Hypothesis Testing

Fundamental of Hypothesis Testing

- There two types of statistical inferences, Estimation and Hypothesis Testing
- Hypothesis Testing: A hypothesis is a **claim** (assumption) about one or more **population** parameters.
  - Average price of a six-pack in the U.S. is μ = $4.90
  - The population mean monthly cell phone bill of this city is: μ = $42
  - The average number of TV sets in U.S. Homes is equal to three; $\mu = 3$

- It Is always about a population parameter, not about a sample statistic
- Sample evidence is used to assess the probability that the claim about the population parameter is true

A. It starts with Null Hypothesis, H_o

$H_0: \mu = 3$ and $\bar{X} = 2.79$

1. We begin with the assumption that H_o is true and any difference between the sample statistic and true population parameter is due to chance and not a real (systematic) difference.
2. Similar to the notion of "innocent until proven guilty"
3. That is, "innocence" is a null hypothesis.

Null Hypo, Continued

4. Refers to the status quo
5. Always contains "=" , "≤" or "≥" sign
6. May or may not be rejected

B. Next we state the Alternative Hypothesis, H_1

1. Is the opposite of the null hypothesis
    1. e.g., The average number of TV sets in U.S. homes is not equal to 3 ($H_1: \mu \neq 3$)
2. Challenges the status quo
3. Never contains the "=" , "≤" or "≥" sign
4. May or may not be proven
5. Is generally the hypothesis that the researcher is trying to prove. Evidence is always examined with respect to $\mathbf{H_1}$, never with respect to $\mathbf{H_o}$.
6. We never "accept" $\mathbf{H_o}$, we either "reject" or "not reject" it

Summary:

- In the process of hypothesis testing, the null hypothesis initially is assumed to be true
- Data are gathered and examined to determine whether the evidence is strong enough with respect to the alternative hypothesis to reject the assumption.
- In another words, the burden is placed on the researcher to show, using sample information, that the null hypothesis is false.
- If the sample information is sufficient enough in favor of the alternative hypothesis, then the null hypothesis is rejected. This is the same as saying if the persecutor has enough evidence of guilt, the "innocence is rejected.
- Of course, erroneous conclusions are possible, type I and type II errors.

Reason for Rejecting H_0

- Illustration: Let say, we **assume** that average age in the US is 50 years (Ho=50). If in fact this is the true (unknown) population mean, it is unlikely that we get a sample mean of 20. So, if we have a sample that produces an average of 20, them we **reject** that the null hypothesis that average age is 50. (note that we are rejecting our assumption or claim). (would we get 20 if the true population mean was 50? NO. That is why we reject 50)

How Is the Test done?

- **We use the distribution of a Test Statistic**, such as Z or t as the criteria.

A. Rejection Region Method:

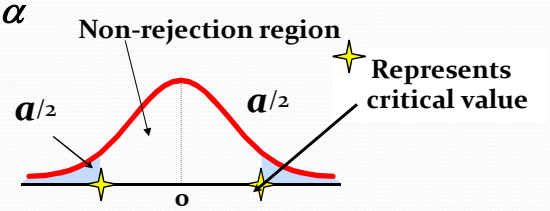
- Divide the distribution into rejection and non-rejection regions
- Defines the unlikely values of the sample statistic if the null hypothesis is true, the critical value(s)
  - Defines rejection region of the sampling distribution
- Rejection region(s) is designated by α , (level of significance)
  - Typical values are .01, .05, or .10
- α is selected by the researcher at the beginning
- α provides the critical value(s) of the test

Rejection Region or Critical Value Approach:

Level of significance = α

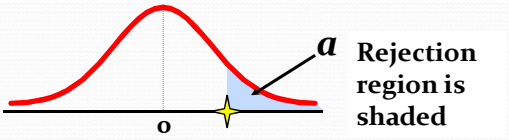
H_0: $\mu = 12$
H_1: $\mu \neq 12$

Two-tail test



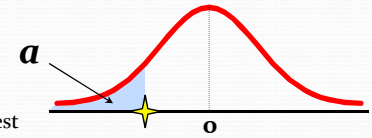
H_0: $\mu \leq 12$
H_1: $\mu > 12$

Upper-tail test



H_0: $\mu \geq 12$
H_1: $\mu < 12$

Lower-tail test



Steps to Hypothesis Testing

1. State the H_0 and H_1 clearly
2. Identify the test statistic (two-tail, one-tail, and Z or t distribution
3. Depending on the type of risk you are willing to take, specify the level of significance,
4. Find the decision rule, critical values, and rejection regions.

If –Critical Value <actual value (sample statistic) <+CriticalValue, then do not reject the H_0

Collect the data and do the calculation for the actual values of the test statistic from the sample