

Exercise 4.1

Q.1 Identify whether the following algebraic expressions are polynomials (Yes or No).

(i) $3x^2 + \frac{1}{x} - 5$

No (Because of $\frac{1}{x}$) Ans.

(ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$

No (Because \sqrt{x} or $(x)^{\frac{1}{2}}$) Ans.

(iii) $x^2 - 3x + \sqrt{2}$

Yes (Because no variable has power in fraction). Ans

(iv) $\frac{3x}{2x-1} + 8$

No (Because of $\frac{1}{2x-1}$) Ans

Q.2 State whether each of the following expressions is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x} + 5}$

Irrational Ans

(ii) $\frac{x^3 - 2x^3 + \sqrt{3}}{2 + 3x - x^2}$

Rational Ans

(iii) $\frac{x^2 + 6x + 9}{x^2 - 9}$

Rational Ans

(iv) $\frac{2\sqrt{x} + 3}{2\sqrt{x} - 3}$

Irrational Ans

Q.3 Reduce the following expression to the lowest form.

(i) $\frac{120x^2y^3z^5}{30x^3yz^2}$

Solution: $\frac{120x^2y^3z^5}{30x^3yz^2}$

$$= \frac{120x^2y^3z^5}{30x^3yz^2}$$

$$= 4x^{2-3}y^{3-1}z^{5-2}$$

$$= 4x^{-1}y^2z^3$$

$$= \frac{4y^2z^3}{x} \text{ Ans}$$

(ii) $\frac{8a(x+1)}{2(x^2-1)}$

Solution: $\frac{8a(x+1)}{2(x^2-1)}$

$$= \frac{8a(x+1)}{2(x^2-1)}$$

$$= \frac{4a(x+1)}{(x-1)(x+1)}$$

$$= \frac{4a}{x-1} \text{ Ans}$$

(iii) $\frac{(x+y)^2 - 4xy}{(x-y)^2}$

Solution: $\frac{(x+y)^2 - 4xy}{(x-y)^2}$

$$\therefore (x+y)^2 = x^2 + y^2 + 2xy$$

$$\therefore (x-y)^2 = x^2 + y^2 - 2xy$$

$$= \frac{x^2 + y^2 + 2xy - 4xy}{x^2 + y^2 - 2xy}$$

$$= \frac{x^2 + y^2 - 2xy}{x^2 + y^2 - 2xy}$$

$$= \frac{(x-y)^2}{(x-y)^2}$$

= 1 **Ans**

(iv) $\frac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x-y)(x^2 + xy + y^2)}$

Solution: $\frac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x-y)(x^2 + xy + y^2)}$

$$(a^3 + b^3) = (a-b)(a^2 + ab + b^2)$$

$$= \frac{\cancel{(x^3 - y^3)}(x^2 - 2xy + y^2)}{\cancel{(x^3 - y^3)}}$$

$$= x^2 - 2xy + y^2$$

$$\therefore (x-y)^2 = x^2 - 2xy + y^2$$

$$= (x-y)^2 \text{ Ans}$$

(v) $\frac{(x+2)(x^2 - 1)}{(x+1)(x^2 - 4)}$

Solution: $\frac{(x+2)(x^2 - 1)}{(x+1)(x^2 - 4)}$

$$= \frac{(x+2)[(x)^2 - (1)^2]}{(x+1)[(x)^2 - (2)^2]}$$

$$= \frac{\cancel{(x+2)}(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x-2)\cancel{(x+2)}}$$

$$= \frac{(x-1)}{(x-2)} \text{ Ans}$$

(vi) $\frac{x^2 - 4x + 4}{2x^2 - 8}$

Solution: $\frac{x^2 - 4x + 4}{2x^2 - 8}$

$$\therefore (a-b)^2 = a^2 - 2ab + b^2$$

$$\therefore a^2 - b^2 = (a+b)(a-b)$$

$$= \frac{(x)^2 - 2(x)(2) + (2)^2}{2(x^2 - 4)}$$

$$= \frac{(x-2)^2}{2[(x)^2 - (2)^2]}$$

$$= \frac{(x-2)^2}{2(x+2)(x-2)}$$

$$= \frac{(x-2)\cancel{(x-2)}}{2(x+2)\cancel{(x-2)}}$$

$$= \frac{x-2}{2(x+2)} \text{ Ans}$$

(vii) $\frac{64x^5 - 64x}{(8x^2 + 8)(2x+2)}$

Solution: $\frac{64x^5 - 64x}{(8x^2 + 8)(2x+2)}$

$$= \frac{64x(x^4 - 1)}{8(x^2 + 1).2(x+1)}$$

$$= \frac{64[(x^2)^2 - (1)^2]}{16(x^2 + 1)(x+1)}$$

$$= \frac{64(x^2 - 1)(x^2 + 1)}{16\cancel{(x^2 + 1)}(x+1)}$$

$$= \frac{4x(x-1)(x+1)}{\cancel{(x+1)}}$$

$$= 4x(x-1) \text{ Ans}$$

(viii) $\frac{9x^2 - (x^2 - 4)^2}{4 + 3x - x^2}$

Solution: $\frac{9x^2 - (x^2 - 4)^2}{4 + 3x - x^2}$

$$\begin{aligned}
&= \frac{(3x)^2 - (x^2 - 4)^2}{4 + 3x - x^2} \\
&= \frac{(3x + x^2 - 4)(3x - x^2 + 4)}{4 + 3x - x^2} \\
&= \frac{(x^2 + 3x - 4) \cancel{(-x^2 + 3x + 4)}}{\cancel{(-x^2 + 3x + 4)}} \\
&= x^2 + 3x - 4 \text{ Ans}
\end{aligned}$$

$$\begin{aligned}
&= \frac{9 - 8}{-4} \\
&= \frac{1}{-4} \\
&= -\frac{1}{4} \text{ Ans}
\end{aligned}$$

(b) $\frac{x^2y^2 - 5z^4}{xyz}$ for $x = 4, y = -2$ and $z = -1$

Solution: $\frac{x^2y^3 - 5z^4}{xyz}$

$$\begin{aligned}
&= \frac{(4)^2(-2)^3 - 5(-1)^4}{(4)(-2)(-1)} \\
&= \frac{16(-8) - 5(1)}{8} \\
&= \frac{16(-8) - 5(1)}{8} \\
&= \frac{-128 - 5}{8} \\
&= -\frac{133}{8} \\
&= -16\frac{5}{8} \text{ Ans}
\end{aligned}$$

Q.4 Evaluate

(a) $\frac{x^3y - 2z}{xz}$ for

- (i) $x = 3, y = -1, z = -2$
(ii) $x = -1, y = -9, z = 4$

Solution for 1st part

When $x = 3, y = -1, z = -2$

$$\begin{aligned}
&\frac{x^3y - 2z}{xz} = \\
&= \frac{(3)^3(-1) - 2(-2)}{(3)(-2)} \\
&= \frac{27(-1) + 4}{-6} \\
&= \frac{-27 + 4}{-6} \\
&= \frac{-23}{-6} \\
&= \frac{23}{6} \text{ Ans}
\end{aligned}$$

Solution for 2nd Part.

When $x = -1, y = -9, z = 4$

$$\begin{aligned}
&\frac{x^3y - 2z}{xz} = \\
&= \frac{(-1)^3(-9) - 2(4)}{(-1)(4)} \\
&= \frac{-1(-9) - 8}{-4}
\end{aligned}$$

Q.5 Perform the indicated operation and simplify.

