Practice 5 multiplication of each type of Matrix Multiplication

Python Programme if you want

Matrix $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}$

$C = \begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}$ $D = \begin{bmatrix} -7 & 9 \\ 4 & -3 \end{bmatrix}$

$E = \begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}$ $F = \begin{bmatrix} 2 & -1 \\ 5 & 8 \\ 4 & 0 \end{bmatrix}$

$G = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 0 & -2 \end{bmatrix}$ $H = \begin{bmatrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{bmatrix}$

(1) Dot Product (2) Column Method (3) Row Method (4) Outer product (5) Block Multiplication

(a) A * B
(b) A * G
(c) C * E
(d) C * D
(e) A * H
(f) C * F
(g) E * F
(h) B * E
(i) D * E
(j) G * H
(k) B * F
(l) D * F

(1)

(a) $A * B = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}_{2\times2} * \begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}_{2\times2}$

DOT PRODUCT

$= \begin{bmatrix} -6+(-25) & 0+30 \\ 8+10 & 0+(-12) \end{bmatrix}$

$= \begin{bmatrix} -31 & 30 \\ 18 & -12 \end{bmatrix}_{2\times2}$

* Issues Encountered

(b) $A * G = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} * \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 0 & -2 \end{bmatrix}$

$2*2 \neq 3*2$

(c) $C * E \begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix} * \begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}$

$2\times2 \neq 3\times2$

Similar Issue faced as above on the following Matrices

(f) C×F (i) D×E (L) D×F
(h) B×E (k) B×F

So to Eradicate the multiplication issue following corrections are done:

Original Order	Modified Order
$A_{2\times2} * G_{3\times2}$	$G_{3\times2} * A_{2\times2}$
$C_{2\times2} * E_{3\times2}$	$E_{3\times2} * C_{2\times2}$

Similar Approach was taken for other Matrices as well

C * F	F * C
D * E	E * D
B * E	E * B
B * F	F * B
D * F	F * D

Now my question Set becomes as below

(a) A × B (✓)	(b) G × A (m)	(c) E × C (m)
(d) C × D	(e) A × H	(f) F × C (m)
(g) E × F	(h) E × B (m)	(i) E × D (m)
(j) G × H	(k) F × B (m)	(l) F × D (m)

(*) (m) marks the ones that were modified
(✓) multiplication done for the matrix on Dot product

With the continuation...

1. (b) $G \times A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 0 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ = Resultant Matrix will be of 3×2

(3×2 = 2×2)

DOT PRODUCT

$$= \begin{bmatrix} [3\times3]+[1\times(-4)] & [3\times(-5)]+[1\times2] \\ [2\times3]+[4\times(-4)] & [2\times(-5)]+[4\times2] \\ [0\times3]+[(-2)\times(-4)] & [0\times(-5)]+[(-2)\times2] \end{bmatrix}_{3\times2} = \begin{bmatrix} 9-4 & -15+2 \\ 6-16 & -10+8 \\ 0+8 & 0-4 \end{bmatrix} = \begin{bmatrix} 5 & -13 \\ -10 & -2 \\ 8 & -4 \end{bmatrix}$$

At the end, we will compare the result set across different method to validate that ours answer is correct

* DOT PRODUCT *

① ⓒ. $E * C = \begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}_{2\times2} = \begin{bmatrix} [(-9)\times(-2)]+[2\times4] & [(-9)\times7]+[2\times(-5)] \\ [8\times(-2)]+[(-1)\times4] & [8\times7]+[(-1)\times(-5)] \\ [4\times(-2)]+[(-3)\times4] & [4\times7]+[(-3)\times(-5)] \end{bmatrix}$

$= \begin{bmatrix} [18+8] & [-63+(-10)] \\ [(-16)+(-4)] & [56+5] \\ [(-8)+(-12)] & [28+15] \end{bmatrix} = \begin{bmatrix} 26 & -73 \\ -20 & 61 \\ -20 & 43 \end{bmatrix}$

① ⓓ. $C * D = \begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}_{2\times2} * \begin{bmatrix} -7 & 9 \\ 4 & -3 \end{bmatrix}_{2\times2} = \begin{bmatrix} [(-2)\times(-7)]+[7\times4] & [(-2)\times9]+[7\times(-3)] \\ [4\times(-7)]+[(-5)\times4] & [4\times9]+[(-5)\times(-3)] \end{bmatrix}$

