic choices of firms between ML and FL when data volume is low ($0 < \mathcal{D} < \frac{3t-2v}{2\alpha}$), summarized in **Proposition 3**.

Proposition 3. If the initial product quality is low ($0 < v < \frac{3}{2}t$) and the data volume is low ($0 < \mathcal{D} < \frac{3t-2v}{2\alpha}$), then we have (see appendix for $\delta_1, \delta_2, \delta_3$, and δ_4):

- (a) If $0 < \beta_2 \leq \frac{\alpha}{2}$, then FL cannot be formed.
- (b) If $\beta_1 > \beta_2 > \frac{\alpha}{2}$, then FL can be formed when $\beta_1 - \beta_2 < \min(\delta_1, \delta_2, \delta_3, \delta_4)$; otherwise, ML is formed.

Proof. The detailed proof can be found in the appendix. Below is the outline of the proof. Since the data volume is low ($0 < \mathcal{D} < \frac{3t-2v}{2\alpha}$), high post-training quality cannot be achieved through ML. For low-quality products, equilibrium can take two possible forms: (1) a partially covered market or (2) touching equilibrium (see [Economides 1984](#) for more details). A partially covered market represents a market that is not fully covered, while touching equilibrium represents a fully covered market where the pricing can still increase even if both firms improve their product quality by the same amount. Thus, after ML training, there are two types of equilibrium scenarios. After FL training, there are three types of equilibrium scenarios: (1) a partially covered market, (2) touching equilibrium, and (3) a fully covered market. Therefore, there are 6 (2×3) cases we need to compare.

Additionally, we need to discuss two scenarios for the training effectiveness: (1) If $0 < \beta_2 \leq \frac{\alpha}{2}$, then FL leads to a lower improvement in firm 2's quality compared to ML ($2\beta_2\mathcal{D} \leq \alpha\mathcal{D}$), meaning firm 2 definitely will not participate in FL because the training effectiveness is too low. This corresponds to part (a) of **Proposition 3**. (2) If $\beta_1 > \beta_2 > \frac{\alpha}{2}$, then FL leads to a higher improvement in both firms' quality compared to ML ($2\beta_1\mathcal{D} > \alpha\mathcal{D}$, and $2\beta_2\mathcal{D} > \alpha\mathcal{D}$), meaning both firms might be willing to participate in FL because the training effectiveness is sufficiently high. This corresponds to part (b) of **Proposition 3**. For both scenarios, we must consider the above 6 cases we need to compare. By solving these cases, we can derive the results in **Proposition 3**. \square

This proposition conveys several key points. First, **Proposition 3** (a) illustrates that under FL, if firm 2's training effectiveness is low ($0 < \beta_2 \leq \frac{\alpha}{2}$), then it will find FL unprofitable, resulting in FL not being formed. Intuitively, if FL does not provide better training outcomes than ML, firm 2 not only fails to achieve higher product quality through FL but also inadvertently helps its competitor, firm 1, in developing a superior product. These two negative effects together make firm 2 unwilling to form FL.

The most interesting case occurs in part (b). **Proposition 3** (b) indicates that when both firm 1 and firm 2 have relatively high training effectiveness ($\beta_1 > \beta_2 > \frac{\alpha}{2}$), whether FL forms depends on the training effectiveness of both firms (β_1 and β_2). Specifically, if the difference in training effectiveness between the two firms is very small ($\beta_1 - \beta_2 < \min(\delta_1, \delta_2, \delta_3, \delta_4)$), then FL will be formed; otherwise, ML will be formed.

The rationales are as follows. FL has two opposing effects: a positive market expansion effect and a negative competition intensification effect. If firm 1 and firm 2 have a low difference in training effectiveness, the market expansion effect dominates the competition intensification effect for both firms, leading to FL formation. However, if there is a large difference in training effectiveness, then firm 1's product quality would improve significantly, allowing it to capture more market share at the expense of firm 2. In this case, for firm 2, the competition intensification effect dominates the market expansion effect, making FL unprofitable for firm 2, and thus ML forms.