(i) $\frac{15}{2x-3y} - \frac{4}{3y-2x}$

Solution: $\frac{15}{2x-3y} - \frac{4}{3y-2x}$

$$\begin{aligned}
&= \frac{15}{2x-3y} - \frac{4}{-2x+3y} \\
&= \frac{15}{2x-3y} - \frac{4}{-(2x-3y)} \\
&= \frac{15}{2x-3y} + \frac{4}{2x-3y} \\
&= \frac{19}{2x-3y} \text{ Ans}
\end{aligned}$$

$$(ii) \quad \frac{1+2x}{1-2x} - \frac{1-2x}{1+2x}$$

$$\text{Solution: } \frac{1+2x}{1-2x} - \frac{1-2x}{1+2x}$$

$$= \frac{(1+2x)^2 - (1-2x)^2}{(1-2x)(1+2x)}$$

$$= \frac{(1)^2 + (2x)^2 + 2(2x)(1) - [(1)^2 + (2x)^2 - 2(2x)(1)]}{(1)^2 - (2x)^2}$$

$$= \frac{1+4x^2+4x - [1+4x^2-4x]}{1-4x^2}$$

$$= \frac{1+4x^2+4x-1-4x^2+4x}{1-4x^2}$$

$$= \frac{4x+4x}{1-4x^2}$$

$$= \frac{8x}{1-4x^2} \quad \text{Ans}$$

$$(iii) \quad \frac{x^2-25}{x^2-36} - \frac{x+5}{x+6}$$

$$\text{Solution: } \frac{x^2-25}{x^2-36} - \frac{x+5}{x+6}$$

$$= \frac{(x)^2 - (5)^2}{(x)^2 - (6)^2} - \frac{x+5}{x+6}$$

$$= \frac{(x+5)(x-5)}{(x+6)(x-6)} - \frac{x+5}{x+6}$$

$$= \frac{(x+5)(x-5) - (x-6)(x+5)}{(x+6)(x-6)}$$

$$= \frac{(x+5)[(x-5) - (x-6)]}{x^2 - 6^2}$$

$$= \frac{(x+5)(x-5-x+6)}{x^2 - 36}$$

$$= \frac{(x+5)(1)}{x^2 - 36}$$

$$= \frac{x+5}{x^2 - 36} \quad \text{Ans}$$

$$(iv) \quad \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2 - y^2}$$

$$\text{Solution: } \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2 - y^2}$$

$$= \frac{x(x+y) - y(x-y)}{(x-y)(x+y)} - \frac{2xy}{x^2 - y^2}$$

$$= \frac{x^2 + xy - xy + y^2}{(x)^2 - (y)^2} - \frac{2xy}{x^2 - y^2}$$

$$= \frac{x^2 + y^2}{x^2 - y^2} - \frac{2xy}{x^2 - y^2}$$

$$= \frac{x^2 + y^2 - 2xy}{x^2 - y^2}$$

$$= \frac{(x-y)^2}{x^2 - y^2}$$

$$= \frac{(x-y)(x-y)}{(x+y)(x-y)}$$

$$= \frac{x-y}{x+y} \quad \text{Ans}$$

$$(v) \quad \frac{x-2}{x^2+6x+9} - \frac{x+2}{2x^2-18}$$

$$\text{Solution: } \frac{x-2}{x^2+6x+9} - \frac{x+2}{2x^2-18}$$

$$= \frac{x-2}{(x)^2 + 2(3)(x) + 3^2} - \frac{x+2}{2(x^2 - 9)}$$

$$= \frac{x-2}{(x+3)^2} - \frac{x+2}{2[(x)^2 - (3)^2]}$$

$$= \frac{x-2}{(x+3)^2} - \frac{x+2}{2(x-3)(x+3)}$$

$$= \frac{x-2}{(x+3)(x+3)} - \frac{x+2}{2(x+3)(x-3)}$$

$$= \frac{2(x-3)(x-2) - (x+3)(x+2)}{2(x+3)(x+3)(x-3)}$$

$$= \frac{2(x^2 - 2x - 3x + 6) - (x^2 + 2x + 3x + 6)}{2(x+3)(x+3)(x-3)}$$

$$= \frac{2(x^2 - 5x + 6) - (x^2 + 5x + 6)}{2(x+3)(x+3)(x-3)}$$

$$= \frac{2x^2 - 10x + 12 - x^2 - 5x - 6}{2(x+3)^2(x-3)}$$

$$= \frac{x^2 - 15x + 6}{2(x+3)^2(x-3)} \text{ Ans}$$

(vi) $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$

Solution: $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$

$$= \frac{(x+1)-(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{x+1-x+1}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)} - \frac{4}{x^4-1}$$

$$= \frac{2x^2+2-2x^2+2}{(x^2)^2-(1)^2} - \frac{4}{x^4-1}$$

$$= \frac{4}{x^4-1} - \frac{4}{x^4-1}$$

$$= \frac{4-4}{x^4-1}$$

$$= \frac{0}{x^4-1}$$

$$= 0 \text{ Ans}$$

Q.6 Perform the indicated operation and simplify.

(i) $(x^2 - 49) \cdot \frac{5x+2}{x+7}$

Solution: $(x^2 - 49) \cdot \frac{5x+2}{x+7}$

$$= [(x)^2 - (7)^2] \cdot \frac{5x+2}{x+7}$$

$$= (x+7)(x-7) \frac{(5x+2)}{(x+7)}$$

$$= (x-7)(5x+2) \text{ Ans}$$

(ii) $\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+2(x)(3)+(3)^2}$

Solution: $\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+2(x)(3)+(3)^2}$

$$= \frac{4(x-3)}{(x^2)-(3)^2} \div \frac{2(9-x^2)}{(x+3)^2}$$

$$= \frac{\cancel{4}(x-3)}{\cancel{(x-3)}(x+3)} \times \frac{(x+3)^2}{2(9-x^2)}$$

$$= \frac{4}{x+3} \times \frac{(x+3)^2}{2(3+x)(3-x)}$$

$$= \frac{\cancel{4}x(x+3)^2}{\cancel{2}(x+3)^2(3-x)}$$

$$= \frac{2}{3-x} \text{ Ans}$$

(iii) $\frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$

Solution: $\frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$

$$= \frac{(x^2)^3-(y^2)^3}{x^2-y^2} \div (x^4+x^2y^2+y^4)$$

$$= \frac{\cancel{(x^2-y^2)}[(x^2)^2+x^2y^2+(y^2)^2]}{\cancel{(x^2-y^2)}} \div (x^4+x^2y^2+y^4)$$

$$= \left(\cancel{x^4+x^2y^2+y^4} \right) \times \frac{1}{\left(\cancel{x^4+x^2y^2+y^4} \right)}$$

$$= 1 \text{ Ans}$$

(iv) $\frac{x^2-1}{x^2+2x+1} \cdot \frac{x+5}{1-x}$

Solution: $\frac{x^2-1}{x^2+2x+1} \cdot \frac{x+5}{1-x}$

$$= \frac{(x+1)(x-1)}{(x^2+2(x)(1)+(1)^2)} \times \frac{x+5}{-(x-1)}$$

$$= \frac{(x+1)\cancel{(x-1)}}{(x+1)^2} \times \frac{(x+5)}{-\cancel{(x-1)}}$$

$$= -\frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x+1)}$$

$$= -\frac{(x+5)}{x+1} \text{ Ans}$$

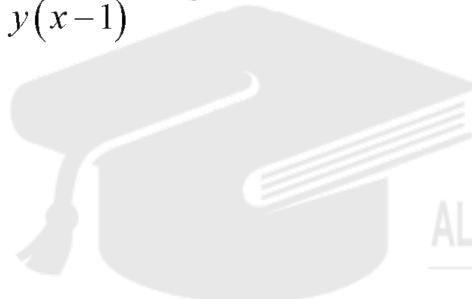
(v) $\frac{x^2 + xy}{y(x+y)} \cdot \frac{x^2 + xy}{y(x+y)} \div \frac{x^2 - x}{xy - 2y}$

Solution: $\frac{x^2 + xy}{y(x+y)} \cdot \frac{x^2 + xy}{y(x+y)} \div \frac{x^2 - x}{xy - 2y}$

$$= \frac{x(\cancel{x+y})}{y(\cancel{x+y})} \cdot \frac{x(\cancel{x+y})}{y(\cancel{x+y})} \div \frac{x(x-1)}{y(x-2)}$$

$$= \frac{x \cancel{x}}{y \cancel{x}} \times \frac{\cancel{x}(x-2)}{\cancel{x}(x-1)}$$

$$= \frac{x(x-2)}{y(x-1)} \text{ Ans}$$



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Exercise 4.2

Q.1 Solve

- (i) If $a+b=10$ and $a-b=6$, then find the value of (a^2+b^2)

Solution:

$$2(a^2+b^2) = (a+b)^2 + (a-b)^2$$

$$2(a^2+b^2) = (10)^2 + (6)^2$$

$$2(a^2+b^2) = 100 + 36$$

$$2(a^2+b^2) = 136$$

$$(a^2+b^2) = \frac{136}{2}^{68}$$

$$(a^2+b^2) = 68 \text{ Ans}$$

- (ii) If $a+b=5$, $a-b=\sqrt{17}$, then find the value of ab .

