Chapter 4 Classification

2022-12-09

Exercise 1

$$(X) = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}} 1 - p\left(x\right) = 1 - \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}} = \frac{1}{1+e^{\beta_0+\beta_1 x}} \frac{p\left(x\right)}{1-p\left(x\right)} = e^{\beta_0+\beta_1 x}$$

Exercise 2

$$p_k\left(x\right) = \frac{\pi_k \dfrac{1}{\sqrt{2\pi}\sigma} \exp\left(-\dfrac{1}{2\sigma^2}\left(x-\mu_k\right)^2\right)}{\sum_{l=1}^{k} \pi_l \dfrac{1}{\sqrt{2\pi}\sigma} \exp\left(-\dfrac{1}{2\sigma^2}\left(x-\mu_l\right)^2\right)} \tag{1}$$

Taking the log of (1) from both sides

$$\log\left(p\left(x\right)\right) = \log\left(\pi_k \exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_k\right)^2\right)\right) - \log\left(\sum_{l=1}^{k} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_l\right)^2\right)\right) \tag{2}$$

Since the second term in (2) is independent of k, finding k for which (1) is largest is equivalent to find k for which (3) is largest

$$\delta_k\left(x\right) = \log\left(\pi_k\right) + \log(\exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_k\right)^2\right) \tag{3}$$

\begin{aligned}\delta _{k}\left(x\right) =\log \left(\pi _{k}\right) + -\dfrac{1}{2\sigma ^{2}}\left(x-\mu _{k}\right) ^{2} \end{aligned}

\begin{aligned}\delta _{k}\left(x\right) =\log \left(\pi _{k}\right) -\dfrac{1}{2\sigma ^{2}}x^{2}+\dfrac{1}{\sigma ^{2}}x\mu _{k}-\dfrac{1}{2\sigma ^{2}}\mu k^{2} \left(4\right) \end{aligned}

Again, as the second term is independent of k, finding k for which (4) is largest is equivalent to find k for which (5) is largest

\begin{aligned}\delta _{k}\left(x\right) =x\dfrac{\mu _{k}}{\sigma ^{2}}-\dfrac{\mu k^{2}}{2\sigma ^{2}}+\log \left(\pi _{k}\right) \left(5\right) \end{aligned}

Exercise 3

\begin{aligned}p _{k}\left(x\right) =\dfrac{\pi _{k}\dfrac{1}{\sqrt{2\pi }\sigma _{k}}\exp \left(-\dfrac{1}{2\sigma _{k}^{2}}\left(x-\mu _{k}\right) ^{2}\right) }{\sum ^{k}_{l=1}\pi _{l}\dfrac{1}{\sqrt{2\pi }\sigma _{l}}\exp \left(-\dfrac{1}{2\sigma _{l}^{2}}\left(x-\mu _{l}\right) ^{2}\right) }\left(1\right) \end{aligned}

Taking the log of (1) from both sides

\begin{aligned}\log \left(p\left(x\right) \right) =\log \left(\pi _{k}\dfrac{1}{\sqrt{2\pi }\sigma _{k}}\exp \left(-\dfrac{1}{2\sigma _{k}^{2}}\left(x-\mu _{k}\right) ^{2}\right) \right) - \log \left(\sum ^{K}_{l=1}\pi _{l}\dfrac{1}{\sqrt{2\pi }\sigma _{l}}\exp \left(-\dfrac{1}{2\sigma _{l}^{2}}\left(x-\mu _{l}\right) ^{2}\right) \right) \left(2\right) \end{aligned}

Since the second term in (2) is independent of k, finding k for which (1) is largest is equivalent to find k for which (3) is largest

\begin{aligned}\delta _{k}\left(x\right) =\log \left(\pi _{k}\right) +\log \left(\dfrac{1}{\sigma _{k}}\right) +\log \left(\exp \left(-\dfrac{1}{2\sigma _{k}^{2}}\left(x-\mu _{k}\right) ^{2}\right) \right) \left(3\right) \end{aligned}

\begin{aligned}\delta_{k}\left(x\right) =\log \left(\pi _{k}\right) +\log \left(\sigma _{k}-1\right) + -\dfrac{1}{2\sigma _{k}^{2}}\left(x-\mu _{k}\right) ^{2}\end{aligned}

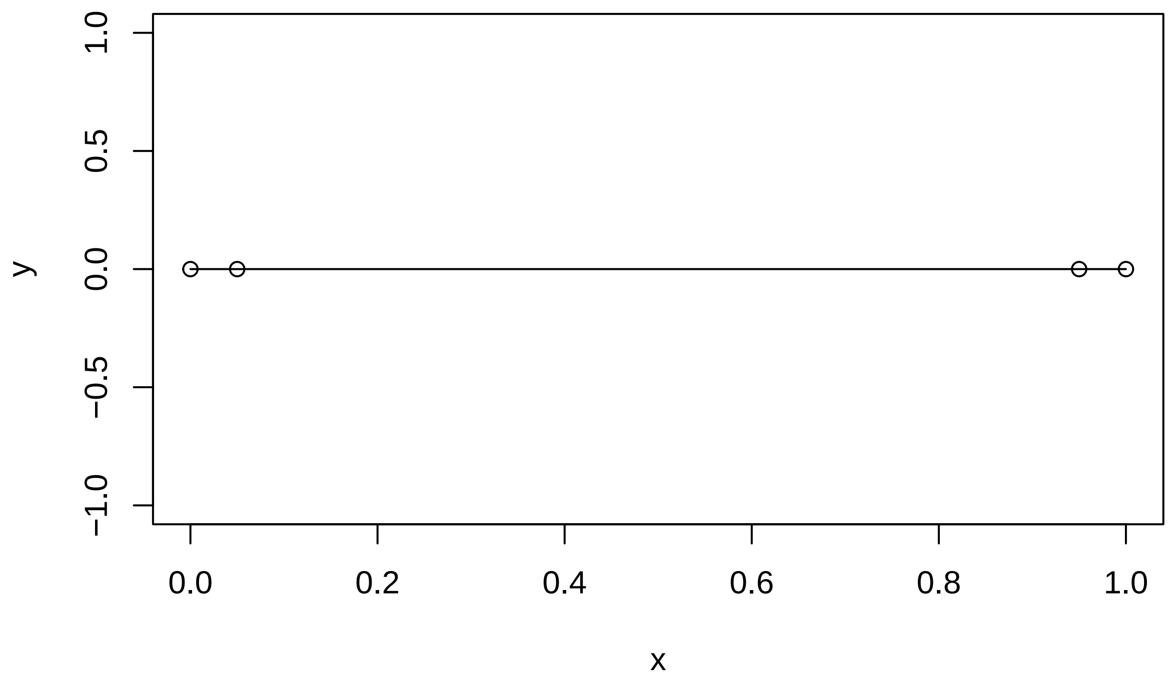
\begin{aligned}\delta _{k}\left(x\right) =-\dfrac{1}{2\sigma k^{2}}x^{2}+\dfrac{\mu _{k}}{\sigma _{k}^{2}}x+\mu _{k}^{2}+\log \left(\pi _{k}\right) -\log \left(\sigma _{k}\right) \end{aligned}

Here in this case, we can see that the Bayes classifier is in fact quadratic.

Exercise 4

a

```
x = c(0, 1)
y = c(0, 0)
plot(x, y, type = 'l')
xx = c(0, 0.05, 0.95, 1)
yy = c(0, 0, 0, 0)
points(xx, yy)
```



When p = 1, the hypercube of X is a line. It's clear that if X is in the range [0.05, 0.95], we will use observations in the range [X - 0.05, X + 0.05] and the fraction (the length of the line segment) of the available observations we will use is

\begin{aligned}\left(\left(x+0.05\right) -\left(x-0.05\right) \right) =0.01 \end{aligned}

If X is less than 0.05 or greater than 0.95, we will use observations in the range [0, X + 0.05] and [X - 0.05, 1] respectively. Hence, the fraction of the available observations we will use is (X + 0.05) and (1.05 - X) respectively. In other words, if we call f(X) is the fraction of the available observations, we can represent f(X) as below:

$$f\left(X\right) = \begin{cases} X + 0.05 & X \in [0, 0.05] \\ 0.1 & X \in [0.05, 0.95] \\ 1.05 - X & X \in [0.95, 1] \end{cases}$$

Also, we know that X is uniformly distributed, therefore

$$f\left(\overline{X}\right) = \begin{cases} \overline{X} + 0.05 = 0.025 + 0.05 = 0.075 & X \in [0, 0.05] \\ 0.1 & X \in [0.05, 0.95] \\ 1.05 - \overline{X} = 0.075 & X \in [0.95, 1] \end{cases}$$

Finally, on average, the fraction of the available observations we use to make the prediction will be

$$\overline{f}(X) = 0.05 \times 0.075 + 0.9 \times 0.1 + 0.05 \times 0.075 = 0.0975 = 9.75\%$$

b

With p = 2 features, our hypercube is now a square and the fraction of the available observations will be calculated as the area of the square

$$\overline{f}(X) = 0.0975 \times 0.0975 = 0.0975^2$$

c

$$\overline{f}(X) = 0.0975^{100}$$

d

When p approaches a very postive large number, the fraction of the available observations is

$$\lim_{n \to +\infty} \overline{f}\,(x) = 0.0975^n = 0$$

e

The length of each side of the hypercube is 0.0975

Exercise 5

a

We expect QDA to perform better on the training set and LDA to perform better on the test set.

b

We expect QDA to perform better both on the training set on the test set.

c

As n increases, the variance tends to raise as well in general so we can expect the test prediction accuracy of QDA relative to LDA to improve.

d

False because QDA might overfit the training data and yield poor generalisation.

Exercise 6

a

$$p(X) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2}} \approx 0.3775$$

b

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2$$

Plug all numbers we have into the above equation, we get

$$X_1 = 50 hours$$

Exercise 7

We have

$$\hat{p}\left(Y = k | X = x\right) = \frac{\pi_k f_k\left(x\right)}{\sum_{l=1}^{k} \pi_l f_l\left(x\right)}$$

And we need to calculate

$$\hat{p}\left(Y = 1 | X = 4\right) = \frac{\pi_1 f_1\left(x\right)}{\pi_0 f_0\left(x\right) + \pi_1 f_1\left(x\right)} \tag{1}$$

In which

$$f_0 = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(x-\mu_0)^2/2\sigma_0^2}$$

$$f_0 = \frac{1}{\sqrt{2\pi} \times 6} e^{-(4-0)^2/2\times 6^2} = \frac{1}{6\sqrt{2\pi}} e^{2/9}$$

Similarly,

$$f_1 = \frac{1}{6\sqrt{2\pi}} e^{-1/2}$$

Also,

$$\pi_0 = 0.2 \; and \; \pi_1 = 0.8$$

Plug all components we have into (1)

$$\hat{p}\left(Y = 1 | X = 4\right) = \frac{4e^{-1/2}}{1e^{-2/9} + 4e^{-1/2}} \approx 0.7519$$

Exercise 8

When K = 1, the KNN model takes just the observation itself as the closet neighbour observation so the error rate on the training set will definitely be 0%. This means the test error rate from KNN model is 36% which is higher than the test error rate from the logistic regression. In this case, in order to classify new observations, we should prefer the logistic regression.

Exercise 9

a

$$probability(default) = \frac{odds}{1 + odds} \approx 0.27$$

b

$$odds = \frac{prob}{1 - prob} \approx 0.19$$

Exercise 10

$$log\left(\frac{P(Y = k|X = x)}{P(Y = K|X = x)}\right) = \log\left(\frac{\pi k}{\pi_K}\right) - \frac{1}{2}\left(\mu_k + \mu_K\right)^T \Sigma^{-1}\left(\mu_k - \mu_K\right) + x^T \Sigma^{-1}\left(\mu_k - \mu_K\right)$$

Since p = 1,

$$\Sigma^{-1} = \sigma^{2^{-1}} \ \ and \ \ x^T = x$$

And therefore we can re-write the first equation as

$$\log\left(\frac{\pi_k}{\pi_K}\right) - \frac{1}{2\sigma^2}\left(\mu_k^2 - \mu_K^2\right) + \frac{1}{\sigma^2}\left(\mu_k - \mu_K\right) x$$

In which

$$a_k = \log\left(\frac{\pi_k}{\pi_K}\right) - \frac{1}{2\sigma^2}\left(\mu_k^2 - \mu_K^2\right) \ \ and \ \ b_{kj} = \frac{1}{\sigma^2}\left(\mu_k - \mu_K\right) x$$

Exercise 11

$$log\left(\frac{P(Y = k|X = x)}{P(Y = K|X = x)}\right) = \log\left(\frac{\pi k}{\pi_K}\right) - \frac{1}{2}\left(x - \mu_k\right)^T \Sigma_k^{-1}\left(x - \mu_k\right) + \frac{1}{2}\left(x - \mu_K\right)^T \Sigma_K^{-1}\left(x - \mu_K\right)$$

$$= \log\left(\frac{\pi k}{\pi_K}\right) - \frac{1}{2}\left(x^T\Sigma_k^{-1} - \mu_k^T\Sigma_k^{-1}\right)\left(x - \mu_k\right) + \frac{1}{2}\left(x^T\Sigma_K^{-1} - \mu_K^T\Sigma_K^{-1}\right)\left(x - \mu_K\right)$$

$$= \log\left(\frac{\pi k}{\pi_K}\right) - \frac{1}{2}\left(x^T\Sigma_k^{-1} - x^T\Sigma_k^{-1}\mu_k - \mu_k^T\Sigma_k^{-1}x + \mu_k^T\Sigma_k^{-1}\mu_k\right) + \frac{1}{2}\left(x^T\Sigma_K^{-1}x - x^T\Sigma_K^{-1}\mu_K - \mu_K^T\Sigma_K^{-1}x + \mu_K^T\Sigma_K^{-1}\mu_K\right)$$

$$= \frac{1}{2}\left(\Sigma_K^{-1} - \Sigma_k^{-1}\right) x^2 + \left(\Sigma_k^{-1}\mu_k - \Sigma_K^{-1}\mu_K\right) x + \frac{1}{2}\left(\Sigma_K^{-1}\mu_K^2 - \Sigma_k^{-1}\mu_k^2\right) + \log\left(\frac{\pi_k}{\pi_K}\right)$$

$$= a_k + \sum_{j=1}^{p} b_{kj}x_j + \sum_{j=1}^{p}\sum_{l=1}^{p} c_{kjl}x_jx_l$$

With

$$a_k = \frac{1}{2}\left(\Sigma_K^{-1}\mu_K^2 - \Sigma_k^{-1}\mu_k^2\right) + \log\left(\frac{\pi_k}{\pi_K}\right)$$

$$b_{kj} = \Sigma_k^{-1}\mu_k - \Sigma_K^{-1}\mu_K$$

and

$$c_{kjl} = \frac{1}{2}\left(\Sigma_K^{-1} - \Sigma_k^{-1}\right)$$

Exercise 12

a

$$\hat{\beta}_0 + \hat{\beta}_1 x$$

b

Recall that we have

$$odds = \frac{prob}{1 - prob}$$

Doing a bit of manipulation, the odds of orange versus apple in your friend's model is

$$\frac{exp(\hat{\alpha}_{orange0} + \hat{\alpha}_{orange1}x)}{exp(\hat{\alpha}_{apple0} + \hat{\alpha}_{apple1}x)}$$

And the log odds of orange versus apple in your friend's model is

$$\hat{\alpha}_{orange0} + \hat{\alpha}_{orange1}x - \hat{\alpha}_{apple0} - \hat{\alpha}_{apple1}x = (\hat{\alpha}_{orange0} - \hat{\alpha}_{apple0}) - (\hat{\alpha}_{orange1} + \hat{\alpha}_{apple1})x$$

c

$$\hat{\alpha}_{orange0} - \hat{\alpha}_{apple0} = \hat{\beta}_0 = 2$$

$$\hat{\alpha}_{orange1} - \hat{\alpha}_{apple0} = \hat{\beta}_1 = -1$$

d

$$\hat{\beta}_0 = \hat{\alpha}_{orange0} - \hat{\alpha}_{apple0} = -1.8$$

$$\hat{\beta}_1 = \hat{\alpha}_{orange1} - \hat{\alpha}_{apple0} = -2.6$$

e

Because the log odds of your model is the same as your friend's model, so

$$\log\left(\frac{P_{you}(orange)}{P_{you}(apple)}\right) = \log\left(\frac{P_{friend}(orange)}{P_{friend}(apple)}\right)$$

$$\frac{P_{you}(orange)}{P_{you}(apple)} = \frac{P_{friend}(orange)}{P_{friend}(apple)}$$

$$\frac{P_{you}(orange)}{P_{you}(apple) + P_{you}(orange)} = \frac{P_{friend}(orange)}{P_{friend}(apple) + P_{friend}(orange)}$$

The denominator is 1, hence

$$P_{you}(orange) = P_{friend}(orange)$$

In other words, the fraction of the time you expect the predicted class labels from your model to agree with those from your friend's model is 2000/2000 (100%)

