```
options(repos="https://cran.rstudio.com" )
```

```
library(ggplot2)
library(softImpute)
```

```
## Loading required package: Matrix
```

```
## Loaded softImpute 1.4-1
```

Exercise 1

a

$$\frac{1}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} (x_{ij} - x_{i'j})^2 = \frac{1}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj} + \bar{x}_{kj} - x_{i'j})^2$$

$$= \frac{1}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} ((x_{ij} - \bar{x}_{kj}) - (x_{i'j} - \bar{x}_{kj}))^2$$

$$= \frac{1}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} \left((x_{ij} - \bar{x}_{kj})^2 - 2(x_{ij} - \bar{x}_{kj})(x_{i'j} - \bar{x}_{kj}) + (x_{i'j} - \bar{x}_{kj})^2 \right)$$

$$= \frac{1}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})^2 - \frac{2}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})(x_{i'j} - \bar{x}_{kj}) + \frac{1}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} (x_{i'j} - \bar{x}_{kj})^2$$

$$= \frac{|C_k|}{|C_k|} \sum_{i\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})^2 - \frac{2}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})(x_{i'j} - \bar{x}_{kj}) + \frac{|C_k|}{|C_k|} \sum_{i'\in C_k} \sum_{j=1}^{p} (x_{i'j} - \bar{x}_{kj})^2$$

$$= \sum_{i\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})^2 - \frac{2}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})(x_{i'j} - \bar{x}_{kj}) + \sum_{i\in C_k} \sum_{j=1}^{p} (\bar{x}_{kj} - x_{ij})^2$$

$$= 2 \sum_{i\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})^2 - \frac{2}{|C_k|} \sum_{i,i'\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})(x_{i'j} - \bar{x}_{kj})$$

$$= 2 \sum_{i\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})^2 - 0 = 2 \sum_{i\in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})^2$$
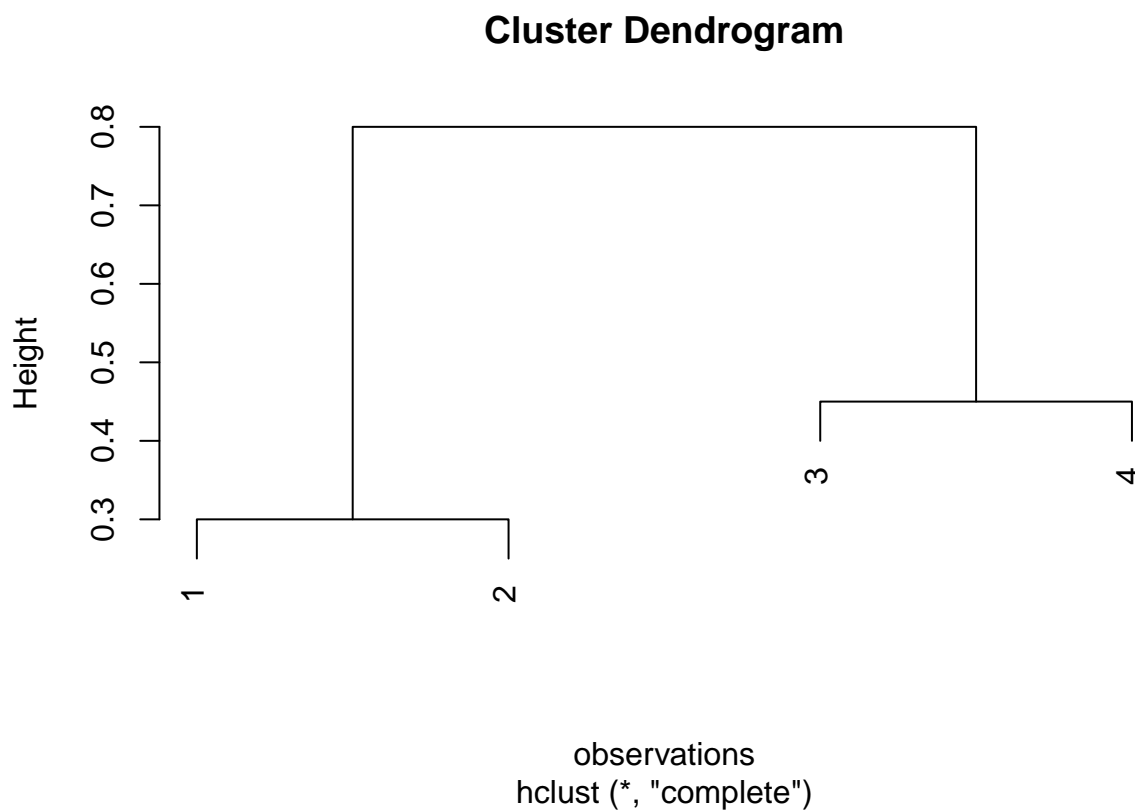
b

The sum of squared distance of each observation from the cluster mean, which is the value of RHS, decreases after each iteration. Hence, the clustering algorithm decreases the objective at each iteration.

Exercise 2

a

```
observations = as.dist(matrix(c(0,0.3,0.4,0.7,
                                0.3, 0, 0.5, 0.8,
                                0.4, 0.5, 0, 0.45,
                                0.7, 0.8, 0.45, 0), nrow = 4))

plot(hclust(observations, method="complete"))
```
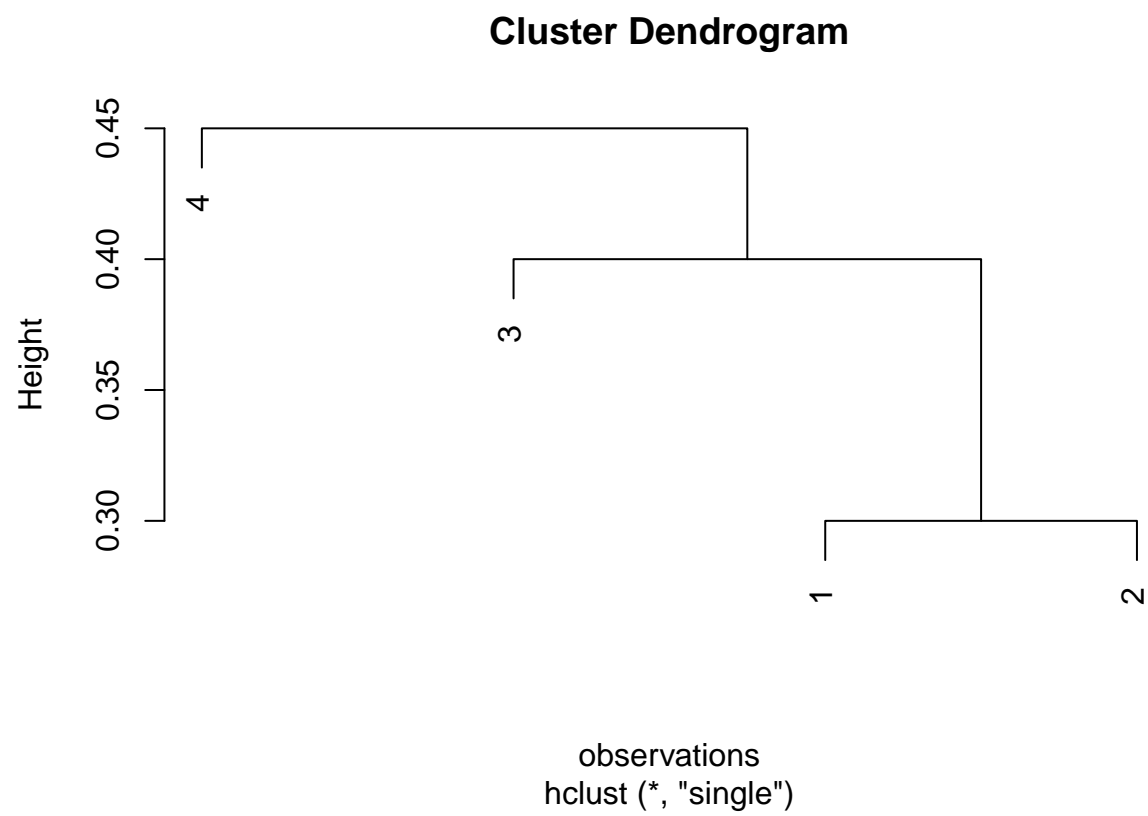
Cluster Dendrogram

observations
hclust (*, "complete")

b

```
plot(hclust(observations, method="single"))
```

Cluster Dendrogram

Height

0.45 0.40 0.35 0.30

4

3

1

2

observations
hclust (*, "single")

c

```
plot(hclust(observations, method="complete"))
abline(h = 0.7, col = "red")
```

Cluster Dendrogram

observations
hclust (*, "complete")
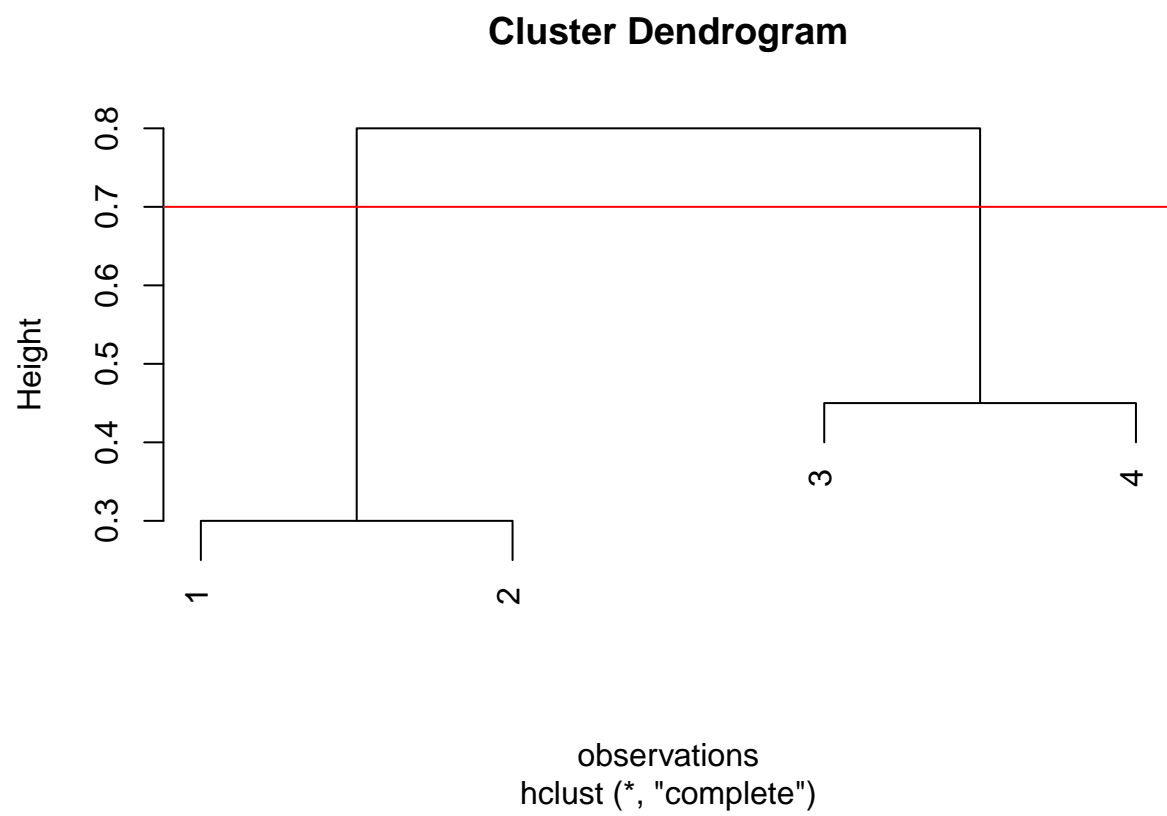
d

```
plot(hclust(observations, method="single"))
abline(h = 0.44, col = "red")
```

Cluster Dendrogram

observations
hclust (*, "single")
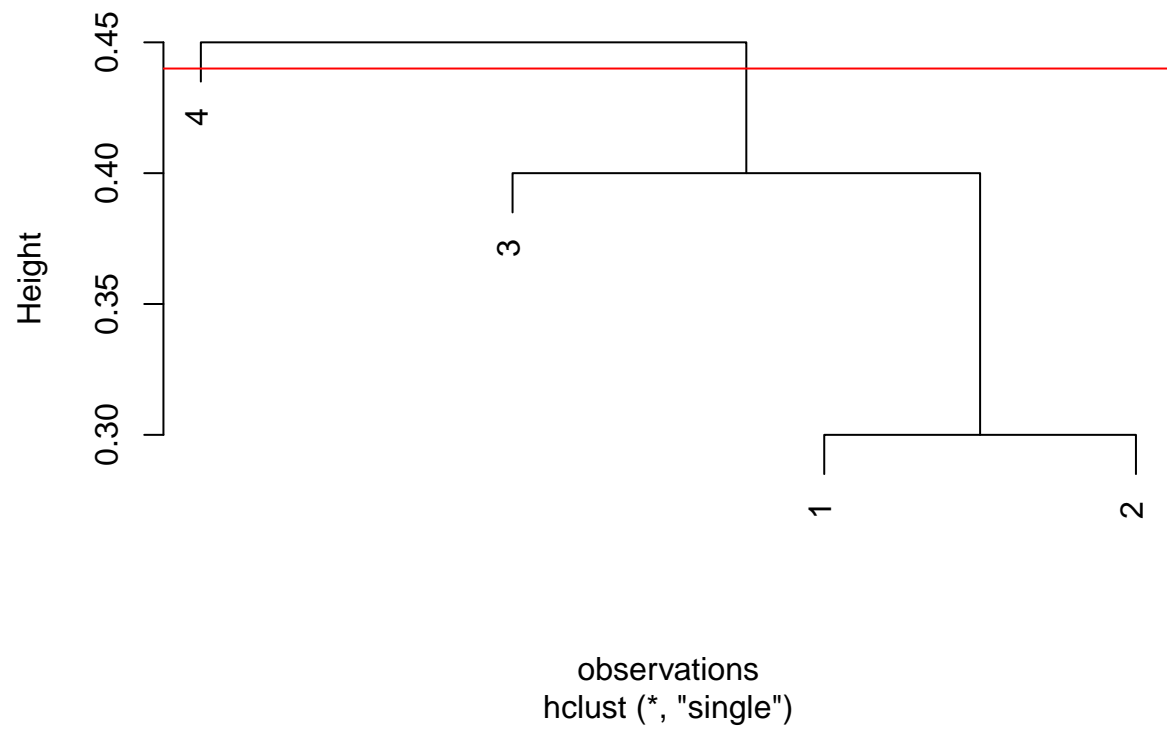
e

```
plot(hclust(observations, method = 'complete'), labels = c(2, 1, 4, 3))
```

Cluster Dendrogram
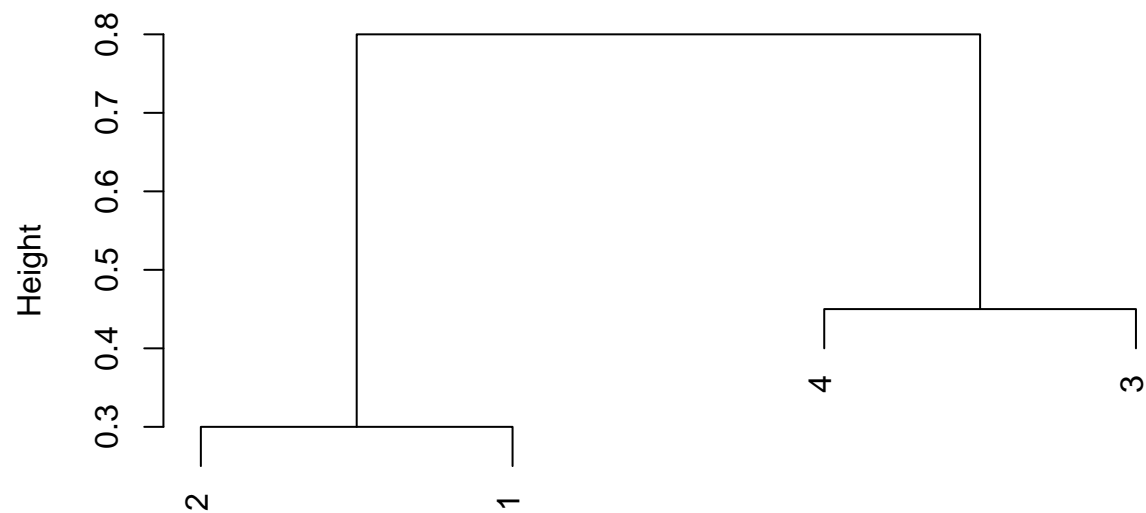
observations
hclust (*, "complete")

Exercise 3

a

```
data = data.frame(X1 = c(1,1,0,5,6,4), X2 = c(4,3,4,1,2,0))
plot(data[,'X1'], data[,'X2'])
```
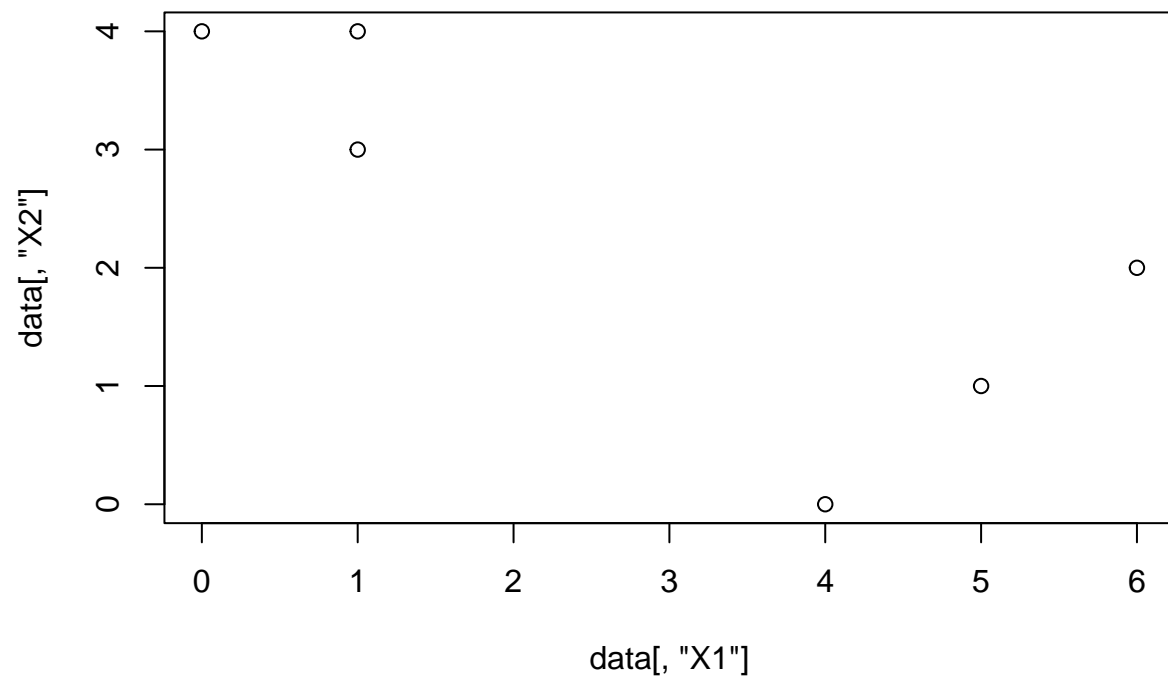
b

```
set.seed(0)
random_sample = sample(1:nrow(data), 3)

plot(data[,'X1'], data[,'X2'])

points(data[random_sample, 'X1'], data[random_sample, 'X2'], col = 'red')
points(data[-random_sample, 'X1'], data[-random_sample, 'X2'], col = 'green')
```
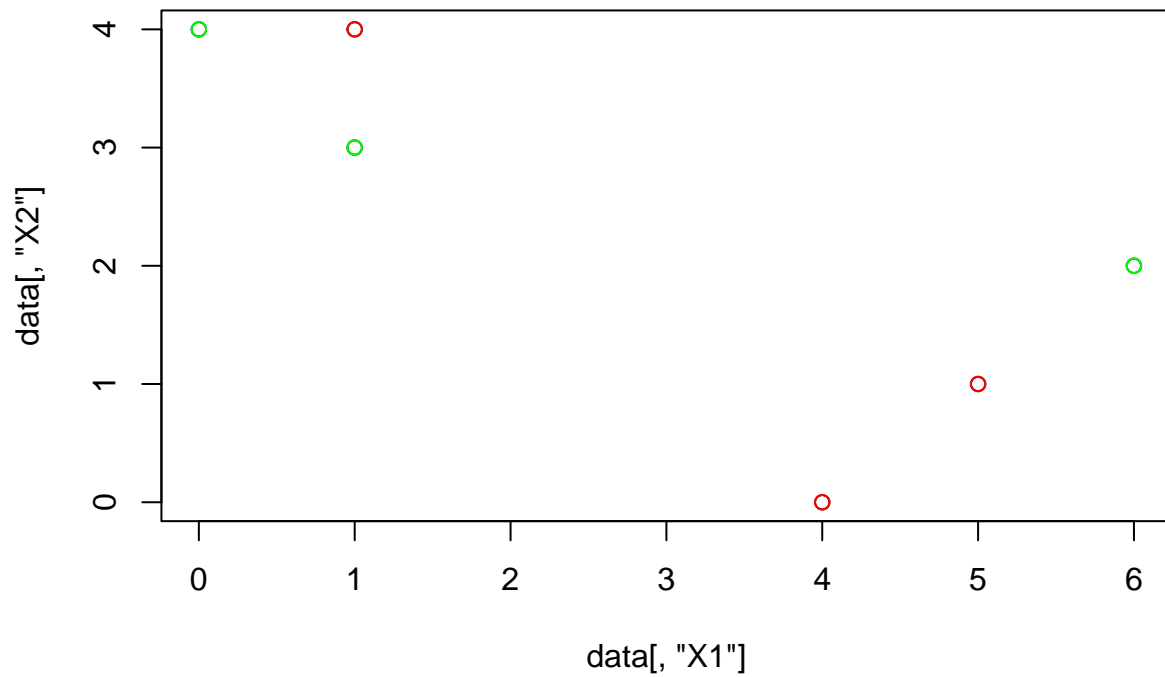
c

```
set.seed(0)
random_sample = sample(1:nrow(data), 3)

centroid_1 = c(sum(data[random_sample, 1])/3, sum(data[random_sample, 2])/3)
centroid_2 = c(sum(data[-random_sample, 1])/3, sum(data[-random_sample, 2])/3)

plot(data[,'X1'], data[,'X2'])
points(centroid_1[1], centroid_1[2], col="red", pch = 2)
points(centroid_2[1], centroid_2[2], col="green", pch = 2)

points(data[random_sample, 'X1'], data[random_sample, 'X2'], col = 'red')
points(data[-random_sample, 'X1'], data[-random_sample, 'X2'], col = 'green')
```

d
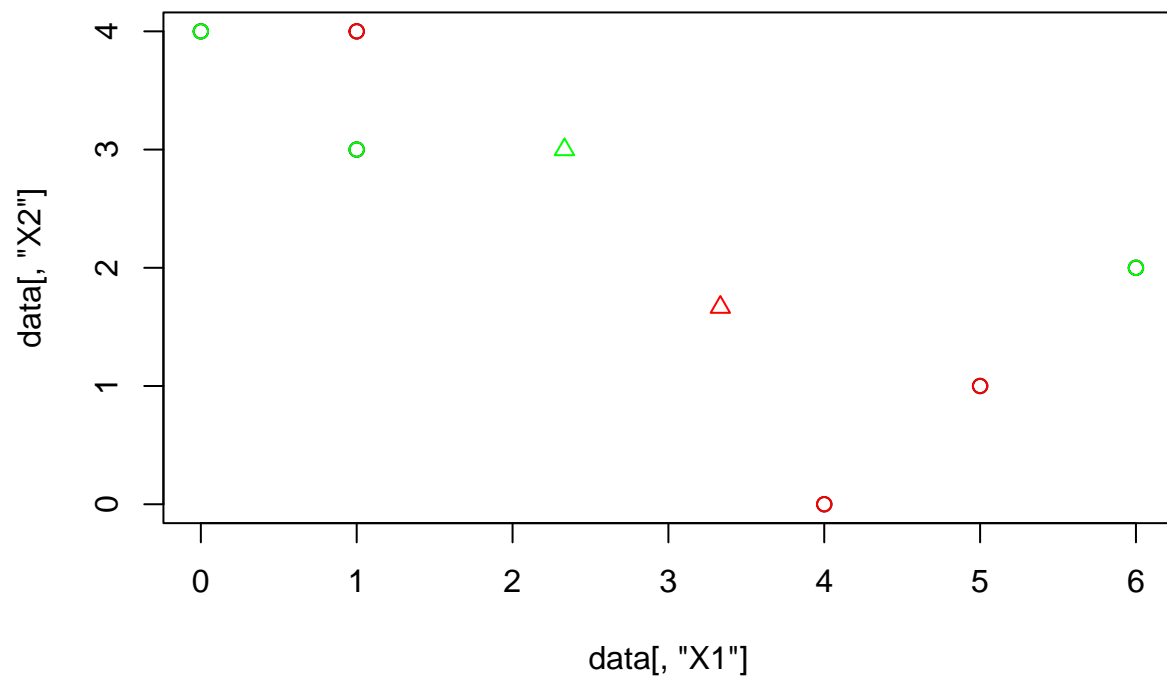
```r
for (i in 1:6){
  euclidean_dist_1 = sqrt((data[i, 1] - centroid_1[1])^2 + (data[i, 2] -centroid_1[2])^2)
  euclidean_dist_2 = sqrt((data[i,1] - centroid_2[1])^2 + (data[i, 2] - centroid_2[2])^2)

  if (euclidean_dist_1 < euclidean_dist_2) {
    data$cluster[i] = 1
  }
  else {
    data$cluster[i] = 2
  }
}

data
```

```
##   X1 X2 cluster
## 1  1  4       2
## 2  1  3       2
## 3  0  4       2
## 4  5  1       1
## 5  6  2       1
## 6  4  0       1
```

```
plot(data[,'X1'], data[,'X2'])
points(centroid_1[1], centroid_1[2], col="red", pch = 2)
points(centroid_2[1], centroid_2[2], col="green", pch = 2)

points(data[data$cluster == 1, 'X1'], data[data$cluster == 1, 'X2'], col = 'red')
points(data[data$cluster == 2, 'X1'], data[data$cluster == 2, 'X2'], col = 'green')
```
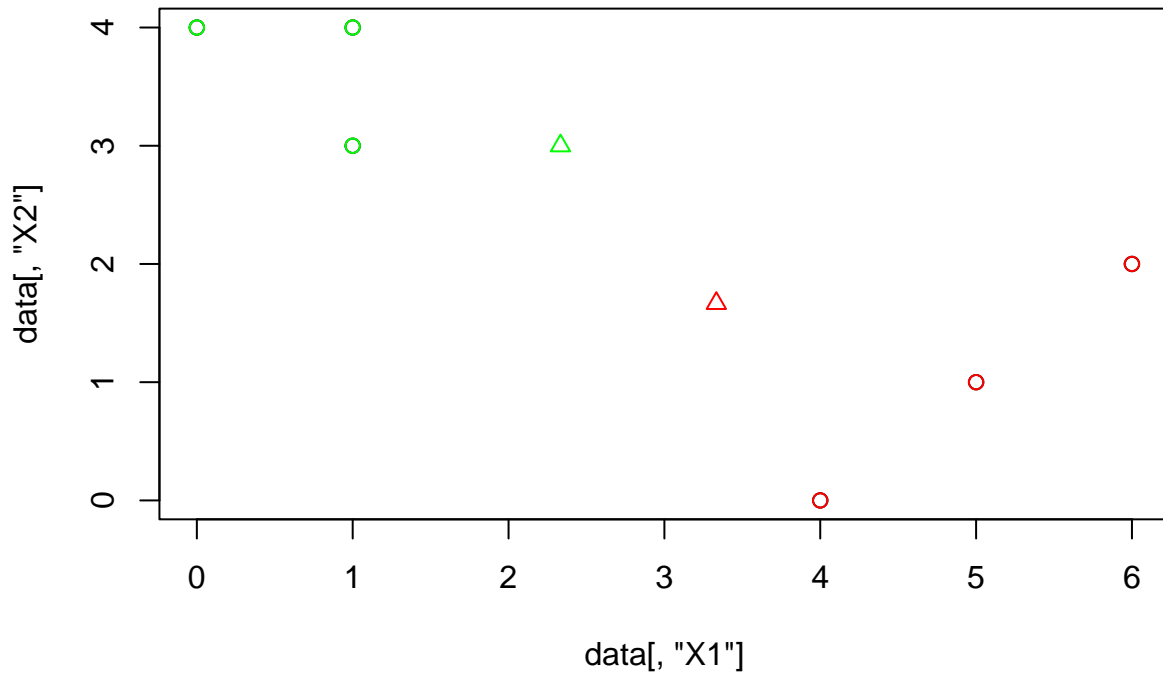

