Department of Electronic and Telecommunication Engineering

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EN2570 - Digital Signal Processing

Design of a Finite Duration Impulse Response Bandpass Digital Filter

(For Prescribed Specifications Using the windowing method in conjunction with the Kaiser window)

Project Report

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Abstract

Design procedure of a Finite Duration Impulse Response(FIR) bandpass Digital Filter which satisfies a set of prescribed specifications, is described in this report where windowing method in conjunction with the Kaiser window is used for the designing procedure. Operation of the filter was analyzed with a combination of sine functions. The design was implemented and tested using `MATLAB R2018a` of the MathWorks Inc. Therefore implementation is not guaranteed to work on the previous version of the software.

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**** PDF is clickable***

Note:
Additionally all the materials related to Task can also be found at

1 Introduction

This report describes the design procedure of an FIR bandpass digital filter.

2 Method

2.1 Filter Implementation

Filter implementation consists of the steps mentioned below. Subsections of this section of the report describes each one of them for designing an FIR bandpass filter.

1. Identifying the prescribed filter specifications
2. Derivation of the filter Parameters
3. Derivation of the Kaiser Window Parameters

2.1.1 Prescribed Filter specifications

Following table describes the desired specifications of the bandpass filter which need to be implemented. The notation used here is the same as the notation used in the reference material[1] and they will be used throughout the report.

Table 1: Prescribed Filter specifications

Parameter	Symbol	Value
Maximum passband ripple(*desired*)	$\tilde{A}_p$	0.09 dB
Minimum stopband attenuation(*desired*)	$\tilde{A}_a$	48 dB
Lower passband edge	ω_{p1}	400 rad/s
Upper passband edge	ω_{p2}	800 rad/s
Lower stopband edge	ω_{a1}	250 rad/s
Upper stopband edge	ω_{a2}	900 rad/s
Sampling frequency	ω_s	2600 rad/s

Following figure illustrates the aforementioned specifications for an idealized frequency responses of Bandpass filter. δ in the figure has the following relationship with peak to peak passband ripple(*practical*) A_p and the minimum stopband attenuation(*practical*) A_a.

$$\tilde{A}_p \geq A_p = 20\log\left(\frac{1+\delta}{1-\delta}\right) \tag{1}$$

$$\tilde{A}_a \leq A_a = -20\log(\delta) \tag{2}$$

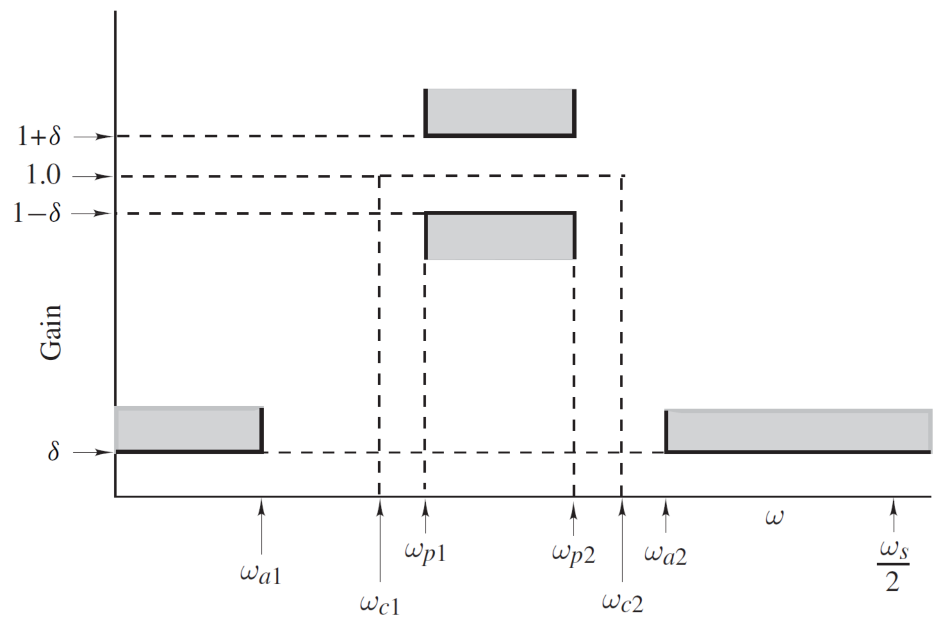

Figure 1: Idealized frequency response of a Bandpass filter[1]

2.1.2 Derivation of filter Parameters

According to the given specifications following parameters are calculated.

Table 2: Derivation of filter Parameters

Parameter	Symbol	Calculation	Value
Lower transition width	B_{t1}	$\omega_{p1} - \omega_{a1}$	150 rad/s
Upper transition width	B_{t2}	$\omega_{a2} - \omega_{p2}$	100 rad/s
Critical transition width	B_t	$\min(B_{t1}, B_{t2})$	100 rad/s
Lower cutoff frequency	ω_{c1}	$\omega_{p1} - B_t/2$	350 rad/s
Upper cutoff frequency	ω_{c2}	$\omega_{p2} + B_t/2$	850 rad/s
Sampling period	T	$2\pi/\omega_s$	0.0024 s

2.1.3 Derivation of the Kaiser Window Parameters

Following equation represents the Kaiser window which will be used to truncate the Infinite duration Impulse Response to obtain the Finite duration Impulse Response for our filter design.

$$w_K(nT) = \begin{cases} \frac{I_0(\beta)}{I_0(\alpha)} & for|n| \leq \frac{N-1}{2} \\ 0 & Otherwise \end{cases} \tag{3}$$

where α is an independent parameter and $I_0(x)$ is the zeroth-order modified Bessel function of the first kind.

