

# Time Series Momentum Replication

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## 1 Introduction

In the paper “Time Series Momentum” by Tobias J. Moskowitz, Yao Hua Ooi, and Lasse Heje Pedersen 2012 [1], published in 2011, the authors describe the importance of how the time series momentum affects different financial accounts and the appropriate time length to consider in an investment. When studying the investment activities of hedgers and speculators, it appears that speculators benefit more from the time series momentum than hedgers do. Speculators may earn a better profit because they tend to invest during extreme markets, whereas hedgers are more conservative. Hence, speculators perform better during the times when hedgers do not. The authors describe “time series momentum” to be very similar among very different asset classes and market, which means that there may be a trend that needs to be followed closely to understand the long-term effect of it. It is important to note that the article emphasizes that the time where this trend needs to be considered is the last 12 months as this is where an investor may be able to analyze if the trend is positive or negative. After analyzing this, the investor may potentially predict whether this particular investment opportunity could be worthwhile. Furthermore, the authors mentioned that typically the 12-month time series momentum profits are most likely to be always positive. It is imperative to distinguish these time series momentum to correctly assess the financial investment opportunity.

The major factor of time series momentum is to understand that this trend only focuses on its own past returns within the last 12 month and it is not compared to the trends of other investments. Time series momentum targets only one risky asset at a time instead of comparing itself with its peers, which is referred to cross-sectional relation. The paper provided a few references that support this idea of time series momentum only focusing on one financial opportunity at time. The list of references that were used are as follows: Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999), Berk, Green, and Naik, (1999); Johnson, (2002); Ahn, Conrad, and Dittmar, (2003); Liu and Zhang, (2008); Sagi and Seasholes, (2007).

The market has a few reactions that are described as under-reaction and delayed over-reaction. The under-reaction occurs within the time series momentum, which later turns into the delayed over-reaction that occurs in the long-term period. However, there were a few issues raised with this theory. The first one that the authors found was that the correlations of the strategies that evolve around the time trends are larger than the correlations of the capital itself. This implies that there is some common factor within the time series momentum that is also within the financial assets. The second issue that was described was that various financial markets are generating a similar time series trend. Lastly, the authors concluded that they could not find a connection between time series momentum and investor's sentiment.

The authors relied on the information from Lo and Mackinlay (1990) and Lewellen (2002) to get a better understanding of how the time series momentum relates to cross-sectional momentum and if there are potentially some relations between them that may be useful to analyze. They found that the time series momentum drives the positive auto-co variance in the long-term effect and the cross-sectional momentum. There are two more returns that were not as strong, which were the serial cross-correlation and variation in mean returns. It was also seen that the negative serial cross-correlation was insignificant in the cross-sectional momentum. The financial earnings usually improve due to auto-co variances in both time series and cross-sectional momentum. This shows that there is a relationship between the two momentum. Furthermore, the authors were able to conclude that there was a strong correlation between time series and cross-sectional momentum when this theory was applied to different financial accounts. The analysis of individual stocks was used from different set of securities, where a robust connection was drawn between the time series and cross-sectional momentum.

Commodity Futures Trading Commission (CFTC) was used to analyze weekly position data for both speculators and hedgers. It was seen that speculators dominate in a positive time trend and change their trading activity when the trend starts to decrease. This is the opposite of what the hedgers do. Additionally, the authors described how spot price predictability and "roll yield" differ. Spot price changes usually operate by information shocks, while roll yield operates by liquidity. Roll yield, in long-term, also may not be affected by changing the spot price. Both concepts have their differences, however they both affect the time series momentum. The authors describe that the hypothesis of a "random walk" may be rejected based on their findings. This hypothesis suggests that future trends should not be followed by what happened in the past. For example, if a stock price went up at some point, it should not mean that it will go up again in the future. Furthermore, the authors talk about a financial account of different types having a stable and robust time series momentum, which yields about 2.5 times the Sharpe ratio. Another constraint on the "random walk" hypothesis was the time period of less than a year. The authors believe that this is a tight time-frame, which makes it harder to predict positive earnings.

## 2 Hypotheses and Test

The hypotheses of this paper is that time series momentum when analyzed during the last 12 months may predict whether the asset will continue to increase or decrease. To confirm this is true, the research focuses on verifying the data against each individual asset's performance. Additionally, the authors argue that the time series momentum and the cross-sectional momentum do have a relationship. This means that when comparing different assets together, one may find that they follow the same time

trend. Concluding tests were made to analyze when spectators and hedgers made the most profit. It was found that spectators and hedgers have an opposite connection, where spectators benefit from hedgers because they do not react the same to the market shock.

### 3 Literature Review

1. The paper "The Capital Asset Pricing Model: Some Empirical Tests" by Fischer Black, Michael C. Jensen, Myron Scholes (1972) [2] describes the portfolio evaluation models that are based on asset pricing model or have some sort of a connection to it. There were a few guidelines incorporated during this methodology, which were that investors were willing to take a financial risk and would choose a time for investment based on mean and variance. Another requirement was that there were no taxes or transaction costs, and that investors would have similar views on the criteria of the security returns. Lastly, the investors who were borrowing the money to make an investment would have to do so with a risk less rate of interest. Based on these requirements, a model would be generated that would distinguish the anticipated risk premiums of an investment and their "systematic risk".
2. The paper "Introducing libeemd: A program package for performing the ensemble empirical mode decomposition" by P.J.J. Luukko, J. Helske, E. Rasanen (2016) [3] summarizes how empirical mode decomposition (EMD) is a strategy for decomposing and analyzing time series data in a financial market. This methodology uses the time series data and inputs it into categories of intrinsic mode functions (IMFs). Then, the Hilbert spectrum analysis (HSA) is used to analyze the frequencies of the IMFs, which makes EMD very useful for understanding the time series data. This process is referred to as the Hilbert-Huang transform (HHT). The paper provides a code library that helps with implementation of current and future derivatives of EMD. The EMD process involves decomposing a signal, a part of Fourier series, which generates a sum of simple elements. Because of the Fourier series, it allows to track the local frequencies of data, which are called intrinsic mode functions (IMFs). This local frequency has two criteria: the difference of zero crossings and the number of local extrema must be more than one, and the "local mean" of the function must be zero.
3. The paper "Value and momentum everywhere" by Asness, Clifford S. and Moskowitz, Tobias J. and Pedersen, Lasse Heje (2013) [4] talks about the two capital market trends, which are the return on an asset and how it relates to it's long-term value and the return on an asset that it's most recent performance, which is referred to the "momentum" effect. These strategies are closely monitored as they provide useful insight of the financial accounts. To examine these theories more, the returns were analyzed between eight diverse markets and asset classes. The authors explained that value and momentum return premia was observed between all markets that were studied, even in the government bonds. Furthermore, the three-factor model was discussed and how it can group the common global risks.
4. The paper "A Five-Factor Asset Pricing Model" by Fama, Eugene F and French, Kenneth R (2015) [5] discusses a five-factor model is more beneficial than a three-factor model of Fama and French (FF 1993). This is because the five-factor model evolves around size, value, profitability, and investment patterns in financial market. However, the five-factor model still has drawbacks. For example, sometimes it doesn't track the low average returns on small stocks efficiently. The three-factor model has some repetitiveness when it is being used for the average returns in the market. The authors used the dividend discount model to describe that the average stock returns have a relationship with ethe book-to-market equity ratio. This is referred to as B/M. It was determined that although small stock portfolios may have a low average return, they still matter in the tests of the five-factor model.
5. The paper "The Credit Risk Premium" by Attakrit Asvanunt and Scott Richardson (2017) [6] states that it is the first to document the presence of a risk surrounding the corporate bonds. Corporate bonds are an imperative part of the fixed-income accounts. To analyze the risk around the corporate bonds, a long time series of the returns was used between corporate and Treasury bonds. There had to be a way to distinguish the long-term corporate versus government bond

returns. But it is important to note that to do so, the data-set needed to have the same cash flow maturity profiles. This paper also highlights the fact that investors need to understand the credit risk and whether it is beneficial in a long-term setting. Additionally, there may be less risky investments that are more worthwhile to invest in rather than a riskier investment. The focus in this research is how much the actual profit an investor would make from being exposed to a credit risk. Based on this data, a significant Sharpe ratio for credit returns was deducted. There was a difference between long-term corporate bonds and government bonds as the corporate bonds usually have a lower duration. The long-term government bonds may suffer from an “over-hedging” issue.

