Department of Electronic and Telecommunication Engineering

University of Moratuwa, Sri Lanka

EN2570 - Digital Signal Processing

Design of a Finite Duration Impulse Response Bandpass Digital Filter

(For Prescribed Specifications Using the windowing method in conjunction with the Kaiser window)

Project Report

Submitted by

Thalagala B.P. 180631J

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Abstract

Design procedure of a Finite Duration Impulse Response(FIR) bandpass Digital Filter which satisfies a set of prescribed specifications, is described in this report where windowing method in conjunction with the Kaiser window is used for the designing procedure. Operation of the filter was analyzed with a combination of sinusoidal signals. The design was implemented and tested using `MATLAB R2018a` of the MathWorks Inc.

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Note:
All the materials and executable `MATLAB R2018a` *Live Script related to the project can also be found at* *https://github.com/bimalka98/Digital-Signal-Processing*

1 Introduction

This report describes the design procedure of an FIR bandpass digital filter which satisfies a predefined set of specifications as shown in the Table 1. ***Kaiser windowing*** method is used, due to its excellent capability to control filter's ripple ratio and main-lobe width to facilitate a given set of specification by simply varying the related parameters. And most importantly a method is available to calculate those parameters through empirical formulae. Therefore this Kaiser Window is heavily used to design filters with prescribed specifications.

`MATLAB R2018a` of the MathWorks Inc. is used to implement and analyze the digital filter. Filtering was done in the frequency domain rather than in the time domain since the time domain convolution is computationally expensive. Frequency domain analysis was done using ***Fast Fourier Transform*** algorithms(*using built-in* `fft()` *and* `ifft()` *functions*). Performance of the filter was analyzed using a combination of sinusoidal signal, whose frequency components lie in the middle of the three main regions(*lower stopband, passband and upper stopband*) of the bandpass filter.

2 Method

First the digital filter is implemented through the procedure described in the **section 2.1**. Then its performance is analyzed as described in the **section 2.2**.

2.1 Filter Implementation

Digital filter implementation consists of the steps that are mentioned below. Subsections of this section of the report describes each one of them thoroughly for designing the FIR bandpass filter using Kaiser Window method.

1. Identifying the prescribed filter specifications
2. Derivation of the filter Parameters
3. Derivation of the Kaiser Window Parameters
4. Derivation of The Ideal Impulse Response
5. Truncating the Ideal Impulse Response to obtain Finite Impulse Response i.e Windowing

2.1.1 Prescribed Filter specifications

Following table describes the desired specifications of the bandpass filter which need to be implemented. The notation used here is the same as the notation used in the reference material[1] and they will be used throughout the report rather than numerical values.

Parameter	Symbol	Value
Maximum passband ripple(*desired*)	$\tilde{A}_p$	0.09 dB
Minimum stopband attenuation(*desired*)	$\tilde{A}_a$	48 dB
Lower passband edge	ω_{p1}	400 rad/s
Upper passband edge	ω_{p2}	800 rad/s
Lower stopband edge	ω_{a1}	250 rad/s
Upper stopband edge	ω_{a2}	900 rad/s
Sampling frequency	ω_s	2600 rad/s

Table 1: Prescribed Filter specifications

Following Fig.1 illustrates the aforementioned specifications graphically for an idealized frequency response of a Bandpass filter. δ in the figure has the following relationship with peak to peak passband ripple(*practical*) A_p and the minimum stopband attenuation(*practical*) A_a.