$= \begin{bmatrix} [14+28] & [(-18)+(-21)] \\ [(-28)+(-20)] & [36+15] \end{bmatrix} = \begin{bmatrix} 42 & -39 \\ -48 & 51 \end{bmatrix}$

① ⓔ. $A * H = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}_{2\times2} \times \begin{bmatrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{bmatrix}_{2\times3} = \begin{bmatrix} [(3\times7)+(-5\times8)] & [(3\times4)+(-5\times1)] & [(3\times9)+(-5\times5)] \\ [(-4\times7)+(2\times8)] & (-4\times4)+(2\times1) & [(-4\times9)+(2\times5)] \end{bmatrix}$

$= \begin{bmatrix} [21-40] & [12-5] & [27-25] \\ [-28+16] & [-16+2] & [-36+10] \end{bmatrix} = \begin{bmatrix} -19 & 7 & 2 \\ -12 & -14 & -26 \end{bmatrix}$

① ⓕ. $F * C = \begin{bmatrix} 2 & -1 \\ 5 & 8 \\ 4 & 0 \end{bmatrix}_{3\times2} * \begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}_{2\times2} = \begin{bmatrix} [(2\times-2)+(-1\times4)] & [(2\times7)+(-1\times-5)] \\ [(5\times-2)+(8\times4)] & [(5\times7)+(8\times-5)] \\ [(4\times-2)+(0\times4)] & [(4\times7)+(0\times-5)] \end{bmatrix} = \begin{bmatrix} (-4+-4) & (14+5) \\ (-10+32) & (35-40) \\ (-8+0) & (28+0 \end{bmatrix}$

$= \begin{bmatrix} -8 & 19 \\ 22 & -5 \\ -8 & 28 \end{bmatrix}$

① ⓖ. $E * F = \begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}_{3\times2} * \begin{bmatrix} 2 & -1 \\ 5 & 8 \\ 4 & 0 \end{bmatrix}_{3\times2}$ → Changing the orders will not give us the candidature to multiply as well.

$3\times2 \neq 3\times2$

① ⓗ. $B * E \sim E * B = \begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}_{2\times2} = \begin{bmatrix} [(-9\times-2)+(2\times5)] & [(-9\times0)+(2\times-6)] \\ [(8\times-2)+(-1\times5)] & [(8\times0)+(-1\times-6)] \\ [4\times-2)+(-3\times5)] & [(4\times0)+(-3\times-6)] \end{bmatrix}$

$= \begin{bmatrix} 18+10 & 0-12 \\ -16-5 & 0+6 \\ -8-15 & 0+18 \end{bmatrix} = \begin{bmatrix} 28 & -12 \\ -21 & 6 \\ -23 & 18 \end{bmatrix}$

* DOT PRODUCT *

①ⓘ. $E * D = \begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -7 & 9 \\ 4 & -3 \end{bmatrix}_{2\times2} = \begin{bmatrix} [(-9\times-7)+(2\times4)] & [(-9\times9)+(2\times-3)] \\ [(8\times-7)+(-1\times4)] & [(8\times9)+(-1\times-3)] \\ [(4\times-7)+(-3\times4)] & [(4\times9)+(-3\times-3)] \end{bmatrix}$

$= \begin{bmatrix} 63+8 & -81-6 \\ -56-4 & 72+3 \\ -28-12 & 36+9 \end{bmatrix} = \begin{bmatrix} 71 & -87 \\ -60 & 75 \\ -40 & 45 \end{bmatrix}$

1.ⓙ. $G * H = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 0 & -2 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{bmatrix}_{2\times3} = \begin{bmatrix} [(3\times7)+(1\times8)] & [(3\times4)+(1\times1)] & [(3\times9)+(1\times5)] \\ [(2\times7)+(4\times8)] & [(2\times4)+(4\times1)] & [(2\times9)+(4\times5)] \\ [(0\times7)+(-2\times8)] & [(0\times4)+(-2\times1)] & [(0\times9)+(-2\times5)] \end{bmatrix}$