6. The paper “An Augmented q-factor Model with Expected Growth” by Kewei Hou, Haitao Mo, Chen Xue, and Lu Zhang (2020) [7] discusses that an investor may be able to make a greater return on an investment with a firm that already has expectations to succeed. Likewise, when a firm has a lower expectation of profiting, then most likely it will not bring as greater of a return. Furthermore, this creates a trend in the financial market as the positive return in one time period is expected to repeat again in the next period. In order to test this hypotheses, cross-sectional forecasting regression was used. This involved the Tobin’s q, operating cash flows, and the change in return on equity. It was determined that high Tobin’s q would generate a low investment-to-assets changes, whereas a high cash flow would return a high-investment-to-assets changes. The q-factor model is very similar to the Fama-French 6-factor model and the Stambaugh-Yuan model.
7. The paper “Short and Long-Horizon Behavioral Factors” by Kent Daniel, David Hirshleifer, and Lin Sun (2020) [8] analyzes a theoretically motivated factor model that uses the cross-section of the U.S. equity returns. This model generates two important factors, which are the long and short-horizon mispricing. The long-horizon mispricing has to do with buying the equity again when it is being mispriced, whereas the short-horizon earnings evolves around investors not being aware of what is going on with the short-term market. The model used in this paper focused on different forms of mispricing, which eventually get corrected either in short or long-term periods. The example that was given in the text was about investors not being aware of the public discussions regarding the quarterly earnings, which caused a form of mispricing. As a result, stock prices may not reflect the true and correct earnings potentials if an investor is not following the latest news.
8. The paper “An Operational Definition of a Statistically Meaningful Trend” by Andreas C. Bryhn and Peter H. Dimberg (2011) [9] talked about the linear trend analysis of time series, and how it is a regular process that is used in financial markets. The data needs to be analyzed and understood, so that it can be significant enough to be considered as a trend. For example, if there is an outlier within the data, it can still be examined as a trend. Statistical meaningfulness was a term that was highlighted in this paper, which may mean two different things: statistically meaningful or not statistically meaningful. It depends on what kind of p value is produced because if it is low then there is not enough representation of the variables that are being used. To cure this, a bigger and better representation of the data needs to be accounted for. Monte Carlo simulation was used in this research to calculate the trends in time series, whereas correlations were calculated by means of linear regression using Matlab and Microsoft Excel.
9. The paper “The devil in HML’s details” by Asness, Clifford and Frazzini, Andrea (2013) [10] discussed the high-minus-low (HML) investment strategies, and where they are being used. Specifically, HML was broken apart into four categories: the calculation, the rub, the tweak, and the caveat. HML accounts usually use the book-to-price ration (B/P) and have long/short term stocks. The time period that is used to analyze HML is anywhere from 6 to 18 months. There is a relationship amid the value and momentum strategies, which may improve its portfolio. Momentum and value have to be analyzed together for maximum performance. There is an issue that was raised with HML strategy, which was it may decrease the negative correlation between true value and momentum.
10. The paper “Risk, Return, and Equilibrium Empirical Tests” by Eugene F. Fama and James D. MacBeth (1973) [11] analyses the connection between average return and the risk of a common

stock in the New York Stock Exchange. “Two-parameter” portfolio model was used to test this hypothesis, which means that the investors are ready to invest their money and they do not have transactions costs or fees while doing so. Additionally, there was a set return on all assets of a certain percentage. The investors were also assumed to be ready to utilize their investment capabilities based on a maximum profit. All these requirements lead to an “efficient set theorem”, where the portfolio of an investor would always bring in the most profit. By using this portfolio model, it was concluded that there is a positive trade off between a profit return and risk.



Figure 1: This plot illustrates the cumulative returns for equity against the backtest [12]

## 4 Data and Preliminaries

The data that was used in the “Time Series Momentum” paper was from 12 cross-currency pairs, nine developed equity indexes, and 13 developed government bond futures. These ranged from January 1965 through December 2009. The reason why these were chosen is because they are the most liquid assets as the ones that are not liquid may skew the data. Using this data-set, the authors were able to calculate the greatest return of the asset that was the most liquid futures contract. This lead to calculating the daily returns, which showed that the equity indexes had a close relationship with the returns of the underlying cash indexes in excess of the government bonds. Furthermore, “far” futures were utilized to test the data-set. It showed that the time series momentum earnings are bigger for the far contracts. Table 1 shows the overview of the statistics of the additional returns on the future contracts. The third column shows when the time series of returns starts for every asset. The next columns show the time series arithmetic mean and the standard deviation of each asset. This table illustrates that depending on the asset, the sample mean varies across the data. There are both positive and negative excess returns. Based on Table 1 findings, commodities and equity indexes have a bigger standard deviation than the currencies and bonds. However, there is discrepancies between commodities and equity indexes too as some of the categories vary from negative to positive. This shows that there is a wide ranging standard deviations, which shows that there is a diversified portfolio. This makes it more difficult to come up with a conclusion that fits all the data.

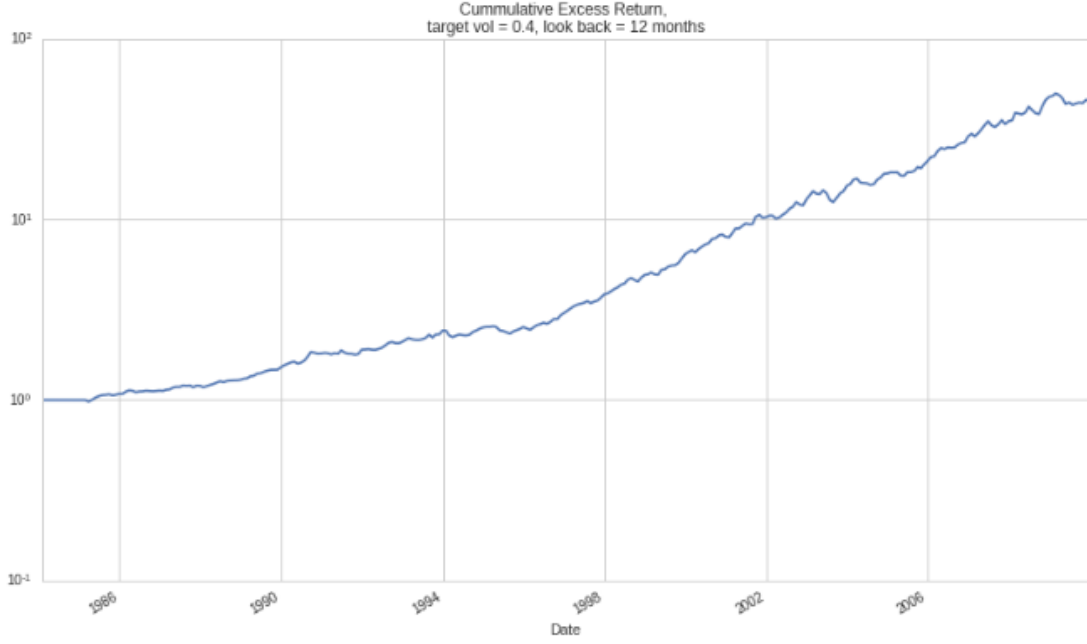


Figure 2: This plot illustrates the cumulative excess returns based on the previous 12 months of data [12].

