Written assessment, September 2, 2024

Last name, First name ______________________________

Exercise 1 (value 9)

A factory has available B units of raw material and has to choose the production mix. A unit of product a consumes r_a units of raw material while a unit of product b consumes r_b units of raw material. The number of units of each product must be integer. The profits for one unit of a and b are p_a and p_b, respectively. For technical reasons, it is possible to produce b only if at least A units of product a are produced. Moreover, the ratio between the unit of b produced and the units of a produced has to be below α/β due to some market forecast. Write an integer linear program to help the factory maximize the profit.

Exercise 2 (value 8)

Consider the following tableau of a minimization problem:

x_1	x_2	x_3	x_4	$-z$
0	0	5	4	-7
1	0	7/2	3/7	3/2
0	1	1	-1/2	2/5

- Write down the current solution and its cost.
- Is the current solution optimal? Why?
- Which are the basic variables?
- Derive the Gomory's cut for the last row and insert it in the tableau without performing a pivot operation.
- Is the solution in the new tableau primal feasible? Is it dual feasible? Explain.

Exercise 3 (value 9)

Consider the following minimization problem in tableau form:

x_1	x_2	x_3	x_4	$-z$
2	-3	0	0	-3
5	-2	0	1	4
3	-1	1	0	2

- Solve the problem with the simplex method and report the solution.
- Add the following constraint to the tableau obtained in the previous step and take it into a basic form:
  $2x_1 + 2x_2 = 1$
- Continue the optimization with the new constraint; report the optimal solution and its cost.

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Solution sketch

Exercise 1

$x_a, x_b =$ units produced for products a and b, respectively
$\delta = 1$ if at least A units of product a are produced, 0 otherwise
$M =$ large number.

$$
\begin{aligned}
\max\ z = p_a x_a + p_b x_b& \\
r_a x_a + r_b x_b &\leq B \\
x_a &\geq A\delta \\
x_b &\leq M\delta \\
\beta x_b &\leq \alpha x_a \\
x_a, x_b &\geq 0, integer \\
\delta &\in \{0,1\}.
\end{aligned}
$$

Exercise 2

- The current solution is $x = (3/2, 2/5, 0, 0)$ and its cost is 7.
- The current solution is optimal since there are no negative reduced costs and it is a minimization.
- The current basic variables are x_1 and x_2.
- The Gomory's cut is $1/2x_4 \geq 2/5$. The new tableau is:

x_1	x_2	x_3	x_4	x_5	$-z$
0	0	5	4	0	-7
1	0	7/2	3/7	0	3/2
0	1	1	-1/2	0	2/5
0	0	0	-1/2	1	-2/5

- The new tableau is dual-feasible since all the reduced costs are greater than or equal to 0. It is however not primal-feasible since there are variables with a negative value (x_5).

Exercise 3

- Variable x_2 has negative reduced costs and can enter the basis with any value, so the problem is unbounded ($-\infty$).

- The tableau with the extra constraint is:

x_1	x_2	x_3	x_4	$-z$
2	-3	0	0	-3
5	-2	0	1	4
3	-1	1	0	2
2	**2**	0	0	1

which becomes:

x_1	x_2	x_3	x_4	$-z$
5	0	0	0	-3/2
7	0	0	1	5
4	0	1	0	5/2
1	1	0	0	1/2

alternatively :

x_1	x_2	x_3	x_4	$-z$
0	-5	0	0	-4
0	-7	0	1	3/2
0	-4	1	0	1/2
1	**1**	0	0	1/2

x_1	x_2	x_3	x_4	$-z$
5	0	0	0	-3/2
7	0	0	1	5
4	0	1	0	5/2
1	1	0	0	1/2

- The optimal solution is $x = (0, 1/2, 5/2, 5)$ with cost $3/2$.