Department of Electronic and Telecommunication Engineering

University of Moratuwa, Sri Lanka

EN2570 - Digital Signal Processing

Design of a Finite Duration Impulse Response Bandpass Digital Filter

(For Prescribed Specifications Using the windowing method in conjunction with the Kaiser window)

Project Report

Submitted by

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Submitted on

March 5, 2021

Abstract

Design procedure of a Finite Duration Impulse Response(FIR) bandpass Digital Filter which satisfies a set of prescribed specifications, is described in this report where windowing method in conjunction with the Kaiser window is used for the designing procedure. Operation of the filter was analyzed with a combination of sine functions. The design was implemented and tested using `MATLAB R2018a` of the MathWorks Inc. Therefore implementation is not guaranteed to work on the previous version of the software.

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**** PDF is clickable***

Note:
Additionally all the materials related to Task can also be found at

1 Introduction

This report describes the design procedure of an FIR bandpass digital filter.

2 Method

2.1 Filter Implementation

Filter implementation consists of the steps mentioned below. Subsections of this section of the report describes each one of them for designing an FIR bandpass filter.

1. Identifying the prescribed filter specifications
2. Derivation of the filter Parameters
3. Derivation of the Kaiser Window Parameters
4. Derivation of The Ideal Impulse Response
5. Truncating the Ideal Impulse Response to obtain Finite Impulse Response: Windowing

2.1.1 Prescribed Filter specifications

Following table describes the desired specifications of the bandpass filter which need to be implemented. The notation used here is the same as the notation used in the reference material[1] and they will be used throughout the report.

Parameter	Symbol	Value
Maximum passband ripple(*desired*)	$\tilde{A}_p$	0.09 dB
Minimum stopband attenuation(*desired*)	$\tilde{A}_a$	48 dB
Lower passband edge	ω_{p1}	400 rad/s
Upper passband edge	ω_{p2}	800 rad/s
Lower stopband edge	ω_{a1}	250 rad/s
Upper stopband edge	ω_{a2}	900 rad/s
Sampling frequency	ω_s	2600 rad/s

Table 1: Prescribed Filter specifications

Following figure illustrates the aforementioned specifications for an idealized frequency responses of Bandpass filter. δ in the figure has the following relationship with peak to peak passband ripple(*practical*) A_p and the minimum stopband attenuation(*practical*) A_a.

$$\tilde{A}_p \geq A_p = 20\log\left(\frac{1+\delta}{1-\delta}\right) \tag{1}$$

$$\tilde{A}_a \leq A_a = -20\log(\delta) \tag{2}$$

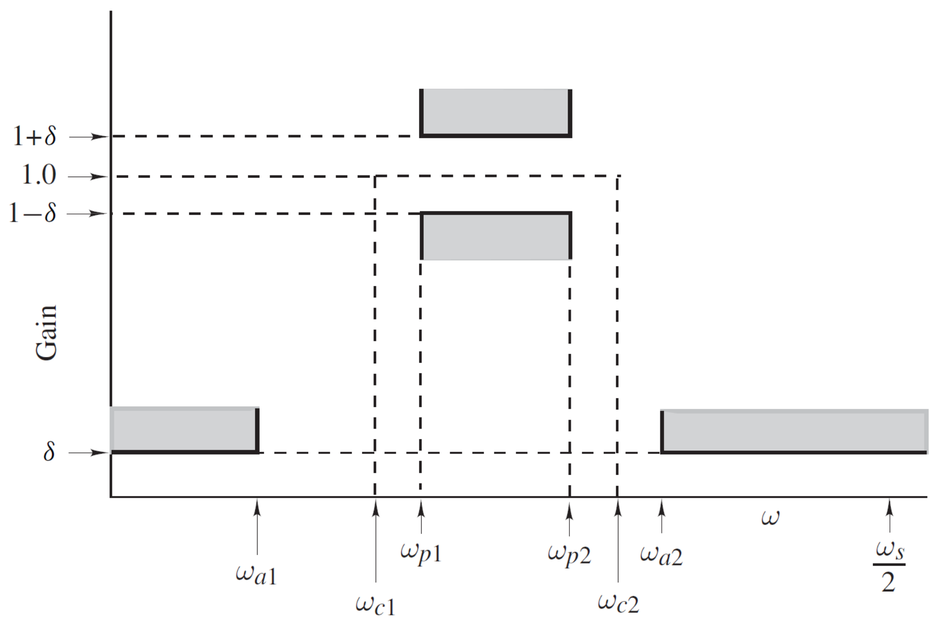

Figure 1: Idealized frequency response of a Bandpass filter[1]

2.1.2 Derivation of filter Parameters

According to the given specifications following parameters are calculated.