Solution:

$$4ab = (a+b)^2 - (a-b)^2$$

$$4ab = (5)^2 - (\sqrt{17})^2$$

$$4ab = 25 - 17$$

$$4ab = 8$$

$$ab = \frac{8}{4}$$

$$ab = 2$$

$$ab = 2 \text{ Ans}$$

- Q.2 If $a^2+b^2+c^2=45$ and $a+b+c=-1$, then find the value of $ab+bc+ca$.

Solution: $a^2+b^2+c^2 = 45$

$$a+b+c = -1$$

$$ab+bc+ca = ?$$

We know that

$$(a+b+c)^2 = a^2+b^2+c^2 + 2(ab+bc+ca)$$

$$(-1)^2 = 45 + 2(ab+bc+ca)$$

$$1 = 45 + 2(ab+bc+ca)$$

$$1 - 45 = 2(ab+bc+ca)$$

$$-44 = 2(ab+bc+ca)$$

$$\cancel{\frac{44}{2}}^{22} = (ab+bc+ca)$$

$$(ab+bc+ca) = -22 \text{ Ans}$$

- Q.3 If $m+n+p=10$ and $mn+np+mp=27$, find the value of $m^2+n^2+p^2$

Solution: $m+n+p=10$

$$mn+np+np=27,$$

$$m^2+n^2+p^2=?$$

We know that

$$(m+n+p)^2 = m^2+n^2+p^2+2mn+2np+2mp$$

$$(10)^2 = m^2+n^2+p^2+2(mn+np+mp)$$

$$100 = m^2+n^2+p^2+2(27)$$

$$100 = m^2+n^2+p^2+54$$

$$100 - 54 = m^2+n^2+p^2$$

$$m^2+n^2+p^2 = 46 \text{ Ans}$$

- Q.4 If $x^2+y^2+z^2=78$ and $xy+yz+zx=59$, find the value of $x+y+z$.

Solution: $x^2+y^2+z^2=78$

$$xy+yz+zx=59,$$

$$x+y+z=?$$

We know that

$$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

$$(x+y+z)^2 = 78 + 2(xy+yz+zx)$$

$$(x+y+z)^2 = 78 + 2(59)$$

$$(x+y+z)^2 = 78 + 118$$

$$(x+y+z)^2 = 196$$

Taking square root at both sides

$$\sqrt{(x+y+z)^2} = \pm\sqrt{196}$$

$$x+y+z = \pm 14 \text{ Ans}$$

Q.5 If $x+y+z=12$ and $x^2+y^2=64$, find the value of $xy+yz+zx$.

Solution: $x+y+z=12$

$$x^2+y^2=64$$

$$xy+yz+zx=?$$

We know that

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy+yz+zx)$$

$$(12)^2 = 64 + 2(xy+yz+zx)$$

$$144 - 64 = 2(xy+yz+zx)$$

$$80 = 2(xy+yz+zx)$$

$$\frac{80}{2} = xy+yz+zx$$

$$40 = xy+yz+zx$$

$$xy+yz+zx = 40 \text{ Ans}$$

Q.6 If $x+y=7$ and $xy=12$, then find the value of x^3+y^3

Solution: $x+y=7$

$$xy=12$$

$$x^3+y^3=?$$

We know that

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

$$(7)^3 = x^3 + y^3 + 3(12)(7)$$

$$343 = x^3 + y^3 + 252$$

$$343 - 252 = x^3 + y^3$$

$$91 = x^3 + y^3$$

$$x^3 + y^3 = 91 \text{ Ans}$$

Q.7 If $3x+4y=11$ and $xy=12$, then find the value of $27x^3+64y^3$.

Solution: $3x+4y=11$

$$xy=12$$

$$27x^3+64y^3=?$$

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

$$(3x+4y)^3 = (3x)^3 + (4y)^3 + 3(3x)(4y)(3x+4y)$$

$$(3x+4y)^3 = 27x^3 + 64y^3 + 36xy(3x+4y)$$

$$(11)^3 = 27x^3 + 64y^3 + 36(12)(11)$$

$$1331 = 27x^3 + 64y^3 + 4752$$

$$1331 - 4752 = 27x^3 + 64y^3$$

$$-3421 = 27x^3 + 64y^3$$

$$27x^3 + 64y^3 = -3421 \text{ Ans}$$

Q.8 If $x-y=4$ and $xy=21$, then find the value of x^3-y^3

Solution: $x-y=4$

$$xy=21$$

$$x^3-y^3=?$$

We know that

$$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$$

$$(4)^3 = x^3 - y^3 - 3(21)(4)$$

$$64 = x^3 - y^3 - 252$$

$$64 + 252 = x^3 - y^3$$

$$316 = x^3 - y^3$$

$$x^3 - y^3 = 316 \text{ Ans}$$

Q.9 If $5x-6y=13$ and $xy=6$, then find the value of $b125x^3-216y^3$

Solution: $5x-6y=13$

$$xy=6$$

$$125x^3-216y^3=?$$

We know that

$$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$$

$$(5x-6y)^3 = (5x)^3 - (6y)^3 - 3(5x)(6y)(5x-6y)$$

$$(5x-6y)^3 = 125x^3 - 216y^3 - 90xy(5x-6y)$$

$$(13)^3 = 125x^3 - 216y^3 - 90(6)(13)$$

$$2197 = 125x^3 - 216y^3 - 7020$$

$$2197 + 7020 = 125x^3 - 216y^3$$

$$9217 = 125x^3 - 216y^3$$

$$125x^3 - 216y^3 = 9217 \text{ Ans}$$

$$343 + 21 = x^3 - \frac{1}{x^3}$$

$$364 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 364 \text{ Ans}$$

Q.10 If $x + \frac{1}{x} = 3$ then find the value of

$$x^3 + \frac{1}{x^3}$$

$$\text{Solution: } x + \frac{1}{x} = 3$$

$$x^3 + \frac{1}{x^3} = ?$$

We know that

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$(3)^3 = x^3 + \frac{1}{x^3} + 3(3)$$

$$27 = x^3 + \frac{1}{x^3} + 9$$

$$27 - 9 = x^3 + \frac{1}{x^3}$$

$$18 = x^3 + \frac{1}{x^3}$$

$$x^3 + \frac{1}{x^3} = 18 \text{ Ans}$$

Q.11 If $x - \frac{1}{x} = 7$, then find the value

$$\text{of } x^3 - \frac{1}{x^3}$$

$$\text{Solution: } x - \frac{1}{x} = 7$$

$$x^3 - \frac{1}{x^3} = ?$$

We know that

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$(7)^3 = x^3 - \frac{1}{x^3} - 3(7)$$

$$343 = x^3 - \frac{1}{x^3} - 21$$

Q.12 If $\left[3x + \frac{1}{3x}\right] = 5$, then find the value

$$\text{of } \left[27x^3 + \frac{1}{27x^3}\right]$$

$$\text{Solution: } \left[3x + \frac{1}{3x}\right] = 5$$

$$\left[27x^3 + \frac{1}{27x^3}\right] = ?$$

We know that

$$\left(3x + \frac{1}{3x}\right)^3 = (3x)^3 + \left(\frac{1}{3x}\right)^3 + 3\left(3x\right)\left(\frac{1}{3x}\right)\left(3x + \frac{1}{3x}\right)$$

$$(5)^3 = 27x^3 + \frac{1}{27x^3} + 3\left(3x + \frac{1}{3x}\right)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 3(5)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 15$$

$$125 - 15 = 27x^3 + \frac{1}{27x^3}$$

$$110 = 27x^3 + \frac{1}{27x^3}$$

$$27x^3 + \frac{1}{27x^3} = 110 \text{ Ans}$$

Q.13 If $\left(5x - \frac{1}{5x}\right) = 6$, then find the

$$\text{value of } \left(125x^3 - \frac{1}{125x^3}\right)$$

$$\text{Solution: } \left(5x - \frac{1}{5x}\right) = 6$$

$$\left(125x^3 - \frac{1}{125x^3}\right) = ?$$

We know that

$$\left(5x - \frac{1}{5x}\right)^3 = (5x)^3 - \left(\frac{1}{5x}\right)^3 - 3\left(5x\right)\left(\frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right)$$

$$(6)^3 = 125x^3 - \frac{1}{125x^3} - 3(6)$$

$$216 = 125x^3 - \frac{1}{125x^3} - 18$$

$$216 + 18 = 125x^3 - \frac{1}{125x^3}$$

$$234 = 125x^3 - \frac{1}{125x^3}$$

$$125x^3 - \frac{1}{125x^3} = 234 \text{ Ans}$$

Q.14 Factorize

(i) $x^3 - y^3 - x + y$

Solution: $x^3 - y^3 - x + y$

$$= (x)^3 - (y)^3 - 1(x - y)$$

$$= (x - y)(x^2 + xy + y^2) - 1(x - y)$$

$$= (x - y)(x^2 + xy + y^2 - 1) \text{ Ans}$$

(ii) $8x^3 - \frac{1}{27y^3}$

Solution: $8x^3 - \frac{1}{27y^3}$

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$

$$= \left[2x - \frac{1}{3y}\right] \left[(2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right]$$

$$= \left(2x - \frac{1}{3y}\right) \left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right) \text{ Ans}$$

Q.15 Find the products, using formula.