$$\tilde{A}_p \geq A_p = 20\log\left(\frac{1+\delta}{1-\delta}\right) \tag{1}$$

$$\tilde{A}_a \leq A_a = -20\log(\delta) \tag{2}$$

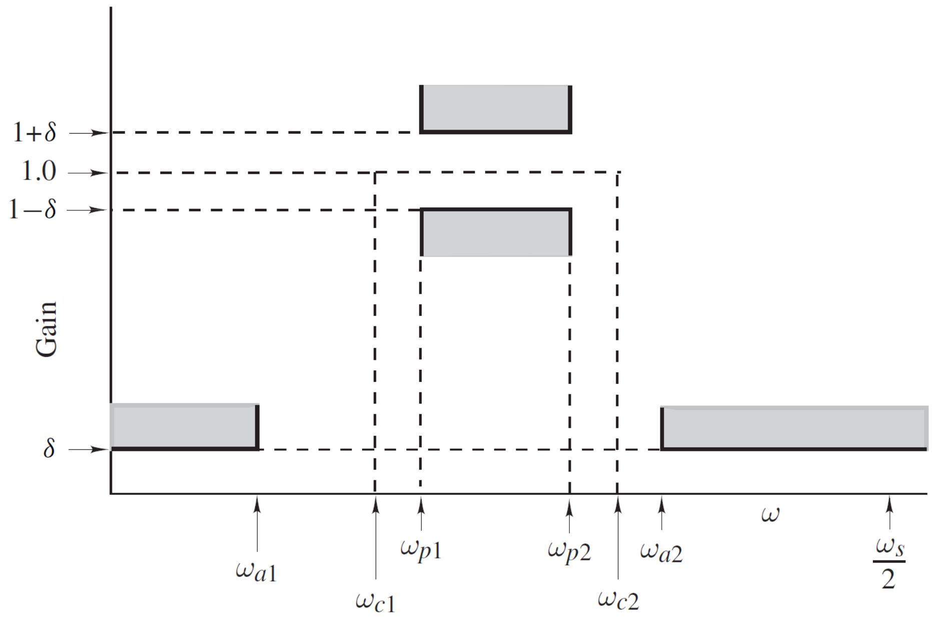

Figure 1: Idealized frequency response of a Bandpass filter[1]

2.1.2 Derivation of filter Parameters

According to the given specifications following parameters are calculated; which then will be used to derive the parameters of the Kaiser window.

Parameter	Symbol	Calculation	Value
Lower transition width	B_{t1}	$\omega_{p1} - \omega_{a1}$	150 rad/s
Upper transition width	B_{t2}	$\omega_{a2} - \omega_{p2}$	100 rad/s
Critical transition width	B_t	$\min(B_{t1}, B_{t2})$	100 rad/s
Lower cutoff frequency	ω_{c1}	$\omega_{p1} - B_t/2$	350 rad/s
Upper cutoff frequency	ω_{c2}	$\omega_{p2} + B_t/2$	850 rad/s
Sampling period	T	$2\pi/\omega_s$	0.0024 s

Table 2: Derivation of filter Parameters

2.1.3 Derivation of the Kaiser Window Parameters

Following equation represents the Kaiser window which will be used to truncate the Infinite duration Impulse Response to obtain the Finite duration Impulse Response for our filter design. Further explanations of the same steps can be found in the *sections 9.4.5* and *9.4.6* in the reference material[1].

$$w_K(nT) = \begin{cases} \frac{I_0(\beta)}{I_0(\alpha)} & for|n| \leq \frac{N-1}{2} \\ 0 & Otherwise \end{cases} \tag{3}$$

where α is an independent parameter and $I_0(x)$ is the zeroth-order modified Bessel function of the first kind.

$$\beta = \alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2} \qquad\qquad I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

Now we have to calculate the required parameters as follows,

a.) Choose δ in Eqs. (1) and (2) such that $\delta = \min(\tilde{\delta_p}, \tilde{\delta_a})$ where,

$$\begin{aligned}\tilde{\delta_p} &= \frac{10^{0.05\tilde{A}_p}-1}{10^{0.05\tilde{A}_p}+1}\\ &= \frac{10^{0.05*0.09}-1}{10^{0.05*0.09}+1}\\ &= 5.181\times10^{-3}\end{aligned} \qquad\qquad \begin{aligned}\tilde{\delta_a} &= 10^{-0.05\tilde{A}_a}\\ &= 3.981\times10^{-3}\end{aligned}$$

$$\therefore\ \delta = 3.981\times10^{-3}$$

b.) With the required δ defined, the actual stopband loss(attenuation) A_a in dB can be calculated using Eq. 2.