Parameter	Symbol	Calculation	Value
Lower transition width	B_{t1}	$\omega_{p1} - \omega_{a1}$	150 rad/s
Upper transition width	B_{t2}	$\omega_{a2} - \omega_{p2}$	100 rad/s
Critical transition width	B_t	$\min(B_{t1}, B_{t2})$	100 rad/s
Lower cutoff frequency	ω_{c1}	$\omega_{p1} - B_t/2$	350 rad/s
Upper cutoff frequency	ω_{c2}	$\omega_{p2} + B_t/2$	850 rad/s
Sampling period	T	$2\pi/\omega_s$	0.0024 s

Table 2: Derivation of filter Parameters

2.1.3 Derivation of the Kaiser Window Parameters

Following equation represents the Kaiser window which will be used to truncate the Infinite duration Impulse Response to obtain the Finite duration Impulse Response for our filter design.

$$w_K(nT) = \begin{cases} \frac{I_0(\beta)}{I_0(\alpha)} & for|n| \leq \frac{N-1}{2} \\ 0 & Otherwise \end{cases} \tag{3}$$

where α is an independent parameter and $I_0(x)$ is the zeroth-order modified Bessel function of the first kind.

$$\beta = \alpha\sqrt{1 - \left(\frac{2n}{N-1}\right)^2} \qquad I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

Now we have to calculate the required parameters as follows,

a.) Choose δ in Eqs. (1) and (2) such that $\delta = \min(\tilde{\delta_p}, \tilde{\delta_a})$ where,

$$\begin{aligned}\tilde{\delta_p} &= \frac{10^{0.05\tilde{A}_p} - 1}{10^{0.05\tilde{A}_p} + 1} \\ &= \frac{10^{0.05*0.09} - 1}{10^{0.05*0.09} + 1} \\ &= 5.181 \times 10^{-3}\end{aligned} \qquad \begin{aligned}\tilde{\delta_a} &= 10^{-0.05\tilde{A}_a} \\ &= 3.981 \times 10^{-3}\end{aligned}$$

$$\therefore \ \delta = 3.981 \times 10^{-3}$$

b.) With the required δ defined, the actual stopband loss(attenuation) A_a in dB can be calculated using Eq. 2.

$$\begin{aligned}\tilde{A_a} \leq A_a &= -20\log(\delta) \\ &= -20\log\big(3.981 \times 10^{-3}\big) \\ A_a &= 48\ dB\end{aligned}$$

c.) Choose parameter α as,

$$\alpha = \begin{cases} 0 & for\ A_a \leq 21\ dB \\ 0.5842(A_a - 21)^{0.4} + 0.07886(A_a - 21) & for\ 21 < A_a \leq 50\ dB \\ 0.1102(A_a - 8.7) & for\ A_a > 50\ dB \end{cases}$$

$$\begin{aligned}\therefore\ \alpha &= 0.5842(A_a-21)^{0.4}+0.07886(A_a-21)\\ &= 0.5842(48-21)^{0.4}+0.07886(48-21)\\ &= 4.3125\end{aligned}$$

d.) Choose parameter D as,

$$D=\begin{cases}0.9222 & for\ A_a\le 21\ dB\\ \frac{A_a-7.95}{14.36} & for\ A_a>21\ dB\end{cases}$$

$$\begin{aligned}\therefore\ D &= \frac{A_a-7.95}{14.36}\\ &= \frac{48-7.95}{14.36}\\ &= 2.7890\end{aligned}$$

e.) Then select the lowest odd value of N that would satisfy the inequality,

$$\begin{aligned}N &\ge \frac{\omega_s D}{B_t}+1\\ &\ge \frac{2600*2.79}{100}+1 \qquad\qquad \therefore\ N=75\\ &\ge 73.51\end{aligned}$$

2.1.4 Derivation of The Ideal Impulse Response

Note : Here subscript 'd' implies "desired", as it is the ideal response of the filter. Subscript d will be omitted to indicate a given expression is no longer ideal.