Traders are usually identified by the Commodity Futures Trading Commission (CFTC) as commercial or non-commercial, which the paper "Time Series Momentum" refers to as hedgers and speculators, respectively. The following formula is used to calculate the net speculator position for each asset:

$$\text{Net Speculator position} = \frac{\text{Speculator long positions} - \text{Speculator short positions}}{\text{Open interest}} \quad (1)$$

This equation calculates net long or short term return on investment. Usually speculators and hedgers positions are the opposite of each other, which derives a sum of zero for their positions. The standard deviation shown in Table 1 has different ranges, but it is still very useful data. The equation used to calculate volatility is as follows:

$$\sigma_t^2 = 261 \sum_{i=0}^{\infty} (1 - \delta) \delta^i (r_{t-1-i} - \bar{r}_t)^2 \quad (2)$$

The variable 261 scales the variance to be yearly, the weights  $(1 - \delta) \delta^i$  add up to one, and  $\bar{r}_t$  is the exponentially weighted average return. The sum  $\sum_{i=0}^{\infty} (1 - \delta) \delta^i = \delta / (1 - \delta)$  is set to equal 60 days. This equation is used in the same manner across different assets and is the most simplest.

In order to calculate the predicted price, the following equation was used:

$$r_t^s / \sigma_{t-1}^s = \alpha + \beta_h r_{t-h}^s / \sigma_{t-h-1}^s + \epsilon_t^s \quad (3)$$

To make sure that the assets were scaled, the equation is divided by their volatility. An alternative to this equation is to look at the sign of the past excess return, which is the following equation:

$$r_t^s / \sigma_{t-1}^s = \alpha + \beta_h \text{sign}(r_{t-h}^s) + \epsilon_t^s \quad (4)$$

The right and left hand side of this equation is independent of the volatility because of the sign function. This equation showed that during the first year, there was a robust return and then it was poorer for the next four years. The conclusion here was that during the first 12 months, there was a positive

time series momentum. However, that changed after this time period, and only minor reversals were observed during the next four years.

Figure 1 and 2 shows plots of the cumulative returns and cumulative excess returns, respectively. This is a graph based on the years and it is increasing between the years of 1986 and 2006. This research supports the hypothesis of the time series momentum as the previous 12 months worth of data may shape the future of the market.

#### 4.1 Time Series Momentum Trading Techniques

The authors then went on to analyze the time series momentum trading strategies, which involved the "look-back period" and the "holding period". The "look-back period" is defined as the length of the time it took to create the portfolio in months and the "holding period" is the length of time after the portfolio has been created. With this, each asset  $s$  and time in month  $t$  was able to show the additional profit over the past  $k$  months. This return was distinguished by whether it was positive or negative for holding period of  $h$  months. The standard deviation that was used was  $1/\sigma_{t-1}^s$  for each month, which was called the ex ante volatility. Ex ante volatility is helpful because it simplifies the process to collect the strategies throughout the assets with different volatility levels and it provides a stable volatility for a time series momentum.

The trading strategies produced a single time series of monthly profits, which made sure that there was no overlapping in months. The methodology here was produced from Jegadeesh and Titman (1993), which focused on showing the monthly return on the financial accounts. The time variable  $t$  was calculated on the sign of the previous return from time  $t - k - 1$  to  $t - 1$ . After this, the time variable was changed to be  $t - k - 2$  to  $t - 2$  to calculate the previous return. This was calculated until the time- $t$  return based on the concluding return is still being used from  $t - k - h$  to  $t - h$ . The average of all the returns from all the financial accounts generated the time series momentum strategy returns of  $r_t^{TSMOM(k,h)}$ . This helped calculate the uncommon production by figuring out the alphas from the following regression:

$$r_t^{TSMOM(k,h)} = \alpha + \beta_1 MKT_t + \beta_2 BOND_t + \beta_3 GSCI_t + sSMB_t + hHml_t + mUMD_t + \epsilon_t \quad (5)$$

Table 2 illustrates the monthly returns of the financial assets based on the holding period of 12 months based on each year separately. The ratios range from negative to positive and the time series momentum explains that based on this data an investor can predict the future behavior of the market. Additionally, this table may be useful for speculators and hedgers to analyze their financial risks. Based on the research from the authors, speculators and hedgers have an opposite relationship.

#### 4.2 Time Series Momentum Factor

The ex ante annualized volatility that was used in the time series momentum factor was 40%, where the position size was  $40\%/\sigma_{t-1}$ , and the  $\sigma_{t-1}$  is the estimate of the ex ante standard deviation. This percentage was chosen because it makes it simpler to distinguish differences between financial accounts, and it replicates the risk of an average individual stock. The equation used for TSMOM return of any instrument  $s$  at time  $t$  is as follows:

$$r_{t,t+1}^{TSMOM} = \text{sign}(r_{t-12,t}^s) \frac{40\%}{\sigma_t^s} r_{t,t+1}^s \quad (6)$$

This equation was used to calculate every instrument during every month from January 1985 to December 2009. Table 1 illustrates the Sharpe ratios of this methodology.

#### 4.3 Liquidity and Sentiment

TSMOM was tested to analyze if there are returns on investment that influenced by illiquidity. To accomplish this, the authors determined if TSMOM had better results from illiquid assets in the cross-section versus the diversified TSMOM factor within the time series momentum. The illiquidity of the