$$\begin{aligned}\tilde{A_a} \leq A_a &= -20\log(\delta)\\ &= -20\log\big(3.981\times10^{-3}\big)\\ A_a &= 48\ dB\end{aligned}$$

c.) Choose parameter α as,

$$\alpha = \begin{cases}0 & for\ A_a \leq 21\ dB\\ 0.5842(A_a-21)^{0.4}+0.07886(A_a-21) & for\ 21 < A_a \leq 50\ dB\\ 0.1102(A_a-8.7) & for\ A_a > 50\ dB\end{cases}$$

$$\begin{aligned}\therefore\ \alpha &= 0.5842(A_a-21)^{0.4}+0.07886(A_a-21)\\ &= 0.5842(48-21)^{0.4}+0.07886(48-21)\\ &= 4.3125\end{aligned}$$

d.) Choose parameter D as,

$$D = \begin{cases}0.9222 & for\ A_a \leq 21\ dB\\ \frac{A_a-7.95}{14.36} & for\ A_a > 21\ dB\end{cases}$$

$$\begin{aligned}\therefore\ D &= \frac{A_a-7.95}{14.36}\\ &= \frac{48-7.95}{14.36}\\ &= 2.7890\end{aligned}$$

e.) Then select the lowest odd value of N that would satisfy the inequality,

$$\begin{aligned}N &\geq \frac{\omega_s D}{B_t}+1\\ &\geq \frac{2600*2.79}{100}+1\\ &\geq 73.51\end{aligned} \qquad\qquad \therefore\ N = 75$$

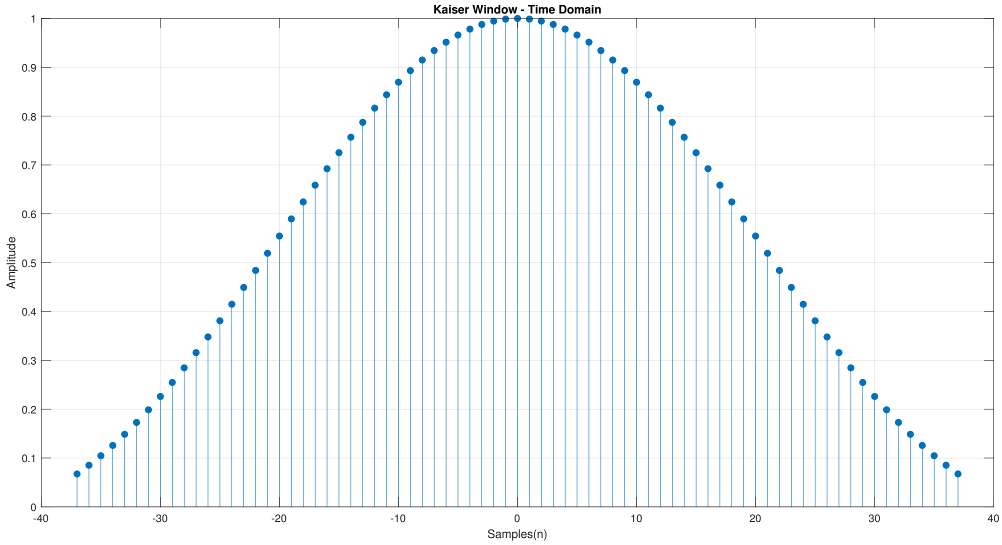

Figure 2: Derived Kaiser Window $w_K(nT)$ using calculated Parameters and Eq. (3)

2.1.4 Derivation of The Ideal Impulse Response

Note : Here subscript 'd' implies "desired", as it is the ideal response of the filter. Subscript d will be omitted to indicate a given expression is no longer ideal.