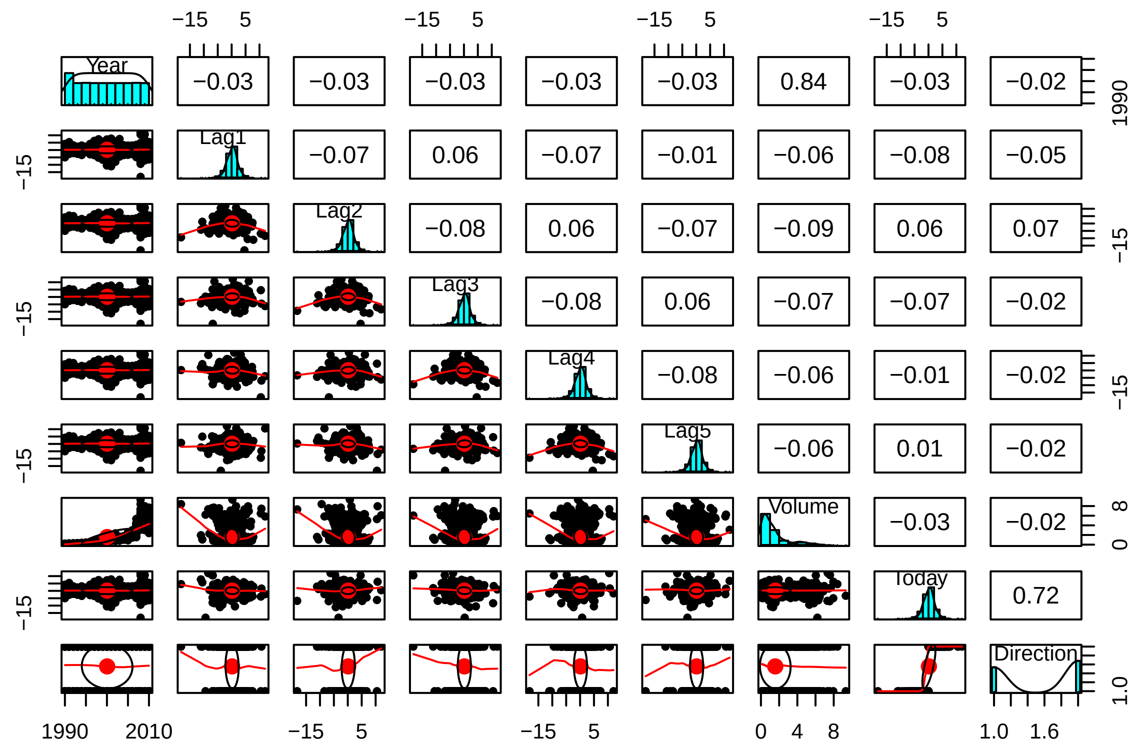
Exercise 13

```
library(ISLR2)
head(Weekly)
```

```
##   Year   Lag1   Lag2   Lag3   Lag4   Lag5    Volume  Today Direction
## 1 1990  0.816  1.572 -3.936 -0.229 -3.484 0.1549760 -0.270      Down
## 2 1990 -0.270  0.816  1.572 -3.936 -0.229 0.1485740 -2.576      Down
## 3 1990 -2.576 -0.270  0.816  1.572 -3.936 0.1598375  3.514        Up
## 4 1990  3.514 -2.576 -0.270  0.816  1.572 0.1616300  0.712        Up
## 5 1990  0.712  3.514 -2.576 -0.270  0.816 0.1537280  1.178        Up
## 6 1990  1.178  0.712  3.514 -2.576 -0.270 0.1544440 -1.372      Down
```

a

```
library(psych)
pairs.panels(Weekly)
```




```
table(Weekly$Direction)/dim(Weekly)[1]
```

```
##
##      Down        Up
## 0.4444444 0.5555556
```

b

Only predictor Lag2 appears to be statistically significant.

```
lgr_model = glm(Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 + Volume,
                data = Weekly, family = binomial)
summary(lgr_model)
```

```
##
## Call:
## glm(formula = Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 +
##     Volume, family = binomial, data = Weekly)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.6949  -1.2565   0.9913   1.0849   1.4579
##
## Coefficients:
```

```
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.26686    0.08593   3.106   0.0019 **
## Lag1        -0.04127    0.02641  -1.563   0.1181
## Lag2         0.05844    0.02686   2.175   0.0296 *
## Lag3        -0.01606    0.02666  -0.602   0.5469
## Lag4        -0.02779    0.02646  -1.050   0.2937
## Lag5        -0.01447    0.02638  -0.549   0.5833
## Volume      -0.02274    0.03690  -0.616   0.5377
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1496.2  on 1088  degrees of freedom
## Residual deviance: 1486.4  on 1082  degrees of freedom
## AIC: 1500.4
##
## Number of Fisher Scoring iterations: 4
```

c

The accuracy is approximately 47%.

```r
lgr_probs = predict(lgr_model)
lgr_preds = rep('Up', dim(Weekly)[1])
lgr_preds[lgr_probs <= 0.5] = 'Down'

mean(lgr_preds == Weekly$Direction)
```

```
## [1] 0.4655647
```

```r
table(lgr_preds, Weekly$Direction)
```

```
##
## lgr_preds Down  Up
##      Down  465 563
##      Up     19  42
```

d

```r
train_mask = Weekly$Year < 2009
train_set = Weekly[train_mask, ]
test_set = Weekly[!train_mask, ]

length(train_set)
```

```
## [1] 9
```

```
lgr_model = glm(Direction ~ Lag2, data = train_set, family = binomial)

lgr_probs = predict(lgr_model, newdata = test_set)
lgr_preds = rep('Up', dim(test_set)[1])
lgr_preds[lgr_probs <= 0.5] = 'Down'

mean(lgr_preds == test_set$Direction)
```

```
## [1] 0.4423077
```

```
table(lgr_preds, test_set$Direction)
```

```
##
## lgr_preds Down Up
##      Down   41 56
##      Up      2  5
```

e

```
library(MASS)
```

```
##
## Attaching package: 'MASS'

## The following object is masked from 'package:ISLR2':
##
##     Boston
```

```
lda_model = lda(Direction ~ Lag2, data = train_set)

lda_preds = predict(lda_model, newdata = test_set)$class

mean(lda_preds == test_set$Direction)
```

```
## [1] 0.625
```

```
table(lda_preds, test_set$Direction)
```

```
##
## lda_preds Down Up
##      Down    9  5
##      Up     34 56
```

f

```r
qda_model = qda(Direction ~ Lag2, data = train_set)

qda_preds = predict(qda_model, newdata = test_set)$class

mean(qda_preds == test_set$Direction)
```

```
## [1] 0.5865385
```

```r
table(qda_preds, test_set$Direction)
```

```
##
## qda_preds Down Up
##      Down    0  0
##      Up     43 61
```

g

```r
library(class)
knn_pred = knn(as.matrix(train_set$Lag2), as.matrix(test_set$Lag2),
               train_set$Direction, k = 1)

mean(knn_pred == test_set$Direction)
```

```
## [1] 0.5
```

```r
table(knn_pred, test_set$Direction)
```

```
##
## knn_pred Down Up
##     Down   21 30
##     Up     22 31
```

h

```r
library(e1071)
nb_model = naiveBayes(Direction ~ Lag2, train_set)
nb_preds = predict(nb_model, newdata = test_set)

mean(nb_preds == test_set$Direction)
```

```
## [1] 0.5865385
```

```r
table(nb_preds, test_set$Direction)
```

```
##
## nb_preds Down Up
##     Down    0  0
##     Up     43 61
```

i

Using one predictor Lag2, QDA and Naive Bayes models appear to provide the best results on the given data.

```r
Weekly[, 2:6]
```

```
##        Lag1   Lag2   Lag3   Lag4   Lag5
## 1     0.816  1.572 -3.936 -0.229 -3.484
## 2    -0.270  0.816  1.572 -3.936 -0.229
## 3    -2.576 -0.270  0.816  1.572 -3.936
## 4     3.514 -2.576 -0.270  0.816  1.572
## 5     0.712  3.514 -2.576 -0.270  0.816
## 6     1.178  0.712  3.514 -2.576 -0.270
## 7    -1.372  1.178  0.712  3.514 -2.576
## 8     0.807 -1.372  1.178  0.712  3.514
## 9     0.041  0.807 -1.372  1.178  0.712
## 10    1.253  0.041  0.807 -1.372  1.178
## 11   -2.678  1.253  0.041  0.807 -1.372
## 12   -1.793 -2.678  1.253  0.041  0.807
## 13    2.820 -1.793 -2.678  1.253  0.041
## 14    4.022  2.820 -1.793 -2.678  1.253
## 15    0.750  4.022  2.820 -1.793 -2.678
## 16   -0.017  0.750  4.022  2.820 -1.793
## 17    2.420 -0.017  0.750  4.022  2.820
## 18   -1.225  2.420 -0.017  0.750  4.022
## 19    1.171 -1.225  2.420 -0.017  0.750
## 20   -2.061  1.171 -1.225  2.420 -0.017
## 21    0.729 -2.061  1.171 -1.225  2.420
## 22    0.112  0.729 -2.061  1.171 -1.225
## 23    2.480  0.112  0.729 -2.061  1.171
## 24   -1.552  2.480  0.112  0.729 -2.061
## 25   -2.259 -1.552  2.480  0.112  0.729
## 26   -2.428 -2.259 -1.552  2.480  0.112
## 27   -2.708 -2.428 -2.259 -1.552  2.480
## 28   -2.292 -2.708 -2.428 -2.259 -1.552
## 29   -4.978 -2.292 -2.708 -2.428 -2.259
## 30    3.547 -4.978 -2.292 -2.708 -2.428
## 31    0.260  3.547 -4.978 -2.292 -2.708
## 32   -2.032  0.260  3.547 -4.978 -2.292
## 33   -1.739 -2.032  0.260  3.547 -4.978
## 34   -1.693 -1.739 -2.032  0.260  3.547
## 35    1.781 -1.693 -1.739 -2.032  0.260
## 36   -3.682  1.781 -1.693 -1.739 -2.032
## 37    4.150 -3.682  1.781 -1.693 -1.739
## 38   -2.487  4.150 -3.682  1.781 -1.693
## 39    2.343 -2.487  4.150 -3.682  1.781
## 40    0.606  2.343 -2.487  4.150 -3.682
## 41    1.077  0.606  2.343 -2.487  4.150
## 42   -0.637  1.077  0.606  2.343 -2.487
## 43    2.260 -0.637  1.077  0.606  2.343
## 44    1.716  2.260 -0.637  1.077  0.606
## 45   -0.284  1.716  2.260 -0.637  1.077
## 46    1.508 -0.284  1.716  2.260 -0.637
```

```
## 47    -0.913  1.508 -0.284  1.716  2.260
## 48    -2.349 -0.913  1.508 -0.284  1.716
## 49    -1.798 -2.349 -0.913  1.508 -0.284
## 50     5.393 -1.798 -2.349 -0.913  1.508
## 51     1.156  5.393 -1.798 -2.349 -0.913
## 52     2.077  1.156  5.393 -1.798 -2.349
## 53     4.751  2.077  1.156  5.393 -1.798
## 54     2.702  4.751  2.077  1.156  5.393
## 55    -0.924  2.702  4.751  2.077  1.156
## 56     1.318 -0.924  2.702  4.751  2.077
## 57     1.209  1.318 -0.924  2.702  4.751
## 58    -0.363  1.209  1.318 -0.924  2.702
## 59    -1.635 -0.363  1.209  1.318 -0.924
## 60     2.106 -1.635 -0.363  1.209  1.318
## 61     0.037  2.106 -1.635 -0.363  1.209
## 62     1.343  0.037  2.106 -1.635 -0.363
## 63     0.999  1.343  0.037  2.106 -1.635
## 64    -1.348  0.999  1.343  0.037  2.106
## 65     0.470 -1.348  0.999  1.343  0.037
## 66    -1.329  0.470 -1.348  0.999  1.343
## 67    -0.892 -1.329  0.470 -1.348  0.999
## 68     1.370 -0.892 -1.329  0.470 -1.348
## 69     3.269  1.370 -0.892 -1.329  0.470
## 70    -2.668  3.269  1.370 -0.892 -1.329
## 71     0.754 -2.668  3.269  1.370 -0.892
## 72    -1.188  0.754 -2.668  3.269  1.370
## 73    -1.745 -1.188  0.754 -2.668  3.269
## 74     0.787 -1.745 -1.188  0.754 -2.668
## 75     1.649  0.787 -1.745 -1.188  0.754
## 76     1.044  1.649  0.787 -1.745 -1.188
## 77    -0.856  1.044  1.649  0.787 -1.745
## 78     1.641 -0.856  1.044  1.649  0.787
## 79    -0.015  1.641 -0.856  1.044  1.649
## 80    -0.398 -0.015  1.641 -0.856  1.044
## 81     2.228 -0.398 -0.015  1.641 -0.856
## 82     0.320  2.228 -0.398 -0.015  1.641
## 83    -1.601  0.320  2.228 -0.398 -0.015
## 84    -1.416 -1.601  0.320  2.228 -0.398
## 85     1.129 -1.416 -1.601  0.320  2.228
## 86    -0.521  1.129 -1.416 -1.601  0.320
## 87    -1.205 -0.521  1.129 -1.416 -1.601
## 88     0.052 -1.205 -0.521  1.129 -1.416
## 89     2.897  0.052 -1.205 -0.521  1.129
## 90    -2.115  2.897  0.052 -1.205 -0.521
## 91     1.853 -2.115  2.897  0.052 -1.205
## 92     0.401  1.853 -2.115  2.897  0.052
## 93    -2.614  0.401  1.853 -2.115  2.897
## 94    -1.694 -2.614  0.401  1.853 -2.115
## 95    -0.245 -1.694 -2.614  0.401  1.853
## 96     1.034 -0.245 -1.694 -2.614  0.401
## 97     1.417  1.034 -0.245 -1.694 -2.614
## 98     0.668  1.417  1.034 -0.245 -1.694
## 99     5.018  0.668  1.417  1.034 -0.245
## 100    3.169  5.018  0.668  1.417  1.034
```