$$\beta = \alpha\sqrt{1 - \left(\frac{2n}{N-1}\right)^2} \qquad I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

Now we have to calculate the required parameters as follows,

a.) Choose δ in Eqs. (1) and (2) such that $\delta = \min(\tilde{\delta_p}, \tilde{\delta_a})$ where,

$$\begin{aligned} \tilde{\delta_p} &= \frac{10^{0.05\tilde{A}_p} - 1}{10^{0.05\tilde{A}_p} + 1} \\ &= \frac{10^{0.05*0.09} - 1}{10^{0.05*0.09} + 1} \\ &= 5.181 \times 10^{-3} \end{aligned} \qquad \begin{aligned} \tilde{\delta_a} &= 10^{-0.05\tilde{A}_a} \\ &= 3.981 \times 10^{-3} \end{aligned}$$

$$\therefore\ \delta = 3.981 \times 10^{-3}$$

b.) With the required δ defined, the actual stopband loss(attenuation) A_a in dB can be calculated using Eq. 2.

$$\begin{aligned} \tilde{A_a} \leq A_a &= -20\log(\delta) \\ &= -20\log\big(3.981 \times 10^{-3}\big) \\ A_a &= 48\ dB \end{aligned}$$

c.) Choose parameter α as,

$$\alpha = \begin{cases} 0 & for\ A_a \leq 21\ dB \\ 0.5842(A_a - 21)^{0.4} + 0.07886(A_a - 21) & for\ 21 < A_a \leq 50\ dB \\ 0.1102(A_a - 8.7) & for\ A_a > 50\ dB \end{cases}$$

$$\begin{aligned}\therefore\ \alpha &= 0.5842(A_a-21)^{0.4}+0.07886(A_a-21)\\ &= 0.5842(48-21)^{0.4}+0.07886(48-21)\\ &= 4.3125\end{aligned}$$

d.) Choose parameter D as,

$$D=\begin{cases}0.9222 & for\ A_a\le 21\ dB\\ \frac{A_a-7.95}{14.36} & for\ A_a>21\ dB\end{cases}$$

$$\begin{aligned}\therefore\ D &= \frac{A_a-7.95}{14.36}\\ &= \frac{48-7.95}{14.36}\\ &= 2.79\end{aligned}$$

e.) Then select the lowest odd value of N that would satisfy the inequality,

$$\begin{aligned}N &\ge \frac{\omega_s D}{B_t}+1\\ &\ge \frac{2600*2.79}{100}+1\\ &\ge 73.51\end{aligned} \qquad\qquad \therefore\ N=75$$

2.1.4 Derivation of The Ideal Impulse Response

Note : Here subscript 'd' implies "desired", as it is the ideal response of the filter. Subscript d will be omitted to indicate a given expression is no longer ideal.

The frequency response of an ideal bandpass filter with cutoff frequencies ω_{c1} and ω_{c2} is given by,

$$H_d(e^{j\omega T})=\begin{cases}1 & for\ -\omega_{c2}\le\omega\le-\omega_{c1}\\ 1 & for\ \omega_{c1}\le\omega\le\omega_{c2}\\ 0 & Otherwise\end{cases}$$

Using the Inverse Fourier Transform, impulse response of the above $H(e^{j\omega T})$ is calculated.

$$\begin{aligned}h_d(nT) &= \frac{1}{\omega_s}\int_{-\omega_s/2}^{\omega_s/2} H(e^{j\omega T})e^{j\omega nT}\ d\omega\\ &= \frac{1}{\omega_s}\left[\int_{-\omega_{c2}}^{-\omega_{c1}} e^{j\omega nT}\ d\omega+\int_{\omega_{c1}}^{\omega_{c2}} e^{j\omega nT}\ d\omega\right]\\ &= \frac{1}{\omega_s}\left[\frac{e^{j\omega nT}}{jnT}\Bigg|_{-\omega_{c2}}^{-\omega_{c1}}+\frac{e^{j\omega nT}}{jnT}\Bigg|_{\omega_{c1}}^{\omega_{c2}}\right]\\ &= \frac{1}{j\omega_s nT}\left[e^{-j\omega_{c1}nT}-e^{-j\omega_{c2}nT}+e^{j\omega_{c2}nT}-e^{j\omega_{c1}nT}\right];where\ \omega_s T=2\pi\\ &= \frac{1}{\pi n}\left[\frac{(e^{j\omega_{c2}nT}-e^{-j\omega_{c2}nT})}{2j}-\frac{(e^{j\omega_{c1}nT}-e^{-j\omega_{c1}nT})}{2j}\right];rearanging\\ &= \frac{1}{\pi n}\left[\sin(\omega_{c2}nT)-\sin(\omega_{c1}nT)\right];from\ Euler's\ Eq.\end{aligned}$$

$$\therefore\ h_d(nT)=\begin{cases}\frac{1}{\pi n}\left[\sin(\omega_{c2}nT)-\sin(\omega_{c1}nT)\right] & \forall n\ne 0\\ \\ \frac{2}{\omega_s}(\omega_{c2}-\omega_{c1}) & for\ n=0\end{cases} \tag{4}$$

2.1.5 Truncating the Ideal Impulse Response to obtain Finite Impulse Response: Windowing

By multiplying the ideal impulse response $h_d(nT)$ in Eq. (4) with the Kaiser window $w_K(nT)$ in Eq. (3), the ideal infinite impulse response can be truncated to obtain the finite impulse response $h(nT)$ for practical implementation.

$$h(nT) = w_K(nT).h_d(nT) \tag{5}$$

3 Results

4 Discussion

5 Conclusion

Bibliography

[1] Andreas Antoniou. *Digital Signal Processing*. McGraw-Hill Professional, US, 2005.