The frequency response of an ideal bandpass filter with cutoff frequencies ω_{c1} and ω_{c2} is given by,

$$H_d(e^{j\omega T}) = \begin{cases} 1 & for \; -\omega_{c2} \leq \omega \leq -\omega_{c1} \\ 1 & for \quad \omega_{c1} \leq \omega \leq \omega_{c2} \\ 0 & Otherwise \end{cases}$$

Using the Inverse Fourier Transform, impulse response of the above $H(e^{j\omega T})$ is calculated.

$$\begin{aligned} h_d(nT) &= \frac{1}{\omega_s} \int_{-\omega_s/2}^{\omega_s/2} H(e^{j\omega T}) e^{j\omega nT} \, d\omega \\ &= \frac{1}{\omega_s} \left[\int_{-\omega_{c2}}^{-\omega_{c1}} e^{j\omega nT} \, d\omega + \int_{\omega_{c1}}^{\omega_{c2}} e^{j\omega nT} \, d\omega \right] \\ &= \frac{1}{\omega_s} \left[\frac{e^{j\omega nT}}{jnT} \bigg|_{-\omega_{c2}}^{-\omega_{c1}} + \frac{e^{j\omega nT}}{jnT} \bigg|_{\omega_{c1}}^{\omega_{c2}} \right] \\ &= \frac{1}{j\omega_s nT} \left[e^{-j\omega_{c1}nT} - e^{-j\omega_{c2}nT} + e^{j\omega_{c2}nT} - e^{j\omega_{c1}nT} \right] ; where \; \omega_s T = 2\pi \\ &= \frac{1}{\pi n} \left[\frac{(e^{j\omega_{c2}nT} - e^{-j\omega_{c2}nT})}{2j} - \frac{(e^{j\omega_{c1}nT} - e^{-j\omega_{c1}nT})}{2j} \right] ; rearanging \\ &= \frac{1}{\pi n} \left[\sin(\omega_{c2}nT) - \sin(\omega_{c1}nT) \right] ; from \; Euler's \; Eq. \end{aligned}$$

$$\therefore \; h_d(nT) = \begin{cases} \frac{1}{\pi n} \left[\sin(\omega_{c2}nT) - \sin(\omega_{c1}nT) \right] & \forall n \neq 0 \\ \\ \frac{2}{\omega_s} (\omega_{c2} - \omega_{c1}) & for \; n = 0 \end{cases} \tag{4}$$