The above discussion also suggests that, to facilitate the formation of FL, FL designers should focus on improving FL's training effectiveness through technological advancements such as Personalized Federated Learning (to ensure that $\beta_1 > \beta_2 > \frac{\alpha}{2}$). Intuitively, if FL does not significantly improve product quality, no firm will be willing to participate in FL. Moreover, FL designers should ideally select firms with relatively high data homogeneity for FL cooperation, as this would also help ensure that the training effectiveness of FL is not too low (because data heterogeneity reduces FL's training effectiveness). Furthermore, FL designers should ensure the fairness of FL, ensuring that the training effectiveness differences between participants are not too large (i.e., $\beta_1 - \beta_2 < \min(\delta_1, \delta_2, \delta_3, \delta_4)$).

4.3. Consumer Welfare

In this subsection, we focus on how initial product quality (v), data volume (\mathcal{D}), and training effectiveness (β_1 and β_2) impact consumer welfare. We analyze scenarios where the initial product quality is low ($0 < v < \frac{3}{2}t$), the data volume is low ($0 < \mathcal{D} < \frac{3t-2v}{2\alpha}$), and FL has good training effectiveness ($\beta_1 > \beta_2 > \frac{\alpha}{2}$).² From the prior analysis, we know that FL can form if the difference in training effectiveness between the two firms is very small ($\beta_1 - \beta_2 < \min(\delta_1, \delta_2, \delta_3, \delta_4)$). Therefore, we can conclude that under these conditions, FL enhances product quality and expands the market, and these two positive effects are undeniably beneficial for consumers.

However, if there is a significant difference in training effectiveness between two firms ($\beta_1 - \beta_2 \geq \min(\delta_1, \delta_2, \delta_3, \delta_4)$), then FL, while beneficial for consumers (if FL were formed), actually cannot be formed due to the competition intensification effect dominating market expansion effect. Therefore, in this case, consumers actually cannot benefit from FL. Overall, these insights suggest that the formation of FL can enhance consumer welfare by improving product quality and expanding the market. Therefore, FL designers and policymakers should seek to promote FL formation.

5. Extension

In this section, we will extend the baseline model and demonstrate the robustness of our results. Specifically, we will investigate the situation of convex transportation cost, concave product improvement, and privacy risk cost.

5.1. Convex transportation cost

First, we consider the case where consumers' transportation cost is convex, in which case Equation (1) becomes:

$$\begin{cases} u_1 = v_1 - p_1 - \frac{1}{2}t(x-0)^2, \\ u_2 = v_2 - p_2 - \frac{1}{2}t(1-x)^2, \end{cases} \quad (4)$$

In this case, through the same analysis as before, we can easily obtain results similar to those of the main model. The primary reason lies in the fact that whether FL can be established between firms mainly depends on the initial product quality (v), data volume (\mathcal{D}), and the difference in training effectiveness between the two firms (β_1 and β_2). The form of the transportation cost does not affect the qualitative conclusions.

5.2. Concave product improvement

Second, we assume that the product improvement function is concave, indicating diminishing marginal improvement in the product's quality. That is, $f(x)$, $g_1(x)$, and $g_2(x)$ in main model becomes concave. For example, $f(x) = \alpha\sqrt{x}$, $g_1(x) = \beta_1\sqrt{x}$, and $g_2(x) = \beta_2\sqrt{x}$.

In this case, applying the same analysis as before, we can easily derive results similar to those in the main model. The key reason is that the formation of FL between firms primarily depends on the initial product quality, the data volume, and the difference in training effectiveness between the two firms. The specific form of the product improvement function does not alter the qualitative conclusions. Furthermore, when the product improvement function is concave, we find that FL becomes more difficult to establish. This is because, under a concave product improvement function, although FL increases the amount of data used to improve model performance, the improvement in model performance is smaller compared to a linear function. As a result, both firms are less willing to establish FL.