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## 101   -1.011   3.169   5.018   0.668   1.417
## 102    0.906  -1.011   3.169   5.018   0.668
## 103   -0.807   0.906  -1.011   3.169   5.018
## 104   -1.613  -0.807   0.906  -1.011   3.169
## 105    0.565  -1.613  -0.807   0.906  -1.011
## 106    0.338   0.565  -1.613  -0.807   0.906
## 107   -0.255   0.338   0.565  -1.613  -0.807
## 108    0.309  -0.255   0.338   0.565  -1.613
## 109   -2.001   0.309  -0.255   0.338   0.565
## 110    0.346  -2.001   0.309  -0.255   0.338
## 111    1.345   0.346  -2.001   0.309  -0.255
## 112   -1.896   1.345   0.346  -2.001   0.309
## 113   -0.483  -1.896   1.345   0.346  -2.001
## 114    0.682  -0.483  -1.896   1.345   0.346
## 115    2.906   0.682  -0.483  -1.896   1.345
## 116   -1.687   2.906   0.682  -0.483  -1.896
## 117    0.858  -1.687   2.906   0.682  -0.483
## 118    0.853   0.858  -1.687   2.906   0.682
## 119   -1.433   0.853   0.858  -1.687   2.906
## 120    0.958  -1.433   0.853   0.858  -1.687
## 121    0.321   0.958  -1.433   0.853   0.858
## 122   -0.450   0.321   0.958  -1.433   0.853
## 123   -0.900  -0.450   0.321   0.958  -1.433
## 124   -1.486  -0.900  -0.450   0.321   0.958
## 125   -0.054  -1.486  -0.900  -0.450   0.321
## 126    2.062  -0.054  -1.486  -0.900  -0.450
## 127    0.692   2.062  -0.054  -1.486  -0.900
## 128    0.241   0.692   2.062  -0.054  -1.486
## 129   -0.967   0.241   0.692   2.062  -0.054
## 130    3.064  -0.967   0.241   0.692   2.062
## 131   -1.256   3.064  -0.967   0.241   0.692
## 132    0.246  -1.256   3.064  -0.967   0.241
## 133   -1.205   0.246  -1.256   3.064  -0.967
## 134   -0.002  -1.205   0.246  -1.256   3.064
## 135    0.540  -0.002  -1.205   0.246  -1.256
## 136    0.599   0.540  -0.002  -1.205   0.246
## 137    0.798   0.599   0.540  -0.002  -1.205
## 138   -2.029   0.798   0.599   0.540  -0.002
## 139   -0.936  -2.029   0.798   0.599   0.540
## 140   -1.903  -0.936  -2.029   0.798   0.599
## 141    2.253  -1.903  -0.936  -2.029   0.798
## 142    0.576   2.253  -1.903  -0.936  -2.029
## 143    1.106   0.576   2.253  -1.903  -0.936
## 144   -0.263   1.106   0.576   2.253  -1.903
## 145    1.161  -0.263   1.106   0.576   2.253
## 146    0.999   1.161  -0.263   1.106   0.576
## 147    0.823   0.999   1.161  -0.263   1.106
## 148    0.442   0.823   0.999   1.161  -0.263
## 149    0.387   0.442   0.823   0.999   1.161
## 150    1.741   0.387   0.442   0.823   0.999
## 151   -0.342   1.741   0.387   0.442   0.823
## 152   -0.923  -0.342   1.741   0.387   0.442
## 153   -1.529  -0.923  -0.342   1.741   0.387
## 154    1.888  -1.529  -0.923  -0.342   1.741
```

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## 155   -0.238   1.888  -1.529  -0.923  -0.342
## 156    0.612  -0.238   1.888  -1.529  -0.923
## 157    2.313   0.612  -0.238   1.888  -1.529
## 158   -0.969   2.313   0.612  -0.238   1.888
## 159   -2.330  -0.969   2.313   0.612  -0.238
## 160    2.110  -2.330  -0.969   2.313   0.612
## 161    0.616   2.110  -2.330  -0.969   2.313
## 162    0.834   0.616   2.110  -2.330  -0.969
## 163    0.078   0.834   0.616   2.110  -2.330
## 164   -0.533   0.078   0.834   0.616   2.110
## 165   -1.427  -0.533   0.078   0.834   0.616
## 166    0.102  -1.427  -0.533   0.078   0.834
## 167    1.607   0.102  -1.427  -0.533   0.078
## 168   -2.653   1.607   0.102  -1.427  -0.533
## 169    0.723  -2.653   1.607   0.102  -1.427
## 170    0.482   0.723  -2.653   1.607   0.102
## 171   -0.622   0.482   0.723  -2.653   1.607
## 172    1.429  -0.622   0.482   0.723  -2.653
## 173    0.976   1.429  -0.622   0.482   0.723
## 174   -0.029   0.976   1.429  -0.622   0.482
## 175   -0.622  -0.029   0.976   1.429  -0.622
## 176   -0.800  -0.622  -0.029   0.976   1.429
## 177    0.884  -0.800  -0.622  -0.029   0.976
## 178   -0.393   0.884  -0.800  -0.622  -0.029
## 179    0.509  -0.393   0.884  -0.800  -0.622
## 180   -0.527   0.509  -0.393   0.884  -0.800
## 181    0.303  -0.527   0.509  -0.393   0.884
## 182    0.230   0.303  -0.527   0.509  -0.393
## 183    0.123   0.230   0.303  -0.527   0.509
## 184    0.325   0.123   0.230   0.303  -0.527
## 185    1.337   0.325   0.123   0.230   0.303
## 186    0.960   1.337   0.325   0.123   0.230
## 187    0.174   0.960   1.337   0.325   0.123
## 188    0.082   0.174   0.960   1.337   0.325
## 189   -0.626   0.082   0.174   0.960   1.337
## 190   -0.262  -0.626   0.082   0.174   0.960
## 191    0.798  -0.262  -0.626   0.082   0.174
## 192   -0.210   0.798  -0.262  -0.626   0.082
## 193    1.996  -0.210   0.798  -0.262  -0.626
## 194   -1.327   1.996  -0.210   0.798  -0.262
## 195    0.984  -1.327   1.996  -0.210   0.798
## 196   -1.766   0.984  -1.327   1.996  -0.210
## 197    1.266  -1.766   0.984  -1.327   1.996
## 198   -0.599   1.266  -1.766   0.984  -1.327
## 199    0.099  -0.599   1.266  -1.766   0.984
## 200    0.395   0.099  -0.599   1.266  -1.766
## 201   -0.207   0.395   0.099  -0.599   1.266
## 202    0.528  -0.207   0.395   0.099  -0.599
## 203    0.214   0.528  -0.207   0.395   0.099
## 204   -0.199   0.214   0.528  -0.207   0.395
## 205    0.740  -0.199   0.214   0.528  -0.207
## 206    1.066   0.740  -0.199   0.214   0.528
## 207   -0.040   1.066   0.740  -0.199   0.214
## 208    0.838  -0.040   1.066   0.740  -0.199
```

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## 209   -1.857   0.838  -0.040   1.066   0.740
## 210    0.079  -1.857   0.838  -0.040   1.066
## 211   -0.530   0.079  -1.857   0.838  -0.040
## 212   -0.346  -0.530   0.079  -1.857   0.838
## 213   -0.285  -0.346  -0.530   0.079  -1.857
## 214    0.366  -0.285  -0.346  -0.530   0.079
## 215    0.990   0.366  -0.285  -0.346  -0.530
## 216   -2.225   0.990   0.366  -0.285  -0.346
## 217   -3.216  -2.225   0.990   0.366  -0.285
## 218    0.298  -3.216  -2.225   0.990   0.366
## 219   -0.206   0.298  -3.216  -2.225   0.990
## 220    0.325  -0.206   0.298  -3.216  -2.225
## 221    0.733   0.325  -0.206   0.298  -3.216
## 222   -0.685   0.733   0.325  -0.206   0.298
## 223   -0.822  -0.685   0.733   0.325  -0.206
## 224    2.427  -0.822  -0.685   0.733   0.325
## 225    0.530   2.427  -0.822  -0.685   0.733
## 226    0.612   0.530   2.427  -0.822  -0.685
## 227   -0.317   0.612   0.530   2.427  -0.822
## 228   -0.048  -0.317   0.612   0.530   2.427
## 229   -3.414  -0.048  -0.317   0.612   0.530
## 230    0.768  -3.414  -0.048  -0.317   0.612
## 231    0.751   0.768  -3.414  -0.048  -0.317
## 232    1.025   0.751   0.768  -3.414  -0.048
## 233   -0.231   1.025   0.751   0.768  -3.414
## 234    1.137  -0.231   1.025   0.751   0.768
## 235   -0.255   1.137  -0.231   1.025   0.751
## 236    1.061  -0.255   1.137  -0.231   1.025
## 237    0.377   1.061  -0.255   1.137  -0.231
## 238    2.183   0.377   1.061  -0.255   1.137
## 239   -0.593   2.183   0.377   1.061  -0.255
## 240   -0.597  -0.593   2.183   0.377   1.061
## 241    0.643  -0.597  -0.593   2.183   0.377
## 242   -2.445   0.643  -0.597  -0.593   2.183
## 243    0.661  -2.445   0.643  -0.597  -0.593
## 244   -1.645   0.661  -2.445   0.643  -0.597
## 245    3.076  -1.645   0.661  -2.445   0.643
## 246   -0.897   3.076  -1.645   0.661  -2.445
## 247    1.910  -0.897   3.076  -1.645   0.661
## 248   -2.425   1.910  -0.897   3.076  -1.645
## 249    0.015  -2.425   1.910  -0.897   3.076
## 250   -0.190   0.015  -2.425   1.910  -0.897
## 251   -1.989  -0.190   0.015  -2.425   1.910
## 252    0.223  -1.989  -0.190   0.015  -2.425
## 253   -1.399   0.223  -1.989  -0.190   0.015
## 254    2.649  -1.399   0.223  -1.989  -0.190
## 255    0.224   2.649  -1.399   0.223  -1.989
## 256   -0.122   0.224   2.649  -1.399   0.223
## 257    0.307  -0.122   0.224   2.649  -1.399
## 258    1.148   0.307  -0.122   0.224   2.649
## 259   -0.255   1.148   0.307  -0.122   0.224
## 260    1.207  -0.255   1.148   0.307  -0.122
## 261    1.756   1.207  -0.255   1.148   0.307
## 262    0.587   1.756   1.207  -0.255   1.148
```

```
## 263    0.106   0.587   1.756   1.207  -0.255
## 264    1.274   0.106   0.587   1.756   1.207
## 265   -0.551   1.274   0.106   0.587   1.756
## 266    0.855  -0.551   1.274   0.106   0.587
## 267    1.215   0.855  -0.551   1.274   0.106
## 268    1.100   1.215   0.855  -0.551   1.274
## 269   -0.052   1.100   1.215   0.855  -0.551
## 270    1.140  -0.052   1.100   1.215   0.855
## 271    0.555   1.140  -0.052   1.100   1.215
## 272   -0.145   0.555   1.140  -0.052   1.100
## 273    1.223  -0.145   0.555   1.140  -0.052
## 274    1.051   1.223  -0.145   0.555   1.140
## 275    1.044   1.051   1.223  -0.145   0.555
## 276   -1.210   1.044   1.051   1.223  -0.145
## 277    0.859  -1.210   1.044   1.051   1.223
## 278    1.692   0.859  -1.210   1.044   1.051
## 279   -0.858   1.692   0.859  -1.210   1.044
## 280    2.252  -0.858   1.692   0.859  -1.210
## 281    1.830   2.252  -0.858   1.692   0.859
## 282   -0.902   1.830   2.252  -0.858   1.692
## 283    2.133  -0.902   1.830   2.252  -0.858
## 284    0.633   2.133  -0.902   1.830   2.252
## 285   -1.120   0.633   2.133  -0.902   1.830
## 286    1.682  -1.120   0.633   2.133  -0.902
## 287   -0.709   1.682  -1.120   0.633   2.133
## 288   -0.685  -0.709   1.682  -1.120   0.633
## 289    0.739  -0.685  -0.709   1.682  -1.120
## 290    0.159   0.739  -0.685  -0.709   1.682
## 291    0.668   0.159   0.739  -0.685  -0.709
## 292    1.568   0.668   0.159   0.739  -0.685
## 293    1.863   1.568   0.668   0.159   0.739
## 294   -0.278   1.863   1.568   0.668   0.159
## 295    0.461  -0.278   1.863   1.568   0.668
## 296   -0.329   0.461  -0.278   1.863   1.568
## 297    0.345  -0.329   0.461  -0.278   1.863
## 298    0.506   0.345  -0.329   0.461  -0.278
## 299   -1.321   0.506   0.345  -0.329   0.461
## 300    1.875  -1.321   0.506   0.345  -0.329
## 301    0.364   1.875  -1.321   0.506   0.345
## 302    1.240   0.364   1.875  -1.321   0.506
## 303   -0.017   1.240   0.364   1.875  -1.321
## 304    1.168  -0.017   1.240   0.364   1.875
## 305    1.730   1.168  -0.017   1.240   0.364
## 306   -0.185   1.730   1.168  -0.017   1.240
## 307   -0.712  -0.185   1.730   1.168  -0.017
## 308    0.650  -0.712  -0.185   1.730   1.168
## 309    0.127   0.650  -0.712  -0.185   1.730
## 310   -2.416   0.127   0.650  -0.712  -0.185
## 311    1.665  -2.416   0.127   0.650  -0.712
## 312    1.600   1.665  -2.416   0.127   0.650
## 313    2.288   1.600   1.665  -2.416   0.127
## 314    3.229   2.288   1.600   1.665  -2.416
## 315   -1.278   3.229   2.288   1.600   1.665
## 316    1.713  -1.278   3.229   2.288   1.600
```

```
## 317   -2.232   1.713  -1.278   3.229   2.288
## 318   -1.687  -2.232   1.713  -1.278   3.229
## 319    1.252  -1.687  -2.232   1.713  -1.278
## 320    1.433   1.252  -1.687  -2.232   1.713
## 321   -0.787   1.433   1.252  -1.687  -2.232
## 322    1.605  -0.787   1.433   1.252  -1.687
## 323   -2.920   1.605  -0.787   1.433   1.252
## 324    1.313  -2.920   1.605  -0.787   1.433
## 325    1.301   1.313  -2.920   1.605  -0.787
## 326   -1.810   1.301   1.313  -2.920   1.605
## 327    1.630  -1.810   1.301   1.313  -2.920
## 328    2.579   1.630  -1.810   1.301   1.313
## 329    1.435   2.579   1.630  -1.810   1.301
## 330   -1.384   1.435   2.579   1.630  -1.810
## 331    0.626  -1.384   1.435   2.579   1.630
## 332   -1.108   0.626  -1.384   1.435   2.579
## 333    0.149  -1.108   0.626  -1.384   1.435
## 334    0.568   0.149  -1.108   0.626  -1.384
## 335   -1.967   0.568   0.149  -1.108   0.626
## 336   -1.711  -1.967   0.568   0.149  -1.108
## 337   -1.154  -1.711  -1.967   0.568   0.149
## 338   -0.443  -1.154  -1.711  -1.967   0.568
## 339    4.181  -0.443  -1.154  -1.711  -1.967
## 340   -0.059   4.181  -0.443  -1.154  -1.711
## 341    0.470  -0.059   4.181  -0.443  -1.154
## 342    0.274   0.470  -0.059   4.181  -0.443
## 343   -2.255   0.274   0.470  -0.059   4.181
## 344    0.566  -2.255   0.274   0.470  -0.059
## 345    3.791   0.566  -2.255   0.274   0.470
## 346    0.954   3.791   0.566  -2.255   0.274
## 347   -0.122   0.954   3.791   0.566  -2.255
## 348    2.225  -0.122   0.954   3.791   0.566
## 349   -0.114   2.225  -0.122   0.954   3.791
## 350    1.450  -0.114   2.225  -0.122   0.954
## 351   -1.393   1.450  -0.114   2.225  -0.122
## 352    0.407  -1.393   1.450  -0.114   2.225
## 353    3.844   0.407  -1.393   1.450  -0.114
## 354    0.930   3.844   0.407  -1.393   1.450
## 355    1.506   0.930   3.844   0.407  -1.393
## 356    1.107   1.506   0.930   3.844   0.407
## 357   -2.301   1.107   1.506   0.930   3.844
## 358   -1.482  -2.301   1.107   1.506   0.930
## 359    2.776  -1.482  -2.301   1.107   1.506
## 360    1.058   2.776  -1.482  -2.301   1.107
## 361   -1.158   1.058   2.776  -1.482  -2.301
## 362    1.533  -1.158   1.058   2.776  -1.482
## 363    2.195   1.533  -1.158   1.058   2.776
## 364   -0.728   2.195   1.533  -1.158   1.058
## 365    2.030  -0.728   2.195   1.533  -1.158
## 366    0.432   2.030  -0.728   2.195   1.533
## 367    2.396   0.432   2.030  -0.728   2.195
## 368   -0.830   2.396   0.432   2.030  -0.728
## 369   -1.366  -0.830   2.396   0.432   2.030
## 370    1.789  -1.366  -0.830   2.396   0.432
```

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## 371   -1.466   1.789  -1.366  -0.830   2.396
## 372   -1.144  -1.466   1.789  -1.366  -0.830
## 373   -1.303  -1.144  -1.466   1.789  -1.366
## 374   -2.065  -1.303  -1.144  -1.466   1.789
## 375   -2.672  -2.065  -1.303  -1.144  -1.466
## 376    3.889  -2.672  -2.065  -1.303  -1.144
## 377   -0.127   3.889  -2.672  -2.065  -1.303
## 378    6.219  -0.127   3.889  -2.672  -2.065
## 379    1.453   6.219  -0.127   3.889  -2.672
## 380    0.603   1.453   6.219  -0.127   3.889
## 381    2.083   0.603   1.453   6.219  -0.127
## 382    0.148   2.083   0.603   1.453   6.219
## 383    1.147   0.148   2.083   0.603   1.453
## 384    4.110   1.147   0.148   2.083   0.603
## 385    0.608   4.110   1.147   0.148   2.083
## 386   -1.268   0.608   4.110   1.147   0.148
## 387    3.338  -1.268   0.608   4.110   1.147
## 388   -0.026   3.338  -1.268   0.608   4.110
## 389   -0.151  -0.026   3.338  -1.268   0.608
## 390    2.566  -0.151  -0.026   3.338  -1.268
## 391    0.889   2.566  -0.151  -0.026   3.338
## 392   -1.436   0.889   2.566  -0.151  -0.026
## 393   -3.506  -1.436   0.889   2.566  -0.151
## 394    2.523  -3.506  -1.436   0.889   2.566
## 395   -2.606   2.523  -3.506  -1.436   0.889
## 396    3.289  -2.606   2.523  -3.506  -1.436
## 397   -0.553   3.289  -2.606   2.523  -3.506
## 398    2.879  -0.553   3.289  -2.606   2.523
## 399   -0.557   2.879  -0.553   3.289  -2.606
## 400    2.096  -0.557   2.879  -0.553   3.289
## 401    0.202   2.096  -0.557   2.879  -0.553
## 402   -2.360   0.202   2.096  -0.557   2.879
## 403   -0.267  -2.360   0.202   2.096  -0.557
## 404   -2.869  -0.267  -2.360   0.202   2.096
## 405    1.409  -2.869  -0.267  -2.360   0.202
## 406    0.091   1.409  -2.869  -0.267  -2.360
## 407    3.742   0.091   1.409  -2.869  -0.267
## 408   -0.798   3.742   0.091   1.409  -2.869
## 409    2.972  -0.798   3.742   0.091   1.409
## 410   -3.090   2.972  -0.798   3.742   0.091
## 411   -0.693  -3.090   2.972  -0.798   3.742
## 412   -1.090  -0.693  -3.090   2.972  -0.798
## 413    4.120  -1.090  -0.693  -3.090   2.972
## 414   -4.856   4.120  -1.090  -0.693  -3.090
## 415    3.646  -4.856   4.120  -1.090  -0.693
## 416   -0.408   3.646  -4.856   4.120  -1.090
## 417    2.369  -0.408   3.646  -4.856   4.120
## 418    3.283   2.369  -0.408   3.646  -4.856
## 419    0.754   3.283   2.369  -0.408   3.646
## 420    1.384   0.754   3.283   2.369  -0.408
## 421    1.463   1.384   0.754   3.283   2.369
## 422    0.605   1.463   1.384   0.754   3.283
## 423    1.224   0.605   1.463   1.384   0.754
## 424    2.859   1.224   0.605   1.463   1.384
```

