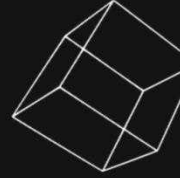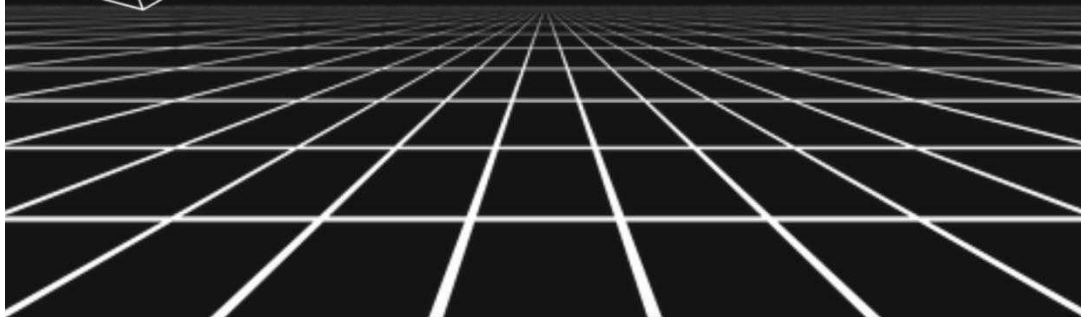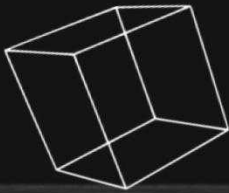Is the Universe Made of Math?

MATHEMATICAL
UNIVERSE

BY MANJUNATH.R

The Mathematical Universe

"The only true wisdom is in knowing you know nothing."

— Socrates

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We Humans, a curious beings developed from the **Darwin's principle of natural selection**, are accustomed into an inquisition. The question is not 'do we know everything from the triumph of the Higgs boson to the underlying discomfort of ultimate question of life, the universe, and everything?' or it is 'do we know enough?' But how the creative principle resides in mathematics? There's something very mathematical about our gigantic Cosmos, and that the more carefully we look, the more equations are built into nature: From basic arithmetic to the calculation of rocket trajectories, math provides a good understanding of the equations that govern the world around us. **Our universe isn't just described by math**, but that universe is a "**grand book**" written in the language of mathematics. We find it very appropriate that mathematics has played a striking role in our growing understanding of the events around us, and of our own existence.

Science knows no country, because knowledge belongs to humanity, and is the torch which illuminates the world. Science is the highest personification of the nation because that nation will remain the first which carries the furthest the works of thought and intelligence.

Louis Pasteur

"Our universe may be one of an infinite number of parallel universes, each connected to the others by an infinite series of **wormholes**."

3   Laws of Universe:

- You cannot get something for nothing because matter and energy are conserved.
- You cannot return to the same energy state because there is always an increase in entropy.
- Absolute zero is unattainable.

Equivalence principle:

The laws of nature in an accelerating frame are equivalent to the laws in a gravitational field.

Matter-energy → curvature of space-time

Geometry → field theory → classical theory → quantum theory

"But the creative principle resides in mathematics. In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed." **— Albert Einstein**

⇨   **Newton's laws of motion** tie into almost everything we see in everyday life.

- **Law 1 (law of inertia):** An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.
- **Law 2:** Force equals mass times acceleration (**F = ma**).
- **Law 3:** For every action, there is an equal and opposite reaction.

⇨ As a remarkable consequence of the **uncertainty principle of quantum mechanics** (which implies that certain pairs of quantities, such as the energy and time, cannot both be predicted with complete accuracy), the empty space is filled with what is called vacuum energy.

Because E=MC²:

- Mass is just energy in disguise.
- A small amount of mass can equal a large amount of energy.

If the mass of the star < 1.4 solar masses

- Electrons prevent further collapse.
- The core will thus continue to collapse and form a **white dwarf**.

If the mass of the star > 1.4 solar masses but mass < 3 solar masses

- Electrons + protons combine to form neutrons.
- Neutrons prevent further collapse.
- The core will thus continue to collapse and form a **neutron star**.

If the mass of the star > 3 solar masses

- Gravity wins! Nothing prevents collapse.
- The core will thus continue to collapse and form a black hole.

*Any object with a physical radius smaller than its **Schwarzschild radius** will be a black hole.*

All the laws of physics that we know, breaks down –

- Below this time: (**Planck Time**)
- Below this length: (**Planck Length**)
- Above this temperature: (**Planck Temperature**)

Density parameter (Ω): The ratio of the total amount of matter in the universe divided by the minimum amount of matter needed to cause the **big crunch**.

- **$\Omega < 1$:** the Universe will continue to expand forever.
- **$\Omega > 1$:** the Universe will eventually halt its expansion and recollapse.
- **$\Omega = 1$:** the Universe contains enough matter to halt the expansion but not enough to recollapse it.

If a black hole has a mass less than the **Planck mass**, its quantum mechanical size could be outside its event horizon. This wouldn't make sense, **Planck mass** is the smallest possible black hole.

Absolute zero: (-273°C) − the lowest possible temperature, at which substances contain no heat energy and all vibrations stop — almost.

⇨ If the two quarks would have occupied precisely the same point with the same properties, they would not have stayed in the same position for long. And quarks would have not formed separate, **well-defined protons and neutrons**. And nor would these, together with electrons have formed separate, well-defined atoms. And the world would have collapsed before it ever reached its present size.

- When 2 similar waves are added, the resultant wave is bigger (**constructive interference**).

- When 2 dissimilar waves are added, they cancel each other out (**destructive interference**).

The different frequencies of light appear as different colors.

Proton charge + Electron charge = 0

Just what it is if **electromagnetism** would not dominate over **gravity** and for the universe to remain electrically neutral.

- It's not their energy; it's their zero rest mass that makes **photons** to travel at the speed of light.
- Just like a dozen is 12 things, a mole is simply **Avogadro's number of particles**.

Undisturbed space + rigid mass = distorted space.

3 dimensions of space + one dimension of time = **single four-dimensional continuum** (space-time).

Since **h** − which is one of the most **fundamental numbers in physics**, ranking alongside the speed of light "**c**" and confined most of these radical departures from life-as-usual to the microscopic realm − is incredibly small (i.e., **6 × 10 to the power of − 34** − a decimal point followed by 33 zeros and a 6 − of a joule second), the frequency of the photon is always greater than its energy, so it do not take many quanta to radiate even ten thousand megawatts.

What is GRAVITY?

- **Newtonian view:** Force tells mass how to accelerate. Accelerated mass tells what gravity is.

- **Einsteinian view:** Mass tells space how to curve. Curved space tells what gravity is.

All objects emit **electromagnetic radiation** according to their temperature. Colder objects emit waves with very low frequency (such as **radio or microwaves**), while hot objects emit visible light or even **ultraviolet** and higher frequencies.

- Longer half-life of nucleus → **Slow Radioactive Decay**.
- **Shorter half-life of nucleus** → Fast Radioactive Decay.

".. Physics at the atomic and subatomic level ..."

… Weird things are possible:

- Energy is quantized (**E = nhυ**).
- Momentum is quantized (**L = nℏ**).
- Charge is quantized (**Q = ne**).

Because

$$E = h\upsilon, \qquad c = \lambda\upsilon, \qquad E = \frac{hc}{\lambda} = pc$$

$$\lambda = \frac{h}{p}$$

Every particle or quantum entity may be partly described in terms not only of particles, but also of waves.

The Thermodynamic Laws think big: they dictate energy behavior…

- **1 Law:** Energy is conserved; its form can be converted.
- **2 Law:** Energies can flow, equilibrate.
- **3 Law:** "Driving force" for equilibration uniquely defined.

- **0 Law:** Thermal equilibrium is transitive.

The Life of a Star:

"More mass

More pressure and temperature

Faster Fusion

Shorter life"

"Less mass

Less pressure and temperature

Slower Fusion

Longer life"

String Theory (A theory that tries to adjust / harmonize / reconcile **General Relativity** and **Quantum mechanics**):

- Different vibrations → Different particles.
- String combinations → Particle interactions.

The universe is made of

- 21% Dark Matter.
- 74% Dark Energy.
- 4% Normal Matter.

MATTER UNDER EXTREME CONDITIONS

$$\text{Nuclei} + \text{heat} + \text{pressure} \rightarrow \text{quark-gluon plasma}$$

- **Hydrogen atom:** Diameter about a Billionth of an inch.
- **Electron:** Diameter at least 1000 times smaller than that of proton.
- **Proton:** Diameter about 60,000 times smaller than H atom.
- **Probability distribution** is the only way to locate an electron in an atom.

The Gas laws deal with how gases behave with respect to pressure, volume, temperature …

- **Boyle's law:**

Volume and pressure are inversely proportional.

- **Charles' law:**

Volume is proportional to temperature.

- **Pressure law:**

Pressure is proportional to temperature.

All three combined:

$$\frac{PV}{T} = \text{constant}$$

n → number of neutrons

p → number of protons

- **If (n / p) ≈1 (atomic number between 1 and 20):**

Nucleus has equal number of protons and neutrons to become stable.

- **If (n / p) >1 (atomic number between 20 and 83):**

Number of protons increase and repulsion between them also increases. To balance this force number of neutrons also increases.

- **If (n / p) >1 (atomic number > 83):**

Nucleus having atomic number higher than 83 has great number of protons and repulsion force between protons. Since the amount of force is too high, number of neutrons cannot balance them and nucleus stays unstable. Thus, we can say that nuclei having atomic number greater than 83 generally undergo **transmutation**, **alpha decay** or **beta decay**.

- Weak nuclear forces + **Maxwell equations** → Electro weak theory.
- Electro weak theory + Quantum Chromodynamics (**QCD**) → Standard Model of particle physics.
- **Standard Model** of particle physics → explains everything except **gravity**.

4 NUMBERS describe the characteristics of electrons and their orbitals:

- **Principal quantum number:** a number that describes the average distance of the orbital from the nucleus and the energy of the electron in an atom.
- **Angular momentum quantum number:** a number that describes the shape of the orbital.
- **Magnetic quantum number:** a number that describes how the various orbitals are oriented in space.
- **Spin quantum number:** a number that describes the direction the electron is spinning in a magnetic field — either clockwise or counterclockwise.

The square of the periods of the planets (**the times for them to complete one orbit**) is proportional to the cubes of their average distance from the Sun. A consequence of this is that the

inner planets move rapidly in their orbits. Venus, Earth and Mars move progressively less rapidly about the Sun. And the **outer planets**, such as Jupiter and Saturn, move stately and slow.

Wavelength of UV radiation < Wavelength of IR radiation < Wavelength of microwave radiation

- Molecule dissociates (when it absorbs UV radiation).
- Molecule vibrates (when it absorbs IR radiation).
- Molecule rotates (when it absorbs microwave radiation).

⇨ If the expansion of space had overwhelmed the pull of gravity in the beginning − stars, galaxies and humans would never have been able to form. If, on the other hand, gravity had been 5% stronger− stars and galaxies might have formed, but they would have quickly collapsed in on themselves and each other to form a sphere of roughly infinite density.

Neutrons have a mass of 939.56 MeV.

If the mass of a neutron was a seventh of a percent more than it is, stars like most of those we can see would not have existed. If the neutron mass was 0.085% less than it is, the Universe would have been full of neutrons and nothing else.

If we cut the surface of a sphere up into faces, edges and vertices, and let F be the number of faces, E the number of edges and V the number of vertices, we will always get:

$$V − E + F = 2.$$

Fibonacci numbers − 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...

- Each number is the sum of the previous two.

- The ratio between the numbers = 1.618034 (**golden ratio**).
- From **pinecones to the Hurricane Sandy**, Fibonacci reflects various patterns found in nature.

The paths of anything you throw have the same shape, called an **upside-down parabola**.

When we observe how objects move around in **gravitationally curved trajectories** in space, we discover another recurring shape: **the ellipse**.

All material particles have properties such as charge and spin.

Space itself has properties such as dimensions.

- These properties are purely mathematical.

Equations aren't the only hints of mathematics that are built into nature: **there are also numbers involving not only motion and gravity**, but also areas as disparate as classical physics, quantum mechanics, and astronomy.

Strong force → force that is responsible for binding together the fundamental particles of matter to form larger particles.

- **If stronger:** No hydrogen would have formed; atomic nuclei for most life-essential elements would have been unstable; thus, there would have been no life chemistry.
- **If weaker:** No elements heavier than hydrogen would have formed: again, no life chemistry.

Weak force → force that is responsible for the radioactive decay of atoms.

- **If stronger:** Too much hydrogen would have been converted to helium in the big bang; hence, stars would have converted too much matter into heavy elements making life chemistry impossible.
- **If weaker:** Too little helium would have been produced from big bang; hence, stars would have converted too little matter into heavy elements making life chemistry impossible.

Electromagnetic force → force that is responsible for most of the interactions we see in our environment today.

- **If stronger:** Chemical bonding would have been disrupted; elements more massive than boron would have been unstable to fission.
- **If weaker:** Chemical bonding would have been insufficient for life chemistry.

c = 299,792,458 meters per second − serves as the single limiting velocity in the universe, being an upper bound to the propagation speed of signals and to the speeds of all material particles.

Ratio of electromagnetic force to gravitational force

- **If larger:** All stars would have been at least 40% more massive than the sun; hence, stellar burning would have been too brief and too uneven for life support.
- **If smaller:** All stars would have been at least 20% less massive than the sun, thus incapable of producing heavier elements.

Ratio of electron to proton mass

- **If larger or smaller:** Chemical bonding would have been insufficient for life chemistry.

Mass of the neutrino

- **If smaller:** Galaxy clusters, galaxies, and stars would have not formed.
- **If greater:** Galaxy clusters and galaxies would have been too dense.

Ratio of exotic matter to ordinary matter

- **If larger:** the universe would have collapsed before the formation of solar-type stars.
- **If smaller:** no galaxies would have formed.

Number of effective dimensions in the early universe

- **If larger or smaller:** Quantum mechanics, gravity, and relativity could not have coexisted; thus, life would have been impossible.
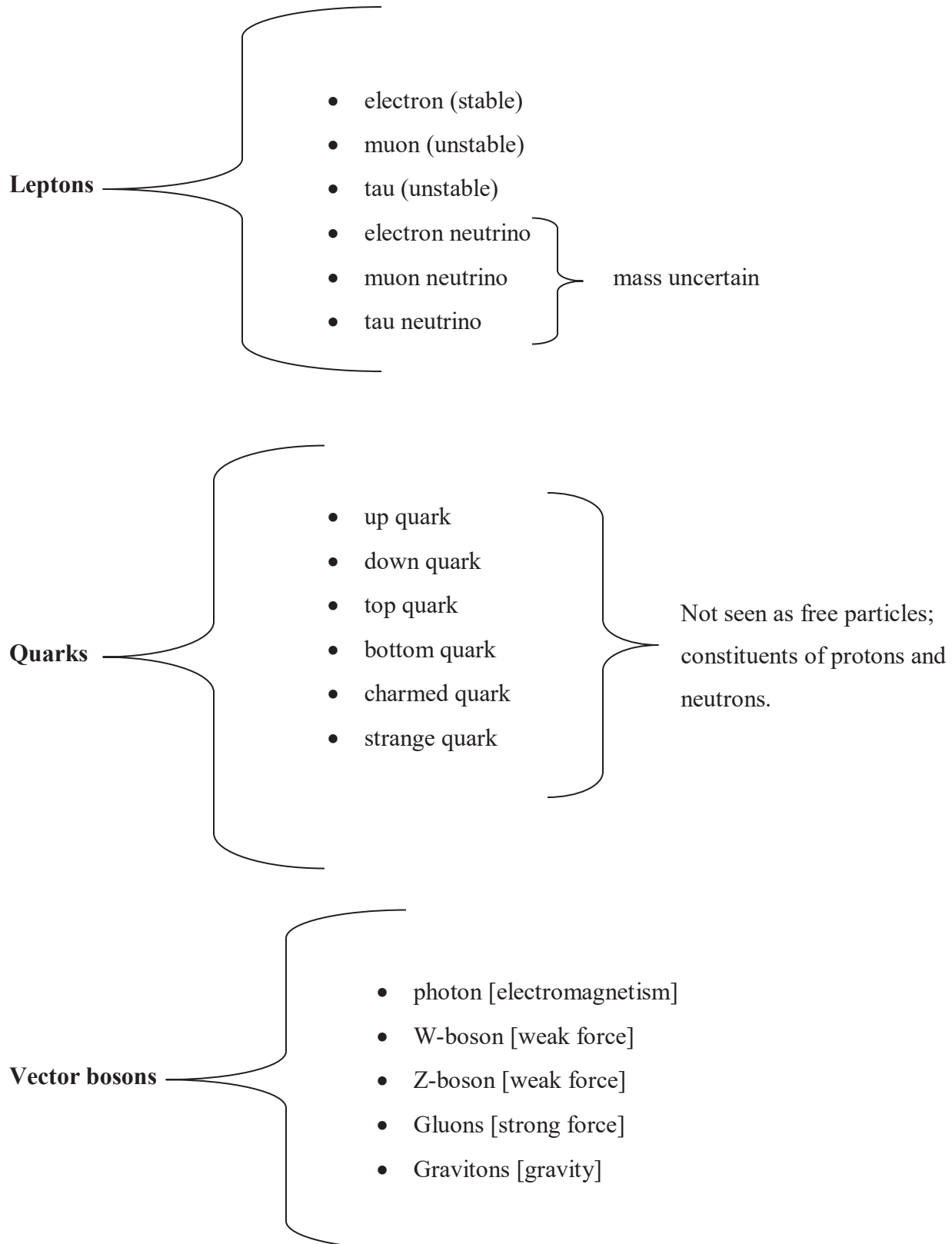
Entropy level of the universe

- **If larger:** Stars would have not formed within proto-galaxies.
- **If smaller:** No proto-galaxies would have formed.

Polarity of the water molecule

- **If greater:** Heat of fusion and vaporization would have been too high for life.
- **If smaller:** Heat of fusion and vaporization would have been too low for life; liquid water would not have worked as a solvent for life chemistry; ice would not have floated, and a runaway freeze-up would have resulted.

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

The force decreases with increasing distance between the charged particles; when the distance is doubled, the force falls by a factor of 4.

Leptons

- electron (stable)
- muon (unstable)
- tau (unstable)
- electron neutrino
- muon neutrino
- tau neutrino

mass uncertain

Quarks

- up quark
- down quark
- top quark
- bottom quark
- charmed quark
- strange quark

Not seen as free particles; constituents of protons and neutrons.