e

```
centroid_1 = c(sum(data[data$cluster == 1,1])/3,sum(data[data$cluster == 1,2])/3)
centroid_2 = c(sum(data[data$cluster == 2,1])/3,sum(data[data$cluster == 2,2])/3)

plot(data[,'X1'], data[,'X2'])
points(centroid_1[1], centroid_1[2], col="red", pch = 2)
points(centroid_2[1], centroid_2[2], col="green", pch = 2)

points(data[data$cluster == 1, 'X1'], data[data$cluster == 1, 'X2'], col = 'red')
points(data[data$cluster == 2, 'X1'], data[data$cluster == 2, 'X2'], col = 'green')
```
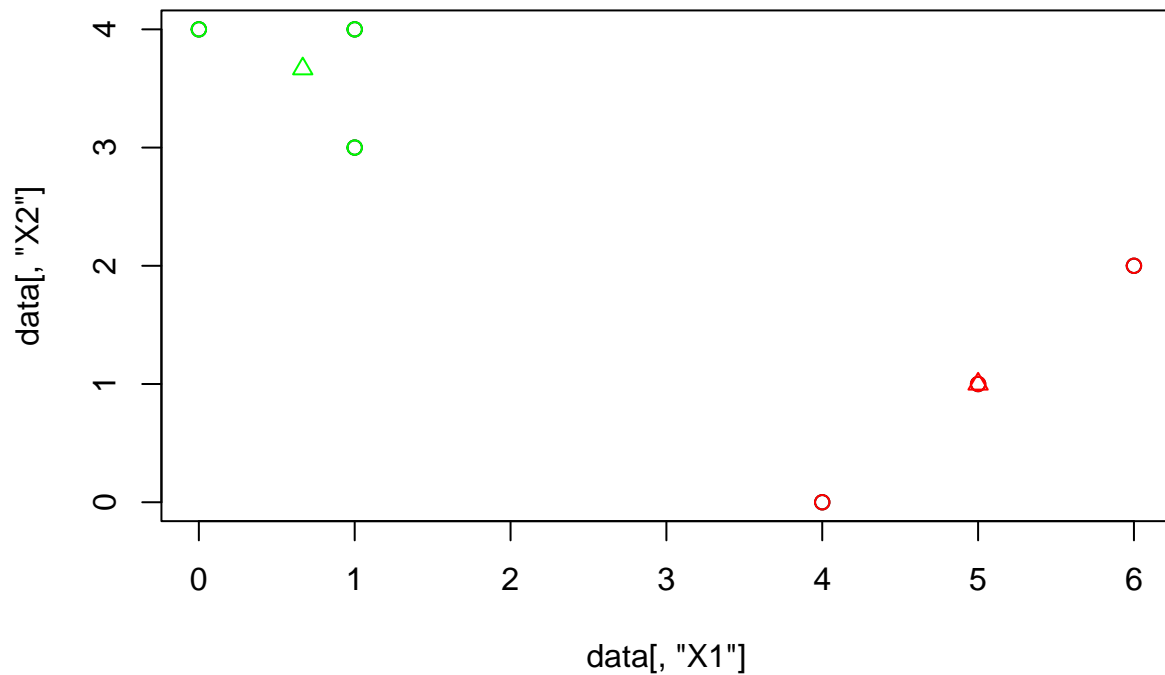
Exercise 4

a/ There is not enough information to tell. It depends on the distances of of the observations.

b/ There is not enough information to tell. Although complete linkage records the maximal distance and single linkage records the minimal distance but these two distances could be the same and they fuse at the same height.

Exercise 5

```r
socks = c(8, 11, 7, 6, 5, 6, 7, 8)
computers = c(0, 0, 0, 0, 1, 1, 1, 1)
shops = matrix(, nrow = 8)
for (i in 1:8){
  shops[i] = paste('shop', i)
}

eight_shoppers = data.frame(cbind(socks, computers))
row.names(eight_shoppers) = shops
eight_shoppers
```

```
##        socks computers
```

```
## shop 1      8         0
## shop 2     11         0
## shop 3      7         0
## shop 4      6         0
## shop 5      5         1
## shop 6      6         1
## shop 7      7         1
## shop 8      8         1
```

```
plot(eight_shoppers)
```
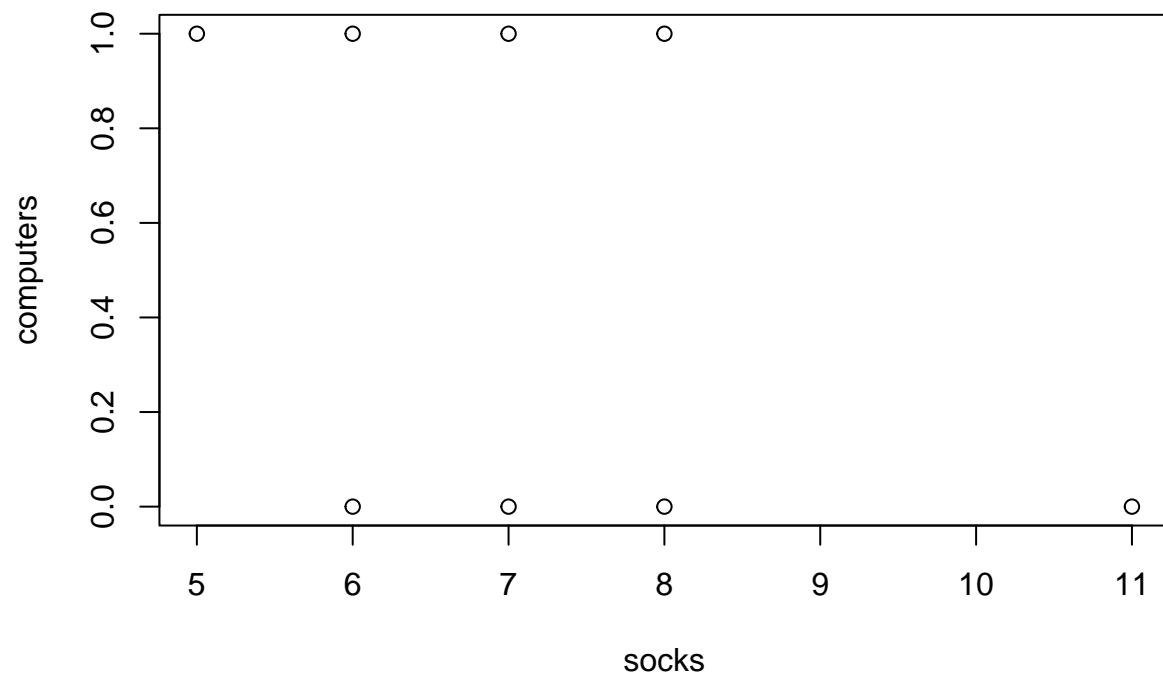

KMeans

```
two_kmeans = kmeans(eight_shoppers, centers = 2)
plot(eight_shoppers, col = two_kmeans$cluster + 1, cex = 2, pch = 20)
```
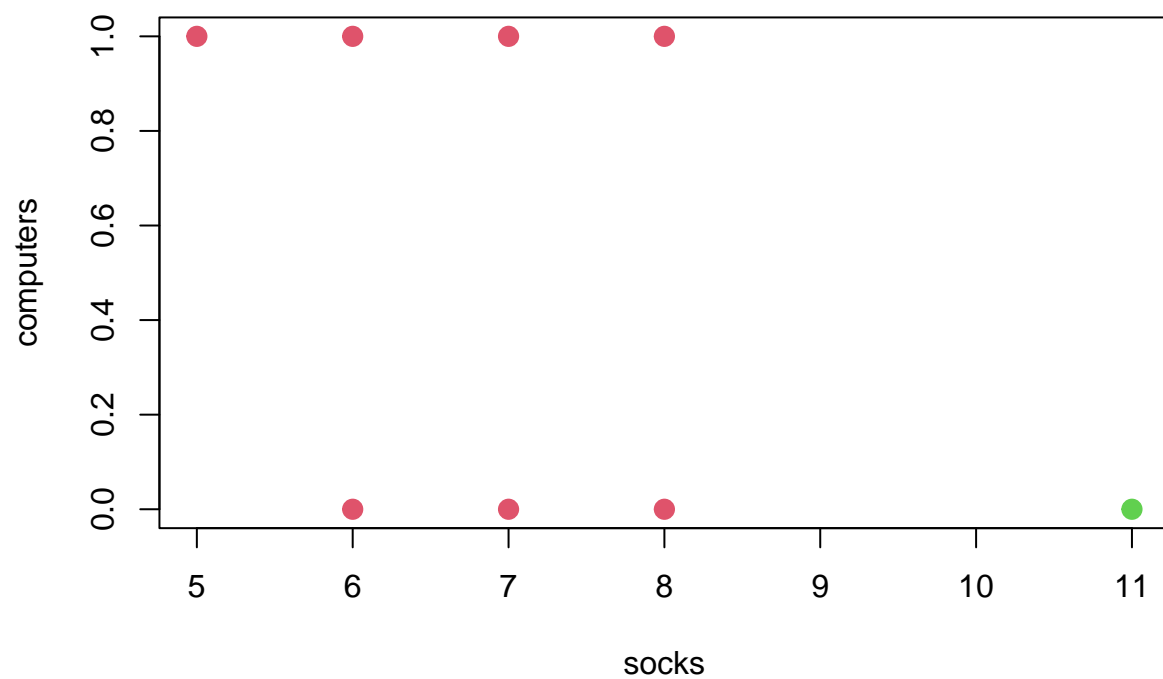
```
scaled_es = scale(eight_shoppers)
sclaed_two_km = kmeans(scaled_es, centers = 2)
plot(scaled_es, col = sclaed_two_km$cluster + 1, cex = 2, pch = 20)
```
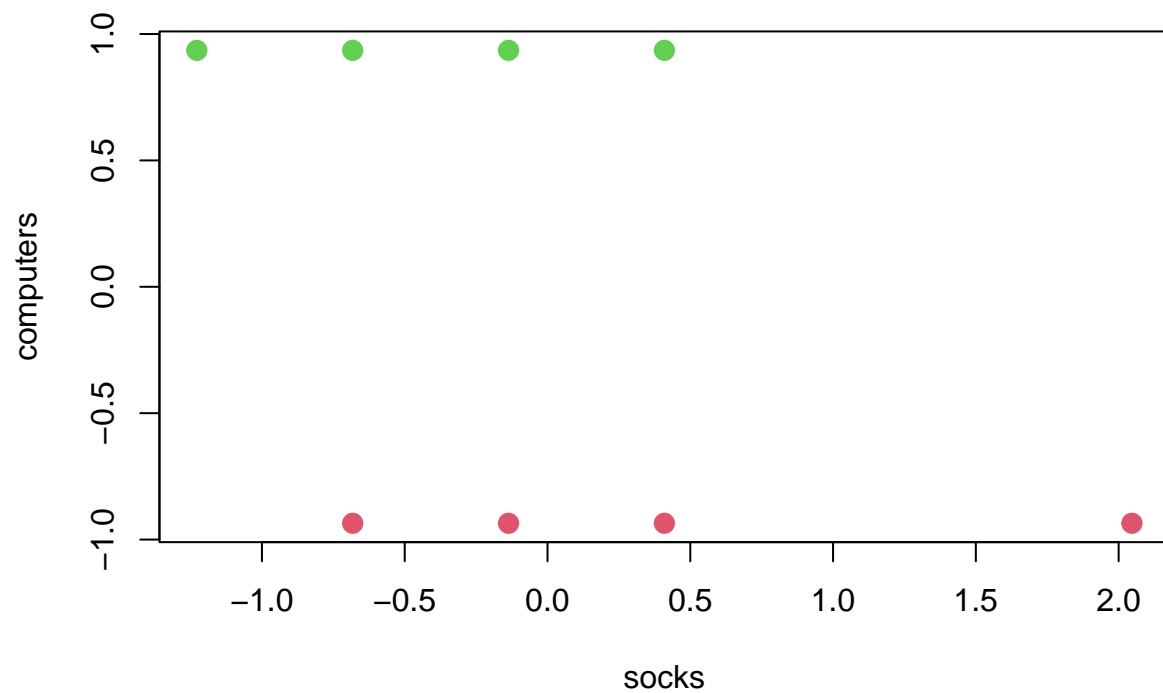
Hierarchical Clustering

```
euclidean_dist = dist(scaled_es, method = 'euclidean')
eu_clusters = cutree(hclust(euclidean_dist), 2)
plot(eight_shoppers, col = eu_clusters + 1, cex = 2, pch = 20)
```
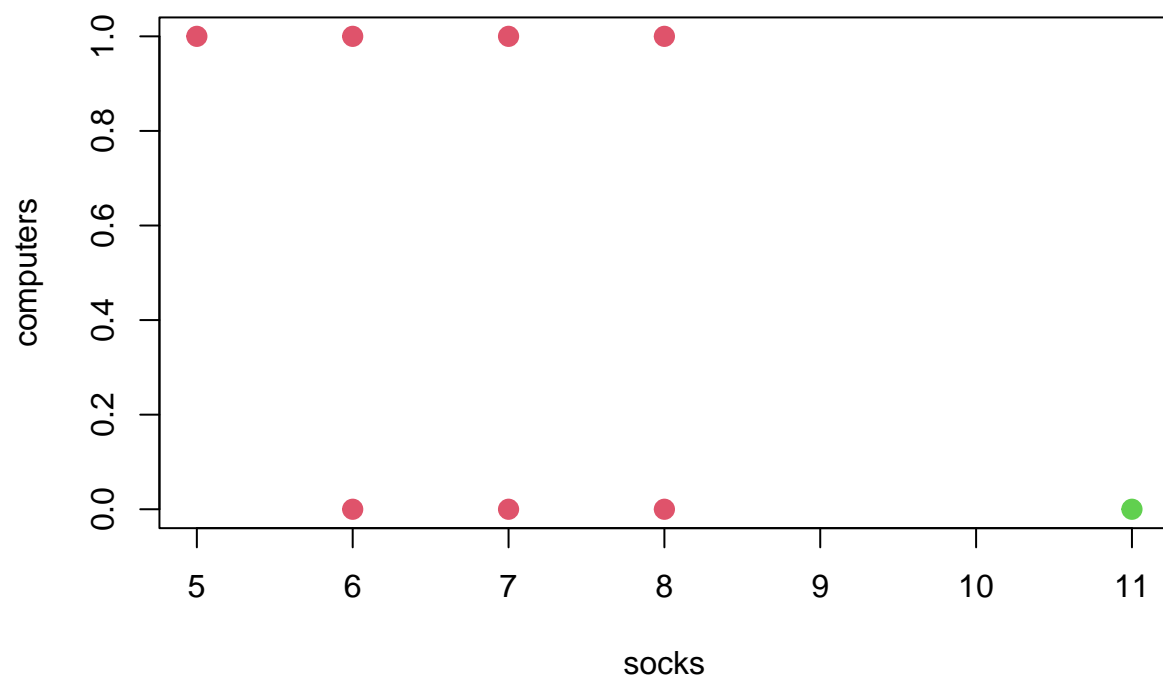
```
library(factoextra)
```

```
## Welcome! Want to learn more? See two factoextra-related books at https://goo.gl/ve3WBa
```

```
cor_dist = get_dist(scaled_es, method = 'pearson')
cor_clusters = cutree(hclust(cor_dist), 2)
plot(eight_shoppers, col = cor_clusters + 1, cex = 2, pch = 20)
```
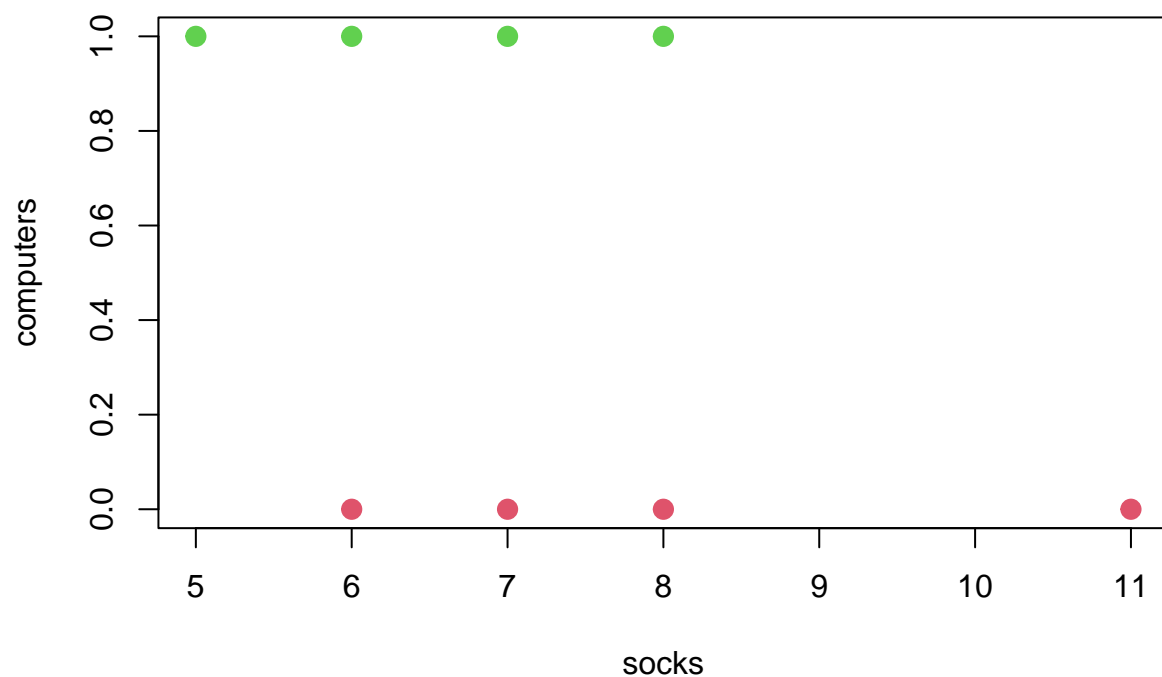
Exercise 6

```
head(USArrests)
```

```
##            Murder Assault UrbanPop Rape
## Alabama      13.2     236       58 21.2
## Alaska       10.0     263       48 44.5
## Arizona       8.1     294       80 31.0
## Arkansas      8.8     190       50 19.5
## California    9.0     276       91 40.6
## Colorado      7.9     204       78 38.7
```

```
pca_usa = prcomp(scale(USArrests))
score_vectors = pca_usa$x
head(score_vectors)
```

```
##                    PC1        PC2         PC3          PC4
## Alabama     -0.9756604  1.1220012 -0.43980366  0.154696581
## Alaska      -1.9305379  1.0624269  2.01950027 -0.434175454
## Arizona     -1.7454429 -0.7384595  0.05423025 -0.826264240
## Arkansas     0.1399989  1.1085423  0.11342217 -0.180973554
## California  -2.4986128 -1.5274267  0.59254100 -0.338559240
## Colorado    -1.4993407 -0.9776297  1.08400162  0.001450164
```

```
bind_data = data.frame(scale(USArrests), data.frame(score_vectors))
head(bind_data)
```

```
##                 Murder   Assault   UrbanPop        Rape         PC1        PC2
## Alabama     1.24256408 0.7828393 -0.5209066 -0.003416473 -0.9756604  1.1220012
## Alaska      0.50786248 1.1068225 -1.2117642  2.484202941 -1.9305379  1.0624269
## Arizona     0.07163341 1.4788032  0.9989801  1.042878388 -1.7454429 -0.7384595
## Arkansas    0.23234938 0.2308680 -1.0735927 -0.184916602  0.1399989  1.1085423
## California  0.27826823 1.2628144  1.7589234  2.067820292 -2.4986128 -1.5274267
## Colorado    0.02571456 0.3988593  0.8608085  1.864967207 -1.4993407 -0.9776297
##                    PC3          PC4
## Alabama    -0.43980366  0.154696581
## Alaska      2.01950027 -0.434175454
## Arizona     0.05423025 -0.826264240
## Arkansas    0.11342217 -0.180973554
## California  0.59254100 -0.338559240
## Colorado    1.08400162  0.001450164
```

```
coef(lm(Murder ~ PC1 + PC2 + PC3 + PC4, data = bind_data))
```

```
##   (Intercept)           PC1           PC2           PC3           PC4
## -1.375410e-16 -5.358995e-01  4.181809e-01 -3.412327e-01  6.492278e-01
```

And these are equal to the loadings we want

```
pca_usa = prcomp(scale(USArrests))
pca_usa$rotation
```

```
##                 PC1        PC2        PC3         PC4
## Murder   -0.5358995  0.4181809 -0.3412327  0.64922780
## Assault  -0.5831836  0.1879856 -0.2681484 -0.74340748
## UrbanPop -0.2781909 -0.8728062 -0.3780158  0.13387773
## Rape     -0.5434321 -0.1673186  0.8177779  0.08902432
```

Exercise 7

$$d^2(r_i, r_j) = 2(p-1)(1 - r_{ij})$$

```
scaled_usa = t(scale(t(USArrests)))
correlation = cor(t(scaled_usa))
squared_dist = as.matrix(dist(scaled_usa)**2)
n = dim(scaled_usa)[1]

dim(correlation)
```

```
## [1] 50 50
```

```
dim(squared_dist)
```

```
## [1] 50 50
```

```
n
```

```
## [1] 50
```

```
head(data.frame(squared_dist / (1 - correlation)))
```

```
##           Alabama Alaska Arizona Arkansas California Colorado Connecticut
## Alabama       NaN      6       6        6          6        6           6
## Alaska          6    NaN       6        6          6        6           6
## Arizona         6      6     NaN        6          6        6           6
## Arkansas        6      6       6      NaN          6        6           6
## California      6      6       6        6        NaN        6           6
## Colorado        6      6       6        6          6      NaN           6
##           Delaware Florida Georgia Hawaii Idaho Illinois Indiana Iowa Kansas
## Alabama          6       6       6      6     6        6       6    6      6
## Alaska           6       6       6      6     6        6       6    6      6
## Arizona          6       6       6      6     6        6       6    6      6
## Arkansas         6       6       6      6     6        6       6    6      6
## California       6       6       6      6     6        6       6    6      6
## Colorado         6       6       6      6     6        6       6    6      6
##           Kentucky Louisiana Maine Maryland Massachusetts Michigan Minnesota
## Alabama          6         6     6        6             6        6         6
## Alaska           6         6     6        6             6        6         6
## Arizona          6         6     6        6             6        6         6
## Arkansas         6         6     6        6             6        6         6
## California       6         6     6        6             6        6         6
## Colorado         6         6     6        6             6        6         6
##           Mississippi Missouri Montana Nebraska Nevada New.Hampshire
## Alabama             6        6       6        6      6             6
## Alaska              6        6       6        6      6             6
## Arizona             6        6       6        6      6             6
## Arkansas            6        6       6        6      6             6
## California          6        6       6        6      6             6
## Colorado            6        6       6        6      6             6
##           New.Jersey New.Mexico New.York North.Carolina North.Dakota Ohio
## Alabama            6          6        6              6            6    6
## Alaska             6          6        6              6            6    6
## Arizona            6          6        6              6            6    6
## Arkansas           6          6        6              6            6    6
## California         6          6        6              6            6    6
## Colorado           6          6        6              6            6    6
##           Oklahoma Oregon Pennsylvania Rhode.Island South.Carolina
## Alabama          6      6            6            6              6
## Alaska           6      6            6            6              6
## Arizona          6      6            6            6              6
## Arkansas         6      6            6            6              6
## California       6      6            6            6              6
## Colorado         6      6            6            6              6
```

```
##               South.Dakota Tennessee Texas Utah Vermont Virginia Washington
## Alabama                  6         6     6    6       6        6          6
## Alaska                   6         6     6    6       6        6          6
## Arizona                  6         6     6    6       6        6          6
## Arkansas                 6         6     6    6       6        6          6
## California               6         6     6    6       6        6          6
## Colorado                 6         6     6    6       6        6          6
##               West.Virginia Wisconsin Wyoming
## Alabama                   6         6       6
## Alaska                    6         6       6
## Arizona                   6         6       6
## Arkansas                  6         6       6
## California                6         6       6
## Colorado                  6         6       6
```

