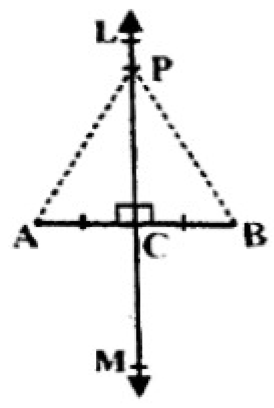
THEOREM 12.1.1

Any point on the right bisector of a line segment is equidistant from its end points.

Solution:



Given:

A line $\overline{LM}$ intersects the line segment $\overline{AB}$ at point C such that $\overline{LM} \perp \overline{AB}$ and $\overline{AC} \cong \overline{BC}$. P is a point on $\overline{LM}$.

To Prove:

$\overline{PA} \cong \overline{PB}$

Construction:

Take a point P on $\overline{LM}$. Join P to the points A and B.

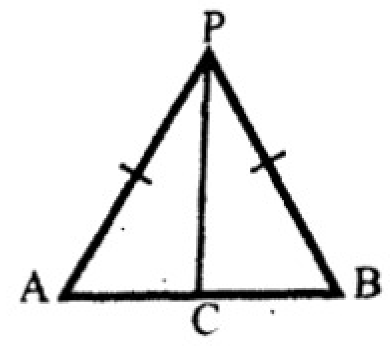
Proof

Statements	Reasons
In $\Delta ACP \leftrightarrow \Delta BCP$ $\overline{AC} \cong \overline{BC}$	given

$\angle ACP \cong \angle BCP$	given ($\overline{PC} \perp \overline{AB}$)
$\overline{PC} \cong \overline{PC}$	Common
$\Delta ACP \cong \Delta BCP$	S.A.S. postulate
Hence $\overline{PA} \cong \overline{PB}$	corresponding sides of congruent triangles

THEOREM 12.1.2

Any point equidistant from the end points of a line segment is on the right bisector of it.



Given:

$\overline{AB}$ is a line segment and P is a point such that $\overline{PA} \cong \overline{PB}$

To prove:

The point P is on right bisector of $\overline{AB}$

Construction:

Joint P to C, the midpoint of $\overline{AB}$

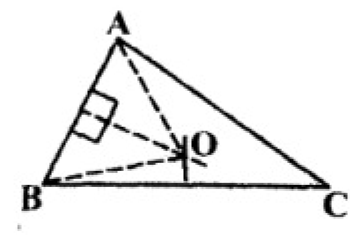
Proof:

Statements	Reasons
In $\Delta ACP \leftrightarrow \Delta BCP$	
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PC} \cong \overline{PC}$	Common

$\overline{AC} \cong \overline{BC}$	Construction.
$\therefore\ \Delta ACP \cong \Delta BCP$	S. S. S. ≅ S. S. S.
$\therefore \angle ACP \cong \angle BCM$ (i)	corresponding angles of congruent triangles
but $m\angle ACP \cong \angle BCP = 180°$ (ii)	Supplementary angles from (i) and (ii)
$\therefore\ m\angle ACP = \angle BCP = 90°$	
$\overline{PC} \perp \overline{AB}$ (iii)	$mACP = 90°$ (proved)
Also $\overline{CA} \cong \overline{AB}$ (iv)	construction
$\therefore\ \overline{PC}$ is right bisector of $\overline{AB}$.	from (iii) and (iv)
or P is on right bisector of $\overline{AB}$.	

THEOREM 12.1.3

The right bisectors of the sides of a triangle are concurrent.



Given

ABC is triangle.

To prove

The right bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are concurrent.

Construction

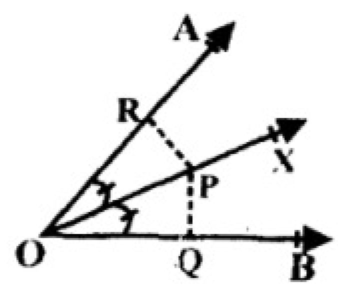
Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which meet each other at the point O. Join O to A, B and C.

Statements	Reasons
In $\overline{OA} \cong \overline{OB}$ …… (i)	Each point on right bisector of a segment is equidistant from its end points
$\overline{OB} \cong \overline{OC}$ …… (ii)	from (i)
$\overline{OA} \cong \overline{OC}$ …… (iii)	from (i) and (ii)
Point O is on the right bisector of $\overline{CA}$. …… (iv)	O is equidistant from A and C
But point O is on the right bisector	Construction

of $\overline{AB}$ and of $\overline{BC}$ …… (v) thus the right bisectors of the three sides of a triangle are concurrent	from (iv) and (v)

THEOREM 12.1.4

Any point on the bisector of an angle is equidistant from its arms.