Vector bosons

- photon [electromagnetism]
- W-boson [weak force]
- Z-boson [weak force]
- Gluons [strong force]
- Gravitons [gravity]

The **greater the distance** d to the galaxy, the **higher the velocity** v with which it receded from us, according to the formula:

$$v = \text{Hubble parameter} \times d$$

The momentum p of something with **intrinsic mass** m_0 moving with velocity v is simply given by: $p = m_0 v$ as long as $v \ll c$.

$$\beta = \frac{v}{c} \quad \textbf{Lorentz factor} = \frac{1}{\sqrt{1-\beta^2}}$$

At $v = 0.42c$, **Lorentz factor** = 1.10 − which means the **effects of Einsteinian relativity** become noticeable.

Energy of the photon = Work Function of the surface + Kinetic energy of the emitted electron

$$h\upsilon = W + \frac{m_0 v^2}{2}$$

If $h\upsilon < W$:

No photoelectric emission

The **Sagnac effect** occurs to a pair of light beams travelling in a circuit in the opposite direction. If the circuit is rotating, the beams will complete the circuit in different duration.

The effect was first demonstrated by **Georges Sagnac** in 1913.

$$m = \frac{m_0}{\sqrt{1-\beta^2}} \qquad L = L_0\sqrt{1 - \beta^2} \qquad \Delta t = \frac{\Delta t_0}{\sqrt{1-\beta^2}}$$

If v = c:

$$m \to \infty$$

$$L \to 0$$

$$\Delta t \to \infty$$

Neutron ↔ proton + electron + antineutrino (**beta decay**)

Proton + electron ↔ neutrino + neutron (**electron capture**)

Proton + antineutrino ↔ positron + neutron (**inverse beta decay**)

- Closed Universe → positively curved
- Open Universe → negatively curved
- Flat Universe → uncurved

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

The momentum and the position of a particle cannot be simultaneously measured with unlimited precision.

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$$

The areal velocity of a planet revolving around the sun in elliptical orbit remains constant which implies one-half its angular momentum divided by its mass remains constant. A consequence of this is that the Planet sweeps out equal areas in equal times.

Curvature of space-time + cosmological constant = density of matter

Black hole temperature:

$$T = \frac{\hbar c^3}{8\pi G k_B M}$$

Thus, a smaller black hole is hotter, and consequently radiates more.

The time that the black hole takes to dissipate is:

$$t_{ev} = \frac{5120\pi G^2 M^3}{\hbar c^4}$$

Thus, a black hole of one solar mass ($M_{sun} = 2.0 \times 10^{30}$ kg) takes more than 10^{67} years to evaporate — much longer than the current age of the universe at 14×10^9 years.

$$L = \sigma T^4 A$$

- L = luminosity
- σ = Stefan-Boltzmann constant
- A = surface area
- T = temperature in Kelvin

A consequence of this is that:

- The larger a star is, the more energy it puts out, and the more luminous it is.
- The star with a higher temperature will be more luminous than the star with lower temperatures

If A stands for the surface area of a black hole (area of the event horizon), then the black hole entropy is given by:

$$S_B = \frac{k_B A}{4L_{Planck}^2}$$

- The larger a black hole is, the more entropy it possess.

Wien's Law:

The wavelength of peak emission is inversely proportional to the temperature of the emitting object.

$$\lambda_{max} = \frac{b}{T}$$

b is a constant of proportionality called Wien's displacement constant, equal to $2.897771955...\times 10^{-3}$ mK

Thus, hotter objects emit most of their radiation at shorter wavelengths; hence they will appear to be bluer.

Stellar Radiation Pressure:

$$P_{radiation} = \frac{4\sigma T^4}{3c}$$

Thus, a doubling of temperature means an increase of radiation pressure by a factor of 16.

The nuclear radius R can be approximated by the following formula:

$$R = r_0 \sqrt[3]{A}$$

A = Atomic mass number (the number of protons Z, plus the number of neutrons N) and $r_0 = 1.25$ fm $= 1.25 \times 10^{-15}$ m.

Thus, size of nucleus depends on the mass number of nucleus.

If electrons were bosons, rather than fermions, then they would not obey the **Pauli Exclusion Principle**. There would be no life chemistry.

$$F = \frac{Gm_1 m_2}{r^2}$$

G represents the gravitational constant, which has a value of 6.674×10^{-11} N (m/kg)^2. Because G is small, gravitational force is very small unless large masses are involved.

Eddington limit:

$$L_{Edd} = \frac{4\pi G M m_p c}{\sigma_T}$$

σ_T is the Thomson scattering cross-section for the electron and m_p is the mass of a proton.

If a star exceeds this limit, its luminosity would be so high that it would blow off the outer layers of the star.

Viral Theorem for star:

Thermal energy + gravitational potential energy $= \dfrac{1}{2}$ gravitational potential energy

Thermal energy $= -\dfrac{1}{2}$ gravitational potential energy

$$K = -\frac{1}{2}\,U$$

As a consequence of this is that:

The thermal energy increases if the gravitational potential energy becomes more negative.

- Wavelength of light $\ll$ size of particle : **Geometrical scattering**

- Wavelength of light $\sim$ size of particle : **Mie scattering**

- Wavelength of light $\gg$ size of particle : **Rayleigh scattering**

$k_B T \ll KE_{Fermi}$: the electron gas is fully degenerate

$k_B T \sim KE_{Fermi}$: the electron gas is partially degenerate

$k_B T \gg KE_{Fermi}$: the electron gas is non-degenerate

The spin of the neutron, proton and electron are all $\frac{1}{2}$. If beta decay involves just a neutron becoming a proton and an electron, spin is not conserved.

$$\text{Neutron} \rightarrow \text{proton} + \text{electron}$$

$$\text{Half integral} \rightarrow \text{integral}$$

$$\frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$$

Hence, the above reaction cannot take place if spin is to be conserved.

The electrostatic repulsion between two protons is $\frac{e^2}{4\pi\varepsilon_0 r^2}$ while the gravitational attraction between them is $\frac{Gm_p^2}{r^2}$. The ratio of these two forces is $\frac{e^2}{4\pi\varepsilon_0 Gm_p^2}$. This expression is independent of distance between them, so the relative strength of the forces is the same throughout all space.

If

$$\frac{mv^2}{2} > \frac{GMm}{r}$$

Object of mass m will escape the gravitational field of mass M.

In classical physics, it is possible to exactly specify both position and momentum simultaneously.

Quantum mechanics:

If we try to localize a particle spatially, we lose information about its momentum.

A light year is the distance traveled by light in a year:

1 light year = (speed of light) × (1 year) = 3×10^{10} cms^{-1} × 3×10^{7} s = 9×10^{17} cm.

- Water freezes at 273 K ($\equiv 0^{\circ}$C)
- Water boils at 373 K ($\equiv 100^{\circ}$C)

Intrinsic energy of proton = KE of quarks+ PE of quarks + intrinsic energy of quarks

Any reaction that can take place in nature must follow this rule, no matter what force is responsible for the reaction:

- In any reaction the total charge of all the particles entering the reaction must be the same as the total charge of all the particles after the reaction.

Because $E = mc^2$ (the equation that represents the correlation of energy to matter: essentially, energy and matter are but two different forms of the same thing) and due to the fuzziness of quantum theory (that implies: photon carries mass proportional to its frequency i.e., $h\upsilon = mc^2$), some of the most incredible mysteries of the quantum realm (a jitter in the amorphous haze of the subatomic world) get far less attention than Schrödinger's famous cat. Virtual particle-antiparticle pairs of mass ($\Delta m = \dfrac{h\Delta\upsilon}{c^2}$) are continually created out of energy ΔE of the empty space consistent with the Heisenberg's uncertainty principle of quantum mechanics (which implies:

$$\Delta mc^2 \times \Delta t \geq \frac{\hbar}{2}$$

where: Δt stands for time during which virtual particle-antiparticle pairs of energy ($\Delta mc^2 = h\Delta\upsilon$) appear together, move apart, then come together and annihilate each other giving energy back to the space without violating the law of energy conservation (which states that energy can neither be created nor destroyed; rather, it can only be transformed from one form to another).

Spontaneous births and deaths of roiling frenzy of particles so called virtual matter – antimatter pairs momentarily occur everywhere, all the time − violate the Energy-momentum relationship: $E = \sqrt{m_0^2 c^4 + p^2 c^2}$ − is the conclusion that mass and energy are interconvertible; they are two different forms of the same thing. The word virtual particles literally mean that these particles are not observed directly, but their indirect effects are measured to a remarkable degree of accuracy. Their properties and consequences are well established and well understood consequences of quantum mechanics.

Hubble's law — Consequence of the expansion of the space through which light is travelling.

$$\left. \frac{m_{proton}c^2}{k_B} = T \right\}$$ Temperature below which proton is effectively removed from the universe

For particles moving at speeds very close to that of light:

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$pc \gg m_0 c^2$:

$$E = pc$$

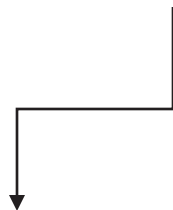Hence the matter behaves similarly to the radiation.

- The angles in a triangle when added together sum up to $180°$.
- The circumference of a circle divided by its diameter is a fixed number called π.
- In a right angled triangle the lengths of the sides are related by $c = \sqrt{a^2 + b^2}$ where c is the length of the side opposite to the right angle.

- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
- 1 keV is a thousand eV
- 1 MeV is a million eV
- 1 GeV is a thousand million eV
- 1 TeV is a million million eV

Particles can only spin at a rate that is a multiple of $\dfrac{h}{2\pi}$.

- Fermions (**quarks and leptons**) spin at $\dfrac{1}{2} \times \dfrac{h}{2\pi}$.

- Bosons (**photons and gluons**) spin at $1 \times \dfrac{h}{2\pi}$ or $2 \times \dfrac{h}{2\pi}$.

Euler's formula:

$$e^{\pi i} + 1 = 0$$

Connects the five fundamental constants of mathematics (e, π, i, 0, 1).

[Imaginary number $i = \sqrt{-1}$].

Maxwell equations	electromagnetism
Schrödinger equation	quantum mechanics
Balmer equation	Interpretation of atomic spectra
Yang-Mills equation	SU(2) gauge symmetry of isospin
Dirac equation	relativistic quantum mechanics
Higgs field equation	symmetry breaking
Einstein equations	relativity
The logistic map	chaotic dynamics

Noether's Theorem (1918):

"For every continuous symmetry there is a corresponding conserved quantity [such as electric charge] and vice versa."

"Even if there is only one possible unified theory, it is just a set of rules and equations…

What is it that breathes fire into the equations and makes a universe for them to describe?"

"Why does the universe go to the bother of existing?"

– Stephen Hawking

$$(i\,\gamma_\mu\,d^\mu - m\,)\,\psi = 0$$

The Dirac Equation that predicts the existence of antimatter

where:

- i = imaginary number
- γ_μ = Pauli matrices
- d^μ = derivative in 4 dimensions
- m = fermion mass
- ψ = wave function

Bayes' Theorem:

$$P(H \mid E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$
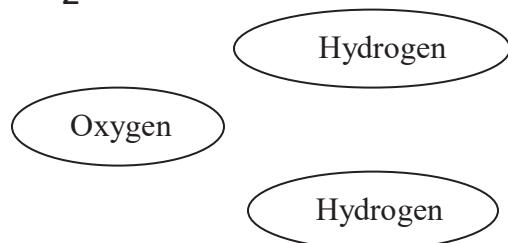
H represents a hypothesis and E the evidence.

- $P(H \mid E)$ – the probability of H given E is true
- $P(E \mid H)$ – the probability of E given H is true
- $P(E)$ – the probability of E
- $P(H)$ – the probability of H

The number 0 is the neutral element of addition:

$$1 + 0 = 1$$
$$23 + 0 = 23$$

Adding 0

Nothing happens

$$F = \mathbf{G} \times \frac{m_1 m_2}{r^2}$$

Constant that controls the strength of gravity

H_2O

Hydrogen

Oxygen

Hydrogen

Consisting of one oxygen atom and two hydrogen atoms, water molecule plays a special role in the chemistry of life.

The Δ^{++} is more massive than the sum of the masses of the proton and the pion (π^+). This means that it is energetically possible for the Δ^{++} to decay into a proton and a pion.

General relativity — tell us about the geometry of spacetime, but not the topology.

The Planck mass $\sqrt{\dfrac{\hbar c}{G}}$ is roughly 24,000,000,000,000,000,000,000 (2.4×10^{22}) times the mass of the electron.

Observations
↓
Hypothesis
↓
Experiment
↓
Laws
↓
Theory

- Planck's law is accurate at all wavelengths.

- Wien's Law is a good approximation at short wavelengths.

- The Rayleigh-Jeans Law is a good approximation at large wavelength.

Five Equations That Changed the World:

- $F = \dfrac{GMm}{d^2}$ (Newton's Law of Universal Gravitation)

- $P + \rho \times \dfrac{1}{2}v^2 = \text{constant}$ (Bernoulli's Law of Hydrodynamic Pressure)

- $\nabla \times E = -\dfrac{\partial B}{\partial t}$ (Faraday's Law of Induction)

- $E = mc^2$ (Albert Einstein's mass–energy equivalence)

- $S_{universe} > 0$ (Clausius's Law of Thermodynamics)

Fine structure constant — Constant characterizing the strength of interaction between charged particles.

Titius–Bode law:

The distance from the Sun to the nth planet is $0{\cdot}075 \times 2^n + 0{\cdot}4$ astronomical units.

$$c = \frac{1}{\sqrt{\text{Vacuum permeability} \times \text{Vacuum permittivity}}}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Determined by the electromagnetic properties of free space – μ_0 and ε_0

Quantum mechanics + general theory of relativity $\rightarrow$ quantum theory of gravity

If the density perturbations were much weaker, then galaxies may never have coalesced. Without galaxies there would be no buildup of heavy elements, and it is unlikely that planets, and life, would have emerged.

In the presence of gravity, time slows down — the stronger the effect of gravity the more that time slows down.

$$\text{Einstein curvature tensor} = \frac{8\pi G}{c^4} \times \text{Energy-momentum stress tensor}$$

- Entropy of Universe = entropy of visible Universe + entropy of dark matter + entropy of black holes
- The total energy of the star = internal energy due to thermal motion and radiation + gravitational potential energy

- Stars with mass > 0.08 M_{sun} burn hydrogen.
- Stars with mass > 0.5 M_{sun} burn hydrogen and helium.
- Stars with mass in the range of 1−8 M_{sun} continue nucleosynthesis up till the production of carbon.
- Stars with mass > 10 M_{sun} synthesize all the elements up to iron and nickel.

Rate of energy production in the **pp-process** of hydrogen burning $\propto$ (Temperature)^4

Rate of energy production in the **CNO-process** of hydrogen burning $\propto$ (Temperature)^{18}

Superstrings

Supersymmetry

Quantum Gravity

Grand Unified Theories
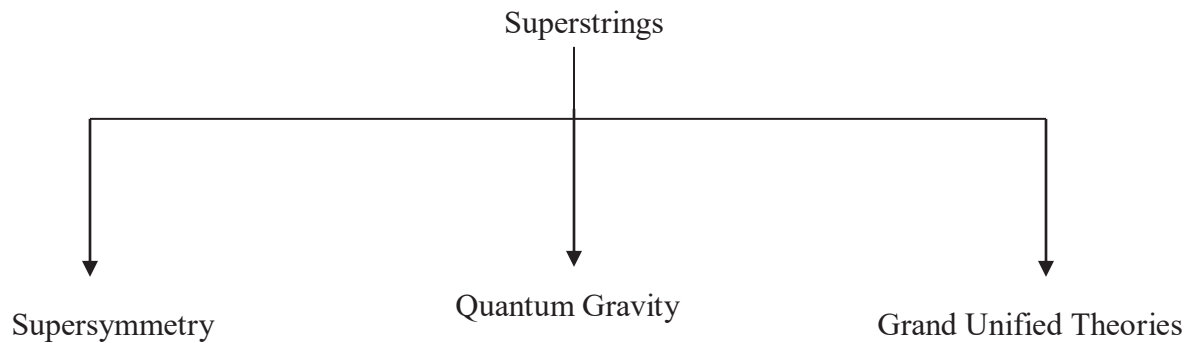
- Size of our universe $\sim 10^{26}$ m
- The distance Earth–Sun is $\sim 1.5 \times 10^{11}$ m
- The radius of the Sun is $\sim 7 \times 10^8$ m
- The radius of the Earth is $\sim 6.4 \times 10^6$ m
- Rocks, Humans, . . . ~ 1 m
- Grains of sand $\sim 10^{-3}$ m
- Viruses $\sim 10^{-7}$ m
- Simple molecules $\sim 10^{-9}$ m
- Atoms $\sim 10^{-10}$ m

$$\text{Redshift} = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} - 1$$

always positive, i.e. observed radiation is redder than the emitted one.

Implies:

Universe is in expansion

$$\text{Temperature of relativistic particles} \propto \frac{1}{\text{scale factor of the universe}}$$

$$\text{Temperature of non-relativistic particles} \propto \frac{1}{(\text{scale factor of the universe})^2}$$

Electromagnetic wave	Wavelength
Radio	~ 10cm or larger
Microwave	~ 1cm
Infrared	~ 10^{-3} cm
Visible	~ 10^{-5} cm
Ultraviolet	~ 10^{-6} cm
X-rays	~ 10^{-8} cm
Gamma-rays	~ 10^{-9} cm or smaller

Principle of equivalence:

m in (m × a) and the m in $\dfrac{GMm}{r^2}$ are identical.

Inertial mass and gravitational mass are identical.

Gravity will affect anything carrying energy

Root of the construction of Einstein's equations which describe gravity

Light intensity drops as $\dfrac{1}{(\text{distance})^2}$

- In an open universe (**negative curvature**): the angles in a triangle add up to less than 180°.

- In a closed universe (**positive curvature**): the angles in a triangle add up to more than 180°.

- In a flat universe (**zero curvature**): the angles in a triangle add up to 180°.

The energy of the universe is constant.

The entropy of the universe tends to a maximum.

