Written assessment, February, 6, 2025

Last name, First name ______________________________

Exercise 1 (value 9)

A company must plan the production of a single type of good for the next T periods. The demand d_t of the product for each period $t \in T$ is kwown. The company can produce up to P units in each period. Units not sold in a period can be stored in a depot and used in the next periods. The depot is initially empty. The cost for storing units is negligible, while the production cost of each unit in a period is c_s if the production of the period is smaller than or equal to ϑ units and $c_l > c_s$ if the production is larger than ϑ units. Write a Mixed Integer Linear Program to plan the production in each period, together with the quantities to be stored. (Hint: use two variables to represent the production of a period, one for productions up to ϑ units, productions higher than ϑ units). PRODUCTS$\Longrightarrow$UNITS

Exercise 2 (value 8)

Consider the following linear program:

$$
\begin{aligned}
\max\ & x_1 + x_2 \\
s.t.\ & 2x_1 \le 5 \\
& x_2 \le 3 \\
& x_1, x_2 \ge 0 \text{ integer}
\end{aligned}
$$

- Relax the integrality constraints, draw the feasible region and solve the problem graphically (report the solution and its value)
- Write the tableau corresponding to the optimal solution in basic form
- Find a Gomory's cut and add it to the tableau, compute and report the new feasible solution
- Is the new solution optimal for the given problem? Motivate your answer

Exercise 3 (value 8)

Consider the following ILP:

$$
\begin{aligned}
\max\ & 2x_1 + x_2 \\
s.t.\ & x_1 - x_2 \ge 0 \\
& 2x_1 + 3x_2 \le 35 \\
& x_1 - 2x_2 \le 0 \\
& x_1 \le 9 \\
& x_1, x_2 \ge 0 \text{ integer}
\end{aligned}
$$

Solve the problem using the standard branch-and-bound algorithm. Solve all the relaxed problems encountered via the graphical method. (Explore at most five nodes of the tree)

Written assessment, January 16, 2025

Solution sketch

Exercise 1

Variables

xs_t = units produced in period t, if production $\leq \vartheta$ units

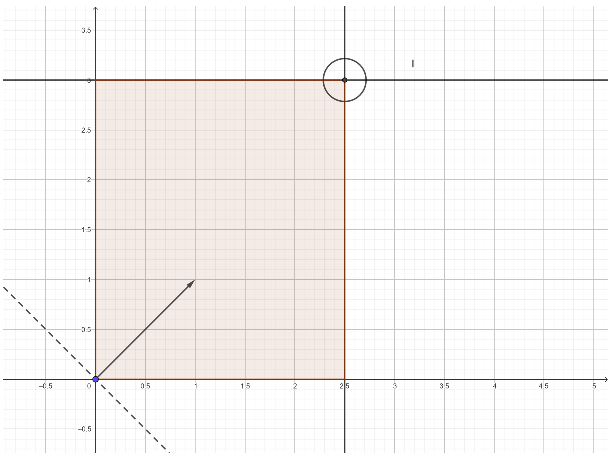
xl_t = units produced in period t, if production $\geq \vartheta$ units

I_t = units stored in the depot at the end of period t

$\delta_t = 1$ if at most ϑ units are produced in period t; 0 otherwise

$$
\begin{aligned}
\min\ & c_s \sum_{t=1}^{T} xs_t + c_l \sum_{t=1}^{T} xl_t \\
& xs_1 + xl_1 = I_1 + d_1 \\
& xs_t + xl_t + I_{t-1} = I_t + d_t && t = 2, \ldots, T \\
& xs_t \leq \vartheta \delta_t && t = 1, \ldots, T \\
& xl_t \leq P(1 - \delta_t) && t = 1, \ldots, T \\
& xs_t \geq 0, \text{integer} && t = 1, \ldots, T \\
& xl_t \geq 0, \text{integer} && t = 1, \ldots, T \\
& I_t \geq 0, \text{integer} && t = 1, \ldots, T \\
& \delta_t \in \{0, 1\} && t = 1, \ldots, T
\end{aligned}
$$