$= \begin{bmatrix} 21+8 & 12+1 & 27+5 \\ 14+32 & 8+4 & 18+20 \\ 0+-16 & 0-2 & 0-10 \end{bmatrix} = \begin{bmatrix} 29 & 13 & 32 \\ 46 & 12 & 38 \\ -16 & -2 & -10 \end{bmatrix}$

1.ⓚ. $F * B = \begin{bmatrix} 2 & -1 \\ 5 & 8 \\ 4 & 0 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}_{2\times2} = \begin{bmatrix} [(2\times-2)+(-1\times5)] & [(2\times0)+(-1\times-6)] \\ [(5\times-2)+(8\times5)] & [(5\times0)+(8\times-6)] \\ [(4\times-2)+(0\times5)] & [(4\times0)+(0\times-6)] \end{bmatrix}$

$= \begin{bmatrix} -4-5 & 0+6 \\ -10+40 & 0-48 \\ -8+0 & 0+0 \end{bmatrix} = \begin{bmatrix} -9 & 6 \\ 30 & -48 \\ -8 & 0 \end{bmatrix}$

1.ⓛ. $F * D = \begin{bmatrix} 2 & -1 \\ 5 & 8 \\ 4 & 0 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -7 & 9 \\ 4 & -3 \end{bmatrix}_{2\times2} = \begin{bmatrix} [(2\times-7)+(-1\times4)] & [(2\times9)+(-1\times-3)] \\ [(5\times-7)+(8\times4)] & [(5\times9)+(8\times-3)] \\ [(4\times-7)+(0\times4)] & [(4\times9)+(0\times-3)] \end{bmatrix}$

$= \begin{bmatrix} [-14-4] & [18+3] \\ [-35+32] & [45-24] \\ [-28+0] & [36+0] \end{bmatrix} = \begin{bmatrix} -18 & 21 \\ -3 & 21 \\ -28 & 36 \end{bmatrix}$

* Column Product *

2@. $A * B = \underset{2\times2}{\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}} \times \underset{2\times2}{\begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}} = \begin{cases} Col1 = -2\begin{bmatrix} 3 \\ -4 \end{bmatrix} + 5\begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ 8 \end{bmatrix} + \begin{bmatrix} -25 \\ 10 \end{bmatrix} = \begin{bmatrix} -31 \\ 18 \end{bmatrix} \\ Col2 = 0\begin{bmatrix} 3 \\ -4 \end{bmatrix} + -6\begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 30 \\ -12 \end{bmatrix} = \begin{bmatrix} 30 \\ -12 \end{bmatrix} \end{cases}$

$= \begin{bmatrix} -31 & 30 \\ 18 & -12 \end{bmatrix}$

2(b). $\underset{3\times2}{\begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 0 & -2 \end{bmatrix}} \times \underset{2\times2}{\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}} = \begin{cases} Col1 = 3\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + -4\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix} + \begin{bmatrix} -4 \\ -16 \\ 8 \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \\ 8 \end{bmatrix} \\ Col2 = -5\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -15 \\ -10 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 8 \\ -4 \end{bmatrix} = \begin{bmatrix} -13 \\ -2 \\ -4 \end{bmatrix} \end{cases}$

$= \begin{bmatrix} 5 & -13 \\ -10 & -2 \\ 8 & -4 \end{bmatrix}$

2.(c). $\underset{3\times2}{\begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}} \times \underset{2\times2}{\begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}} = \begin{cases} Col1 = -2 \times \begin{bmatrix} -9 \\ 8 \\ 4 \end{bmatrix} + 4\begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 18 \\ -16 \\ -8 \end{bmatrix} + \begin{bmatrix} 8 \\ -4 \\ -12 \end{bmatrix} = \begin{bmatrix} 26 \\ -20 \\ -20 \end{bmatrix} \\ Col2 = 7\begin{bmatrix} -9 \\ 8 \\ 4 \end{bmatrix} + -5\begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} -63 \\ 56 \\ 28 \end{bmatrix} + \begin{bmatrix} -10 \\ 5 \\ 15 \end{bmatrix} = \begin{bmatrix} -73 \\ 61 \\ 43 \end{bmatrix} \end{cases}$