Theory

External Reality Theory (ERT)

External reality exists completely independent of human beings

Mathematical Universe Theory (MUT)

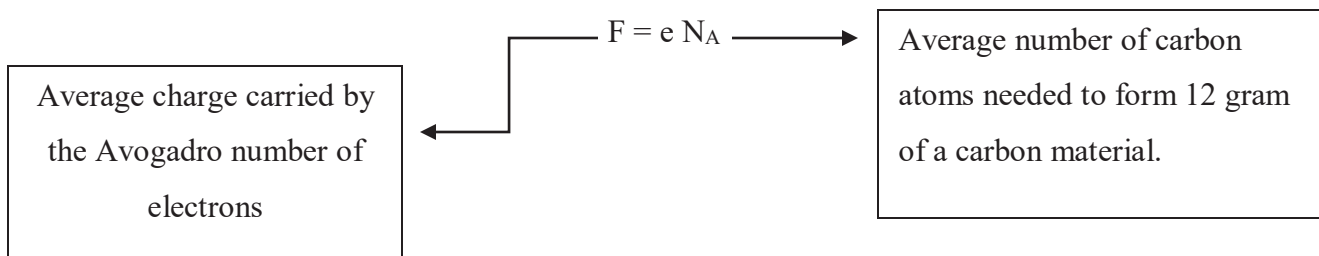External physical reality is a mathematical structure

G

When associated with c and with the reduced Planck's constant $\hbar$, it leads to the definition of the Planck's time:

$$t_{Planck} = \sqrt{\frac{\hbar G}{c^5}} = 5.4 \times 10^{-44} \text{ seconds}$$

The shortest possible time interval that can be measured.

Faraday constant:

$$F = e \, N_A$$

Average charge carried by the Avogadro number of electrons

Average number of carbon atoms needed to form 12 gram of a carbon material.

The nuclear charge Q can be approximated by the following formula:

$$Q = Ze$$

Z = Atomic number (the number of protons).

Thus, charge of nucleus depends on the number of protons.

$$Q = \frac{\text{number of protons}}{N_A} \times F$$

$$Q = \text{number of moles of proton} \times F$$

The strong coupling constant defines the strength of the force that holds protons and neutrons together.

The Universe is made up of three things:

- Vacuum
- Matter
- Photons

Total energy density of the universe:

$$\rho = \rho_{\text{vacuum}} + \rho_{\text{matter}} + \rho_{\text{radiation}}$$

$\rho_{\text{vacuum}} = \dfrac{\Lambda c^2}{8\pi G}$ is constant and independent of time. The cosmological constant "Λ" has negative pressure equal and opposite to its energy density and so causes the expansion of the universe to accelerate.

Matter:

$$E = mc^2$$

For N particles:

$$E = Nmc^2$$

$$\rho_{matter} \propto \frac{Nmc^2}{R^3}$$

Radiation:

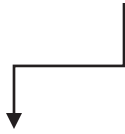$$E = \frac{hc}{\lambda} \quad \text{and} \quad \lambda \propto R$$

For N photons:

$$E \propto \frac{Nhc}{R}$$

$$\rho_{radiation} \propto \frac{Nhc}{R^4}$$

- $\rho_{vacuum} \propto R^0$
- $\rho_{matter} \propto R^{-3}$
- $\rho_{radiation} \propto R^{-4}$

$$\frac{m_{proton}}{m_{electron}} = 1836.15267389$$

Changing their values changes the physical phenomena

When the Universe was at the Planck temperature $\sqrt{\dfrac{\hbar c^5}{G k_B^2}}$ and the mean energy of photons was close to the Planck energy $\sqrt{\dfrac{\hbar c^5}{G}}$.

If the thermal kinetic energy of stellar gas cloud wins over gravity:

$$\frac{3}{2} N k_B T > \frac{3 G M^2}{5 R} \longrightarrow \text{Expansion}$$

If

$$\frac{3 G M^2}{5 R} > \frac{3}{2} N k_B T \rightarrow \textbf{gravity wins: } \text{collapse!}$$

Solar mass $= 2 \times 10^{30}$ kg − of which about 70% is hydrogen, 28% helium, and 2% consists of other elements. Only about a seventh part of that hydrogen mass is available at any time for hydrogen fusion in the core of the Sun.

Because relativistic mass $= \dfrac{\text{rest mass}}{\sqrt{1-\dfrac{v^2}{c^2}}}$: the faster an particle moves, the more kinetic energy it possess. But according to **E= mc²**, kinetic energy adds to an particle's mass, so the faster an particle moves, the harder it is to further increase the particle's speed. This effect is really significant only for particles moving at speeds close to the speed of light. As a particle approaches the speed of light, its mass increases ever more quickly to infinite, so it take infinite amount of energy to speed it up further. This is the reason that any material particle is forever confined by relativity to move at speeds slower than the speed of light. Only photons that have no intrinsic mass move at the speed of light.

$$E^2 = p^2c^2 + m_0{}^2c^4$$

$m_0 = 0$:

$$E = pc$$

The **Compton wavelength of a particle** characterizes the length scale at which the wave property of a given particle starts to show up. In an interaction that is characterized by a length scale larger than the Compton wavelength, particle behaves classically (i.e., no observation of wave nature). For interactions that occur at a length scale comparable than the Compton wavelength, the wave nature of the particle begins to take over from classical physics.

At Planck length ($\sqrt{\dfrac{\hbar G}{c^3}}$), the gravitational force is as strong as the other forces and space-time is "foamy" − filled with tiny bubbles and wormholes appearing and disappearing into the vacuum.

Rayleigh scattering law: The amount of scattering of light is inversely proportional to the fourth power of the wavelength.

$$I \propto \frac{1}{\lambda^4}$$

Thus, Rayleigh scattering is more intense at shorter wavelengths.

Supersymmetry — The positive zero point energy of the boson field exactly cancels the negative zero point energy of the fermion field.

$h \rightarrow 6.62607004 \times 10^{-34} \text{ m}^2 \text{ kg/s}$ — Because h is too small: **Quantum mechanics is for little things**.

Einstein's Relativity:

Motion and Gravity make Clocks tick slowly.

Gravity pulls everything in, but a mysterious force called **dark energy** tries to push it all back together again. Our fate relies on which force will win the desire to succeed.

Because of

CP violation (violation of charge conjugation parity symmetry)

there was more matter than antimatter right after the Big Bang.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Tachyons (if they exist) have $v > c$. This means that m is imaginary!

General theory of relativity describes gravity, ignoring quantum mechanics.

Math in Nature:

| Hexagon | Bee Hive |
| Concentric Circles | Ripples of a pond when a stone hits the surface of the water. |

Doppler Effect:

- When a wave source moves toward an observer, its waves appear to have a shorter wavelength.
- When a wave source moves away from an observer, its waves appear to have a longer wavelength.

In more than three space dimensions, planetary orbits would be unstable and planets would either fall into the sun or escape its attraction altogether

What goes up must get down ⟶ **Newton's law of gravity**

What goes up need not descend− if it is shot upward faster than the **escape velocity** ($\sqrt{\dfrac{2GM}{R}}$)

Since gravity weakens with distance, the earth pulls on your head with less force than it pulls on your feet, which are a meter or two closer to the earth's center. The difference is so tiny we cannot feel it, but an astronaut near the surface of a black hole would be literally torn apart.

Because:

$$2\pi r = n\lambda$$

Only orbits with circumferences corresponding to a whole number of electron wavelengths could survive without destructive interference.

Because:

$$r = \frac{3GM}{c^2}$$

The photon spheres can only exist in the space surrounding an extremely compact object (a black hole or possibly an "ultracompact" neutron star).

- **In phase** → wave crests and troughs reinforce each other.
- **Out of phase** → wave crests and troughs cancel out.

- The energy above which (**Grand unification energy** ≈ 10^{16}GeV), the electro-magnetic force, weak force, and strong force become indistinguishable from each other.
- Since the graviton has no mass of its own, the **gravitational force of attraction** between the sun and every planet is long range.
- The proton and neutron masses are so similar; they differ only by the replacement of an up quark with a down quark.

Because:

$$\frac{E}{B} = c$$

Electric and magnetic fields turn into each other in a wavelike motion, creating an electromagnetic field that travels at the speed of light.

When **two black holes** collide, they merge, and the area of the final black hole is greater than the sum of the areas of the original holes.

Inside the nucleus of an atom, a proton is never permanent a proton and a neutron is never permanently a neutron. They keep on changing into each other. A neutron emits a pi meson and become proton and a proton absorbs a pi meson and become a neutron.

Neutron $\rightarrow$ proton $+ \pi^-$

Proton $+ \pi^- \rightarrow$ neutron

There is no escape from a black hole in **classical theory**, but quantum theory enables energy and information to escape.

The **inherent goal of unification** is to show that all of these forces are, in fact, manifestations of a single force. We can't perceive this unity at the low energies of our everyday lives, or even in our most powerful accelerators at CERN. But close to the Big Bang temperatures, at inconceivably high energies… If the forces unify, the proton can be unstable, and eventually decay …

Proton → positron + neutral pion

Neutral pion → 2 gamma ray photons

- Accelerated massive bodies give off gravitational waves just as bound electrons in an atom emit electromagnetic radiation
- A rotating neutron star (a tiny, burnt out star) generates regular pulses of radio waves.
- Quantum mechanics says that the position of a particle is uncertain, and therefore that there is some possibility that a particle will be within an energy barrier rather than outside of it. The process of moving from outside to inside without traversing the distance between is known as **quantum tunneling**, and it is very important for the fusion reactions in stars like the Sun.

According to string theory, our universe is made up of small vibrating strings whose size is 10^{23} times smaller than protons. A **String** can do something aside from moving − it can vibrate in different ways. One mode of vibration makes the string appear as a photon, another as a neutron, and so on...

Because

$$dM = \frac{k}{8\pi}\,dA + \Omega dJ + \Phi dQ$$

M stands for mass, k for surface gravity, A for area of the event Horizon, J for angular momentum, Ω for angular velocity, Q for charge and Φ for the electrostatic potential

the size and shape of the black hole depends only on its mass, charge and rate of rotation, and not on the nature of the star that had collapsed to form it.

The electromagnetic and weak interactions lose their symmetry below 100 GeV

Entropy (a thermodynamic measure of untidiness in a system and a measure of how much information a system contains) is defined as:

$$S = k_B \ln \{\text{number of states}\}$$

which, for N particles of the same type, is

$$S = k_B \ln \{(\text{no of one-particle states}) \, N\}$$

$$S = k_B N \ln \{\text{a not-too-big number}\}$$

$$S = k_B N$$

This means: the more particles, the more disorder.

Hund's rule: every orbital in a subshell is singly occupied with one electron before any one orbital is doubly occupied, and all electrons in singly occupied orbitals have the same spin.

Because:

Kinetic energy of the emitted electron = photon energy − 13.6 eV

photons need an energy > 13.6 eV to ionize hydrogen atom.

$11 \times 11 \rightarrow 121$

$111{,}111{,}111 \times 111{,}111{,}111 \rightarrow 12{,}345{,}678{,}987{,}654{,}321$

Palindrome number − a number that reads the same forwards or backwards

If particle A enters the ergosphere of a Kerr black hole, then it splits into particles B and C.

$$E_A = E_B + E_C$$

Particle C with Energy $E_C < 0$ (negative energy) → falls into the black hole.

Particle B with Energy $E_B > E_A$ → escapes

The added negative energy particle will slow down the spinning of the Kerr black hole and reduce its energy and therefore its mass.

Waves over time → As the waves decay, their amplitude becomes smaller and, eventually, the waves disappear

Because:

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

Electricity and magnetism are related.

"This difficulty [the inconsistency of the laws of gravity with a finite mean density of matter] also arises in the general theory of relativity. However, I have shown that this can be overcome through the introduction of the so-called "λ–term" to the field equations… I showed that these equations can be satisfied by a spherical space of constant radius over time, in which matter has a density ρ that is constant over space and time."

– **Albert Einstein**

German astronomer and mathematician **Johannes Kepler** who came from a poor Protestant family in Germany − established that the planet orbit the sun in a elliptical path, meaning that the planet to Sun distance is constantly changing as the planet goes around its orbit. Kepler's laws of motion tell how the planet moves on this way. Later **Isaac Newton's universal law of gravitation** explained why the planets move in this way.
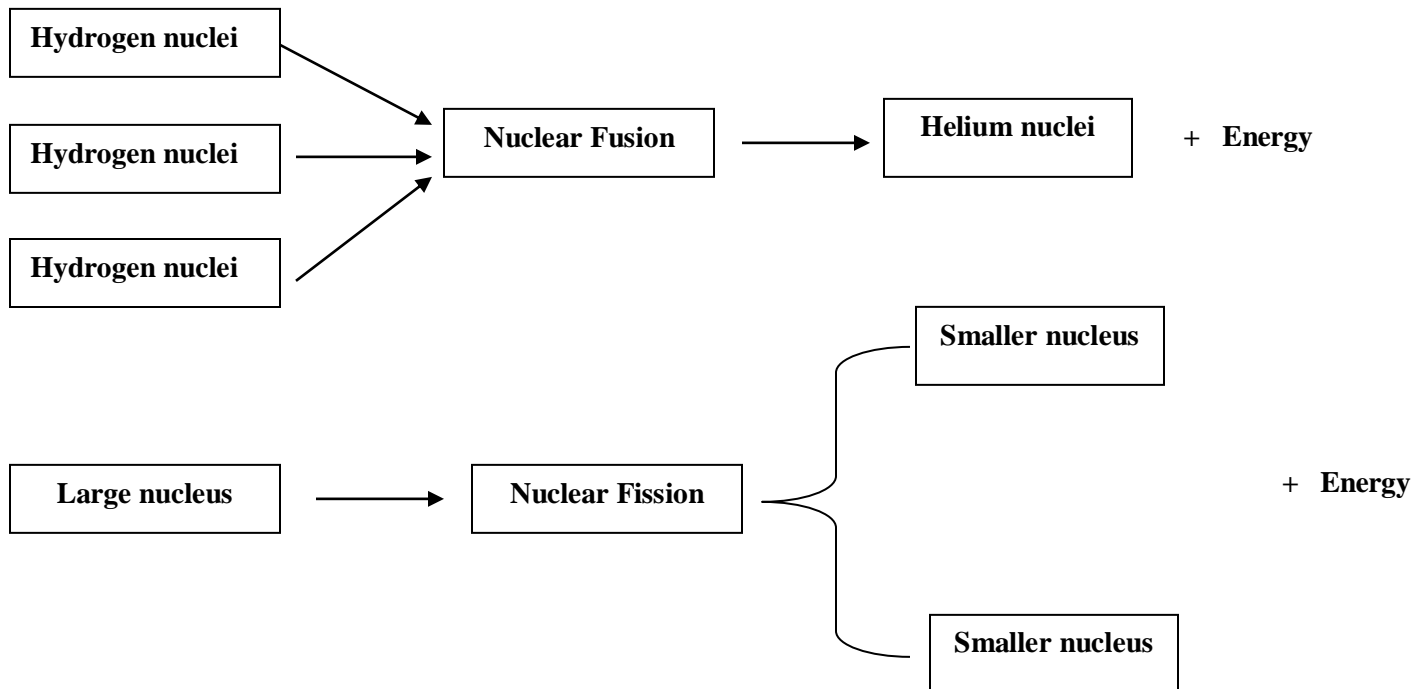
Tycho's model

Planets orbit around the Sun and the Sun orbit around the Earth at the center of the Universe.

Electromagnetic wave

The undulating strength of the electric and magnetic disturbance − propagating through space − carrying electromagnetic radiant energy

The **expansion of the Universe** can be compared to the expanding surface of a balloon that is being inflated. As more air is blown into it, we would see the surface area of the balloon expanding and every point on its surface getting further and further away from one other.

- In a bound atom of hydrogen the negatively charged electron moves round the positively charged nuclei. In high temperature plasma the nuclei and electrons are no longer bound.
- Motion of stars in galaxies reveals the existence of hypothetical form of mass thought to account for approximately 85% of the mass in the universe and about 27% of its total mass–energy density, or "**dark matter**," whose nature remains unknown.

Hydrogen nuclei		
Hydrogen nuclei	→ **Nuclear Fusion** →	**Helium nuclei** + **Energy**
Hydrogen nuclei		

Large nucleus	→ **Nuclear Fission** →	**Smaller nucleus**
		Smaller nucleus

+ **Energy**

Atomism	The world view that everything is built up from two fundamental principles: atom (**fundamental indivisible component**) and void.

The sum of multiple waves → superposition **(The resulting wave form is stable in time and space)**

Complementarity Principle
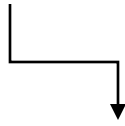
Wave and particle or **position and momentum** cannot be observed at the same time.

Ontology:

What the underlying structure of reality is?

Paradigm → framework for thinking about the nature of reality

So far as we know, all the fundamental laws of physics, such as Newton's equations, are reversible. Then were does irreversibility come from? It comes from order going to disorder, but we do not understand this until we know the origin of the order. Why is it that the situations we find ourselves in every day are always out of equilibrium?

– Richard Feynman

Aristotle (384−322 B.C)	The earth is spherical in shape
Aristarchus (312-230 B.C)	The Universe is Sun-centered
Johannes Kepler (1571−1630)	Planets more around the Sun in Orbits which are not circular but elliptical
Nicolaus Copernicus (1473−1543)	The Sun is at the centre of the Solar System
Galileo Galilei (1564−1642)	The Sun has both hot high temperature and dark low temperature spots

Arithmetic and number theory	patterns of number and counting
Geometry	patterns of shape
Calculus	patterns of motion
Logic	patterns of reasoning
Probability theory	patterns of chance
Topology	patterns of closeness and position

Gravity and Distance:

$$F_1 = \frac{GMm}{r^2}$$

If the distance between the masses triples, the gravitational force decreases by three squared, or nine:

$$F_3 \text{ (force at thrice the distance)} = \frac{GMm}{(3r)^2} = \frac{GMm}{9r^2} = \frac{1}{9}F_1$$

Increasing the distance by twenty times would decrease the gravitational attraction by four hundred times:

$$F_{20} \text{ (force at twenty times the distance)} = \frac{GMm}{(20r)^2} = \frac{GMm}{400r^2} = \frac{1}{400}F_1$$

Since the Moon's mass is 7.35×10^{22} kg i.e., about 1.2 percent of Earth's mass, it has a much weaker gravitational pull on us. This means our weight would be less on the Moon than on Earth. In fact, we would weigh about one-sixth what we weigh on Earth.

