Exercise 1.3

Q1. Which of the following matrices are conformable for addition?

A = $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$, C = $\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$,

D = $\begin{bmatrix} 2+1 \\ 3 \end{bmatrix}$, E = $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, f = $\begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$

Solution:

Matrices of same order are conformable for addition.

So, according to this definition;

(i) Matrices A and E are conformable for addition (because both have order 2-by-2).

(ii) Matrices B and D are conformable for addition (because both have order 1-by-1).

(iii) Matrices C and F are conformable for addition (because both have order 3-by-2).

Q2. Find the additive inverse of following matrices.

A = $\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$, C = $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$,

D = $\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$, E = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, F = $\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Solution:

The additive inverse of a matrix is obtained by changing the sign of each entity.

So, according to the definition;

(i) Additive inverse of A = - A = $\begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$

(ii) Additive inverse of B = - B = $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$

(iii) Additive inverse of C = - C = $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$

(vi) Additive inverse of D = - D = $\begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$

(v) Additive inverse of E = - E = $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(vi) Additive inverse of F = - F = $\begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$

Q3. If

A = $\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$, **B =** $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, **C =** $\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$, **D =** $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$,

then find:

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **(ii)** $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **(iii)** $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ **(v)** **2A** **(vi)** **(-1) B**

(vii) **(-2) C** **(viii)** **3D** **(ix)** **3C**

Solution:

(i) **A +** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$

So, **A +** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$

(ii) $\mathbf{B} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1+(-2) \\ (-1)+3 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

So, $\mathbf{B} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(iii) $\mathbf{C} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+(-1) & (-1)+1 & 2+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$

So, $\mathbf{C} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$

(iv) $\mathbf{D} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$

So, $\boldsymbol{D} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$

(v) **2A**

$= 2 \times \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 \times (-1) & 2 \times 2 \\ 2 \times 2 & 2 \times 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

So, $2A = \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

(vi) (-1) B

$= (-1) \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} (-1) \times 1 \\ (-1) \times -1 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

So, $(-1)B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(vii) (-2) C

$= (-2) \times [1 \quad -1 \quad 2]$

$= [(-2) \times 1 \quad (-2) \times -1 \quad (-2) \times 2]$

$= [-2 \quad 2 \quad -4]$

So, $(-2)C = [-2 \quad 2 \quad -4]$

(viii) 3D

$= 3 \times \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 1 & 3 \times 2 & 3 \times 3 \\ 3 \times -1 & 3 \times 0 & 3 \times 2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$

So, $3D = \begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$

(ix) **3C**

$= (3) \times [1 \quad -1 \quad 2]$

$= [(3)\times 1 \quad (3)\times -1 \quad (3)\times 2]$

$= [3 \quad -3 \quad 6]$

So, **(3)C =** $[3 \quad -3 \quad 6]$

Q4. **Perform the indicated operations and simplify the following.**

(i) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(iii) $[2 \quad 3 \quad 1] + [1 \quad 0 \quad 2] - [2 \quad 2 \quad 2]$

(iv) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

(vi) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Solution:

(i) $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1+0+1 & 0+2+1 \\ 0+3+1 & 1+0+0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

(ii) $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1+0-1 & 0+2-1 \\ 0+3-1 & 1+0-0 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} \end{bmatrix}$

(iii) $= \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} + (\begin{bmatrix} 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 2 \end{bmatrix})$

$= \begin{bmatrix} 2+1-2 & 3+0-2 & 1+2-2 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$

(iv) $= \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 0+3 & 1+3 & 2+3 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{3} & \mathbf{4} & \mathbf{5} \end{bmatrix}$

(v) $= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 2+0 & 3+(-2) \\ 2+(-2) & 3+(-1) & 1+0 \\ 3+0 & 1+2 & 2+(-1) \end{bmatrix}$

$= \begin{bmatrix} \mathbf{2} & \mathbf{2} & \mathbf{1} \\ \mathbf{0} & \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{3} & \mathbf{1} \end{bmatrix}$

(vi) $= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+2+1 & 2+1+1 \\ 0+0+1 & 1+0+1 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{4} & \mathbf{4} \\ \mathbf{2} & \mathbf{2} \end{bmatrix}$

Q5. For the matrices A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, **B =** $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$ **and C =** $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

verify the following rules,

(i) $A + C = C + A$ **(ii)** $A + B = B + A$ **(iii)** $B + C = C + B$

(iv) $A + (B + A) = 2A + B$ **(v)** $(C - B) + A + C + (A - B)$

(vi) $2A + B = A + (A + B)$ **(vii)** $(C - B) - A = (C - A) - B$

(viii) $(A + B) + C = A + (B + C)$ **(ix)** $A + (B - C) = (A - C) + B$

(x) $2A + 2B = 2(A + B)$

Solution:

(i) $A + C = C + A$

L.H.S = A + C

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + (-1) & 2 + 0 & 3 + 0 \\ 2 + 0 & 3 + (-2) & 1 + 3 \\ 1 + 1 & -1 + 1 & 0 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix} \qquad \text{------------(1)}$$

R.H.S = C + A

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 + 1 & 0 + 2 & 0 + 3 \\ 0 + 2 & (-2) + 3 & 3 + 1 \\ 1 + 1 & 1 + -1 & 2 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix} \qquad \text{------------(2)}$$

From "1" and "2", it is proved that: **A + C = C + A**

(ii) A + B = B + A

Solution:

$$L.H.S = A + B$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(-1) & 2+(-1) & 3+1 \\ 2+2 & 3+(-2) & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} \quad \text{------------(1)}$$

$$R.H.S = B + A$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & (-1)+2 & 1+3 \\ 2+2 & (-2+3) & 2+1 \\ 3+1 & 1+(-1) & 3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} \quad \text{------------(2)}$$

From "1" and "2", it is proved that: **A + B = B + A**

(iii) B + C = C + B

Solution:

$$L.H.S = B + C$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(-1) & -1+0 & 1+0 \\ 2+(-2) & -2+(-2) & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 0 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix} \quad \text{------------(1)}$$

R.H.S = C + B

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 0+(-1) & 0+1 \\ 0+2 & (-2)+(-2) & 3+2 \\ 1+3 & 1+1 & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 0 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix} \quad \text{------------(2)}$$

From "1" and "2", it is proved that: **B + C = C + B**

(iv) A + (B + A) = 2A + B

Solution:

L.H.S = A + (B + A)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1+1+1 & 2+(-1)+2 & 3+1+3 \\ 2+2+2 & 3+(-2)+3 & 1+2+1 \\ 1+3+1 & -1+1+(-1) & 0+3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix} \quad \text{------------(1)}$$

R.H.S = 2A + B

$$= \quad 2 \times \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \quad \begin{bmatrix} 2 \times 1 & 2 \times 2 & 2 \times 3 \\ 2 \times 2 & 2 \times 3 & 2 \times 1 \\ 2 \times 1 & 2 \times (-1) & 2 \times 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \quad \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \quad \begin{bmatrix} 2+1 & 4+(-1) & 6+1 \\ 4+2 & 6+(-2) & 2+2 \\ 2+3 & (-2)+1 & 0+3 \end{bmatrix}$$

$$= \quad \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix} \qquad \text{------------(2)}$$

From "1" and "2", it is proved that: **A + (B + A) = 2A**

(v) (C – B) + A = C + (A – B)

Solution:

L.H.S = (C-B) + A

$$= \quad \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \quad \begin{bmatrix} -1-1 & 0-(-1) & 0-1 \\ 0-2 & -2-(-2) & 3-2 \\ 1-3 & 1-1 & 2-3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \quad \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \quad \begin{bmatrix} -2+1 & 1+2 & -1+3 \\ -2+2 & 0+3 & 1+1 \\ -2+1 & 0+(-1) & -1 \end{bmatrix}$$

$$= \quad \begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix} \qquad \text{------------(1)}$$

R.H.S = C + (A - B)

$$= \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix} + \left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} \quad \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$= \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix} + \begin{bmatrix}1-1 & 2-(-1) & 3-1\\ 2-2 & 3-(-2) & 1-2\\ 1-3 & -1-1 & 0-3\end{bmatrix}$$

$$= \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix} + \begin{bmatrix}0 & 3 & 2\\ 0 & 5 & -1\\ -2 & -2 & -3\end{bmatrix}$$

$$= \begin{bmatrix}-1+0 & 0+3 & 0+2\\ 0+0 & -2+5 & 3+(-1)\\ 1+(-2) & 1+(-2) & 2+(-3)\end{bmatrix}$$

$$= \begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix} \quad \text{------------(2)}$$

From "1" and "2", it is proved that: **(C – B) + A + C + (A – B)**

(vi) 2A + B = A + (A + B)

Solution:

L.H.S = 2A + B

$$= 2 \times \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}2\times1 & 2\times2 & 2\times3\\ 2\times2 & 2\times3 & 2\times1\\ 2\times1 & 2\times(-1) & 2\times0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}2+1 & 4+(-1) & 6+1\\ 4+2 & 6+(-2) & 2+2\\ 2+3 & (-2)+1 & 0+3\end{bmatrix}$$

$$= \begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix} \quad \text{------------(1)}$$

R.H.S = A + (A + B)

$$= \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\0 & 2 & -1\end{bmatrix} + \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\0 & 2 & -1\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$

$$= \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\0 & 2 & -1\end{bmatrix} + \begin{bmatrix}1+(-1) & 2+(-1) & 3+1\\2+2 & 3+(-2) & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$$

$$= \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\0 & 2 & -1\end{bmatrix} + \begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

$$= \begin{bmatrix}1+2 & 2+1 & 3+4\\2+4 & 3+1 & 1+3\\1+4 & -1+0 & 0+3\end{bmatrix}$$

$$= \begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix} \qquad \text{------------(2)}$$