ASSET_CLASS	FUTURES	Start	Mean	Std	Skew	Kurt	Sharpe Ratio
BOND	AUSTRALIA 10-YEAR BOND	9/22/1987	-0.0244	0.0137	-0.3368	4.2927	-1.7838
BOND	AUSTRALIA 3-YEAR BOND	12/4/1989	-0.0183	0.0137	-0.2034	3.8708	-1.3344
BOND	CANADA 10-YEAR BOND	9/19/1989	0.0098	0.064	-0.2109	2.3455	0.1538
BOND	EURO BOBL	10/8/1991	0.0016	0.0339	-0.2887	1.8962	0.0484
BOND	EURO BUND	11/27/1990	0.0098	0.0529	-0.2032	1.7357	0.1861
BOND	EURO BUXL	10/6/1998	0.0038	0.0997	-0.1955	2.0124	0.0382
BOND	EURO SCHATZ	3/11/1997	-0.0134	0.0146	-0.3324	3.6816	-0.9156
BOND	JAPAN 10-YEAR BOND	10/22/1985	0.0074	0.0542	-0.5901	8.9518	0.136
BOND	LONG GILT	1/3/1984	-0.0127	0.0755	0.0872	3.7265	-0.1688
BOND	US 10-YEAR NOTE	1/3/1984	0.0156	0.0689	0.076	3.2148	0.2269
BOND	US 2-YEAR NOTE	6/27/1990	-0.0081	0.0185	-0.0891	4.0655	-0.4388
BOND	US 5-YEAR NOTE	5/24/1988	0.0012	0.0431	-0.0979	2.7085	0.0284
BOND	US LONG BOND	1/3/1984	0.0274	0.1034	-0.0237	1.9634	0.2646
COMMODITIES	ALUMINIUM	7/25/1997	-0.0217	0.2131	-0.1772	2.471	-0.1016
COMMODITIES	BRENT CRUDE	6/27/1988	0.1729	0.3557	-0.4146	11.5905	0.486
COMMODITIES	COCOA	1/3/1984	-0.039	0.3035	0.1705	2.5969	-0.1285
COMMODITIES	COFFEE	1/3/1984	-0.0273	0.3724	0.5869	9.6552	-0.0733
COMMODITIES	COPPER	7/23/1997	0.1133	0.2807	0.0752	4.565	0.4035
COMMODITIES	CORN	1/3/1984	-0.0908	0.2332	0.0768	2.9448	-0.3892
COMMODITIES	COTTON	1/3/1984	-0.0294	0.2484	0.1078	2.5545	-0.1183
COMMODITIES	GASOIL	7/5/1989	0.1447	0.3357	-0.3526	10.7393	0.431
COMMODITIES	GOLD	1/3/1984	-0.0255	0.1615	0.1759	7.768	-0.1577
COMMODITIES	HEATING OIL	7/2/1986	0.1641	0.3659	-0.3195	9.5853	0.4483
COMMODITIES	LEAN HOGS	4/3/1986	-0.0217	0.2333	-0.0693	1.2697	-0.0929
COMMODITIES	LIVE CATTLE	1/3/1984	0.0026	0.1446	-0.0978	1.3376	0.0178
COMMODITIES	NATURAL GAS	4/5/1990	-0.0052	0.5312	0.5047	5.5002	-0.0097
COMMODITIES	NICKEL	7/25/1997	0.1594	0.3935	0.1149	3.363	0.405
COMMODITIES	PLATINUM	1/30/1984	0.0353	0.2315	-0.044	7.4779	0.1523
COMMODITIES	RBOB GASOLINE	10/5/2005	0.0703	0.4422	-0.069	1.8821	0.1589
COMMODITIES	SILVER	1/3/1984	-0.0217	0.279	-0.3456	6.2958	-0.0779
COMMODITIES	SOY MEAL	1/3/1984	0.0533	0.2439	-0.031	2.5715	0.2186
COMMODITIES	SOY OIL	1/3/1984	-0.0333	0.2397	0.2251	1.8114	-0.1388
COMMODITIES	SOYBEANS	1/3/1984	-0.0084	0.2277	-0.1572	2.628	-0.0368
COMMODITIES	SUGAR	1/3/1984	0.0181	0.3685	-0.0212	4.0372	0.0492
COMMODITIES	WHEAT	1/3/1984	-0.0603	0.2573	0.1473	2.7786	-0.2345
COMMODITIES	WTI CRUDE	1/3/1984	0.1346	0.3759	-0.3765	10.9173	0.358
COMMODITIES	ZINC	7/25/1997	0.0092	0.3078	-0.086	3.3653	0.03
CURRENCIES	AUSTRALIAN DOLLAR	1/14/1987	0.0146	0.1197	-0.6075	11.0146	0.1223
CURRENCIES	CANADIAN DOLLAR	4/7/1986	-0.0082	0.0724	0.0903	8.6474	-0.1137
CURRENCIES	EURO	5/21/1998	0.0021	0.1021	0.0266	1.3833	0.0201
CURRENCIES	JAPANESE YEN	5/28/1986	-0.0301	0.1143	0.6129	6.7518	-0.2636
CURRENCIES	NEW ZEALAND	5/9/1997	0.0167	0.1391	-0.2998	3.8182	0.12
CURRENCIES	NORWAY	5/20/2002	0.0472	0.1346	-0.1743	2.9403	0.3508
CURRENCIES	SWEDEN	5/20/2002	0.0367	0.1348	0.3729	5.9377	0.2724
CURRENCIES	SWITZERLAND	4/8/1986	-0.0137	0.1186	0.1735	2.0546	-0.1157
CURRENCIES	UK	5/29/1986	-0.0028	0.1015	-0.2066	3.498	-0.0278
EQUITY INDEXES	AEX (NETHERLANDS)	1/4/1989	0.0085	0.25	-7.0447	238.3628	0.034
EQUITY INDEXES	DAX (GERMANY)	11/27/1990	0.0389	0.237	0.0231	6.0168	0.1642
EQUITY INDEXES	FTSE 100 (UK)	3/1/1988	0.0093	0.1881	0.0019	5.7149	0.0497
EQUITY INDEXES	FTSE/MIB (ITALY)	3/24/2004	0.001	0.2246	0.0756	8.9781	0.0044
EQUITY INDEXES	IBEX 35 (SPAIN)	7/2/1992	0.0745	0.2373	-0.1767	4.3022	0.314
EQUITY INDEXES	S&P 500 (US)	9/11/1997	-0.0035	0.2192	0.215	10.1467	-0.0159
EQUITY INDEXES	SPI 200 (AUSTRALIA)	5/4/2000	0.0307	0.1674	-0.3216	6.2255	0.1834
EQUITY INDEXES	TOPIX (JAPAN)	5/21/1990	-0.0553	0.2428	0.2691	9.5202	-0.2279

Table 1: This table illustrates the mean return and standard deviation of the futures contracts from January 1965 to December 2009. The Sharpe Ratio is also shown, which shows how well the financial assets will perform compared to its risk.[12]

futures was used from the daily dollar trading volume from broker feeds. The assets had to be put in order of how well they performed in their daily trading volume. Then, this was divided by the standard deviation to compute the standard normalized rank of each category. The results would produce either



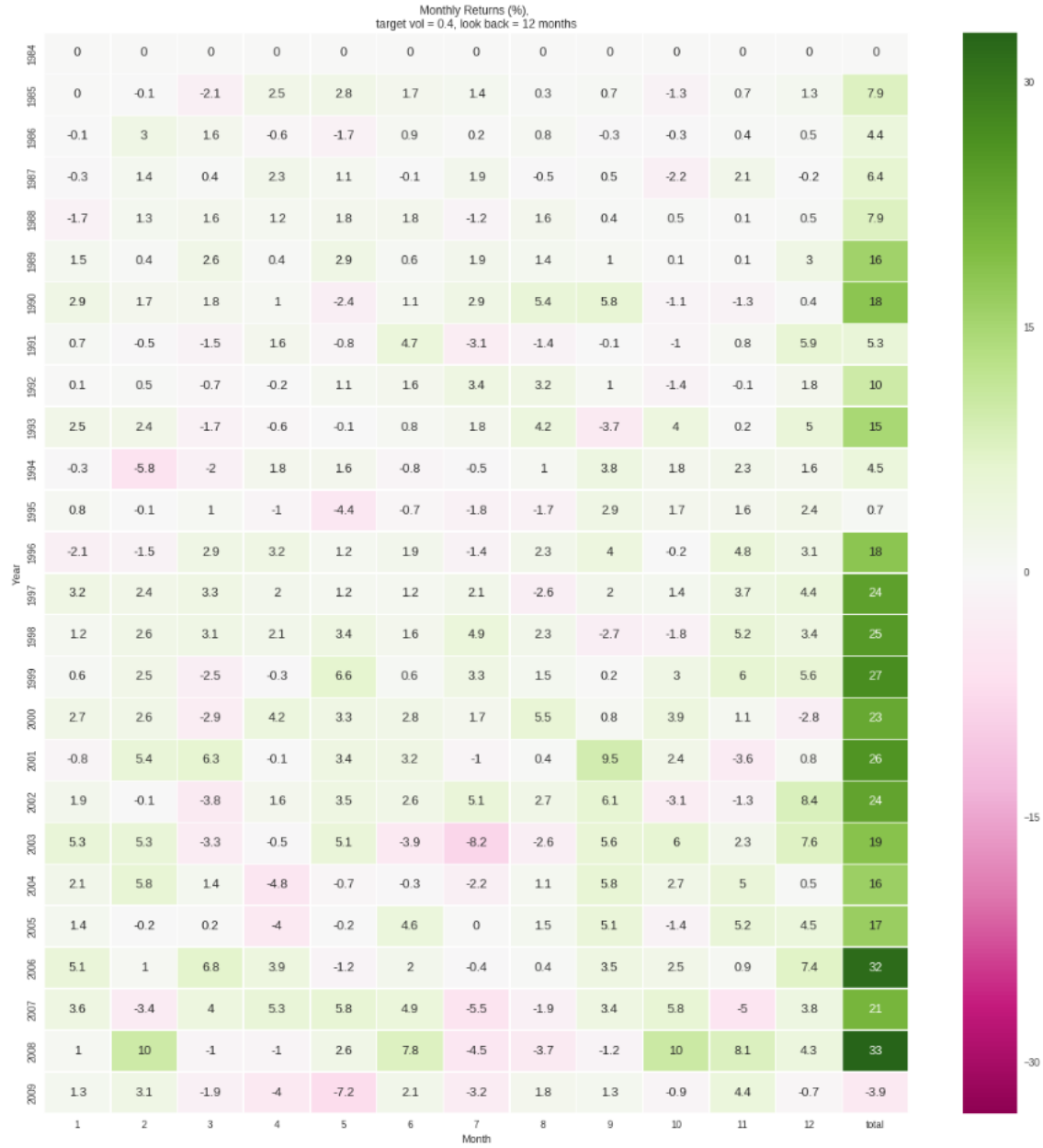


Table 2: This table illustrates the monthly returns on assets based on the previous 12 months of the assets [12].

a positive or a negative value, which would indicate that the contract is more or less illiquid. Next, the authors analyzed the effect of TSMOM on the returns regarding the time series of liquidity. They did not find any important conclusions here between TSMOM profitability and market volatility.