The frequency response of an ideal bandpass filter with cutoff frequencies ω_{c1} and ω_{c2} is given by,

$$H_d(e^{j\omega T})=\begin{cases}1 & for\ -\omega_{c2}\le\omega\le-\omega_{c1}\\ 1 & for\ \omega_{c1}\le\omega\le\omega_{c2}\\ 0 & Otherwise\end{cases}$$

Using the Inverse Fourier Transform, impulse response of the above $H(e^{j\omega T})$ is calculated.

$$\begin{aligned}h_d(nT) &= \frac{1}{\omega_s}\int_{-\omega_s/2}^{\omega_s/2} H(e^{j\omega T})e^{j\omega nT}\ d\omega\\ &= \frac{1}{\omega_s}\left[\int_{-\omega_{c2}}^{-\omega_{c1}} e^{j\omega nT}\ d\omega+\int_{\omega_{c1}}^{\omega_{c2}} e^{j\omega nT}\ d\omega\right]\\ &= \frac{1}{\omega_s}\left[\frac{e^{j\omega nT}}{jnT}\bigg|_{-\omega_{c2}}^{-\omega_{c1}}+\frac{e^{j\omega nT}}{jnT}\bigg|_{\omega_{c1}}^{\omega_{c2}}\right]\\ &= \frac{1}{j\omega_s nT}\left[e^{-j\omega_{c1}nT}-e^{-j\omega_{c2}nT}+e^{j\omega_{c2}nT}-e^{j\omega_{c1}nT}\right];where\ \omega_s T=2\pi\\ &= \frac{1}{\pi n}\left[\frac{(e^{j\omega_{c2}nT}-e^{-j\omega_{c2}nT})}{2j}-\frac{(e^{j\omega_{c1}nT}-e^{-j\omega_{c1}nT})}{2j}\right];rearanging\\ &= \frac{1}{\pi n}\left[\sin(\omega_{c2}nT)-\sin(\omega_{c1}nT)\right];from\ Euler's\ Eq.\end{aligned}$$

$$\therefore\ h_d(nT)=\begin{cases}\frac{1}{\pi n}\left[\sin(\omega_{c2}nT)-\sin(\omega_{c1}nT)\right] & \forall n\ne 0\\ \\ \frac{2}{\omega_s}(\omega_{c2}-\omega_{c1}) & for\ n=0\end{cases}\tag{4}$$

2.1.5 Truncating the Ideal Impulse Response to obtain Finite Impulse Response: Windowing

By multiplying the ideal impulse response $h_d(nT)$ in Eq. (4) with the Kaiser window $w_K(nT)$ in Eq. (3), the ideal infinite impulse response can be truncated to obtain the finite impulse response $h(nT)$ for practical implementation.

$$h(nT) = w_K(nT).h_d(nT) \tag{5}$$

Obtaining the transfer function in the $\mathcal{Z}$ domain using $\mathcal{Z}$ Transformation,

$$\begin{aligned} H'(z) &= \mathcal{Z}\left\{h(nT)\right\} \\ &= \mathcal{Z}\left\{w_K(nT).h_d(nT)\right\} \end{aligned} \tag{6}$$

The response is anti-causal it need to be shifted in the time domain to make it causal and get the final filter response$H(Z)$, in $\mathcal{Z}$ domain it is represented as follows.

$$H(z) = z^{-\left(\frac{N-1}{2}\right)}.H'(z) \tag{7}$$

2.2 Filter Performance Evaluation

Performance of the filter was evaluated by using the following excitation $x(nT)$ which is a combination of three sinusoidal signals. Frequencies of these three sinusoidal signals are specified as follows to cover all three bands in the filter.

$$x(nT) = \sum_{i=1}^{3} sin(w_i nT)$$

Parameter	Symbol	Calculation	Value
Middle frequency of the lower stopband	ω_1	$\frac{0+\omega_{a1}}{2}$	125 rad/s
Middle frequency of the passband	ω_2	$\frac{\omega_{p1}+\omega_{p2}}{2}$	600 rad/s
Middle frequency of the upper stopband	ω_3	$\frac{\omega_{a2}+\omega_s/2}{2}$	1100 rad/s