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## 425   -0.338   2.859   1.224   0.605   1.463
## 426    2.488  -0.338   2.859   1.224   0.605
## 427   -1.072   2.488  -0.338   2.859   1.224
## 428    1.085  -1.072   2.488  -0.338   2.859
## 429   -1.320   1.085  -1.072   2.488  -0.338
## 430    1.182  -1.320   1.085  -1.072   2.488
## 431   -1.147   1.182  -1.320   1.085  -1.072
## 432    0.053  -1.147   1.182  -1.320   1.085
## 433    0.157   0.053  -1.147   1.182  -1.320
## 434   -1.770   0.157   0.053  -1.147   1.182
## 435    2.112  -1.770   0.157   0.053  -1.147
## 436   -1.348   2.112  -1.770   0.157   0.053
## 437    0.165  -1.348   2.112  -1.770   0.157
## 438    2.957   0.165  -1.348   2.112  -1.770
## 439    1.167   2.957   0.165  -1.348   2.112
## 440    1.562   1.167   2.957   0.165  -1.348
## 441    1.926   1.562   1.167   2.957   0.165
## 442   -3.872   1.926   1.562   1.167   2.957
## 443   -1.765  -3.872   1.926   1.562   1.167
## 444   -2.786  -1.765  -3.872   1.926   1.562
## 445   -2.451  -2.786  -1.765  -3.872   1.926
## 446    1.740  -2.451  -2.786  -1.765  -3.872
## 447   -5.004   1.740  -2.451  -2.786  -1.765
## 448   -5.184  -5.004   1.740  -2.451  -2.786
## 449    3.611  -5.184  -5.004   1.740  -2.451
## 450    1.093   3.611  -5.184  -5.004   1.740
## 451    2.417   1.093   3.611  -5.184  -5.004
## 452   -4.034   2.417   1.093   3.611  -5.184
## 453   -1.816  -4.034   2.417   1.093   3.611
## 454    7.317  -1.816  -4.034   2.417   1.093
## 455    1.349   7.317  -1.816  -4.034   2.417
## 456    2.615   1.349   7.317  -1.816  -4.034
## 457    3.854   2.615   1.349   7.317  -1.816
## 458   -1.340   3.854   2.615   1.349   7.317
## 459    3.361  -1.340   3.854   2.615   1.349
## 460    2.473   3.361  -1.340   3.854   2.615
## 461   -1.308   2.473   3.361  -1.340   3.854
## 462   -0.874  -1.308   2.473   3.361  -1.340
## 463    1.849  -0.874  -1.308   2.473   3.361
## 464    3.219   1.849  -0.874  -1.308   2.473
## 465    0.241   3.219   1.849  -0.874  -1.308
## 466    3.731   0.241   3.219   1.849  -0.874
## 467   -2.496   3.731   0.241   3.219   1.849
## 468   -1.453  -2.496   3.731   0.241   3.219
## 469    4.444  -1.453  -2.496   3.731   0.241
## 470   -3.145   4.444  -1.453  -2.496   3.731
## 471   -0.748  -3.145   4.444  -1.453  -2.496
## 472    0.739  -0.748  -3.145   4.444  -1.453
## 473   -0.072   0.739  -0.748  -3.145   4.444
## 474    2.999  -0.072   0.739  -0.748  -3.145
## 475    1.499   2.999  -0.072   0.739  -0.748
## 476    0.363   1.499   2.999  -0.072   0.739
## 477   -1.269   0.363   1.499   2.999  -0.072
## 478    0.851  -1.269   0.363   1.499   2.999
```

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## 479    4.223   0.851  -1.269   0.363   1.499
## 480   -2.177   4.223   0.851  -1.269   0.363
## 481    2.870  -2.177   4.223   0.851  -1.269
## 482   -1.597   2.870  -2.177   4.223   0.851
## 483    0.735  -1.597   2.870  -2.177   4.223
## 484   -0.535   0.735  -1.597   2.870  -2.177
## 485   -0.561  -0.535   0.735  -1.597   2.870
## 486   -2.139  -0.561  -0.535   0.735  -1.597
## 487    1.990  -2.139  -0.561  -0.535   0.735
## 488   -2.569   1.990  -2.139  -0.561  -0.535
## 489    3.803  -2.569   1.990  -2.139  -0.561
## 490   -2.050   3.803  -2.569   1.990  -2.139
## 491    5.771  -2.050   3.803  -2.569   1.990
## 492    0.867   5.771  -2.050   3.803  -2.569
## 493    1.105   0.867   5.771  -2.050   3.803
## 494   -4.359   1.105   0.867   5.771  -2.050
## 495   -2.080  -4.359   1.105   0.867   5.771
## 496   -2.140  -2.080  -4.359   1.105   0.867
## 497    2.106  -2.140  -2.080  -4.359   1.105
## 498    0.673   2.106  -2.140  -2.080  -4.359
## 499    0.872   0.673   2.106  -2.140  -2.080
## 500    0.665   0.872   0.673   2.106  -2.140
## 501   -0.411   0.665   0.872   0.673   2.106
## 502   -1.201  -0.411   0.665   0.872   0.673
## 503   -4.348  -1.201  -0.411   0.665   0.872
## 504    0.427  -4.348  -1.201  -0.411   0.665
## 505    4.148   0.427  -4.348  -1.201  -0.411
## 506   -6.632   4.148   0.427  -4.348  -1.201
## 507    4.348  -6.632   4.148   0.427  -4.348
## 508    4.708   4.348  -6.632   4.148   0.427
## 509    0.536   4.708   4.348  -6.632   4.148
## 510    1.885   0.536   4.708   4.348  -6.632
## 511    1.858   1.885   0.536   4.708   4.348
## 512   -0.378   1.858   1.885   0.536   4.708
## 513    1.177  -0.378   1.858   1.885   0.536
## 514   -1.134   1.177  -0.378   1.858   1.885
## 515    0.282  -1.134   1.177  -0.378   1.858
## 516    2.626   0.282  -1.134   1.177  -0.378
## 517    0.748   2.626   0.282  -1.134   1.177
## 518   -1.891   0.748   2.626   0.282  -1.134
## 519    1.643  -1.891   0.748   2.626   0.282
## 520   -1.624   1.643  -1.891   0.748   2.626
## 521   -5.634  -1.624   1.643  -1.891   0.748
## 522    4.721  -5.634  -1.624   1.643  -1.891
## 523   -2.615   4.721  -5.634  -1.624   1.643
## 524   -2.958  -2.615   4.721  -5.634  -1.624
## 525   -0.946  -2.958  -2.615   4.721  -5.634
## 526    5.686  -0.946  -2.958  -2.615   4.721
## 527   -1.001   5.686  -0.946  -2.958  -2.615
## 528    4.975  -1.001   5.686  -0.946  -2.958
## 529    4.301   4.975  -1.001   5.686  -0.946
## 530   -1.891   4.301   4.975  -1.001   5.686
## 531    1.186  -1.891   4.301   4.975  -1.001
## 532  -10.538   1.186  -1.891   4.301   4.975
```

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## 533    5.748 -10.538   1.186  -1.891   4.301
## 534    1.247   5.748 -10.538   1.186  -1.891
## 535   -1.363   1.247   5.748 -10.538   1.186
## 536   -0.815  -1.363   1.247   5.748 -10.538
## 537   -0.986  -0.815  -1.363   1.247   5.748
## 538   -2.056  -0.986  -0.815  -1.363   1.247
## 539    7.202  -2.056  -0.986  -0.815  -1.363
## 540   -1.375   7.202  -2.056  -0.986  -0.815
## 541    0.515  -1.375   7.202  -2.056  -0.986
## 542   -1.569   0.515  -1.375   7.202  -2.056
## 543    0.910  -1.569   0.515  -1.375   7.202
## 544    1.671   0.910  -1.569   0.515  -1.375
## 545    2.102   1.671   0.910  -1.569   0.515
## 546   -1.973   2.102   1.671   0.910  -1.569
## 547   -4.074  -1.973   2.102   1.671   0.910
## 548    3.031  -4.074  -1.973   2.102   1.671
## 549    0.609   3.031  -4.074  -1.973   2.102
## 550    1.351   0.609   3.031  -4.074  -1.973
## 551    0.987   1.351   0.609   3.031  -4.074
## 552    0.951   0.987   1.351   0.609   3.031
## 553   -1.727   0.951   0.987   1.351   0.609
## 554   -1.920  -1.727   0.951   0.987   1.351
## 555   -1.166  -1.920  -1.727   0.951   0.987
## 556   -0.843  -1.166  -1.920  -1.727   0.951
## 557   -1.916  -0.843  -1.166  -1.920  -1.727
## 558   -2.471  -1.916  -0.843  -1.166  -1.920
## 559    1.656  -2.471  -1.916  -0.843  -1.166
## 560   -1.242   1.656  -2.471  -1.916  -0.843
## 561    3.415  -1.242   1.656  -2.471  -1.916
## 562   -4.255   3.415  -1.242   1.656  -2.471
## 563    0.127  -4.255   3.415  -1.242   1.656
## 564   -1.897   0.127  -4.255   3.415  -1.242
## 565   -1.978  -1.897   0.127  -4.255   3.415
## 566    4.156  -1.978  -1.897   0.127  -4.255
## 567   -4.215   4.156  -1.978  -1.897   0.127
## 568   -0.473  -4.215   4.156  -1.978  -1.897
## 569    1.097  -0.473  -4.215   4.156  -1.978
## 570   -1.661   1.097  -0.473  -4.215   4.156
## 571    1.556  -1.661   1.097  -0.473  -4.215
## 572    1.819   1.556  -1.661   1.097  -0.473
## 573    0.924   1.819   1.556  -1.661   1.097
## 574   -0.404   0.924   1.819   1.556  -1.661
## 575   -2.572  -0.404   0.924   1.819   1.556
## 576   -1.006  -2.572  -0.404   0.924   1.819
## 577   -4.277  -1.006  -2.572  -0.404   0.924
## 578   -0.938  -4.277  -1.006  -2.572  -0.404
## 579   -0.062  -0.938  -4.277  -1.006  -2.572
## 580   -6.720  -0.062  -0.938  -4.277  -1.006
## 581   -0.930  -6.720  -0.062  -0.938  -4.277
## 582    1.799  -0.930  -6.720  -0.062  -0.938
## 583   -2.749   1.799  -0.930  -6.720  -0.062
## 584    4.880  -2.749   1.799  -0.930  -6.720
## 585    5.026   4.880  -2.749   1.799  -0.930
## 586    0.810   5.026   4.880  -2.749   1.799
```

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## 587    1.082   0.810   5.026   4.880  -2.749
## 588   -1.653   1.082   0.810   5.026   4.880
## 589    3.716  -1.653   1.082   0.810   5.026
## 590   -1.089   3.716  -1.653   1.082   0.810
## 591   -1.348  -1.089   3.716  -1.653   1.082
## 592    0.340  -1.348  -1.089   3.716  -1.653
## 593   -4.000   0.340  -1.348  -1.089   3.716
## 594    0.905  -4.000   0.340  -1.348  -1.089
## 595   -0.079   0.905  -4.000   0.340  -1.348
## 596   -2.760  -0.079   0.905  -4.000   0.340
## 597    2.107  -2.760  -0.079   0.905  -4.000
## 598   -0.397   2.107  -2.760  -0.079   0.905
## 599   -0.415  -0.397   2.107  -2.760  -0.079
## 600    0.707  -0.415  -0.397   2.107  -2.760
## 601   -1.992   0.707  -0.415  -0.397   2.107
## 602   -2.369  -1.992   0.707  -0.415  -0.397
## 603    1.976  -2.369  -1.992   0.707  -0.415
## 604   -4.334   1.976  -2.369  -1.992   0.707
## 605   -4.217  -4.334   1.976  -2.369  -1.992
## 606  -11.050  -4.217  -4.334   1.976  -2.369
## 607    7.780 -11.050  -4.217  -4.334   1.976
## 608    2.924   7.780 -11.050  -4.217  -4.334
## 609    1.892   2.924   7.780 -11.050  -4.217
## 610   -1.664   1.892   2.924   7.780 -11.050
## 611    2.900  -1.664   1.892   2.924   7.780
## 612   -1.576   2.900  -1.664   1.892   2.924
## 613    3.045  -1.576   2.900  -1.664   1.892
## 614    1.637   3.045  -1.576   2.900  -1.664
## 615    1.027   1.637   3.045  -1.576   2.900
## 616   -0.947   1.027   1.637   3.045  -1.576
## 617    1.655  -0.947   1.027   1.637   3.045
## 618   -3.041   1.655  -0.947   1.027   1.637
## 619    1.941  -3.041   1.655  -0.947   1.027
## 620    1.409   1.941  -3.041   1.655  -0.947
## 621    0.990   1.409   1.941  -3.041   1.655
## 622   -2.295   0.990   1.409   1.941  -3.041
## 623   -1.573  -2.295   0.990   1.409   1.941
## 624    0.506  -1.573  -2.295   0.990   1.409
## 625   -0.978   0.506  -1.573  -2.295   0.990
## 626   -2.315  -0.978   0.506  -1.573  -2.295
## 627    0.726  -2.315  -0.978   0.506  -1.573
## 628   -1.299   0.726  -2.315  -0.978   0.506
## 629    3.848  -1.299   0.726  -2.315  -0.978
## 630    2.874   3.848  -1.299   0.726  -2.315
## 631    0.159   2.874   3.848  -1.299   0.726
## 632   -1.497   0.159   2.874   3.848  -1.299
## 633   -0.114  -1.497   0.159   2.874   3.848
## 634   -2.149  -0.114  -1.497   0.159   2.874
## 635   -1.044  -2.149  -0.114  -1.497   0.159
## 636    1.275  -1.044  -2.149  -0.114  -1.497
## 637   -4.342   1.275  -1.044  -2.149  -0.114
## 638   -0.269  -4.342   1.275  -1.044  -2.149
## 639   -1.718  -0.269  -4.342   1.275  -1.044
## 640    4.891  -1.718  -0.269  -4.342   1.275
```