Given

A point P is on $\overline{OX}$, the bisector of $\angle AOB$.

To Prove

$\overline{PQ} \cong \overline{PR}$ i.e., P is equidistant from $\overline{OA}$ and $\overline{OB}$.

Construction

Draw $\overline{PR} \perp \overline{OA}$ and $\overline{PQ} \perp \overline{OB}$

Proof

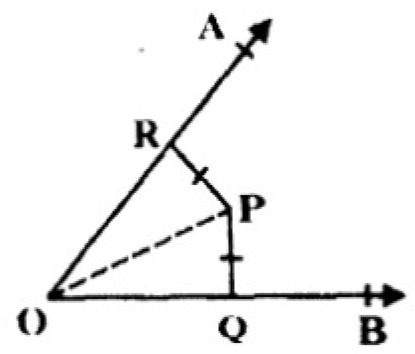
Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	common
$\angle PRO \cong \angle PQO$	construction
$\angle POQ \cong \angle POR$	given
$\therefore \Delta POQ \cong \Delta POR$	S.A.A. $\cong$ S.A.A.

Hence $\overline{PQ} \cong \overline{PR}$	corresponding sides of congruent triangles

THEOREM 12.1.5

Converse of 12.1.4

Any point inside an angle, equidistant from its arms, is on the bisector of it.



Given

Any point P lies inside $\angle AOB$ such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overline{OB}$ and $\overline{PR} \perp \overline{OA}$

To Prove

Point P is on the bisector of $\angle AOB$.

Construction

Join P to O.

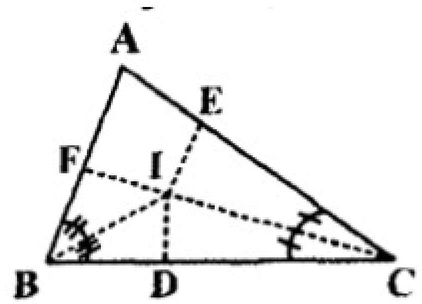
Proof

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$ $\overline{PO} \cong \overline{PO}$	given (right angles) common

$\overline{PQ} \cong \overline{PR}$	given
$\therefore \Delta POQ \cong \Delta POR$	H.S. $\cong$ H.S.
And $\angle POQ \cong \angle POR$ (i)	corresponding angles of congruent triangles
i.e., P is on the bisector of $\angle AOB$.	from (i) proved

THEOREM 12.1.6

The bisectors of the angles of a triangle are concurrent.



Given
ABC is the triangle

To Prove
The bisectors of $\angle A, \angle B$ and $\angle C$ are concurrent.

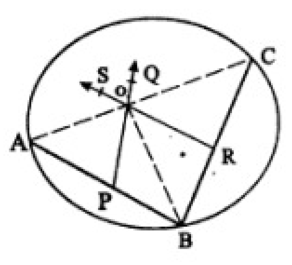
Construction
Draw the bisectors of $\angle B$ and $\angle C$ which intersect at point I. From I, draw $\overline{IF} \perp \overline{AB}$, $\overline{IE} \perp \overline{CA}$, and $\overline{ID} \perp \overline{BC}$.

Proof:

Statements	Reasons
$\overline{ID} \cong \overline{IF}$ Similarly, $\overline{ID} \cong \overline{IE}$	Any point on bisector of an angle is equidistant from its arms
$\therefore \overline{IE} \cong \overline{IF}$ So, the point I is on the bisector of $\angle A$ …… (i)	Each is congruent to $\overline{ID}$ (proved)
Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$ …… (ii)	
Thus the bisectors of $\angle A, \angle B$ and $\angle C$ are concurrent.	from (i) and (ii)

EXERCISE 12.1

Q1. Prove that the center of a circle is on the right bisectors of each of its chords.



Solution:

Given:

A, B, C are three non-Collinear points.

Required:

To find the Centre of the circle passing through A, B, C

Construction:

(i) Join B to A.

(ii) Take PQ right bisector of AB and RS right bisector of BC. They intersect at 0.

(iii)Join O to A, B, C. O is the Centre of the circle.