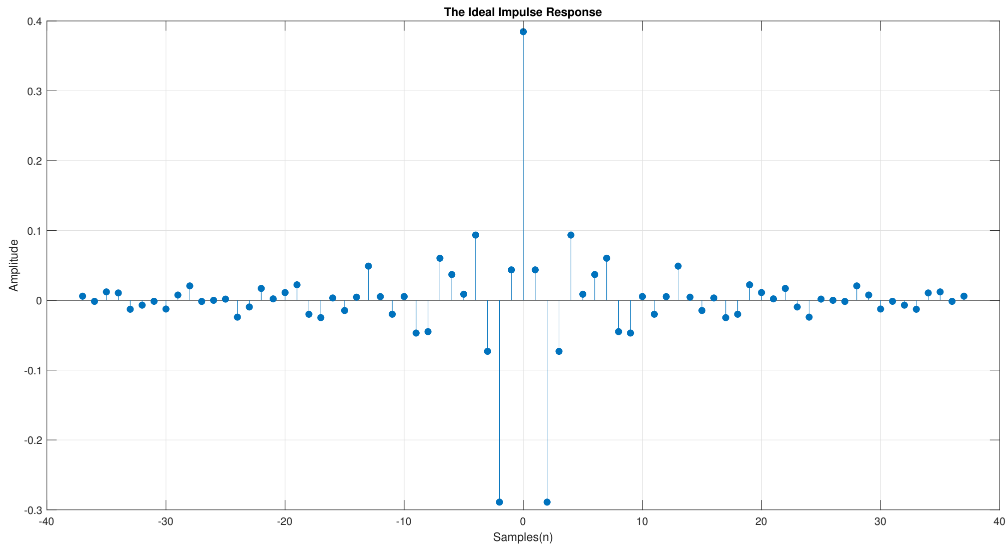

Figure 3: Anti-Causal Ideal Impulse Response $h_d(nT)$

2.1.5 Truncating the Ideal Impulse Response to obtain Finite Impulse Response: Windowing

By multiplying the ideal impulse response $h_d(nT)$ in Eq. (4) with the Kaiser window $w_K(nT)$ in Eq. (3), the ideal infinite impulse response can be truncated to obtain the finite impulse response $h(nT)$ for practical implementation.

$$h(nT) = w_K(nT).h_d(nT) \tag{5}$$

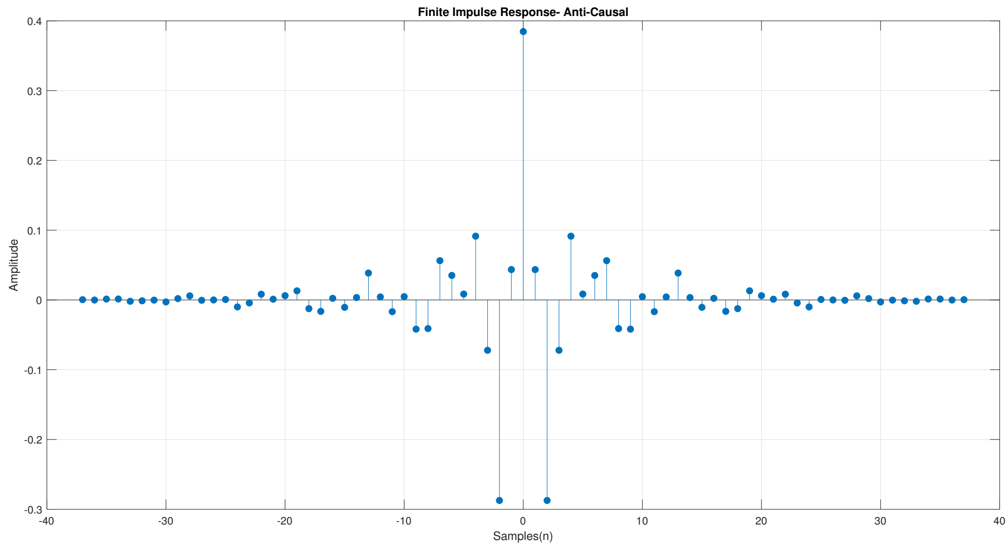

Figure 4: Anti-Causal Finite Impulse Response $h(nT)$

Obtaining the transfer function in the $\mathcal{Z}$ domain using $\mathcal{Z}$ Transformation,

$$\begin{aligned} H'(z) &= \mathcal{Z}\left\{h(nT)\right\} \\ &= \mathcal{Z}\left\{w_K(nT).h_d(nT)\right\} \end{aligned} \tag{6}$$

The response is anti-causal it need to be shifted in the time domain to make it causal and get the final filter response$H(Z)$, in $\mathcal{Z}$ domain it is represented as follows. Then the equation is evaluated at $e^{j\omega}$ to get the frequency response of the filter

$$H(z) = z^{-\left(\frac{N-1}{2}\right)}.H'(z) \tag{7}$$

2.2 Filter Performance Evaluation

Performance of the filter was evaluated by using the following excitation $x(nT)$ which is a combination of three sinusoidal signals. Frequencies of these three sinusoidal signals are specified as follows to cover all three bands(*lower stopband, passband and upper stopband*) in the bandpass filter.

$$x(nT) = \sum_{i=1}^{3} sin(w_i nT)$$

Parameter	Symbol	Calculation	Value
Middle frequency of the lower stopband	ω_1	$\frac{0+\omega_{a1}}{2}$	125 rad/s
Middle frequency of the passband	ω_2	$\frac{\omega_{p1}+\omega_{p2}}{2}$	600 rad/s
Middle frequency of the upper stopband	ω_3	$\frac{\omega_{a2}+\omega_s/2}{2}$	1100 rad/s