Exercise 8

a

```
scaled_usarrest = scale(USArrests)
pca_usarrest = prcomp(scaled_usarrest)
std_1 = pca_usarrest$sdev
var_1 = std_1**2
pve_1 = var_1 / sum(var_1)
pve_1
```

```
## [1] 0.62006039 0.24744129 0.08914080 0.04335752
```

b

```
loadings = pca_usarrest$rotation
pve_2 = rep(NA, 4)
for (i in 1:4){
  scores = scaled_usarrest %*% loadings[, i]
  pve = sum(scores**2) / sum(scaled_usarrest**2)
  pve_2[i] = pve
}

pve_2
```

```
## [1] 0.62006039 0.24744129 0.08914080 0.04335752
```

Exercise 9

a

```
library(factoextra)
clusterability = get_clust_tendency(USArrests, n = 45)
clusterability$hopkins_stat
```

```
## [1] 0.5791842
```

```
hc_usarrest = hclust(dist(USArrests), method = 'complete')
```

b

```
plot(hc_usarrest, xlab = 'States', cex = 0.6, main = 'Complete Linkage')
```
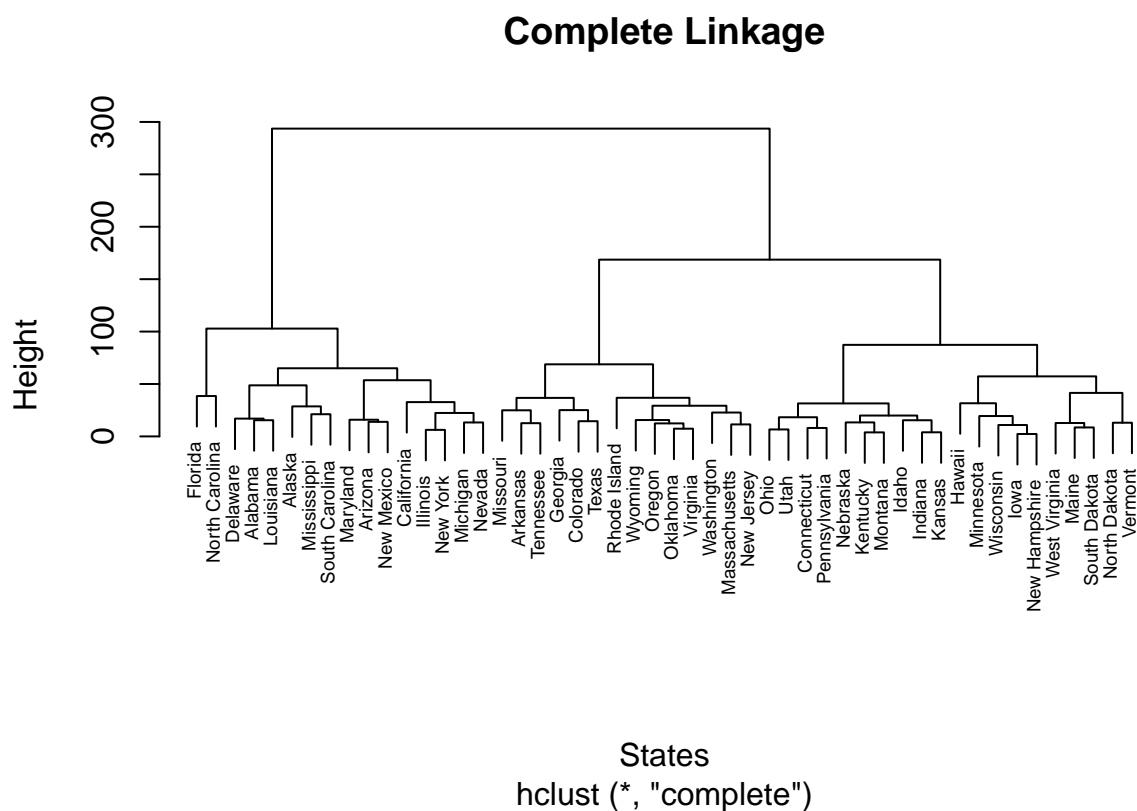


```
data.frame(cutree(hc_usarrest, k = 3))
```

```
##                 cutree.hc_usarrest..k...3.
## Alabama                                  1
## Alaska                                   1
## Arizona                                  1
## Arkansas                                 2
## California                               1
## Colorado                                 2
```

```
## Connecticut                    3
## Delaware                       1
## Florida                        1
## Georgia                        2
## Hawaii                         3
## Idaho                          3
## Illinois                       1
## Indiana                        3
## Iowa                           3
## Kansas                         3
## Kentucky                       3
## Louisiana                      1
## Maine                          3
## Maryland                       1
## Massachusetts                  2
## Michigan                       1
## Minnesota                      3
## Mississippi                    1
## Missouri                       2
## Montana                        3
## Nebraska                       3
## Nevada                         1
## New Hampshire                  3
## New Jersey                     2
## New Mexico                     1
## New York                       1
## North Carolina                 1
## North Dakota                   3
## Ohio                           3
## Oklahoma                       2
## Oregon                         2
## Pennsylvania                   3
## Rhode Island                   2
## South Carolina                 1
## South Dakota                   3
## Tennessee                      2
## Texas                          2
## Utah                           3
## Vermont                        3
## Virginia                       2
## Washington                     2
## West Virginia                  3
## Wisconsin                      3
## Wyoming                        2
```

c

```
hc_scaled_usarrest = hclust(dist(scale(USArrests)), method = 'complete')
plot(hc_scaled_usarrest, xlab = 'States', cex = 0.6, main = 'Complete Linkage')
```

Complete Linkage

States
hclust (*, "complete")

```
data.frame(cutree(hc_scaled_usarrest, 3))
```

```
##                 cutree.hc_scaled_usarrest..3.
## Alabama                                     1
## Alaska                                      1
## Arizona                                     2
## Arkansas                                    3
## California                                  2
## Colorado                                    2
## Connecticut                                 3
## Delaware                                    3
## Florida                                     2
## Georgia                                     1
## Hawaii                                      3
## Idaho                                       3
## Illinois                                    2
## Indiana                                     3
## Iowa                                        3
## Kansas                                      3
## Kentucky                                    3
## Louisiana                                   1
## Maine                                       3
## Maryland                                    2
## Massachusetts                               3
## Michigan                                    2
```
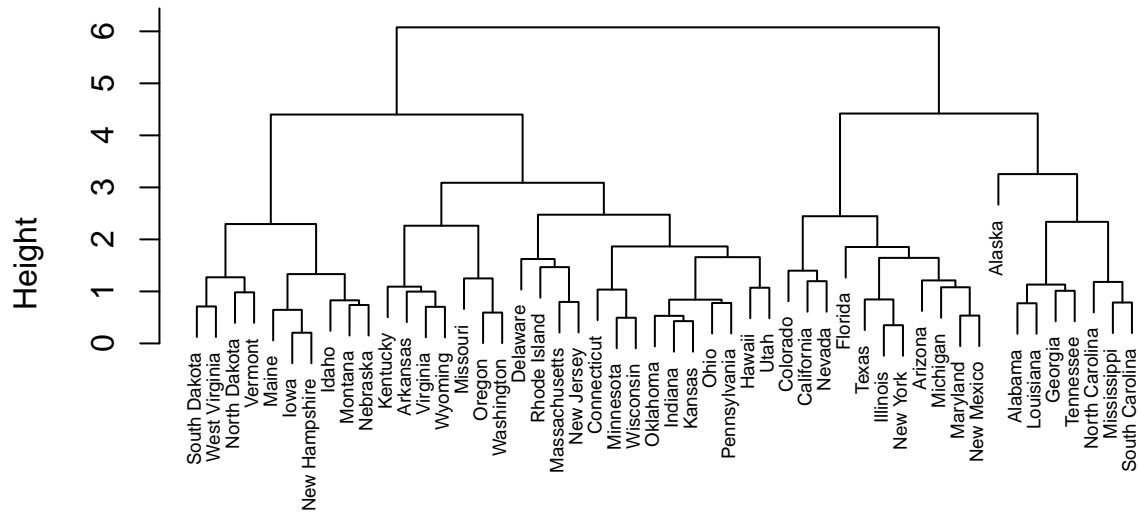
```
## Minnesota                          3
## Mississippi                        1
## Missouri                           3
## Montana                            3
## Nebraska                           3
## Nevada                             2
## New Hampshire                      3
## New Jersey                         3
## New Mexico                         2
## New York                           2
## North Carolina                     1
## North Dakota                       3
## Ohio                               3
## Oklahoma                           3
## Oregon                             3
## Pennsylvania                       3
## Rhode Island                       3
## South Carolina                     1
## South Dakota                       3
## Tennessee                          1
## Texas                              2
## Utah                               3
## Vermont                            3
## Virginia                           3
## Washington                         3
## West Virginia                      3
## Wisconsin                          3
## Wyoming                            3
```

d

Scaling data before performing hierarchical clustering outputs a more balanced dendrogram and also avoid bias by some variables.

Exercise 10

a

```
set.seed(42)
generated_data = matrix(rnorm(60*50), ncol = 50)
generated_data[1:20, ] = generated_data[1:20, ]
generated_data[21:40, ] = generated_data[21:40, ] - 2
generated_data[41:60, ] = generated_data[41:60, ] + 2
dim(generated_data)
```

```
## [1] 60 50
```

```
get_clust_tendency(generated_data, n = 50)
```

```
## $hopkins_stat
## [1] 0.6507836
##
## $plot
```
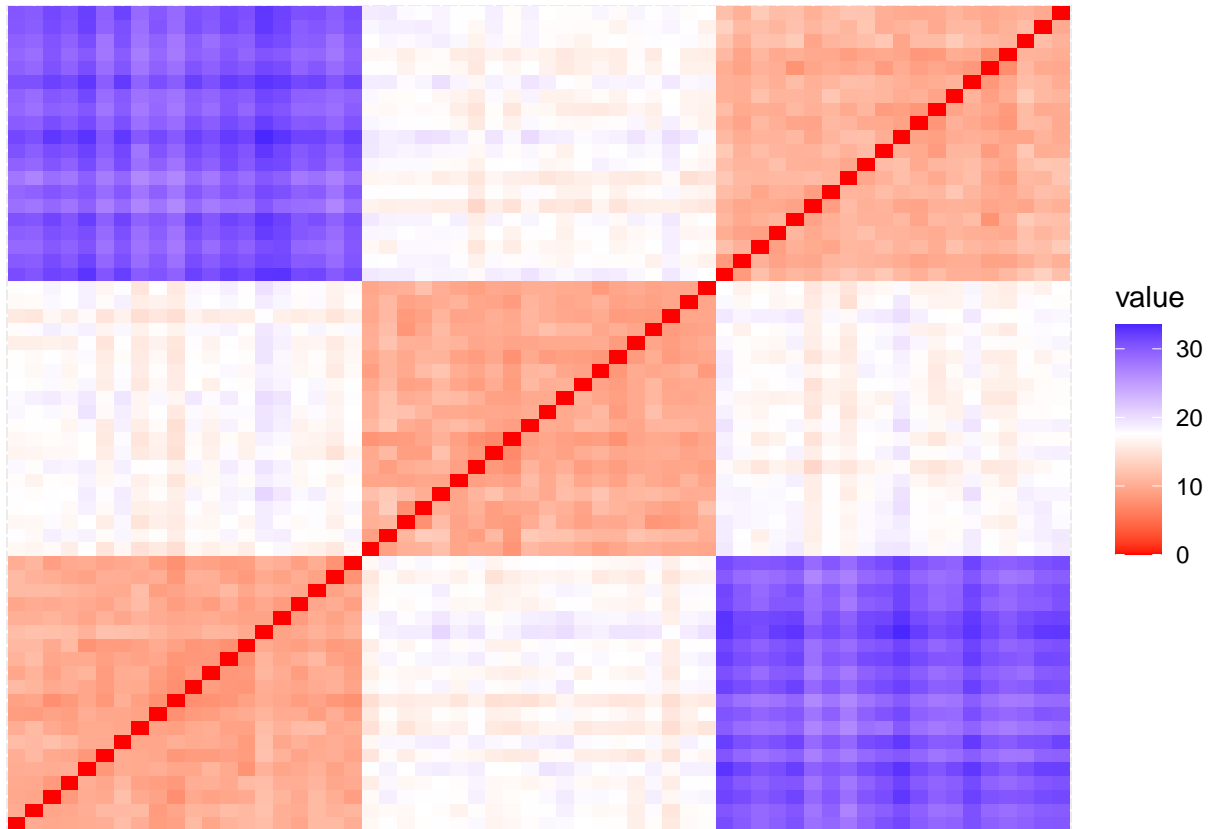

b

```
scaled_generated_data = scale(generated_data)
svd_generated_data = svd(generated_data)

M = 2
u = svd_generated_data$u[, 1:M]
d = svd_generated_data$d[1:M]
v = svd_generated_data$v[, 1:M]

reconstructed_data = u %*% (d * t(v))
get_clust_tendency(reconstructed_data, n = 50)$hopkins
```

```
## [1] 0.9236313
```

```
plot(reconstructed_data, col = c(rep(1, 20), rep(2, 20), rep(3, 20)))
```
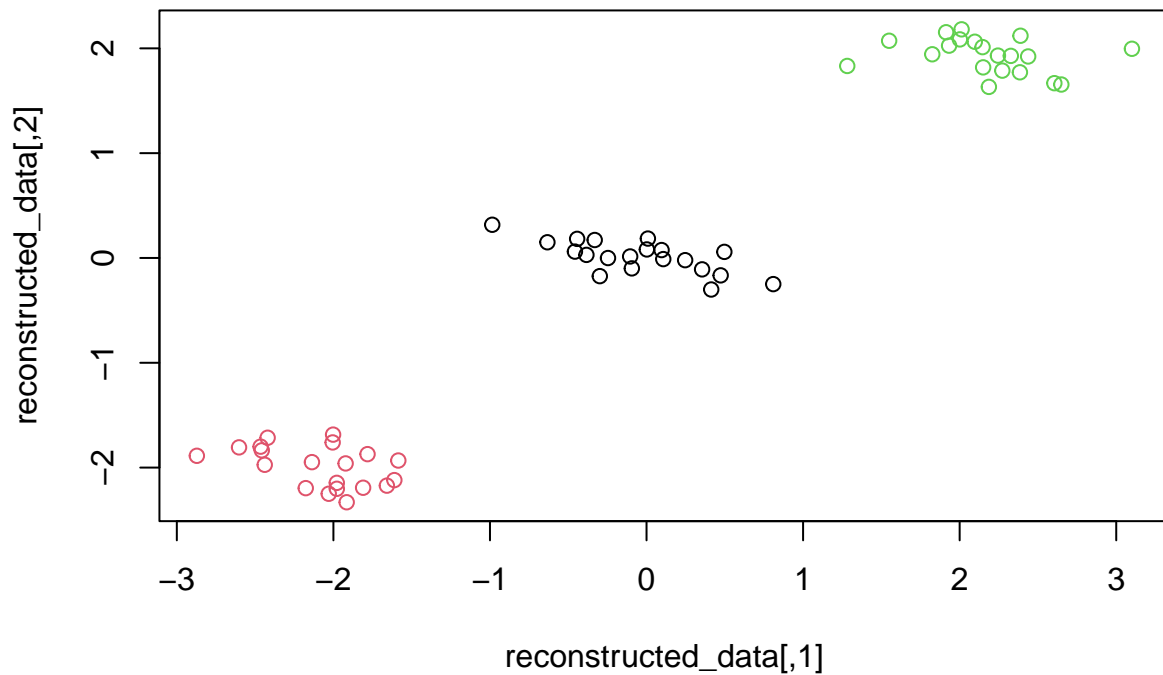
After being constructed, instances in each class became identical so we merely only see 3 dots on the plot.

c

```
three_kmeans = kmeans(generated_data, 3, nstart = 20)
three_kmeans$cluster
```

```
##  [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## [39] 3 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
```

```
table(three_kmeans$cluster)
```

```
##
##  1  2  3
## 20 20 20
```

d

```
two_kmeans = kmeans(generated_data, 2, nstart = 20)
two_kmeans$cluster
```

```
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
## [39] 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

```
table(two_kmeans$cluster)
```

```
##
##  1  2
## 40 20
```

There are 20 misclassifications in total, the algorithm class number 3 as 1.

e

```
four_kmeans = kmeans(generated_data, 4, nstart = 20)
four_kmeans$cluster
```

```
## [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [39] 1 1 3 3 3 3 2 2 3 3 2 2 2 2 2 2 2 2 2 2 3 3 3
```

```
table(four_kmeans$cluster)
```

```
##
##  1  2  3  4
## 20 11  9 20
```

There are around 10 misclassifications in total, the algorithm split class number 3 as 2 classes. Since this is an unsupervised learning task, it's unfairly to judge the model to be correct or incorrect. In these cases, it's incorrect compared to the original data because we forced it to cluster that way.

f

```
three_kmeans = kmeans(reconstructed_data, 3, nstart = 20)
three_kmeans$cluster
```

```
## [1] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## [39] 3 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

```
get_clust_tendency(reconstructed_data, n = 50)$hopkins
```

```
## [1] 0.9236313
```

```
table(three_kmeans$cluster)
```

```
##
##  1  2  3
## 20 20 20
```

```
score_vectors = t(d %*% t(u))
variance = apply(score_vectors, 2, var)
variance / sum(variance)
```

```
## [1] 1
```

All instances were clustered correctly. The first two principal components reserve almost 100% of total variance.

g

```
set.seed(42)
X1 = matrix(rnorm(60*50), ncol = 50)
X1[1:20, ] = X1[1:20, ]
X1[21:40, ] = X1[21:40, ] - 2
X1[41:60, ] = X1[41:60, ] + 2

kmeans_no_scaling = kmeans(X1, 3, nstart = 20)
kmeans_no_scaling$cluster
```

```
##  [1] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [39] 1 1 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
```

```
table(kmeans_no_scaling$cluster)
```

```
##
##  1  2  3
## 20 20 20
```

```
kmeans_scaling = kmeans(scale(X1), 3, nstart = 20)
kmeans_scaling$cluster
```

```
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [39] 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
```

```
table(kmeans_scaling$cluster)
```

```
##
##  1  2  3
## 20 20 20
```

There is no significant difference between scaling and non-scaling before performing Kmeans in this particular case. (In practice, it really depends on the data characteristics to scale or not to scale or not to scale before clustering).

Exercise 11

First look

```
library(MASS)
boston = Boston
head(boston)
```

```
##      crim zn indus chas  nox    rm  age    dis rad tax ptratio  black lstat
## 1 0.00632 18  2.31    0 0.538 6.575 65.2 4.0900   1 296    15.3 396.90  4.98
## 2 0.02731  0  7.07    0 0.469 6.421 78.9 4.9671   2 242    17.8 396.90  9.14
## 3 0.02729  0  7.07    0 0.469 7.185 61.1 4.9671   2 242    17.8 392.83  4.03
## 4 0.03237  0  2.18    0 0.458 6.998 45.8 6.0622   3 222    18.7 394.63  2.94
## 5 0.06905  0  2.18    0 0.458 7.147 54.2 6.0622   3 222    18.7 396.90  5.33
## 6 0.02985  0  2.18    0 0.458 6.430 58.7 6.0622   3 222    18.7 394.12  5.21
##   medv
## 1 24.0
## 2 21.6
## 3 34.7
## 4 33.4
## 5 36.2
## 6 28.7
```

```
library(psych)
```

```
##
## Attaching package: 'psych'
```

```
## The following objects are masked from 'package:ggplot2':
##
##     %+%, alpha
```

```
pairs.panels(boston)
```
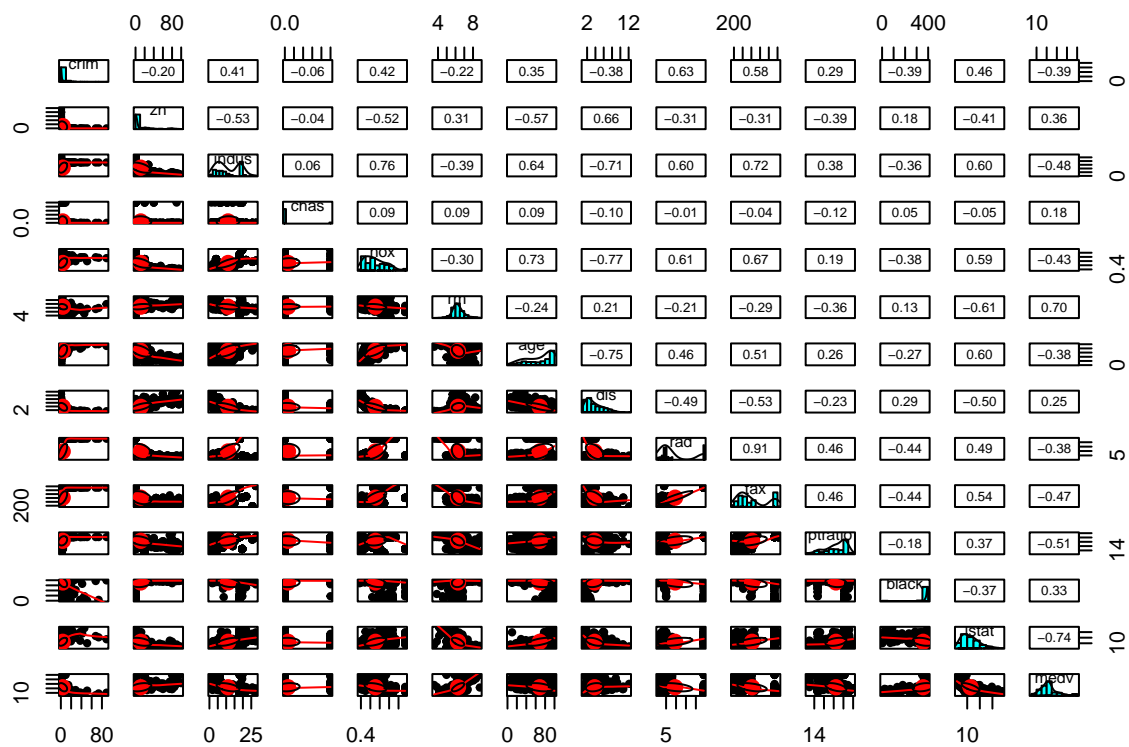
table(boston$zn)

```
## 
##     0 12.5 17.5   18   20   21   22   25   28   30   33   34   35   40   45 52.5
##   372   10    1    1   21    4   10   10    3    6    4    3    3    7    6    3
##    55   60   70   75   80 82.5   85   90   95  100
##     3    4    3    3   15    2    2    5    4    1
```

table(boston$chas)

```
## 
##   0   1
## 471  35
```

hist(boston$black)