Figure 4 and 5 show graphs of the cumulative returns volatility to the benchmark and cumulative returns on logarithmic scale, respectively. Both the plots follow an increasing market trend, although there are some drops in the market. The logarithmic scale shows a more smooth plot.

#### 4.4 Data Used to Replicate

The data that was used to follow the time series momentum analyses was used from the following link: <https://github.com/anthonyng2/Time-Series-Momentum/tree/master/TSMOM/data>. [12]

The code to replicate the analysis was used from the following link: <https://github.com/anthonyng2/Time-Series-Momentum/blob/master/TSMOM/TS%20MOM.ipynb> [12].

Table 1 in this paper replicates Table 1 that was analyzed in the time series momentum paper. Table 1 illustrates the annualized performance of each individual asset. It shows the calculated mean and standard deviation (referred to as volatility). This table can be used to examine each asset by itself or cross reference different assets to compare them to each other. Table 2 shows the monthly returns of assets based on the previous 12 months. This is the key analysis of the time series momentum because based on this data one can predict how the market will continue to either increase or decrease.

Figure 1 shows the cumulative returns of the assets, while figure 2 shows the cumulative excess returns based on the past 12 months. Figure 3 is a plot of the financial assets that are below the market value and shows the dips of the market from 1984 to 2009. Both figure 4 and figure 5 are shown for comparison. Figure 4 shows the cumulative returns volatility based on the benchmark scale versus figure 5 shows the cumulative returns based on the logarithmic scale.

## 5 Time Series Vs. Cross-Sectional Momentum

The authors wanted to determine any overlaps and differences between time series and cross-sectional momentum as they have already found that both of these methods had important relationship. They regressed on the XSMOM strategy in order to see the outcome of the TSMOM strategy among all assets. The authors were able to see that there was a significant connection between the beta of TSMOM on XSMOM, which was equal to 0.66 with t-statistic of 15.17 and R-square of 44%. Nevertheless, the alpha between the two elements showed that TSMOM was not fully represented by XSMOM. This means that both TSMOM and XSMOM are similar but definitely not the same. The authors were able to show that TSMOM is related to XSMOM in all different asset categories, even where TSMOM wasn't part of the strategy of the asset. This research showed that TSMOM still had a dominating alpha, but was not fully explained by the cross-sectional momentum strategies.

When this TSMOM methodology was used on returns for assets individually, the authors found a stronger and repeated pattern. The alphas appeared to still show a positive sign for every asset category. Additionally, a cross-asset relationship between TSMOM and XSMOM was observed in the individual categories. The concluding outcomes showed an important correlation in the time series and cross-sectional momentum, which agreed to the results from Asness, Moskowitz, and Pedersen (2010). This research can be seen through Table 1 and Table 2 as they can be used to analyze the relationship between time series and cross-sectional momentum. Table 1 breaks the details between the assets separately, whereas table 2 gives a total overview of the ratios based on the total assets.

### 5.1 Formal Decomposition

The authors derived a more formal way of describing the relationship between the time series and the cross-sectional momentum. For the cross-sectional momentum, the weight of instrument  $i$  is written as  $w_t^{XS,i} = (1/N)r_{t-12,t}^i - r_{t-12,t}^{EW}$ . This is taken from the previous 12-month additional return over the equal-weighted average return, which is described as  $r_{t-12,t}^{EW} = (1/N) \sum_{i=1}^N r_{t-12,t}^i$ . This results in the following equation that calculates the return to the portfolio:

$$r_{t,t+1}^{XS} = \sum_{i=1}^N W_t^{XS,i} r_{t,t+1}^i \quad (7)$$

The monthly expected return is  $\mu^i = E(r_{t,t+1}^i) = E(r_{t-12,t}^i)/12$ , where  $\mu = [\mu^1, \dots, \mu^N]'$ ,  $R_{t,s} = [r_{t,s}^1, \dots, r_{t,s}^N]'$ , and  $\Omega = E[(R_{t-12,t} - 12\mu)(R_{t,t+1} - \mu)']$ . Thus, the equation for the expected return to cross-sectional momentum (XSMOM) can be written as follows:

$$E[r_{t,t+1}^{XS}] = \frac{tr(\Omega)}{N} - \frac{1'\Omega 1}{N^2} + 12\sigma_\mu^2 = \frac{N-1}{N^2} tr(\Omega) - \frac{1}{N^2} [1'\Omega 1 - tr(\Omega)] + 12\sigma_\mu^2 \quad (8)$$

where  $tr$  is the trace of a matrix,  $1$  is an  $(N \times 1)$  vector of ones, and  $\sigma_\mu^2$  is the cross-sectional variance of the mean monthly returns  $\mu^i$ . This finding of the cross-sectional momentum profits can be broken down to an auto-covariance element between one year returns and one month returns, a cross-covariance element illustrating the temporal leads and lags between stocks, and the cross-sectional difference in the unconditional mean returns. Additionally, the authors found that the cross-sectional momentum earnings do not need to only be positive. They can also be negative and show that the past high returns of an asset may predict a lower future returns of a different asset. This is what will define a momentum. The returns to the time series momentum can be written as follows:

$$E(r_{t,t+1}^{TS}) = E\left(\sum_{i=1}^N W_t^{TS,i} r_{t,t+1}^i\right) = \frac{tr(\Omega)}{N} + 12 \frac{\mu' \mu}{N} \quad (9)$$

The time series momentum profits are mainly generated by the time series predictability, when the average squared mean returns of a financial account is minor. The time series momentum profits can be broken down into auto-covariance term that highlights the cross-sectional momentum. This shows that there is a relationship between the TSMOM and XSMOM profitability, where the data can prove this theory.

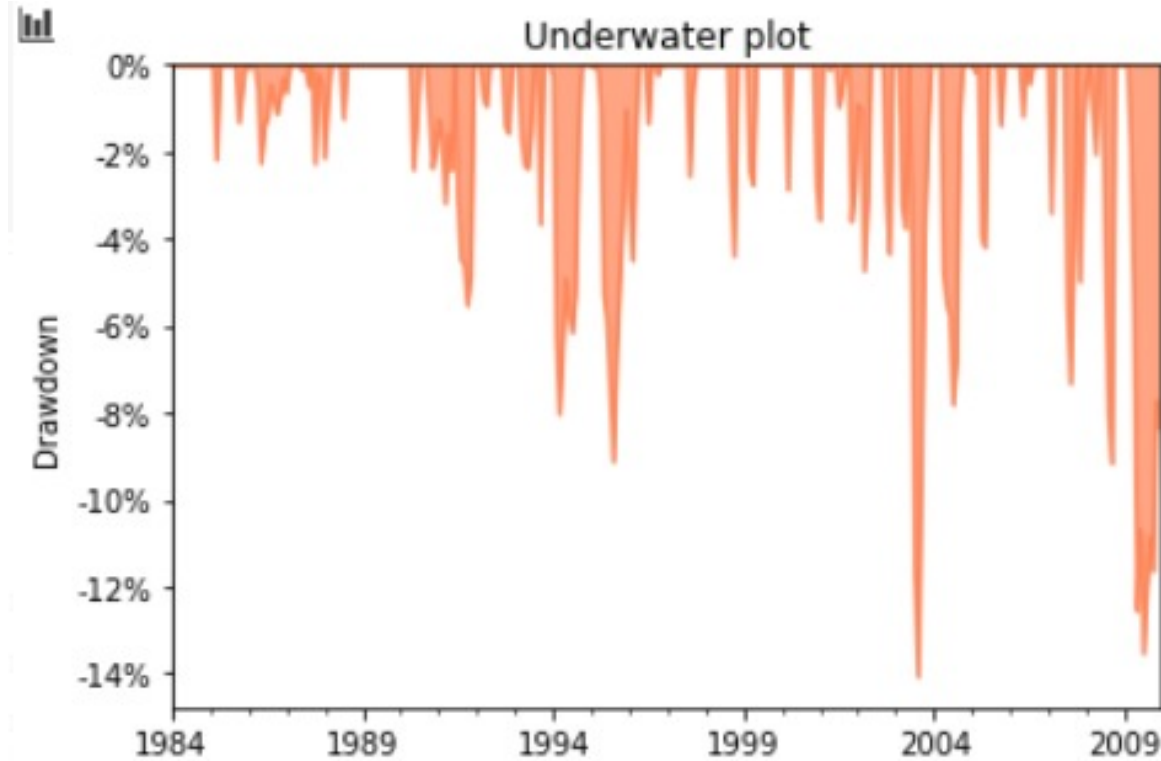


Figure 3: This plot shows the underwater graph of the assets from 1984 to 2009 [12].