Spontaneous generation theory

Different kinds of nonliving matter give rise to different kinds of living creatures

(Rotting meat gives rise to flies while old rags give rise to mice)

Albert Einstein's theory:

The entire universe can expand or contract − just like the overall stretching or shrinking of an elastic sheet

Max Tegmark's 4 distinct types of parallel universes:

- Parallel universes with the same laws of physics but different initial conditions
- Parallel universes with the same equations of physics but perhaps different constants of nature
- Parallel universes superimposed in the same physical space but mutually isolated and evolving independently
- Parallel universes with different mathematical structures

365 → the number of days in a year

$10^2 + 11^2 + 12^2$	Sum of 3 consecutive squares
$13^2 + 14^2$	Sum of the next 2 squares

28 → the number of days in the lunar month

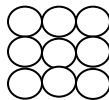|	
$1 + 2 + 4 + 7 + 14$	Sum of all of its divisors
$1^3 + 3^3$	Sum of the cubes of the first two odd numbers

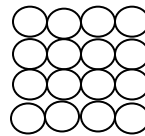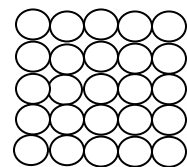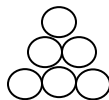**Square numbers:** Numbers that can be arranged in a square

| 1 | 4 | 9 | 16 | 25 |

Triangular numbers: Numbers that can be arranged in a triangle
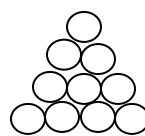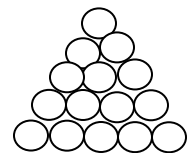
| 1 | 3 | 6 | 10 | 15 |

Palindromic numbers

Numbers reading the same forwards and backwards

121

523323325

Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve independence

– Hermann Minkowski, 1908

Object moves at constant velocity in an inertial frame ⟺ Object experiences zero net force

In string theory:

- Laws of physics
- Particle spectrum
- Nature of forces

Dictated by shape + size (**geometry**) of dimensions

$$\sqrt{\frac{\hbar G}{c^3}} \rightarrow \text{Planck length and } \sqrt{\frac{\hbar G}{c^5}} \rightarrow \text{Planck time are the smallest possible units.}$$

$$\sqrt{\frac{\hbar c^5}{G k_B^2}} \rightarrow \text{Planck temperature is the hottest possible temperature.}$$

$$\sqrt{\frac{\hbar c}{G}} \rightarrow \text{Planck mass, however, is not the smallest possible mass. Many things weigh less, like,}$$

for example, an electron or a proton. The Planck mass is big because

$$G = 6.67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \text{ (relatively very weak)}$$

Spatial dimensions ≥ 4:

> Electrons fall on the nuclei and therefore the atomic structure of matter does not exist.
>
> The atomic matter and therefore life are possible only in 3-dimensional space

- If the electron charge were increased by a factor ~3 no nuclei with atomic number > 5 would exist and no living organisms would be possible

So one reason why scientists do mathematical theories is because they surprise us, they become smarter than us, and eventually we become the students of the theory.

− **Donald Hoffman**

Entropy change

$$\Delta S \geq 0$$

- $= 0$ (reversible process)
- > 0 (irreversible process)

We are the cosmos made conscious and life is the means by which the universe understands itself…

Brian Cox

Boltzmann factor: $e^{-\frac{E}{k_B T}}$

Express the probability of the system **occupying the state of** energy E at absolute temperature T

- Plasma → ions + electrons
- When electrons interact with the electric (Coulombic) fields of ions − electrons would be deflected and a portion of their kinetic energy would be lost.

Final KE of electrons = initial KE of electrons − energy of emitted X-ray photons

The 'lost' kinetic energy is emitted as X-ray photons (**Bremsstrahlung radiation**)

The KE per electron is $\frac{3\,k_B T}{2}$. When an electron gives up all its kinetic energy, then the **Bremsstrahlung** photon is emitted with highest frequency υ:

$$\upsilon = \frac{3\,k_B T}{2h}$$

$\lambda T = \frac{2c_2}{3}$ (where c_2 is the second radiation constant)

Since boat displaces a weight of water equal to its own weight:

It floats in water   **Principle of flotation**

Temperature > **Curie temperature**

Ferromagnetic → Paramagnetic

(**Magnetic materials lose their ferromagnetic properties**)

Temperature > **Néel temperature**

Antiferromagnetic → Paramagnetic

(**Magnetic materials lose their antiferromagnetic properties**)

Temperature < Debye temperature ($T_D = \dfrac{h\upsilon_D}{k_B}$, where υ_D → Debye frequency)

The temperature of a crystal's highest normal mode of vibration

The heat capacity of crystals increases linearly with **(Temperature) ³**

We are the representatives of the cosmos; we are an example of what hydrogen atoms can do, given 15 billion years of cosmic evolution.

Carl Sagan

The **decay width** (г) of a decaying particle with a **lifetime** (τ) is given by: $г = \dfrac{h}{2\pi\tau}$

$$г \propto \frac{1}{\tau}$$

Electron + proton → neutrino + neutron (**inverse beta decay**)

Takes place in stars of extremely high density

Albert Einstein Did Not Win the Nobel Prize for His Theory of Relativity in 1921 but For the Photoelectric Effect

Albert Einstein's general theory of relativity suggests that the sun's gravity bends the path of light from distant stars. It's a testable prediction, but only during a total solar eclipse.

- Albert Einstein's general theory of relativity was proved in 1919 during a solar eclipse.

- It took 15 years for Albert Einstein's Relativity theory to make him famous.

Einstein presented his general theory of relativity in 1916, but for an entire century nobody could find physical proof of black holes. In 2016, scientists finally detected gravitational waves that emitted from two black holes colliding, proving that such weird things not only exist, but that Einstein was right all along.

BIG BANG THEORY

Describes earliest moments of Universe

Beginning of Universe

Beginning of TIME

Flatness problem: Why is the density in the Universe almost critical?

Horizon problem: Why is the large scale of the Universe so smooth?

Supergiant star ⟶ **Core collapse** ⟶ **Neutrinos**

They are among the most abundant particles in the gigantic Universe, and still are hard to detect. They're similar to electrons, but they have no electrical charge and their mass is almost zero, so they interact very little with normal matter as they stream through the Universe at near light-speed. Billions of them are zipping through human body right now. Hence, they are also called "**ghost particles**."

How does the universe work?	Why does it work that way?
• Observe the universe • Precise measurements • Is it testable? **Experimental physics**	• Look for abstract ideas • Mathematical models • Unified descriptions **Theoretical physics**

- In 1962, The U.S. Blew Up A **Hydrogen Bomb** In Space That Was 100 Times More Powerful Than Hiroshima.
- Astronauts on the **International Space Station** Witness around 15 Sunrises and 15 Sunsets Every Day.
- In 1977, We Received A Signal From **Deep Space** That Lasted 72 Seconds. We Still Don't Know How Or Where It Came From.

Geocentric Model of the Universe	Heliocentric Model of the Universe
(Ptolemaic Model)	(Copernican Model)
↓	↓
Earth centered model of the universe	**Sun centered model of the universe**
(Earth at the center)	(Sun at the center)

A cosmic mystery of immense proportions, once seemingly on the verge of solution, has deepened and left astronomers and astrophysicists more baffled than ever. The crux ... is that the vast majority of the mass of the universe seems to be missing.

[Reporting a Nature article discrediting explanation of invisible mass being due to neutrinos]
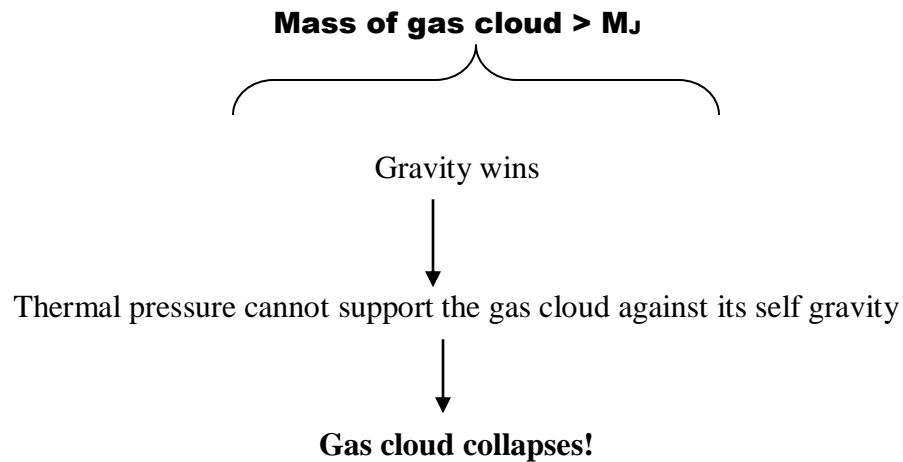
— William J. Broad

If you leave at Age of 15 in a Spaceship at Speed of Light and Spends **5 Years in Space**, when you get back on Earth you will 20 Years old. But all of your Friends who were 15 when you Left, will be 65 Years Old at that Time.

If you fall into black hole, you will able to see both the Universe beginning and ending due to **Time Dilation**.

Jeans mass:

$$M_J = \frac{3k_B TR}{2Gm}$$

k_B = Boltzmann constant, T = temperature in Kelvin, R = radius of gas cloud, m = mass of gas particle and G = gravitational constant

Mass of gas cloud > M_J

Gravity wins

Thermal pressure cannot support the gas cloud against its self gravity

Gas cloud collapses!

Virial Theorem: $2K + U = 0$

- If $2K > U$: the gas pressure will dominate over gravity
- If $2K < U$: the gas cloud will collapse

Low mass star → cooler, fainter, long lifetime

High mass star → hotter, brighter, short lifetime

Describes the interaction of a charged particle (**of charge q and mass m**) with an electromagnetic field

Lorentz force law for the force F acting on the particle:

$$F = q (E + v \times B)$$

where E and B are electric and magnetic field vectors and v is the velocity of the particle.

$$\% \text{ mass increase} = \frac{m - m_0}{m_0} \times 100 = (\textbf{Lorentz factor} - 1) \times 100$$

Intensity:

$$I = \varepsilon_0 c^2 (E \times B)$$

Since: $c = \dfrac{E}{B}$

$$I = \varepsilon_0 c E^2$$

$$\boxed{I \propto E^2}$$

- The Sun's Mass Takes Up 99.86% Of The Solar System
- One Million Earths Can Fit Inside The Sun
- There Are More Trees On Earth Than Stars In The Milky Way
- The Sunset On Mars Appears Blue
- There Are More Stars In The Universe Than Grains of Sands on Earth
- One Day on Venus Is Longer Than One Year on Earth.
- Venus is the hottest planet in the solar system and has an average surface temperature of around 450° C.

$$\Delta A \times \Delta B \geq \frac{\hbar}{2}$$

where A and B are two conjugate variables in quantum mechanics

- position and momentum
- angular orientation and angular momentum
- energy and time

You can get an idea of human nature only when you can see the relationship of the individual human being to the whole cosmos.

Rudolf Steiner

Planck mass = 1.2×10^{19} GeV

about 22μ gram − much heavier than any mass of existing elementary particles

The **mass** m of a particle moving with a velocity v is given by $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$ where: m_0 = rest mass of particle and c = speed of light.
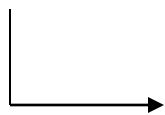
$$\sqrt{c^2 - v^2} = \frac{m_0 c}{m}$$

$$\frac{\sqrt{c^2 - v^2}}{v} = \frac{m_0 c}{mv}$$

$$\sqrt{\frac{c^2}{v^2} - 1} = \frac{\lambda}{\lambda_C}$$

$$\boxed{\lambda = \frac{\lambda_C}{\sqrt{\dfrac{c^2}{v^2} - 1}}}$$

$\lambda_C \rightarrow$ **Compton Wavelength of the particle**

The length scale at which relativistic quantum field theory becomes crucial for its accurate description

If $\lambda = \lambda_C$:

$$\boxed{v = \frac{c}{\sqrt{2}} = 0.70710678118 \; \mathbf{c}}$$

Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{2} = 1.41421356237$$

Total energy of the particle:

$$E = \gamma m_0 c^2 = \sqrt{2}\ m_0 c^2 = 1.41421356237\ m_0 c^2$$

Kinetic energy of the particle:

$$KE = (\gamma m_0 c^2 - m_0 c^2) = (\sqrt{2} - 1)\ m_0 c^2 = 0.41421356237\ m_0 c^2$$

Momentum of the particle:

$$p = \gamma m_0 v = \sqrt{2}\ m_0 \frac{c}{\sqrt{2}} = m_0 c$$

$$\%\ \textbf{mass increase} = \frac{m - m_0}{m_0} \times 100 = (\gamma - 1) \times 100$$

$$\%\ \textbf{mass increase} = (\sqrt{2} - 1) \times 100 = 41.4213562373$$

Photon frequency required to increase the energy of the particle from $m_0 c^2$ to $\sqrt{2}\ m_0 c^2$:

$$h\upsilon = (\sqrt{2} - 1)\ m_0 c^2 = 0.41421356237\ m_0 c^2$$

$$\upsilon = 0.41421356237 \; \frac{m_0 c^2}{h}$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m^2 c^2 - m^2 v^2 = m_0{}^2 c^2$$

$$mv \, dv + v^2 dm = c^2 dm$$

$$\frac{dm}{dv} = \frac{mv}{c^2 - v^2} = \frac{2m_0}{c}$$

$$\frac{E}{KE} = \frac{\gamma m_0 c^2}{m_0 c^2 \, (\gamma - 1)} = \frac{\sqrt{2}}{(\sqrt{2}-1)} = 3.41421356237$$

$$E = 3.41421356237 \; KE$$

- If a particle travels at a velocity $v = 0.70710678118$ **c, then** its wavelength $\dfrac{h}{mv}$ will be equal to its Compton wavelength $\dfrac{h}{m_0 c}$

- Its total energy will be $1.41421356237 \ m_0 c^2$

- Its kinetic energy will be $0.41421356237 \ m_0 c^2$

For electron:

$$m_0 c^2 = 8.187105776925 \times 10^{-14} \text{ J}$$

$$\boxed{\mathbf{KE} = 3.39121024936 \times 10^{-14} \text{ J}}$$

For proton:

$$m_0 c^2 = 1.503277484 \times 10^{-10} \text{ J}$$

$$\boxed{\mathbf{KE} = 6.2267792187 \times 10^{-11} \text{ J}}$$

- Potential difference that should be applied so that electron's energy increases from $m_0 c^2$ to mc^2:

$$V = \frac{mc^2 - m_0 c^2}{e} = \frac{\gamma m_0 c^2 - m_0 c^2}{e} = \frac{m_0 c^2 \ (\gamma - 1)}{e}$$

$$V = \frac{8.187105776925 \times 10^{-14}}{1.60217662 \times 10^{-19}} \times (\sqrt{2} - 1)$$

$$\boxed{\mathbf{V = 2.11662697297 \times 10^5 \ Volts}}$$

- Potential difference that should be applied so that proton's energy increases from m_0c^2 to mc^2:

$$V = \frac{mc^2 - m_0c^2}{e} = \frac{\gamma m_0c^2 - m_0c^2}{e} = \frac{m_0c^2\,(\gamma-1)}{e}$$

$$V = \frac{1.503277484 \times 10^{-10}}{1.60217662 \times 10^{-19}} \times (\sqrt{2} - 1)$$

$$\boxed{V = 3.8864499338 \times 10^8 \text{ Volts}}$$

- Its mass increase by 41.4213562373%

- **Photon frequency** required to increase the energy of the electron from m_0c^2 to $\sqrt{2}\,m_0c^2$:

$$\upsilon = 0.41421356237\ \frac{m_0c^2}{h} = 0.41421356237 \times \frac{8.187105776925 \times 10^{-14}}{6.62607004 \times 10^{-34}}$$

$$\boxed{\upsilon = 5.1179812904 \times 10^{19} \text{ Hz}}$$

- **Photon frequency** required to increase the energy of the proton from m_0c^2 to $\sqrt{2}\,m_0c^2$:

$$\upsilon = 0.41421356237\ \frac{m_0c^2}{h} = 0.41421356237 \times \frac{1.503277484 \times 10^{-10}}{6.62607004 \times 10^{-34}}$$

$$\boxed{\upsilon = 9.397394203 \times 10^{22} \text{ Hz}}$$

The rate of change of electron's mass with change in its velocity:

$$\frac{dm}{dv} = \frac{2m_0}{c} = \frac{2 \times 9.109\,383\,7015 \times 10^{-31}}{3 \times 10^8}$$

$$\boxed{\frac{dm}{dv} = 6.07292246767 \times 10^{-39} \text{ kg/m/s}}$$

The rate of change of proton's mass with change in its velocity:

$$\frac{dm}{dv} = \frac{2m_0}{c} = \frac{2 \times 1.67262192369\,51 \times 10^{-27}}{3 \times 10^8}$$

$$\boxed{\frac{dm}{dv} = 1.11508128246 \times 10^{-35} \text{ kg/m/s}}$$

In a sense cosmology contains all subjects because it is the story of everything, including biology, psychology and human history. In that single sense it can be said to contain an explanation also of time's arrow. But this is not what is meant by those who advocate the cosmological explanation of irreversibility. They imply that in some way the time arrow of cosmology imposes its sense on the thermodynamic arrow. I wish to disagree with this view. The explanation assumes that the universe is expanding. While this is current orthodoxy, there is no certainty about it. The red-shifts might be due to quite different causes. For example, when light passes through the expanding clouds of gas it will be red-shifted. A large number of such clouds might one day be invoked to explain these red shifts. It seems an odd procedure to attempt to 'explain' everyday occurrences, such as the diffusion of milk into coffee, by means of theories of the universe which are themselves less firmly established than the phenomena to be explained. Most people believe in explaining one set of things in terms of others about which they are more certain, and the explanation of normal irreversible phenomena in terms of the cosmological expansion is not in this category.

— Peter Theodore Landsberg

Binary Stars – a pair of stars in orbit around their common center of gravity.

- **Apparent Magnitude** – a star's brightness as it appears to Earth
- **Absolute Magnitude** – how bright a star actually is.

$$\text{(Apparent Magnitude} - \text{Absolute Magnitude)} = 5\log_{10}\left(\frac{\text{Distance between the star and the earth}}{10}\right)$$

Distance modulus

Measured in parsecs

- **Red Giants** – a large, cool star of high luminosity
- **Super giants** – a very large, very bright red giant star
- **Nebulae** – clouds of dust and gases in space

- Hydrogen in core is depleted.
- Core contracts and heats up.
- Heat causes the outer layers to expand.
- Expanding causes the layers to cool.

A star's color reveals its surface temperature.