Histogram of boston$black
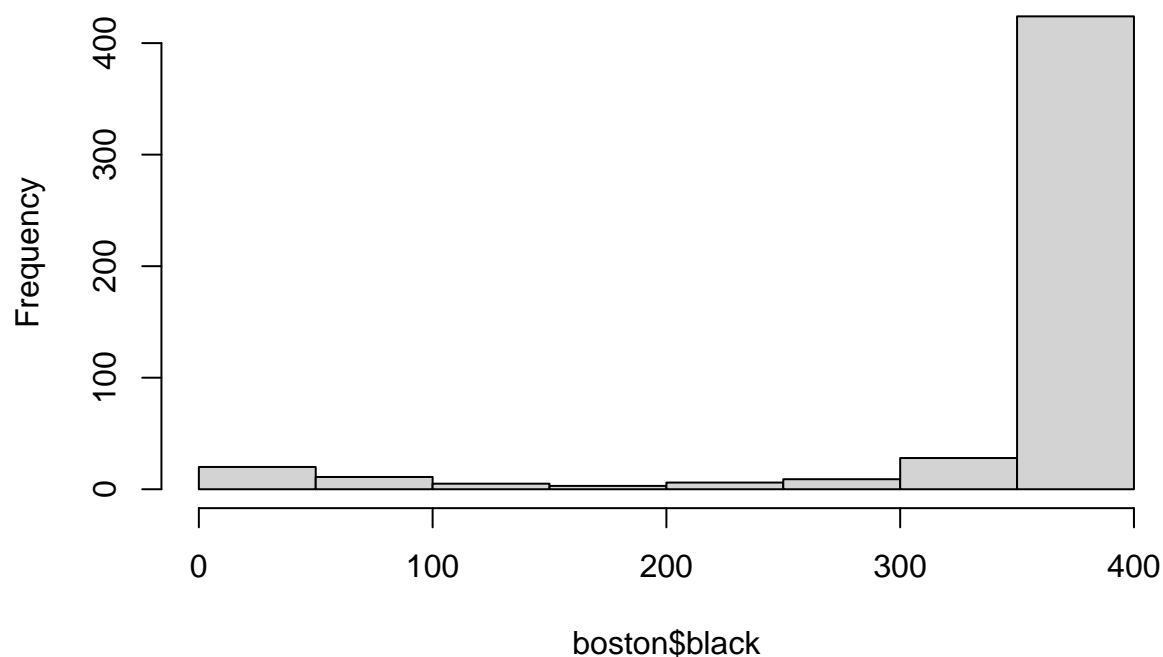
Writing the function

```r
scaled_boston = scale(boston)

fill_nan = function(nan_rate, M){

  set.seed(42)
  row_nan_index = sample(nrow(boston), nan_rate * nrow(boston))
  col_nan_index = sample(ncol(boston), nan_rate * nrow(boston), replace = T)
  nan_index = cbind(row_nan_index, col_nan_index)
  Xna = scaled_boston
  Xna[nan_index] = NA

  imputation = softImpute(Xna, rank.max = M, maxit = 100, trace.it = F)
  imputed_Xhat = complete(Xna, imputation)
  correlation = cor(imputed_Xhat[nan_index], scaled_boston[nan_index])

  return(correlation)

}
```

```r
nan_rates = seq(0.05, 0.3, by = 0.05)
M_range = c(1:8)
correlations = matrix(nrow = length(M_range), ncol = length(nan_rates))
```

```
for (m in 1:length(M_range)){
  for (n in 1:length(nan_rates)){
    correlations[m, n] = fill_nan(nan_rates[n], M_range[m])
  }
}
```

```
## Warning in simpute.als(x, J, thresh, lambda, maxit, trace.it, warm.start, :
## Convergence not achieved by 100 iterations
```

```
correlations
```

```
##              [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,]   0.3679410 0.6597288 0.5368808 0.6557710 0.5242510 0.5198488
## [2,]   0.6210335 0.6264166 0.6153261 0.7226254 0.5962005 0.6225547
## [3,]   0.5968443 0.6264086 0.6191804 0.6976242 0.6680883 0.6386526
## [4,]  -0.1366559 0.5554989 0.5788634 0.6541163 0.3230895 0.5333878
## [5,]   0.7570159 0.6089703 0.6224969 0.6784240 0.3591517 0.5952954
## [6,]   0.6622354 0.3993243 0.3075116 0.5875837 0.2655345 0.4514147
## [7,]   0.5778245 0.7167376 0.7220754 0.7196409 0.2945373 0.4155679
## [8,]   0.6078424 0.6559724 0.6778938 0.6740666 0.3100012 0.5679190
```

```
results = data.frame(correlations)
colnames(results) = nan_rates
rownames(results) = M_range
results
```

```
##          0.05       0.1      0.15       0.2      0.25       0.3
## 1   0.3679410 0.6597288 0.5368808 0.6557710 0.5242510 0.5198488
## 2   0.6210335 0.6264166 0.6153261 0.7226254 0.5962005 0.6225547
## 3   0.5968443 0.6264086 0.6191804 0.6976242 0.6680883 0.6386526
## 4  -0.1366559 0.5554989 0.5788634 0.6541163 0.3230895 0.5333878
## 5   0.7570159 0.6089703 0.6224969 0.6784240 0.3591517 0.5952954
## 6   0.6622354 0.3993243 0.3075116 0.5875837 0.2655345 0.4514147
## 7   0.5778245 0.7167376 0.7220754 0.7196409 0.2945373 0.4155679
## 8   0.6078424 0.6559724 0.6778938 0.6740666 0.3100012 0.5679190
```

Exercise 12

```
set.seed(15)

X = data.matrix(scale(USArrests))

nomit = 20
ina = sample(seq(50), nomit)
inb = sample(4, nomit, replace = T)
Xna = X
nan_index = cbind(ina, inb)
Xna[nan_index] = NA
#View(Xna)
```

```
xbar = colMeans(Xna, na.rm = T)
Xhat = Xna
Xhat[nan_index] = xbar[inb]
pca_Xhat = prcomp(Xhat)
```

```
M = 1
pcs = pca_Xhat$rotation
reconstructed_usarrest = pca_Xhat$x[, 1:M] %*% t(pcs[, 1:M])
Xhat[nan_index] = reconstructed_usarrest[nan_index]

cor(Xhat[nan_index], X[nan_index])
```

```
## [1] 0.6436628
```

```
Xhat[1:5, ]
```

```
##                Murder   Assault   UrbanPop         Rape
## Alabama    1.24256408 0.2986381 -0.5209066 -0.003416473
## Alaska     0.50786248 1.1068225  0.6343882  2.484202941
## Arizona    0.07163341 1.4788032  0.9989801  1.042878388
## Arkansas   0.23234938 0.2308680 -1.0735927 -0.184916602
## California 0.27826823 1.0144567  1.7589234  2.067820292
```

```
X[1:5, ]
```

```
##                Murder   Assault   UrbanPop         Rape
## Alabama    1.24256408 0.7828393 -0.5209066 -0.003416473
## Alaska     0.50786248 1.1068225 -1.2117642  2.484202941
## Arizona    0.07163341 1.4788032  0.9989801  1.042878388
## Arkansas   0.23234938 0.2308680 -1.0735927 -0.184916602
## California 0.27826823 1.2628144  1.7589234  2.067820292
```

Exercise 13

a

```
gene_expression = read.csv('Ch12Ex13.csv', header = F)
dim(gene_expression)
```

```
## [1] 1000   40
```

```
gene_exp = gene_expression
head(data.frame(t(gene_exp)))
```

```
##            X1         X2         X3         X4        X5          X6
## V1 -0.9619334 -0.2925257  0.2587882 -1.1521320 0.1957828  0.03012394
## V2  0.4418028 -1.1392670 -0.9728448 -2.2131680 0.5933059 -0.69101430
```

```
## V3 -0.9750051  0.1958370  0.5884858 -0.8615249  0.2829921 -0.40342580
## V4  1.4175040 -1.2811210 -0.8002581  0.6309253  0.2471472 -0.72985900
## V5  0.8188148 -0.2514393 -1.8203980  0.9517719  1.9786680 -0.36409860
## V6  0.3162937  2.5119970 -2.0589240 -1.1657240 -0.8710180  1.12534900
##             X7         X8         X9        X10        X11        X12
## V1  0.08541773  1.1166100 -1.2188570  1.2673690 -0.7447816 -1.1312190
## V2 -1.11305400  1.3417000 -1.2772790 -0.9183490  0.7960816  0.5335755
## V3 -0.67796880  0.1032784 -0.5589246 -1.2535000  0.5785438  1.4864370
## V4 -0.56292900  0.3909632 -1.3444930 -1.0671140 -1.1282110  0.4258659
## V5  0.93819440 -1.9274910  1.1591150 -0.2406378 -1.0709430  0.0888090
## V6  0.11880910  0.4516918 -1.5010440  1.1638890 -0.4067883  0.7972244
##             X13        X14        X15        X16          X17        X18
## V1 -0.7163585  0.25265240  0.15204570 -0.3076564 -0.953017300 -0.6482428
## V2  1.2288400 -0.80692600  1.35350900 -1.7647020 -0.005029033  0.4271488
## V3  0.7424597 -0.02958559 -1.40699500  1.0916400 -1.038571000 -1.3429060
## V4  0.6872972  1.11636500  0.03240639  0.1803118 -1.837990000  0.9419534
## V5 -1.1260010 -0.90541840 -1.62868700 -0.4416847  1.569250000 -0.1211663
## V6 -1.0550180  0.20431880 -1.13196800 -0.1568716 -0.009161650  0.9620230
##             X19        X20          X21        X22        X23         X24
## V1  1.2243140  0.1998116 -0.578483700 -0.9423007 -0.2037282 -1.66647500
## V2  0.2104399 -1.6661850  1.184845000  0.9942695  0.9267470 -0.26466990
## V3 -0.7959764  2.1244430 -0.549933300  1.9947980 -0.3763795  1.57523400
## V4  1.1248100 -0.3005170 -0.003648889  0.3709738 -0.7325388 -1.39710600
## V5  1.1553080  0.5888714  1.001318000 -0.7142866  1.8866900  0.39683010
## V6 -0.8460150 -0.1146617  2.105999000  0.8372602 -0.7453392  0.06545948
##             X25        X26        X27        X28         X29        X30
## V1 -0.48445510 -0.7410727  1.1606160  1.0120670 -0.07207847 -1.1367820
## V2 -0.23024890 -0.5044715 -1.9908390  0.7439787 -0.55686990 -0.4793098
## V3 -0.02104012 -2.1555440  1.3795230 -0.1293130  2.25004900  0.8972060
## V4  0.06435464  0.3962633 -0.6098107  0.4066957 -0.45575000 -0.6498382
## V5 -0.59093700 -1.6545070  0.4353578  0.1086210 -0.06782660 -1.4518100
## V6 -0.57092840 -0.2634981 -0.3231758 -0.2648087 -0.95359030 -1.1350710
##             X31         X32        X33        X34       X35       X36
## V1  0.90062470  0.85177040  0.7277152  0.73650210 -0.3521296 0.7055155
## V2  2.06205000  0.37778360 -0.7743120  1.38566600 -0.3824607 1.0981800
## V3 -1.21895300  0.21763380  0.1120140 -0.91505140 -1.1933990 0.8365308
## V4  0.01622336 -0.16940640 -0.9092526  1.36000300 -1.6648650 0.9107369
## V5  1.27469300  1.11291200 -0.9868791 -0.06501109  1.0769630 1.1745880
## V6  1.19447000 -0.00252012  0.3201157 -1.19094700  1.1025290 0.2985715
##             X37        X38        X39          X40       X41        X42
## V1  1.3003580  0.03825201 -0.9792838  0.793761200  0.7865069 -0.3104631
## V2  0.8196898  0.51903840 -1.3143180  0.257026900  0.3486049  0.1587819
## V3  1.4909570 -1.72751400  0.3668783  0.130600800 -1.0811150 -1.6264910
## V4  1.6847820 -0.08545033  0.4034652 -1.053028000  2.2591350  0.2745213
## V5 -1.5091130  0.12830170  1.6198630 -0.003775532 -2.1895390  0.5036481
## V6 -1.0628270  0.05773285  0.2613481 -0.272794500 -1.7603010 -0.2892407
##             X43        X44        X45          X46        X47        X48
## V1  1.6988850 -0.7945937  0.3484377 -2.265401000 -0.1622053  1.1308650
## V2 -0.1849131  1.9501980  1.7314190 -0.007093533  0.5052937  1.3759370
## V3  0.6093191 -0.3837559  0.2647067 -1.075917000 -1.7049200 -0.0900827
## V4 -1.4088060 -1.2348060 -0.1200581  1.561510000 -0.4455920 -0.8677340
## V5 -1.6956000  0.7846392  1.3021910 -0.755885500 -1.2869090 -1.0437160
## V6 -0.2335841  1.5307520 -0.7390133  0.782638400 -0.5288534  0.4844302
##             X49        X50        X51        X52        X53        X54
```

```
## V1 -0.4555460 -0.89916630  0.7268389 -0.8094409  0.2670851 -1.73726400
## V2  2.1665240  0.20454060 -0.3685858  1.7146650  0.8315306 -1.19672700
## V3  0.2846788 -0.44306170 -0.2991162 -0.7739517  0.2410810 -0.33758430
## V4  0.1486715 -0.02878769 -2.1404600  0.1123969 -0.0127944  0.04702676
## V5 -0.2973123  0.92165180 -1.1371960 -0.0648509  0.9892672 -1.10023300
## V6 -1.2627040  0.54301250  2.8763590  0.1866578 -0.2051988 -0.64072270
##            X55        X56         X57        X58         X59        X60
## V1 -1.4114250 -0.4535512 -1.03549100  1.3621430  0.91745670 -0.7851422
## V2 -0.2008006  1.3652010 -0.02816577 -1.7599680 -0.60092050  0.6210088
## V3  1.2491340  0.7086719  1.31850700  3.4585510 -0.19494920  0.4287486
## V4 -0.1636117 -1.1190370  0.28884440  0.9281802 -2.42395200  1.5578780
## V5  0.9485218  0.9693492  0.08228936  0.8664452  1.37227500  2.2344000
## V6 -1.0486260  0.1893329  0.61254480 -0.7754966  0.03994449  0.2251232
##           X61        X62         X63         X64        X65        X66
## V1  0.5735182  0.9181962  0.25628730  0.35196660  1.1743370 -0.4808464
## V2 -1.4049840 -1.9040840 -0.49077710 -0.53516840 -0.3543507 -1.8018610
## V3 -1.4925220 -1.6837010 -0.51787610 -0.14914790 -1.0865270  0.1936917
## V4 -0.5461443 -1.4602030 -0.89908060  0.06304456 -1.5281560  0.2055542
## V5 -1.1168170 -0.6797440 -1.53350700 -2.89261400 -0.3418698  0.9418432
## V6  0.5580270  0.1835040 -0.09222776  0.43571790  0.3996504  0.9158531
##           X67        X68        X69         X70        X71        X72
## V1 -0.4188297  0.9551128 -1.2890070  0.18619740 -0.0313255  0.4670973
## V2 -0.1418732 -1.5314570  0.7477240 -0.92686240  0.5263548 -0.4034640
## V3  0.3299104  0.4924870  2.1759870 -1.25683000  0.9514733  0.3939671
## V4  0.5758395  1.1781920 -0.7784894 -0.95757000  1.0815600 -1.6184980
## V5 -0.3478696 -0.5770620  0.7215801  0.29784280 -0.2526780  0.9781416
## V6 -1.1790890 -0.3400896  0.9899070 -0.04238906  1.0584370 -0.5118950
##            X73         X74        X75        X76        X77        X78
## V1  1.02419800  0.26735850  0.2318261  0.7475925  1.2170690  0.3833583
## V2 -0.03102188  0.52402870  0.9513305  1.4340410  1.2118140 -0.6788450
## V3 -0.32433080 -0.92529480  0.2995610  1.2164600  1.5856290  0.4057452
## V4  0.17423530 -0.09508596  1.9908410  0.3648151 -1.1398580 -1.1499270
## V5 -0.58038460 -0.58170830 -1.5710630 -0.6049416  1.3319010 -1.0145910
## V6 -0.03142884  1.05488000  0.4179885 -1.2640020  0.1641926 -0.1634669
##           X79        X80        X81        X82         X83        X84
## V1 -0.9880528 -0.1568529  1.7355350 -0.3522983  0.68864000  1.2244060
## V2 -3.2404900  0.7563530  2.1365810  1.0324020 -2.35089800  0.5101664
## V3 -0.7834583 -1.2806740 -2.3165160  0.1149000 -0.08096164 -0.5867379
## V4 -1.9433230  0.1490092  1.0647530  0.4098704 -0.40001420 -1.1497010
## V5  0.5031153 -1.8229000  0.5401993 -0.5840188  0.44569040  1.1858170
## V6  0.2736059 -1.5335130  0.9354490 -1.8014130 -0.24748690 -0.4182793
##           X85          X86        X87        X88         X89        X90
## V1  0.7942963 -0.006402398  0.2191506 -0.8864638  0.43976030 -0.8863898
## V2  1.3862890 -1.803563000 -1.5265380 -1.2968710 -1.76747100  0.8653283
## V3  1.3564550  0.498294300  0.4036648  0.5480717  0.69000350 -0.2874406
## V4 -0.9410000  0.132999300 -0.4920219  0.1117493 -0.05991141 -0.9795920
## V5  1.2413720 -0.061116390 -0.8418074 -1.6037720  1.06274300  0.2365342
## V6  0.2047216 -1.358270000 -1.2724030  0.8922390  1.11962200 -1.6155420
##           X91        X92        X93         X94        X95          X96
## V1 -0.8538185 -0.9899943 -0.6508777  1.05394700 -0.3908780 -0.070586400
## V2 -0.7724048 -0.1108457 -1.2336450  1.33016200 -0.8954067 -0.001279981
## V3 -0.5189004  1.2504620 -0.2117770  0.13995980  0.9855528  0.509503300
## V4  0.4825913 -0.3595148  0.9955770 -0.09369478 -0.8902725 -0.435388500
## V5 -1.5292930 -0.2550374  0.6582017 -0.28517250  0.8590990  1.143988000
```

```
## V6   0.4822002   0.1779343   1.4186230  -0.73515100   0.2762970   0.190480600
##               X97          X98          X99         X100         X101         X102
## V1  -0.4620508   0.5409083   0.93163500  -0.2092743   0.6173500  -0.40507750
## V2   1.0757590   0.2767932   2.23140100  -0.3168590  -0.2957037  -0.28130540
## V3  -0.5419024   1.3830800  -0.32775510  -0.4726869  -0.9065880   0.78888650
## V4  -1.2461570   1.0898610  -0.48744410   0.6866396  -0.0684791  -0.46905400
## V5  -0.7997656   0.3928749  -0.03855792  -0.1208278   0.3232668  -0.22437800
## V6  -0.4432717  -0.9223270   1.39441400  -1.2949840   0.5998703   0.03136775
##               X103         X104         X105         X106         X107         X108
## V1   1.05310400   0.60228420   1.0174610   0.6081673   0.20673600  -1.897727000
## V2   0.04116174  -0.71458950   2.5235860   0.2333733   0.05322626   0.007317735
## V3   0.22819240   0.45257110  -1.8601430   0.3917267  -1.41747600  -0.180270500
## V4   0.44929780   0.03740589   1.2075540   0.2961469   0.41207790   0.562098800
## V5  -0.50357200  -1.31198500   0.5197868   0.4956351  -0.03756048   0.858883800
## V6   1.81875000  -0.29274310   0.9539449   0.5256714   1.20084900  -0.692958100
##               X109         X110         X111         X112         X113         X114
## V1  -0.6825828   0.48133840  -0.46303100  -0.2797417  -0.41369010   1.61876700
## V2  -1.8961240   0.48109130   0.33200650  -0.6454991  -0.36926990  -0.41602600
## V3   0.4983746   0.60260130  -0.48203080  -0.1262490   0.87453700  -1.24503200
## V4   0.5236366  -0.08827406  -1.19981400   0.6185945  -0.96451720   0.85109940
## V5   0.4154162   0.77529580  -0.05204835   0.8967021   0.07418931   0.05677277
## V6   2.1722120   0.02746565   0.18247120   0.1520082  -1.91515700   0.55562350
##               X115         X116         X117         X118         X119         X120
## V1  -0.7210557  -0.4530932   0.01425716   0.21576460   0.1888702  -0.05014849
## V2   0.5756340  -0.8793241  -0.80955900   0.49266980  -0.8038873  -0.34468900
## V3  -1.3491260   1.0169440  -1.82499900  -0.43763530  -1.3785990  -1.82193300
## V4  -1.0541750  -0.5175424  -0.31215420  -1.57004500   2.0073630  -0.65701850
## V5   0.1820294   0.8278325  -0.13969690   0.36427890   0.9059131  -0.90735480
## V6  -1.6512530   0.1413645   1.43152800  -0.01345597  -1.1657840  -0.12280930
##               X121         X122         X123         X124         X125         X126
## V1  -1.4954200   0.3678378   0.517144000  -0.4843355   0.6748556  -0.7624486
## V2   0.1926790   0.4648077  -0.930604400   1.3390210   0.1679515  -1.2953840
## V3  -0.7954872  -0.5672741   1.293843000   1.1592320   0.2204652  -0.8536304
## V4  -1.7215390   0.3921659   1.685983000   0.1726345  -1.9600530   0.4156749
## V5   1.1434090  -1.0474820   0.003567543  -0.7305390  -0.8770991  -0.9429300
## V6   0.5228269  -0.7645738   0.155037600   0.3167954   0.3694030  -0.6483243
##               X127         X128         X129         X130         X131         X132
## V1   0.3860738  -0.6640033  -1.72434400   1.1563190   0.6935066   0.1431564
## V2  -0.8979096  -0.1991474   0.01347128  -0.1953723  -2.0050820   0.2780552
## V3   1.3390690  -1.3226000  -0.37137370  -0.3137379  -1.2637410   0.8111377
## V4  -0.8714537   0.8115584  -1.03360100  -0.6197514  -1.2648330  -0.3879487
## V5   0.7661303  -2.7576300  -0.69335700   1.5795720   0.5000376   0.3151851
## V6   0.5713007   0.2472737  -1.51559300   2.2265150  -0.8660515   0.5985495
##               X133         X134         X135         X136         X137         X138
## V1   1.4928140  -1.6321530   0.1278460  -2.4036640   1.4439280  -0.8788930
## V2  -0.7009808   0.6449347   0.6981556   0.9972710  -1.6432840  -0.8007625
## V3  -1.5669140  -1.4504440   0.9797361   0.3701666   0.1733165  -0.2735915
## V4  -1.1228460   0.8806802   1.9065370   0.2442495  -1.1353730   1.4046550
## V5   0.7065280   1.4578610  -0.6694734  -1.1484960  -0.6382386  -0.9991880
## V6  -0.1737050   0.0325864  -0.1017262   1.4844910   1.4493400  -0.5752857
##               X139         X140         X141         X142         X143         X144
## V1  -1.30643800  -0.8771990  -1.1643800  -1.9823480  -0.98994420  -0.1516846
## V2   0.24273400  -0.1057190  -0.1790229  -1.4190720   0.63089030   0.4198107
## V3   0.04525164   1.5708060  -0.6983562  -0.4272629   0.08298622   1.4543630
```