$= \begin{bmatrix} 26 & -73 \\ -20 & 61 \\ -20 & 43 \end{bmatrix}$

2.(d). $\underset{2\times2}{\begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}} \times \begin{bmatrix} -7 & 9 \\ 4 & -3 \end{bmatrix} = \left.\begin{cases} Col1 = -7\begin{bmatrix} -2 \\ 4 \end{bmatrix} + 4\begin{bmatrix} 7 \\ -5 \end{bmatrix} = \begin{bmatrix} 14 \\ -28 \end{bmatrix} + \begin{bmatrix} 28 \\ -20 \end{bmatrix} = \begin{bmatrix} 42 \\ -48 \end{bmatrix} \\ Col2 = 9\begin{bmatrix} -2 \\ 4 \end{bmatrix} + -3\begin{bmatrix} 7 \\ -5 \end{bmatrix} = \begin{bmatrix} -18 \\ 36 \end{bmatrix} + \begin{bmatrix} -21 \\ 15 \end{bmatrix} = \begin{bmatrix} -39 \\ 51 \end{bmatrix} \end{cases}\right\} = \begin{bmatrix} 42 & -39 \\ -48 & 51 \end{bmatrix}$

2.(e). $\underset{2\times2}{\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}} \times \underset{2\times3}{\begin{bmatrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{bmatrix}} = \left.\begin{cases} Col1 = 7\begin{bmatrix} 3 \\ -4 \end{bmatrix} + 8\begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 21 \\ -28 \end{bmatrix} + \begin{bmatrix} -40 \\ 16 \end{bmatrix} = \begin{bmatrix} -19 \\ -12 \end{bmatrix} \\ Col2 = 4\begin{bmatrix} 3 \\ -4 \end{bmatrix} + 1\begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ -16 \end{bmatrix} + \begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ -14 \end{bmatrix} \\ Col3 = 9\begin{bmatrix} 3 \\ -4 \end{bmatrix} + 5\begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 27 \\ -36 \end{bmatrix} + \begin{bmatrix} -25 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -26 \end{bmatrix} \end{cases}\right\} = \begin{bmatrix} -19 & 7 & 2 \\ -12 & -14 & -26 \end{bmatrix}$

2.(f). $\underset{3\times2}{\begin{bmatrix} 2 & -1 \\ 5 & 8 \\ 4 & 0 \end{bmatrix}} \times \underset{2\times2}{\begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}} = \left.\begin{cases} Col1 = -2\begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ -10 \\ -8 \end{bmatrix} + \begin{bmatrix} -4 \\ 32 \\ 0 \end{bmatrix} = \begin{bmatrix} -8 \\ 22 \\ -8 \end{bmatrix} \\ Col2 = 7\begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} + -5\begin{bmatrix} -1 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 14 \\ 35 \\ 28 \end{bmatrix} + \begin{bmatrix} 5 \\ -40 \\ 0 \end{bmatrix} = \begin{bmatrix} 19 \\ -5 \\ 28 \end{bmatrix} \end{cases}\right\} = \begin{bmatrix} -8 & 19 \\ 22 & -5 \\ -8 & 28 \end{bmatrix}$

2.(g). Invalid rank matrix as per Page 3

* Column Product *

2.(h). $E*B = \underset{3\times2}{\begin{bmatrix}-9 & 2\\ 8 & -1\\ 4 & -3\end{bmatrix}} \times \underset{2\times2}{\begin{bmatrix}-2 & 0\\ 5 & -6\end{bmatrix}} = \begin{cases} Col\,1 = -2\begin{bmatrix}-9\\8\\4\end{bmatrix} + 5\begin{bmatrix}2\\-1\\-3\end{bmatrix} = \begin{bmatrix}18\\-16\\-8\end{bmatrix} + \begin{bmatrix}10\\-5\\-15\end{bmatrix} = \begin{bmatrix}28\\-21\\-23\end{bmatrix} \\ Col\,2 = 0\begin{bmatrix}-9\\8\\4\end{bmatrix} + -6\begin{bmatrix}2\\-1\\-3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}-12\\6\\18\end{bmatrix} = \begin{bmatrix}-12\\6\\18\end{bmatrix} \end{cases} = \underset{3\times2}{\begin{bmatrix}28 & -12\\ -21 & 6\\ -23 & 18\end{bmatrix}}$