Sun	Yellow	5,500 K

Stars are classified by their spectra as:

- O, B, A, F, G, K, and M spectral types

O B A F G K M
hottest to coolest
bluish to reddish

O	hotter than 25,000 K
B	11,000 - 25,000 K
A	7500 - 11,000 K
F	6000 - 7500 K
G	5000 - 6000 K
K	3500 - 5000 K
M	cooler than 3500 K

Hertzsprung - Russell diagram → Graph of **luminosity** (or absolute magnitude) versus **temperature** (or spectral class)

How Far Away?	Distance
How Bright?	Luminosity
How Hot?	Spectral Type
How Massive?	Mass

Many of the stars in our universe come in pairs. Ordinary stars orbiting around a black hole will appear to "wobble" in the sky.

Parallax Formula:

$$P = \frac{1}{d}$$

- P = the parallax angle of star (in arcseconds)
- d = the distance to star (in parsecs)

The closer the star, the more its apparent position shifts as observed from earth.
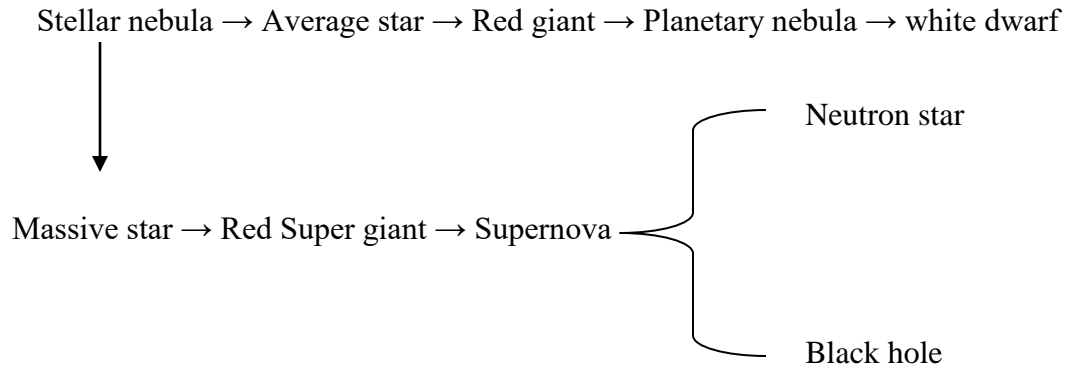
1 parsec = 3.26 light years

Luminosity and Inverse square Law:

$$\text{Apparent brightness} = \frac{\text{Luminosity}}{4 \times (\text{distance})^2}$$

$$\text{Apparent brightness} \propto \frac{1}{(\text{distance})^2}$$

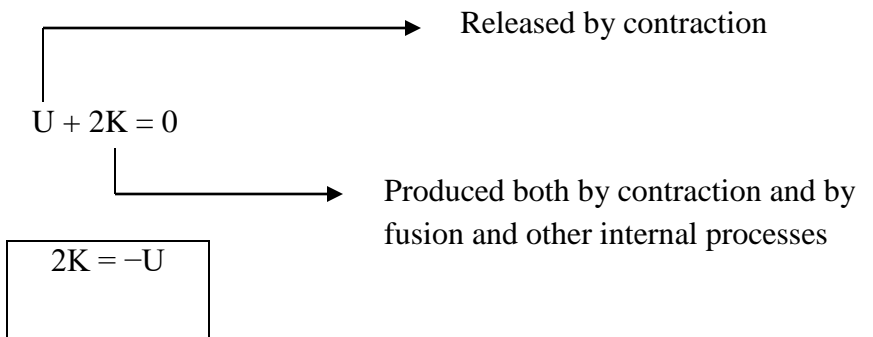**Red color** → cooler → lower energy radiation → lower luminosity

Blue color → hotter → higher energy radiation → higher luminosity

Stellar nebula → Average star → Red giant → Planetary nebula → white dwarf

Massive star → Red Super giant → Supernova ⟨ Neutron star / Black hole

Life Cycle of a star

- A **quasar** is an enormously bright, distant galaxy with a giant black hole at its center.

- A **black dwarf** is a theoretical stellar remnant, specifically a white dwarf that has cooled sufficiently that it no longer emits significant heat or light.

- A **supernova** is a powerful and luminous stellar explosion.

The virial theorem:

Released by contraction

$$U + 2K = 0$$

Produced both by contraction and by fusion and other internal processes

$$2K = -U$$

The negative gravitational energy of a star is equal to twice its thermal energy.

- A decrease in total energy E of the star leads to a decrease in U but an increase in K and hence temperature T increases, i.e. when a star loses its total energy, it heats up.

The rate of change of the total energy of star

$\downarrow$

(rate of nuclear energy generation in the deep interior − rate of energy loss in the form of radiation from the surface)

In a state of thermal equilibrium:

rate of nuclear energy generation in the deep interior = rate of energy loss in the form of radiation from the surface

---

The total energy of star remains constant

---

4 possible sources of energy generation in stars:

- cooling

- contraction

- chemical reactions

- nuclear reactions

If the temperature in the contracting core reaches values close to 10^{10} Kelvin, the energy of the photons becomes large enough to break up the heavy nuclei into lighter ones − in particular ^{56}Fe is disintegrated into α particles and neutrons:

$$^{56}\text{Fe} + \gamma \leftrightarrow 13 \text{ α particles} + 4 \text{ neutrons} \quad (\textbf{Photo-disintegration})$$
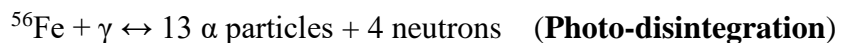
For ultrarelativistic particles (v ~ c) the rest mass of the particle may be neglected and the equation:

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

take the form:

$$E \simeq k_B T = pc$$

$$\lambda \simeq \frac{hc}{k_B T}$$

Gas pressure:

$$P_{gas} \propto \rho T$$

Radiation pressure:

$$P_{rad} \propto T^4$$

Degeneracy pressure → resistance of electrons (or neutrons) against compression into a smaller volume.

$$P_{deg} \propto \rho^{5/3} \quad \text{(Non-relativistic case)}$$

$$P_{deg} \propto \rho^{4/3} \quad \text{(Relativistic case)}$$

$M < 0.3 M_{sun}$	Star completely convective
$0.3 M_{sun} < M < 1.5 M_{sun}$	• Core radiative • Envelope convective
$M > 1.5 M_{sun}$	• Core convective • Envelope radiative

- **Brown dwarfs** → heavier than a planet (13 × mass of Jupiter) and lighter than a star.

Hydrogen burning: $4p + 2e^- \rightarrow {}^4He + 2\nu_e$

- proceeds by pp chains and CNO cycle

- no heavier elements formed because no stable isotopes with mass number A = 8

- neutrinos from proton → neutron conversion

- typical temperature 10^7 K ($\sim$ 1 keV)

Helium burning: ${}^4He + {}^4He + {}^4He \leftrightarrow {}^8Be + {}^4He \rightarrow {}^{12}C$

- triple alpha reaction builds up Be with concentration $\sim 10^9$

  ${}^{12}C + {}^4He \rightarrow {}^{16}O$
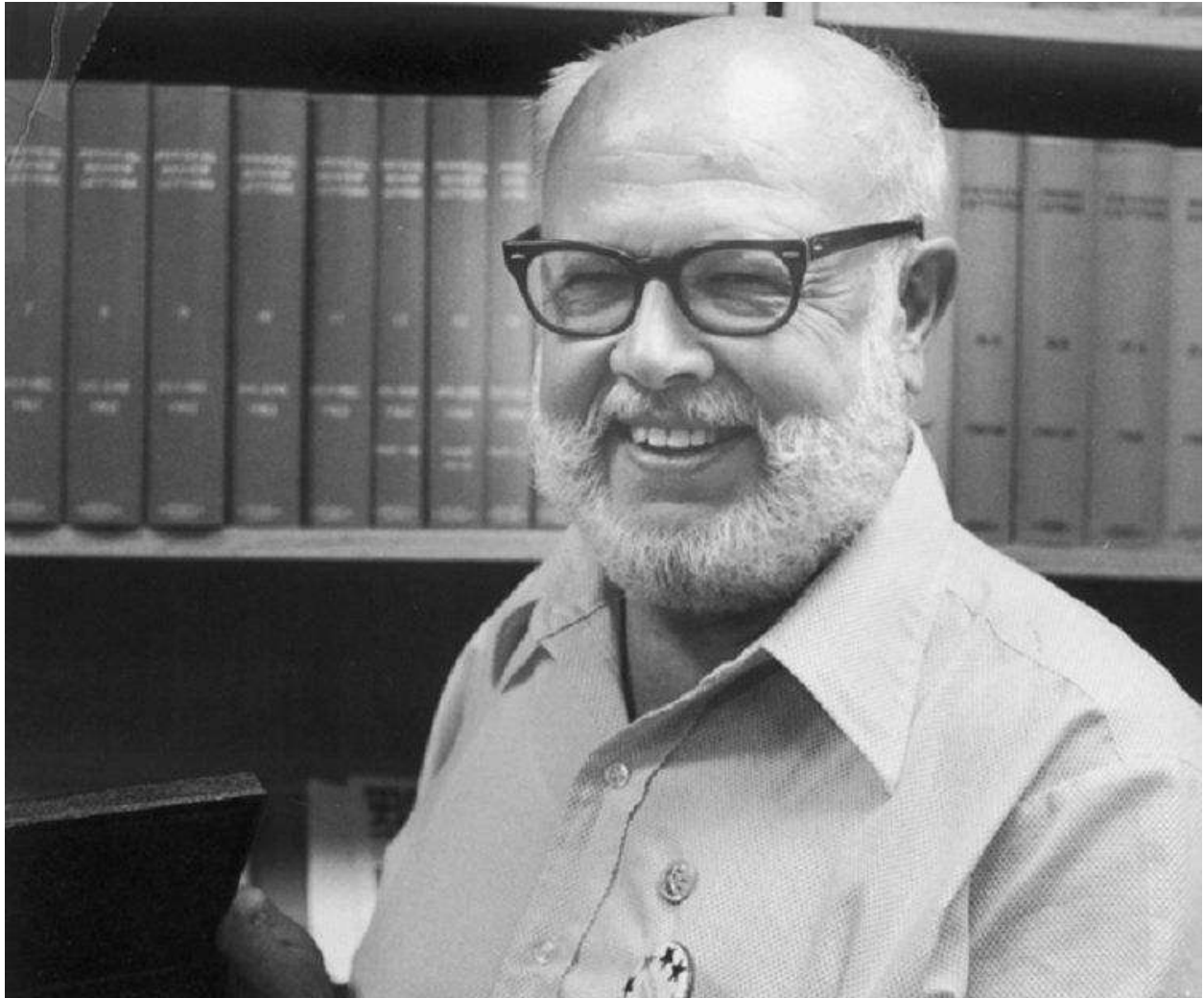
  ${}^{16}O + {}^4He \rightarrow {}^{20}Ne$

- typical temperature 10^8 K ($\sim$ 10 keV)

Carbon burning:

- Many reactions like ${}^{12}C + {}^{12}C \rightarrow {}^{20}Ne + {}^4He$ etc.

- typical temperature 10^9 K ($\sim$ 100 keV)

A star generally cannot reach hydrostatic equilibrium if its surface is too cool.

William Alfred Fowler was an American nuclear physicist, later astrophysicist, who, with Subrahmanyan Chandrasekhar won the 1983 Nobel Prize in Physics. He is known for his theoretical and experimental research into nuclear reactions within stars and the energy elements produced in the process.

- If the distance between nuclei $< 10^{-15}$m, the strong nuclear force overpowers the electromagnetic repulsion.

Hydrostatic equilibrium keeps the fusion process at a constant rate:

- If the nuclear fusion process speeds up:

  More energy would be produced and pressure would increase. This increased pressure would cause the core to expand and cool, and the fusion rate would slow down to normal.

- If the core temperature drops:

  The nuclear fusion process slows down

  The pressure would decrease and the core would contract

  As the core shrinks

  Temperature would increase

  Fusion rate would return to normal

The force of electrostatic repulsion between 2 protons is:

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

and the potential energy is:

$$E_P = \frac{e^2}{4\pi\varepsilon_0 r}$$

So if $\frac{3}{2} k_B T = \frac{e^2}{4\pi\varepsilon_0 r}$, the two protons can exceed the electrostatic force of repulsion and fuse together.

$$r_{min} = r = \frac{e^2}{6\pi\varepsilon_0 k_B T}$$

$r_{min} \rightarrow$ the distance of closest approach at which the nuclear attractive force becomes dominant to bind the two protons together.

- At low velocities, the electrostatic force of repulsion prevents the collision of protons.
- At high velocities, protons come close enough for the strong nuclear attractive force to bind them together.

There is a small (but non-zero) probability that two protons can overcome their repulsion and fuse even if their velocities are too low. This is called **quantum mechanical tunneling**.

$$\textbf{Probability} \propto \exp\left(-\frac{r_{min}}{\lambda}\right)$$

$$\frac{mv^2}{2} = \frac{3}{2} k_B T$$

$$\lambda = \frac{h}{\sqrt{3mk_B T}}$$

$$\textbf{Probability} \propto \exp\left(-\frac{r_{min}}{\lambda}\right)$$

$$\textbf{Probability} \propto \exp\left(-\frac{\text{Fine structure constant}}{\pi} \times \sqrt{\frac{mc^2}{3k_BT}}\right)$$

Probability is highest at fast speeds v (or high temperatures T)

- $\upsilon = \dfrac{k_BT_{core}}{h}$ is the frequency corresponding to the core temperature of the star.

- For a photon traveling from the center of Sun to the surface, with constant mean free path "ℓ" and assuming no destruction and recreation, the time taken for it to traverse this path is: $t = \dfrac{R_{sun}^2}{c\ell}$, where c is the speed of light and R_{sun} is the radius of the sun.

For ℓ = 3.01×10^{-5} m:

$$t = 5.36 \times 10^{13} \text{ s} = 1.70 \times 10^6 \text{ yr.}$$

If the photon didn't interact at all with matter, then $t = \dfrac{R_{sun}}{c}$.

Hawking Radiation:

- Proposed by Professor Stephen Hawking in 1974.
- Reduces mass and energy of a black hole.

Our Moon is constantly moving away from Earth whereas the Andromeda Galaxy is constantly moving towards Earth. Millions of years from now, people will see a tiny Moon but a huge galaxy in the night sky.

Thermodynamics

1. Energy is conserved

2. Randomness increases

3. Absolute Zero temperature is Unattainable

Because:

$$T_{BH} \propto \frac{1}{M}$$

In 2012, the Quantum Physicists **Dr. S. Haroche** and **Dr. D. Wineland** received Nobel Prize for their experiments which showed that "A Particle can be at two different locations at the same time." These Experiments indirectly showed that "**Parallel Universe do Exist!**"

Neutron stars are so dense that one teaspoon of their contents weighs more than **Mount Everest**.

- Tiny Black Hole is hot
- Big Black Hole is cold

60% Dark Energy (we don't know what it is)

35% Cold dark matter (we don't know what it is)

5% Nuclei and electrons (visible as stars ~0.5%)

Venus looked just like earth 2 billion years ago

Objects that travel with v << c obey this relation:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

- Objects moving away from observer → frequency decreases → wavelength increases (**red shift**)

- Objects moving towards observer → frequency increases → wavelength decreases (**blue shift**)

Einstein Theory → 4 dimensions (length, width, depth, and time)

String theory → 4 dimensions + 7 other dimensions

11th dimension holds the universe together

The black hole no hair theorem:

Mass, charge, and angular momentum are the only properties a black hole can possess

The Sky is Dark at Night → there must be some limit to the observable Universe.

Density parameter $(\Omega) = \dfrac{density}{critical\ density}$

- $\Omega > 1 \rightarrow$ closed universe

- $\Omega = 1 \rightarrow$ flat universe

- $\Omega < 1 \rightarrow$ open universe

Gamma radiation $\rightarrow \lambda < 0.001$ nm, $E_\gamma > 1.24$ MeV

$\downarrow$

Produced in nuclear reactions and other very high energy processes

X-rays $\rightarrow 0.001$ nm $< \lambda < 10$ nm, 124 eV $< E_{x\text{-ray}} < 1.24$ MeV

$\downarrow$

Produced in supernovae remnants and the solar corona, as well as in the hot gas between galaxy clusters

Thomson Scattering ($h\upsilon \ll m_0c^2$)

The photon and electron just both bounce off each other, changing their direction, but there is no exchange of energy

Compton scattering ($h\upsilon > m_0c^2$)

A photon of high energy collides with a stationary electron and transfers part of its energy and momentum to the electron, decreasing its frequency in the process

The maximum energy gained by photons via **inverse Compton scattering** is proportional to its initial energy multiplied by the square of twice the Lorentz factor (where the **Lorentz factor** is given by $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$ and v is the velocity of the electron):

$$E_{max} = (h\upsilon)_{max} \propto 4\,\gamma^2\,h\upsilon_0$$

- $1\ eV = 1.602 \times 10^{-19}\ J = 1.602 \times 10^{-12}\ erg$

Brown dwarf
- Too big to be a planet
- Too small to be a star

Pulsars → Rotating neutron stars emitting beams of particles and electromagnetic radiation
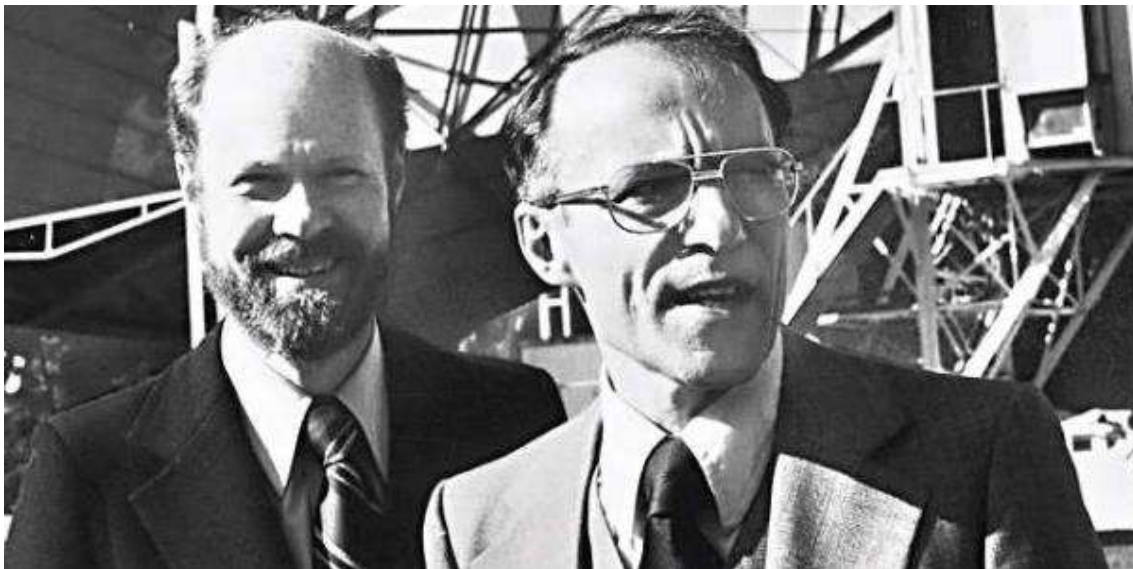
- **Gravity** warps space and time
- Even photons with no mass can have their trajectories bent → **gravitational lensing**