## 5.2 TSMOM and cross-sectional momentum

The authors did more testing to see how TSMOM and XSMOM work together and what benefits they can provide. They first used XSMOM to analyze if TSMOM can record some returns to the cross-sectional momentum among all financial accounts together and separately. They were able to find that TSMOM described the cross-sectional momentum well between all accounts, which included commodities, equity indexes, bonds, and currencies. Furthermore, the alphas of XSMOM were not much different from zero, which means that TSMOM also focuses on profits of XSMOM. The authors also regress the Fama-French cross-sectional momentum factor for personal US equities, UMD, on their TSMOM portfolio. It is important to note that there was no overlap in the data that was used for

this research. More research was completed regarding two popular hedge fund indexes, which were the "Manged Futures" hedge fund index and "Global Macro" hedge fund index. It was concluded that TSMOM described some of the leading elements in the asset pricing, which was a part of the cross-sectional momentum. In the hedge fund strategy returns, TSMOM was an important characteristics of asset price behavior. Furthermore, TSMOM appears to be a basic implementable factor that represented the production criterion. Figure 3 is a graph of the underwater plot that shows the dips of the market. This is particularly useful for speculators and hedgers to analyze. Speculator tend to have greater interest when the market takes big turns.

## 6 Who Trades with Time Series Momentum?

Based on the research, the authors found that speculators profit more by using the time series momentum trends than the hedgers do. This is because speculators and hedgers follow the opposite positions. Speculators have less than average positions following negative returns, and more than average positions following positive 12-month returns. Additionally, speculators benefit more using the trends because they are related to positive abnormal returns.

Time series momentum appear to show accurate trends for about 12 months, and then they usually reverse after that. A more in-depth look was taken to see if these trends can be related to the evolution of trading stands. This can lead to knowing what the difference is in TSMOM and trading positions. The example that was given in the paper was if the under-reaction to the news was the source of TSMOM, then most likely a reverse cycle wouldn't be seen after a year. However, if there was an over-reaction in the market, then there may be a reverse cycle as it tends to return to the fundamentals. Furthermore, the research showed that speculators stop following the trends where positive returns from TSMOM no longer are available. Likewise, hedgers start to have more interest in their positions and follow the trend closely during this time. Hence, if the over-reaction in the market is happening due to this situation, then it is most likely because of hedgers and not speculators. This shows that there is a strong relationship between hedgers and speculators based on the time series momentum trend. Speculators are inclined to make better profit at the cost of hedgers.

The joint dynamics of time series momentum returns were also closely analyzed to understand the trading patterns. The vector auto regressive (VAR) model was used to calculate the difference in net speculator positions. A greater time period was used of 24 months in this research to see when the reversal of the trend would occur. The Cholesky decomposition was used to confirm that the time series momentum was mostly benefited by speculators. But the research showed that the speculators only had profit from TSMOM for about a year, while hedgers were not benefiting. The authors describe the reason for speculators to make money during this time was because they were providing liquidity to hedgers. This was analyzed by reviewing the predictability of returns in trading positions.

The authors used "roll return" or "roll yield" to describe the previous return of each futures contract, which was calculated by the following equation:

$$\text{Price Change}_{t-12,t} = \frac{\text{Price}_t - \text{Price}_{t-12}}{\text{Price}_{t-12}} - r_{t-12,t}^f \quad (10)$$

The prices are calculated to the nearest-to-expiration futures prices and  $r_{t-12,t}^f$  is the risk-free interest rate over the 12-month period, where the roll return is the following equation

$$\text{Futures return}_{t-12,t} = \text{Price Change}_{t-12,t} + \text{Roll Return}_{t-12,t} \quad (11)$$

The roll return is more important in the commodity markets versus the convenience yield futures. This equation is calculated to the nearest-to-expiration or next-to-nearest expiration contract. The conclusion here is that the hedgers' price has an influence on the roll returns. Because hedgers usually rely on the future investments, it also affects the future prices, which assist the roll yield as each futures contract expires at the spot price. Sometimes hedgers are not working with a full length futures, which causes the positive roll return and a "normal backwardation" (Keynes).

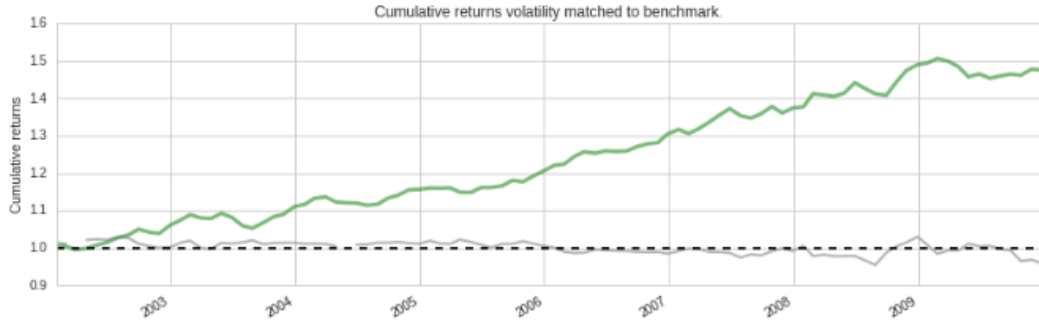


Figure 4: This graph shows the cumulative returns on volatility to benchmark scale from 2003 to 2009 [12].

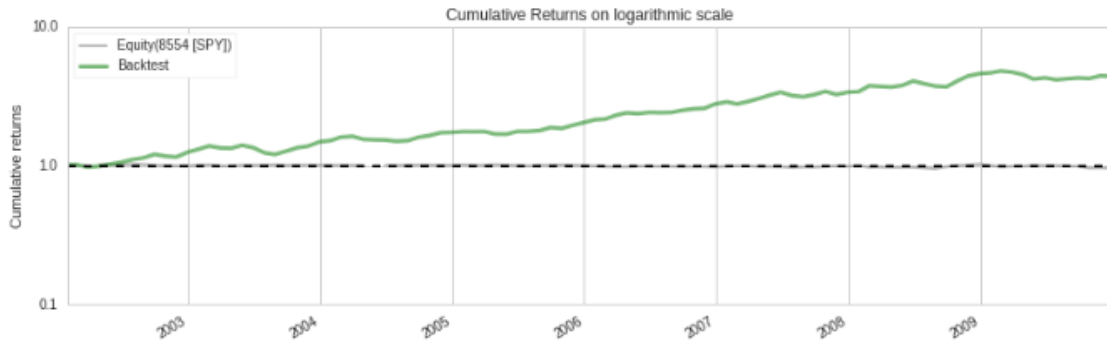


Figure 5: This graph shows the cumulative returns on logarithmic scale from 2003-2009 [12].

## 7 Conclusion

This research paper agrees with the research that was in the "Time Series Momentum" paper as it replicates the data that shows that time trends do matter in the financial market. The time series momentum was found to be a key trend in trading strategies. It is different from cross-sectional momentum, but has a relationship that can guide investors on the right path of understanding where the market stands. Decomposing the time series and the cross-sectional momentum profits was a necessity to analyze the dominant force of the positive auto-covariance among a security's excess return in a given time frame. Additionally, the over-reaction and under-reaction of the speculators and hedgers was analyzed in the time series momentum. Speculators were found to benefit more from during the time series momentum trend, however this gives them a change to provide liquidity to hedgers. Time series momentum provides the ability to test the random walk hypothesis and a number of leading observations and rational asset pricing methodologies.