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## 641   -2.058   4.891  -1.718  -0.269  -4.342
## 642   -1.539  -2.058   4.891  -1.718  -0.269
## 643   -3.712  -1.539  -2.058   4.891  -1.718
## 644   -1.972  -3.712  -1.539  -2.058   4.891
## 645   -1.800  -1.972  -3.712  -1.539  -2.058
## 646    0.069  -1.800  -1.972  -3.712  -1.539
## 647   -0.080   0.069  -1.800  -1.972  -3.712
## 648   -6.839  -0.080   0.069  -1.800  -1.972
## 649   -7.992  -6.839  -0.080   0.069  -1.800
## 650    0.600  -7.992  -6.839  -0.080   0.069
## 651    1.337   0.600  -7.992  -6.839  -0.080
## 652    5.137   1.337   0.600  -7.992  -6.839
## 653    2.215   5.137   1.337   0.600  -7.992
## 654    1.302   2.215   5.137   1.337   0.600
## 655   -2.635   1.302   2.215   5.137   1.337
## 656   -2.418  -2.635   1.302   2.215   5.137
## 657   -0.460  -2.418  -2.635   1.302   2.215
## 658   -4.992  -0.460  -2.418  -2.635   1.302
## 659   -2.132  -4.992  -0.460  -2.418  -2.635
## 660   -3.238  -2.132  -4.992  -0.460  -2.418
## 661    4.339  -3.238  -2.132  -4.992  -0.460
## 662    5.874   4.339  -3.238  -2.132  -4.992
## 663    1.499   5.874   4.339  -3.238  -2.132
## 664    0.369   1.499   5.874   4.339  -3.238
## 665   -0.690   0.369   1.499   5.874   4.339
## 666    1.687  -0.690   0.369   1.499   5.874
## 667    2.277   1.687  -0.690   0.369   1.499
## 668    0.619   2.277   1.687  -0.690   0.369
## 669   -2.572   0.619   2.277   1.687  -0.690
## 670   -2.494  -2.572   0.619   2.277   1.687
## 671    0.706  -2.494  -2.572   0.619   2.277
## 672   -2.273   0.706  -2.494  -2.572   0.619
## 673    3.791  -2.273   0.706  -2.494  -2.572
## 674    2.089   3.791  -2.273   0.706  -2.494
## 675   -2.780   2.089   3.791  -2.273   0.706
## 676   -4.478  -2.780   2.089   3.791  -2.273
## 677   -0.662  -4.478  -2.780   2.089   3.791
## 678   -3.040  -0.662  -4.478  -2.780   2.089
## 679    0.627  -3.040  -0.662  -4.478  -2.780
## 680    1.591   0.627  -3.040  -0.662  -4.478
## 681   -0.828   1.591   0.627  -3.040  -0.662
## 682   -1.458  -0.828   1.591   0.627  -3.040
## 683    0.528  -1.458  -0.828   1.591   0.627
## 684    7.503   0.528  -1.458  -0.828   1.591
## 685   -3.605   7.503   0.528  -1.458  -0.828
## 686    1.778  -3.605   7.503   0.528  -1.458
## 687   -1.200   1.778  -3.605   7.503   0.528
## 688    2.911  -1.200   1.778  -3.605   7.503
## 689    0.585   2.911  -1.200   1.778  -3.605
## 690    3.479   0.585   2.911  -1.200   1.778
## 691    0.358   3.479   0.585   2.911  -1.200
## 692    1.167   0.358   3.479   0.585   2.911
## 693   -1.173   1.167   0.358   3.479   0.585
## 694    3.254  -1.173   1.167   0.358   3.479
```

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## 695    2.508   3.254  -1.173   1.167   0.358
## 696    0.086   2.508   3.254  -1.173   1.167
## 697    0.716   0.086   2.508   3.254  -1.173
## 698   -1.955   0.716   0.086   2.508   3.254
## 699    0.971  -1.955   0.716   0.086   2.508
## 700    1.262   0.971  -1.955   0.716   0.086
## 701   -0.483   1.262   0.971  -1.955   0.716
## 702    0.540  -0.483   1.262   0.971  -1.955
## 703   -1.855   0.540  -0.483   1.262   0.971
## 704   -0.261  -1.855   0.540  -0.483   1.262
## 705    1.338  -0.261  -1.855   0.540  -0.483
## 706    0.241   1.338  -0.261  -1.855   0.540
## 707    1.505   0.241   1.338  -0.261  -1.855
## 708    1.327   1.505   0.241   1.338  -0.261
## 709   -0.270   1.327   1.505   0.241   1.338
## 710    1.735  -0.270   1.327   1.505   0.241
## 711   -3.807   1.735  -0.270   1.327   1.505
## 712    3.310  -3.807   1.735  -0.270   1.327
## 713    0.797   3.310  -3.807   1.735  -0.270
## 714    0.121   0.797   3.310  -3.807   1.735
## 715   -1.002   0.121   0.797   3.310  -3.807
## 716    2.119  -1.002   0.121   0.797   3.310
## 717    0.238   2.119  -1.002   0.121   0.797
## 718   -0.272   0.238   2.119  -1.002   0.121
## 719   -1.435  -0.272   0.238   2.119  -1.002
## 720    2.214  -1.435  -0.272   0.238   2.119
## 721    0.312   2.214  -1.435  -0.272   0.238
## 722    1.191   0.312   2.214  -1.435  -0.272
## 723    1.352   1.191   0.312   2.214  -1.435
## 724    0.664   1.352   1.191   0.312   2.214
## 725    1.149   0.664   1.352   1.191   0.312
## 726    1.207   1.149   0.664   1.352   1.191
## 727    1.602   1.207   1.149   0.664   1.352
## 728    0.151   1.602   1.207   1.149   0.664
## 729   -0.913   0.151   1.602   1.207   1.149
## 730    1.028  -0.913   0.151   1.602   1.207
## 731    0.267   1.028  -0.913   0.151   1.602
## 732   -0.148   0.267   1.028  -0.913   0.151
## 733    0.073  -0.148   0.267   1.028  -0.913
## 734    1.041   0.073  -0.148   0.267   1.028
## 735   -3.137   1.041   0.073  -0.148   0.267
## 736   -0.963  -3.137   1.041   0.073  -0.148
## 737   -0.155  -0.963  -3.137   1.041   0.073
## 738    3.046  -0.155  -0.963  -3.137   1.041
## 739   -0.218   3.046  -0.155  -0.963  -3.137
## 740   -0.413  -0.218   3.046  -0.155  -0.963
## 741    0.528  -0.413  -0.218   3.046  -0.155
## 742   -2.920   0.528  -0.413  -0.218   3.046
## 743   -0.777  -2.920   0.528  -0.413  -0.218
## 744   -0.273  -0.777  -2.920   0.528  -0.413
## 745   -0.195  -0.273  -0.777  -2.920   0.528
## 746    2.480  -0.195  -0.273  -0.777  -2.920
## 747    0.162   2.480  -0.195  -0.273  -0.777
## 748    1.245   0.162   2.480  -0.195  -0.273
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## 749   -0.128   1.245   0.162   2.480  -0.195
## 750   -0.052  -0.128   1.245   0.162   2.480
## 751   -0.798  -0.052  -0.128   1.245   0.162
## 752   -1.117  -0.798  -0.052  -0.128   1.245
## 753   -1.026  -1.117  -0.798  -0.052  -0.128
## 754   -1.379  -1.026  -1.117  -0.798  -0.052
## 755    1.429  -1.379  -1.026  -1.117  -0.798
## 756   -3.426   1.429  -1.379  -1.026  -1.117
## 757    0.078  -3.426   1.429  -1.379  -1.026
## 758    3.151   0.078  -3.426   1.429  -1.379
## 759    0.858   3.151   0.078  -3.426   1.429
## 760    0.529   0.858   3.151   0.078  -3.426
## 761    0.924   0.529   0.858   3.151   0.078
## 762    0.412   0.924   0.529   0.858   3.151
## 763   -1.634   0.412   0.924   0.529   0.858
## 764    1.927  -1.634   0.412   0.924   0.529
## 765   -0.827   1.927  -1.634   0.412   0.924
## 766   -1.242  -0.827   1.927  -1.634   0.412
## 767   -1.124  -1.242  -0.827   1.927  -1.634
## 768    3.145  -1.124  -1.242  -0.827   1.927
## 769    3.183   3.145  -1.124  -1.242  -0.827
## 770    1.544   3.183   3.145  -1.124  -1.242
## 771   -1.168   1.544   3.183   3.145  -1.124
## 772    1.052  -1.168   1.544   3.183   3.145
## 773    0.720   1.052  -1.168   1.544   3.183
## 774   -0.266   0.720   1.052  -1.168   1.544
## 775    0.522  -0.266   0.720   1.052  -1.168
## 776    1.334   0.522  -0.266   0.720   1.052
## 777    0.148   1.334   0.522  -0.266   0.720
## 778   -2.123   0.148   1.334   0.522  -0.266
## 779   -0.141  -2.123   0.148   1.334   0.522
## 780   -1.406  -0.141  -2.123   0.148   1.334
## 781    0.299  -1.406  -0.141  -2.123   0.148
## 782    2.704   0.299  -1.406  -0.141  -2.123
## 783    0.189   2.704   0.299  -1.406  -0.141
## 784   -0.308   0.189   2.704   0.299  -1.406
## 785    0.814  -0.308   0.189   2.704   0.299
## 786    0.887   0.814  -0.308   0.189   2.704
## 787   -1.803   0.887   0.814  -0.308   0.189
## 788   -0.869  -1.803   0.887   0.814  -0.308
## 789   -1.532  -0.869  -1.803   0.887   0.814
## 790    0.128  -1.532  -0.869  -1.803   0.887
## 791    0.706   0.128  -1.532  -0.869  -1.803
## 792   -3.266   0.706   0.128  -1.532  -0.869
## 793    0.831  -3.266   0.706   0.128  -1.532
## 794    0.411   0.831  -3.266   0.706   0.128
## 795    1.253   0.411   0.831  -3.266   0.706
## 796   -1.477   1.253   0.411   0.831  -3.266
## 797    3.053  -1.477   1.253   0.411   0.831
## 798    0.799   3.053  -1.477   1.253   0.411
## 799   -0.230   0.799   3.053  -1.477   1.253
## 800    0.175  -0.230   0.799   3.053  -1.477
## 801    1.573   0.175  -0.230   0.799   3.053
## 802   -2.086   1.573   0.175  -0.230   0.799
```

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## 803    0.241  -2.086   1.573   0.175  -0.230
## 804    1.458   0.241  -2.086   1.573   0.175
## 805    1.325   1.458   0.241  -2.086   1.573
## 806    0.469   1.325   1.458   0.241  -2.086
## 807    0.041   0.469   1.325   1.458   0.241
## 808   -0.629   0.041   0.469   1.325   1.458
## 809    0.324  -0.629   0.041   0.469   1.325
## 810   -0.868   0.324  -0.629   0.041   0.469
## 811   -1.198  -0.868   0.324  -0.629   0.041
## 812    1.072  -1.198  -0.868   0.324  -0.629
## 813    1.926   1.072  -1.198  -0.868   0.324
## 814   -0.288   1.926   1.072  -1.198  -0.868
## 815   -1.827  -0.288   1.926   1.072  -1.198
## 816    1.112  -1.827  -0.288   1.926   1.072
## 817   -2.678   1.112  -1.827  -0.288   1.926
## 818   -0.780  -2.678   1.112  -1.827  -0.288
## 819   -0.588  -0.780  -2.678   1.112  -1.827
## 820    1.595  -0.588  -0.780  -2.678   1.112
## 821    1.813   1.595  -0.588  -0.780  -2.678
## 822    1.195   1.813   1.595  -0.588  -0.780
## 823    1.097   1.195   1.813   1.595  -0.588
## 824    1.601   1.097   1.195   1.813   1.595
## 825   -0.250   1.601   1.097   1.195   1.813
## 826   -0.451  -0.250   1.601   1.097   1.195
## 827    0.631  -0.451  -0.250   1.601   1.097
## 828    0.106   0.631  -0.451  -0.250   1.601
## 829   -1.606   0.106   0.631  -0.451  -0.250
## 830    2.977  -1.606   0.106   0.631  -0.451
## 831    0.168   2.977  -1.606   0.106   0.631
## 832   -2.029   0.168   2.977  -1.606   0.106
## 833    1.762  -2.029   0.168   2.977  -1.606
## 834   -1.534   1.762  -2.029   0.168   2.977
## 835    0.234  -1.534   1.762  -2.029   0.168
## 836    1.598   0.234  -1.534   1.762  -2.029
## 837    0.170   1.598   0.234  -1.534   1.762
## 838   -0.171   0.170   1.598   0.234  -1.534
## 839   -0.451  -0.171   0.170   1.598   0.234
## 840    2.016  -0.451  -0.171   0.170   1.598
## 841   -0.329   2.016  -0.451  -0.171   0.170
## 842   -0.620  -0.329   2.016  -0.451  -0.171
## 843    0.049  -0.620  -0.329   2.016  -0.451
## 844   -0.492   0.049  -0.620  -0.329   2.016
## 845    1.719  -0.492   0.049  -0.620  -0.329
## 846   -0.051   1.719  -0.492   0.049  -0.620
## 847    1.156  -0.051   1.719  -0.492   0.049
## 848   -2.604   1.156  -0.051   1.719  -0.492
## 849   -1.875  -2.604   1.156  -0.051   1.719
## 850    1.036  -1.875  -2.604   1.156  -0.051
## 851    0.630   1.036  -1.875  -2.604   1.156
## 852   -2.788   0.630   1.036  -1.875  -2.604
## 853   -0.061  -2.788   0.630   1.036  -1.875
## 854   -0.563  -0.061  -2.788   0.630   1.036
## 855    2.065  -0.563  -0.061  -2.788   0.630
## 856   -0.372   2.065  -0.563  -0.061  -2.788
```

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## 857   -2.314  -0.372   2.065  -0.563  -0.061
## 858    0.331  -2.314  -0.372   2.065  -0.563
## 859    3.085   0.331  -2.314  -0.372   2.065
## 860    0.063   3.085   0.331  -2.314  -0.372
## 861   -0.986   0.063   3.085   0.331  -2.314
## 862    2.807  -0.986   0.063   3.085   0.331
## 863   -0.554   2.807  -0.986   0.063   3.085
## 864    1.229  -0.554   2.807  -0.986   0.063
## 865   -0.922   1.229  -0.554   2.807  -0.986
## 866    1.597  -0.922   1.229  -0.554   2.807
## 867   -0.370   1.597  -0.922   1.229  -0.554
## 868    1.603  -0.370   1.597  -0.922   1.229
## 869    1.029   1.603  -0.370   1.597  -0.922
## 870    1.188   1.029   1.603  -0.370   1.597
## 871    0.218   1.188   1.029   1.603  -0.370
## 872    0.639   0.218   1.188   1.029   1.603
## 873   -0.947   0.639   0.218   1.188   1.029
## 874    1.217  -0.947   0.639   0.218   1.188
## 875    1.470   1.217  -0.947   0.639   0.218
## 876   -0.018   1.470   1.217  -0.947   0.639
## 877   -0.303  -0.018   1.470   1.217  -0.947
## 878    0.940  -0.303  -0.018   1.470   1.217
## 879    1.224   0.940  -0.303  -0.018   1.470
## 880   -1.144   1.224   0.940  -0.303  -0.018
## 881    0.534  -1.144   1.224   0.940  -0.303
## 882   -0.606   0.534  -1.144   1.224   0.940
## 883    1.491  -0.606   0.534  -1.144   1.224
## 884   -0.016   1.491  -0.606   0.534  -1.144
## 885   -0.582  -0.016   1.491  -0.606   0.534
## 886    1.843  -0.582  -0.016   1.491  -0.606
## 887   -0.713   1.843  -0.582  -0.016   1.491
## 888    1.216  -0.713   1.843  -0.582  -0.016
## 889   -0.299   1.216  -0.713   1.843  -0.582
## 890   -4.412  -0.299   1.216  -0.713   1.843
## 891    1.130  -4.412  -0.299   1.216  -0.713
## 892   -1.133   1.130  -4.412  -0.299   1.216
## 893    3.544  -1.133   1.130  -4.412  -0.299
## 894   -1.062   3.544  -1.133   1.130  -4.412
## 895    1.612  -1.062   3.544  -1.133   1.130
## 896    0.630   1.612  -1.062   3.544  -1.133
## 897    2.168   0.630   1.612  -1.062   3.544
## 898    0.655   2.168   0.630   1.612  -1.062
## 899    0.773   0.655   2.168   0.630   1.612
## 900    0.015   0.773   0.655   2.168   0.630
## 901    1.122   0.015   0.773   0.655   2.168
## 902   -0.461   1.122   0.015   0.773   0.655
## 903    1.360  -0.461   1.122   0.015   0.773
## 904   -1.866   1.360  -0.461   1.122   0.015
## 905    1.674  -1.866   1.360  -0.461   1.122
## 906   -1.980   1.674  -1.866   1.360  -0.461
## 907    0.053  -1.980   1.674  -1.866   1.360
## 908    1.802   0.053  -1.980   1.674  -1.866
## 909    1.441   1.802   0.053  -1.980   1.674
## 910   -1.185   1.441   1.802   0.053  -1.980
```

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## 911   -4.899  -1.185   1.441   1.802   0.053
## 912   -1.775  -4.899  -1.185   1.441   1.802
## 913    1.436  -1.775  -4.899  -1.185   1.441
## 914   -0.530   1.436  -1.775  -4.899  -1.185
## 915    2.312  -0.530   1.436  -1.775  -4.899
## 916   -0.364   2.312  -0.530   1.436  -1.775
## 917   -1.387  -0.364   2.312  -0.530   1.436
## 918    2.112  -1.387  -0.364   2.312  -0.530
## 919    2.796   2.112  -1.387  -0.364   2.312
## 920    0.066   2.796   2.112  -1.387  -0.364
## 921    2.020   0.066   2.796   2.112  -1.387
## 922    0.270   2.020   0.066   2.796   2.112
## 923   -3.917   0.270   2.020   0.066   2.796
## 924    2.309  -3.917   0.270   2.020   0.066
## 925   -1.669   2.309  -3.917   0.270   2.020
## 926   -3.706  -1.669   2.309  -3.917   0.270
## 927    0.347  -3.706  -1.669   2.309  -3.917
## 928   -1.237   0.347  -3.706  -1.669   2.309
## 929    2.807  -1.237   0.347  -3.706  -1.669
## 930    1.588   2.807  -1.237   0.347  -3.706
## 931   -2.440   1.588   2.807  -1.237   0.347
## 932    1.125  -2.440   1.588   2.807  -1.237
## 933   -0.402   1.125  -2.440   1.588   2.807
## 934   -4.522  -0.402   1.125  -2.440   1.588
## 935   -0.752  -4.522  -0.402   1.125  -2.440
## 936   -5.412  -0.752  -4.522  -0.402   1.125
## 937    0.409  -5.412  -0.752  -4.522  -0.402
## 938    4.871   0.409  -5.412  -0.752  -4.522
## 939   -4.596   4.871   0.409  -5.412  -0.752
## 940    1.405  -4.596   4.871   0.409  -5.412
## 941    0.231   1.405  -4.596   4.871   0.409
## 942   -1.661   0.231   1.405  -4.596   4.871
## 943   -2.800  -1.661   0.231   1.405  -4.596
## 944   -0.404  -2.800  -1.661   0.231   1.405
## 945    3.212  -0.404  -2.800  -1.661   0.231
## 946   -1.075   3.212  -0.404  -2.800  -1.661
## 947    4.195  -1.075   3.212  -0.404  -2.800
## 948   -2.742   4.195  -1.075   3.212  -0.404
## 949    4.314  -2.742   4.195  -1.075   3.212
## 950    0.540   4.314  -2.742   4.195  -1.075
## 951    1.149   0.540   4.314  -2.742   4.195
## 952   -1.812   1.149   0.540   4.314  -2.742
## 953    2.670  -1.812   1.149   0.540   4.314
## 954   -3.467   2.670  -1.812   1.149   0.540
## 955    1.777  -3.467   2.670  -1.812   1.149
## 956   -2.835   1.777  -3.467   2.670  -1.812
## 957   -0.048  -2.835   1.777  -3.467   2.670
## 958   -3.096  -0.048  -2.835   1.777  -3.467
## 959   -3.001  -3.096  -0.048  -2.835   1.777
## 960   -1.211  -3.001  -3.096  -0.048  -2.835
## 961   -1.854  -1.211  -3.001  -3.096  -0.048
## 962    1.710  -1.854  -1.211  -3.001  -3.096
## 963   -0.232   1.710  -1.854  -1.211  -3.001
## 964    0.203  -0.232   1.710  -1.854  -1.211
```