(i) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

Solution: $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$$= (x^2 + y^2) \left[(x^2)^2 - (x^2)(y^2) + (y^2)^2 \right]$$

$$\left[(x^2)^3 + (y^2)^3 \right]$$

$$= x^6 + y^6 \text{ Ans}$$

(ii) $(x^3 - y^3)(x^6 + x^3y^3 + y^6)$

Solution: $(x^3 - y^3)(x^6 + x^3y^3 + y^6)$

$$(x^3 - y^3) \left[(x^3)^2 + (x^3)(y^3) + (y^3)^2 \right]$$

$$\left[(x^3)^3 - (y^3)^3 \right]$$

$$= x^9 - y^9 \text{ Ans}$$

(iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)$

$$(x^2 + xy + y^2)(x^4 - x^2y^2 + y^4)$$

$$= [(x - y)(x^2 + xy + y^2)][(x + y)(x^2 - xy + y^2)]$$

$$= [(x^2 + y^2)(x^4 - x^2y^2 + y^4)]$$

$$= [(x^3 - y^3)(x^3 + y^3)][(x^2)^3 + (y^2)^3]$$

$$= [(x^3)^2 - (y^3)^2][(x^6 + y^6)]$$

$$= [(x^6 - y^6)(x^6 + y^6)]$$

$$= [(x^6)^2 - (y^6)^2]$$

$$= x^{12} - y^{12} \text{ Ans}$$

(iv) $(2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)(4x^4 - 2x^2 + 1)$

Solution:

$$\begin{aligned} & (2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)(4x^4 - 2x^2 + 1) \\ &= [(2x^2 - 1)(4x^4 + 2x^2 + 1)][(2x^2 + 1)(4x^4 - 2x^2 + 1)] \\ &= [(2x^2)^3 - (1)^3][(2x^2)^3 + (1)^3] \\ &= (8x^6 - 1)(8x^6 + 1) \\ &= (8x^6)^2 - (1)^2 \\ &= 64x^{12} - 1 \text{ Ans} \end{aligned}$$



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$$\begin{aligned}
 &= \frac{3}{4} \times 4 \times \sqrt[3]{2} \\
 &= 3 \times \sqrt[3]{2} \\
 &= 3\sqrt[3]{2} \text{ Ans}
 \end{aligned}$$

Exercise 4.3

Q.1 Express each of the following surd in the simplest form:

(i) $\sqrt{180}$

Solution: $\sqrt{180}$

$$= (180)^{\frac{1}{2}}$$

$$= (2 \times 2 \times 3 \times 3 \times 5)^{\frac{1}{2}}$$

$$= (2^2 \times 3^2 \times 5)^{\frac{1}{2}}$$

$$= 2^{\frac{2 \times 1}{2}} \times 3^{\frac{2 \times 1}{2}} \times 5^{\frac{1}{2}}$$

$$= 2 \times 3 \times \sqrt{5}$$

$$= 6\sqrt{5} \text{ Ans}$$

(ii) $3\sqrt{162}$

Solution: $3\sqrt{162}$

$$3(\sqrt{81 \times 2})$$

$$= 3(\sqrt{9^2} \times \sqrt{2})$$

$$= 3 \times 9(\sqrt{2})$$

$$= 27\sqrt{2} \text{ Ans}$$

(iii) $\frac{3}{4}\sqrt[3]{128}$

Solution: $\frac{3}{4}\sqrt[3]{128}$

$$= \frac{3}{4}(\sqrt[3]{64 \times 2})$$

$$= \frac{3}{4}(\sqrt[3]{4^3 \times 2})$$

$$= \frac{3}{4} \left[\sqrt[3]{4^3} \times \sqrt[3]{2} \right]$$

(iv) $\sqrt[5]{96x^6y^7z^8}$

Solution: $\sqrt[5]{96x^6y^7z^8}$

$$= \sqrt[5]{32 \times 3 \times x^5y^5z^5 \times x^1y^2z^3}$$

$$= \sqrt[5]{2^5 \times 3 \times x^5y^5z^5 \times xy^2z^3}$$

$$= \sqrt[5]{2^5 x^5 y^5 z^5} \times \sqrt[5]{3xy^2 z^3}$$

$$= \sqrt[5]{2^5} \times \sqrt[5]{x^5} \times \sqrt[5]{y^5} \times \sqrt[5]{z^5} \times \sqrt[5]{3xy^2 z^3}$$

$$= 2xyz\sqrt[5]{3xy^2 z^3} \text{ Ans}$$

Q.2 Simplify

(i) $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

Solution: $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

$$= \frac{\sqrt{9 \times 2}}{\sqrt{3} \times \sqrt{2}}$$

$$= \frac{\sqrt{3^2} \times \sqrt{2}}{\sqrt{3} \times \sqrt{2}}$$

$$= \frac{3}{\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{(\sqrt{3})^2}$$

$$= \frac{\cancel{3}\sqrt{3}}{\cancel{3}}$$

$$= \sqrt{3} \text{ Ans}$$

$$(ii) \quad \frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$$

Solution:

$$\begin{aligned} &= \frac{\sqrt{21}\sqrt{9}}{\sqrt{63}} \\ &= \frac{\sqrt{21}\sqrt{3^2}}{\sqrt{9 \times 7}} \\ &= \frac{\sqrt{21} \times 3}{\sqrt{3^2} \times \sqrt{7}} \\ &= \frac{\sqrt{21} \times 3}{3\sqrt{7}} \\ &= \frac{\cancel{3}\sqrt{21}}{\cancel{3}\sqrt{7}} \\ &= \frac{\sqrt{21}}{\sqrt{7}} \\ &= \frac{\sqrt{7 \times 3}}{\sqrt{7}} \\ &= \frac{\cancel{\sqrt{7}} \times \sqrt{3}}{\cancel{\sqrt{7}}} \\ &= \sqrt{3} \text{ Ans} \end{aligned}$$

$$(iii) \quad = \sqrt[5]{243x^5y^{10}z^{15}}$$

Solution:

$$\begin{aligned} &= \sqrt[5]{243x^5y^{10}z^{15}} \\ &= \sqrt[5]{3^5x^5(y^2)^5(z^3)^5} \\ &= \sqrt[3]{3^5} \times \sqrt[5]{x^5} \times \sqrt[5]{(y^2)^5} \times \sqrt[3]{(z^3)^5} \\ &= 3 \times x \times y^2 \times z^3 \\ &= 3xy^2z^3 \text{ Ans} \end{aligned}$$

$$(iv) \quad \frac{4}{5}\sqrt[3]{125}$$

Solution:

$$\begin{aligned} &= \frac{4}{5}\sqrt[3]{5 \times 5 \times 5} \\ &= \frac{4}{5}\sqrt[3]{5^3} \end{aligned}$$

$$= \frac{4}{\cancel{5}} \times \cancel{5}$$

= 4 **Ans**

$$(v) \quad \sqrt{21} \times \sqrt{7} \times \sqrt{3}$$

Solution:

$$\begin{aligned} &= \sqrt{21} \times \sqrt{7} \times \sqrt{3} \\ &= \sqrt{7 \times 3} \times \sqrt{7} \times \sqrt{3} \\ &= \sqrt{7 \times 3 \times 7 \times 3} \\ &= \sqrt{7 \times 7 \times 3 \times 3} \\ &= \sqrt{7^2} \times \sqrt{3^2} \\ &= 7 \times 3 \\ &= 21 \text{ Ans} \end{aligned}$$