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## V4 -0.22591920  0.1305281 -1.1798270 -0.8287056 -1.15548400  1.1039100
## V5 -1.38500300  0.7151270  0.2393708  0.1164812 -1.02590500 -2.0844040
## V6  0.58918850  0.4185815  0.4925463  1.0144310 -0.62424140  0.2209360
##           X145       X146       X147       X148       X149       X150
## V1  0.9125068  0.4076698 -1.24218400 -0.6426944  1.9302440  0.41019940
## V2 -0.7764052 -0.7766187 -1.02016800  0.1675418 -1.2476040 -1.25803300
## V3  1.2653660 -0.3608104 -0.97668930 -0.3899430  0.4179004  0.09682857
## V4 -1.4156660  1.3297000 -0.30290130 -0.1316080 -1.7035420  1.38522200
## V5 -1.8122210 -0.3242543 -0.89641940  0.7556421  1.8048030 -1.55047800
## V6 -0.0910377 -1.2079560  0.01371212 -0.4980104  0.0460844  0.37290590
##           X151        X152        X153       X154       X155       X156
## V1 -1.2913490  2.635045000  0.48707230  0.8538923  1.0884430  0.2260140
## V2  0.2400204  0.353725500 -0.48136910 -1.8481330 -0.5639334 -0.9702911
## V3  0.2339679 -0.003517701 -2.09142600  0.1092008 -1.1144220 -0.1114619
## V4  2.3391040  0.837357300 -1.78501500  0.7459048 -2.0678390  0.2407248
## V5 -0.1006846  0.412195300 -0.04477948  0.9657262  1.8636050 -1.8330800
## V6  0.9191041 -0.236758500 -2.33938700 -2.0791570  0.1913018  0.2160934
##           X157       X158        X159       X160       X161       X162
## V1  0.06819884 -0.9848155 -1.31085400  2.46405500 -0.6654281  0.9128626
## V2 -0.39756060  0.4190441 -0.01006552  0.35446430 -0.5498684  0.7311192
## V3 -0.14425940 -1.0055710  0.59858450  0.04301727  0.3614288  0.1363050
## V4  0.15093980  2.0494320  0.60241900 -0.22885420  0.6126973 -0.1312517
## V5  0.40602520 -0.7067314  1.92739200  1.56517100 -0.3946728  0.9314308
## V6  0.63376110  0.4215432 -1.21821300  0.71239570 -0.7512104  1.1823030
##           X163        X164       X165       X166       X167        X168
## V1  0.9646642  1.60800300  1.8354000  0.7024627  1.2178540 -1.12365400
## V2  0.3131712  1.54005300  0.3020303 -0.4090037  0.4545041  0.05235751
## V3 -1.7615780  0.62316040 -0.2490861  0.0931224  0.0229701 -1.18183300
## V4  0.4665767  0.57798890 -0.3166106  0.5354361 -0.2197846  1.09475100
## V5 -1.0403870 -2.65494000  0.3902921  1.0485080  1.1419210 -0.70480120
## V6 -0.8570967 -0.05695113  1.1292040 -0.2947133 -0.8233888  0.89486220
##           X169       X170       X171        X172       X173       X174
## V1  0.66833010  1.2164110  0.2345754 -0.41869660  0.2382201 -0.5505882
## V2 -1.42383400  1.3386510 -0.8700074  0.06074699  1.6868450 -0.4636450
## V3  0.07072333  0.5695853  1.6760930  0.80302500 -0.8561230  0.0453635
## V4  0.70718960 -0.9456515  1.2953760  0.25916510  0.5060116 -0.4098308
## V5  1.69613000 -1.3620540 -0.2491367 -0.12232100 -0.1344096 -1.2326190
## V6  0.70765600 -1.4202680  1.1612050  0.06785694 -1.7542550 -0.6405494
##           X175       X176        X177       X178       X179       X180
## V1 -0.5006028  1.1638970  2.15553700 -1.70915700 -1.6008230 -1.0385530
## V2  0.8789056 -1.2026520  1.04723800  0.51774040 -0.6395313 -1.1219100
## V3 -0.2268530 -1.3320630 -0.01731969  0.01618453  0.6276257  0.1584738
## V4  0.4353847 -0.1634118  0.36454670  1.52512500  0.4550406 -1.5683790
## V5  0.4087219 -0.5481565  0.81250140  0.26735680  1.2525150 -0.1881953
## V6 -1.1429400 -0.2104632  0.46736480  0.34131480  0.2800277  0.1845084
##           X181        X182       X183       X184       X185       X186
## V1  0.3230942 -0.88884720  0.3936790  0.23654150 -0.4304968 -0.5479331
## V2 -2.2410490  1.50202600 -1.1055610  0.40788130 -0.4890390 -1.3027890
## V3  1.4929800  1.63485800 -1.2592600 -0.24929270  0.9591378  0.5413121
## V4 -1.5286610 -1.15030700 -1.2018340  0.99323330 -0.2236106 -0.2864519
## V5  1.4145550 -0.70014050  0.2640331 -0.07365947  0.9538490 -0.6290312
## V6  1.2747880 -0.09585824 -1.2253620  1.30803300  0.8227360 -0.5796001
##           X187       X188       X189       X190       X191        X192
## V1 -1.3222520  0.6821267  2.1627890 -0.4166696 -1.3573180 -0.67122650
```

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## V2 -1.0216100  0.3571815 -0.6410703 -1.0150700 -0.1112854 -0.19208350
## V3  1.1307880 -0.4524806 -1.0477440 -1.1722400 -0.8142653 -0.31416140
## V4  0.8840154  0.5370022  0.7846147 -1.1445630  0.6577703 -0.36567650
## V5 -0.1265069  0.9402107 -1.9146730 -0.1169741 -0.6252379 -0.05588486
## V6  0.5179410 -1.1952850  0.5119641  0.1889160  0.3592326  0.84711950
##          X193       X194       X195       X196        X197       X198
## V1  0.6499182  0.7712912  2.6766320 -1.3708710  0.05775915 -0.1970675
## V2 -0.9098442  0.3896078 -0.2300779  0.1880270 -2.02523200 -1.4142510
## V3  0.5482280 -0.5860856 -1.7803350  1.6570300 -0.37748300  0.8145690
## V4 -0.5705280 -0.3272804 -1.6177600  0.8657194 -0.49447370 -2.2809550
## V5  2.0218380  0.3362576 -0.8811360  0.3408286 -0.58955000  0.2839716
## V6 -0.7952944 -2.0204710 -0.4292995  0.1155440 -0.77952710  1.0952000
##          X199        X200       X201       X202       X203       X204
## V1 -1.26151800 -0.66244260 -1.3323520  0.2773236  1.0855340 -1.6427180
## V2  0.74006570  0.51105520 -0.6017003  0.8644934 -0.2686663  0.5592865
## V3  0.74347440 -0.09654323  2.5024620  0.4667759 -0.5246236  0.7846686
## V4 -0.71055840 -2.52758200 -2.3787080 -0.8342667  0.1862210  1.2123780
## V5 -0.02389975  1.67695300 -1.0655620  0.1913468  0.7720300 -1.5651940
## V6 -0.14755910  0.98706780 -0.2244157 -1.2561530 -0.8643795  0.4788151
##          X205        X206        X207        X208       X209        X210
## V1 -0.4571803  1.46117200 -1.672533000  1.56109600 -1.4933710 -1.18244900
## V2  0.2349121 -0.02596937 -0.978812000  0.04057291 -0.1019076  0.85845760
## V3 -0.9840732  0.40642220 -0.173296700  1.00210500  0.3752330  0.04294817
## V4 -0.1309608  0.90291440  1.137603000 -0.46751350  0.5960912  0.65747480
## V5  0.7295496 -0.03023288  0.213379100 -1.15158100 -0.3418922 -1.57094400
## V6 -0.1282879  0.38416310  0.003405224 -0.05511658 -0.7187573 -1.05775300
##          X211       X212       X213        X214        X215       X216
## V1 -0.35658690 -0.9156401  0.8494855 -0.48960720  0.72798320 -0.6193132
## V2 -0.93088360  0.5304504 -0.5882823 -0.17093680  1.36979400  0.2180647
## V3  0.04077397 -0.2333500  0.5097610 -0.45826410  0.01695745 -0.8412001
## V4  0.40643550  1.4791270  0.8379988  0.83054840  0.83370520 -0.1347093
## V5  0.84786230  0.5338417 -1.8441160  0.05790073 -1.49370100 -1.7692170
## V6  0.60537650 -1.4906520  0.3710475 -1.82320800  0.73242350 -1.0501610
##          X217       X218        X219        X220       X221       X222
## V1  0.34373550  1.8136250  1.45322700  0.41242940  0.2019760  1.6843470
## V2  0.05119915  0.5308562  1.01660400  0.54781750 -0.1102824  0.2162949
## V3  0.07174996  0.3594092 -1.08166800 -0.56172980 -0.3523520 -0.5164344
## V4  1.76303400 -0.5225572  1.45863600  1.70080700  0.1893671  0.3463855
## V5 -0.30640240  0.0795156  0.02276871 -0.04721615 -1.6672890  1.8403310
## V6  0.02341662 -0.2246502 -0.35530030 -0.02599257  0.5850439  0.3998404
##          X223        X224       X225       X226        X227       X228
## V1 -0.9648245  0.66215520 -0.5533440  2.4442050 -0.30462980 -0.1173699
## V2 -1.2344210  0.95606100  0.4651511  0.5596903  0.80426040 -1.2353420
## V3 -0.7926150  0.85186230 -1.4618940 -0.4236916  0.06669258  1.6683560
## V4 -0.7034260 -0.74435630  0.1585469 -0.1020359 -1.70678400  1.5135010
## V5 -1.0081060  0.07277533  0.3585296 -1.1675420  0.72308600 -1.2271920
## V6  0.5379084 -1.93835300 -0.0207608 -2.0588870  0.88895410 -0.6747004
##          X229       X230       X231        X232        X233       X234
## V1  0.96942880  0.5909396 -1.2089580 -0.30192840  0.40126050 -1.6883680
## V2 -0.28879390  1.0062870 -0.1287105 -0.05506845 -0.27685800  0.3425124
## V3  0.05499602 -0.9057176  0.9834239 -0.03812302  0.05022081 -0.9258410
## V4 -0.55739840  0.5517037  1.1544840  0.19203890 -1.51447800 -1.3288930
## V5  0.49200580  1.5337940 -1.7538840  0.70690200  0.36964930  0.3851013
## V6 -0.61397970 -0.7493878  0.8891834 -0.65683680 -0.24172990  1.1565550
```

```
##            X235       X236        X237        X238       X239       X240
## V1  -0.7129925 -0.2543003 -1.00156700 -0.92410040  0.4402456 -0.3268166
## V2   1.4552800 -0.7595871  0.03912816 -0.06762163 -0.1193565 -0.7336143
## V3  -0.3517457 -1.5443590 -1.78031600  0.79060980 -0.1780753  0.2364432
## V4   0.4161088  0.6361211 -0.67748650 -0.32176600 -0.3934993 -1.6906880
## V5   0.5630007 -0.6944060 -0.22400670 -1.28114600  0.9088429 -0.4288735
## V6  -0.3543722  0.8790678 -1.83328800 -1.62774600  0.6711114 -1.5302670
##            X241       X242       X243        X244       X245       X246
## V1  -0.27447590  1.2841420  0.3042118  1.27368300  1.0334480  0.6653842
## V2  -1.06721800 -1.2034860 -0.3444994 -0.02185040  1.7219700  0.6426474
## V3   1.80712600 -2.3784280 -1.2018370 -0.07517866 -0.1246015 -0.3938512
## V4  -0.03216948 -1.4382780 -0.5470993 -0.59724890 -1.1365630  1.0691640
## V5  -0.09212174 -1.0920240 -0.2800943  0.98313080  0.4451820 -1.1825490
## V6   0.37773190  0.2421823 -0.4333329  0.86695300 -1.1727040  0.7408518
##            X247       X248       X249       X250        X251       X252
## V1  -0.3133281  0.4640767  0.4786059  2.5362360 -0.53506310  1.3681060
## V2  -0.6237715 -1.9712850  0.5027203 -0.5343934 -0.07051842  0.7692553
## V3   1.8457240  1.0599840  1.8255240 -0.5880530  0.75329020  0.4051955
## V4  -0.8607243 -0.3014736  0.5172248  0.5444529 -1.18906800  0.4125468
## V5  -0.4749345  1.1182920 -0.4271225  1.2741870  1.43638300 -1.8910110
## V6   0.2679777  0.9674240 -1.4113240 -1.1405270 -1.28334600 -1.0527970
##            X253        X254        X255        X256        X257       X258
## V1   0.1418443 -0.78281500  1.88151900  0.55292290 -1.84485700 -0.5503079
## V2   1.1723740 -0.21792890 -0.63006460  0.87818210  0.08528255 -1.0881580
## V3   1.1516240  0.05853952 -0.74814230  0.13359300  0.80610460 -0.9066625
## V4  -1.5196510 -0.75457980 -0.08250428 -0.09731776  0.23286710 -0.8228929
## V5   1.1011060  0.41506570  1.09693100  0.91541980  1.35730600  0.0109238
## V6   1.4901900  1.02270000  2.31283600  0.87644720  0.22074080 -0.5398211
##            X259       X260       X261       X262       X263       X264
## V1  -0.02601476 -2.1592500  1.77060000  0.7604735 -0.4845428 -0.4727639
## V2   0.43202800  2.1377990  0.06206283 -0.7067467 -1.0600450  0.8942918
## V3  -2.12659600 -1.1431540  1.46708100  0.5369896  2.0626930  0.1590567
## V4  -0.89568700  0.2701725  0.31386530 -0.0746449  0.3868924  1.1871890
## V5  -0.53804480  1.6497980 -0.49419120  0.4874801  1.3949000  2.3248850
## V6   1.23188300 -2.1020000 -0.40810600 -1.0270330 -0.5130171  1.6086860
##            X265        X266       X267        X268        X269        X270
## V1   0.33167210 -0.97230080 -0.5312442  0.06203674  0.10353000  1.169799000
## V2  -0.05484585 -0.23914980 -0.8948782 -0.33837440 -0.01032162 -0.032070810
## V3  -0.19680350  1.06479700 -0.7260831 -0.55725480  0.79324820  0.000556495
## V4   0.56705140 -0.29233850  1.4670220  1.82601800 -1.22490600  3.222134000
## V5  -0.62575470  0.39524000 -1.9555940  0.36909190  0.40127840 -0.806898000
## V6   0.62646790 -0.05898864  0.8955435 -1.29057100  0.10331280  2.201916000
##            X271       X272       X273        X274       X275       X276
## V1   0.39418920 -0.1341024 -0.3798756  1.27866700 -0.6993323  0.56425050
## V2   0.09224979 -0.9399536  0.3990168 -1.00947500  0.0123165 -1.46827500
## V3  -0.30988800 -0.3694581 -1.0342370 -0.08989707 -0.7189023  0.52185770
## V4  -1.28198700 -0.7691559  2.0029780  0.95878780  0.2171809 -0.90865010
## V5   0.34971550 -0.2001564 -0.8957085 -0.62475510 -0.3601385  0.01782081
## V6  -2.05532600  0.8073041  1.4083790  0.47613020  0.9555429  1.42774400
##            X277       X278       X279       X280        X281        X282
## V1   0.3029420  0.2154412  0.9210030 -0.8875519 -0.002893014 -0.19517360
## V2  -1.9902220 -0.8175837 -0.7066146 -1.9269280 -2.053876000  1.61617400
## V3  -0.6384750 -0.9893796  0.3786798  0.8298065 -0.522334900  0.30470280
## V4  -1.1826170  0.8713705 -0.1948007  0.3391615  0.428839100 -0.53888670
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## V5 -0.5113048 -1.7017180  1.1245160 -1.2430180 -1.957618000 -2.37565400
## V6 -0.4761818  1.4728750  0.9346260  0.3399947  0.334580500  0.05547521
##              X283         X284         X285         X286         X287         X288
## V1 -2.23467900  0.47832300  1.6226920 -1.1960740  1.6595090  0.2045454
## V2  0.58795650 -0.82029580  1.1023720 -0.2393882  0.6094410  0.4640065
## V3 -0.71570530  1.79216400  0.2873148 -0.5667964  1.6603520  0.1622794
## V4 -1.48324300 -1.94745600  1.4122420 -0.2957817  2.0989240  0.1370094
## V5 -1.25420200 -0.01859973  0.6726376 -0.3082732 -0.3575282  0.7190656
## V6  0.05020565 -0.58276670 -1.6515470 -0.3272143 -1.2338600 -0.4949269
##              X289         X290         X291         X292         X293         X294
## V1  1.3859760 -0.85781180  0.6858621 -0.5816625 -0.6845529  0.37847290
## V2  0.5605362  0.25204420 -1.1996800 -0.8787443 -1.1682300  1.53977600
## V3 -1.1043860 -0.03362419  0.6077916 -0.2712148  0.4963991 -1.01655200
## V4 -0.6797990  0.93472830 -0.3002944  0.4720448 -1.8970770  0.05360819
## V5 -0.4067890  1.19198500 -1.2564090 -0.0822743  1.3297180 -0.70241220
## V6 -0.3713676 -0.46753590  2.2809630 -0.6687052 -0.1408628  0.16686200
##              X295         X296         X297         X298         X299         X300
## V1  1.3129570  0.31402150 -0.50218520  1.25211500 -0.3031554  0.81123190
## V2 -0.2851237 -0.04079823 -0.01625638 -0.04848325 -0.6334486  0.03624687
## V3  1.7132700  0.60523810  0.13533020  1.20943600 -1.8361040 -0.23000400
## V4 -0.8800088 -0.68336590  0.02146303  1.51612200 -0.8109516 -1.53911600
## V5  0.0212486  1.76061900  0.75506570 -0.14839040  0.4497824  0.72197660
## V6  0.6291781 -0.28606420  0.60633880 -0.60823670  0.1556035  0.65835190
##              X301         X302         X303         X304         X305         X306
## V1 -1.2593790  0.7629873 -0.80345350 -0.3793839 -1.3664940  3.5192990
## V2  0.5749691  0.1498169  1.69310700  1.2327420  0.1796436 -0.5731206
## V3  0.5974023 -1.7851450  0.89073240 -0.6564379  1.3591110  1.8899900
## V4  0.7879319  1.6200830  0.49166450 -1.7115360 -0.9260267 -1.3681350
## V5  1.0901100 -1.3925720  0.00664823 -0.7485457 -0.7594184 -1.6342910
## V6 -1.4646660  0.3683140 -1.04663000  0.5405553  0.1106738  1.5884050
##              X307         X308         X309         X310         X311         X312
## V1 -1.2131480  0.1943573  0.09436564 -0.8784914  0.2604359  0.6577070
## V2 -0.2280570 -1.1320420  1.08431000  0.3861999 -0.6843459  0.4163541
## V3 -0.2789920 -1.5795350  1.03163000 -0.2453823  0.7445604  0.7556631
## V4  0.2347019  1.1352010  0.84735820  0.9163824  2.7361650 -1.2690780
## V5  0.9370522 -1.4999160 -1.16782800 -1.5347530 -0.9041653 -0.2681330
## V6  1.8188900  1.8141080 -0.21258120 -0.2532264  0.2495322  1.2734230
##              X313         X314         X315         X316         X317         X318
## V1  0.7657965 -2.23536100 -0.9028261 -0.1663838 -0.06549942 -0.5056380
## V2 -0.2495510  0.27414130 -0.6857168  0.5706372  1.12713500  0.1495988
## V3  1.3372350 -1.30251300 -0.9235105 -1.3347820 -0.06956319  2.3455950
## V4  1.2795460  0.08352601 -0.1092170  0.3106137 -1.61046300  0.8000875
## V5 -1.4788020 -0.41512930 -0.3754292 -0.6649979  0.10709030 -1.8571880
## V6 -0.8004672 -0.42184550 -0.1152687 -2.6679550 -0.38207810 -0.4997538
##              X319         X320         X321         X322         X323         X324
## V1  0.52028670  0.03570148 -0.9106896  0.9143431 -0.3367091  0.09476286
## V2 -1.33768500 -1.67320300  1.2585620 -0.1120024 -0.5533295  1.29870800
## V3  0.13462640  1.59479500  1.8022800  1.6441210  0.6297007 -1.52857900
## V4  0.01188452  1.69739400 -2.1383550  0.2508170 -0.4464984 -0.51197980
## V5  0.36841520 -1.14019300  0.3238824  0.4526929 -0.5289564  0.27116420
## V6  0.15282940 -0.05444751  1.8323840  0.6469526 -1.0467030  0.67739050
##              X325         X326         X327         X328         X329         X330
## V1  2.1272210  0.1965845 -0.96760470 -0.2848972  0.16444600 -1.11187100
## V2 -0.1915555  0.4387526  0.66025780  0.5497897 -0.08515008 -0.41519390
```

```
## V3 -1.0257340 -0.9280888  1.02915700  0.6170152 -0.64596450 -0.74656670
## V4 -2.1014010  0.6618323  0.12318710  1.0366070  1.45092000  0.07824663
## V5 -0.3960868 -0.8867958 -0.08031208 -0.3621152 -0.35766230  0.17883070
## V6  0.5831273 -2.5969410 -1.20297600 -0.7030547 -1.53327300 -0.90457060
##           X331       X332       X333       X334       X335       X336
## V1 -1.9293650 -0.4702609  0.5343878 -0.92105000  0.6843744  1.5220790
## V2 -2.0349970 -0.4524589  0.4871902 -0.93276870  1.3965300  0.3169721
## V3 -1.0637490  0.3324192  0.2587047 -0.79406430 -1.3544710 -0.3719902
## V4 -0.8773658  0.2361255 -1.5995550  0.98557780  0.2685025 -2.1195970
## V5  1.8047820 -0.7090793 -1.9708220 -1.26944800 -1.4125840  0.1915366
## V6  0.5061262 -0.2979953 -1.1991100  0.01513415  0.5298037 -0.6923434
##           X337       X338       X339       X340       X341       X342       X343
## V1  0.1946133  1.6972250  0.2780682 -0.5710105 -0.1578794  2.4883390  1.0713690
## V2  0.9011202 -0.4891004 -0.7275357 -0.5379159  0.2818978 -0.6989417 -2.2614270
## V3 -0.6815565 -0.1795790  0.4197927 -1.2405960  0.1625803  0.5815245 -0.2139025
## V4 -1.7076330 -1.2426100 -1.7106840 -0.4813206 -0.9986958  0.3969532 -0.4379595
## V5  1.8375160 -0.8733917  0.3202050  0.3563706  1.0123350  0.7718480 -1.0966790
## V6  0.2321325 -0.2594792  0.4603884 -2.0540550 -0.3638829  0.8751343 -0.7879652
##           X344       X345       X346        X347       X348       X349
## V1  0.34221910  0.2213274  0.06836274  0.30742160  1.5086340  0.61584220
## V2  0.29811940 -0.7581436  0.18629020 -0.08389878  1.2055090  0.95458680
## V3 -1.96355100  0.5113012 -1.38746100  2.50007700 -0.1287118  0.09942238
## V4  0.94239310  1.2075250 -0.33181930 -0.17500900 -0.5341338 -1.36841400
## V5  0.02735992  0.1244414 -0.21257180 -0.79533700  0.8148195  0.56483300
## V6  0.50511750 -0.7209427 -1.36358900 -0.94813240  0.5002368 -1.45654200
##           X350        X351        X352       X353        X354        X355
## V1  0.08981202 -2.39845300  0.04780729  0.3361434  0.43484570  0.21102360
## V2  1.54913100 -0.08340362  0.61356430 -1.0012690  0.13549570 -0.41512630
## V3 -1.00729000 -0.93991830  1.57023400  0.2829105  0.03467589  0.62329740
## V4  0.33215090 -0.95409340 -0.86626240  0.0902038 -0.41621510 -1.46881200
## V5 -0.71286880 -0.62305820 -1.02147600  0.4743566 -0.17487740  0.38394710
## V6  0.78219230  0.07226298  0.43916210  2.1374820 -0.14985550 -0.03654078
##          X356       X357       X358       X359       X360       X361
## V1  0.1107935 -1.9933230  1.5050160  0.3541571  0.4348378  0.7682439
## V2 -0.4236961  0.8708227 -1.6605360  0.2267495  2.0514070  0.3924741
## V3  0.3161812 -0.2910256 -0.8664185 -0.3697527  1.7571830 -0.1013137
## V4  1.4855760  0.9754167  1.9614960  0.4159644  0.5821256  1.1705000
## V5  0.1062618  2.0524810 -1.5860430 -0.9494970 -0.9524536 -0.1023902
## V6 -1.0235100  1.6410690 -1.1585990 -0.2127382 -0.2349001 -0.2674624
##           X362       X363       X364       X365        X366        X367
## V1  0.40968840 -1.9766520 -1.0875790  1.0648640  2.120203000 -0.50583750
## V2 -0.27472520 -1.2210640 -0.9488587 -0.5339416  0.677345200  1.31303900
## V3  0.04289738 -0.5319394 -0.5683756  0.1139007  0.404035700  1.37693600
## V4 -0.16455230  0.9343454 -0.1761716 -0.1811301 -0.194182900 -0.05362125
## V5 -1.02517400  1.0502830  0.4511632  1.3142290 -0.881880800 -1.71250200
## V6  1.35744800  0.5664258  1.2097560  0.1591247  0.004252387 -0.86347690
##          X368       X369       X370       X371        X372       X373
## V1  0.4069871  2.0400260 -0.1939354  1.3702230 -0.51535730 -0.7182328
## V2  0.2693072  1.1382520  1.2288370  0.7140187  1.08545900  1.5342830
## V3  0.7965078 -0.6829798 -1.1591060 -2.7023980 -0.53834210 -0.1189226
## V4 -1.8089610 -0.3113817 -0.1932489 -1.6767650  0.04397581  0.5063283
## V5  1.1290490 -0.2429087  0.3672703 -0.4101741 -1.43909200 -0.5912901
## V6  1.2934520  1.6649400  0.4904138 -2.3413250  0.47110280  0.5853374
##          X374       X375        X376       X377       X378       X379
```

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## V1 -0.1772423 -0.40157650  0.829883600 -0.7858066 -0.9385012  0.41780680
## V2 -1.8398010  0.04050863 -2.150074000  0.2168707 -0.5978683 -0.07602373
## V3  1.6146560  0.31156600  0.292749300 -1.4037420 -0.6257315 -0.02420947
## V4  1.0461510  0.30779070 -0.339706600 -1.0179510  0.8265614  2.08431400
## V5 -2.4904860 -0.82977470 -0.455490300  1.7501670 -0.2344639 -1.29938300
## V6 -0.2694558 -0.05145059  0.001035457 -0.1165226 -0.4049239 -0.42704750
##             X380        X381        X382        X383        X384        X385
## V1  2.098098000 -0.4488623 -1.09528800 -0.5289728 -0.4847939  0.12775700
## V2 -1.059674000 -0.5475545  0.09534096  1.3688660  0.4420535 -2.07520600
## V3  0.006136925  0.2961484  1.01809400  2.4716370 -0.6567950  0.07741261
## V4  1.477442000  0.4635137  2.46252500  0.8572509  1.1561680  1.57798400
## V5 -0.969845100  1.7435320  0.28062840  0.5914403 -0.3236406  0.65345170
## V6  1.121721000 -0.6584189  0.57939830 -1.8483380  2.3468410 -0.79218200
##            X386       X387       X388       X389       X390        X391
## V1 -0.0746731 -0.3591621 -2.0484810 -0.7511800 -1.5813830 -0.28977610
## V2 -0.3078097 -1.4541300  1.3096160  1.4700870  1.5104290 -0.05765530
## V3 -0.8428604  0.2469591 -0.3787776  1.8662720  1.0027260  1.17785900
## V4  0.0184765 -0.2943356  1.4882260  0.3315873  0.8497741  0.65960570
## V5  0.3078103 -0.7890811 -0.3546811  0.8646890  0.8917980 -0.24806580
## V6  1.0377160  0.7380478  1.6527280  0.4198822  0.1919998  0.09016752
##            X392       X393        X394       X395        X396       X397
## V1 -0.6919545 -0.1719879 -0.22256400  0.9192413 -1.11006500 -0.6608890
## V2  0.1380716  1.0093440 -0.09618196  2.0780370  0.04750139  1.1655110
## V3 -0.5891046 -0.5855291  0.27813550 -1.2316610 -0.09925392  1.4969090
## V4 -0.8728608  0.4288513 -0.87673070 -1.7102080  0.17777230  0.9401438
## V5 -0.7054614  0.8143653  0.16709050  0.3734085  0.01880045  1.0458940
## V6 -1.5944000 -0.7423833  0.28572910 -1.7135720 -0.29211240  1.7342750
##            X398       X399       X400        X401        X402       X403
## V1 -0.8737749 -0.1328097  0.30548910  2.16384200  0.88037850  0.5051163
## V2  0.5927729 -0.3230584 -0.07651839 -1.64903800  0.06120169  0.1821510
## V3  1.4938860  1.7910110 -0.80010100  1.67551200 -1.23188900  0.2108499
## V4  0.3799607 -0.4045980 -1.97778900 -1.09972300  0.54654880 -0.7902651
## V5  0.6559319  0.9263784  1.51408100  0.05771982 -1.13596700  1.0561170
## V6 -1.2936600  0.1148715  1.21641400 -1.71798700  1.29480800 -0.6169977
##            X404       X405       X406       X407      X408       X409       X410
## V1  0.1105902 -0.2928586  0.77232000  2.1287940 0.5479680  1.5321290  1.6664310
## V2 -0.6582669 -0.8660844 -0.06337660  0.0327901 1.5322630  0.4553214 -0.5640012
## V3  0.2532629 -1.0043580  0.05610082  1.0488220 0.1660880  1.2705590  1.4342830
## V4  0.2857857  1.7964350  1.34765800 -0.1673823 1.1662630  0.2910317  0.6658383
## V5  1.1968600 -1.1874710  0.79798410 -1.2062990 0.3733508 -0.3061602  1.4156150
## V6  0.8212279  2.0856380  0.52905230 -1.0906750 0.4721306 -0.3010973  0.7646785
##            X411       X412       X413       X414       X415       X416
## V1 -0.47800000 -0.3686816 -0.5161661  0.3969377 -0.3148718 -0.6846898
## V2 -0.76494190  0.5090528  1.5682140  2.1039460  0.8333607 -1.5322490
## V3 -0.07773389  0.3264012  0.2439700 -0.1019252 -1.8215120 -0.3644836
## V4 -0.18144240 -1.6128700  0.1503741 -0.1372630 -2.1702120 -0.8129608
## V5  0.32945470  0.7742416  1.0121540 -2.1537460  0.7127172 -0.7368040
## V6 -0.50462820 -0.3152287  0.8392895  0.5394922  0.3221748  0.4206429
##            X417        X418       X419        X420       X421       X422
## V1  1.63224800 -0.68453380 -0.3130770 -0.64003270  0.6353044  0.02484837
## V2  1.08233600 -0.03639905 -0.5524403 -1.41916300  0.0779916  0.84032340
## V3 -0.63450940  1.63324000  0.5280031  2.49300100  0.7032355  0.20051690
## V4  0.06329308  1.18874000 -0.4134103 -1.96834500  0.2010990  0.33713880
## V5 -1.17019300  0.38721530  0.1654828  0.08188284  0.6982220  0.40647810
```