```
## 965    2.857   0.203  -0.232   1.710  -1.854
## 966    0.145   2.857   0.203  -0.232   1.710
## 967   -0.462   0.145   2.857   0.203  -0.232
## 968   -0.725  -0.462   0.145   2.857   0.203
## 969   -3.159  -0.725  -0.462   0.145   2.857
## 970    0.756  -3.159  -0.725  -0.462   0.145
## 971    0.270   0.756  -3.159  -0.725  -0.462
## 972   -3.331   0.270   0.756  -3.159  -0.725
## 973   -9.399  -3.331   0.270   0.756  -3.159
## 974  -18.195  -9.399  -3.331   0.270   0.756
## 975    4.596 -18.195  -9.399  -3.331   0.270
## 976   -6.781   4.596 -18.195  -9.399  -3.331
## 977   10.491  -6.781   4.596 -18.195  -9.399
## 978   -3.898  10.491  -6.781   4.596 -18.195
## 979   -6.198  -3.898  10.491  -6.781   4.596
## 980   -8.389  -6.198  -3.898  10.491  -6.781
## 981   12.026  -8.389  -6.198  -3.898  10.491
## 982   -2.251  12.026  -8.389  -6.198  -3.898
## 983    0.418  -2.251  12.026  -8.389  -6.198
## 984    0.926   0.418  -2.251  12.026  -8.389
## 985   -1.698   0.926   0.418  -2.251  12.026
## 986    6.760  -1.698   0.926   0.418  -2.251
## 987   -4.448   6.760  -1.698   0.926   0.418
## 988   -4.518  -4.448   6.760  -1.698   0.926
## 989   -2.137  -4.518  -4.448   6.760  -1.698
## 990   -0.730  -2.137  -4.518  -4.448   6.760
## 991    5.173  -0.730  -2.137  -4.518  -4.448
## 992   -4.808   5.173  -0.730  -2.137  -4.518
## 993   -6.868  -4.808   5.173  -0.730  -2.137
## 994   -4.540  -6.868  -4.808   5.173  -0.730
## 995   -7.035  -4.540  -6.868  -4.808   5.173
## 996   10.707  -7.035  -4.540  -6.868  -4.808
## 997    1.585  10.707  -7.035  -4.540  -6.868
## 998    6.168   1.585  10.707  -7.035  -4.540
## 999    3.255   6.168   1.585  10.707  -7.035
## 1000   1.669   3.255   6.168   1.585  10.707
## 1001   1.522   1.669   3.255   6.168   1.585
## 1002  -0.388   1.522   1.669   3.255   6.168
## 1003   1.303  -0.388   1.522   1.669   3.255
## 1004   5.893   1.303  -0.388   1.522   1.669
## 1005  -4.988   5.893   1.303  -0.388   1.522
## 1006   0.467  -4.988   5.893   1.303  -0.388
## 1007   3.623   0.467  -4.988   5.893   1.303
## 1008   2.279   3.623   0.467  -4.988   5.893
## 1009   0.651   2.279   3.623   0.467  -4.988
## 1010  -2.640   0.651   2.279   3.623   0.467
## 1011  -0.253  -2.640   0.651   2.279   3.623
## 1012  -2.446  -0.253  -2.640   0.651   2.279
## 1013  -1.929  -2.446  -0.253  -2.640   0.651
## 1014   6.967  -1.929  -2.446  -0.253  -2.640
## 1015   4.134   6.967  -1.929  -2.446  -0.253
## 1016   0.839   4.134   6.967  -1.929  -2.446
## 1017   2.329   0.839   4.134   6.967  -1.929
## 1018  -0.632   2.329   0.839   4.134   6.967
```

```
## 1019   2.195  -0.632   2.329   0.839   4.134
## 1020   0.273   2.195  -0.632   2.329   0.839
## 1021  -1.218   0.273   2.195  -0.632   2.329
## 1022   2.591  -1.218   0.273   2.195  -0.632
## 1023   2.452   2.591  -1.218   0.273   2.195
## 1024  -2.239   2.452   2.591  -1.218   0.273
## 1025  -1.836  -2.239   2.452   2.591  -1.218
## 1026   4.514  -1.836  -2.239   2.452   2.591
## 1027   1.511   4.514  -1.836  -2.239   2.452
## 1028  -0.743   1.511   4.514  -1.836  -2.239
## 1029  -4.021  -0.743   1.511   4.514  -1.836
## 1030   3.195  -4.021  -0.743   1.511   4.514
## 1031   2.261   3.195  -4.021  -0.743   1.511
## 1032  -0.192   2.261   3.195  -4.021  -0.743
## 1033   0.010  -0.192   2.261   3.195  -4.021
## 1034   1.328   0.010  -0.192   2.261   3.195
## 1035   0.039   1.328   0.010  -0.192   2.261
## 1036  -0.356   0.039   1.328   0.010  -0.192
## 1037   2.178  -0.356   0.039   1.328   0.010
## 1038  -1.010   2.178  -0.356   0.039   1.328
## 1039   2.680  -1.010   2.178  -0.356   0.039
## 1040  -0.782   2.680  -1.010   2.178  -0.356
## 1041  -3.897  -0.782   2.680  -1.010   2.178
## 1042  -1.639  -3.897  -0.782   2.680  -1.010
## 1043  -0.715  -1.639  -3.897  -0.782   2.680
## 1044   0.874  -0.715  -1.639  -3.897  -0.782
## 1045   3.130   0.874  -0.715  -1.639  -3.897
## 1046  -0.422   3.130   0.874  -0.715  -1.639
## 1047   3.097  -0.422   3.130   0.874  -0.715
## 1048   0.991   3.097  -0.422   3.130   0.874
## 1049   0.862   0.991   3.097  -0.422   3.130
## 1050   0.577   0.862   0.991   3.097  -0.422
## 1051   0.987   0.577   0.862   0.991   3.097
## 1052   1.381   0.987   0.577   0.862   0.991
## 1053  -0.188   1.381   0.987   0.577   0.862
## 1054   2.110  -0.188   1.381   0.987   0.577
## 1055  -2.513   2.110  -0.188   1.381   0.987
## 1056  -6.388  -2.513   2.110  -0.188   1.381
## 1057   2.232  -6.388  -2.513   2.110  -0.188
## 1058  -4.226   2.232  -6.388  -2.513   2.110
## 1059   0.158  -4.226   2.232  -6.388  -2.513
## 1060  -2.252   0.158  -4.226   2.232  -6.388
## 1061   2.509  -2.252   0.158  -4.226   2.232
## 1062   2.374   2.509  -2.252   0.158  -4.226
## 1063  -3.646   2.374   2.509  -2.252   0.158
## 1064  -5.032  -3.646   2.374   2.509  -2.252
## 1065   5.416  -5.032  -3.646   2.374   2.509
## 1066  -1.213   5.416  -5.032  -3.646   2.374
## 1067   3.548  -1.213   5.416  -5.032  -3.646
## 1068  -0.096   3.548  -1.213   5.416  -5.032
## 1069   1.819  -0.096   3.548  -1.213   5.416
## 1070  -3.779   1.819  -0.096   3.548  -1.213
## 1071  -0.700  -3.779   1.819  -0.096   3.548
## 1072  -0.663  -0.700  -3.779   1.819  -0.096
```

```
## 1073   3.750  -0.663  -0.700  -3.779   1.819
## 1074   0.456   3.750  -0.663  -0.700  -3.779
## 1075   1.446   0.456   3.750  -0.663  -0.700
## 1076   2.050   1.446   0.456   3.750  -0.663
## 1077  -0.212   2.050   1.446   0.456   3.750
## 1078   1.650  -0.212   2.050   1.446   0.456
## 1079   0.948   1.650  -0.212   2.050   1.446
## 1080   0.586   0.948   1.650  -0.212   2.050
## 1081   0.015   0.586   0.948   1.650  -0.212
## 1082   3.599   0.015   0.586   0.948   1.650
## 1083  -2.173   3.599   0.015   0.586   0.948
## 1084   0.043  -2.173   3.599   0.015   0.586
## 1085  -0.861   0.043  -2.173   3.599   0.015
## 1086   2.969  -0.861   0.043  -2.173   3.599
## 1087   1.281   2.969  -0.861   0.043  -2.173
## 1088   0.283   1.281   2.969  -0.861   0.043
## 1089   1.034   0.283   1.281   2.969  -0.861
```

j

QDA

```r
qda_model = qda(Direction ~ Lag4 + Lag5, data = train_set)

qda_preds = predict(qda_model, newdata = test_set)$class

mean(qda_preds == test_set$Direction)
```

```
## [1] 0.5192308
```

```r
table(qda_preds, test_set$Direction)
```

```
##
## qda_preds Down Up
##      Down    9 16
##      Up     34 45
```

Naive Bayes

```r
nb_model = naiveBayes(Direction ~ Lag3 + Lag4 + Lag5, train_set)
nb_preds = predict(nb_model, newdata = test_set)

mean(nb_preds == test_set$Direction)
```

```
## [1] 0.5
```

```r
table(nb_preds, test_set$Direction)
```

```
##
## nb_preds Down Up
##     Down    8 17
##     Up     35 44
```

KNN

```r
knn_pred = knn(as.matrix(train_set$Lag2), as.matrix(test_set$Lag2),
               train_set$Direction, k = 5)

mean(knn_pred == test_set$Direction)
```

```
## [1] 0.5288462
```

```r
table(knn_pred, test_set$Direction)
```

```
##
## knn_pred Down Up
##     Down   16 22
##     Up     27 39
```

Exercise 14

```r
head(Auto)
```

```
##   mpg cylinders displacement horsepower weight acceleration year origin
## 1  18         8          307        130   3504         12.0   70      1
## 2  15         8          350        165   3693         11.5   70      1
## 3  18         8          318        150   3436         11.0   70      1
## 4  16         8          304        150   3433         12.0   70      1
## 5  17         8          302        140   3449         10.5   70      1
## 6  15         8          429        198   4341         10.0   70      1
##                        name
## 1 chevrolet chevelle malibu
## 2         buick skylark 320
## 3        plymouth satellite
## 4             amc rebel sst
## 5               ford torino
## 6          ford galaxie 500
```

a

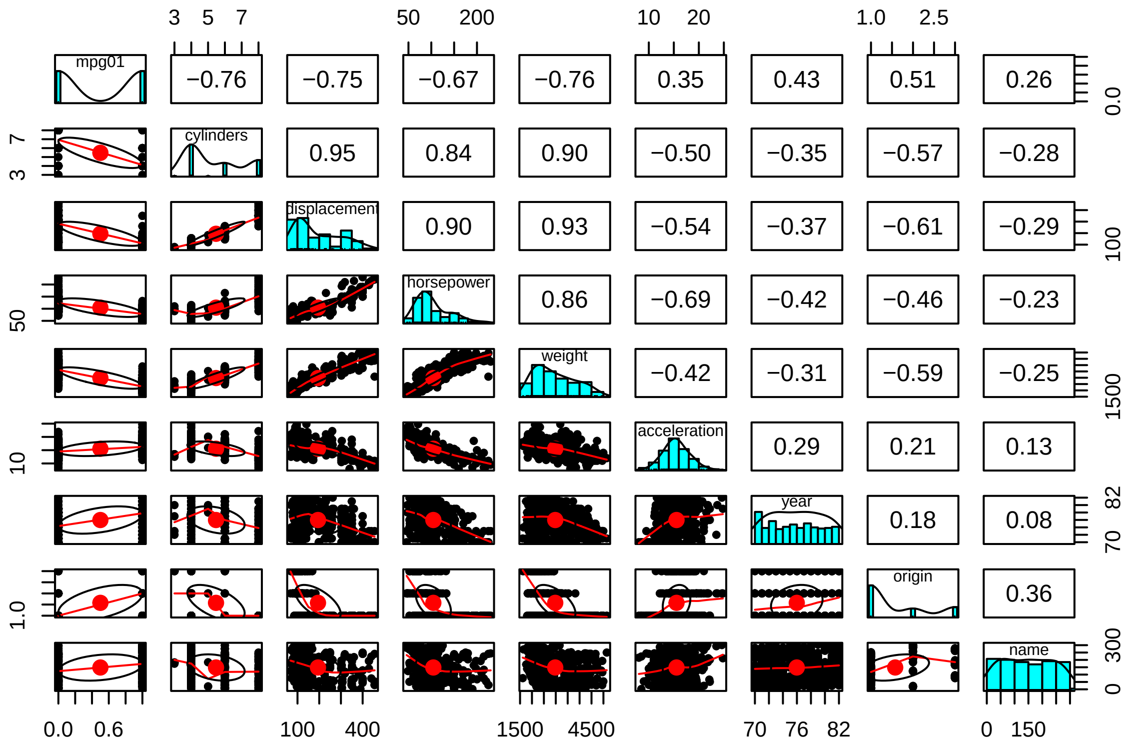
```
mpg01 = rep(0, dim(Auto)[1])
mpg01[Auto$mpg > median(Auto$mpg)] = 1
new_auto = data.frame(mpg01, Auto[, 2:9])
head(new_auto)
```