Table 3: Frequencies for Filter Performance Evaluation

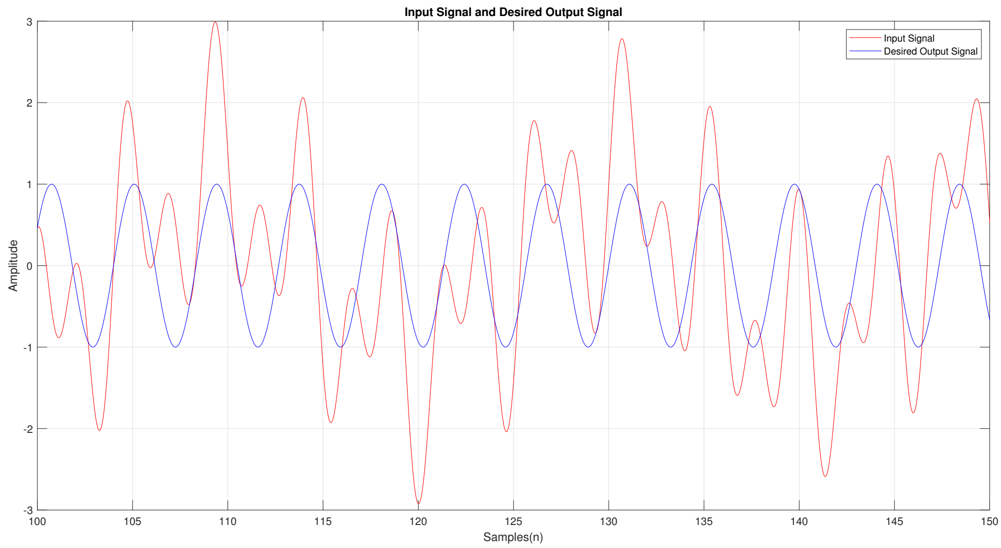

Figure 5: Input Signal

3 Results

3.1 FIR Filter Characteristics

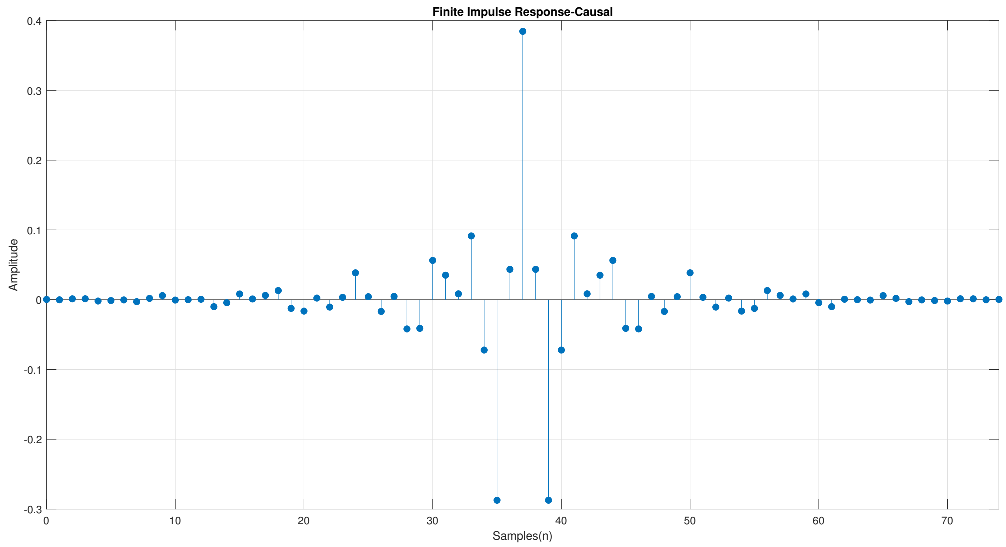

Figure 6: Causal Impulse Response of the Filter

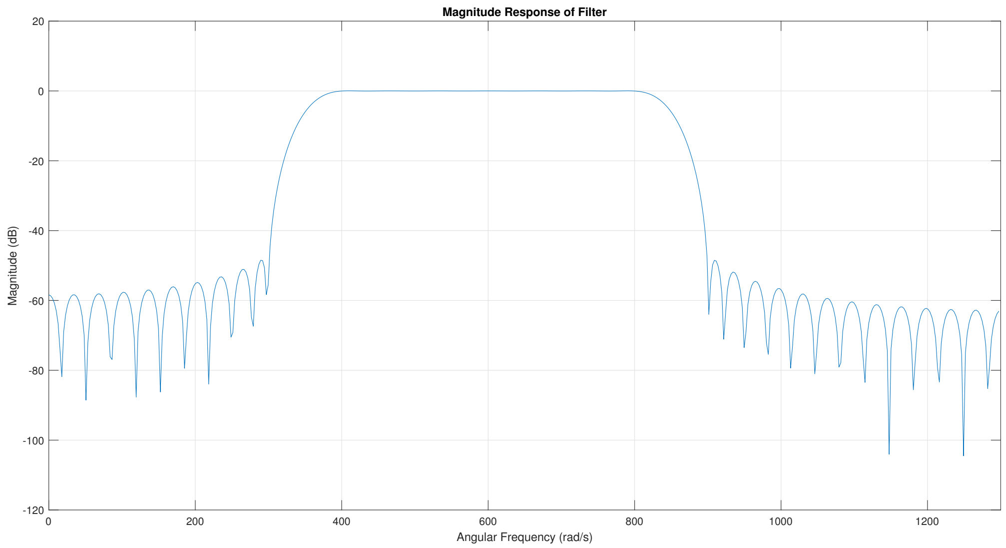