Proof:

Statements	Reasons
In $\overline{OA} \cong \overline{OB}$ (i)	O is on right bisector of AB
$\overline{OB} \cong \overline{OC}$ (ii)	O is on right bisector of BC
$\overline{OA} \cong \overline{OB} \cong \overline{OC}$ (iii)	From (i), (ii)
Hence O is equidistant from A, B, C.	
Therefore, O is the required Centre of the circle.	

Q2. Where will be the Centre of a circle passing through three non-collinear points? And why?

Solution

Given

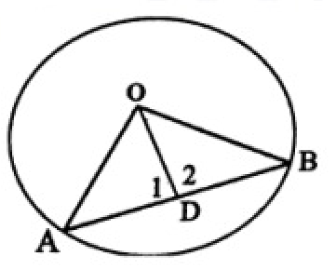
O is the Centre of a circle. AB is any chord of the circle.

To prove

O is right bisector of AB.

Construction

Take midpoint D of AB and join D to O.



Proof

Statements	Reasons
In $\Delta AOD \leftrightarrow \Delta BOD$	
$(\overline{OA}) \cong (\overline{OB})$	Radius of same circle
$(\overline{OD}) \cong (\overline{OD})$	Common
$(\overline{AD}) \cong (\overline{BD})$	Construction
$\Delta AOD \cong \Delta BOD$	S.S.S. $\cong$ S.S.S.
But $m\angle 1 = m\angle 2 = 180°$	Supplementary angles
$m\angle 1 + m\angle 2 = 180°$ $2m\angle 1 = 180°$ $m\angle 1 = 90°$	From (i)
DO is right bisector of AB.	
i.e. O is on the right bisector of AB.	

Q3. Three villages P, Q and R are not on the same line. The people of these villages want to make a Children Park at such a place which is equidistant from these three villages. After fixing the place of Children Park, prove that the park is equidistant from three villages.

Solution:

Given

P, Q, R, are three villages on the same straight line

To prove:

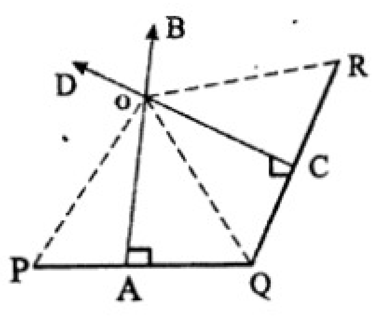
To find the point equidistant from P, Q, R.

Construction

(i) Join Q to P and R.

(ii) Take AB right bisector of PQ and CD right bisector of QR. AB and CD intersect at O.

(iii) Join O to P, Q, R. O is the place of children park



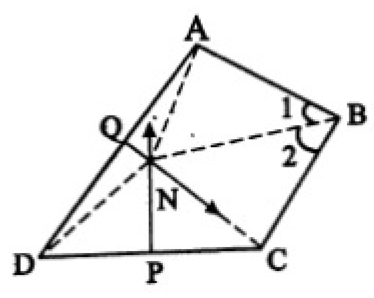
Proof

Statements	Reasons
$\overline{OP} = \overline{QR} = \overline{OR}$ (i)	O is on the right bisector PQ.
$\overline{OQ} = \overline{OR}$ (ii)	O is on the right bisector QR.
$\therefore \overline{OP} = \overline{OQ} = \overline{OR}$	From (i) and (ii)
Hence O is equidistant from P, Q, R	

EXERCISE 12.2

Q1. In a quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}$, $\overline{CD}$ meet each other at point N. Prove that $\overline{BN}$ is a bisector of ∠ABC.

Solution



Given
In the quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$
$\overline{NP}$ is right bisector of $\overline{CD}$ and $\overline{NQ}$ is right bisector of $\overline{AD}$.
They meet at N.

To Prove:
BN is a bisector of ∠ABC

Construction:
Join N to A, B, C, D.

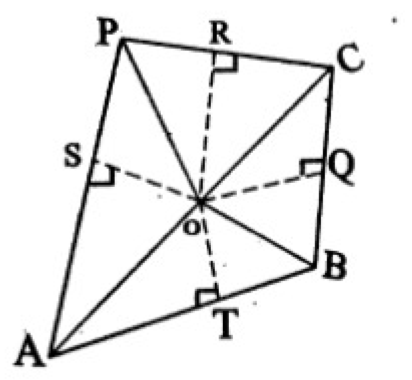
Proof:

Statements	Reasons
$\overline{ND} \cong \overline{NC}$ (i)	N is on right bisector of $\overline{DC}$
$\overline{ND} \cong \overline{NA}$ (ii)	N is on right bisector of $\overline{AC}$
$\overline{NA} \cong \overline{NC}$ (iii)	From (i), (ii)
In ΔBNA ↔ ΔBNC	
$\overline{NA} \cong \overline{NC}$	From (iii)
$\overline{AB} \cong \overline{CD}$	Given
$\overline{BN} \cong \overline{BN}$	Common

ΔBNA ↔ ΔBNC	S.S.S ≅ S.S.S.
Hence ∠1 ∠2	Corresponding angles of congruent triangles.
Hence $\overline{BN}$ is bisector of ∠ABC.	