2.(i). $E*D = \underset{3\times2}{\begin{bmatrix}-9 & 2\\ 8 & -1\\ 4 & -3\end{bmatrix}} \times \underset{2\times2}{\begin{bmatrix}-7 & 9\\ 4 & -3\end{bmatrix}} = \begin{cases} col\,1 = -7\begin{bmatrix}-9\\8\\4\end{bmatrix} + 4\begin{bmatrix}2\\-1\\-3\end{bmatrix} = \begin{bmatrix}63\\-56\\-28\end{bmatrix} + \begin{bmatrix}8\\-4\\-12\end{bmatrix} = \begin{bmatrix}71\\-60\\-40\end{bmatrix} \\ Col\,2 = 9\begin{bmatrix}-9\\8\\4\end{bmatrix} + -3\begin{bmatrix}2\\-1\\-3\end{bmatrix} = \begin{bmatrix}-81\\72\\36\end{bmatrix} + \begin{bmatrix}-6\\3\\9\end{bmatrix} = \begin{bmatrix}-87\\75\\45\end{bmatrix} \end{cases} = \underset{3\times2}{\begin{bmatrix}71 & -87\\ -60 & 75\\ -40 & 45\end{bmatrix}}$

2.(j). $G*H = \underset{3\times2}{\begin{bmatrix}3 & 1\\ 2 & 4\\ 0 & -2\end{bmatrix}} \times \underset{2\times3}{\begin{bmatrix}7 & 4 & 9\\ 8 & 1 & 5\end{bmatrix}} = \begin{cases} Col\,1 = 7\begin{bmatrix}3\\2\\0\end{bmatrix} + 8\begin{bmatrix}1\\4\\-2\end{bmatrix} = \begin{bmatrix}21\\14\\0\end{bmatrix} + \begin{bmatrix}8\\32\\-16\end{bmatrix} = \begin{bmatrix}29\\46\\-16\end{bmatrix} \\ Col\,2 = 4\begin{bmatrix}3\\2\\0\end{bmatrix} + 1\begin{bmatrix}1\\4\\-2\end{bmatrix} = \begin{bmatrix}12\\8\\0\end{bmatrix} + \begin{bmatrix}1\\4\\-2\end{bmatrix} = \begin{bmatrix}13\\12\\-2\end{bmatrix} \\ col\,3 = 9\begin{bmatrix}3\\2\\0\end{bmatrix} + 5\begin{bmatrix}1\\4\\-2\end{bmatrix} = \begin{bmatrix}27\\18\\0\end{bmatrix} + \begin{bmatrix}5\\20\\-10\end{bmatrix} = \begin{bmatrix}32\\38\\-10\end{bmatrix} \end{cases} = \underset{3\times3}{\begin{bmatrix}29 & 13 & 32\\ 46 & 12 & 38\\ -16 & -2 & -10\end{bmatrix}}$

2.(k). $F*B = \underset{3\times2}{\begin{bmatrix}2 & -1\\ 5 & 8\\ 4 & 0\end{bmatrix}} \times \underset{2\times2}{\begin{bmatrix}-2 & 0\\ 5 & -6\end{bmatrix}} = \begin{cases} Col1 = -2\begin{bmatrix}2\\5\\4\end{bmatrix} + 5\begin{bmatrix}-1\\8\\0\end{bmatrix} = \begin{bmatrix}-4\\-10\\-8\end{bmatrix} + \begin{bmatrix}-5\\40\\0\end{bmatrix} = \begin{bmatrix}-9\\+30\\-8\end{bmatrix} \\ Col\,2 = 0\begin{bmatrix}2\\5\\4\end{bmatrix} + -6\begin{bmatrix}-1\\8\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}6\\-48\\0\end{bmatrix} = \begin{bmatrix}6\\-48\\0\end{bmatrix} \end{cases} = \underset{3\times2}{\begin{bmatrix}-9 & 6\\ 30 & -48\\ -8 & 0\end{bmatrix}}$