```
##   mpg01 cylinders displacement horsepower weight acceleration year origin
## 1     0         8          307        130   3504         12.0   70      1
## 2     0         8          350        165   3693         11.5   70      1
## 3     0         8          318        150   3436         11.0   70      1
## 4     0         8          304        150   3433         12.0   70      1
## 5     0         8          302        140   3449         10.5   70      1
## 6     0         8          429        198   4341         10.0   70      1
##                        name
## 1 chevrolet chevelle malibu
## 2         buick skylark 320
## 3        plymouth satellite
## 4             amc rebel sst
## 5               ford torino
## 6          ford galaxie 500
```

b

cylinders, displacement, horsepower and weight are features that seem most likely to be useful in predicting mpg01.

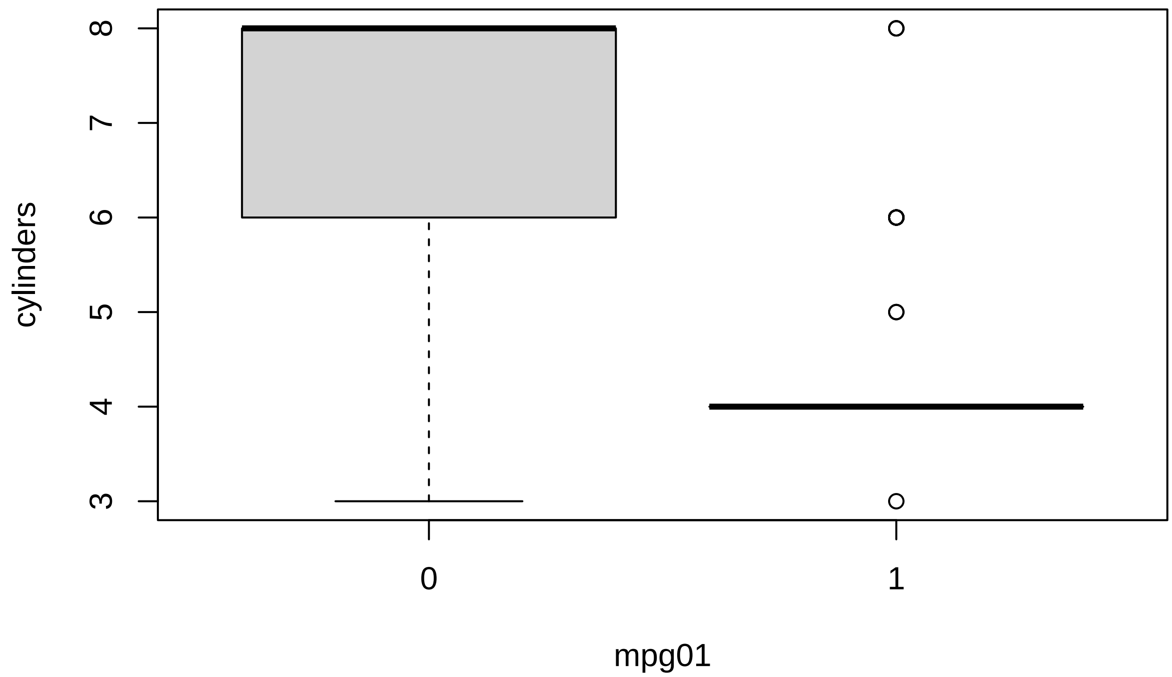
```
library(psych)
pairs.panels(new_auto)
```



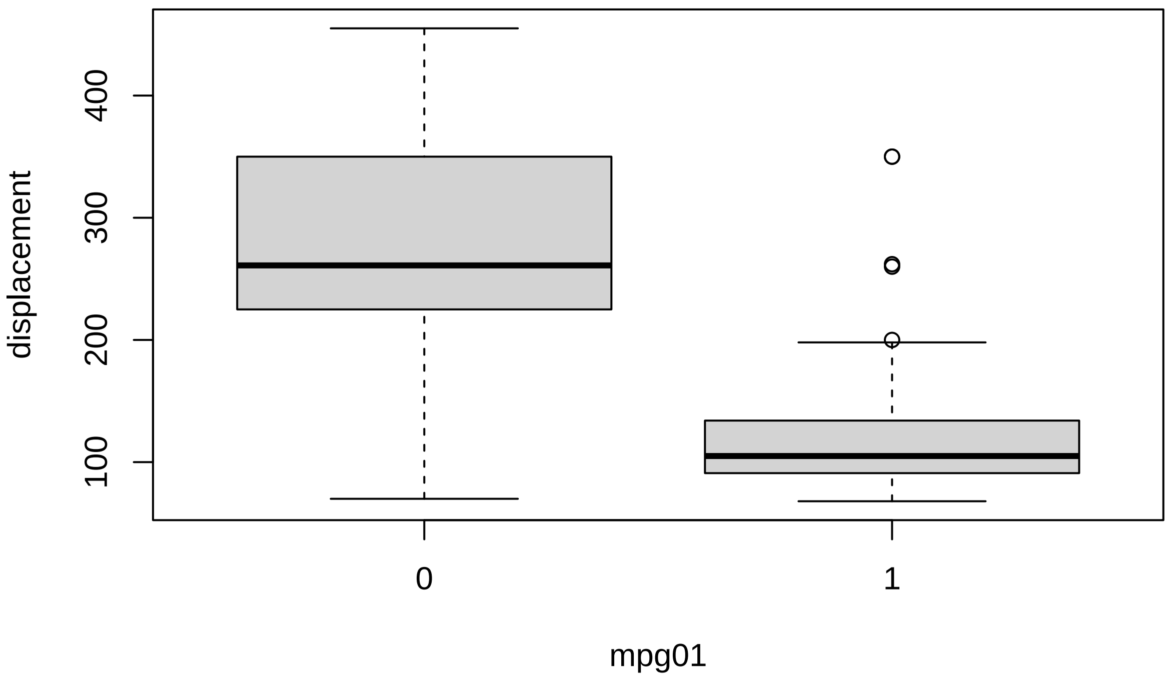
```
attach(new_auto)
```

```
## The following object is masked _by_ .GlobalEnv:
##
##     mpg01
```

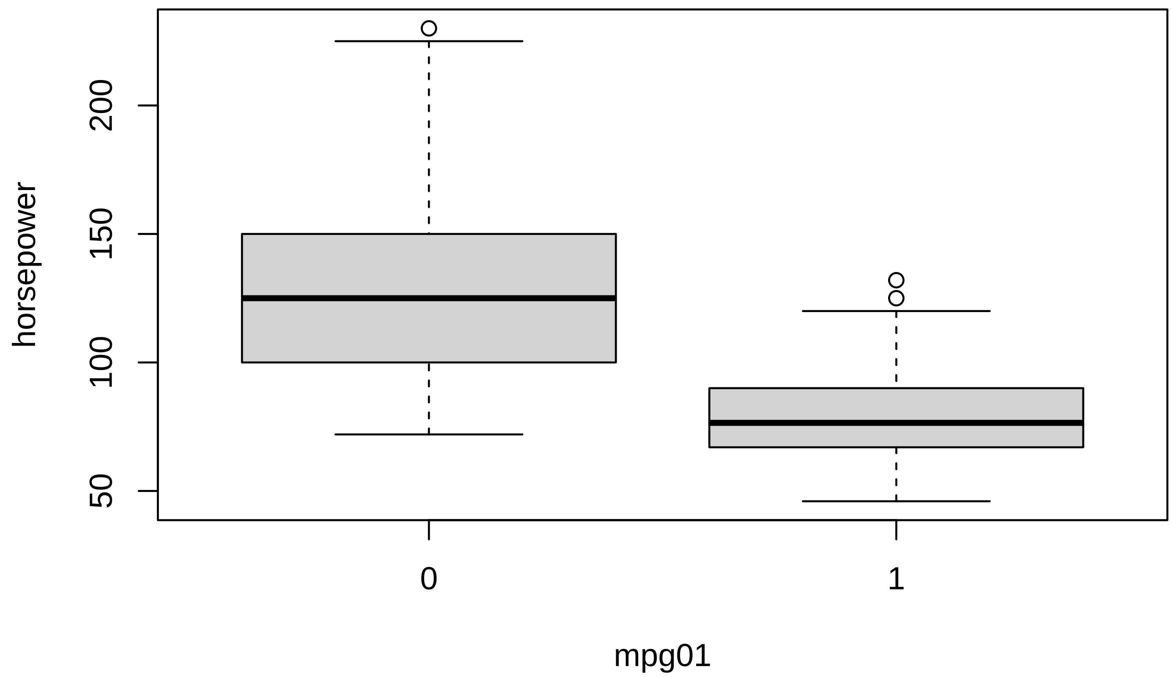
```
boxplot(cylinders ~ mpg01)
```



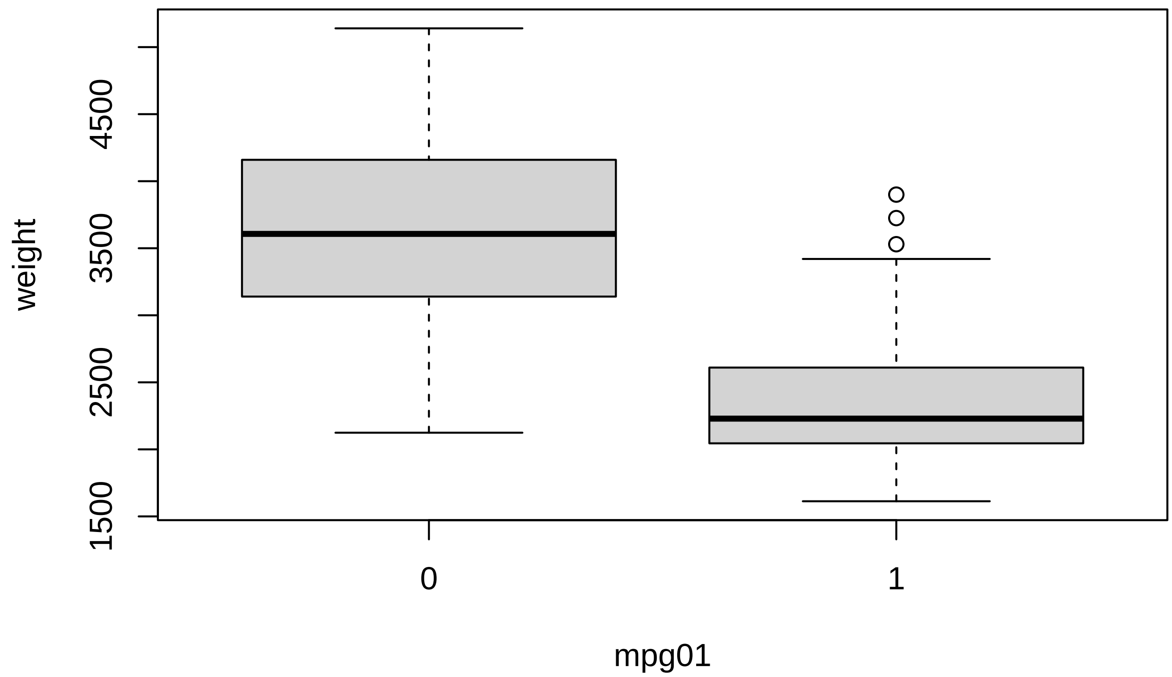

```
boxplot(displacement ~ mpg01)
```




```
boxplot(horsepower ~ mpg01)
```




```
boxplot(weight ~ mpg01)
```



c

```
set.seed(1)
train_rate = 0.6
train_indices = sample(dim(Auto)[1],
                       round(dim(Auto)[1] * train_rate),
                       replace = F)
train_set = new_auto[train_indices, ]
test_set = new_auto[-train_indices, ]
```

```
mean(train_set$mpg01)
```

```
## [1] 0.4978723
```

```
mean(test_set$mpg01)
```

```
## [1] 0.5031847
```

d

```r
library(MASS)
lda_model = lda(mpg01 ~ cylinders + displacement + horsepower + weight,
                data = train_set)
test_preds = predict(lda_model, test_set[, 2:9])$class
table(test_preds, test_set$mpg01)
```

```
##
## test_preds  0  1
##          0 63  6
##          1 15 73
```

```r
1 - mean(test_preds == test_set$mpg01)
```

```
## [1] 0.133758
```

e

```r
qda_model = qda(mpg01 ~ cylinders + displacement + horsepower + weight,
                data = train_set)
test_preds = predict(qda_model, test_set[, 2:9])$class
table(test_preds, test_set$mpg01)
```

```
##
## test_preds  0  1
##          0 68  9
##          1 10 70
```

```r
1 - mean(test_preds == test_set$mpg01)
```

```
## [1] 0.1210191
```

f

```r
lgr_model = glm(mpg01 ~ cylinders + displacement + horsepower + weight,
                data = train_set, family = binomial)
test_probs = predict(lgr_model, test_set[, 2:9], type = 'response')
test_preds = test_probs > 0.5
table(test_preds, test_set$mpg01)
```

```
##
## test_preds  0  1
##      FALSE 67  8
##      TRUE  11 71
```

```r
1 - mean(test_preds == test_set$mpg01)
```

```
## [1] 0.1210191
```

g

```r
library(e1071)
nb_model = naiveBayes(mpg01 ~ cylinders + displacement + horsepower + weight,
                      data = train_set)
test_preds = predict(nb_model, test_set[, 2:9])
table(test_preds, test_set$mpg01)
```

```
##
## test_preds  0  1
##          0 67  8
##          1 11 71
```

```r
1 - mean(test_preds == test_set$mpg01)
```

```
## [1] 0.1210191
```

h

In this case, among three values 1, 10 and 100, K = 10 seems to be the best.

```r
library(class)
knn_preds = knn(train_set[c('cylinders', 'displacement', 'horsepower', 'weight')],
                test_set[c('cylinders', 'displacement', 'horsepower', 'weight')],
                train_set$mpg01, k = 1)

table(knn_preds, test_set$mpg01)
```

```
##
## knn_preds  0  1
##         0 68 14
##         1 10 65
```

```r
1 - mean(knn_preds == test_set$mpg01)
```

```
## [1] 0.1528662
```

```r
knn_preds = knn(train_set[c('cylinders', 'displacement', 'horsepower', 'weight')],
                test_set[c('cylinders', 'displacement', 'horsepower', 'weight')],
                train_set$mpg01, k = 10)

table(knn_preds, test_set$mpg01)
```

```
##
## knn_preds  0  1
##         0 66  7
##         1 12 72
```

```
1 - mean(knn_preds == test_set$mpg01)
```

```
## [1] 0.1210191
```

```
knn_preds = knn(train_set[c('cylinders', 'displacement', 'horsepower', 'weight')],
                test_set[c('cylinders', 'displacement', 'horsepower', 'weight')],
                train_set$mpg01, k = 100)

table(knn_preds, test_set$mpg01)
```

```
##
## knn_preds  0  1
##         0 68 11
##         1 10 68
```

```
1 - mean(knn_preds == test_set$mpg01)
```

```
## [1] 0.133758
```

Exercise 15

a

```
Power = function(){
  x = 2
  a = 3
  print(x^3)
}
Power()
```

```
## [1] 8
```

b

```
Power2 = function(x, a){
  print(x^a)
}
Power2(3, 8)
```

```
## [1] 6561
```

c

```r
Power2(10, 3)
```

```
## [1] 1000
```

```r
Power2(8, 17)
```

```
## [1] 2.2518e+15
```

```r
Power2(131, 3)
```

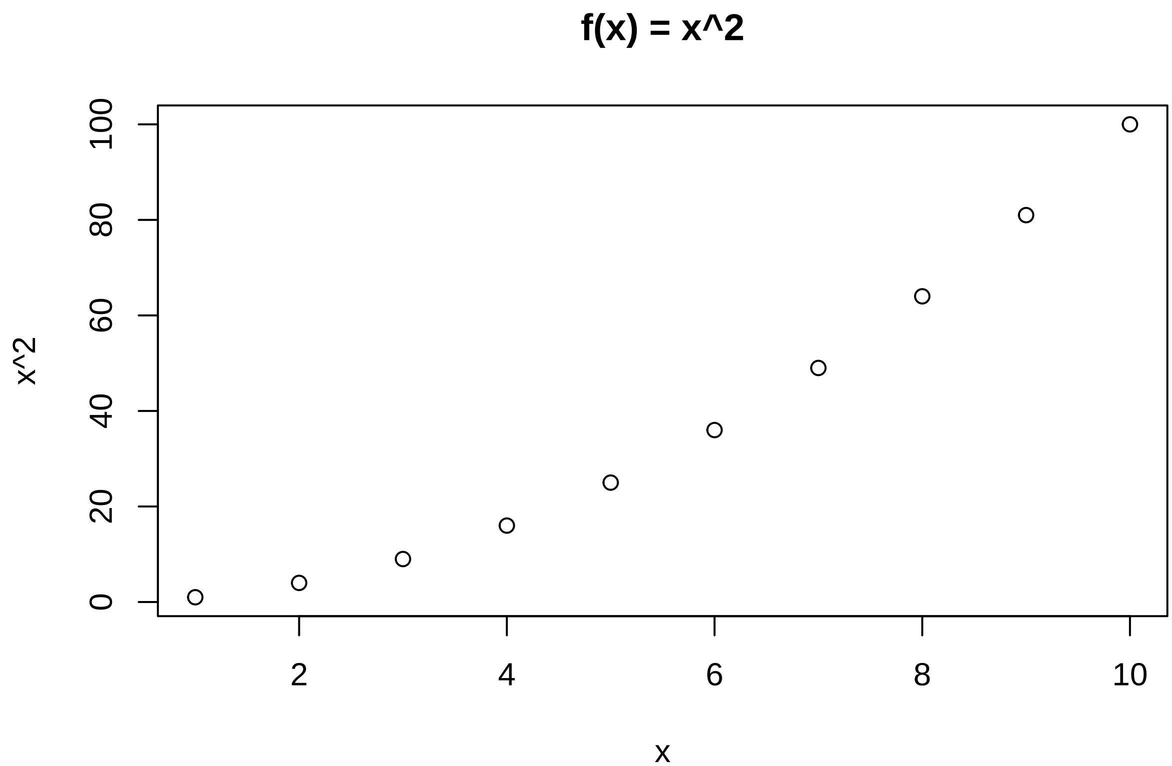
```
## [1] 2248091
```

d

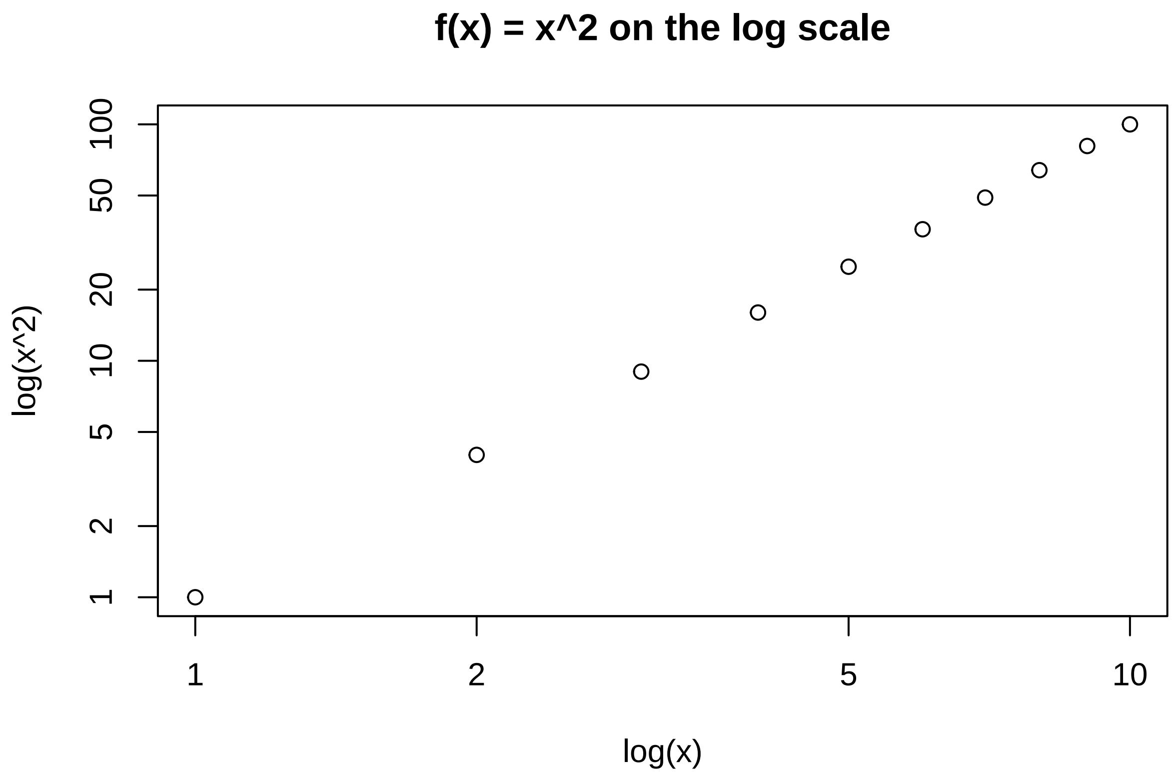
```r
Power3 = function(x, a){
  return(x^a)
}
```

e