$\lambda \ll R$	$\sigma \approx \pi R^2$	**Geometrical scattering**
$\lambda \approx R$	$\sigma \propto \dfrac{1}{\lambda}$	**Mie scattering**
$\lambda \gg R$	$\sigma \propto \dfrac{1}{\lambda^4}$	**Rayleigh scattering**

$$\text{Energy of radiation} \propto \frac{1}{\text{(scale factor of the universe)}}$$

$$\text{Energy density of radiation} \propto \frac{1}{\text{(scale factor of the universe)}^4}$$

$$\text{Relative brightness} = \frac{\text{absolute brightness}}{\text{(distance)}^2}$$
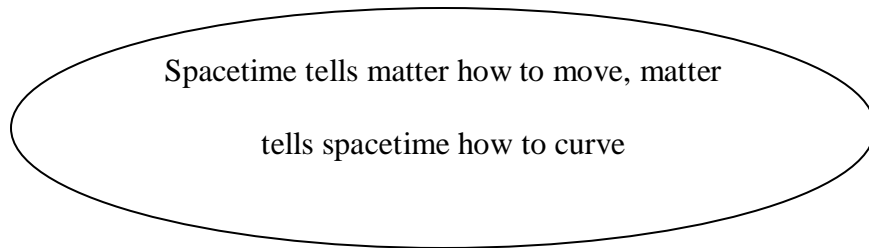

In 1964, **Arno Penzias and Robert Wilson**, two engineers at Bell Labs in New Jersey discovered Cosmic Background radiation when trying to get rid of noise from an antenna aimed at telecommunications satellites.

$$\frac{d(photons)}{dt} = \text{Metric} + \text{Compton Scattering}$$

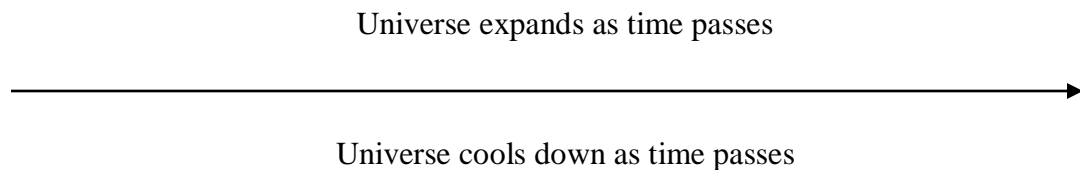$$\frac{d(electrons+hadrons)}{dt} = \text{Metric} + \text{Compton Scattering} + \text{Weak Interaction}$$

$$\frac{d(neutrinos)}{dt} = \text{Metric} + \text{Weak Interaction}$$

Spacetime tells matter how to move, matter

tells spacetime how to curve

Special Relativity:

- The speed of light is the same for any observer

Universe expands as time passes

Universe cools down as time passes

- If A couples to B, and B to C, A should couple to C.

Spacetime curvature ⟵ $G_{\mu\nu} = \dfrac{8\pi G}{c^4} T_{\mu\nu}$ ⟶ **Energy-momentum density of matter**

Gravitation = Geometry of space-time

At scale $L \sim \sqrt{\dfrac{G\hbar}{c^3}}$, energy fluctuations become so large that even spacetime geometry is no longer smooth at all.

3 types of geometries for our universe:

- Hyperbolic (negative curvature)
- Elliptic (positive curvature)
- Euclidean (zero curvature)

Photon + Hydrogen atom → proton + electron   (**photodissociation**)

Proton + electron → Photon + Hydrogen atom   (**radiative recombination**)
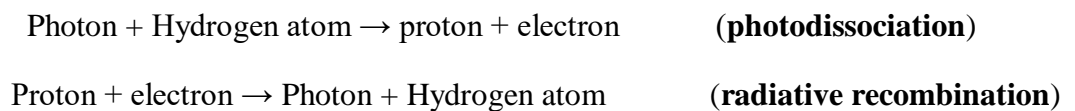
General theory of relativity { Equivalence between inertial mass and gravitational mass

Big Bang	Birth of the Universe
Planck Era	String Theory / Quantum Cosmology
Inflation Era	Symmetry Breaking → Exponential Expansion
Quark Era	Free Quarks in Thermal Equilibrium
Hadron Era	Matter Anti Matter Asymmetry
Lepton Era	Rapid Expansion/cooling (leptons/photons equilibrium)

Radiation Era	Nucleosynthesis, Decoupling
Matter Era	Structure Formation, first galaxies
Acceleration Era	Acceleration phase of the Universe

Newton Theory:

Weight is proportional to Mass

Einstein Theory:

Energy is proportional to Mass

Neither explained origin of Mass

- **Electroweak theory** predicted a heavy version of the photon called the Z^0 which was discovered in 1983.
- **Quantum field theory** which postulates that matter is composed out of elementary particles bound together by forces, mediated by exchange of other elementary particles.

An alien star passed through our solar system 70,000 years ago. Astronomers have discovered that a red dwarf came 0.8 light-years from the sun around the time modern humans are thought to have first spread across Asia. This is the "**closest known flyby of a star**" in our solar system.

Mercury	0
Venus	0
Earth	1
Mars	2
Jupiter	66
Saturn	62
Uranus	27
Neptune	13

There are more than 171 moons in our solar system

If the earth was the size of a marble you would need seven miles of space to build a scale model of the solar system.

$$S = k_B \ln W$$

For Black hole:

$$S = \frac{k_B A}{4L_{Planck}^2}$$

$$\frac{A}{4L_{Planck}^2} = \ln W$$

For The Smallest Known Black Hole

If $A = L_{Planck}^2$:

$$W = 1.28402541669$$

Black hole entropy is the amount of entropy that must be assigned to a black hole in order for it to comply with

the laws of thermodynamics as they are interpreted by observers external to that black hole.

— **Jacob Bekenstein**

- 33 light-years away there is an **exoplanet** completely covered burning ice.
- You can't cry on space because your tears won't ever fall.
- There is a **water reservoir** floating in space that is equivalent to 140 trillion times all the water in the world's ocean.

$$\text{Electron relativistic mass} = \frac{\text{electron rest mass}}{\sqrt{1-\frac{v^2}{c^2}}}$$

If Electron relativistic mass = proton rest mass:

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{\text{electron rest mass}}{\text{proton rest mass}} = \frac{1}{1836.1567343}$$
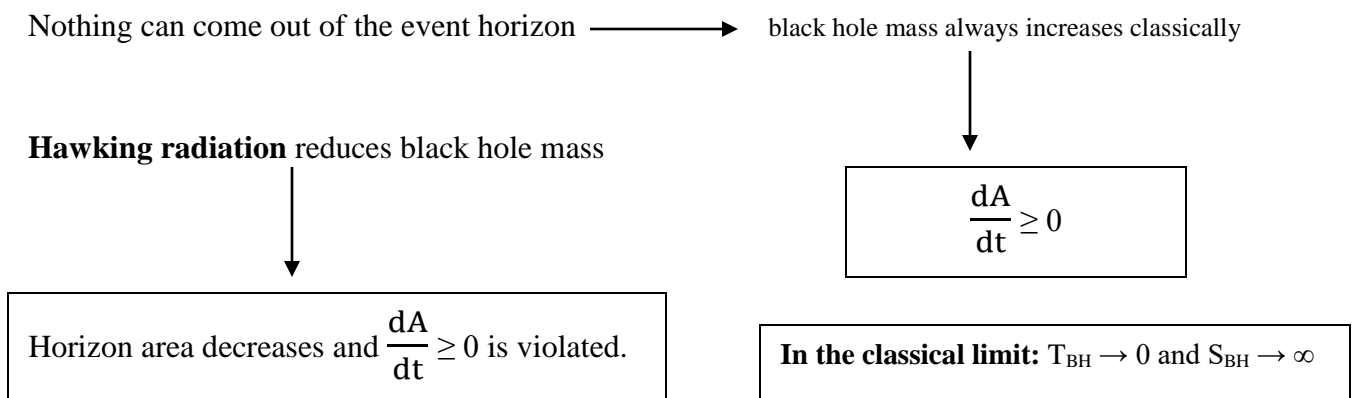
$$v = 0.99999985169c$$

We don't have any real pictures of the Milky Way galaxy. Most non-illustrated images of the entire Milky Way spiral are actually of another spiral galaxy called **Messier 74**. It's impossible to take a full photo of the Milky Way's spiral structure because it's about 100,000 light-years across, and we're stuck on the inside.

Pluto was named by an 11-year-old British girl. One day, Venetia Burney was having breakfast with her grandpa, who was reading about the new planet and wondering what it would be called. She had read about Greek and Roman legends, so she asked him, "**Why not call it Pluto?**" Grandpa later passed on her suggestion to an astronomer at Oxford University, and the rest is history.

Einstein was once asked how it felt to be the smartest man alive. Einstein's reply was "**I don't know, you'll have to ask Nikola Tesla**."
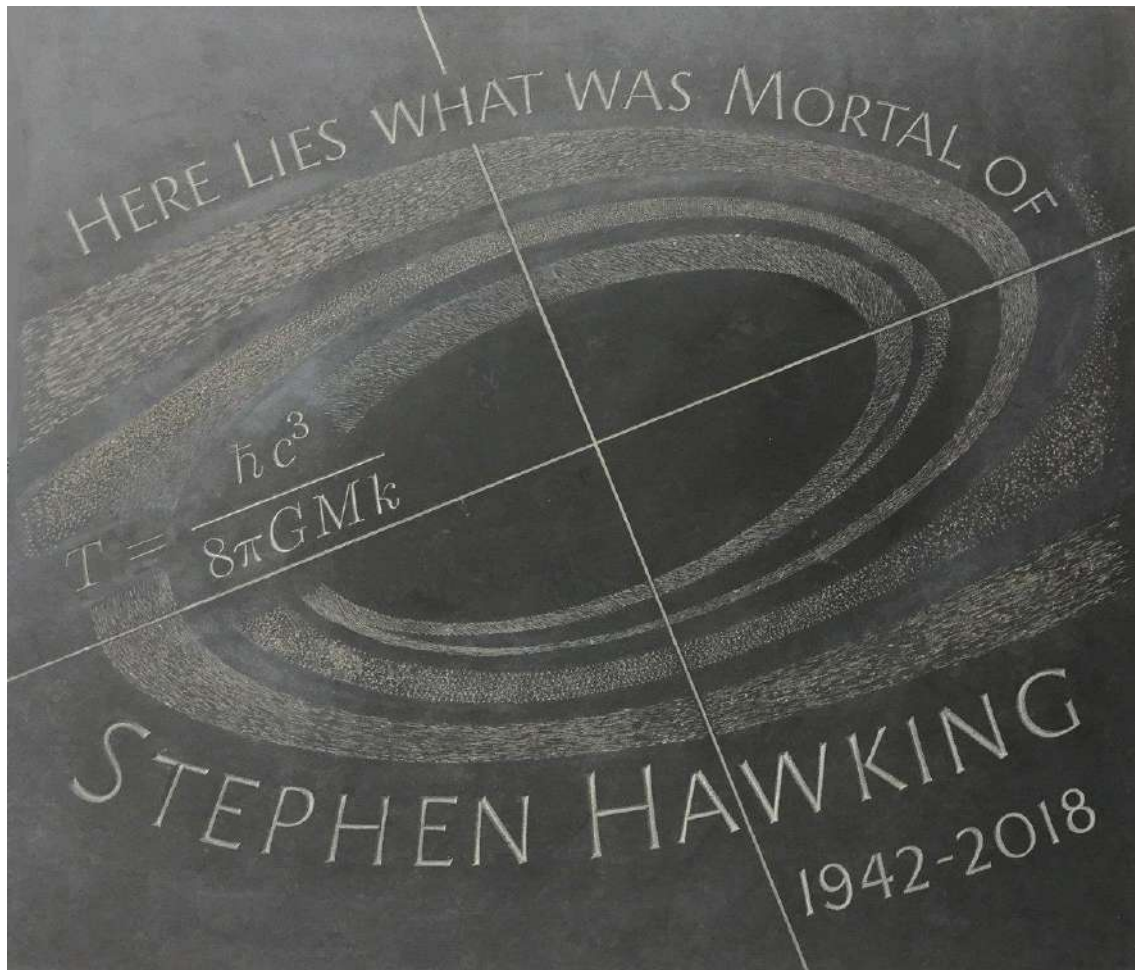
A tombstone of the crypt with Hawking's ashes with the formula ($T_{BH} = \dfrac{\hbar c^3}{8\pi GM k_B}$) for the black

hole temperature, placed in the flooring at Westminster Abbey in London.

Nothing can come out of the event horizon $\longrightarrow$ black hole mass always increases classically

Hawking radiation reduces black hole mass

$$\boxed{\dfrac{dA}{dt} \geq 0}$$

Horizon area decreases and $\dfrac{dA}{dt} \geq 0$ is violated.

In the classical limit: $T_{BH} \to 0$ and $S_{BH} \to \infty$

No hair theorem:

Black holes are characterized by 3 quantities:

<div style="border:1px solid black; padding:10px; text-align:center;">

mass, charge, and angular momentum

</div>

For black hole with 10 solar mass:

Temperature $\sim 6 \times 10^9$ K **(very low temperature)**

Entropy $\sim 10^{79} k_B$ **(huge entropy)**

<div style="border:1px solid black; padding:10px; text-align:center;">

Entropy of sun $\sim 10^{58}$

</div>

<div style="border:1px solid black; padding:10px; text-align:center;">

Spin parameter "a" cannot exceed **black hole mass** "M": for a > M the Kerr metric describes a naked singularity.

</div>

<div style="border:1px solid black; padding:10px; text-align:center;">

**Hawking 1975:** Black hole background + Quantum Field theory $\rightarrow$ Black hole emits radiation!!

Hawking 1976: Black hole as a quantum pure state + hawking radiation $\rightarrow$ Unitarily of Quantum Mechanics is broken!!

</div>

- **Solar Mass Black Hole:** $T_{BH} \sim 10^{-7}$ K, $t_{evap} \sim 10^{66}$ yr

- **Black Hole from Large Hadron Collider:** $T_{BH} \sim 10^{44}$ K, $t_{evap} \sim 10^{-80}$ s

Information is indeed lost	Information is stored in Hawking radiation	Evaporation stops at Planck scale
Quantum Mechanics is wrong	General Relativity is wrong	Statistical Mechanics is wrong

Quantum Field Theory is valid up to 10^{-16}cm

- Particles collide in three dimensional space.

- As the particles approach in a particle accelerator, their gravitational attraction increases steadily.

- When the particles are extremely close, they may enter space with more dimensions.

- The extra dimensions would allow gravity to increase more rapidly so a black hole can form.

- Such a black hole would immediately evaporate, sending out a unique pattern of thermal radiation.
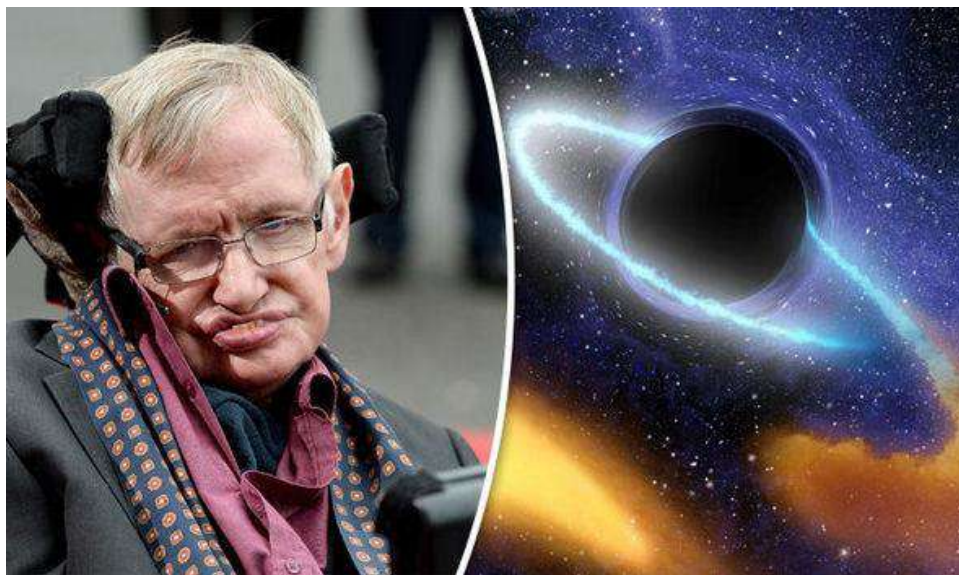
"This was a golden age, in which we solved most of the major problems in black hole theory even before there was any observational evidence for black holes. In fact, we were so successful with the classical general theory of relativity that I was at a bit of a loose end in 1973 after the publication with George Ellis of our book The Large Scale Structure of Space–Time. My work with Penrose had shown that general relativity broke down at singularities, so the obvious next step would be to combine general relativity—the theory of the very large—with quantum theory—the theory of the very small. In particular, I wondered, can one have atoms in which the nucleus is a tiny primordial black hole, formed in the early universe? My investigations revealed a deep and previously unsuspected relationship between gravity and thermodynamics, the science of heat, and resolved a paradox that had been argued over for thirty years without much progress: how could the radiation left over from a shrinking black hole carry all of the information about what made the black hole? I discovered that information is not lost, but it is not returned in a useful way—like burning an encyclopedia but retaining the smoke and ashes.

To answer this, I studied how quantum fields or particles would scatter off a black hole. I was expecting that part of an incident wave would be absorbed, and the remainder scattered. But to my great surprise I found there seemed to be emission from the black hole itself. At first, I thought this must be a mistake in my calculation. But what persuaded me that it was real was that the emission was exactly what was required to identify the area of the horizon with the entropy of a black hole. This entropy, a measure of the disorder of a system, is summed up in this simple formula which expresses the entropy in terms of the area of the horizon, and the three fundamental constants of nature, c, the speed of light, G, Newton's constant of gravitation, and ℏ, Planck's constant. The emission of this thermal radiation from the black hole is now called Hawking radiation and I'm proud to have discovered it."