## 8 Future Research

Time series momentum trends focus on when the market is up or down and may guide an investor on a correct path of when to invest their money. When monitoring the time trend in the previous 12 months, a prediction may be made of whether the market will continue to rise or fall. However, future research may also focus on the correct timing of short-term trading so that an investor may profit the most. For example, would it be more beneficial to analyze the long-term effect of the time trend in the market versus the short-term? If the trend is showing that there is a positive trend in the market, then is it better to wait before trading or better to trade sooner and invest the capital into another asset that has just started recovering from a downfall? Future research may have more

of an oversight of other elements that have an impact on the time series momentum. From the most recent pandemic, no one was able to predict something like that happening. The market took many different turns during this time, however can time series momentum research include outside factors like a pandemic affecting the future market?

## References

- [1] T. J. Moskowitz, Y. H. Ooi, and L. H. Pedersen, “Time series momentum,” *Journal of financial economics*, vol. 104, no. 2, pp. 228–250, 2012.
- [2] F. Black, M. C. Jensen, M. Scholes, *et al.*, “The capital asset pricing model: Some empirical tests,” *Studies in the theory of capital markets*, vol. 81, no. 3, pp. 79–121, 1972.
- [3] P. J. Luukko, J. Helske, and E. Räsänen, “Introducing libeemd: A program package for performing the ensemble empirical mode decomposition,” *Computational Statistics*, vol. 31, no. 2, pp. 545–557, 2016.
- [4] C. S. Asness, T. J. Moskowitz, and L. H. Pedersen, “Value and momentum everywhere,” *The Journal of Finance*, vol. 68, no. 3, pp. 929–985, 2013.
- [5] E. F. Fama and K. R. French, “A five-factor asset pricing model,” *Journal of financial economics*, vol. 116, no. 1, pp. 1–22, 2015.
- [6] A. Asvanunt and S. Richardson, “The credit risk premium,” *The Journal of Fixed Income*, vol. 26, no. 3, pp. 6–24, 2017.
- [7] K. Hou, H. Mo, C. Xue, and L. Zhang, “An augmented q-factor model with expected growth,” *Review of Finance*, 2020.
- [8] K. Daniel, D. Hirshleifer, and L. Sun, “Short-and long-horizon behavioral factors,” *The Review of Financial Studies*, vol. 33, no. 4, pp. 1673–1736, 2020.
- [9] A. C. Bryhn and P. H. Dimberg, “An operational definition of a statistically meaningful trend,” *PLoS One*, vol. 6, no. 4, 2011.
- [10] C. Asness and A. Frazzini, “The devil in hml’s details,” *The Journal of Portfolio Management*, vol. 39, no. 4, pp. 49–68, 2013.
- [11] E. F. Fama and J. D. MacBeth, “Risk, return, and equilibrium: Empirical tests,” *Journal of political economy*, vol. 81, no. 3, pp. 607–636, 1973.
- [12] A. Ng, “Time-series-momentum.” <https://github.com/anthonyng2/Time-Series-Momentum>, 2016.

## 9 Appendices

<https://github.com/anthonyng2/Time-Series-Momentum/blob/master/TSMOM/TS%20MOM.ipynb>.

```
[19]: import numpy as np
import pandas as pd
import datetime
import pyfolio as pf
import matplotlib.pyplot as plt
import matplotlib
import seaborn as sns
import pytz

# SB Updates: inconsistencies with empyrical and pyfolio current version
import empyrical
```

## 1 1a. Individual Futures Performance

```
[22]: std_index = res.resample('BM').last().index  
      mth_index = pd.DataFrame(index=std_index)  
      mth_index_vol = pd.DataFrame(index=std_index)  
      summary_stats = pd.DataFrame(index=['Asset', 'Start', 'Mean', 'Std', \
```

```

[23]: for oo in res.columns:
        returns = res[oo]
        returns.dropna(inplace=True)

        first_date = returns.index[0].strftime("%Y-%m-%d")    # store this to show
        ↪when data series starts

        ret_index = (1 + returns).cumprod()
        ret_index[0] = 1

        # equation (1) ex ante vol estimate
        day_vol = returns.ewm(ignore_na=False,
                                adjust=True,
                                com=60,
                                min_periods=0).std(bias=False)
        vol = day_vol * np.sqrt(261) # annualise

        ret_index = pd.concat([ret_index, vol], axis=1)
        ret_index.columns = [oo, 'vol']

        # convert to monthly
        ret_m_index = ret_index.resample('BM').last().ffill()

        # SB Updates: `ix` is deprecated to we use `iloc` instead
        ret_m_index.iloc[0][oo] = 1

        mth_index = pd.concat([mth_index, ret_m_index[oo]], axis=1)
        tmp = ret_m_index['vol']
        tmp.name = oo + "_Vol"
        mth_index_vol = pd.concat([mth_index_vol, tmp], axis=1)

        tmp_mean = ret_index[oo].pct_change().mean()*252
        tmp_std = ret_index[oo].pct_change().std()*np.sqrt(252)
        tmp_skew = ret_index[oo].pct_change().skew()
        tmp_kurt = ret_index[oo].pct_change().kurt()
        sr = tmp_mean / tmp_std

        dict = {'Asset': oo,
                'Start': first_date,
                'Mean': np.round(tmp_mean,4),
                'Std': np.round(tmp_std,4),
                'Skew': np.round(tmp_skew,4),
                'Kurt': np.round(tmp_kurt,4),
                'Sharpe Ratio': np.round(sr,4),
                }
        summary_stats[oo] = pd.Series(dict)

```



```
[24]: summary_stats = summary_stats.transpose()
futures_list = pd.read_csv("./data/futures_list.csv")
# futures_list
# summary_stats
all = summary_stats.reset_index().merge(futures_list)
all.sort_values(by=["ASSET_CLASS", "FUTURES"], inplace=True)
del all['Asset'], all['index']
```

### 1.0.1 Individual Futures Contracts Performance

These are annualized performance.

```
[25]: all.set_index(['ASSET_CLASS', 'FUTURES']).style.set_properties(**{'text-align': 'right'})

# SB Updates: saved to output file
# all.to_csv('AssetClass_Futures_Sorted.csv')
```

## 2 1b. Trading Strategy - TSMOM with Volatility Scaling (1984 - 2009)

```
[26]: pnl = pd.DataFrame(index=std_index)
leverage = pd.DataFrame(index=std_index)
strategy_cumm_rtns = pd.DataFrame(index=std_index)
```

```
[27]: for oo in mth_index:
    df = pd.concat([mth_index[oo], mth_index_vol[oo+"_Vol"]], axis=1)
    df['returns'] = df[oo].pct_change(look_back)

    df['pnl'] = 0.
    df['leverage'] = 0.
    try:
        for k, v in enumerate(df['returns']):
            if k <= look_back:
                # skip the first 12 observations
                continue
            if df['returns'].iloc[k-1] < tolerance:
                # negative returns, sell and hold for 1 mth, then close position
                if vol_flag == 1:
                    df['pnl'].iloc[k] = (df[oo].iloc[k - 1] / df[oo].iloc[k] -
→ 1) * \
                                target_vol / df[oo+"_Vol"].iloc[k - 1]
                    df['leverage'].iloc[k] = target_vol / df[oo+"_Vol"].iloc[k]
→ - 1]
```

```

        else:
            df['pnl'].iloc[k] = (df[oo].iloc[k - 1] / df[oo].iloc[k] -
→1)

            df['leverage'].iloc[k] = 1.
        elif df['returns'].iloc[k-1] > tolerance:
            # positive returns, buy and hold for 1 mth, then close position
            if vol_flag == 1:
                df['pnl'].iloc[k] = (df[oo].iloc[k] / df[oo].iloc[k - 1] -
→1) * \
                                target_vol / df[oo+"_Vol"].iloc[k - 1]
                df['leverage'].iloc[k] = target_vol / df[oo+"_Vol"].iloc[k
→- 1]
            else:
                df['pnl'].iloc[k] = (df[oo].iloc[k] / df[oo].iloc[k - 1] -
→1)

                df['leverage'].iloc[k] = 1.

    except: pass
    # convert to cumulative index
    pnl = pd.concat([pnl, df['pnl']], axis=1)
    leverage = pd.concat([leverage, df['leverage']], axis=1)

    ret_index = (1 + df['pnl'][13:]).cumprod()
    ret_index[0] = 1
    strategy_cumm_rtms = pd.concat([strategy_cumm_rtms, ret_index], axis=1)