```
## V6 -1.35644600 -1.50125100  0.1784296 -0.41181440 -0.6187303 -1.63275100
##           X423         X424         X425        X426         X427        X428
## V1  0.2276222 -0.24606820  0.02422383 -1.2445820 -1.205975000  0.2730940
## V2 -1.7098560  0.27776000  0.74011220 -1.2731700  1.131976000  0.2353140
## V3 -0.2754812 -0.83790370  0.42449710  0.5148536 -2.015570000 -2.0370870
## V4  0.4166243 -1.84686200 -0.42824140 -0.2494561  0.699065700 -0.3257139
## V5 -0.8686266 -0.00735442  0.65007990 -1.4461420  0.003029073 -0.9236295
## V6  0.9341833  1.86242700 -1.97749500 -2.1059600  0.199934300 -1.2673570
##           X429         X430        X431        X432        X433        X434         X435
## V1  1.7294140 -1.0649510 -0.1137367 -0.6437515  0.9406484  1.9188100 -1.0060460
## V2 -0.1315957 -0.5609298  1.5933470 -0.5486336 -0.0732610  1.7021250  0.9338393
## V3  1.2592850 -0.1231664  0.4727724  0.4004884 -1.9123980  0.5758911 -1.2719060
## V4  0.3945505 -0.2801631 -0.7259151  0.3699031 -0.5407352 -0.1884987  0.8697955
## V5  0.7246028  0.6640256 -0.6433904  0.3201251  0.7114017  0.7172262  0.6598879
## V6 -2.1820020 -0.6698680  0.5870458  1.0811650 -1.0623410  1.0255220 -0.4184728
##           X436         X437        X438        X439        X440        X441
## V1  0.5470826 0.03384706 -0.3158289  0.2885446 -0.3443922  0.11663140
## V2 -0.3862376 1.00292500 -2.0881460 -0.8136357 -0.7267547  0.03470017
## V3 -1.1339270 0.17038040  0.4535735 -0.9379423 -0.7456550 -0.72396930
## V4  0.2244141 0.56946140  0.2266684  1.5788870  1.6837770  0.62320550
## V5 -0.4134207 0.39480110 -0.5724395 -0.8757870 -0.8423195 -1.42916600
## V6 -0.9386603 1.03023400 -0.7877408  1.5975220 -1.0196850  1.40493500
##           X442         X443         X444          X445         X446        X447
## V1  0.54296600  1.1439480 -1.284549000  0.919811800 -0.84609020 -1.4193040
## V2 -0.55705550 -0.2362688  0.736392700 -0.105802400 -0.75361480 -1.4418830
## V3 -0.01206468 -0.1305829  0.001629246  0.687943700 -0.02073712 -0.8397088
## V4 -0.73723520  1.3806770  1.536579000 -0.491879300  2.34497800 -0.4089882
## V5  0.28636380 -1.7118380 -0.806239600 -0.003757133 -1.39976400 -0.2536973
## V6  2.57139800 -0.5794973 -0.179727400 -0.534349700  0.76727510  1.5523350
##           X448         X449        X450         X451        X452        X453
## V1 -2.66569800  0.8351061  2.3774470  0.01396971 -1.6334240 0.4889325
## V2 -0.20707460 -0.2265637  0.3626464  0.17576590  1.4923150 0.1638138
## V3 -0.31441280 -0.5500358  2.5929540 -0.00993352  1.7580610 1.6736690
## V4 -0.70968860  0.5254089  0.3444975 -0.62538400  1.0731640 1.4897620
## V5  0.27483610 -1.6867140 -0.3593319  1.38730200 -1.0043810 0.4332542
## V6  0.02285501  0.5463822 -2.4429080 -1.44009400  0.9598972 0.3015147
##           X454         X455        X456         X457        X458        X459
## V1 -1.0191790  1.0905530 -1.1394700 -0.01572564  0.2974913  3.2005900
## V2  1.2365950 -0.7807749 -0.5744266  0.16061600 -0.1670844  0.6793468
## V3 -0.3237728 -0.2260029  0.1162977 -1.17864800 -0.1399882  0.7102212
## V4 -2.0920980  0.8389208  0.8400870 -1.22273100 -0.2747392  0.6836523
## V5 -0.2580840 -0.5177061 -0.6226180 -0.26469250 -0.6102036 -0.3375595
## V6 -0.1894557 -0.1378549  0.7674300  0.87364600  0.3316684  1.5725850
##           X460         X461        X462        X463        X464         X465
## V1  0.089244240  0.57097230  0.5286856 -0.4409048 -0.6727934  2.15431300
## V2  0.716958600  0.47541400  1.7341580  2.0690680  1.4833070  1.10317700
## V3 -0.976966000  0.75178760  0.6219122  0.4029645 -0.7190340  0.30963540
## V4 -0.516211900 -1.21744100 -0.7519543  0.5675399 -1.4553760  0.12142050
## V5  0.009141821 -0.03640826 -0.9473156  0.6318221 -0.9603362 -1.03710300
## V6 -0.826931000  0.72614520  0.1275074  0.6185451  1.7654530 -0.01826594
##           X466         X467        X468        X469         X470        X471
## V1  0.5938527 -0.3840638  0.7197833  1.7081730  1.07521600  0.7774204
## V2 -1.6248000 -0.4959813 -0.3333905 -1.5837970  0.30823760  0.7114921
## V3  1.4942030 -0.2619450 -2.5961320 -0.1126323 -0.07422831 -2.2514280
```

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## V4 -1.0601430 -0.1515672  0.1884617  2.8804510 -0.63166630 -0.7328429
## V5 -1.4763880 -0.6292733  1.0838560  0.2359088  0.38932470  1.7315600
## V6  1.4627660 -0.6776270 -0.5291840 -0.2161507 -0.48248020 -0.6870706
##             X472        X473         X474       X475       X476       X477
## V1  0.05220005 -1.08369800 -0.008856207  2.0306430 -1.1133290  1.1382720
## V2  1.69759200 -0.62600460  0.148606400 -1.0268400  0.1013403 -0.7889823
## V3  0.40557050 -0.43639930  0.938411100 -1.3663760  0.5044141 -0.1802634
## V4 -1.23228600  0.79519470  0.098035980 -0.5202770 -0.2167409 -0.5235305
## V5 -0.46176420 -0.01895182  1.048168000 -0.5497962  0.9319698 -1.7060310
## V6 -0.23411120 -1.26217400  1.557804000 -0.8276935 -0.5685562 -0.2139040
##             X478       X479       X480        X481       X482       X483
## V1 -0.61489260 -2.1935370 -1.2156860 -0.04532636  1.5988130  0.9275422
## V2  0.68824880  1.4501360 -0.4752253  1.11471000  1.0836380  3.0840000
## V3 -0.77954000  0.4667678  0.3681907  0.46234120  1.5671400  0.9200424
## V4 -0.08022832  0.1969645  1.0103000 -1.42047600 -1.0473790  1.2468530
## V5  0.55442680 -0.2672956 -0.2980582  0.70778160  0.6293519 -2.9974150
## V6  0.60494560 -0.2887226 -0.7916070  0.21024590 -0.9369790 -0.7569724
##             X484       X485       X486       X487       X488       X489
## V1  0.82928460  1.0246040 -0.4762381  1.6388680 -0.6320510 -1.3796190
## V2  0.02417865 -0.7905065 -0.1518179  0.7077418 -1.6676360  0.6135681
## V3 -0.55613480  0.2484038 -0.1295370 -2.7505660  1.1688640 -0.4714252
## V4  1.64340800 -1.1333150 -1.6954240 -0.1897425  0.1158406 -0.4470733
## V5  1.65555500  1.0653380 -1.2073010  0.8183150 -1.3377580  0.4349437
## V6 -0.55840080 -1.3961560 -0.1417092  0.3391864 -1.7105550  0.1406125
##             X490       X491       X492        X493       X494       X495
## V1 -0.25745570  1.6799730 -2.5458580  0.01207878  1.9609250 -0.3859055
## V2  0.35803980 -2.8482750  1.4902010 -1.42257500 -1.4074330 -1.0923560
## V3  0.03266384 -0.7353572  1.8181600 -0.05547517 -0.5321590 -0.9597622
## V4 -1.76976100 -1.2949520  1.4715870 -1.90951500  0.4089532  1.1194310
## V5  1.49755300  1.5159440  1.4874240 -1.48691500 -0.1724197 -0.4347803
## V6 -0.10861610 -0.4985722 -0.3166144  1.45014500 -1.4640370  1.3305350
##             X496       X497       X498       X499        X500        X501
## V1  0.91075750 -1.4481310 -1.1216150 -0.9736180 -0.067186920 -2.13984200
## V2 -0.35844320 -1.8329100  0.1753791  0.3303850  0.008946448  0.36136880
## V3  0.01657976  1.5902990 -0.5994464 -0.0587262 -0.365809600  0.14661790
## V4 -0.30673540  0.9573508  0.1065530 -1.9779110 -0.419763200 -0.09934484
## V5 -0.46169770  0.7634770  2.0826220  0.8830630 -0.932659400  0.62447850
## V6  0.51759950  0.1142917  0.1105824 -0.5063493  0.737224200  1.22132800
##             X502        X503       X504       X505       X506       X507
## V1 -1.2634790  0.08330797  0.1883251  0.1898142  0.2176154 -0.6488141
## V2  0.1597477 -1.12816300  0.6406271  0.9467517  0.6822415 -1.6112440
## V3 -1.8666330  0.42291880 -1.5962520 -0.8218646 -0.1201119  0.6594105
## V4 -0.8964610  0.93405270  0.1669182 -1.7418470 -1.6934460  0.1563209
## V5 -0.1393987  0.45860320  1.3270990  0.8953817 -0.4794835 -0.8737230
## V6  0.2076094  0.04198025 -0.6514519 -2.2654420 -1.2403830 -0.6098107
##             X508         X509       X510       X511       X512       X513
## V1  0.2547744 -0.468830000 -1.1925100  0.1212282 -1.0359330 -0.1918883
## V2 -0.4101517 -0.007731573  2.0175290  0.1276652 -0.4510595  0.1004386
## V3 -0.2135455 -0.800041900 -1.9104360 -1.8804670 -1.0849310  2.0316600
## V4  0.7933506  0.833600000  2.5787800 -0.1809626  1.8719870  1.0771280
## V5 -1.5378780  0.387908600  0.2453103 -0.2131559  0.4178902  0.1785653
## V6  1.3560690 -0.606067700  1.2385450  0.7364041  0.3204820 -0.3435443
##             X514       X515       X516       X517       X518       X519
## V1 -0.97054630 0.05777194 -2.1106930  0.9781763  0.3710217  0.7209110
```