5.3. Privacy risk cost

Third, because FL still has the risk of privacy leaks, we incorporate the cost of privacy risk into the model. Therefore, under ML, since the data is trained locally, the profits of both firms are given by (taking the full market coverage case as an example, \hat{x} is the location of the indifferent consumer)

² Other cases can be analyzed in a similar manner.

$$\begin{cases} \pi_1 = p_1 \hat{x}, \\ \pi_2 = p_2 (1 - \hat{x}), \end{cases} \quad (5)$$

However, under FL, since the data is trained globally and has the risk of privacy leaks, the profits of both firms are given by

$$\begin{cases} \pi_1 = p_1 \hat{x} - c\mathcal{D}, \\ \pi_2 = p_2 (1 - \hat{x}) - c\mathcal{D}, \end{cases} \quad (6)$$

where c is a weight parameter that captures how sensitive a player is to the cost of contributing. Unlike the concave utility function, we represent the cost as a linear function of \mathcal{D} , since the privacy risk rises in direct proportion to the amount of information shared.³

In this case, we find that FL becomes more difficult to establish. This is because, when considering privacy costs, although FL may lead to improvements in the product and higher market demand, the associated privacy costs must be borne. As a result, both firms are less willing to form FL.

6. Conclusion

In this paper, we examine the factors influencing the formation of Federated Learning (FL) in competitive markets, focusing on the interplay between initial product quality, data volume, and training effectiveness. Our results show that while FL offers significant potential for improving model-based product quality and expanding market demand, its adoption is hindered by concerns over the competitive dynamics and training effectiveness differences between firms.

Our analysis reveals that FL is more likely to form when the data volume is low and firms' training effectiveness differences are small. However, when there is a significant difference in training effectiveness, firms are less likely to participate in FL due to concerns of competitive disadvantage. Specifically, firms with weaker training effectiveness may perceive FL as disproportionately benefiting their competitors, leading them to opt for ML instead.

We also discussed the implications of FL formation for consumer welfare, highlighting that FL can enhance product quality and market demand, which ultimately benefits consumers, but only under specific conditions. In light of these findings, we suggest that FL designers should aim to improve training effectiveness and encourage collaboration among firms with more homogeneous data. Additionally, they must ensure fairness by keeping the training effectiveness differences between participants small.

In addition, we explored three important extensions that impact the dynamics of FL adoption. First, we considered the case of convex transportation costs, which affects consumer behavior and can influence market coverage. Second, we examined the impact of a concave product improvement function, which suggests diminishing returns to product quality improvements through FL, making firms less willing to adopt it. Finally, we incorporated the cost of privacy risks into the model, highlighting that while FL may improve product offerings and market demand, the associated privacy concerns discourage firms from fully embracing this technology. These extensions provide a more comprehensive understanding of the factors influencing FL adoption and its potential limitations.

This study's model has certain limitations that open avenues for future research. First, it focuses on the interaction between two firms, and extending the model to include competition among multiple firms would offer deeper insights into the dynamics of FL in more complex market structures. Second, the model does not account for potential data poisoning attacks, where one firm could intentionally corrupt shared data to gain an advantage or disrupt the collaborative learning process. Incorporating data poisoning into the analysis would provide a more realistic view of the challenges that may arise in federated settings. Furthermore, the strategic location choice of firms could also be considered. In this paper, we assume firms are located at the two ends, and it would be interesting to explore whether FL influences the firms' strategic location decisions. These extensions would enhance the model's applicability and robustness, contributing to a more comprehensive understanding of FL adoption.

Declaration of Competing Interest

All the authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at [doi:10.1016/j.ject.2024.12.003](https://doi.org/10.1016/j.ject.2024.12.003).

³ For example, if the firm is required to compensate victims of a data breach, the total compensation will be a linear function of the number of victims.

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