Q.3 Simplify by combining similar terms.

$$(i) \quad \sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$$

Solution:

$$\begin{aligned} &= \sqrt{45} - 3\sqrt{20} + 4\sqrt{5} \\ &= \sqrt{9 \times 5} - 3\sqrt{5 \times 4} + 4\sqrt{5} \\ &= \sqrt{3^2} \times \sqrt{5} - 3\sqrt{2^2} \times \sqrt{5} + 4\sqrt{5} \\ &= 3\sqrt{5} - 3 \times 2\sqrt{5} + 4\sqrt{5} \\ &= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5} \\ &= \sqrt{5}(3 - 6 + 4) \\ &= \sqrt{5}(3 - 2) \\ &= \sqrt{5}(1) \\ &= \sqrt{5} \text{ Ans} \end{aligned}$$

$$(ii) \quad 4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300}$$

Solution:

$$\begin{aligned} &= 4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300} \\ &= 4\sqrt{4 \times 3} + 5\sqrt{9 \times 3} - 3\sqrt{25 \times 3} + \sqrt{100 \times 3} \\ &= 4 \times 2\sqrt{3} + 5 \times 3\sqrt{3} - 3 \times 5\sqrt{3} + 10\sqrt{3} \\ &= 8\sqrt{3} + 15\sqrt{3} - 15\sqrt{3} + 10\sqrt{3} \\ &= 8\sqrt{3} + 15\sqrt{3} - 15\sqrt{3} + 10\sqrt{3} \end{aligned}$$

$$\begin{aligned}
&= \sqrt{3}(8 + 15 - 15 + 10) \\
&= \sqrt{3}(8 + 10) \\
&= \sqrt{3}(18) \\
&= 18\sqrt{3} \text{ Ans}
\end{aligned}$$

(ii) $(\sqrt{5} + \sqrt{3})^2$

Solution: $(\sqrt{5} + \sqrt{3})^2$

$$\begin{aligned}
&= (\sqrt{5})^2 + 2(\sqrt{5})(\sqrt{3}) + (\sqrt{3})^2 \\
&= 5 + 2\sqrt{5 \times 3} + 3 \\
&= 8 + 2\sqrt{15} \text{ Ans}
\end{aligned}$$

(iii) $\sqrt{3}(2\sqrt{3} + 3\sqrt{3})$

Solution: $\sqrt{3}(2\sqrt{3} + 3\sqrt{3})$

$$\begin{aligned}
&= \sqrt{3} \times \sqrt{3}(2 + 3) \\
&= (\sqrt{3})^2 \times (5) \\
&= 3(5) \\
&= 15 \text{ Ans}
\end{aligned}$$

(iii) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$

Solution: $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$

$$\begin{aligned}
&= (\sqrt{5})^2 - (\sqrt{3})^2 \\
&= 5 - 3 \\
&= 2 \text{ Ans}
\end{aligned}$$

(iv) $2(6\sqrt{5} - 3\sqrt{5})$

Solution: $2(6\sqrt{5} - 3\sqrt{5})$

$$\begin{aligned}
&= 2 \times \sqrt{5}(6 - 3) \\
&= 2 \times \sqrt{5}(3) \\
&= 6\sqrt{5} \text{ Ans}
\end{aligned}$$

(iv) $\left(\sqrt{2} + \frac{1}{\sqrt{3}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{3}}\right)$

Solution: $\left(\sqrt{2} + \frac{1}{\sqrt{3}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{3}}\right)$

$$\begin{aligned}
&= (\sqrt{2})^2 - \left(\frac{1}{\sqrt{3}}\right)^2 \\
&= 2 - \frac{(1)^2}{(\sqrt{3})^2} \\
&= 2 - \frac{1}{3} \\
&= \frac{6-1}{3} \\
&= \frac{5}{3} \text{ Ans}
\end{aligned}$$

Q.4 Simplify

(i) $(3 + \sqrt{3})(3 - \sqrt{3})$

Solution: $(3 + \sqrt{3})(3 - \sqrt{3})$

$$\begin{aligned}
&= (3)^2 - (\sqrt{3})^2 \\
&= 9 - 3 \\
&= 6 \text{ Ans}
\end{aligned}$$

(v) $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})(x + y)(x^2 + y^2)$

Solution: $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})(x + y)(x^2 + y^2)$

$$\begin{aligned}
&= \left[(\sqrt{x})^2 - (\sqrt{y})^2 \right] (x + y)(x^2 + y^2) \\
&= (x - y)(x + y)(x^2 + y^2) \\
&= \left[(x)^2 - (y)^2 \right] (x^2 + y^2) \\
&= (x^2 - y^2)(x^2 + y^2)
\end{aligned}$$

$$= \left[(x^2)^2 - (y^2)^2 \right]$$
$$= x^4 - y^4 \text{ Ans}$$



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Exercise 4.4

Q.1 Rationalize the denominator of the following

$$(i) \frac{3}{4\sqrt{3}}$$

$$\text{Solution: } \frac{3}{4\sqrt{3}}$$

$$\begin{aligned} &= \frac{3}{4\sqrt{3}} \\ &= \frac{3}{4\sqrt{3}} \times \frac{4\sqrt{3}}{4\sqrt{3}} \\ &= \frac{3(4\sqrt{3})}{(4\sqrt{3})^2} \\ &= \frac{12\sqrt{3}}{16(\sqrt{3})^2} \end{aligned}$$

$$= \frac{12\sqrt{3}}{16 \times 3}$$

$$= \frac{12\sqrt{3}}{48}$$

$$= \frac{\sqrt{3}}{4} \text{ Ans}$$

$$(ii) \frac{14}{\sqrt{98}}$$

$$\text{Solution: } \frac{14}{\sqrt{98}}$$

$$= \frac{14}{\sqrt{98}}$$

$$= \frac{14}{\sqrt{98}} \times \frac{\sqrt{98}}{\sqrt{98}}$$

$$\begin{aligned} &= \frac{14(\sqrt{98})}{(\sqrt{98})^2} \\ &= \frac{14(\sqrt{7 \times 7 \times 2})}{98} \\ &= \frac{14 \times 7 \times \sqrt{2}}{98} \\ &= \frac{98 \times \sqrt{2}}{98} \\ &= \sqrt{2} \text{ Ans} \end{aligned}$$

$$(iii) \frac{6}{\sqrt{8}\sqrt{27}}$$

$$\text{Solution: } \frac{6}{\sqrt{8}\sqrt{27}}$$

$$\begin{aligned} &= \frac{6}{\sqrt{8}\sqrt{27}} \\ &= \frac{6}{\sqrt{8}\sqrt{27}} \times \frac{\sqrt{8}\sqrt{27}}{\sqrt{8}\sqrt{27}} \\ &= \frac{6(\sqrt{8}\sqrt{27})}{(\sqrt{8})^2(\sqrt{27})^2} \end{aligned}$$

$$= \frac{6(\sqrt{4 \times 2})(\sqrt{9 \times 3})}{8 \times 27}$$

$$= \frac{6 \times 2\sqrt{2} \times 3\sqrt{3}}{216}$$

$$= \frac{6 \times 3 \times 2(\sqrt{2 \times 3})}{216}$$

$$= \frac{36\sqrt{6}}{216^6}$$

$$= \frac{\sqrt{6}}{6} \text{ Ans}$$

$$(iv) \quad \frac{1}{3+2\sqrt{5}}$$

$$\text{Solution: } \frac{1}{3+2\sqrt{5}}$$

$$= \frac{1}{3+2\sqrt{5}}$$

$$= \frac{1}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$= \frac{3-2\sqrt{5}}{(3)^2 - (2\sqrt{5})^2}$$

$$= \frac{3-2\sqrt{5}}{9-4.5}$$

$$= \frac{3-2\sqrt{5}}{9-20}$$

$$= \frac{3-2\sqrt{5}}{-11} \text{ Ans}$$

$$(vi) \quad \frac{2}{\sqrt{5}-\sqrt{3}}$$

$$\text{Solution: } \frac{2}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{2}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{2}$$

$$= \sqrt{5} + \sqrt{3} \text{ Ans}$$

$$(v) \quad \frac{15}{\sqrt{31}-4}$$

$$\text{Solution: } \frac{15}{\sqrt{31}-4}$$

$$= \frac{15}{\sqrt{31}-4}$$

$$= \frac{15}{\sqrt{31}-4} \times \frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$= \frac{15(\sqrt{31}+4)}{(\sqrt{31})^2 - (4)^2}$$

$$= \frac{15(\sqrt{31}+4)}{31-16}$$

$$= \frac{15(\sqrt{31}+4)}{15}$$

$$= \sqrt{31} + 4 \text{ Ans}$$

$$(vii) \quad \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\text{Solution: } \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{(\sqrt{3}-1)^2}{3-1}$$

$$= \frac{(\sqrt{3})^2 - 2(\sqrt{3})(1) + (1)^2}{2}$$

$$= \frac{3-2\sqrt{3}+1}{2}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}} \\ = 2 - \sqrt{3} \text{ Ans}$$