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## V2 -0.34854440 0.73621040  0.5504080  1.2074370 -1.3909340  0.2783987
## V3  0.13458640 0.28401890 -0.4448820  2.0463520  0.3389809 -1.1435010
## V4  0.62351330 0.56712340 -0.7033314 -0.6460037 -0.6635445 -1.0628700
## V5  0.67774070 2.09814900 -0.6661024  1.2156340 -1.1092250 -0.7558977
## V6  0.02292801 1.50943200  0.7492082 -1.7372810 -1.0181680  1.0813150
##          X520       X521       X522        X523       X524        X525
## V1  1.8795950 -0.3963161  0.11032290 -0.593140900  0.41962180 -0.54573870
## V2 -0.3223220 -0.5617144 -1.01431800  1.135995000 -1.44331200 -0.69482910
## V3  0.1294422 -2.4690530 -1.68355200  0.005904135  0.97008570  0.69463540
## V4 -0.4286695 -0.5192278  0.74316690 -1.591537000  1.96292800  0.22979530
## V5 -0.1299135 -1.0097500 -0.04314962  1.142942000  0.03658388  0.06258478
## V6 -0.5202699 -0.0861674 -0.26107750 -1.223266000 -0.83833380 -0.41940210
##          X526       X527       X528        X529       X530        X531
## V1  1.16092100  0.6398178 -0.12202040  0.18464500 -0.5178060  0.06798835
## V2  1.00319400  1.7161750  0.77638740  0.78872030 -1.6281070  1.44978700
## V3  0.09923566 -0.4823343  0.07014508 -0.03637904  0.1920146 -0.90846370
## V4  0.50672530  0.8182592  0.19879030  0.40581710 -0.4010099  1.42102900
## V5 -1.52193300 -0.5118575 -0.41364200 -1.92177000 -1.9741470 -0.95556550
## V6 -0.35218370 -0.5165821 -0.20863090 -0.98173930  0.5495045  0.46509640
##          X532       X533       X534       X535       X536        X537
## V1 -0.18479720 -1.4036920  0.2297407 -0.8890813 -0.1604012 -0.24213680
## V2 -0.97090230  0.6306979  0.8793904 -1.1288440  0.2642909  0.09119491
## V3 -0.54152590 -0.2678852  0.5159698 -1.5024530 -1.2012990  1.05005200
## V4  0.45443380 -1.5111500  2.4043100 -0.5130415  0.9378763  0.84074580
## V5 -0.02482901 -2.0562570 -0.8691745 -0.2489190 -0.4366070  1.52336500
## V6  1.15817200  0.5104555  2.2187550  0.2762023 -1.9712850 -1.25711300
##          X538       X539       X540        X541       X542       X543
## V1 -0.0288372 -0.3162155 -0.4161601 -1.023896000  1.0994950  0.8177125
## V2  0.7407056  0.6239700 -0.6135165 -1.079649000  1.5156170 -1.4425200
## V3 -1.8179080  1.2038210 -2.3671230 -0.351686200  1.6314280  0.1412495
## V4  0.5018589  0.9199908  0.6249441 -0.513275100  1.1080620  0.5724393
## V5 -0.7533826 -1.5252850  1.4191440 -0.000947965 -0.1101461 -0.3567389
## V6  1.6098100  0.6752692  0.9205426 -1.481664000 -0.9054373 -2.0668090
##          X544        X545       X546       X547       X548       X549
## V1  0.1688755  0.03866544  1.0781750  0.3794273 -1.0781750  0.1887635
## V2  0.6633210  0.61555170 -0.6288748  0.3060544  0.8342304 -0.7912308
## V3  0.3128470  1.33609000 -2.0171770  0.6096412 -0.5321044  1.3641020
## V4 -0.7758223  0.73595240 -0.5885695 -1.5051480  0.8566616  0.2378845
## V5 -0.6587543 -0.63913400  1.0976220 -0.2423401  1.3788790 -1.2575400
## V6  0.7667750 -0.50076200 -1.6164080  2.4330880  0.7478653 -1.0368670
##          X550       X551       X552       X553       X554       X555
## V1  1.58313400 -0.2501123  0.3429942 -0.1247019 -0.9933148 -1.0214390
## V2 -1.09237700 -1.5532120 -1.2838460  0.3161424  2.3684080  2.1706100
## V3 -0.35978620  0.8392144 -0.6729147 -0.9820842  1.1641340  0.6665313
## V4  0.09178289 -0.1659412  0.5988490  1.4540750 -0.7171956 -0.7029480
## V5 -0.03875510  1.3315790 -0.2033978  0.2341045 -1.3688570  0.2686157
## V6  2.66298600 -0.2684837 -0.9746849 -0.2479719 -0.1721546  0.4834036
##          X556       X557       X558        X559        X560        X561
## V1  0.8406430  0.8495731  0.4696189 -1.26965500 -1.10512000 -1.89794600
## V2 -0.6423378  0.3786972  1.0244130  0.16718500  0.03918725 -0.07645971
## V3  0.6280234 -0.6563325 -0.5925556 -0.55665380 -0.10433760 -1.54412800
## V4 -0.4716626  0.8546558 -0.1158771 -0.06085357 -0.19405970 -0.97875340
## V5  0.6705049  2.4220310  2.1825780  1.25031600  0.57240920  1.01830500
## V6 -1.1358270 -0.4609647 -0.6467009  0.13894230  0.92072690 -0.24223550
```

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##             X562       X563      X564       X565       X566       X567
## V1  0.49178730 -0.7047223  1.7775760 -0.2453890 -2.1112520 -0.5853145
## V2  0.57275140  1.1011720  0.5017193 -0.3935715 -1.5208590  0.1807918
## V3 -1.64967500 -0.7704407  0.3253949 -0.6949702  1.0279280  1.4313220
## V4 -0.05463533  0.7582618  0.1825550  0.7439966  2.7559460  1.0843680
## V5 -0.59156280  0.5054007 -0.9751750  0.7338776  0.4104823  2.0942920
## V6 -1.60752300 -0.5980343 -1.3270420  1.0741090 -1.3300910 -2.0540400
##             X568       X569      X570        X571       X572      X573
## V1 -0.5179037  0.51391910 -0.6264269  0.192142600 -1.4272780  0.4009910
## V2 -0.9096948 -0.75871750  0.3039065  1.262771000  0.2006099 -0.7951840
## V3 -0.7324259 -0.67660640  0.8860777 -0.750620000  0.8846798  1.9345980
## V4 -1.7477870 -0.09645950  0.6439996 -2.086338000  0.2907691 -1.5229140
## V5  2.0871290 -0.82375080  0.1000389  1.152850000 -1.8243440  0.3229758
## V6 -0.1958721 -0.07416629  0.2518938 -0.009857253 -0.1149836  3.0695570
##             X574       X575      X576       X577       X578      X579
## V1  1.16156500  0.3263468 -1.0149820  0.1012584  0.9918933  1.1026510
## V2  0.04252877  1.4969870  0.8667814  0.7975445 -1.4542530 -0.4827237
## V3  0.61207260  0.4713294 -0.2610514  1.1299070  0.2041338  0.3505303
## V4 -0.07151252  0.5477470  1.1114850 -1.0284830  0.1993044  1.0556830
## V5 -0.36410450 -0.9549067  0.8884445 -1.6580990  1.1472120 -0.4577943
## V6  1.83875600  0.6461170 -0.9366838  0.4781078  1.1452240 -0.4273727
##            X580       X581       X582       X583        X584        X585
## V1  0.2803750 -0.4061774  1.2864130 -0.2540306 -1.144449000  0.26641090
## V2  1.4916200  0.6633941 -1.7857790 -0.9227106 -0.227761400 -1.07232300
## V3  0.9118860  0.7992924  1.1580260  2.0332650  0.127050900  0.84621370
## V4 -0.5701386  1.5111110  0.6690709  0.5137191 -0.003134947 -0.44117080
## V5 -0.1966638 -0.0707168 -0.4040265 -1.0305150 -0.960833400  0.07728886
## V6  0.3741048  0.4099875 -0.6284446  0.5834180  0.385103700 -0.62698710
##            X586        X587       X588        X589       X590       X591
## V1  0.63685570 -1.40211500 -1.5118250 -0.30119290  0.4913611 -0.8981565
## V2  0.74610250 -0.72983240  0.9430092 -0.03294286 -0.6144495 -0.6656298
## V3  0.23634910  1.01609100  0.3190095 -0.82764930 -0.9232574  1.0133620
## V4  0.05214117  0.05787203 -0.7253567  0.96147630 -0.4421924  0.6110421
## V5  0.50593420  0.02962757  0.6329926 -0.38573030  1.3471190 -1.5587000
## V6 -0.09066295 -0.46698020 -0.2990406 -0.20585230  0.1923548  1.8769870
##            X592        X593        X594       X595       X596       X597
## V1 -1.5750550 -0.50325670 -0.62290840 -0.6848300 -0.2215168  0.3880508
## V2 -0.3686736 -0.08977567  0.69964420 -1.8612150  1.1036960  0.6601621
## V3 -1.3824250  0.55384250 -0.06247139  0.8383506 -0.1997640 -0.3584096
## V4 -0.1240718 -1.17960900  1.02095200  0.9145370  1.3139550  1.3025490
## V5 -0.4990927 -0.14704500  0.76122790  2.1707980 -1.9116880  0.2021227
## V6 -0.5309078  0.10076050  1.46377300 -0.9878187 -0.7750924 -0.8182505
##            X598       X599       X600        X601       X602        X603
## V1 -0.8063165 -0.5472130  1.21349400 -0.41701320  0.1645932 -1.56962700
## V2  0.4125648 -1.6066360 -0.80502150 -1.07921800  0.6849344  1.88450500
## V3 -0.3486333 -2.0351190 -0.05612577  1.71345900  0.1596853 -1.07950500
## V4 -0.2932863  0.7778321 -1.43231100 -0.07087143  1.4432610  0.98448100
## V5 -1.7792290 -0.9655684 -0.51258250  1.08667500 -0.3357023 -0.12886200
## V6  0.3118295 -0.5209121 -1.93202700 -0.89131100  0.1390800  0.02060816
##            X604       X605       X606       X607       X608        X609
## V1  1.4666760 -1.4960850 -1.1502890 -2.0230090 -0.6816524  0.50701040
## V2 -1.4708200 -0.2220740  0.9418470  0.5649013 -1.0645810  1.45202500
## V3  2.0598970 -0.3429949  1.3852620  0.1064419 -0.5910395 -0.02762624
## V4  0.2078608  0.1955679 -1.1404600  0.6337866  0.9585200  1.18105100
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## V5   1.8847790  0.3571957 -0.5770318  0.1959914 -0.3929905  0.27102590
## V6   0.1419429 -0.0449160 -0.5140478  0.6973594 -0.7420970 -1.43025900
##            X610       X611       X612       X613       X614        X615
## V1   0.7471119  0.1759890 -0.4375236  0.9427262  0.89104500 -1.69178600
## V2  -1.4442300 -0.7540000  1.0500130  0.7126508 -0.89986950 -0.02041056
## V3   0.2181389 -0.1213729 -1.1118810  0.2007320  0.03208482 -1.35753700
## V4  -2.3914590 -0.1163748  0.4419058  1.0399490 -0.20437800 -1.21195100
## V5  -0.1596646  0.3693712 -0.7695378 -1.1480600 -0.49204100  1.14309500
## V6   2.5133010  1.2110730  0.2826131 -0.7901080  0.65734720 -1.84648700
##            X616       X617       X618       X619       X620        X621
## V1  -0.71214740 -0.3322689  0.9830335 -0.9880418  0.2655464 -0.7281862
## V2   1.26967600  0.1163017 -0.2811018  0.4093702  0.4978427 -2.4443610
## V3  -1.19683000  0.3595671  1.5789370  1.7256160  0.1475785 -0.6751603
## V4  -1.32810400 -0.3294724  0.7891090  0.1246891 -1.0096460 -0.3957392
## V5  -0.04162245 -0.8540480 -0.1371882 -0.1128139 -1.0435300  0.9393475
## V6  -1.82396800  0.4622229  0.6869990 -0.3256601  1.8287760  0.3293567
##            X622       X623       X624       X625       X626       X627        X628
## V1   1.3754050 -0.5992643 -0.2721732  0.4053248  0.8467341  0.5181260  1.5794140
## V2  -0.8632323  0.3783489  1.7826310  0.4933589  0.2921253  0.5720211 -0.2142836
## V3   0.4854132 -1.2128820 -0.2885697 -0.9169194  0.7738001 -0.9626799 -1.0717100
## V4  -0.2721355 -2.1659370  0.2480974 -0.5193774 -0.1157790 -2.1544880 -1.7612040
## V5   1.9209150 -1.3254290  0.2466200  2.4996680 -0.9721239 -0.8510522 -1.0934000
## V6  -0.7603890  0.1051114  2.0692510 -0.4180339  0.6600434 -1.1988130  1.5906450
##            X629       X630       X631       X632        X633       X634
## V1  -0.11295170 -0.4846529 -2.3778540 -0.6360480  0.03566377 -0.1091669
## V2  -0.23395550  1.0122130  0.2788531 -0.6701360  0.10638400  0.5952879
## V3  -0.04975481 -1.0169690 -1.2227720  0.7567572  1.77241700 -0.8885107
## V4   0.65314460  0.5926625  0.1260705  0.0227272  0.89023390 -1.0052860
## V5  -0.48121300  0.1952381 -1.9611610  0.7915466  0.77607060  0.6245696
## V6   0.63372450 -0.5493850 -1.0120850 -0.3862465  0.29676750  0.3554542
##            X635        X636       X637       X638       X639       X640
## V1  -1.87130500 -0.07933309 -0.3185969  0.1181811  0.5324036 -0.1976839
## V2   0.76276370 -0.34170490  1.6118620 -0.3162932 -1.4495790  0.4506472
## V3   0.86426730  0.20234280 -0.8306344  0.5196840  0.2277111  1.5255380
## V4   0.28241830  0.74626150  0.8921589  0.6661052 -0.7321297  0.4185465
## V5   1.17557100  0.63804550  0.1923568 -1.4613940 -2.4768580 -0.1175913
## V6  -0.02865124  0.63316530  1.0543910  1.2853070  0.8109581  2.8096200
##            X641       X642       X643       X644       X645       X646        X647
## V1   1.66437900  0.3144951  0.6015918 -1.429558 -0.4729991 -0.4878888  1.0273890
## V2   1.12990000 -1.9580080 -0.1638196 -2.081914 -1.0141980  0.1123629 -1.2985970
## V3  -0.77869550  0.4328868  0.5882359  1.411115 -0.7693723 -0.3303089  1.8809720
## V4  -0.82870470  0.2485605  1.3392400  1.224812  1.3933790 -0.3065891  1.0422420
## V5   0.06591268 -0.9798840  1.2566930 -1.744939 -1.1332640 -2.7487500 -1.8019100
## V6   0.31345340  0.8348862 -1.3696330 -1.319397 -0.4857457 -0.1171416 -0.5644251
##            X648       X649        X650        X651       X652       X653
## V1  -1.09576400 -0.9164456  1.05409500 -1.43983400  0.5325641 -1.0469290
## V2  -0.92261100 -1.3127630 -0.14256220 -2.30488300  0.6958684 -1.1559200
## V3   0.33322560 -0.8049920  0.55397670 -0.08775313  1.4391690 -0.5277358
## V4  -1.29328000  1.7889620 -0.04544975 -0.87951820 -0.2140128 -0.6008303
## V5   0.86299920 -0.1945065  0.42499360 -0.19488790  0.4601286 -2.0252250
## V6  -0.01800285 -0.7335065 -0.27462440  0.49849290  0.3677307  0.2019147
##            X654       X655       X656       X657       X658       X659
## V1   0.08738588 -0.1741117 -0.5338263  0.09931775 -1.0201070  0.4300418
## V2  -0.35105530  0.4631775 -1.1193160  1.60862900 -0.2084907  1.9239530
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## V3  1.28355300  2.8237910 -1.1559490  1.60231000  0.9054224  0.4400684
## V4 -0.21032980 -0.0144479 -1.0758010  0.50476530  1.4594560  0.2952224
## V5 -0.73981600 -2.2369570 -1.2165530 -0.78616970 -0.5491949  0.1023547
## V6  0.08463683 -0.5401128 -1.0535180 -1.45885500 -1.3114300 -0.3167251
##           X660       X661       X662       X663       X664       X665
## V1  1.5028480 -0.2707608  0.8715554 -1.2162940 -1.89273900 -0.8223826
## V2 -0.7212617 -0.3149240 -0.6874732  0.8433805 -0.29751080 -1.1096010
## V3  0.9407055  1.8161540 -0.1337061 -0.3492536 -0.46657460 -0.8086896
## V4  0.6297948  1.6946850 -0.2226940  0.2359891  0.91428370  0.5234243
## V5  1.5956280 -0.6468139 -0.5027498 -0.6112727  1.17247200 -1.2243380
## V6  1.9768720  0.3892016  0.1267491  0.6843220  0.04458141 -0.7144213
##           X666       X667       X668        X669        X670       X671
## V1  0.75178200  0.77357450 -1.78753200  0.08881747 -0.18005290 -1.4717150
## V2  0.06158548  0.35727450 -0.92689280  0.46628130 -1.03945400 -1.4216790
## V3 -0.21824400 -0.59335450 -0.57729980  0.32530240 -0.99527380 -1.4708600
## V4  0.37814330 -1.79666200  1.89616800  0.82488970 -2.14964400 -1.9205810
## V5  1.39132500 -0.15436150 -0.30811580 -0.08608494 -0.06808813 -0.5232705
## V6  0.25802420 -0.02407432  0.05033551  0.57687080  1.61681900 -0.9520944
##          X672        X673        X674       X675       X676       X677
## V1  0.4900949 -0.39783320 -0.16967940 -2.2149830  1.0081450  0.5242607
## V2  0.5420073  0.01225306 -0.16568310 -1.7410930  0.3595513  0.9502757
## V3  0.6803407 -0.57846280 -0.55513340  0.3283349 -0.9500335  1.9012790
## V4  2.3728060 -0.30233080 -0.09584185  0.5787717  0.6690592 -1.2140950
## V5 -0.5313430  1.07225900 -0.73809560 -0.6976978  1.5925020  0.1343618
## V6  0.3586832  0.74640480  0.45852660  0.6312851  0.5343223 -1.3336970
##          X678       X679       X680        X681       X682        X683
## V1  0.2648206 -0.3882564 -0.3994519  0.22714620 -0.6111570  0.54065050
## V2 -0.6887496 -0.3945030 -0.5871080 -0.90495260  0.1849971  0.60389680
## V3  0.7317588 -0.7427655  1.5585430 -0.03967166  1.0314800  0.93471720
## V4  0.8839132 -0.8383758 -1.3432140 -0.95448420  0.2505964 -0.59970780
## V5  1.0906850 -0.5324999  1.7221570  0.51397240  0.2124272  0.01457052
## V6 -1.3995650  1.2605490  1.1487430  1.30469700  0.7966427 -0.44945230
##           X684       X685       X686        X687       X688        X689
## V1  0.9099550  1.4066220 -0.7555871 -0.02059243  0.1378585 -0.92154040
## V2  0.8432076 -1.8917840 -0.4139746 -0.34322950 -0.4240921 -1.34687200
## V3 -0.9139735 -1.1394630 -1.7378310 -0.20888160  0.7555976  0.27818980
## V4 -0.1872277 -0.3380756  1.7174420 -0.81692350  1.3264370  2.03131800
## V5  0.2292307  2.2038750 -0.8173428 -1.01976900  0.5789412  0.06127333
## V6  1.2339860 -0.1122689 -0.4333292  1.28368200 -0.2295489  0.35521920
##           X690       X691       X692        X693        X694        X695
## V1  0.57652340  0.2988937 -0.8623010 -0.81013810  0.47255590  0.56849900
## V2 -0.98940680  0.1311460  2.2060070 -0.48802590 -0.05859646  0.38150740
## V3  0.41957370 -0.8584298 -2.1569020 -0.02258244  0.44453840  0.46773880
## V4 -2.31067900 -0.6279129  0.1400342  0.50383340 -1.10495500 -0.40483820
## V5 -0.43416790 -0.8796653  0.4797187 -0.22221410  1.55517800 -1.34656500
## V6 -0.07915782 -0.1058318  2.7064230  1.03371500 -1.53381000 -0.08423313
##           X696       X697        X698       X699        X700        X701
## V1  1.4130980  0.05881738 -0.58344790  0.2701231  1.07804600 -0.14657960
## V2  1.3960940  0.16338360 -0.07448124 -1.0436230  0.01575521 -0.56864840
## V3  0.2336009 -0.08672898 -1.11590300  1.4896610  0.48612490  0.51263230
## V4  1.3719060 -1.10665100  0.02760885 -0.2150529  0.11653490  0.03169142
## V5 -2.7270390 -0.25810980  1.00948700  0.9315255 -0.27541730 -0.15452220
## V6  0.2091286  0.61300160  1.15691400  0.7463084  1.13774400 -1.07044600
##           X702       X703       X704       X705       X706       X707
```

```
## V1 -0.6352536  0.5240847  1.1051460 -0.50101190  0.4241926 -1.0773580
## V2  0.3361345  0.0725406 -1.4115280 -0.06088542 -0.4587048  0.7295546
## V3  0.2547365  0.6165087 -1.5430190 -0.27364440 -0.4209944 -0.6019760
## V4  0.3481929 -1.2470470 -0.7594790 -0.61559990  1.4423770  0.4340126
## V5 -0.5848437  0.4722185 -1.3199030 -0.18951590 -0.1742047  0.3327995
## V6  0.6517613  2.2693690  0.1373407  1.30647500  1.8285910 -0.7353875
##           X708       X709        X710        X711        X712        X713
## V1 -0.6258375  0.8401206 -0.05946661 -0.07721755  0.41670960 -1.37991400
## V2  1.0171460  0.9378846 -0.75360800 -1.25851500  0.41167930 -0.01983617
## V3  0.4050523  0.7368737  0.96426270  1.03105600  0.70746650 -0.09003496
## V4 -0.7593517  0.9965910  0.68580570  1.09422700  0.08717525 -1.01754800
## V5  0.3740898  1.2992370  0.27210490  0.31966820 -0.09691663  0.19350630
## V6 -0.1807326 -0.5415974  0.64686700 -2.50062500 -1.26125500  1.75933400
##           X714       X715       X716       X717        X718        X719
## V1 -1.4821500  0.1220714  1.1958870 -0.2540624 -0.344407300 -1.8097990
## V2 -1.4050400  0.2475681 -0.9419050  0.4729573  1.638864000  1.4750590
## V3 -0.3154756  0.4652751 -1.0833410  0.1091577 -1.962424000  2.1540650
## V4 -0.5148007 -1.9252250 -0.4533493 -0.8085183 -0.009047911  1.6141850
## V5 -0.9337977 -0.5655694  1.3204040 -1.2996470  0.928894200  0.2912830
## V6 -1.2099930 -0.7176182 -0.5418542  1.6504850 -0.590674700 -0.3261695
##           X720       X721       X722        X723        X724       X725
## V1 -1.6308950 -0.2719659 -1.7909010 -0.22849920  1.78424100 -0.1663136
## V2  1.5665310 -0.9187196 -0.2551083 -2.31533000 -1.78057900  2.0119120
## V3 -0.7525802  0.3595398 -0.1667216  1.18612900  0.20382300 -0.4761804
## V4  1.9095890 -0.3571213 -1.0769750  0.02718465 -0.78358380 -0.0250342
## V5  0.5629205  1.1021620  1.0283930 -0.71877530  0.01163696  0.7609504
## V6 -1.3327660  0.9690472  0.1350358 -0.47703960 -2.50859100 -0.1619376
##           X726       X727       X728        X729        X730       X731
## V1  0.8090874 -0.9723293 -1.9525020  1.78642300  1.3129880  1.1086750
## V2  0.7423579  0.3025405 -0.9301512 -0.62373040  1.1397860  1.5250680
## V3  0.7903376 -0.8904054  1.9019430 -0.09844711  1.4229910  0.9588439
## V4 -0.2258267 -0.5865495 -0.9055543  1.25716500 -0.8712608  0.6123897
## V5 -1.4171170 -1.4493820  2.1538970 -0.53429070 -0.4080144 -0.2410641
## V6 -0.8318247 -0.9343191  0.4242188 -0.01222969 -1.7666520 -0.8733687
##            X732       X733       X734        X735       X736        X737
## V1 -0.99320070  0.52609980 -0.7680710  0.52900000  0.1425380 -1.54049300
## V2 -0.08140044 -0.41325520  0.1779261 -1.61176000  0.9343846  0.77976650
## V3 -1.19134200  0.98695040 -0.1504406  0.05980348  1.3170760  0.03187052
## V4  0.61518580  0.22290470 -0.6071368  0.61401370  0.5062400  0.80910630
## V5  0.66034800 -0.09367741  1.0224710  1.70413700  0.2394388 -1.25292200
## V6  1.15450400  1.68182400  0.2677225 -0.80115530 -2.6269890 -0.65212100
##            X738       X739       X740       X741       X742       X743
## V1 -0.77680910 -3.0533000  0.8714743  0.1909621  0.8495726 -0.5634817
## V2  0.94885910 -0.2789342 -1.4534760  0.9851275  0.2641752  0.1818199
## V3 -0.06273985 -2.0269130  0.7009227 -1.7412860 -0.9905284  0.3433529
## V4  1.13028700 -1.0072460 -0.4443174  0.0190286 -1.6766650 -0.9478715
## V5  2.28267200  1.3974780  2.1824530 -0.2321540  0.2893750  0.8156552
## V6 -0.03435950  1.0179790 -1.3030250  0.4336666 -1.2351750  0.3965565
##           X744       X745        X746       X747        X748        X749
## V1  1.2044170  1.7044930 -0.05800861  0.7708767 -0.02769032  1.303131000
## V2 -0.4894290 -0.1439310  1.35427500  0.3486849  0.85329970  1.012087000
## V3  0.2314262 -0.8259445 -0.23929900  2.2504460 -0.62062610  0.129755200
## V4  0.7400705  0.6973516  0.27132150  0.1638040 -0.48666180  2.244177000
## V5 -0.1813321 -0.8407041  0.98389900 -2.4666310 -0.87543890 -1.354803000
```

```
## V6  0.4395390 -0.6411114 -0.21728310  0.5912959  0.26215870  0.009106597
##              X750        X751        X752         X753        X754        X755
## V1 -0.71158590 -0.7711920 -1.3389080  0.408160700  0.2065376 -0.2244763
## V2 -0.92888070  0.7960816 -0.8027995 -0.458245600  0.6862764 -0.5953003
## V3 -0.04307628  0.4355247 -0.8846403  1.068867000  1.3092810 -0.2035059
## V4 -1.12399800  0.9161138  1.0671320  0.004417286  1.7804410 -0.5779668
## V5  0.41689030  1.1412770  0.5517586  0.721086700  0.3683187  0.3393237
## V6 -1.66525200  0.9075290  0.5139403  0.655677200 -1.1047280  0.4778255
##              X756       X757       X758        X759        X760       X761
## V1 -1.5040280  0.2387984 -0.4400945  1.08594800  0.2879373 -0.5925822
## V2 -0.4073832 -0.3809501 -0.5792708 -1.35588700 -0.1130526  1.0127050
## V3  1.1370820  0.9142530  0.8364688 -0.19091880 -0.0598250  1.0046850
## V4 -1.3600310  0.7795765 -1.2892180  0.08769516 -0.3606093  0.4109785
## V5 -0.7018681  0.4449621  0.1894396 -0.31278090 -1.0163420  0.5002854
## V6  0.7826775  0.5481967  1.8524910  0.04527923 -1.1803940  0.2339004
##              X762        X763        X764       X765         X766        X767
## V1 -1.03048200  0.72273890 -0.8148326  0.4146352 -0.19263650 -0.4101087
## V2 -0.18140630 -1.46927500 -1.8954180 -1.0159970 -0.08470884 -0.7886400
## V3 -0.06019712  0.92997480 -0.1652187  2.1365120  0.85672100  1.6181410
## V4  0.18083410  0.01112966  0.5144418 -0.7019133  1.09551700  1.3825580
## V5 -1.35535800 -1.27572500 -0.3823098  0.3622430  1.19049000 -1.2995160
## V6  0.66147330 -0.54481960 -0.1692764 -1.3112270 -0.53577220  0.3319862
##             X768        X769       X770        X771        X772        X773
## V1  0.2063615  0.2846302 -0.6048848  0.58322060  0.2801450  1.3156100
## V2  1.4490190  0.4195211  0.4069906 -0.47603130  0.5650942 -1.1216410
## V3 -1.0808550 -0.4055031 -1.2595490  0.44391150  1.3446370  0.1597877
## V4 -0.4003390  0.2120678  0.8360682  0.82483930  0.5397017  1.8657330
## V5 -0.1057151 -0.5605734  0.5172754 -0.05326433  1.1164460  0.4172165
## V6 -0.3447741  1.0242540 -1.8414760 -1.24144400 -1.6373060 -1.7201420
##              X774       X775        X776       X777       X778        X779
## V1 -1.45267700 -1.1580090 -0.04717828  0.1629123  0.9409331 -0.3928341
## V2 -0.06941162  0.1723395 -0.03840447  1.1775840  1.6365410  0.8535055
## V3 -0.30136400  0.7517435 -0.18311430  0.1244014  1.6974140 -1.9318760
## V4  1.19610900  0.4270450 -1.25229500 -0.1788584 -2.0899160 -0.3603960
## V5  0.19890320 -1.8971870 -0.39514270  1.1005910 -0.5302948  1.6328130
## V6  0.69979000 -0.5018942 -0.30256490 -1.5615480 -0.5194421 -2.1025400
##             X780       X781        X782       X783        X784         X785
## V1 -0.5623320  1.5020730 -0.47726160 -0.7501636 -1.01805900 -1.054689000
## V2 -1.5901010 -2.3989530  0.00748957 -0.4533979  1.75658700  1.527629000
## V3  0.0534175 -0.1369543  0.53292110 -1.0356600 -1.41713500 -0.008123334
## V4 -0.2125989  0.3492872 -1.75377300  0.3246749  0.05752017  0.653986400
## V5  1.3122700  1.9324390 -0.41353610  0.6591747  2.01603300  0.452147700
## V6 -1.4228190  1.2382190 -0.77086480 -0.9605078  0.76794160  0.160960500
##             X786         X787        X788        X789        X790        X791
## V1 -0.6413453  1.404537000 -0.06575134 -0.04939262  0.51082750  0.28506260
## V2  0.1672024 -2.923243000 -2.26275800  0.75918750 -2.18201100  1.71744500
## V3  0.6741640  1.564483000 -0.78121840  0.40078610 -1.02187600 -0.77539050
## V4 -0.5535178  0.008765323 -0.61333510  0.84955200 -0.26617030 -0.12000160
## V5  0.2455877 -0.111009800 -0.01600776  0.29269800  0.01991169  0.52521720
## V6 -1.5028400  0.442969800 -0.38167160  0.20703090  1.38572300  0.09860155
##              X792       X793        X794        X795       X796       X797
## V1 -0.04417974  2.1606190  0.25143790 -0.75780270  2.0291140 -0.7597542
## V2 -2.14771500  0.3521260  0.11103700  0.13863320  1.0077100 -0.2469696
## V3 -0.39741910  0.2117558 -0.17392910  0.05729733 -0.7397226  0.3084581
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## V4  1.12374100 -0.6470156  0.33468950  0.28837090 -0.5330147 -1.9519220
## V5  0.45263280 -0.7744650  0.02650803 -1.16092700 -1.2001700  0.1652025
## V6  0.26257450  0.6402703 -0.60647930  0.37106420 -0.1119981  0.2853102
##            X798       X799       X800       X801        X802       X803
## V1  0.5409988 -2.2667650 -1.3672180  1.04608900  0.34553120  1.1697040
## V2  1.2303340  0.3351331  1.2919580 -0.00000232  0.29339670 -1.2829430
## V3  1.9246910  1.8502340  0.5357880  0.64394840  0.39068870 -0.3883313
## V4  1.1536470 -0.2445792 -1.1852130  1.15978900 -1.15183400 -0.5615776
## V5 -0.8038915 -0.3082748  0.3524884 -0.22268850 -0.09571666  0.4122895
## V6  1.0312170  0.5332829 -0.5455086 -0.65635110  0.81033220  0.4211987
##            X804       X805       X806        X807       X808        X809
## V1  1.6227910 -0.2779612 -1.1629680  0.875151100  0.2441953 -0.02546942
## V2  2.3851250 -0.6402161  0.1330253  0.000731171 -0.3128785 -1.08146200
## V3 -0.3231220 -0.8616966  2.0852240 -1.208710000 -0.9111995 -1.47021900
## V4 -1.5021360  0.2136899 -0.6329638 -0.193097400  0.6052626 -1.96824900
## V5  0.5093718 -0.1478995  0.1502263 -0.191484500  0.3990665  0.74695070
## V6 -0.6077353  0.7436439  0.6531147  0.067913300 -0.8489170  1.98611300
##            X810        X811         X812         X813       X814       X815
## V1 -0.07705133 -2.05056400  0.061337130 -0.494998400  0.4792697  0.7513906
## V2 -0.88072260  1.06301500 -0.007693965  0.899695800 -0.1328603 -0.8659210
## V3 -0.71827440  0.05066276 -0.085321460  1.547390000  0.1351049 -1.3952140
## V4  0.25522250 -0.66032760 -0.491463300  0.821169400  0.7162004  0.8465430
## V5  1.94609300  1.31036200  2.153415000 -1.007771000  1.0421990 -1.3002120
## V6  0.20313280 -0.64995030 -0.346510500 -0.008813462  1.8454400  0.6631713
##            X816        X817       X818        X819       X820       X821
## V1  0.1725892 -0.06084720  0.6369664 -1.46398300 -0.8490696  0.5089401
## V2  1.2354440  0.31041960 -1.0861270  0.06068965  0.1328686 -0.7015299
## V3  0.5901277 -2.40489200 -0.6635350 -0.92661620  0.8078446  1.3067180
## V4 -1.8006350  0.08646449 -0.1420308 -1.33330300 -0.3132353  0.3915749
## V5 -0.6333373 -0.47695200 -1.0186100 -0.70221650  0.2966908  1.0997760
## V6 -0.1318002 -0.10117210 -0.1072861 -1.38827000 -0.1471594  1.8650720
##            X822        X823       X824       X825       X826        X827
## V1  0.63246100  1.16624500  0.3905792 -0.1369166 -1.0938040 -1.66980000
## V2  0.56408430  1.66055100  1.0965050  0.1074717  1.1250320  0.57377220
## V3 -1.15807500 -1.62591800 -0.4920402 -0.6645142 -0.3806519  0.95762060
## V4  0.05309722 -0.20907170 -0.6425849 -0.6470042  1.2361500 -0.62538460
## V5 -0.15190970  0.73383090  0.3584608 -1.4279940  0.8395937 -0.04308182
## V6 -0.63236290 -0.06744242 -0.6367868 -0.1543411 -0.2819171  0.76556300
##            X828       X829       X830       X831       X832       X833       X834
## V1  1.1319470  0.5696478 -0.6294001 -0.9174054  0.9956580 -0.8894882  1.3259770
## V2  0.5224184  0.1789226 -2.2445400 -0.3038185  0.6841635 -0.2811645  1.7309050
## V3 -0.6656139  0.9687541  0.2119665 -1.6380590  0.7332582 -0.7402077  1.5724370
## V4 -1.6288450  1.3573680  0.3894679 -0.3467781  0.9449956 -1.3738040 -0.5963825
## V5  0.6440818  0.6760235 -2.1735250  0.8838816 -0.1854941  2.8919980  0.1861260
## V6 -1.9558020 -1.4544030 -1.3852500 -0.6758029  0.5621920 -0.5095243  0.6678059
##            X835       X836       X837       X838        X839       X840
## V1  0.3409506  1.2620760  2.18965700  1.8667360  0.735091200 -0.7863044
## V2 -2.1888650 -0.7600402 -1.06940600 -0.3468833  0.747870600  2.0382840
## V3  1.4345290 -2.1399500 -0.26000740  0.8890667 -0.000046900 -0.1154721
## V4 -1.2508520 -1.3607290 -0.56223700 -0.1274468  1.348814000 -0.6886890
## V5  0.1880192  1.3466440  0.02146689  1.4304220 -0.004293224  0.9381195
## V6 -0.1879272  0.1767450  0.63761600 -2.2576590  0.199057900 -0.1854638
##            X841        X842      X843      X844      X845       X846
## V1 -0.87697430 -1.22086800  1.6166520  0.4980438  1.6025590  0.63557560
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## V2  1.97521600  0.30111620 -0.8612993  1.0373580  0.7525393 -0.70323090
## V3 -0.31244350 -2.03008000 -1.7457070  0.7333106 -1.2587260  1.47260500
## V4  0.34571960 -0.70179840  2.2433800 -0.6810720  1.9168080  2.02856100
## V5  0.09163018  0.08524112  1.8507470 -0.2031754 -0.3953048  0.78687700
## V6 -1.18368300  0.03775985 -0.9717092  0.4880640  0.6774481  0.04769079
##           X847        X848        X849        X850        X851        X852
## V1 -1.0684320 -1.70792700 -1.0212180 -0.2271061  1.24246400 -0.57890860
## V2  0.3875388  1.49113200 -1.4959810  1.2887030  0.03617525 -1.14156900
## V3  0.3848753 -0.07955294 -0.3921603  1.0179010 -0.03631599 -0.03589313
## V4 -1.8384600 -0.45294380 -0.5077572  0.1102284 -0.30253530  0.08649885
## V5  0.8521534 -1.70619600  0.3852688  0.7796032 -0.57470940  0.15472800
## V6 -0.3675838  1.21097000 -0.2442244 -1.3804900  0.43863430 -2.85388700
##           X853        X854        X855        X856        X857       X858
## V1  1.0320770 -0.1914775 -0.42418610 -2.39161300  1.6460650 -0.7670604
## V2 -0.3376248 -0.9955960  0.48141330 -0.09810114 -0.2862455 -0.6942854
## V3 -0.2376712 -0.5719259 -1.81876500 -0.09325368  0.2771793 -0.9743483
## V4  1.0928260  0.2171549 -2.10662400  0.26388030  0.7020198  1.0914410
## V5 -0.5384941 -1.3410250  1.30557900 -0.55857460  1.5478660  0.9183951
## V6  0.2515526 -1.1737580 -0.08740409  0.13368390 -0.6291021  1.1074970
##           X859        X860        X861        X862        X863        X864
## V1  0.75627100  1.4927100  0.6559360  0.4947172 -0.2531019 -0.6629416
## V2  1.37243800 -0.7199028 -0.5871402 -0.2301099  0.6751816 -0.6684469
## V3 -0.71502450 -0.1288850  1.4849130 -1.6478940 -1.7619860  2.0106260
## V4  0.56629270 -0.3142583 -0.6533059  0.2824051 -0.6321180  1.3596160
## V5 -0.07358233 -0.1839092  0.2658844 -0.3789998  0.7217275  0.6186938
## V6 -0.90303350  1.0812120  0.3158667 -0.1301719 -1.7932870  1.0343390
##           X865        X866        X867        X868        X869        X870
## V1  0.56212560 -0.7792979  0.85896230  0.9349611 -0.7919525  1.35948700
## V2 -0.43320950 -2.1349230  0.42331070  1.6298850  1.2396260 -0.68397870
## V3  0.86868300  0.1114684  0.05130464 -1.0877740  0.2876882  0.02099824
## V4 -0.04466273  0.8202725 -0.91768360  0.9220936 -0.6163602  2.68590200
## V5 -0.58547590 -1.0879170 -1.75132600  0.3854698 -0.6345362 -0.14462080
## V6  0.16338440  1.1892350  0.87561880 -0.6727520  0.6018114  1.32223000
##           X871        X872       X873        X874        X875        X876
## V1  0.01964938 -1.18594000  0.3436321  0.58548780 -1.4302010  1.0190290
## V2  0.48946600  0.06380143 -0.3843236 -0.29998030  1.3473020 -1.0715990
## V3  1.04467300 -0.28633180 -0.3878032  0.90242100  1.0370830  0.3359908
## V4 -1.09400600 -0.66886040  1.6140640  1.48005000 -1.7040400 -0.6521202
## V5 -1.26307000 -0.15689360 -0.5955147 -0.09991788  0.7470431  0.1082125
## V6  0.51618930  0.69027000 -1.3685810  1.43831700  0.6360727  0.9224822
##           X877        X878        X879        X880        X881       X882
## V1  0.3202989 -0.5015762 -0.14137600 -1.43081800  0.58785770  0.8689636
## V2 -0.6365512  0.1928244  0.75964900  2.29652200 -1.42128500 -0.2013740
## V3  1.3198780  0.3688429  0.13059110  0.61613740  0.66638750  0.3112518
## V4 -0.3819877 -0.8499133 -1.35578200  0.33031290 -0.81840580 -0.9247279
## V5 -0.4871094 -0.6795451 -0.07421788  0.05438528  0.06432345  0.1906510
## V6  0.4112999 -0.3703849 -2.13087800 -0.81049950 -0.50650630 -0.9885347
##           X883        X884       X885        X886        X887        X888
## V1  0.4566550 -1.35596000  0.6438078  0.07010119  0.01039156  0.72120000
## V2  0.9638491 -0.77570110  0.1585294 -0.34883300 -0.54838030  0.77487940
## V3 -2.2378900 -1.45969400 -1.0027720 -0.21606290  0.76324840 -0.93189280
## V4  0.4210065 -1.68985600 -0.1812609  0.55590410 -0.81282830 -0.26581540
## V5  0.3972796 -1.36478900  0.1881216 -1.11434500 -1.63478100 -1.27360600
## V6 -0.6870674  0.04437966  1.0682760  0.27123110  0.14394540  0.03291844
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```
##              X889          X890         X891        X892         X893        X894
## V1  0.09381490  -0.56425700  0.007023682  -0.5615375   1.75596000   0.1704266
## V2 -0.22184000   0.09047860  1.192642000   0.1419200  -0.21278350   1.3184110
## V3  0.01157565   0.47858670  0.761878200  -0.9657676  -0.14635880   0.8699514
## V4 -1.32127900  -0.43320920 -1.628415000   1.0756100  -0.77517780   0.9326741
## V5 -2.56522100   0.42477710  0.289176900   0.9804556  -0.25619900  -0.8961302
## V6 -0.69185160   0.07232066 -0.652498300  -1.5236360   0.07961125  -0.4451129
##              X895         X896        X897        X898         X899        X900
## V1  1.0651950   1.5041560   0.6599597  -0.8382010   0.39468910  -0.3796260
## V2 -1.1336620  -0.0477080   0.2852269  -1.2852060   1.47175100  -1.1422990
## V3  0.3034185  -0.2870446  -0.6082962   1.0919430   0.05077218  -1.8132860
## V4  1.7092740  -0.8464260   0.1852383  -0.5735278  -0.54746400   0.3444314
## V5 -1.2186890  -0.8476596  -0.3046626  -2.3599570   0.54547100   0.7928062
## V6 -0.4216412  -1.8095840  -1.9676110  -0.7881309   0.53629840   0.7663268
##              X901         X902          X903        X904        X905         X906
## V1  2.0667800  -1.0297500  -0.123095500   0.1739577   0.5531906  -0.75906870
## V2  0.3633626  -1.1959110   0.004294699  -0.8519134  -2.4435590   1.06288900
## V3  0.8771536  -0.7752540  -1.010106000  -1.3166670   0.1524802   0.06801613
## V4  0.1907076  -2.1040590  -0.159129400  -0.7071567  -0.6160735   0.06387697
## V5 -0.4165517  -0.6203647  -0.894662200  -0.2985566   1.1331770  -0.91923310
## V6 -1.2341210  -0.8998980   0.329413400  -0.7312690  -0.8843110  -0.47503390
##              X907           X908        X909        X910        X911        X912
## V1  0.03582959  -0.003495063  -0.2201904  -1.5576800  -0.2380460   0.4249825
## V2  0.79085350   1.052093000  -0.9968810  -0.7037607  -0.3681636  -1.7465400
## V3  0.04518163  -0.791391800   1.1155480   0.6424789   0.3608139   0.3181208
## V4  0.45232170   0.126482300  -1.0954860  -0.2901112   0.9008221  -1.2346210
## V5 -0.57540010  -0.490108400  -0.8886470  -0.9851917  -0.3871882   0.5876329
## V6  0.24130040  -0.363103900   0.8223490   0.2042809  -0.8663852  -0.3829641
##              X913        X914        X915        X916        X917        X918        X919
## V1 -0.44677970  -1.401855  -0.5474143   0.2479418  -1.3517030   1.2853750   2.0966330
## V2 -0.04093408  -1.235310   0.6300323   1.2568970  -0.5846703  -0.4470747  -0.5008069
## V3  0.72978230   1.187970   0.2512890  -2.5759650  -0.1151626  -0.6429563  -0.8475154
## V4 -0.36835470   2.150188   0.3361185  -0.8847018   1.0862710   0.3616700   1.1918560
## V5  1.56810900  -1.372164  -0.1761917   1.9161020  -0.2328411  -0.7571848  -2.7449650
## V6 -0.43894750   0.978491   1.1719430   0.2166380  -0.1714394   1.0992670   0.9903307
##              X920         X921        X922         X923        X924        X925
## V1 -1.075717000   0.64381860   0.8111071  -0.37336920  -0.1177557   0.9395093
## V2 -0.001746825  -0.65901420   1.1799540   0.48613060   0.3539342  -0.1756950
## V3 -0.666493900  -0.13886830   0.6228956  -0.90527550   1.1388700   1.0030660
## V4  1.132069000  -0.03339338  -2.4046100   1.30177700   0.6351350   0.4178984
## V5 -2.569140000  -0.81189250   0.1870650  -0.04610147  -1.7676710   0.3100866
## V6 -0.680151200   1.14005000  -0.1113911   0.44715620   0.6381620  -0.6288187
##              X926         X927         X928         X929         X930        X931
## V1 -0.2235445  -1.1888660   0.59996120  -0.86330600  -0.66913610   0.7580809
## V2 -0.8509989  -0.4545327  -0.09943265  -0.48793760   0.08262384   0.7490010
## V3 -0.4040670  -1.0849400   0.72923640   0.22050450  -1.81555000  -0.1139596
## V4 -0.6264451   1.1288160   0.14321920   0.09006043  -0.15384180   1.1074450
## V5  1.8700200   1.2728000  -0.08708173  -0.41402380  -0.18234870  -1.2175490
## V6 -0.2607009   0.4875614  -0.55332990  -1.04719700  -1.51010600  -0.4821755
##              X932        X933        X934        X935        X936        X937
## V1  0.9772123   1.4869600   0.8091820   1.04959100  -1.3783170  -1.1663800
## V2  0.1092167  -1.5439850  -1.4668680  -0.07035627  -0.7761757  -0.3774430
## V3 -3.5271880  -1.6484970  -0.9945496   0.46451600  -0.8337270  -1.6394070
## V4 -0.8081458  -0.2615624   1.2777840   1.19100600   0.6637940   0.5450351
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## V5  1.3052290 -0.5503982 -1.1352710  0.39855180 -1.1216050  1.4005010
## V6 -0.4292242  0.7960990 -1.5369740 -1.75292800  1.2465590  2.5163240
##            X938        X939        X940        X941        X942        X943
## V1  0.45053170  0.55398670  0.1715338 -0.8068815  0.9788529  1.171035000
## V2 -0.30446880 -1.16496700  0.2649411  1.4382290  0.3611041  1.890076000
## V3  0.37905400 -0.79608130 -0.5479242 -1.4773170 -0.2877611  0.007019281
## V4  0.08211945 -0.30459970 -0.2311610  0.5374188 -0.3900816  0.434018500
## V5  0.03667063  0.07874646 -0.7591474  1.1312090  1.7284610  0.223172300
## V6  0.51715300  0.80684510 -0.5231862  0.4962162 -1.0458920 -1.021798000
##          X944       X945        X946        X947       X948       X949
## V1 -0.2547510 -0.2617795  1.35700900  1.04819500  2.2288980 -1.4705190
## V2  1.4843550 -0.8888009 -1.15897200 -0.91904630  1.8333610  0.8390895
## V3 -0.6890549  0.6727177 -1.77490300  0.14577950 -1.1626630 -0.8703584
## V4  0.1006756  0.2221468 -0.01326677  0.24744710 -0.3586945  1.2011240
## V5  0.1971776  1.9352300 -0.42308570  0.09124576  2.8974660  0.4940827
## V6  0.2941100 -1.2471210 -0.01751377 -1.19552000  0.1252737  0.7780694
##          X950       X951       X952        X953       X954       X955
## V1 -1.0312500  0.1648135 -1.2438380  0.11630820  1.1975470 -0.6981310
## V2 -1.2340170  2.0648620  0.5036313  2.03000700 -0.4386965  2.4535440
## V3 -0.2182988 -0.5556146  1.9763010 -1.11930200  0.1764839 -1.3966350
## V4 -0.4289292  0.6685300  0.2714859 -0.76132640  1.4367730  0.8436090
## V5 -0.8657282 -0.1622478  0.6278645 -1.49804400 -0.6122457  1.3103240
## V6  0.7334690 -0.9633904 -0.6428605  0.03295067 -0.1634440  0.7794752
##          X956       X957        X958       X959        X960        X961
## V1  0.2936049  0.4372977  0.62776260 -1.1605440 -0.02078842  1.04472400
## V2 -0.3853686  0.5893337  0.06484014  0.3978536  0.54271110  0.65968930
## V3  1.3600840  0.5897531 -1.90301800  1.2936970 -1.81853700  0.69667260
## V4 -0.7570108 -0.9350238 -0.11854870  1.6055960  0.20817770  0.23414160
## V5 -1.8656510 -0.1217740  0.71470850 -0.6779083  0.88708190 -0.85315670
## V6  0.7832159  0.5983993  0.46778180  0.9507378 -0.27497110 -0.06128801
##          X962        X963       X964       X965       X966        X967
## V1  1.311595 -0.08243903  0.9192248 -0.2432699  0.3510944 -1.34038400
## V2 -2.134584  0.10733250  0.8540678 -0.3339704  0.4813534  0.31395550
## V3  1.647552  0.11219570  1.4440520 -0.1890999  0.2060802 -1.28153500
## V4  1.691224 -1.31194400 -1.0930980  0.2809294 -1.7008830 -0.09318068
## V5 -1.454029  0.48780460  1.5153340  0.1993837 -0.2069855 -2.99132300
## V6  1.541222  0.59538340  0.2219548 -1.3326060 -1.0313400 -0.85246530
##          X968        X969       X970       X971       X972        X973
## V1  0.4258461 -0.81435370  0.6933741  0.7055446  0.2783505 -1.20269200
## V2 -0.8146854 -0.77421540 -0.2683207 -0.1464686 -1.0350110 -0.17723320
## V3  1.0981080 -0.01392543 -0.8238425 -1.7704690 -1.6906980 -0.43712290
## V4 -0.2685050 -0.95725120  1.1870160 -0.6908792  1.5661240  0.12405880
## V5 -1.1326310  0.22360490  0.5563960  0.3811930 -0.5446115 -0.03007637
## V6  1.2243210  1.54352100 -0.1802022  0.2650282  0.4016218 -1.35357000
##           X974       X975        X976        X977        X978       X979
## V1  0.64276180 -0.6059740 -1.37725800 -0.92599270  0.567243900  2.5954810
## V2  0.02169295  0.2259434 -0.12136540 -0.04637114  0.870366000 -0.9662976
## V3  0.73246760 -0.7472986  1.51023700  0.72363860  1.113742000  0.3693113
## V4  0.68958520 -0.3168895 -0.49762780  2.02878400  1.931292000  0.7702376
## V5  0.11195120  0.5936792 -0.07755099 -0.55004010  0.272265500 -0.1293229
## V6 -1.20840800 -1.1665810 -0.61792220  0.09041432  0.003351645 -0.6492060
##           X980       X981       X982       X983        X984        X985
## V1 -0.2399845  0.6875364 -0.1517890  0.2275443 -0.66864800  0.03082907
## V2  0.6839356  0.1339006  0.3495957 -0.6581023 -0.40444120 -0.43699100
```

```
## V3 -1.3202210  0.3761377  0.4000536 -0.6284581 -1.33836800 -0.49652120
## V4 -1.8157440 -1.4123190 -1.3607380 -0.3307767 -0.06434358 -0.72996700
## V5 -2.2767930 -1.2754880  0.2317062 -1.9515000 -0.91936490  0.19242690
## V6  0.9771001 -0.2946974 -1.5602000 -0.5160640 -2.76748600  0.15057890
##           X986       X987       X988       X989        X990       X991
## V1  0.02842956 -0.3654551 -2.2080120  0.2970394  2.12970100  1.3250410
## V2 -1.09009700  1.0317910  0.6070065 -0.9019485  0.15011610  0.7408377
## V3 -1.12896700  0.5007854 -2.0308930  1.2481290  0.44280510 -0.4355333
## V4  0.42576600 -0.4546751  2.5248920 -1.6733350  0.63513160 -3.0655290
## V5  1.19815000  1.5677310  0.3428824  1.3815820  0.77316320 -2.3789380
## V6 -0.81674850 -1.9155740 -0.7654394  1.4021570 -0.07170323  0.1016926
##           X992       X993       X994       X995       X996       X997
## V1 -0.11617140 -1.4701460 -0.3792718 -1.4650060  1.0751480 -1.2261250
## V2 -0.16239210 -0.6333746 -0.8955208  2.0344650  3.0032670 -0.5017017
## V3 -0.23591190  1.4466600 -1.1274590  0.4408494 -0.1234407 -0.7174301
## V4  1.59729400  0.7374776 -0.6312482 -0.5304421 -1.0367400 -0.1691128
## V5 -0.08694592 -0.1223420  1.4180290  1.0753370 -1.2706040  0.5995296
## V6  1.01960000 -0.5536003  0.7313706  0.2981279 -1.2770290 -0.9979873
##           X998       X999      X1000
## V1 -3.0563280  1.4506580  0.7179769
## V2  0.4498887  1.3103480  0.7634819
## V3  1.8803620  0.3838369  0.3135760
## V4 -0.7428410 -0.4088601 -0.3264731
## V5  2.2383460 -0.4711108 -0.1587002
## V6 -0.2917377 -1.3923960  0.4681130
```
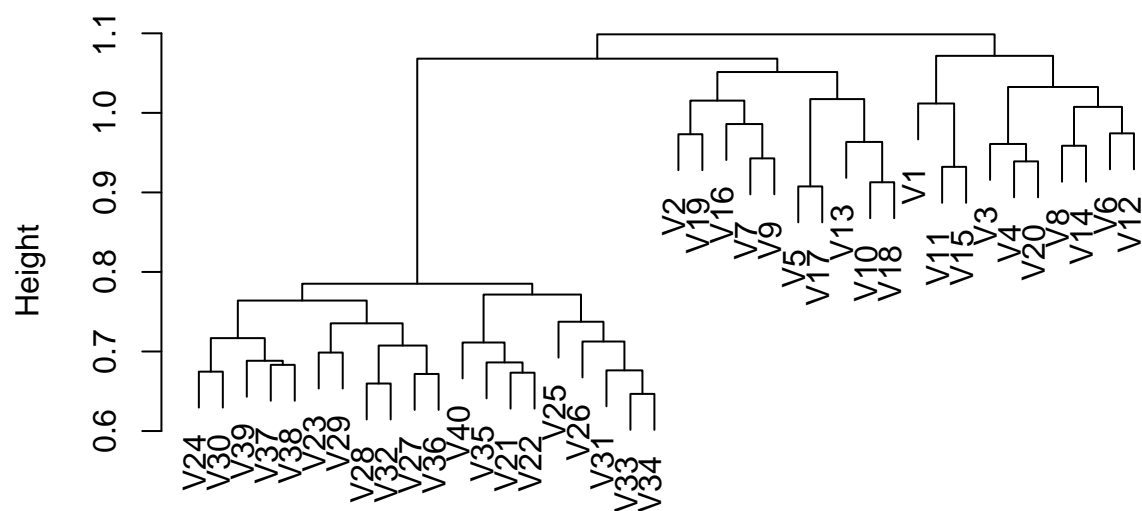
b

```
corr_dist = as.dist(1 - cor(gene_exp))
complete_hc = hclust(corr_dist, method = 'complete')
plot(complete_hc)
```
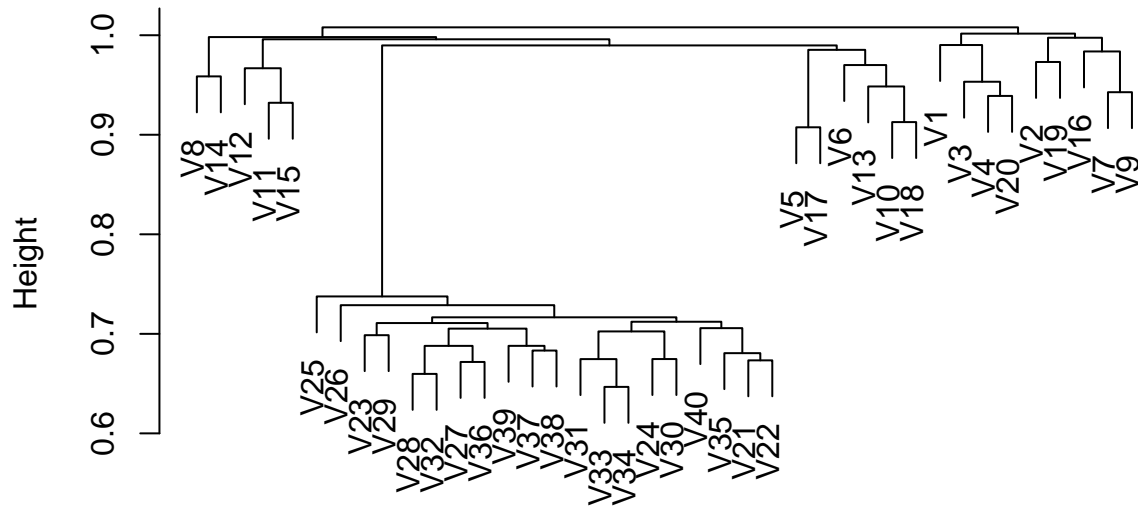
Cluster Dendrogram