— **Stephen Hawking**

- Neutron Star has a hard surface; the curvature is large - but finite.
- Black Hole: No Surface − curvature is infinite at the centre

> Black holes are the harmonic oscillator of quantum gravity.
>
> (**A. Strominger**)

Stephen Hawking-Jacob Bekenstein debate (1972)

> **Stephen Hawking:** the analogy between the area theorem and the second law of thermodynamics is just a matter of coincidence.

Jacob Bekenstein: I am not convinced. Nowhere in nature the second law of thermodynamics is violated. Why black holes would be an exception? I believe that the area of black holes is actually a manifestation of their entropy.

> **John Wheeler** (Jacob Bekenstein's doctoral advisor) **to Beckenstein:** Your idea is just so crazy that it might actually be true.

> > **Stephen Hawking:** If a black hole has entropy, it must have a temperature, and if it has a temperature it must radiate like a blackbody. But if nothing can escape from a black hole, how can it radiate?

1974: end of the debate and discovery of the Hawking radiation. Professor Stephen Hawking admits that **Jacob Bekenstein** was right: Black Holes have entropy, which is proportional to their area.

For the reversible reaction:

$$A + B \leftrightarrow C + D$$
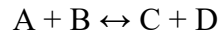
the equilibrium constant expression is written as:

$$K = \frac{[C]\ [D]}{[A]\ [B]}$$

- If $K>1$ then equilibrium favors products
- If $K<1$ then equilibrium favors the reactants

Entropic force:

$$F\ \Delta x = T\ \Delta S$$

$$T = T_U = \frac{\hbar\ a}{2\pi c k_B}$$

$$F\ \Delta x = \frac{\hbar\ a}{2\pi c k_B}\ \Delta S$$

$$F = ma$$

$$\Delta S = 2\pi k_B\ \frac{mc}{\hbar}\ \Delta x$$

Change of entropy on the **holographic screen** due to a linear displacement **Δx** of a particle with mass **m**

$$\boxed{KE = \gamma m_0 c^2 - m_0 c^2}$$

If $KE = m_0 c^2$:

$$\boxed{\gamma = 2}$$

$$\frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} = 2$$

$$\boxed{v = \sqrt{\frac{3}{4}}\, c = 0.86602540378\ \mathbf{c}}$$

So as the particle travels at velocity $v = 0.86602540378\ \mathbf{c}$, the kinetic energy of the particle equals its rest mass energy.

$$\boxed{KE = eV}$$

If $KE = m_0 c^2$:

$$V = \frac{m_0 c^2}{e}$$

For electron:

$$V = \frac{8.187105776925 \times 10^{-14}}{1.602176634 \times 10^{-19}} = 5.10998949978 \times 10^5 \text{ Volts}$$

Potential difference of **$5.10998949978 \times 10^5$ Volts** should be applied so that electron's kinetic energy equals its rest mass energy.

For proton:

$$V = \frac{1.503277484 \times 10^{-10}}{1.602176634 \times 10^{-19}} = 9.3827200578 \times 10^8 \text{ Volts}$$

Potential difference of **9.3827200578×10^8 Volts** should be applied so that proton's kinetic energy equals its rest mass energy.

So Einstein was wrong when he said, "**God does not play dice**." Consideration of black holes suggests, not only that God does play dice, but that he sometimes confuses us by throwing them where they can't be seen.

— Stephen Hawking

$$Q = ne$$

$$\frac{dQ}{dt} = \frac{dn}{dt} \times e$$

$$I = \frac{dn}{dt} \times e$$

If I = Planck current = 3.479×10^{25} A:

$$\frac{dn}{dt} = \frac{I}{e} = \frac{3.479 \times 10^{25}}{1.602176634 \times 10^{-19}}$$

$$\boxed{\frac{dn}{dt} = 2.171421007 \times 10^{44}}$$

In **Planck current**, $2.171421007 \times 10^{44}$ electrons are flowing every second.

$KE = \dfrac{P^2}{2m_0}$	$KE = \dfrac{3}{2} k_B T$	$KE = \dfrac{P^2}{2m_0}$	$KE = eV$
$KE = \dfrac{P}{2}\sqrt{\dfrac{3k_B T}{m_0}}$		$KE = P\sqrt{\dfrac{eV}{2m_0}}$	

Relativistic particle	Non relativistic particle
$Pv_P = mc^2$ where: v_P = phase velocity $(mv)\, v_P = mc^2$ $v v_P = c^2$ **Since v < c:** $v_P > c$	$Pv_P = \dfrac{m_0 v^2}{2}$ $(m_0 v)\, v_P = \dfrac{m_0 v^2}{2}$ $v_P = \dfrac{v}{2}$

$$P v_P = mc^2$$

$$\frac{h v_P}{\lambda} = mc^2$$

$$h\upsilon = mc^2$$

$$\frac{hc}{\lambda_c} = m_0 c^2$$

$$h\upsilon_C = m_0 c^2$$

Compton frequency

$$mc^2 = \frac{m_0 c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$\upsilon = \frac{\upsilon_C}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{\upsilon_C}{\sqrt{1 - \dfrac{v}{v_P}}}$$

The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time.

− Subrahmanyan Chandrasekhar

$$\Delta\lambda = \lambda_C\,(1- \cos\theta)$$

$\Delta\lambda = 0$	$\theta = 0^\circ$
$\Delta\lambda = \lambda_C$	$\theta = 90^\circ$
$\Delta\lambda = \dfrac{\lambda_C}{2}$	$\theta = 120^\circ$
$\Delta\lambda = 2\lambda_C$	$\theta = 180^\circ$

If the planet Earth was just 5% closer to the Sun, it would experience runaway greenhouse effect − with temperatures rising to nearly 900 degrees Fahrenheit. Conversely, if the planet Earth were about 20% farther from the Sun, it would experience runaway glaciations of the kind that has left Mars sterile.
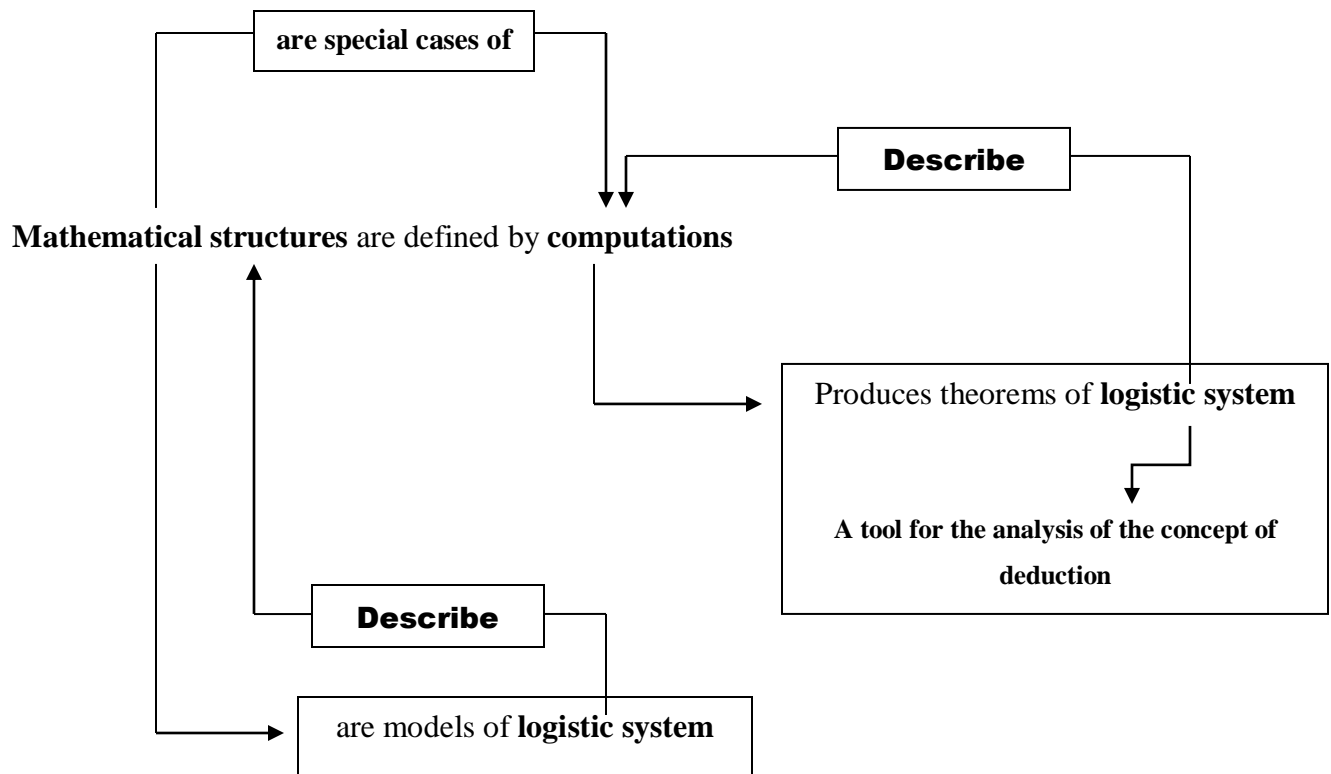
Law of conservation of mass	mass can neither be created nor destroyed but is transformed from one form to another
Law of conservation of energy	energy can neither be created nor destroyed but is transformed from one form to another
Law of conservation of momentum	momentum is neither created nor destroyed but only changed through the action of forces as described by **Newton's laws of motion**

Computable Universe Theory

Our external physical reality is defined by **computable functions**

(The **functions** that can be calculated using a mechanical calculation device given unlimited amounts of time and storage space)

are special cases of

Describe

Mathematical structures are defined by **computations**

Produces theorems of **logistic system**

A tool for the analysis of the concept of deduction

Describe

are models of **logistic system**

Theories of Origin of Life:

1	Life formation on the earth may have been taken place due to supernatural entity
2	Life formation did not take place on earth. It took place somewhere else in the space or on any other planet and carried to the earth
3	Life formation on the earth could have arisen through a series of organic chemical reactions that produced ever more complex biochemical structures
4	Life may have evolved from non-living matter as association with prebiotic molecules under primitive earth conditions

I learned very early the difference between knowing the name of something and knowing something.

— **Richard P. Feynman**

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

If $r = \sqrt{\dfrac{\hbar G}{c^3}}$:

$$F = \frac{e^2}{(\text{Planck charge})^2} \times \text{Planck force}$$

$$F = [\frac{1.602176634 \times 10^{-19}}{1.87554603778 \times 10^{-18}}]^2 \times \text{Planck force}$$

$$F = [\frac{1}{11.7062376144}]^2 \times \text{Planck force}$$

$$F = 0.00729735256 \times \text{Planck force}$$

So as the distance between the 2 electrons equals Planck length, the electric force between two electrons is 0.00729735256 times the Planck force.

If $F = \dfrac{c^4}{G}$:

$$\frac{c^4}{G} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$r = \frac{e}{\text{Planck charge}} \times \text{Planck length}$$

$$r = 0.08542454313 \times \text{Planck length}$$

So as the electric force between two electrons equals the Planck force, the distance between the 2 electrons is 0.08542454313 times the Planck length.

On average, gravity on a neutron star is 2 billion times stronger than gravity on Earth. In fact, it's strong enough to significantly bend radiation from the star in a process known as **gravitational lensing**, allowing astronomers to see some of the back side of the star.

Muons are subatomic particles that exist for just two millionths of a second. Scientists think they could be the key to many mysteries of the Universe.

$$F = \frac{G \times \text{electron rest mass} \times \text{electron rest mass}}{r^2}$$

If $r = \sqrt{\dfrac{\hbar G}{c^3}}$:

$$F = \frac{\hbar c}{\lambda_C^2} \quad \text{(where } \lambda_C \text{ is the reduced Compton wavelength of the electron)}$$

So as the distance between the 2 electrons equals Planck length, the gravitational force between

$$\text{two electrons} \propto \frac{1}{(\text{reduced compton wavelength of the electron})^2}$$

If $F = \dfrac{c^4}{G}$:

$$\dfrac{c^4}{G} = \dfrac{G \times \text{electron rest mass} \times \text{electron rest mass}}{r^2}$$

$$r = \dfrac{r_s}{2}$$

So as the gravitational force between two electrons equals the Planck force, the distance between the 2 electrons is half the Schwarzschild radius of the electron.

Solar radius $= 6.957 \times 10^5$ km

Schwarzschild radius of the sun $= \dfrac{2GM}{c^2} = 3.0 \times 10^3$ m $= 3$ km

Solar radius $= (2.319 \times 10^5) \times$ **Schwarzschild radius of the sun**

Some scientists say that if space is truly infinite, then there is probably an exact copy of you somewhere in the universe.

- $pc \ll m_0c^2 \rightarrow$ Classical particle
- $pc \gg m_0c^2 \rightarrow$ Ultrarelativistic particle
- $3 + 4 = 4 + 3$ (**commutative**)
- $3 - 5 \neq 5 - 3$ (**noncommutative**)
- $EA + T = EAT \neq TEA = T + EA$

Frame dragging is the idea that spacetime is elastic and particles in it will exchange energy. That means spacetime will absorb some of the energy of a spinning particle. Research studies have shown that Earth is dragging spacetime around it as it rotates.

Gravitoelectromagnetism	Electromagnetism
$F_g = m (E_g + v \times 4B_g)$	$F_e = q (E + v \times B)$
$S_g = -\dfrac{c^2}{8\pi G} E_g \times 4B_g$	$S_e = c^2\varepsilon_0 E \times B$

- F_g = the net (**Lorentz**) force acting on particle of mass m due to a gravitoelectromagnetic field
- v = velocity of the particle and c = speed of light in vacuum
- E_g = gravitoelectric field and B_g = gravitomagnetic field
- G = gravitational constant and S_g = gravitoelectromagnetic Poynting vector

- F_g = the net (**Lorentz**) force acting on particle of charge q due to a electromagnetic field
- v = velocity of the particle and c = speed of light in vacuum
- E = electric field and B = magnetic field
- ε_0 = absolute permittivity of free space and S_e = electromagnetic Poynting vector

Non-contact force

A force which acts on an object without coming physically in contact with it

All four known fundamental interactions are non-contact forces

Supercooling is the process of lowering the temperature of a liquid or a gas below its freezing point without it becoming a solid.

Zero-energy universe hypothesis

The total amount of energy in the universe is exactly zero: its amount of positive energy in the form of matter is exactly canceled out by its negative energy in the form of gravity

Lambda-CDM model	Big-Bang cosmological model with a **cosmological constant** and **cold dark matter**

Provides an explanation for the universe's observed accelerating expansion

Provides an explanation for the dynamics of galaxies and galaxy

Due to the highly **elliptical orbit of Pluto**, it sometimes gets closer to the Sun than Neptune. In fact during the years 1979 to 1999, Neptune was the 9th Planet and Pluto was the 8th Planet from the Sun!

Eternal inflation	New universes pop into existence at an unknown rate − creating a complex web of bubble universes within a vast multiverse
Loop quantum gravity	The universe is a network of intersecting quantum threads − each of which carries quantum information about the size and shape of nearby space
Graviphoton	A hypothetical particle which emerges as an excitation of the gravitational field in spacetime dimensions higher than four − as described in **Kaluza–Klein theory**. **classical unified field theory of gravitation and electromagnetism**
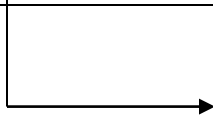
whose physical properties are virtually indistinguishable from a photon

Ekpyrotic model of the universe

Our current universe arose from the collision of two three-dimensional universes traveling in a hidden fourth spatial dimension. This model does not require a singularity at the moment of the **Big Bang**.

Hartle-Hawking model

Universe has no initial boundaries in time or space

Fermions (= matter): quarks and leptons

Bosons (= interactions): gauge fields + Higgs boson (God's particle)

Venus and Uranus are the only planets that rotate **clockwise**. The other six planets in the solar system rotate **counterclockwise**.

Naked singularity → a hypothetical gravitational singularity without an event horizon	

Weak Anthropic Principle	Strong Anthropic Principle
If the world were any different we would not be here	The world had to be as it is in order for us to be here
↓	↓
The emergence of life is possible	The emergence of life is inevitable

absurd universe	Our universe just happens to be the way it is.
unique universe	There is a deep underlying unity in the laws of physics that make it necessary for the Universe being the way it is.
multiverse	The idea of multiple universes. Each of which comprise everything that exists: the entirety of space, time, matter, energy, information, and the physical laws and constants that describe them.
Intelligent design	Life on earth is so complex that it cannot be explained by the scientific theory of evolution and therefore must have been designed by a supernatural entity.
self-explaining universe	No phenomenon can be said to exist until it is observed.
fake universe	We are living in a simulated universe.

$N \rightarrow$ number of spatial dimensions

$T \rightarrow$ number of time dimensions

- If $N > 3$ and $T = 1$: the orbit of a planet about its Sun cannot remain stable.

- If T > 1: the high energetic protons and electrons would be unstable and could decay into particles having greater mass than themselves.
  - A proton can decay into a neutron, a positron and a neutrino
  - An electron can decay into a neutron, an antiproton and a neutrino
- If N = 1 and T = 3: all particles are **tachyons** with imaginary rest mass.

> Only a (**N** + **T**) = (3 + 1) **dimensional universe** can contain dynamic observers who are complex and stable enough to be able to understand and predict all of space and time and their contents (including planets, stars, galaxies and all other forms of matter and energy) to any extent at all

T > 1 or T < 1: insufficient predictability

N > 3: insufficient stability

N< 3: insufficient complexity

|	
1 dimensional universe	made up of only 1 dimension (width)
2 dimensional universe	made up of 2 dimensions (width and breadth)
3 dimensional universe	made up of 3 dimensions (width, breadth and height)
4 dimensional universe	made up of 4 dimensions (width, breadth, height and time)
5 dimensional universe	more challenging to visualize because we ourselves cannot perceive dimensions > 4 around us

Causality Principle: all real events necessarily have a cause

Dark matter could warm certain planets in the place of a sun, allowing life to arise on a sunless planet.