Figure 7: Magnitude Response of the Filter

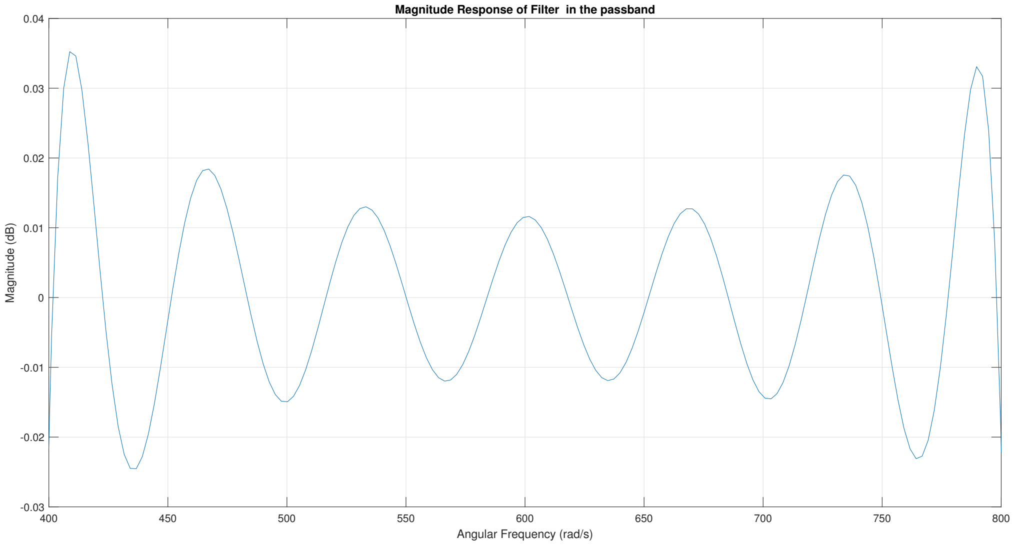

Figure 8: Magnitude Response of the Filter for the frequencies in the Passband

3.2 Input Output Signal Characteristics

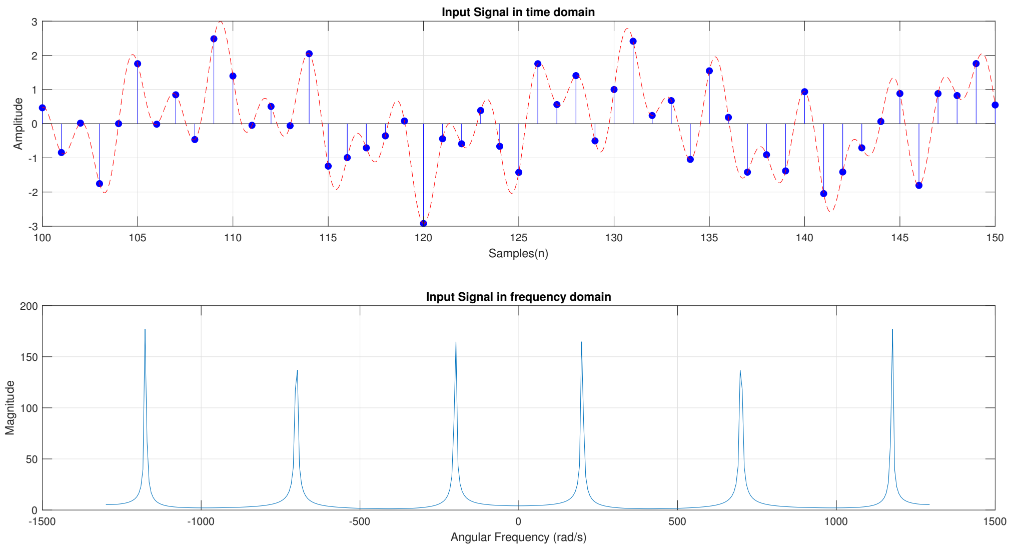