Table 3: Frequencies for Filter Performance Evaluation

3 Results

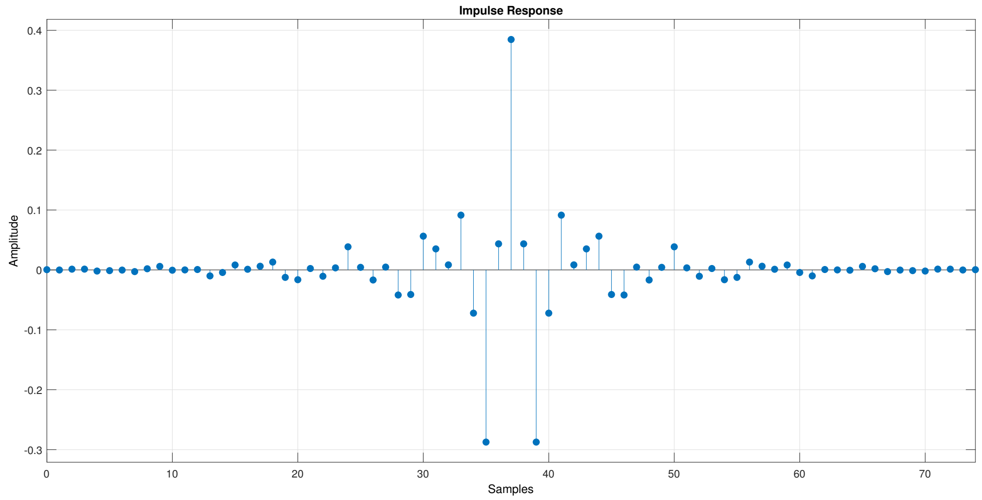

Figure 2: Causal Impulse Response of the Filter

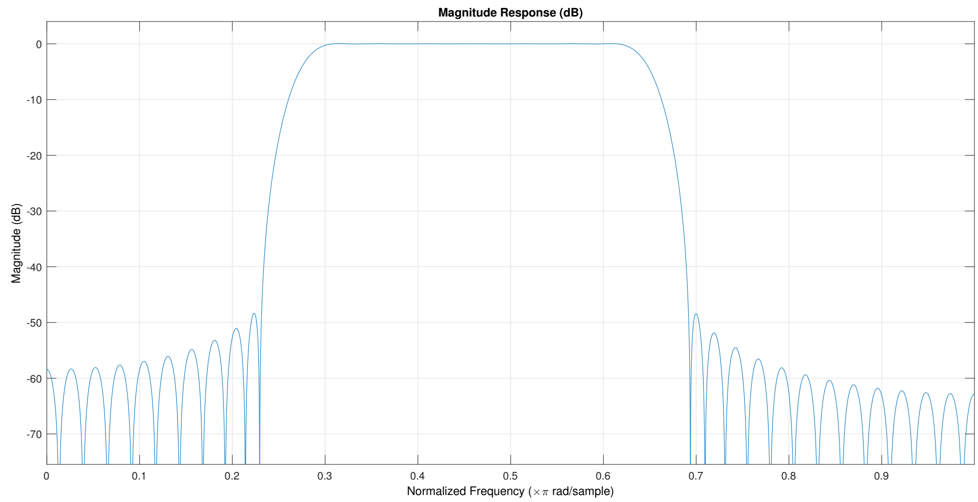

Figure 3: Causal Impulse Response of the Filter

4 Discussion

5 Conclusion

Bibliography

[1] Andreas Antoniou. *Digital Signal Processing*. McGraw-Hill Professional, US, 2005.

```
% Index number = 180631J
% Therefore the required parameters
A = 6;
B = 3;
C = 1;
```

Prescribed Filter specifications

```
tilde_A_p = 0.03+0.01*A;     % Maximum passband ripple
tilde_A_a = 45 + B;          % Minimum stopband attenuation
omega_p1 = C*100 + 300;      % Lower passband edge
omega_p2 = C*100 + 700;      % Upper passband edge
omega_a1 = C*100 + 150;      % Lower stopband edge
omega_a2 = C*100 + 800;      % Upper stopband edge
omega_s  = 2*(C*100 +1200); % Sampling frequency
```