2.(l). $F*D = \underset{3\times2}{\begin{bmatrix}2 & -1\\ 5 & 8\\ 4 & 0\end{bmatrix}} \times \underset{2\times2}{\begin{bmatrix}-7 & 9\\ 4 & -3\end{bmatrix}} = \begin{cases} Col1 = -7\begin{bmatrix}2\\5\\4\end{bmatrix} + 4\begin{bmatrix}-1\\8\\0\end{bmatrix} = \begin{bmatrix}-14\\-35\\-28\end{bmatrix} + \begin{bmatrix}-4\\32\\0\end{bmatrix} = \begin{bmatrix}-18\\-3\\-28\end{bmatrix} \\ Col2 = 9\begin{bmatrix}2\\5\\4\end{bmatrix} + -3\begin{bmatrix}-1\\8\\0\end{bmatrix} = \begin{bmatrix}18\\45\\36\end{bmatrix} + \begin{bmatrix}3\\-24\\0\end{bmatrix} = \begin{bmatrix}21\\21\\36\end{bmatrix} \end{cases} = \begin{bmatrix}-18 & 21\\ -3 & 21\\ -28 & 36\end{bmatrix}$

* Row Method *

3. ⓐ $A*B = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}_{2\times2} \times \begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}_{2\times2}$

Row1 = $3[-2\ 0] + -5[5\ -6] = [-6\ 0] + [-25\ 30] = \begin{bmatrix} -31 & 30 \end{bmatrix}$

Row2 = $-4[-2\ 0] + 2[5\ -6] = [8\ 0] + [10\ -12] = \begin{bmatrix} 18 & -12 \end{bmatrix}$

$= \begin{bmatrix} -31 & 30 \\ 18 & -12 \end{bmatrix}_{2\times2}$

3. ⓑ. $G*A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 0 & -2 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}_{2\times2}$

Row1 = $3[3\ -5] + 1[-4\ 2] = [9\ -15] + [-4\ 2] = \begin{bmatrix} 5 & -13 \end{bmatrix}$

Row2 = $2[3\ -5] + 4[-4\ 2] = [6\ -10] + [-16\ 8] = \begin{bmatrix} -10 & -2 \end{bmatrix}$

Row3 = $0[3\ -5] + -2[-4\ 2] = [0\ 0] + [8\ -4] = \begin{bmatrix} 8 & -4 \end{bmatrix}$

$= \begin{bmatrix} 5 & -13 \\ -10 & -2 \\ 8 & -4 \end{bmatrix}_{3\times2}$

3. ⓒ. $E*C = \begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}_{2\times2}$

Row1 = $-9[-2\ 7] + 2[4\ -5] = [18\ -63] + [8\ -10] = \begin{bmatrix} 26 & -73 \end{bmatrix}$

Row2 = $8[-2\ 7] + -1[4\ -5] = [-16\ 56] + [-4\ 5] = \begin{bmatrix} -20 & 61 \end{bmatrix}$

Row3 = $4[-2\ 7] + -3[4\ -5] = [-8\ 28] + [-12\ 15] = \begin{bmatrix} -20 & 43 \end{bmatrix}$

$= \begin{bmatrix} 26 & -73 \\ -20 & 61 \\ -20 & 43 \end{bmatrix}_{3\times2}$

3. ⓓ. $C*D = \begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}_{2\times2} \times \begin{bmatrix} -7 & 9 \\ 4 & -3 \end{bmatrix}_{2\times2}$

Row1 = $-2[-7\ 9] + 7[4\ -3] = [14\ -18] + [28\ -21] = \begin{bmatrix} 42 & -39 \end{bmatrix}$

Row2 = $4[-7\ 9] + -5[4\ -3] = [-28\ 36] + [-20\ 15] = \begin{bmatrix} -48 & 51 \end{bmatrix}$

$= \begin{bmatrix} 42 & -39 \\ -48 & 51 \end{bmatrix}_{2\times2}$

3. ⓔ. $A*H = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}_{2\times2} \times \begin{bmatrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{bmatrix}_{2\times3}$

Row1 = $3[7\ 4\ 9] + -5[8\ 1\ 5] = [21\ 12\ 27] + [-40\ -5\ -25] = \begin{bmatrix} -19 & 7 & 2 \end{bmatrix}$

Row2 = $-4[7\ 4\ 9] + 2[8\ 1\ 5] = [-28\ -16\ -36] + [16\ 2\ 10] = \begin{bmatrix} -12 & -14 & -26 \end{bmatrix}$