(viii) $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$

Solution: $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$

$$\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \\ = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$

$$= \frac{(\sqrt{5} + \sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2} \\ = \frac{(\sqrt{5})^2 + 2(\sqrt{5})(\sqrt{3}) + (\sqrt{3})^2}{5 - 3}$$

$$= \frac{5 + 2\sqrt{15} + 3}{2}$$

$$= \frac{8 + 2\sqrt{15}}{2}$$

$$= \frac{\cancel{2}(4 + \sqrt{15})}{\cancel{2}}$$

$$= 4 + \sqrt{15} \text{ Ans}$$

Q.2 find the conjugate of $x + \sqrt{y}$

(i) $3 + \sqrt{7}$

Solution

Conjugate $3 - \sqrt{7}$

(ii) $4 - \sqrt{5}$

Solution

Conjugate $4 + \sqrt{5}$

(iii) $2 + \sqrt{3}$

Solution

Conjugate $2 - \sqrt{3}$

(iv) $2 + \sqrt{5}$

Solution

Conjugate $2 - \sqrt{5}$

(v) $5 + \sqrt{7}$

Solution

Conjugate $5 - \sqrt{7}$

(vi) $4 - \sqrt{15}$

Solution

Conjugate $4 + \sqrt{15}$

(vii) $7 - \sqrt{6}$

Solution

Conjugate $7 + \sqrt{6}$

(viii) $9 + \sqrt{2}$

Solution

Conjugate $9 - \sqrt{2}$

Q.3

(i) If $x = 2 - \sqrt{3}$, find $\frac{1}{x}$

Solution: Given that $x = 2 - \sqrt{3}$

$$\frac{1}{x} = \frac{1}{2 - \sqrt{3}}$$

$$= \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= \frac{2 + \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$= \frac{2 + \sqrt{3}}{4 - 3}$$

$$= \frac{2 + \sqrt{3}}{1}$$

$$\frac{1}{x} = 2 + \sqrt{3} \text{ Ans}$$

(ii) If $x = 4 - \sqrt{17}$, find $\frac{1}{x}$

Solution: Given that $x = 4 - \sqrt{17}$

$$\frac{1}{x} = \frac{1}{4 - \sqrt{17}}$$

$$= \frac{1}{4 - \sqrt{17}} \times \frac{4 + \sqrt{17}}{4 + \sqrt{17}}$$

$$\begin{aligned}
&= \frac{4+\sqrt{17}}{(4)^2 - (\sqrt{17})^2} \\
&= \frac{4+\sqrt{17}}{16-17} \\
&= \frac{4+17}{-1} \\
&= -1(4+\sqrt{17}) \\
\frac{1}{x} &= -4-\sqrt{17} \text{ Ans}
\end{aligned}$$

(iii) If $x = \sqrt{3} + 2$, find $x + \frac{1}{x}$

Solution: Given that $x = \sqrt{3} + 2$

$$\begin{aligned}
\frac{1}{x} &= \frac{1}{\sqrt{3}+2} \\
&= \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} \\
&= \frac{\sqrt{3}-2}{(\sqrt{3})^2 - (2)^2} \\
&= \frac{\sqrt{3}-2}{3-4} \\
&= \frac{\sqrt{3}-2}{-1} \\
&= -(\sqrt{3}-2) \\
&= -\sqrt{3}+2
\end{aligned}$$

$$\begin{aligned}
x + \frac{1}{x} &= (\sqrt{3}+2) + (-\sqrt{3}+2) \\
&= \sqrt{3}+2-\sqrt{3}+2 \\
&= 2+2 \\
x + \frac{1}{x} &= 4 \text{ Ans}
\end{aligned}$$

Q.4 Simplify

$$(i) \quad \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$

$$\text{Solution: } \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$\begin{aligned}
&= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} + \frac{(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} \\
&= \frac{(\sqrt{5}-\sqrt{3}) + \sqrt{2}(\sqrt{5}-\sqrt{3})}{5-3} \\
&\quad + \frac{1(\sqrt{5}+\sqrt{3}) - \sqrt{2}(\sqrt{5}+\sqrt{3})}{5-3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\sqrt{5}-\sqrt{3} + \sqrt{10}-\sqrt{6}}{2} + \frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2} \\
&= \frac{\cancel{\sqrt{5}} - \cancel{\sqrt{3}} + \cancel{\sqrt{10}} - \cancel{\sqrt{6}}}{2} + \frac{\cancel{\sqrt{5}} + \cancel{\sqrt{3}} - \cancel{\sqrt{10}} - \cancel{\sqrt{6}}}{2} \\
&= \frac{\cancel{2}\sqrt{5} - \cancel{2}\sqrt{6}}{\cancel{2}} \\
&= \sqrt{5} - \sqrt{6} \text{ Ans}
\end{aligned}$$

$$(ii) \quad \frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2+\sqrt{5}}$$

$$\text{Solution: } \frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2+\sqrt{5}}$$

$$= \frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2+\sqrt{5}}$$

$$\begin{aligned}
&= \left(\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} \right) + \left(\frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \right) \\
&\quad + \left(\frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} \right)
\end{aligned}$$

$$\begin{aligned}
&= \left(\frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2} \right) + \left(\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} \right) \\
&+ \left(\frac{2-\sqrt{5}}{(2)^2 - (\sqrt{5})^2} \right) \\
&= \left(\frac{2-\sqrt{3}}{4-3} \right) + \left(\frac{2(\sqrt{5}+\sqrt{3})}{5-3} \right) + \left(\frac{2-\sqrt{5}}{4-5} \right) \\
&= \left(\frac{2-\sqrt{3}}{1} \right) + \left(\frac{2(\sqrt{5}+\sqrt{3})}{2} \right) + \left(\frac{2-\sqrt{5}}{-1} \right) \\
&= 2-\sqrt{3} + \sqrt{5} + \sqrt{3} - 2 + \sqrt{5} \\
&= 2\cancel{-}\cancel{2} - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} + \sqrt{5} + \sqrt{5} \\
&= \sqrt{5} + \sqrt{5} \\
&= 2\sqrt{5} \text{ Ans}
\end{aligned}$$

Q.5 If $x = 2 + \sqrt{3}$, then find the value of $x - \frac{1}{x}$ and $\left(x - \frac{1}{x}\right)^2$

(i)

Solution: Given that $x = 2 + \sqrt{3}$

$$\begin{aligned}
\frac{1}{x} &= \frac{1}{2+\sqrt{3}} \\
&= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} \\
&= \frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2} \\
&= \frac{2-\sqrt{3}}{4-3} \\
&= \frac{2-\sqrt{3}}{1} \\
&= 2-\sqrt{3}
\end{aligned}$$

To find the value of $x - \frac{1}{x}$

$$\begin{aligned}
x - \frac{1}{x} &= (2+\sqrt{3}) - (2-\sqrt{3}) \\
&= 2\cancel{+}\sqrt{3} - 2\cancel{+}\sqrt{3} \\
&= \sqrt{3} + \sqrt{3} \\
&= 2\sqrt{3}
\end{aligned}$$