```r
x = c(1:10)
y = Power3(x, 2)
plot(x, y, xlab = 'x', ylab = 'x^2', main = 'f(x) = x^2')
```

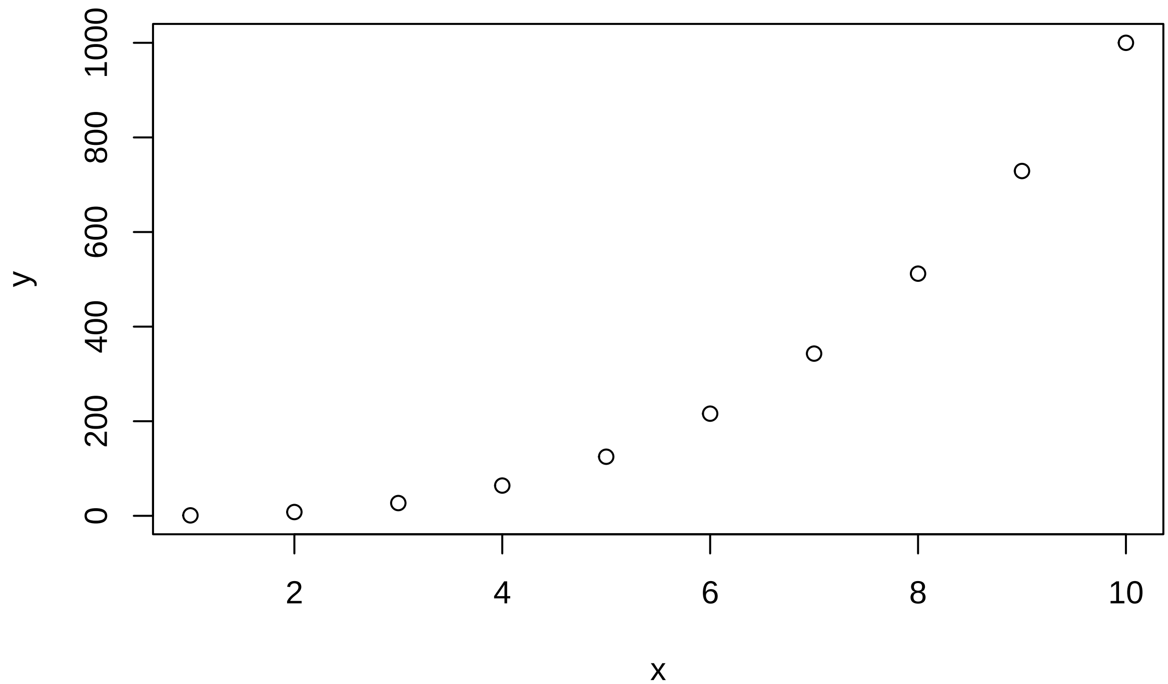



```r
x = c(1:10)
y = Power3(x, 2)
plot(x, y, log = 'xy', xlab = 'log(x)', ylab = 'log(x^2)',
     main = 'f(x) = x^2 on the log scale')
```



f

```
PlotPower = function(x, a){
  y = x^a
  plot(x, y)
}
PlotPower(1:10, 3)
```



Exercise 16

```
head(Boston)
```

```
##      crim zn indus chas   nox    rm  age    dis rad tax ptratio  black lstat
## 1 0.00632 18  2.31    0 0.538 6.575 65.2 4.0900   1 296    15.3 396.90  4.98
## 2 0.02731  0  7.07    0 0.469 6.421 78.9 4.9671   2 242    17.8 396.90  9.14
## 3 0.02729  0  7.07    0 0.469 7.185 61.1 4.9671   2 242    17.8 392.83  4.03
## 4 0.03237  0  2.18    0 0.458 6.998 45.8 6.0622   3 222    18.7 394.63  2.94
## 5 0.06905  0  2.18    0 0.458 7.147 54.2 6.0622   3 222    18.7 396.90  5.33
## 6 0.02985  0  2.18    0 0.458 6.430 58.7 6.0622   3 222    18.7 394.12  5.21
##   medv
## 1 24.0
## 2 21.6
## 3 34.7
## 4 33.4
## 5 36.2
## 6 28.7
```

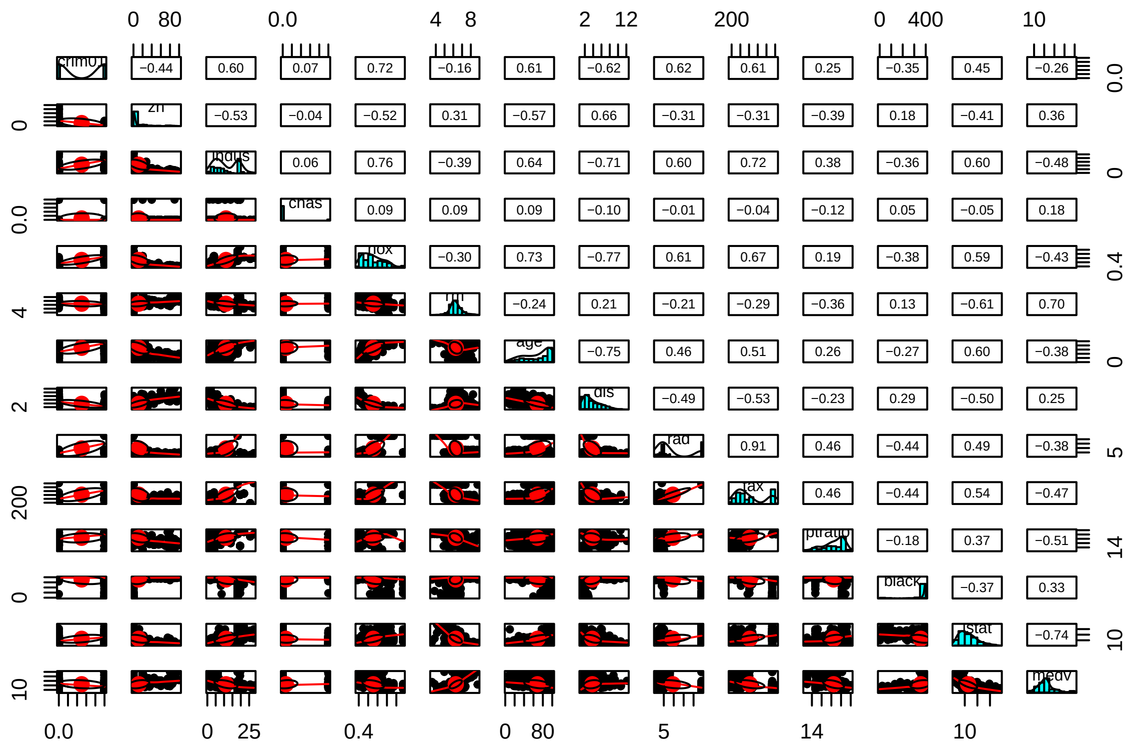
Create the response variable

```
crim01 = rep(0, dim(Boston)[1])
crim01[Boston$crim > median(Boston$crim)] = 1
new_boston = data.frame(crim01, Boston[, 2:14])
head(new_boston)
```

```
##   crim01 zn indus chas   nox    rm  age    dis rad tax ptratio  black lstat
## 1      0 18  2.31    0 0.538 6.575 65.2 4.0900   1 296    15.3 396.90  4.98
## 2      0  0  7.07    0 0.469 6.421 78.9 4.9671   2 242    17.8 396.90  9.14
## 3      0  0  7.07    0 0.469 7.185 61.1 4.9671   2 242    17.8 392.83  4.03
## 4      0  0  2.18    0 0.458 6.998 45.8 6.0622   3 222    18.7 394.63  2.94
## 5      0  0  2.18    0 0.458 7.147 54.2 6.0622   3 222    18.7 396.90  5.33
## 6      0  0  2.18    0 0.458 6.430 58.7 6.0622   3 222    18.7 394.12  5.21
##   medv
## 1 24.0
## 2 21.6
## 3 34.7
## 4 33.4
## 5 36.2
## 6 28.7
```

Exploring

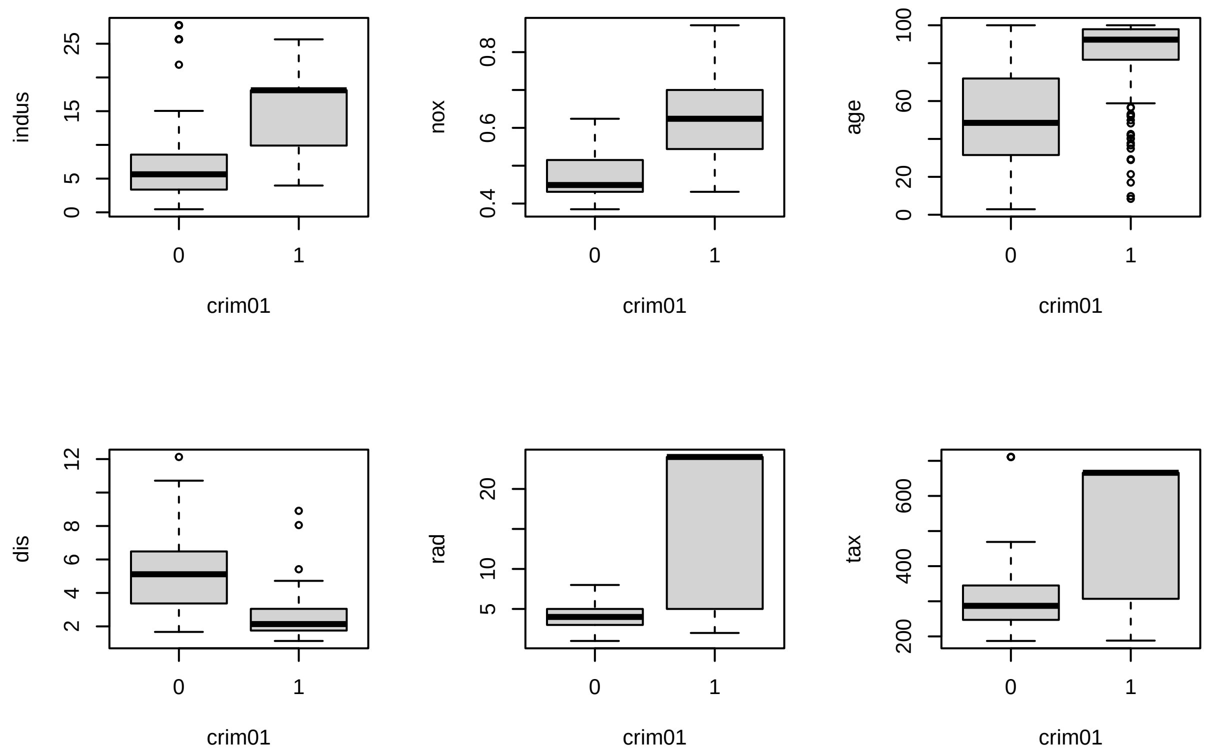
```
library(psych)
pairs.panels(new_boston)
```




```r
attach(new_boston)
```

```
## The following object is masked _by_ .GlobalEnv:
##
##     crim01
```

```r
par(mfrow = c(2, 3))
boxplot(indus ~ crim01)
boxplot(nox ~ crim01)
boxplot(age ~ crim01)
boxplot(dis ~ crim01)
boxplot(rad ~ crim01)
boxplot(tax ~ crim01)
```



Splitting data

```
set.seed(1)
train_rate = 0.6
train_indices = sample(dim(Boston)[1],
                       round(dim(Boston)[1] * train_rate),
                       replace = F)
train_set = new_boston[train_indices, ]
test_set = new_boston[-train_indices, ]
```

```
mean(train_set$crim01)
```

```
## [1] 0.4835526
```

```
mean(test_set$crim01)
```

```
## [1] 0.5247525
```

Logistic regression

```r
lgr_model = glm(crim01 ~ indus + nox + age + dis + rad + tax,
                data = train_set, family = binomial)
test_probs = predict(lgr_model, test_set[, 2:14], type = 'response')
test_preds = test_probs > 0.5
table(test_preds, test_set$crim01)
```

```
##
## test_preds  0  1
##      FALSE 80 10
##      TRUE  16 96
```

```r
1 - mean(test_preds == test_set$crim01)
```

```
## [1] 0.1287129
```

```r
lgr_model = glm(crim01 ~ .,
                data = train_set, family = binomial)
test_probs = predict(lgr_model, test_set[, 2:14], type = 'response')
test_preds = test_probs > 0.5
table(test_preds, test_set$crim01)
```

```
##
## test_preds  0  1
##      FALSE 86 10
##      TRUE  10 96
```

```r
1 - mean(test_preds == test_set$crim01)
```

```
## [1] 0.0990099
```

LDA

```r
lda_model = lda(crim01 ~ indus + nox + age + dis + rad + tax,
                data = train_set)
test_preds = predict(lda_model, test_set[, 2:14])$class
table(test_preds, test_set$crim01)
```

```
##
## test_preds  0  1
##          0 93 28
##          1  3 78
```

```r
1 - mean(test_preds == test_set$crim01)
```

```
## [1] 0.1534653
```

```r
lda_model = lda(crim01 ~ .,
                data = train_set)
test_preds = predict(lda_model, test_set[, 2:14])$class
table(test_preds, test_set$crim01)
```

```
##
## test_preds  0  1
##          0 93 29
##          1  3 77
```

```r
1 - mean(test_preds == test_set$crim01)
```

```
## [1] 0.1584158
```

Naive Bayes

```r
nb_model = naiveBayes(crim01 ~ indus + nox + age + dis + rad + tax,
                      data = train_set)
test_preds = predict(nb_model, test_set[, 2:14])
table(test_preds, test_set$crim01)
```

```
##
## test_preds  0  1
##          0 86 28
##          1 10 78
```

```r
1 - mean(test_preds == test_set$crim01)
```

```
## [1] 0.1881188
```

```r
nb_model = naiveBayes(crim01 ~ .,
                      data = train_set)
test_preds = predict(nb_model, test_set[, 2:14])
table(test_preds, test_set$crim01)
```

```
##
## test_preds  0  1
##          0 87 29
##          1  9 77
```

```r
1 - mean(test_preds == test_set$crim01)
```

```
## [1] 0.1881188
```

KNN

```r
knn_preds = knn(train_set[c('indus', 'nox', 'age', 'dis', 'rad', 'tax')],
                test_set[c('indus', 'nox', 'age', 'dis', 'rad', 'tax')],
                train_set$crim01, k = 1)

table(knn_preds, test_set$crim01)
```

```
##
## knn_preds  0  1
##         0 79  7
##         1 17 99
```

```r
1 - mean(knn_preds == test_set$crim01)
```

```
## [1] 0.1188119
```

```r
knn_preds = knn(train_set[c('indus', 'nox', 'age', 'dis', 'rad', 'tax')],
                test_set[c('indus', 'nox', 'age', 'dis', 'rad', 'tax')],
                train_set$crim01, k = 3)

table(knn_preds, test_set$crim01)
```

```
##
## knn_preds   0   1
##         0  87   6
##         1   9 100
```

```r
1 - mean(knn_preds == test_set$crim01)
```

```
## [1] 0.07425743
```

```r
knn_preds = knn(train_set, test_set,
                train_set$crim01, k = 3)

table(knn_preds, test_set$crim01)
```

```
##
## knn_preds   0   1
##         0  84   6
##         1  12 100
```

```r
1 - mean(knn_preds == test_set$crim01)
```

```
## [1] 0.08910891
```