From "1" and "2", it is proved that: **2A + B = A + (A + B)**

(vii) (C − B) −A = (C−A)   B

Solution:

L.H.S = (C - B) – A

$$= \left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix} \quad \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$= \begin{bmatrix}-1-1 & 0-(-1) & 0-1\\0-2 & -2-(-2) & 3-2\\1-3 & 1-1 & 2-3\end{bmatrix} - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$= \begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix} - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$= \begin{bmatrix}-2-1 & 1-2 & -1-3\\-2-2 & 0-3 & 1-1\\-2-1 & 0-(-1) & -1-0\end{bmatrix}$$

$$= \begin{bmatrix}-3 & -1 & -4\\-4 & -3 & 0\\-3 & 1 & -1\end{bmatrix} \qquad \text{------------(1)}$$

R.H.S = A + (B + C)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1+(-1) & -1+0 & 1+0 \\ 2+(-2) & -2+(-2) & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1 \\ 0 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 2+(-1) & 3+1 \\ 2+0 & 3+(-4) & 1+5 \\ 1+4 & -1+2 & 0+5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix} \quad \text{------------(2)}$$

From "1" and "2", it is proved that: **(A + B) + C = A + (B + C)**

(ix) A + (B – C) = (A – C) + B

Solution:

L.H.S = A + (B – C)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1-(-1) & -1-0 & 1-0 \\ 2-(-2) & -2-(-2) & 2-3 \\ 3-1 & 1-1 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 2+(-1) & 3+1 \\ 2+2 & 3+0 & 1+(-1) \\ 1+2 & -1+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 4 \\ 4 & 3 & 1 \\ 3 & -1 & 1 \end{bmatrix} \quad \text{------------(1)}$$

R.H.S = (A– C) + B

$$= \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} - \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}\right) + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}1-(-1) & 2-0 & 3-0\\2-0 & 3-(-2) & 1-3\\1-1 & -1-1 & 0-2\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}0 & 2 & 3\\2 & 5 & -2\\0 & -2 & -2\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}0+1 & 2+(-1) & 3+1\\2+2 & 5+(-2) & -2+2\\0+3 & -2+1 & -2+3\end{bmatrix}$$

$$= \begin{bmatrix}1 & 1 & 4\\4 & 3 & 0\\3 & -1 & 1\end{bmatrix} \qquad \text{------------(2)}$$

From "1" and "2", it is proved that: **A + (B– C) = (A– C) + B**

(x) 2A + 2B = 2(A + B)

Solution:

L.H.S = 2A + 2B

$$= \left(2 \times \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}\right) + \left(2 \times \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$

$$= \begin{bmatrix}2\times1 & 2\times2 & 2\times3\\2\times2 & 2\times3 & 2\times1\\2\times1 & 2\times(-1) & 2\times0\end{bmatrix} + \begin{bmatrix}2\times1 & 2\times-1 & 2\times1\\2\times2 & 2\times-2 & 2\times2\\2\times3 & 2\times1 & 2\times3\end{bmatrix}$$

$$= \begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix} + \begin{bmatrix}2 & -2 & 2\\4 & -4 & 4\\6 & 2 & 6\end{bmatrix}$$

$$= \begin{bmatrix}2+2 & 4+(-2) & 6+2\\4+4 & 6+(-4) & 2+4\\2+6 & (-2)+2 & 0+6\end{bmatrix}$$

$$= \begin{bmatrix}4 & 2 & 8\\8 & 2 & 6\\8 & 0 & 6\end{bmatrix} \qquad \text{------------(1)}$$

R.H.S = 2(A + B)

$$= 2 \times \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\0 & 2 & -1\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$

$$= 2 \times \begin{bmatrix}1+(-1) & 2+(-1) & 3+1\\2+2 & 3+(-2) & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$$

$$= 2 \times \begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

$$= \begin{bmatrix}2\times2 & 2\times1 & 2\times4\\2\times4 & 2\times1 & 2\times3\\2\times4 & 2\times0 & 2\times3\end{bmatrix}$$

$$= \begin{bmatrix}4 & 2 & 8\\8 & 2 & 6\\8 & 0 & 6\end{bmatrix} \qquad \text{------------(2)}$$

From "1" and "2", it is proved that: **2A + 2B = 2(A + B)**

Q6. If A = $\begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}$ and B = $\begin{bmatrix}0 & 7\\-3 & 8\end{bmatrix}$, find

(i) 3A – 2B **(ii) $2A^t - 3B^t$**

Solution:

(i) 3A – 2B

$$= 3 \times \begin{bmatrix}1 & -2\\3 & 4\end{bmatrix} - 2 \times \begin{bmatrix}0 & 7\\-3 & 8\end{bmatrix}$$

$$= \begin{bmatrix}3\times1 & 3\times-2\\3\times3 & 3\times4\end{bmatrix} - \begin{bmatrix}2\times0 & 2\times7\\2\times(-3) & 2\times8\end{bmatrix}$$

$$= \begin{bmatrix}3 & -6\\9 & 12\end{bmatrix} - \begin{bmatrix}0 & 14\\-6 & 16\end{bmatrix}$$

$$= \begin{bmatrix}3-0 & (-6)-(14)\\9-(-6) & 12-16\end{bmatrix}$$

$$= \begin{bmatrix}3 & -20\\15 & -4\end{bmatrix}$$

So, **3A – 2B** = $\begin{bmatrix} \mathbf{3} & \mathbf{-20} \\ \mathbf{15} & \mathbf{-4} \end{bmatrix}$

(ii) $2A^t - 3B^t$

Solution: $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, $B^t = \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$

$2A^t = 2 \times \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, $3B^t = 3 \times \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$

$2A^t = \begin{bmatrix} 2 \times 1 & 2 \times 3 \\ 2 \times (-2) & 2 \times 4 \end{bmatrix}$, $3B^t = \begin{bmatrix} 3 \times 0 & 3 \times (-3) \\ 3 \times 7 & 3 \times 8 \end{bmatrix}$

$2A^t = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix}$, $3B^t = \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$

$$2A^t - 3B^t = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 6-(-9) \\ -4-21 & 8-24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$$

So, **$2A^t - 3B^t$** = $\begin{bmatrix} \mathbf{2} & \mathbf{15} \\ \mathbf{-25} & \mathbf{-16} \end{bmatrix}$

Q7. If $2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$, then find a and b.

Solution:

Given $2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$

L.H.S = $2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 2 \times 2 & 2 \times 4 \\ 2 \times (-3) & 2 \times a \end{bmatrix} + \begin{bmatrix} 3 \times 1 & 3 \times b \\ 3 \times 8 & 3 \times (-4) \end{bmatrix}$$

$= \left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t$

$= \left(\begin{bmatrix}1+1 & 2+1\\0+2 & 1+0\end{bmatrix}\right)^t$

$= \left(\begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}\right)^t$

$= \begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$ --------- (i)

R.H.S $= A^t + B^t$

$= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t + \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$

$= \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$

$= \begin{bmatrix}1+1 & 0+2\\2+1 & 1+0\end{bmatrix}$

$= \begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$ --------- (ii)

From (i) and (ii), it is proved that: **$(A + B)^t = A^t + B^t$**

(ii) $(A - B)^t = A^t - B^t$

Solution:

L.H.S $= (A - B)^t$

$= \left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t$

$= \left(\begin{bmatrix}1-1 & 2-1\\0-2 & 1-0\end{bmatrix}\right)^t$

$= \left(\begin{bmatrix}0 & 1\\-2 & 1\end{bmatrix}\right)^t$

$= \begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$ --------- (i)

R.H.S $= A^t + B^t$

$= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$

$= \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$

$$= \begin{bmatrix} 1-1 & 0-2 \\ 2-1 & 1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix} \qquad \text{---------- (ii)}$$

From (i) and (ii), it is proved that: **$(A - B)^t = A^t - B^t$**

(iii) To prove $A + A^t$ is symmetric

Solution:

$$A + A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \qquad \text{---------- (i)}$$

Now we will take transpose of A + A'

$$(A + A') = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \qquad \text{---------- (ii)}$$

From 1 and 2 , it is proved that: **$A + A^t = (A + A^t)^t$**

So, it is Symmetric.

(iv) To prove $A - A^t$ is skew symmetric

Solution:

$$A - A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \qquad \text{---------- (i)}$$

Now we will take transpose of A - A'

$$(A - A') = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$= (-1)\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$= (-)\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \quad \text{--------- (ii)}$$

$$= -(A - A')$$

From (i) and (ii), it is obvious that:

$A - A^t$ is skew symmetric

(v) To prove B + B' is symmetric
Solution:

$$B + B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} \quad \text{--------- (i)}$$

Now we will take transpose of B + B'

$$(B + B^t) = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} \quad \text{--------- (ii)}$$

From 1 and 2 , it is proved that: **$B + B^t = (B + B^t)^t$**

So, it is Symmetric.

(vi) To prove B - B' is skew symmetric
Solution:

$$B - B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \text{---------- (i)}$$

Now we will take transpose of B + B'

$$(B + B') = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= (-1)\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= (-)\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \text{---------- (ii)}$$

$$= -(B - B')$$

From 1 and 2 , it is proved that: **B - B^t**

So, it is skew Symmetric.