To find the value of $\left(x - \frac{1}{x}\right)^2$

We know that

$$x - \frac{1}{x} = 2\sqrt{3}$$

Taking square on both sides

$$\begin{aligned}
\left(x - \frac{1}{x}\right)^2 &= (2\sqrt{3})^2 \\
&= 4(\sqrt{3})^2 \\
&= 4(3) \\
&= 12 \text{ Ans}
\end{aligned}$$

(ii) If $x = \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}}$, find the value of

$$x + \frac{1}{x}, x^2 + \frac{1}{x^2} \text{ and } x^3 + \frac{1}{x^3}$$

Solution: Given that $x = \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}}$

$$\frac{1}{x} = \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$x + \frac{1}{x} = \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}} + \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$= \frac{(\sqrt{5} - \sqrt{2})^2 + (\sqrt{5} + \sqrt{2})^2}{(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})}$$

$$= \frac{(\sqrt{5})^2 + (\sqrt{2})^2 - 2\sqrt{5} \times \sqrt{2} + (\sqrt{5})^2 + (\sqrt{2})^2 + 2\sqrt{5} \times \sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2}$$

$$= \frac{5 + 2 - 2\cancel{\sqrt{10}} + 5 + 2 + 2\cancel{\sqrt{10}}}{5 - 2}$$

$$x + \frac{1}{x} = \frac{14}{3}$$

Taking square on both sides

$$\left(x + \frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2$$

$$x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = \frac{196}{9}$$

$$x^2 + \frac{1}{x^2} = \frac{196}{9} - 2$$

$$x^2 + \frac{1}{x^2} = \frac{196 - 18}{9}$$

$$x^2 + \frac{1}{x^2} = \frac{178}{9}$$

$$x^2 + \frac{1}{x^2} = \frac{178}{9}$$

$$\text{To find } x^3 + \frac{1}{x^3}$$

$$x + \frac{1}{x} = \frac{14}{3}$$

Taking cube on both sides

$$\left(x + \frac{1}{x}\right)^3 = \left(\frac{14}{3}\right)^3$$

$$x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = \frac{2744}{27}$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{2744}{27}$$

$$x^3 + \frac{1}{x^3} + 3\left(\frac{14}{3}\right) = \frac{2744}{27}$$

$$x^3 + \frac{1}{x^3} + 14 = \frac{2744}{24}$$

$$x^3 + \frac{1}{x^3} = \frac{2744}{27} - 14$$

$$x^3 + \frac{1}{x^3} = \frac{2744 - 378}{27}$$

$$x^3 + \frac{1}{x^3} = \frac{2366}{27} \text{ Ans}$$

Q.6 Determine the rational numbers

a and b if

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + b\sqrt{3}$$

Solution: Given that

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + b\sqrt{3}$$

$$a + b\sqrt{3} = \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{(\sqrt{3})^2 + (1)^2 - 2\cancel{\sqrt{3}} + (\sqrt{3})^2 + (1)^2 + 2\cancel{\sqrt{3}}}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{2(\sqrt{3})^2 + 2}{(\sqrt{3})^2 - 1}$$

$$= \frac{2[(\sqrt{3})^2 + (1)^2]}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{2(3+1)}{3-1}$$

$$= \frac{2(4)}{2}$$

$$a+b\sqrt{3} = 4$$

$$a+b\sqrt{3} = 4+0\sqrt{3}$$

Comparing both sides

$$a = 4 \quad b\sqrt{3} = 0\sqrt{3}$$

$$b = \frac{0\sqrt{3}}{\sqrt{3}}$$

$$b = 0 \text{ Ans}$$



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Review Exercise 4

Q.1 Multiple type questions?

- (i) is an algebraic ...
(a) Expression (b) Sentence
(c) Equation (d) In-equation

(ii) The degree of polynomial $4x^4 + 3x^2y$ is
(a) 1 (b) 2
(c) 3 (d) 4

(iii) $a^3 + b^3$ is equal to
(a) $(a-b)(a^2 + ab + b^2)$ (b) $(a+b)(a^2 - ab + b^2)$
(c) $(a-b)(a^2 - ab + b^2)$ (d) $(a-b)(a^2 + ab + b^2)$

(iv) $(3+\sqrt{2})(3-\sqrt{2})$ is equal to
(a) 7 (b) -7
(c) -1 (d) 1

(v) Conjugate of surd $a + \sqrt{b}$ is;
(a) $-a + \sqrt{b}$ (b) $a - \sqrt{b}$
(c) $\sqrt{a} + \sqrt{b}$ (d) $\sqrt{a} - \sqrt{b}$

(vi) $\frac{1}{a-b} - \frac{1}{a+b}$ is equal to
(a) $\frac{2a}{a^2 - b^2}$ (b) $\frac{2b}{a^2 - b^2}$
(c) $\frac{-2a}{a^2 - b^2}$ (d) $\frac{-2b}{a^2 - b^2}$

(vii) $\frac{a^2 - b^2}{a+b}$ is equal to
(a) $(a-b)^2$ (b) $(a+b)^2$
(c) $a+b$ (d) $a-b$

(viii) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ is equal to
(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $a-b$ (d) $a+b$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
a	d	b	a	b	b	d	c

Q.2 Fill in the blanks

- (i) The degree of polynomial $x^2y^2 + 3xy + y^3$ is _____
- (ii) $x^2 - 4$ _____
- (iii) $x^3 + \frac{1}{x^3} = \left[x + \frac{1}{x}\right] (\text{_____})$
- (iv) $2(a^2 + b^2) = (a+b)^2 + (\text{_____})^2$
- (v) $\left[x - \frac{1}{x}\right]^2 = \text{_____}$
- (vi) Order of surd $\sqrt[3]{x}$ is _____
- (vii) $\frac{1}{2-\sqrt{3}} = \text{_____}$

ANSWER KEY

- (i) 4
- (ii) $(x-2)(x+2)$
- (iii) $x^2 - 1 + \frac{1}{x^2}$
- (iv) $a-b$
- (v) $x^2 + \frac{1}{x^2} - 2$
- (vi) 3
- (vii) $2 + \sqrt{3}$

Q.3 If $x + \frac{1}{x} = 3$, find

(i) $x^2 + \frac{1}{x^2}$

Solution: Given that $x + \frac{1}{x} = 3$

$$\therefore (a+b)^2 = a^2 + b^2 + 2ab$$

Putting the values

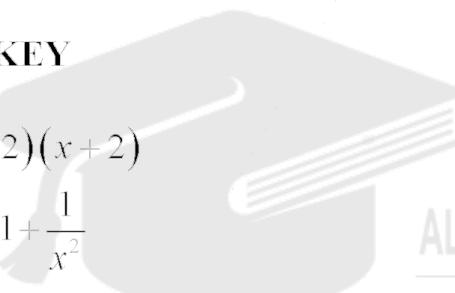
$$\left[x + \frac{1}{x}\right]^2 = (x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right)$$

$$(3)^2 = x^2 + \frac{1}{x^2} + 2$$

$$9 = x^2 + \frac{1}{x^2} + 2$$

$$9 - 2 = x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = 7 \text{ Ans}$$



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(ii) $x^4 + \frac{1}{x^4}$

Solution: Given that $x^2 + \frac{1}{x^2} = 7$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right)$$

$$(7)^2 = x^4 + \frac{1}{x^4} + 2$$

$$49 = x^4 + \frac{1}{x^4} + 2$$

$$x^4 + \frac{1}{x^4} = 49 - 2$$

$$x^4 + \frac{1}{x^4} = 47 \text{ Ans}$$

Q.4 If $x - \frac{1}{x} = 2$ find

(i) $x^2 + \frac{1}{x^2}$

(ii) $x^4 + \frac{1}{x^4}$

Solution (i)

Given that $x - \frac{1}{x} = 2$

$$\therefore (a+b)^2 = a^2 + b^2 + 2ab$$

Putting the values

$$\left(x - \frac{1}{x}\right)^2 = (x)^2 + \left(\frac{1}{x}\right)^2 - 2(x)\left(\frac{1}{x}\right)$$

$$(2)^2 = x^2 + \frac{1}{x^2} - 2$$

$$4 + 2 = x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = 6 \text{ Ans}$$

Solution (ii)

Given that $x^2 + \frac{1}{x^2} = 6$

$$\left(x^2 + \frac{1}{x}\right)^2 = x^4 + \frac{1}{x^4} + 2\left(x^2\right)\left(\frac{1}{x^2}\right)$$

$$(6)^2 = x^4 + \frac{1}{x^4} + 2$$

$$x^4 + \frac{1}{x^4} = 36 - 2$$

$$x^4 + \frac{1}{x^4} = 34 \text{ Ans}$$

Q.5 Find the value of $x^3 + y^3$ and xy if $x+y=5$ and $x-y=3$.