Figure 9: Input signal representation in Time and Frequency Domains

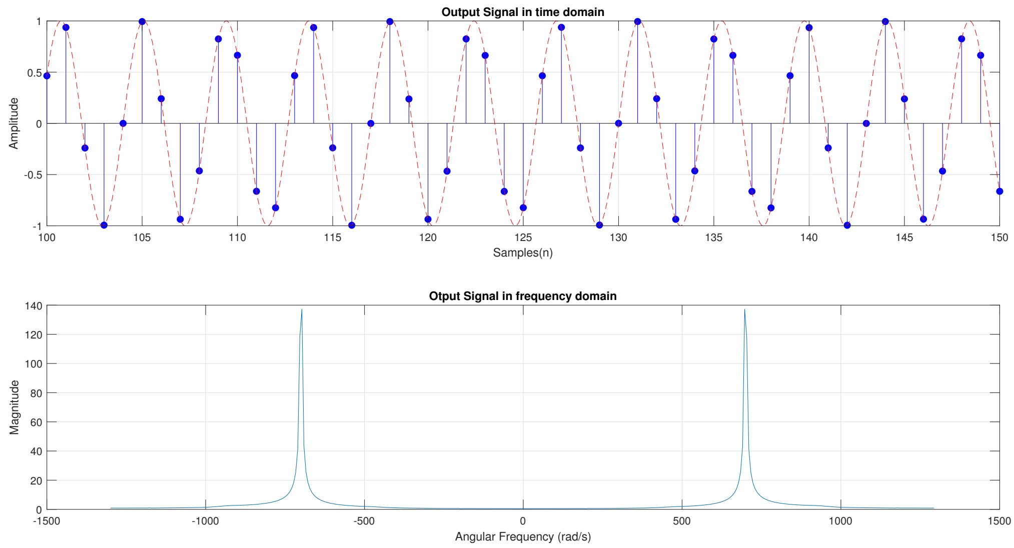

Figure 10: Output signal representation in Time and Frequency Domains

4 Discussion

5 Conclusion

Bibliography

[1] Andreas Antoniou. *Digital Signal Processing*. McGraw-Hill Professional, US, 2005.