

9th Mathematics Exercise 1.6 Notes

Exercise 1.6

Q1. Use matrices, if possible, to solve the following systems of linear equations by:

(i) the matrix inversion method

(ii) the Cramer's rule

$$(i) \quad 2x - 2y = 4$$

$$3x + 2y = 6$$

$$(iii) \quad 4x + 2y = 8$$

$$3x - y = -1$$

$$(v) \quad 3x - 2y = 4$$

$$-6x + 4y = 7$$

$$(vii) \quad 2x - 2y = 4$$

$$-5x - 2y = -10$$

$$(ii) \quad 2x + y = 3$$

$$6x + 5y = 1$$

$$(iv) \quad 3x - 2y = -6$$

$$5x - 2y = -10$$

$$(vi) \quad 4x + y = 9$$

$$-3x - y = -5$$

$$(viii) \quad 3x - 4y = 4$$

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(i) Solution by Matrix Inversion Method:

$$(i) \quad 2x - 2y = 4$$

$$3x + 2y = 6$$

Step 1

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = 2 \times 2 - 3 \times (-2) = 4 + 6 = 10 \neq 0$$

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Step 3

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= M^{-1} \begin{bmatrix} 4 \\ 6 \end{bmatrix} \\ \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{|M|} \text{Adj } M \begin{bmatrix} 4 \\ 6 \end{bmatrix} \\ &= \frac{1}{10} \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} \\ &= \frac{1}{10} \begin{bmatrix} 2 \times 4 + 2 \times 6 \\ -3 \times 4 + 2 \times 6 \end{bmatrix} \\ &= \frac{1}{10} \begin{bmatrix} 8 + 12 \\ -12 + 12 \end{bmatrix} \\ &= \frac{1}{10} \begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \end{aligned}$$

$$\Rightarrow x = 2, y = 0$$

$$(ii) \quad 2x + y = 3$$

$$6x + 5y = 1$$

Step 1

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = 2 \times 5 - 1 \times 6 = 10 - 6 = 4 \neq 0$$

Step 3

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= M^{-1} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\ \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{|M|} \text{Adj } M \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \end{aligned}$$

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$$= \frac{1}{4} [5 \times 3 + (-1) \times 1]$$

$$= \frac{1}{4} \begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$$\Rightarrow x = \frac{7}{2}, y = -4$$

(iii) $4x + 2y = 8$

$$3x - y = -1$$

Step 1

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = 4 \times (-1) - 2 \times 3 = -4 - 6 = -10 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} \text{Adj } M \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$= \frac{1}{-10} \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$= \frac{1}{-10} \begin{bmatrix} (-1) \times 8 + (-2) \times (-1) \\ -3 \times 8 + 4 \times (-1) \end{bmatrix}$$

$$= \frac{1}{-10} \begin{bmatrix} -8 + 2 \\ -24 - 4 \end{bmatrix}$$

$$= \frac{1}{-10} \begin{bmatrix} -6 \\ -28 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{14}{5} \end{bmatrix}$$

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$$\Rightarrow x = \frac{3}{5}, y = \frac{14}{5}$$

$$(iv) 3x - 2y = -6$$

$$5x - 2y = -10$$

Step 1

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix} = 3 \times (-2) - 5 \times (-2) = -6 + 10 = 4 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} \text{Adj } M \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} (-2) \times (-6) + (-2) \times (-10) \\ (-5) \times (-6) + 3 \times (-10) \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -12 - 20 \\ 30 - 30 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -8 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$\Rightarrow x = -2, y = 0$$

$$(v) 3x - 2y = 4$$

$$-6x + 4y = 7$$

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Step 1

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$ is singular because

$$\det M = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix} = 3 \times 4 - (-6) \times (-2) = 12 - 12 = 0$$

So, M is a singular matrix. Hence the system of linear equations has no solution

(vi) $4x + y = 9$

$$-3x - y = -5$$

Step 1

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix} = 4 \times (-1) - -3 \times 1 = -4 + 3 = -1 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} \text{Adj } M \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$= \frac{1}{-1} \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$= \frac{1}{-1} \begin{bmatrix} (-1) \times 9 + (-1) \times (-5) \\ 3 \times 9 + 4 \times (-5) \end{bmatrix}$$

$$= -1 \begin{bmatrix} -9 + 5 \\ 27 - 20 \end{bmatrix}$$

$$= -1 \begin{bmatrix} -4 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

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$$\Rightarrow x = 4, y = -7$$

(vii) $2x - 2y = 4$

$$-5x - 2y = -10$$

Step 1

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix} = 2 \times (-2) - 5 \times 2 = -4 - 10 = -14 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} \text{Adj } M \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} -2 & -2 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} (-2) \times 4 + 2 \times (-10) \\ 5 \times 4 + 2 \times (-10) \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} -8 - 20 \\ 20 - 20 \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} -28 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\Rightarrow x = 2, y = 0$$

(viii) $3x - 4y = 4$

$$x + 2y = 8$$

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Step 1

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = 3 \times 2 - (-4) \times 1 = 6 + 4 = 10 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} \text{Adj } M \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 2 \times 4 + 4 \times 8 \\ (-1) \times 4 + 3 \times 8 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 8 + 32 \\ -4 + 24 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 40 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\Rightarrow x = 4, y = 2$$

(ii) **Solution by Cramer's Rule:**

(i) $2x - 2y = 4$

$$3x + 2y = 6$$

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

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$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = 2 \times 2 - 3 \times (-2) \\ &= 4 + 6 = 10 \neq 0 \end{aligned}$$

$$A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$$

$$\begin{aligned} |A_x| &= \begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix} = 4 \times 2 - 6 \times (-2) \\ &= 8 + 12 = 20 \neq 0 \end{aligned}$$

$$A_y = \begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}$$

$$\begin{aligned} |A_y| &= \begin{vmatrix} 2 & 4 \\ 6 & 3 \end{vmatrix} = 2 \times 6 - 3 \times 4 \\ &= 12 - 12 = 0 \neq 0 \end{aligned}$$

$$x = \frac{|A_x|}{|A|} = \frac{20}{10} = 2$$

$$y = \frac{|A_y|}{|A|} = \frac{0}{10} = 0$$

So, $x = 2$ and $y = 0$

$$(ii) \quad 2x + y = 3$$

$$6x + 5y = 1$$

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = 2 \times 5 - 1 \times 6 \\ &= 10 - 6 = 4 \neq 0 \end{aligned}$$

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$$A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$$

$$\begin{aligned} |A_x| &= \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = 3 \times 5 - 1 \times 1 \\ &= 15 - 1 = 14 \neq 0 \end{aligned}$$

$$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$$

$$\begin{aligned} |A_y| &= \begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix} = 2 \times 1 - 3 \times 6 \\ &= 2 - 18 = -16 \neq 0 \end{aligned}$$

$$x = \frac{|A_x|}{|A|} = \frac{14}{4} = \frac{7}{2}$$

$$y = \frac{|A_y|}{|A|} = \frac{-16}{4} = -4$$

So, $x = \frac{7}{2}$ and $y = -4$

$$(iii) \quad 4x + 2y = 8$$

$$3x - y = -1$$

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = 4 \times (-1) - 3 \times 2 \\ &= -4 - 6 = -10 \neq 0 \end{aligned}$$

$$A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$$

$$\begin{aligned} |A_x| &= \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix} = 8 \times (-1) - 2 \times (-1) \\ &= -8 + 2 = -6 \neq 0 \end{aligned}$$

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$$A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$$

$$\begin{aligned} |A_y| &= \begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix} = 4 \times (-1) - 3 \times 8 \\ &= -4 - 24 = -28 \neq 0 \end{aligned}$$

$$x = \frac{|A_x|}{|A|} = \frac{-6}{-10} = \frac{3}{5}$$

$$y = \frac{|A_y|}{|A|} = \frac{-28}{10} = \frac{14}{5}$$

So, $x = \frac{3}{5}$ and $y = \frac{14}{5}$

$$(iv) 3x - 2y = -6$$

$$5x - 2y = -10$$

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix} = 3 \times (-2) - 5 \times (-2) \\ &= -6 + 10 = 4 \neq 0 \end{aligned}$$

$$A_x = \begin{bmatrix} -6 & -2 \\ -10 & -2 \end{bmatrix}$$

$$\begin{aligned} |A_x| &= \begin{vmatrix} -6 & -2 \\ -10 & -2 \end{vmatrix} = -6 \times (-2) - (-10) \times (-2) \\ &= 12 - 20 = -8 \end{aligned}$$

$$A_y = \begin{bmatrix} 3 & -6 \\ 5 & -10 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}$$

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$$\begin{aligned}
 &= 3 \times (-10) - 5 \times (-6) \\
 &= -30 + 30 = 0 \\
 x &= \frac{|A_x|}{|A|} = \frac{-8}{4} = -2 \\
 y &= \frac{|A_y|}{|A|} = \frac{0}{4} = 0
 \end{aligned}$$

So, $x = -2$ and $y = 0$

$$(v) \quad 3x - 2y = 4$$

$$-6x + 4y = 7$$

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$$

$$\begin{aligned}
 |A| &= \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix} = 3 \times 4 - (-6) \times (-2) \\
 &= 12 - 12 = 0
 \end{aligned}$$

Since it is a singular matrix therefore solution is not possible.

$$(vi) \quad 4x + y = 9$$

$$-3x - y = -5$$

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix} = 4 \times (-1) - (-3) \times 1$$

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$$\begin{aligned}
 &= -4 + 3 = -1 \neq 0 \\
 A_x &= \begin{bmatrix} 9 & 1 \\ -5 & -1 \end{bmatrix} \\
 |A_x| &= \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix} = 9 \times (-1) - (-5) \times 1 \\
 &= -9 + 5 = -4 \neq 0 \\
 A_y &= \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \\
 |A_y| &= \begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix} = 4 \times (-5) - (-3) \times 9 \\
 &= -20 + 27 = 7 \neq 0 \\
 x &= \frac{|A_x|}{|A|} = \frac{-4}{-1} = 4 \\
 y &= \frac{|A_y|}{|A|} = \frac{7}{-1} = -7
 \end{aligned}$$

So, $x = 4$ and $y = -7$

$$(vii) 2x - 2y = 4$$

$$-5x - 2y = -10$$

$$\begin{aligned}
 \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 4 \\ -10 \end{bmatrix} \\
 A &= \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \\
 |A| &= \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix} = 2 \times (-2) - (-5) \times (-2) \\
 &= -4 - 10 = -14 \neq 0 \\
 A_x &= \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix} \\
 |A_x| &= \begin{vmatrix} 4 & -2 \\ -10 & -2 \end{vmatrix} = 4 \times (-2) - (-10) \times 2
 \end{aligned}$$

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$$\begin{aligned}
 &= -8 - 20 = -28 \\
 A_y &= \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix} \\
 |A_y| &= \begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix} = 2 \times (-10) - (-5) \times 4 \\
 &= -20 + 20 = 0 \\
 x &= \frac{|A_x|}{|A|} = \frac{-28}{-14} = 2 \\
 y &= \frac{|A_y|}{|A|} = \frac{0}{-14} = 0
 \end{aligned}$$

So, $x = 2$ and $y = 0$

$$(viii) 3x - 4y = 4$$

$$x + 2y = 8$$

$$\begin{aligned}
 \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 4 \\ 8 \end{bmatrix} \\
 A &= \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \\
 |A| &= \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = 3 \times 2 - 1 \times (-4) \\
 &= 6 + 4 = 10 \neq 0 \\
 A_x &= \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix} \\
 |A_x| &= \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix} = 4 \times 2 - 8 \times (-4) \\
 &= 8 - 32 = 40 \\
 A_y &= \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix} \\
 |A_y| &= \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix} = 3 \times 8 - 1 \times 4
 \end{aligned}$$

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$$\begin{aligned} &= 24 - 4 = 20 \\ x &= \frac{|A_x|}{|A|} = \frac{40}{10} = 4 \\ y &= \frac{|A_y|}{|A|} = \frac{20}{10} = 2 \end{aligned}$$

So, $x = 4$ and $y = 2$

Q2. The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150 cm. Find the dimensions of the rectangle.

Solution:

(i) **Method 1: Matrix Inversion Method:**

Let length of rectangle is x cm and width of rectangle is y cm.

According to the given conditions

$$x = 4y$$

$$\text{or } x - 4y = 0 \quad \text{----- (i)}$$

$$\text{Perimeter} = 150\text{cm}$$

$$\text{Perimeter} = 2(x + y) = 150$$

$$\text{or } x + y = 75 \quad \text{----- (ii)}$$

By solving (i) and (ii), we get

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

The coefficient matrix $M = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 1 & -4 \\ 1 & 1 \end{vmatrix} = 1 \times 1 - 1 \times (-4) = 1 + 4 = 5 \neq 0$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

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$$\begin{aligned}
 \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{M} \operatorname{Adj} M \begin{bmatrix} 0 \\ 75 \end{bmatrix} \\
 &= \frac{1}{5} \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 75 \end{bmatrix} \\
 &= \frac{1}{5} \begin{bmatrix} 1 \times 0 + 4 \times 75 \\ (-1) \times 0 + 1 \times 75 \end{bmatrix} \\
 &= \frac{1}{5} \begin{bmatrix} 0 + 300 \\ 0 + 75 \end{bmatrix} \\
 &= \frac{1}{5} \begin{bmatrix} 300 \\ 75 \end{bmatrix} \\
 &= \begin{bmatrix} 60 \\ 15 \end{bmatrix}
 \end{aligned}$$

$\Rightarrow x = 60, y = 15$

So, length = $x = 60$ cm

width = $y = 15$ cm

(ii) Method 2: By Cramer's rule:

$$x - 4y = 4 \quad ; \quad x + y = 8$$

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$$

$$\begin{aligned}
 |A| &= \begin{vmatrix} 1 & -4 \\ 1 & 1 \end{vmatrix} = 1 \times 1 - (-4) \times 1 \\
 &= 1 + 4 = 5 \neq 0
 \end{aligned}$$

$$A_x = \begin{bmatrix} 0 & -4 \\ 75 & 1 \end{bmatrix}$$

$$\begin{aligned}
 |A_x| &= \begin{vmatrix} 0 & -4 \\ 75 & 1 \end{vmatrix} = 0 \times 1 - (-4) \times (75) \\
 &= 0 + 300 = 300
 \end{aligned}$$

$$A_y = \begin{bmatrix} 1 & 0 \\ 1 & 75 \end{bmatrix}$$

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$$\begin{aligned}|A_y| &= \begin{vmatrix} 1 & 0 \\ 1 & 75 \end{vmatrix} = 1 \times 75 - 0 \times 1 \\ &= 75 - 0 = 75 \\ x &= \frac{|A_x|}{|A|} = \frac{300}{5} = 60 \\ y &= \frac{|A_y|}{|A|} = \frac{75}{5} = 15\end{aligned}$$

$$\Rightarrow x = 60, y = 15$$

So, length = $x = 60$ cm

width = $y = 15$ cm

Q3. Two sides of a rectangle differ by 3.5 cm. Find the dimensions of the rectangle if its perimeter is 67 cm.

Solution:

(I) **Method1: Matrix Inversion Method:**

Let the length of the rectangle be x cm and its width be y cm.

According to the given conditions

$$x - y = 3.5$$

$$\text{or } 10x - 10y = 35 \quad (\text{Hint: multiply both sides by 10})$$

$$\text{and } 2(x + y) = 2x + 2y = 67$$

$$\text{or } 2x + 2y = 67$$

$$\begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

The coefficient matrix $M = \begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}$ is non-singular because

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$$\det M = \begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix} = 10 \times 2 - (-10) \times 2 \\ = 20 + 20 = 40 \neq 0$$

So, M is not a singular matrix

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\text{adj } M}{|M|} \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$
$$= \frac{1}{40} \begin{bmatrix} 2 & 10 \\ -2 & 10 \end{bmatrix} \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$
$$= \frac{1}{40} \begin{bmatrix} 2 \times 35 + 10 \times 67 \\ -2 \times 35 + 10 \times 67 \end{bmatrix}$$
$$= \frac{1}{40} \begin{bmatrix} 70 + 670 \\ -70 + 670 \end{bmatrix}$$
$$= \frac{1}{40} \begin{bmatrix} 740 \\ 600 \end{bmatrix} = \begin{bmatrix} 18.5 \\ 15 \end{bmatrix}$$

$$\Rightarrow x = 18.5, y = 15$$

So, length is x = 18.5 cm and

width is y = 15 cm.

(ii) Method 2: By Cramer's rule:

$$10x - 10y = 35 ; 2x + 2y = 67$$

$$\begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

$$A = \begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix} = 10 \times 2 - (-10) \times 2 \\ = 20 + 20 = 40 \neq 0$$

$$A_x = \begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}$$

9th Mathematics Exercise 1.6 Notes

$$\begin{aligned}|A_x| &= \begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix} \\&= 35 \times 2 - (-10) \times 67 \\&= 70 + 670 = 740\end{aligned}$$

$$\begin{aligned}A_y &= \begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix} \\|A_y| &= \begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix} = 10 \times 67 - 35 \times 2 \\&= 670 - 70 = 600\end{aligned}$$

$$\begin{aligned}x &= \frac{|A_x|}{|A|} = \frac{740}{40} = 18.5 \\y &= \frac{|A_y|}{|A|} = \frac{600}{40} = 15\end{aligned}$$

$$\Rightarrow x = 18.5, y = 15$$

So, length is $x = 18.5$ cm and
width is $y = 15$ cm.

Q4. The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.

Solution:

(i) **Method 1: Matrix Inversion Method:**

Let each equal angle be x° and the third angle be y°

$$\therefore 2x - 16 = y$$

$$\text{or } 2x - y = 16$$

9th Mathematics Exercise 1.6 Notes

and $2x + y = 180$ (Because sum of all the sides of a triangle is 180°)

$$\therefore \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

The coefficient matrix $M = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 2 = 2 + 2 = 4 \neq 0$$

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= M^{-1} \begin{bmatrix} 16 \\ 180 \end{bmatrix} \\ \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{M} \text{Adj } M \begin{bmatrix} 16 \\ 180 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 16 \\ 180 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 1 \times 16 + 1 \times 180 \\ 1 \times 16 + 1 \times 180 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 16 + 180 \\ -32 + 360 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 197 \\ 328 \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix} \end{aligned}$$

$$\Rightarrow x = 49, y = 82$$

$$x + y + z = 180^\circ$$

$$49^\circ + 82^\circ + z = 180^\circ$$

$$z = 180^\circ - 49^\circ - 82^\circ = 49^\circ$$

So, the angles are $49^\circ, 49^\circ, 82^\circ$

(ii) **Method 2: By Cramer's rule:**

9th Mathematics Exercise 1.6 Notes

$$2x - y = 16 \quad ; \quad 2x + y = 180$$

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 2 \\ &= 2 + 2 = 4 \neq 0 \end{aligned}$$

$$A_x = \begin{bmatrix} 16 & -1 \\ 80 & 1 \end{bmatrix}$$

$$\begin{aligned} |A_x| &= \begin{vmatrix} 16 & -1 \\ 80 & 1 \end{vmatrix} \\ &= 16 \times 1 - (-1) \times 80 \end{aligned}$$

$$|A_x| = 16 + 80 = 196$$

$$A_y = \begin{bmatrix} 2 & 16 \\ 2 & 80 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 2 & 16 \\ 2 & 80 \end{vmatrix} = 2 \times 80 - 16 \times 2$$

$$|A_y| = 360 - 32 = 328$$

$$x = \frac{|A_x|}{|A|} = \frac{196}{4} = 49$$

$$y = \frac{|A_y|}{|A|} = \frac{328}{4} = 82$$

$$\Rightarrow x = 49, y = 82$$

$$x + y + z = 180^\circ$$

$$49^\circ + 82^\circ + z = 180^\circ$$

9th Mathematics Exercise 1.6 Notes

$$z = 180^\circ - 49^\circ - 82^\circ = 49^\circ$$

So, the angles are $49^\circ, 49^\circ, 82^\circ$

Q5. One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle.

Solution:

(i) **Method 1: Matrix Inversion Method:**

Let two angles be x° and y°

$$\therefore 2x + 12 = y$$

$$\text{or } 2x - y = -12$$

$$\text{and } x + y = 90 \quad (\text{sum of angles})$$

$$\therefore \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

The coefficient matrix $M = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 1 = 2 + 1 = 3 \neq 0$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{M} \text{Adj } M \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

9th Mathematics Exercise 1.6 Notes

$$\begin{aligned}
 &= \frac{1}{3} \begin{bmatrix} 1 \times (-12) + 1 \times 90 \\ -1 \times (-12) + 2 \times 90 \end{bmatrix} \\
 &= \frac{1}{3} \begin{bmatrix} -12 + 90 \\ 12 + 180 \end{bmatrix} \\
 &= \frac{1}{3} \begin{bmatrix} 78 \\ 192 \end{bmatrix} = \begin{bmatrix} 26 \\ 64 \end{bmatrix}
 \end{aligned}$$

$\Rightarrow x = 26, y = 64$

So, the angles are 26° and 64°

(ii) Method 2: By Cramer's rule:

$$2x - y = 12 ; \quad x + y = 90$$

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{aligned}
 |A| &= \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 1 \\
 &= 2 + 1 = 3 \neq 0
 \end{aligned}$$

$$A_x = \begin{bmatrix} -12 & -1 \\ 90 & 1 \end{bmatrix}$$

$$\begin{aligned}
 |A_x| &= \begin{vmatrix} -12 & -1 \\ 90 & 1 \end{vmatrix} \\
 &= (-12) \times 1 - (-1) \times 90
 \end{aligned}$$

$$|A_x| = -12 + 90 = 78$$

$$A_y = \begin{bmatrix} 90 & 12 \\ -1 & 2 \end{bmatrix}$$

9th Mathematics Exercise 1.6 Notes

$$\begin{aligned}|A_y| &= \begin{vmatrix} 90 & 12 \\ -1 & 2 \end{vmatrix} \\ &= 90 \times 2 - 12 \times (-1) \\ |A_y| &= 180 + 12 = 192\end{aligned}$$

$$\begin{aligned}x &= \frac{|A_x|}{|A|} = \frac{78}{3} = 26 \\ y &= \frac{|A_y|}{|A|} = \frac{192}{3} = 64\end{aligned}$$

$$\Rightarrow x = 26, y = 64$$

So, the angles are 26° and 64°

Q6. Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

(i) Method 1: Matrix Inversion Method:

Let the speeds of two cars be x km/h and y km/h

$$x - y = 6 \quad \text{--- (i)}$$

Distance covered in $4\frac{1}{2}$ hours = $600 - 123 = 477$ km

$$4\frac{1}{2}x + 4\frac{1}{2}y = 477$$

$$\frac{9}{2}x + \frac{9}{2}y = 477$$

$$\text{or } \frac{x}{2} + \frac{y}{2} = 53$$

$$\text{or } x + y = 106 \quad \text{--- (ii)}$$

9th Mathematics Exercise 1.6 Notes

The two equations are (i) and (ii)

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

The coefficient matrix $M = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 \times 1 - 1 \times (-1) = 1 + 1 = 2 \neq 0$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{M} \text{Adj } M \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 \times 6 + 1 \times 106 \\ -1 \times 6 + 1 \times 106 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 6 + 106 \\ -6 + 106 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 112 \\ 100 \end{bmatrix} = \begin{bmatrix} 56 \\ 50 \end{bmatrix}$$

$$\Rightarrow x = 56, y = 50$$

The speeds of the two cars are 56 km/h and 50 km/h

(ii) Method 2: By Cramer's rule:

$$x - y = 6 \quad ; \quad x + y = 106$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 \times 1 - 1 \times (-1)$$

9th Mathematics Exercise 1.6 Notes

$$= 1 + 1 = 2 \neq 0$$

$$A_x = \begin{bmatrix} 6 & -1 \\ 106 & 1 \end{bmatrix}$$

$$\begin{aligned}|A_x| &= \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix} \\&= 6 \times 1 - (-1) \times (106) \\&= 6 + 106 = 112\end{aligned}$$

$$A_y = \begin{bmatrix} 1 & 6 \\ 1 & 106 \end{bmatrix}$$

$$\begin{aligned}|A_y| &= \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix} = 1 \times 106 - 6 \times 1 \\|A_y| &= 106 - 6 = 100\end{aligned}$$

$$x = \frac{|A_x|}{|A|} = \frac{112}{2} = 56$$

$$y = \frac{|A_y|}{|A|} = \frac{100}{5} = 50$$

$$\Rightarrow x = 56, y = 50$$

The speeds of the two cars are 56 km/h and 50 km/h

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