$= \begin{bmatrix} -19 & 7 & 2 \\ -12 & -14 & -26 \end{bmatrix}_{2\times3}$

3. ⓕ. $F*C = \begin{bmatrix} 2 & -1 \\ 5 & 8 \\ 4 & 0 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -2 & 7 \\ 4 & -5 \end{bmatrix}_{2\times2}$

Row1 = $2[-2\ 7] + -1[4\ -5] = [-4\ 14] + [-4\ 5] = \begin{bmatrix} -8 & 19 \end{bmatrix}$

Row2 = $5[-2\ 7] + 8[4\ -5] = [10\ 35] + [32\ -40] = \begin{bmatrix} 22 & -5 \end{bmatrix}$

Row3 = $4[-2\ 7] + 0[4\ -5] = [-8\ 28] + [0\ 0] = \begin{bmatrix} -8 & 28 \end{bmatrix}$

$= \begin{bmatrix} -8 & 19 \\ 22 & -5 \\ -8 & 28 \end{bmatrix}_{3\times2}$

3. ⓖ. $E*B = \begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}_{2\times2}$

Row1 = $-9[-2\ 0] + 2[5\ -6] = [18\ 0] + [10\ -12] = \begin{bmatrix} 28 & -12 \end{bmatrix}$

Row2 = $8[-2\ 0] + -1[5\ -6] = [-16\ 0] + [-5\ 6] = \begin{bmatrix} -21 & 6 \end{bmatrix}$

Row3 = $4[-2\ 0] + -3[5\ -6] = [-8\ 0] + [-15\ 18] = \begin{bmatrix} -23 & 18 \end{bmatrix}$

$= \begin{bmatrix} 28 & -12 \\ -21 & 6 \\ -23 & 18 \end{bmatrix}_{3\times2}$

3. ⓗ. $E*D = \begin{bmatrix} -9 & 2 \\ 8 & -1 \\ 4 & -3 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -7 & 9 \\ 4 & -3 \end{bmatrix}_{2\times2}$

Row1 = $-9[-7\ 9] + 2[4\ -3] = [63\ -81] + [8\ -6] = \begin{bmatrix} 71 & -87 \end{bmatrix}$

Row2 = $8[-7\ 9] + -1[4\ -3] = [-56\ 72] + [-4\ 3] = \begin{bmatrix} -60 & 75 \end{bmatrix}$

Row3 = $4[-7\ 9] + -3[4\ -3] = [-28\ 36] + [-12\ 9] = \begin{bmatrix} -40 & 45 \end{bmatrix}$

$= \begin{bmatrix} 71 & -87 \\ -60 & 75 \\ -40 & 45 \end{bmatrix}_{3\times2}$

3. ⓘ. $G*H = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 0 & -2 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{bmatrix}_{2\times3}$

Row1 = $3[7\ 4\ 9] + 1[8\ 1\ 5] = [21\ 12\ 27] + [8\ 1\ 5] = \begin{bmatrix} 29 & 13 & 32 \end{bmatrix}$

Row2 = $2[7\ 4\ 9] + 4[8\ 1\ 5] = [14\ 8\ 18] + [32\ 4\ 20] = \begin{bmatrix} 46 & 12 & 38 \end{bmatrix}$

Row3 = $0[7\ 4\ 9] + -2[8\ 1\ 5] = [0\ 0\ 0] + [-16\ -2\ -10] = \begin{bmatrix} -16 & -2 & -10 \end{bmatrix}$

$= \begin{bmatrix} 29 & 13 & 32 \\ 46 & 12 & 38 \\ -16 & -2 & -10 \end{bmatrix}_{3\times3}$

3. ⓙ. $F*B = \begin{bmatrix} 2 & -1 \\ 5 & 8 \\ 4 & 0 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}_{2\times2}$

Row1 = $2[-2\ 0] + -1[5\ -6] = [-4\ 0] + [-5\ 6] = \begin{bmatrix} -9 & 6 \end{bmatrix}$

Row2 = $5[-2\ 0] + 8[5\ -6] = [-10\ 0] + [40\ -48] = \begin{bmatrix} 30 & -48 \end{bmatrix}$

Row3 = $4[-2\ 0] + 0[5\ -6] = [-8\ 0] + [0\ 0] = \begin{bmatrix} -8 & 0 \end{bmatrix}$