Solution: Given that $x+y=5$

$$x-y=3$$

As we know that

$$\therefore (x+y)^2 - (x-y)^2 = 4xy$$

Putting the values

$$4xy = (5)^2 - (3)^2$$

$$4xy = 25 - 9$$

$$4xy = 16$$

$$xy = \frac{16}{4}$$

$$xy = 4 \text{ Ans}$$

As we know that

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

Putting the values

$$(5)^3 = x^3 + y^3 + 3 \times 4 \times 5$$

$$125 = x^3 + y^3 + 60$$

$$x^3 + y^3 = 125 - 60$$

$$x^3 + y^3 = 65$$

$$x^3 + y^3 = 65 \text{ Ans}$$

Q.6 If $P = 2 + \sqrt{3}$, find

(i) $P + \frac{1}{P}$

Solution: Given that $P = 2 + \sqrt{3}$

$$\frac{1}{P} = \frac{1}{2 + \sqrt{3}}$$

$$\frac{1}{P} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{4 - 3}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{1}$$

$$\frac{1}{P} = 2 - \sqrt{3}$$

$$P + \frac{1}{P} = 2 + \cancel{\sqrt{3}} + 2 - \cancel{\sqrt{3}}$$

$$P + \frac{1}{P} = 4 \text{ Ans}$$

(ii) $P - \frac{1}{P}$

As we know that

$$\frac{1}{P} = 2 - \sqrt{3} \text{ and}$$

$$P = 2 + \sqrt{3}$$

$$\begin{aligned} P - \frac{1}{P} &= 2 + \sqrt{3} - (2 - \sqrt{3}) \\ &= \cancel{2} + \sqrt{3} - \cancel{2} + \sqrt{3} \\ &= 2\sqrt{3} \text{ Ans} \end{aligned}$$

(iii) $P^2 + \frac{1}{P^2}$

Solution: Given that $P + \frac{1}{P} = 4$

$$\therefore (a+b)^2 = a^2 + b^2 + 2ab$$

$$\left(P + \frac{1}{P}\right)^2 = (P)^2 + \left(\frac{1}{P}\right)^2 + 2(P)\left(\frac{1}{P}\right)$$

$$(4)^2 = P^2 + \frac{1}{P^2} + 2$$

$$16 - 2 = P^2 + \frac{1}{P^2}$$

$$P^2 + \frac{1}{P^2} = 14 \text{ Ans}$$

(iv) $P^2 - \frac{1}{P^2}$

Solution:

$$P^2 - \frac{1}{P^2} = \left(P + \frac{1}{P}\right)\left(P - \frac{1}{P}\right)$$

$$\begin{aligned} P^2 - \frac{1}{P^2} &= (4)(2\sqrt{3}) \\ &= 8\sqrt{3} \text{ Ans} \end{aligned}$$

Q.7 If $q = \sqrt{5} + 2$ find.

(i) $q + \frac{1}{q}$

Solution: Given that $q = \sqrt{5} + 2$

$$\begin{aligned} \frac{1}{q} &= \frac{1}{\sqrt{5} + 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} - 2} \\ &= \frac{\sqrt{5} - 2}{(\sqrt{5})^2 - (2)^2} \\ &= \frac{\sqrt{5} - 2}{5 - 4} \\ &= \sqrt{5} - 2 \end{aligned}$$

$$q + \frac{1}{q} = \sqrt{5} + \cancel{2} + \sqrt{5} - \cancel{2}$$

$$q + \frac{1}{q} = 2\sqrt{5} \text{ Ans}$$

$$(ii) q - \frac{1}{q}$$

Solution: Given that $q = \sqrt{5} + 2$

$$\frac{1}{q} = \sqrt{5} - 2$$

$$q - \frac{1}{q} = \sqrt{5} + 2 - (\sqrt{5} - 2)$$

$$= \cancel{\sqrt{5}} + 2 - \cancel{\sqrt{5}} + 2$$

$$q - \frac{1}{q} = 4 \text{ Ans}$$

$$(iii) q^2 + \frac{1}{q^2}$$

Solution: Given that $q - \frac{1}{q} = 4$

Squaring both sides

$$\left(q - \frac{1}{q}\right)^2 = (4)^2$$

$$q^2 + \frac{1}{q^2} - 2 = 16$$

$$q^2 + \frac{1}{q^2} = 16 + 2$$

$$q^2 + \frac{1}{q^2} = 18 \text{ Ans}$$

$$(iv) q^2 - \frac{1}{q^2}$$

Solution: Given that $q + \frac{1}{q} = 2\sqrt{5}$

$$q - \frac{1}{q} = 4$$

By using formula

$$q^2 - \frac{1}{q^2} = \left(q + \frac{1}{q}\right)\left(q - \frac{1}{q}\right)$$

$$= (2\sqrt{5})(4)$$

$$= 8\sqrt{5} \text{ Ans}$$

Q.8 Simplify

$$(i) \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}}$$

Solution:

$$\begin{aligned} &= \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}} \times \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} + \sqrt{a^2-2}} \\ &= \frac{(\sqrt{a^2+2} + \sqrt{a^2-2})^2}{(\sqrt{a^2+2})^2 - (\sqrt{a^2-2})^2} \\ &= \frac{(\sqrt{a^2+2})^2 + (\sqrt{a^2-2})^2 + 2(\sqrt{a^2+2})(\sqrt{a^2-2})}{a^2+2-a^2+2} \end{aligned}$$

$$= \frac{a^2 + 2 + a^2 - 2 + 2(\sqrt{a^4 - 2a^2 + 2a^2 - 4})}{4}$$

$$= \frac{2a^2 + 2\sqrt{a^4 - 4}}{4}$$

$$= \frac{2(a^2 + \sqrt{a^4 - 4})}{4}$$

$$= \frac{a^2 + \sqrt{a^4 - 4}}{2} \text{ Ans}$$

$$(ii) \frac{1}{a - \sqrt{a^2 - x^2}} - \frac{1}{a + \sqrt{a^2 - x^2}}$$

$$= \left(\frac{1}{a - \sqrt{a^2 - x^2}} \times \frac{a + \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}} \right)$$

$$- \left(\frac{(1)}{(a + \sqrt{a^2 - x^2})(a - \sqrt{a^2 - x^2})} \left(a - \sqrt{a^2 - x^2} \right) \right)$$

$$= \left(\frac{a + \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2} \right) - \left(\frac{a - \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2} \right)$$

$$= \left(\frac{a + \sqrt{a^2 - x^2}}{a - (a^2 - x^2)} \right) - \left(\frac{a - \sqrt{a^2 - x^2}}{a - (a^2 - x^2)} \right)$$

$$= \left(\frac{a + \sqrt{a^2 - x^2}}{a^2 - a^2 + x^2} \right) - \left(\frac{a - \sqrt{a^2 - x^2}}{a^2 - a^2 + x^2} \right)$$

$$\begin{aligned}&= \left(\frac{a + \sqrt{a^2 - x^2}}{x^2} \right) - \left(\frac{a - \sqrt{a^2 - x^2}}{x^2} \right) \\&= \frac{a + \sqrt{a^2 - x^2} - a + \sqrt{a^2 - x^2}}{x^2} \\&= \frac{2\sqrt{a^2 - x^2}}{x^2} \quad \text{Ans}\end{aligned}$$



Unit 4: Algebraic Expressions and Algebraic

Formulas Overview

Algebraic expression:

An algebraic expression is that in which constants or variables or both are combined by basic operations.

Polynomial:

Polynomial means an expression with many terms.

Degree of Polynomial:

Degree of Polynomial means highest power of variable.

Rational expression:

Expression in the form $\frac{p(x)}{q(x)}$, ($q(x) \neq 0$) is called rational expression.

Surd:

An irrational radical with rational radicand is called a surd.

e.g., $\sqrt{3}$, $\sqrt{\frac{2}{5}}$,

Monomial Surd:

A surd which contains a single term is called monomial surd.

Binomial Surd:

A surd which contains sum or difference of two surds is called binomial surd.