Exercise 2



Optimal solution : $x = (5/2, 3, 0, 0)$ $z = 11/2$

x_1	x_2	s_1	s_2		
0	0	-1/2	-1	-11/2	$-z$
1	0	1/2	0	5/2	
0	1	0	1	3	

Gomory's cut:
$1/2 s_1 \geq 1/2$

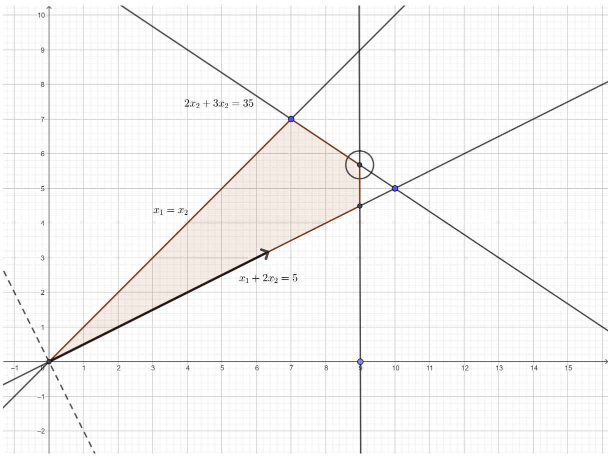
x_1	x_2	s_1	s_2	s_3		
0	0	-1/2	-1	0	-11/2	$-z$
1	0	1/2	0	0	5/2	
0	1	0	1	0	3	
0	0	**-1/2**	0	1	-1/2	

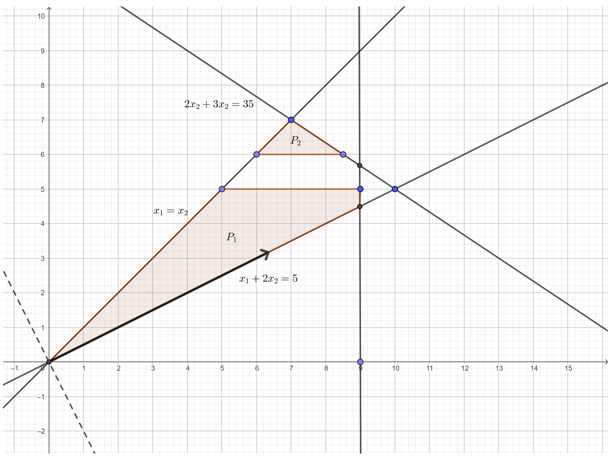
x_1	x_2	s_1	s_2	s_3		
0	0	0	-1	-1	-5	$-z$
1	0	0	0	1	2	
0	1	0	1	0	3	
0	0	1	0	-2	1	

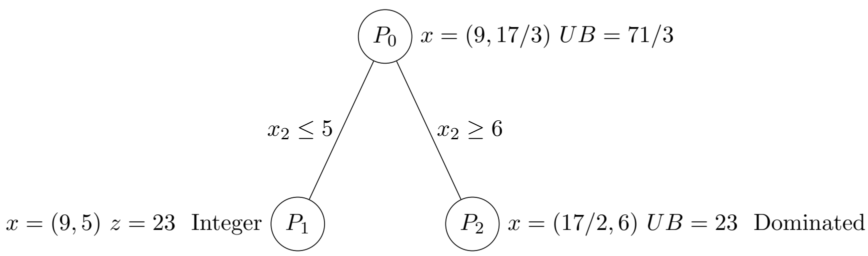
New solution : $x = (2, 3, 1, 0, 0), z = 5$.

It is the optimal solution of the original problem since it is optimal for the relaxed problem (non-positive reduced costs) and it is feasible (integer)

Exercise 3







The optimal solution is in node P_1 with $x = (9, 5)$ and value 23.