Q2. The bisectors of ∠A, B and ∠C of a quadrilateral ABCP meet each other at point O. prove that the bisector of ∠P will also pass through the point O.

Solution:



Given:
ABCP is a quadrilateral.
$\overline{AO}$, $\overline{BO}$, $\overline{CO}$ are bisector of ∠A, ∠B, ∠C, respectively.
P is joined to O.

To prove:
$\overline{PO}$ is bisector of ∠P

Construction:
From O draw
$\overline{QT} \perp \overline{AB}$ $\overline{OQ} \perp \overline{BC}$, $\overline{OR} \perp \overline{PC}$ and $\overline{OS} \perp \overline{AP}$ respectively.

Proof:

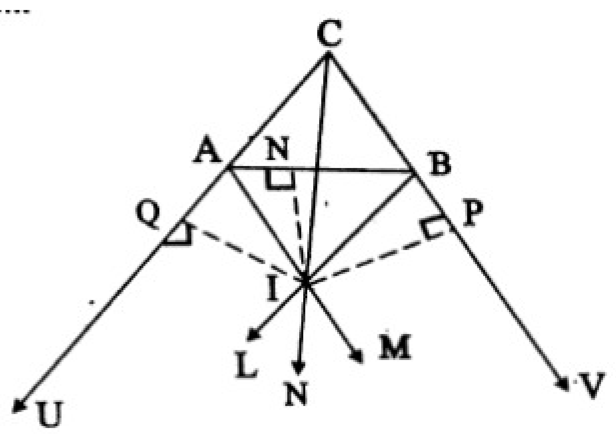
Statements	Reasons
$\overline{OS} \cong \overline{OT}$ (i)	$\overline{AO}$, is bisector of ∠A
$\overline{OT} \cong \overline{OQ}$ (ii)	$\overline{BO}$ is bisector of ∠D
$\overline{OQ} \cong \overline{OR}$ (iii)	$\overline{CO}$ is bisector of ∠C
$\overline{OS} \cong \overline{OR}$	From (I), (ii), (iii)
∴ O is on bisector of ∠P	

Hence $\overline{PO}$ is bisector of ∠P, or Bisector of ∠P also passes through Q.	

EXERCISE 12.3

Q1. Prove that the bisectors of the angles of base of an isosceles triangle intersect each other on its altitude.

Solution



Given

In ΔABC, sides $\overline{CA}$ and $\overline{CB}$ are produced.
$\overline{BL}$ is bisector of $\angle ABV$.
$\overline{AM}$ is bisector of $\angle BAU$.
$\overline{BL}$ and $\overline{AM}$ is intersect at I.
C is joined to I.

To Prove:
C is bisector of $\angle C$

Construction:
Draw $\overline{IP} \perp \overline{CV}$, $\overline{IQ} \perp \overline{CU}$ and $\overline{IN} \perp \overline{AB}$.

Proof

Statements	Reasons
$\overline{IN} \cong \overline{IP}$ (i)	$\overline{BI}$ is bisector of $\angle ABV$
$\overline{IN} \cong \overline{IQ}$ (ii)	$\overline{AI}$ is a bisector of $\angle BAU$
$\overline{IP} \cong \overline{IQ}$ (iii)	From (i) and (ii)
Now $\overline{TP}$ and $\overline{IQ}$ are perpendicular to $\overline{CB}$ and $\overline{CA}$ produced CI is bisector of angles $\angle C$.	

REVIEW EXERCISE

Q1. Which of the following are true and which are false?

(i) Bisection means to divide into two equal parts.

(ii) Right bisection of line segment means to draw perpendicular which passes through the mid-point.

(iii) Any point on the right bisector of a line segment is not equidistant from its end points.

(iv) Any point equidistant from the end points of a line segment is on the right bisector of it.

(v) The right bisectors of the sides of a triangle are not concurrent.

(vi) The bisectors of the angles of a triangle are concurrent.

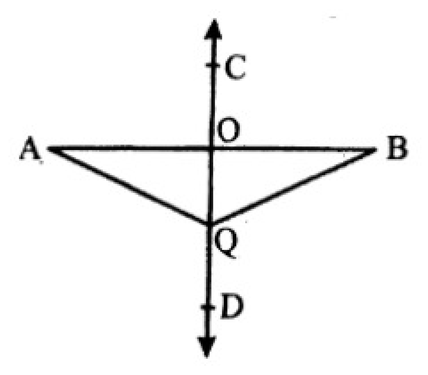
(vii) Any point on the bisector of an angle is not equidistant from its arms.

(viii) Any point inside an angle, equidistant from its arms, is on the bisector of it.

Answers:

(i) T	**(ii)** T	**(iii)** F	**(iv)** T	**(v)** F	**(vi)** T
(vii) F	**(viii)** T				

Q2. If $\overline{CD}$ is a right bisector of line segment $\overline{AB}$, then



(i) $m\overline{OA}$ = ... (ii) $m\,\overline{AQ}$ = ...

Answers:

(i) $m\,\overline{OB}$	**(ii)** $m\,\overline{BQ}$

Q3. Define the following.

(i) Bisector of a line segment:

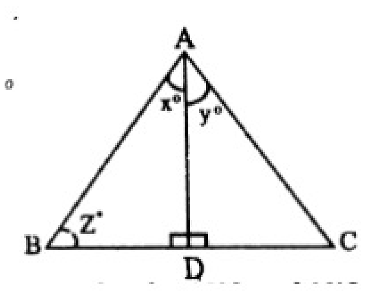
A line passing through the midpoint of a segment is called the bisector of line segment.

(ii) Bisector of an angle:

A ray that bisects an angle is called bisector of the angle.

Q4. The given triangle ABC is equilateral triangle and AD is bisector of angle Az then find the values of unknown x°, y° and z°.

Solution:



ΔABC is equilateral

$m\angle A = m\angle B = m\angle C = 60°$

$\therefore \angle° = 60°$

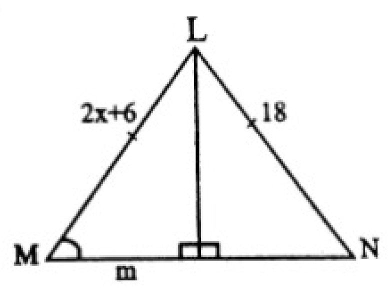
$\overline{AD}$ is bisector of $\angle A$

$x° = y° = m\angle A$

$= \frac{1}{2}(60°) = 30°$

$x° = y° = 30°$

Q5. In the given congruent triangles LMO and LNO, find the unknowns x and m.



Solution:

Corresponding sides of congruent triangles ΔLMO and ΔLNO.

$\overline{LM} \cong \overline{LN}$

$\therefore 2x + 6 = 18$

$\Rightarrow 2x = 18 - 6 = 12$

$x = \frac{12}{2} = 6$

Given that m$\overline{ON} = 12$

Since given triangles are congruent therefore $m\overline{OM} = m\overline{ON} = 12$

$m\overline{OM} = m = 12$

Q6. $\overline{CD}$ is the right bisector of the line segment $\overline{AB}$

(i) if $m\,\overline{AB}$ = 6 cm, then find the $m\,\overline{AL}$ and $m\,\overline{LB}$

(ii) if $m\,\overline{BD}$ = 4 cm, then find the $m\,\overline{AD}$

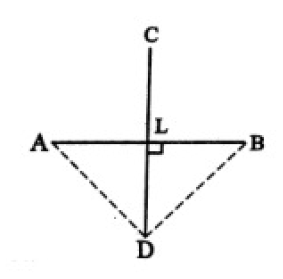
Solution:

$\overline{CD}$ is right bisector

$\therefore \quad \overline{AL} \cong \overline{BL}$

$\therefore \quad m\,\overline{AL} = m\,\overline{BL}$

$= \frac{1}{2}\,m\,\overline{AB} = \frac{1}{2}(6cm) = 3cm$



$m\,\overline{AL} = m\,\overline{BL}$= 3cm

In $\triangle ALD \leftrightarrow \triangle BLD$

$\overline{AL} \cong \overline{BL}$

$\angle BLD \cong \angle BLD$

and $DL \cong DL$

$\Delta ALD \cong \Delta BLD$

So, $m\,\overline{AD} \cong m\,\overline{BQ}$ = 4cm

mAD = 4cm