Derivation of filter Parameters

```
B_t1 = omega_p1 - omega_a1; % Lower transition width
B_t2 = omega_a2 - omega_p2; % Upper transition width
B_t  = min(B_t1,B_t2);      % Critical transition width
omega_c1 = omega_p1-B_t/2;  % Lower cutoff frequency
omega_c2 = omega_p2+B_t/2;  % Upper cutoff frequency
T = 2*pi /omega_s;          % Sampling period
```

Derivation of the Kaiser Window Parameters

```
tilde_delta_p  =  (10^(0.05*tilde_A_p) -1)/(10^(0.05*tilde_A_p)  +1);
tilde_delta_a  = 10^(-0.05*tilde_A_a);
delta = min(tilde_delta_p, tilde_delta_a);

A_a = -20*log10(delta);% Actual stopband attenuation

% Choose parameter alpha as,
if A_a <=21
    alpha = 0;
elseif 21 < A_a && A_a <=50
    alpha = 0.5842*(A_a - 21)^0.4 + 0.07886*(A_a - 21);
else
    alpha = 0.1102*(A_a - 8.7);
end

% Choose parameter D as,
if A_a <= 21
    D = 0.9222;
else
    D = (A_a - 7.95)/14.36;
end

% Select the lowest odd value of N that satisfies the inequality
N = ceil(omega_s*D/B_t + 1);
```

```
if mod(N,2) ==0
    N = N+1; % If calculated N is evn, make it odd by adding 1
end
```

Creating the Kaise Window

```
n = -(N-1)/2:1:(N-1)/2;                              % Range of the Kaizer window
beta = alpha*sqrt(1-(2*n/(N-1)).^2);                 % betas for I(beta)
terms = 50;                                          % Number of terms to be considered for bessel
w_k_nT = ZerothOrderModifiedBessel(beta,terms)...
            /ZerothOrderModifiedBessel(alpha,terms);    % Calculating window coefficients
stem(n,w_k_nT,'filled');
title('Kaiser Window - Time Domain');
xlabel('Samples(n)');
ylabel('Amplitude');
```

Derivation of The Ideal Impulse Response

```
h_d_nT = (sin(omega_c2*n*T) - sin(omega_c1*n*T))./(pi*n); % For each n != 0
h_d_nT((N+1)/2) = (omega_c2 - omega_c1)*(2/omega_s);      % For n = 0
stem(n,h_d_nT,'filled');
title('The Ideal Impulse Response');
xlabel('Samples(n)');
ylabel('Amplitude');
```

Truncating the Ideal Impulse Response to obtain Finite Impulse Response(Anti-Causal)

```
h_nT = h_d_nT.*w_k_nT; % Windowing using Kaiser window
stem(n,h_nT,'filled');
title('Finite Impulse Response- Anti-Causal')
xlabel('Samples(n)');
ylabel('Amplitude');
```

Finite Impulse Response(Causal)

```
n_causal = 0:1:N-1; % Making the range of n positive
stem(n_causal,h_nT,'filled');
title('Finite Impulse Response-Causal')
xlabel('Samples(n)');
ylabel('Amplitude');
grid on;
```

Plotting the magnitude response of filter in the range (0,omega_s/2)

```
%fvtool(h_nT,'magnitude')
[H_ejomegaT, omega] = freqz(h_nT);
omega = (omega/pi)*(omega_s/2); % rad/s = (normalized freq)*(sampling freq/2)
magnitude = 20*log10(abs(H_ejomegaT));
plot(omega, magnitude);
```

```
title('Magnitude Response of Filter');
xlabel('Angular Frequency (rad/s)');
ylabel('Magnitude (dB)');
grid on;
```

Magnitude response of the digital filter for the frequencies in the passband

```
plot(omega, magnitude);
xlim([omega_p1 omega_p2]);
title('Magnitude Response of Filter  in the passband');
xlabel('Angular Frequency (rad/s)');
ylabel('Magnitude (dB)');
grid on;
```

Local Function definitions

```
function value = ZerothOrderModifiedBessel(x,terms)
value = 1;
for k = 1:terms
    value = value + ((1/factorial(k))*(x/2).^k).^2;
end
end
```