Chapter 2: Finite State Automata

Finite Automata with Output

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Informatics Study Program - UNRAM

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Course Outline

Finite State Transducer

Also said as "Automata with Output"
It has similar things to DFA as:

- The finite set Q of states
- Input alphabet Σ
- Output alphabet Γ
- No finale states

The translation of inputs to outputs is written as:

$$F_M : D \to R$$

where: F_M is a function represented by M
D is a subset of Σ
R is a subset of Γ

Mealy Machines

Definition

Mealy machine $M = (Q, \Sigma, \Gamma, \delta, \theta, q_0)$
where

Q is finite set of internal states,
Σ is the input alphabet,
Γ is the ouput alphabet,
$\delta : Q \times \Sigma \to Q$ is the transition function,
$\theta : Q \times \Sigma \to \Gamma$ is the ouput function.
$q_0 \in Q$ is the initial state.

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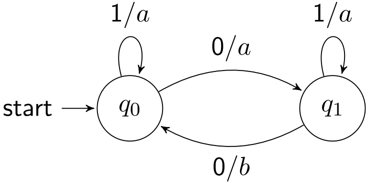
Example:



$$\delta(q_0, 0) = q_1$$
$$\theta(q_0, 0) = a$$

Moore Machines

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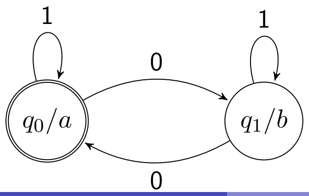


$$\delta(q_0, 0) = q_1$$
$$\theta(q_1) = b$$

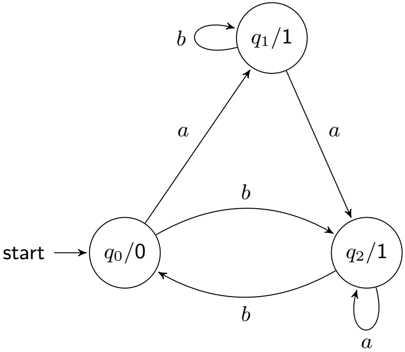
Mealy and Moore

Definition

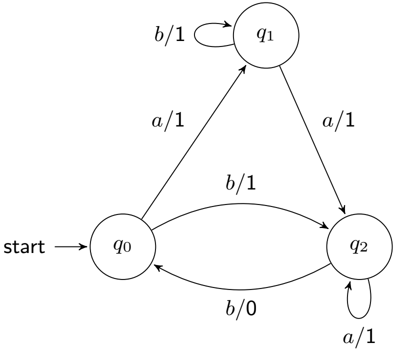
Two finite state transducers M and N are equivalent if they implement the same function,

$$F_M(w) = F_N(w)$$

Moore to Mealy

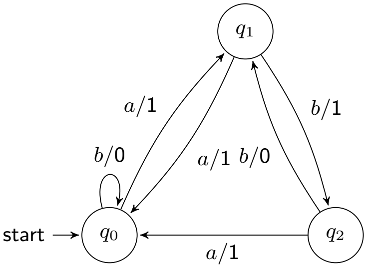


Moore Machine



Mealy Machine

Mealy to Moore



Mealy Machine

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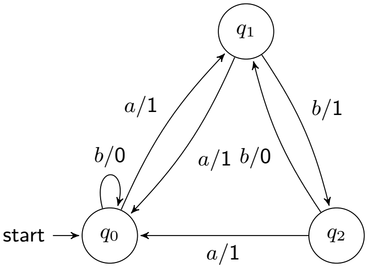
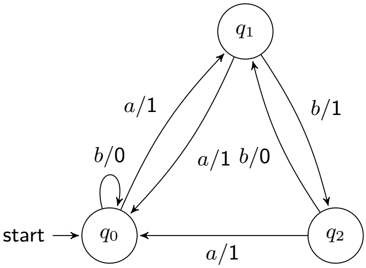


Mealy Machine

The Mealy Transition Table:

q_i	Input/output			
	a		b	
	q_{i+1}	O	q_{i+1}	O
q_0	q_1	1	q_0	0
q_1	q_0	1	q_2	1
q_2	q_0	1	q_1	0

Mealy to Moore



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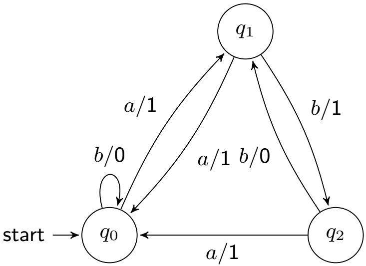
The Moore Transition Table:

q_i	q_{i+1}		Output
	a	b	
q_{00}	q_{11}	q_{00}	0
q_{01}	q_{11}	q_{00}	1
q_{10}	q_{01}	q_{21}	0
q_{11}	q_{01}	q_{21}	1
q_{21}	q_{01}	q_{10}	1

Mealy to Moore



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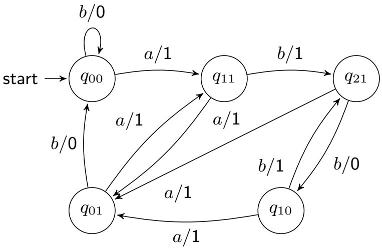

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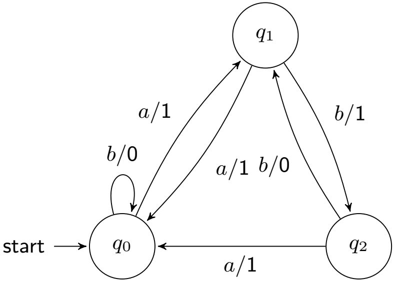
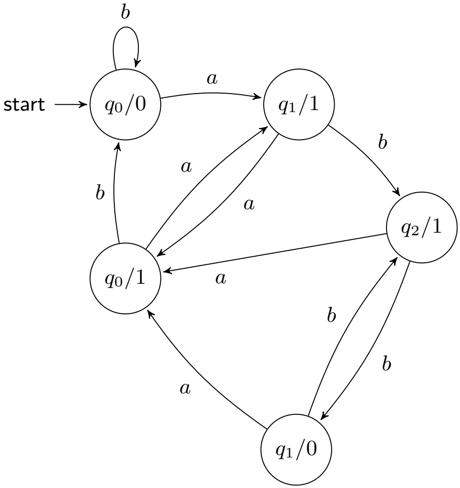


Mealy Machine



Moore Machine