$= \begin{bmatrix} -9 & 6 \\ 30 & -48 \\ -8 & 0 \end{bmatrix}_{3\times2}$

3. ⓚ. $F*D = \begin{bmatrix} 2 & -1 \\ 5 & 8 \\ 4 & 0 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -7 & 9 \\ 4 & -3 \end{bmatrix}_{2\times2}$

Row1 = $2[-7\ 9] + -1[4\ -3] = [-14\ 18] + [-4\ 3] = \begin{bmatrix} -18 & 21 \end{bmatrix}$

Row2 = $5[-7\ 9] + 8[4\ -3] = [-35\ 45] + [32\ -24] = \begin{bmatrix} -3 & 21 \end{bmatrix}$

Row3 = $4[-7\ 9] + 0[4\ -3] = [-28\ 36] + [0\ 0] = \begin{bmatrix} -28 & 36 \end{bmatrix}$

$= \begin{bmatrix} -18 & 21 \\ -3 & 21 \\ -28 & 36 \end{bmatrix}_{3\times2}$

* Outer Product *

4.ⓐ. $A*B = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}_{2\times2} \times \begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}_{2\times2} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}\begin{bmatrix} -2 & 0 \end{bmatrix} + \begin{bmatrix} -5 \\ 2 \end{bmatrix}\begin{bmatrix} 5 & -6 \end{bmatrix} = \begin{bmatrix} 3[-2\ 0] \\ -4[-2\ 0] \end{bmatrix} + \begin{bmatrix} -5[5\ -6] \\ 2[5\ -6] \end{bmatrix}$ (Row multiplication Applied)

$= \begin{bmatrix} -6 & 0 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} -25 & 30 \\ 10 & -12 \end{bmatrix} = \begin{bmatrix} -31 & 30 \\ 18 & -12 \end{bmatrix}$

4.ⓑ. $G*A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 0 & -2 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}_{2\times2} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}[3\ -5] + \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}[-4\ 2] = \begin{bmatrix} 3[3\ -5] \\ 2[3\ -5] \\ 0[3\ -5] \end{bmatrix} + \begin{bmatrix} 1[-4\ 2] \\ 4[-4\ 2] \\ -2[-4\ 2] \end{bmatrix}$

$= \begin{bmatrix} 9 & -15 \\ 6 & -10 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -4 & 2 \\ -16 & 8 \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 5 & -13 \\ -10 & -2 \\ 8 & -4 \end{bmatrix}$

* Block Multiplication *

This will follow the process of Dot/Colorumn/Row multiplication process where the big Matrix will be partioned following the n*m (Row * column) rule

* Outer Product *

4.④. $A*B = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}_{2\times2} \times \begin{bmatrix} -2 & 0 \\ 5 & -6 \end{bmatrix}_{2\times2} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}\begin{bmatrix} -2 & 0 \end{bmatrix} + \begin{bmatrix} -5 \\ 2 \end{bmatrix}\begin{bmatrix} 5 & -6 \end{bmatrix} = \begin{bmatrix} 3[-2\ 0] \\ -4[-2\ 0] \end{bmatrix} + \begin{bmatrix} -5[5\ -6] \\ 2[5\ -6] \end{bmatrix}$ (Row multiplication Applied)

$= \begin{bmatrix} -6 & 0 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} -25 & 30 \\ 10 & -12 \end{bmatrix} = \begin{bmatrix} -31 & 30 \\ 18 & -12 \end{bmatrix}$

4.⑤. $G*A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 0 & -2 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}_{2\times2} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}[3\ -5] + \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}[-4\ 2] = \begin{bmatrix} 3[3\ -5] \\ 2[3\ -5] \\ 0[3\ -5] \end{bmatrix} + \begin{bmatrix} 1[-4\ 2] \\ 4[-4\ 2] \\ -2[-4\ 2] \end{bmatrix}$

$= \begin{bmatrix} 9 & -15 \\ 6 & -10 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -4 & 2 \\ -16 & 8 \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 5 & -13 \\ -10 & -2 \\ 8 & -4 \end{bmatrix}$

* Block Multiplication *

This will follow the process of Dot/Colocumn/Row Multiplication process where the big Matrix will be partioned following the n*m (Row * column) rule