corr_dist
hclust (*, "complete")

```
table(cutree(complete_hc, k = 2))
```

```
##
##  1  2
## 10 30
```

```
average_hc = hclust(corr_dist, method = 'average')
plot(average_hc)
```
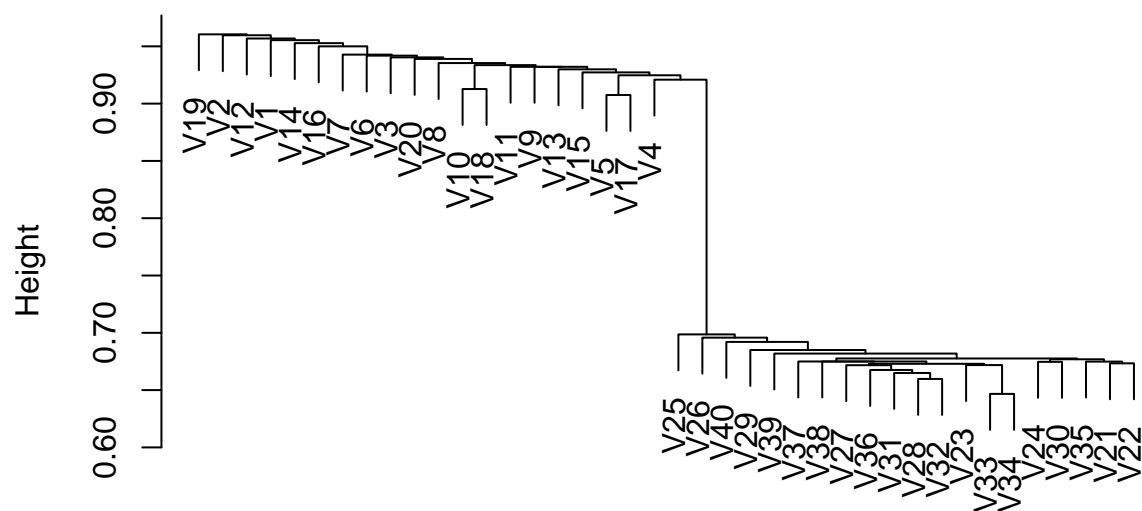
Cluster Dendrogram

corr_dist
hclust (*, "average")

```
table(cutree(average_hc, k = 2))
```

```
##
##  1  2
##  9 31
```

```
single_hc = hclust(corr_dist, method = 'single')
plot(single_hc)
```
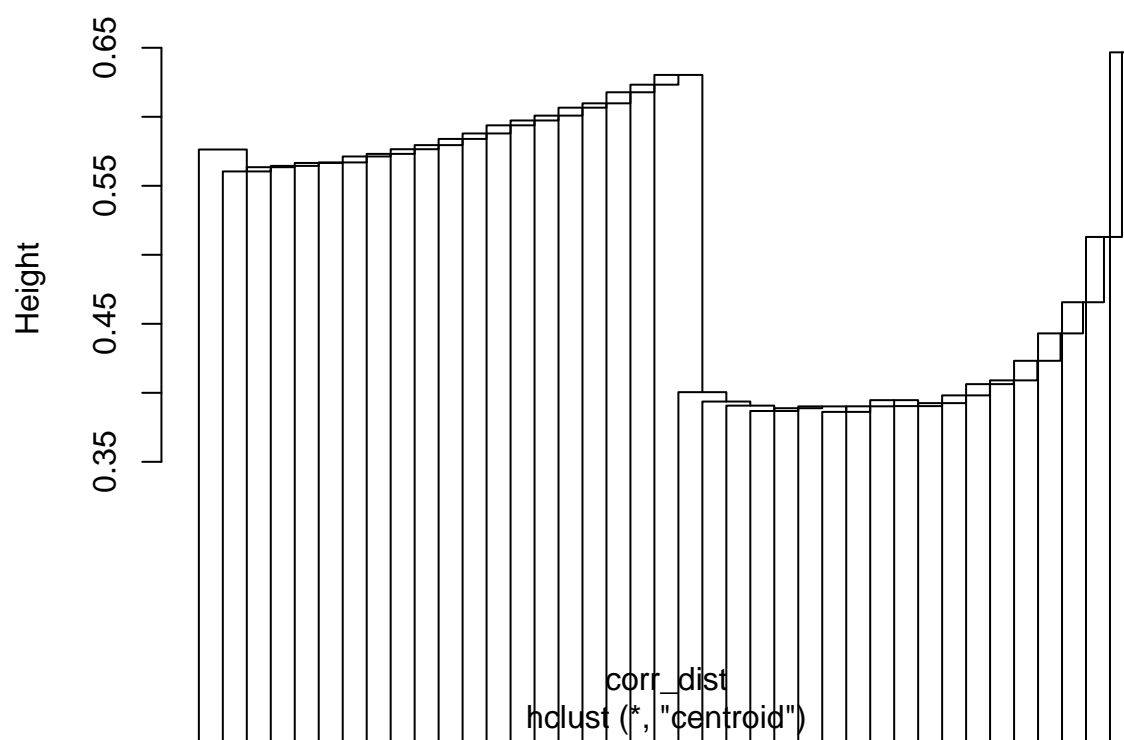
Cluster Dendrogram

corr_dist
hclust (*, "single")

```
table(cutree(single_hc, k = 2))
```

```
##
##  1  2
## 39  1
```

```
centroid_hc = hclust(corr_dist, method = 'centroid')
plot(centroid_hc)
```

Cluster Dendrogram


```
table(cutree(centroid_hc, k = 2))
```

```
##
## 1 2
## 1 39
```

The results are different depending on the type of linkage used.

c

We can transpose the data set (use genes as observations and patients as predictors), perform pca on the transposed data set, then project observations on a 2-dimension space and see which genes differ the most.

```
pca_gene = prcomp(scale(gene_exp))
loadings = pca_gene$rotation
two_dim_gene = as.matrix(gene_exp) %*% as.matrix(loadings[, 1:2])
```

```
plot(two_dim_gene)
```
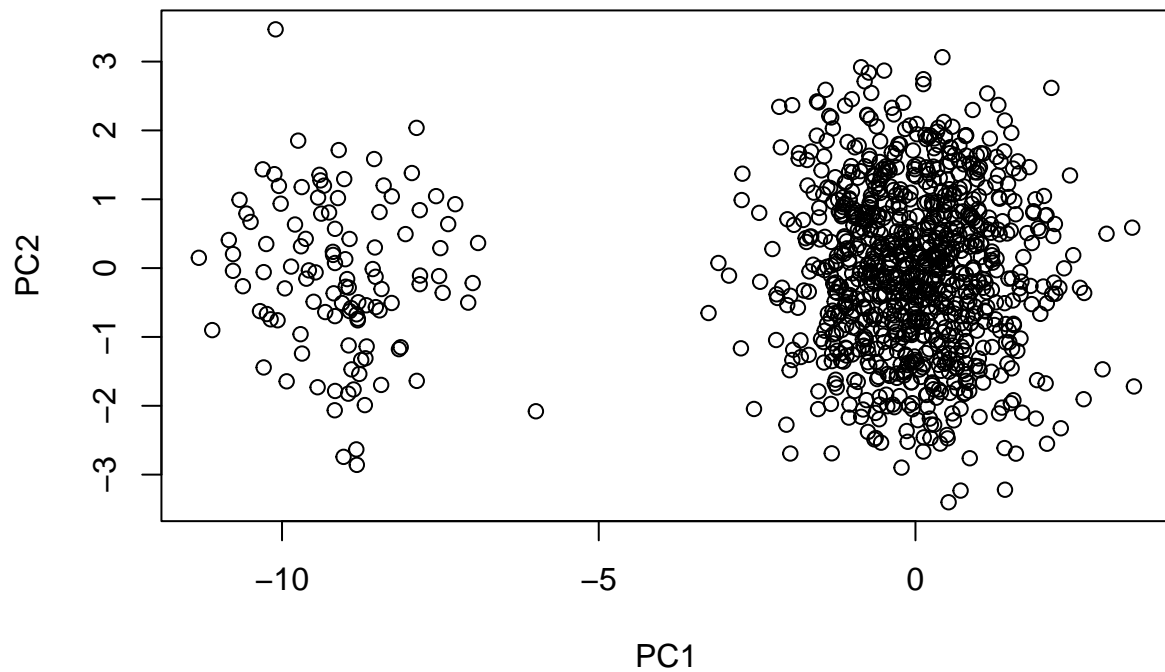
And these are genes that differ the most.

```
which(two_dim_gene[, 1] < -5)
```

```
##   [1]   11   12   13   14   15   16   17   18   19   20  501  502  503  504  505  506  507  508
##  [19]  509  510  511  512  513  514  515  516  517  518  519  520  521  522  523  524  525  526
##  [37]  527  528  529  530  531  532  533  534  535  536  537  538  539  540  541  542  543  544
##  [55]  545  546  547  548  549  550  551  552  553  554  555  556  557  558  559  560  561  562
##  [73]  563  564  565  566  567  568  569  570  571  572  573  574  575  576  577  578  579  580
##  [91]  581  582  583  584  585  586  587  588  589  590  591  592  593  594  595  596  597  598
## [109]  599  600
```