```

```

[28]: # pnl
pnl.columns = res.columns
leverage.columns = leverage.columns
strategy_cumm_rtms.columns = res.columns
df = pnl
df['port_avg'] = df.mean(skipna = 1, axis=1)
Strategy = df['port_avg'].copy()
Strategy.name = "TSMOM with Vol"
dataport_index = (1 + df['port_avg']).cumprod()

# SB Updates: saved to output file
# pnl.to_csv('Strategy_TSMOM_With_Vol.csv')

```

```

[29]: # SB Updates: `pf.empyrical` to `empyrical` everywhere
print ("Annualized Sharpe Ratio = ", empyrical.sharpe_ratio(df['port_avg'],
→period='monthly'))
print ("Annualized Mean Returns = ", empyrical.annual_return(df['port_avg'],
→period='monthly'))

print ("Annualized Standard Deviations = ", empyrical.
→annual_volatility(df['port_avg'], period='monthly'))

```

Annualized Sharpe Ratio = 1.5594286210223922  
Annualized Mean Returns = 0.15820525315443468  
Annualized Standard Deviations = 0.0977748556596916

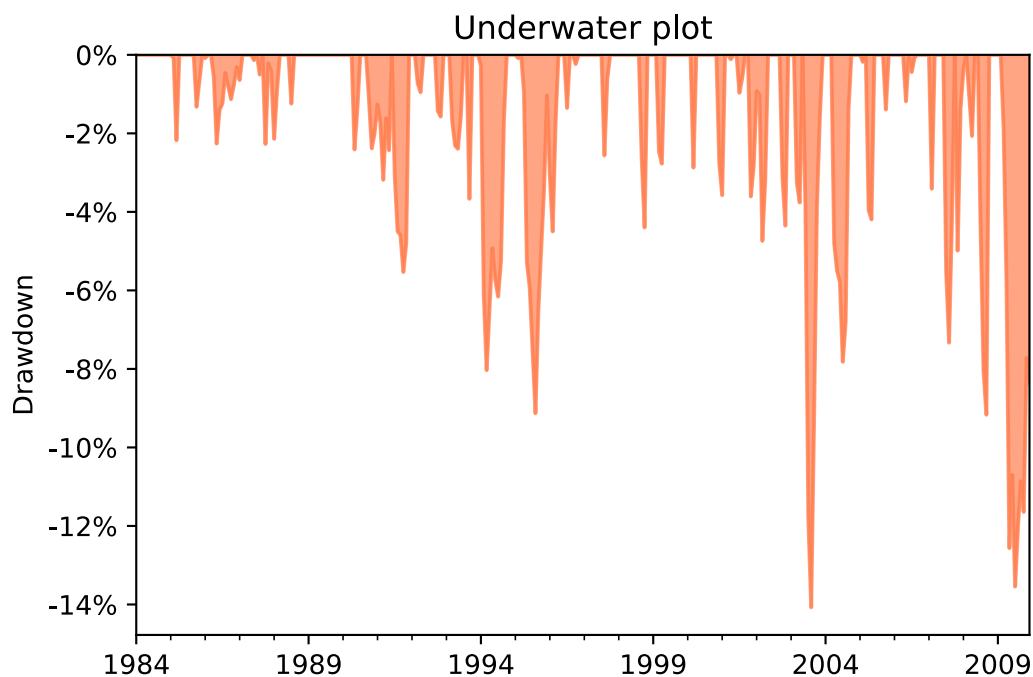
```
[30]: print ("Max Drawdown = ", empirical.max_drawdown(df['port_avg']))  
      print ("Calmar ratio = ", empirical.calmar_ratio(df['port_avg'],  
      ↪period='monthly'))
```

Max Drawdown = -0.1407262029068766  
Calmar ratio = 1.12420608164298

```
[31]: eastern = pytz.timezone('US/Eastern')  
      # df['port_avg'].index  
  
      # SB Updates: added to_datetime conversion  
      df['port_avg'].index = df['port_avg'].index.tz_localize(eastern)  
  
      # pf.create_full_tear_sheet(df['port_avg'])  
      # df['port_avg'].index  
      # df['port_avg'].index = df['port_avg'].index.tz_convert(eastern)  
      # pf.create_full_tear_sheet(df['port_avg'])
```

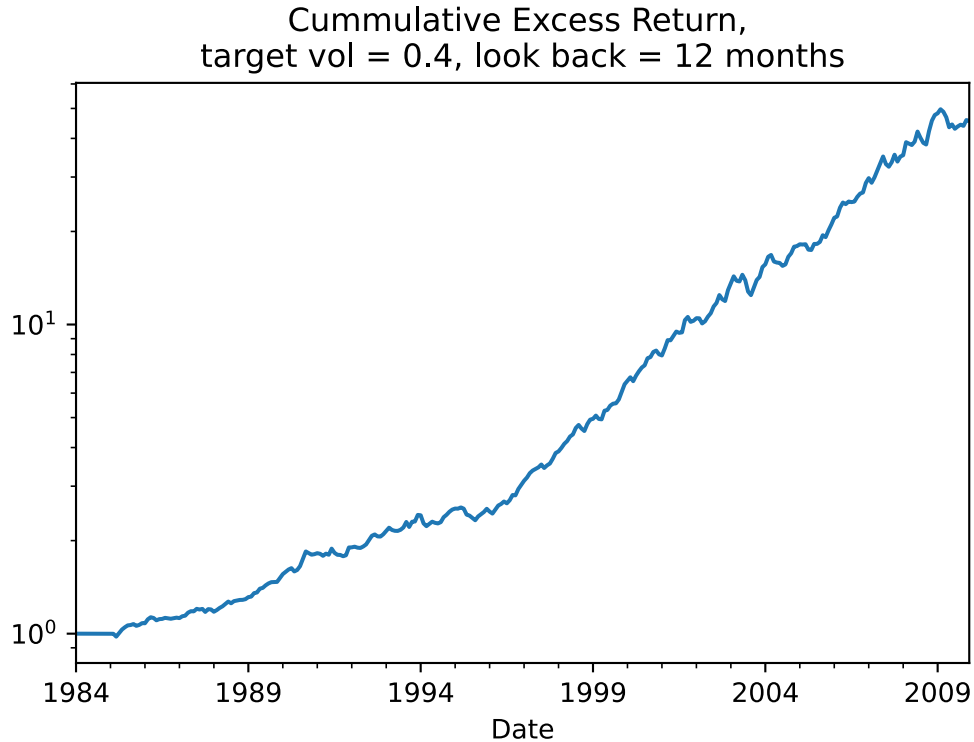
```
[32]: pf.plot_drawdown_underwater(df['port_avg']);
```

[32]:



```
[33]: ax = (1 + df['port_avg']).cumprod().plot(logy=True);
ax.set_title("Cumulative Excess Return, " + \
            "\ntarget vol = " + str(target_vol) + ", look back = " + \
            str(look_back) + " months");
```

[33]:



```
[34]: tmp = df['port_avg'].reset_index()
tmp['Date'] = pd.to_datetime(tmp['Date'], format='%Y-%m-%d')
tmp = tmp.set_index('Date')
tmp['month'] = tmp.index.month
tmp['year'] = tmp.index.year
tmp = np.round(tmp, 3)
res = tmp.pivot('year', 'month', 'port_avg')
res['total'] = np.sum(res, axis=1)
```

```
[17]: fig, ax = plt.subplots(figsize=(20,20));
sns.heatmap(res.fillna(0) * 100,
            annot=True,
            annot_kws={
                "size": 13},
            alpha=1.0,
            center=0.0,
            cbar=True,
```

```

cmap=matplotlib.cm.PiYG,
linewidths=.5,
ax = ax);
ax.set_ylabel('Year');
ax.set_xlabel('Month');
ax.set_title("Monthly Returns (%), " + \
"\ntarget vol = " + str(target_vol) + ", look back = " + \
str(look_back) + " months");
plt.show()

```

[17]:

