Exercise 1.2

Q1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

A = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$, C = $\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$,

D = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, E = $\begin{bmatrix} 0 \end{bmatrix}$, f = $\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

Solution:

Matrix A is a null matrix (because all its entities are zero)

Matrix B is a row matrix (because it has only one row).

Matrix C is a column matrix (because it has only one column).

Matrix D is a diagonal matrix (because its diagonal entities are 1).

Matrix E is a null matrix (because all its entities are 0).

Matrix F is a column matrix (because it has only one column).

Q2. From the following matrices, identify

(a) Square matrices **(b) Rectangular matrices**

(c) Row matrices **(d) Column matrices**

(e) Identity matrices **(f) Null matrices**

(i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$, **(ii)** $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$, **(iii)** $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$,

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, **(v)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, **(vi)** $\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$,

(vii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, **(viii)** $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(ix)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution:

(a) (iii), (iv) and (viii) are square matrices because the number of rows are equal to number of columns.

(b) (i), (ii), (v), (vi) (vii), (ix) are rectangular matrices because their rows and columns are not equal.

(c) (vi) is a row matrix because it has only one row.

(d) (ii) and (vii) are column matrices because they have only one column.

(e) (iv) is an identity matrix as well because its diagonal elements are "1".

(f) (ix) is a null matrix because its each entity is zero.

Q3. From the following matrices, identify diagonal, scalar and unit (identity) matrices.

A = $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$, C = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

D = $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$, E = $\begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$

Solution:

Matrix A is a scalar matrix (because its diagonal entities are same).

Matrix B is a diagonal matrix (because its diagonal entities are non-zero and non-diagonal entities are zero).

Matrix C is an identity matrix (because its diagonal entities are 1).

Matrix D is a diagonal matrix (because its one diagonal entity is non-zero and non-diagonal entities are zero).

Matrix E is a scalar matrix (because its diagonal entities are same).

Q4. Find negative of matrices A, B, C, D and E when:

A = $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, B = $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, C = $\begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$,

D = $\begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$, E = $\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$

Solution:

Negative of a matrix is obtained by inverting (changing) the signs of all its entities.

So,

(i) $-A = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ **(ii)** $-B = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$

(iii) $-C = \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$ **(iv)** $-D = \begin{bmatrix} 3 & -2 \\ 4 & -5 \end{bmatrix}$

(v) $-E = \begin{bmatrix} -1 & 5 \\ -2 & -3 \end{bmatrix}$

Q5. Find the transpose of each of the following matrices:

$A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$,

$D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$, $E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution:

Transpose of a matrix is obtained by converting all the columns of that matrix to the rows and all the rows to the columns.

So, according to the definition;

(i) $A^t = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$ **(ii)** $B^t = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$

(iii) $C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$ **(iv)** $D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

(v) $E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$ **(vi)** $F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Q6. Verify that if $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ then

(i) $(A^t)^t = A$ **(ii)** $(B^t)^t = B$

Solution:

(i) To prove: $(A^t)^t = A$

Given $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

Taking transpose of A^t, we will get

$(A^t)^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = A$

Hence proved: **$(A^t)^t = A$**

(ii) To prove: $(B^t)^t = B$

Given $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

Taking transpose of B^t, we will get

$(B^t)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = B$

Hence proved: **$(B^t)^t = B$**