Cosmic strings

If they exist, would be extremely thin with diameters of the same order of magnitude as that of a **proton**, i.e. **~ 1 fm**, or smaller
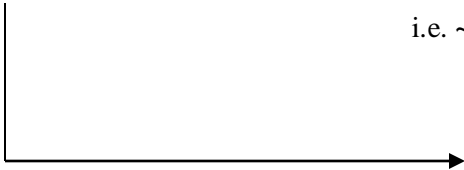
Could provide precious observational signatures of **string theory**

When two **cosmic strings** interact, the **cosmic string loop** is formed

which wiggle and oscillate, producing gravitational waves, then slowly shrink as they lose energy until they disappear
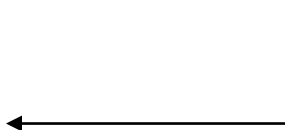
The only thing that can make a bigger atom than hydrogen is a star. The **entire periodic table**, every element you have ever heard of was processed inside the body of a star. The star then unraveled or exploded… and here we are.

We are dead stars

Black hole cosmology

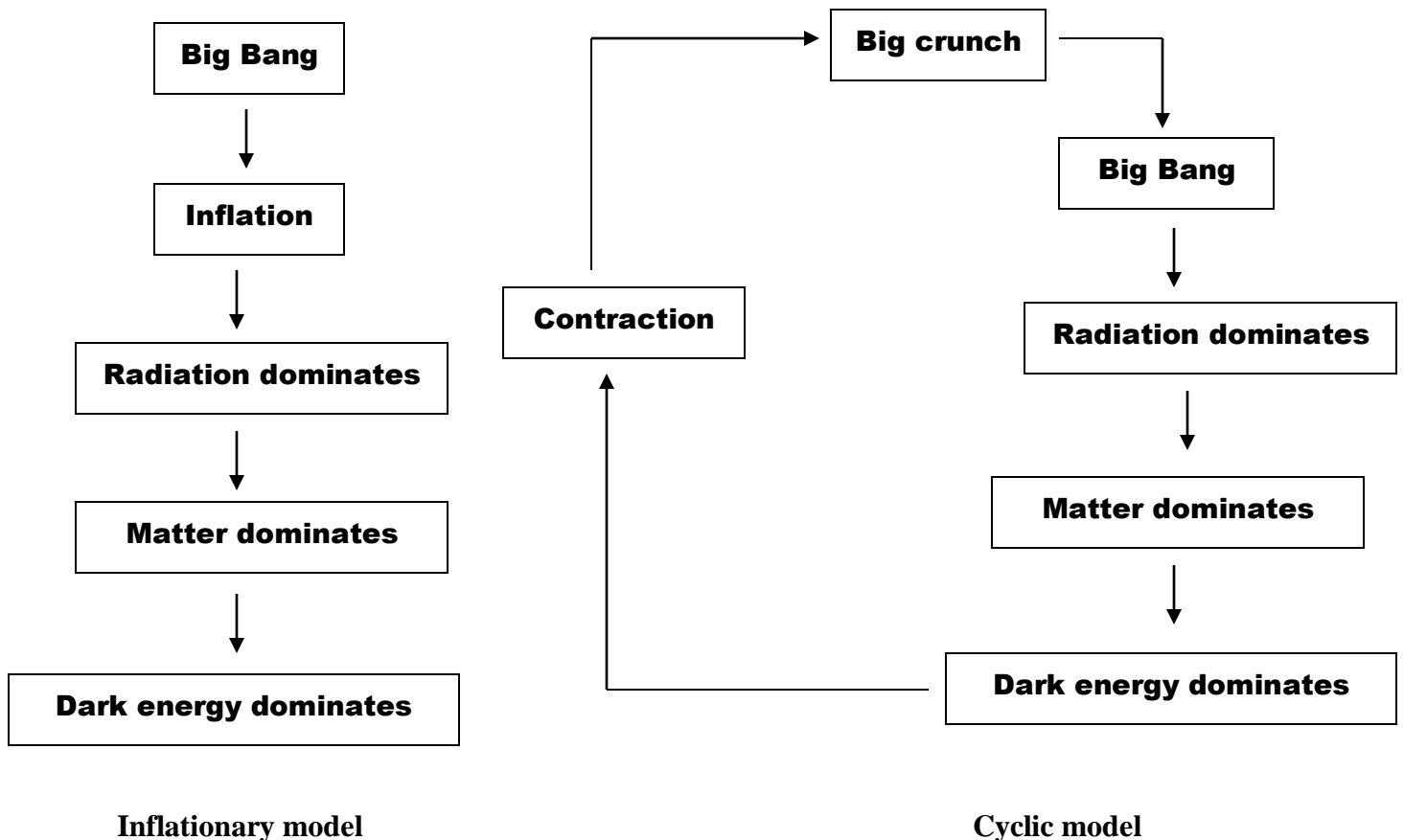The Hubble radius of the observable universe is equal to its Schwarzschild radius
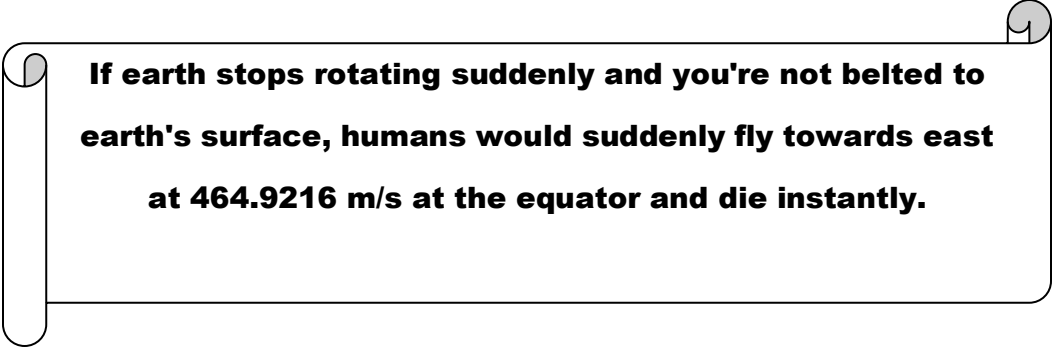
$$\frac{c}{H} = \frac{2GM}{c^2}$$

$$H = \frac{c^3}{2GM}$$

Critical density:

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3c^6}{32\pi G^3 M^2}$$

$$\rho_c \propto \frac{1}{M^2}$$

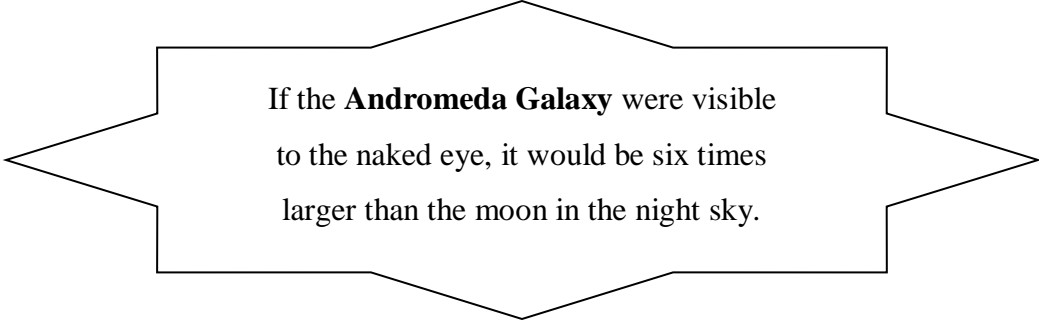|		
Big Bang		**Big crunch**

Inflationary model

Cyclic model

About ten billion years into its life, the **Milky Way galaxy** has now converted about 90% of its initial gas content into stars.

- **Conformal cyclic cosmology:** The universe goes through infinite endless cycles from creation to destruction over and over again.
- **Loop quantum cosmology:** Application of **loop quantum gravity** to eliminate singularities such as the big bang and big crunch singularity.

Eternal recurrence: The idea that all events in the world repeat themselves in the same sequence through an eternal series of cycles

Quantum emergence

Space and time develop out of a primeval state described by a quantum theory of gravity

If the **Andromeda Galaxy** were visible to the naked eye, it would be six times larger than the moon in the night sky.

Phantom energy

Dark energy with $\dfrac{\text{its pressure}}{\text{its energy density}} < -1$

Causes a **Cosmic Doomsday**

The matter of the universe (**from stars and galaxies to atoms and subatomic particles**) and even spacetime itself is progressively torn apart by the expansion of the universe at a certain time in the future − until distances between particles will become infinite.

- Dark energy with $\dfrac{\text{its pressure}}{\text{its energy density}} < -\dfrac{1}{3} \longrightarrow$ **cosmic acceleration**

- If the **expansion is accelerating**:

$$-1 < \textbf{Dark energy with } \dfrac{\text{its pressure}}{\text{its energy density}} < -\dfrac{1}{3}$$

Then the scale factor of the universe grows more rapidly than the horizon. Galaxies disappear beyond the horizon.

The number of atoms in the body of an average male (**60 kg**) is about	10^{27}
The number of atoms making up the earth is about	10^{50}
The number of atoms in the known universe is estimated at	10^{80}

Isenthalpic process	$\Delta H = 0$ (**Enthalpy constant**)
Isentropic process	$\Delta S = 0$ (**Entropy constant**)
Steady state process	$\Delta U = 0$ (**Internal energy constant**)

Weakless universe

A hypothetical universe that contains no weak interactions

General relativity

Clocks run slower in deeper gravitational wells

Rays of light bend in the presence of a gravitational field

Rotating masses drag along the spacetime around them

The universe is expanding, and the far parts of it are moving away from us faster than the speed of light

The properties of planetary orbits are dependent on spacetime structure

Cosmic neutrino background ($T_v = 1.95K$)

(**Low-energy neutrinos** left over from early universe)
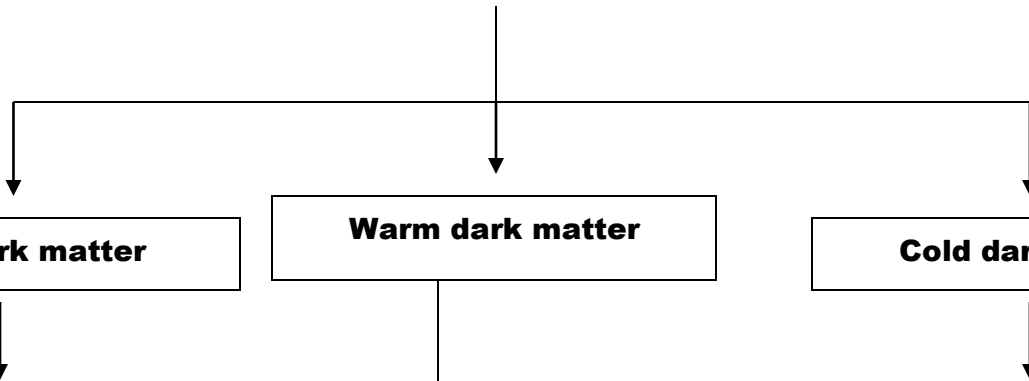
whose thermal energy $\dfrac{3k_B T_v}{2}$ falls below its rest mass energy $m_v c^2$

Redshift

Photons, which have no mass, lengthen their wavelength rather than slow their speed

Dark matter

Hot dark matter

- possess a very small mass
- moves at ultrarelativistic speed

Warm dark matter

Cold dark matter

- more massive
- moves slowly compared to the speed of light

According to **physicists**, Dark Matter binds the universe (**It interacts with gravity**), whereas the Dark Energy loosens up or speed up the expansion of the universe (**It creates space**).

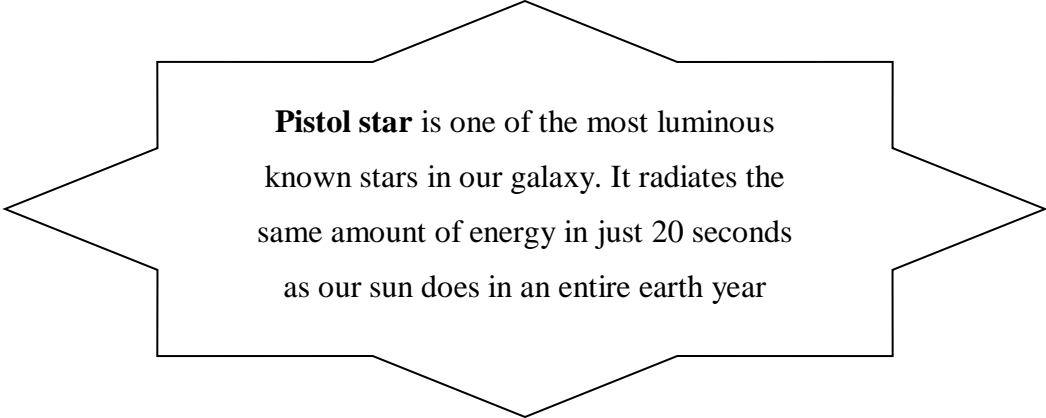- possess mass between **Cold dark matter** and **Hot dark matter**
- moves at speeds perhaps thousands of miles per hour

decoupling	some type of particle stops being destroyed as often as it is created
Compton scattering	photon collision with a charged particle that emits a different photon
baryon acoustic oscillations	density fluctuations stemming from acoustic waves in the early universe
Cosmic Background Imager	radio telescope in Chile (**a country in western South America**) aimed at studying the cosmic microwave background (CMB)
C-Band All Sky Survey	6 cm survey of the entire sky
CMB anisotropies	variations in the spectrum of the CMB
Hubble expansion	ongoing expansion of the universe
CMB lensing	gravitational lensing of the CMB
Cosmic Background Explorer	1989 satellite for surveying the CMB
cosmological simulation	simulation of the history of the universe
dark energy	energy theorized to accelerate the expansion of the universe
dark matter	non-baryonic matter
ordinary matter	baryonic matter
DASI	**Degree Angular Scale Interferometer** (2000−2003 South Pole CMB telescope)
DGP gravity	alternative theory of gravity addressing "**dark energy**"
extragalactic background light	electromagnetic radiation from star formation and active galactic nucleus
epoch of reionization	time when the universe's neutral hydrogen atoms ionized
Galactic Emission Mapping	radio sky survey
gravitational wave background	combination of weak gravitational waves such as from distant source
inflation	theorized rapid expansion of the **very early universe** by 78 orders-of-magnitude within its first 10^{-32} seconds of existence
initial fluctuations	early universe density variations leading to the large scale structure of the current universe
initial fluctuation spectrum	the power spectral density of the early universe
intensity mapping	measure of cosmic matter density

Isotropy	uniformity in all directions
LAMBDA	**Legacy Archive for Microwave Background Data Analysis** (NASA archive center for CMB data)
light cone	the region of space-time from which light can reach you
LiteBIRD	**Light satellite for the studies of B-mode polarization and inflation from cosmic background radiation detection** (planned spacecraft to map CMB polarization)
magnetic energy spectrum	spatial spectral density of the energy associated with magnetic fields
microwave	electromagnetic radiation (frequency 300 MHz to 300 GHz)
millimeter astronomy	observation and analysis at wavelengths on the order of a millimeter
maximum likelihood mapmaking	method of making maps using varying data
Planck	microwave telescope in space
polarimetry	measurement of polarization of electromagnetic radiation
polarization	property of waves that oscillate in a particular direction
primordial gravitational waves	gravitational waves formed in the early universe
QMAP	mid 1990s balloon-based CMB survey
QUIJOTE	survey to measure the polarization of the CMB
Rayleigh scattering	wave scattering by particles smaller than the wavelength
recombination	time when the universe's ionized hydrogen atoms neutralized
relic	particles left over from the events of the early universe
spinning dust emission	microwave emission from spinning dust particles that carry a dipole electric field
electric dipole radiation	electromagnetic radiation generated by an oscillating electric dipole
surface of last scattering	sphere around us from which CMB photons are just reaching us
Sachs-Wolfe effect	gravitational redshift of the CMB
thermal dust emission	glow of dust heated by stars and active galactic nucleus
Topological defects	stable configurations of matter formed at phase transitions in the very early Universe

68% of the universe is dark energy and 27% is dark matter. Both of these are invisible, even with a telescope, which means we are only able to see 5% of the universe.

Pistol star is one of the most luminous known stars in our galaxy. It radiates the same amount of energy in just 20 seconds as our sun does in an entire earth year

What is a star life cycle? A star life cycle is birth, **Nebula** then to its death. A star is born from the death of another star.

Science merely amplifies the capabilities of human beings. Science gives us the ability to do ill and to do good more than we had, and to question science in this respect is like questioning whether people ought to have two hands or just one, because with two hands they could do more evil than they can with just one.

Steven Weinberg

Avogadro's hypothesis states that equal volumes of gases at the same temperature and pressure contain the same number of particles. At **Standard Temperature and Pressure**, one mole (**6.02×10^{23} particles**) of any gas occupies a volume of 22.4 liters

Bernoulli's principle	As speed of the fluid increases, pressure decreases.

A neutron star known as PSR 11748-2446AD, has an equator spinning at 24% the speed of light, which translates to over 70,000 kilometers per second.

Pauli exclusion principle	Fermions	Bosons
prohibits two particles from existing in the same quantum state	✓	✗

We can have a number of bosons coexisting in a single energy level.

Greisen–Zatsepin–Kuzmin limit

Theoretical limit on the energy of cosmic ray protons traveling from other galaxies through the intergalactic medium to our galaxy

5×10^{19} eV or about 8 joules

(The energy of a proton travelling at $\approx$ 99.99999999999999998% the speed of light)

Isothermal process	$\Delta T = 0$ (**Temperature constant**)
Isobaric process	$\Delta P = 0$ (**Pressure constant**)
Isochoric process	$\Delta V = 0$ (**Volume constant**)
Adiabatic process	$\Delta Q = 0$ (**No heat flow between the system and the surroundings**)

$$\Delta G = \Delta H - T \Delta S$$

- If ΔG is negative (< 0), the process is **spontaneous** (exergonic).
- If ΔG is positive (> 0), the process is **non spontaneous** (endergonic).

	ΔH < 0	**ΔH > 0**
ΔS > 0	Spontaneous at all T ($\Delta G < 0$)	Spontaneous at high T (**when TΔS is large**)
ΔS < 0	Spontaneous at low T (**when TΔS is small**)	Non Spontaneous at all T ($\Delta G > 0$)

The universe has no centre and is constantly expanding every second - making it impossible to reach the edge

- The universe is so vast that when you look into the night sky, you're looking back in time.
- At 25 km tall and 600 km wide, **Olympus Mons** on mars is the tallest mountain on any of the planets of the solar system. That's 3 times the size of Everest.
- **Sagittarius B** is a vast molecular cloud of gas and dust floating near the centre of the Milky Way. It contains 10-billion-billion-billion liters of alcohol.
- There are probably more than **170 billion galaxies** in the observable universe.

The orbit of Neptune's moon **Triton** is slowly decaying, meaning it is getting closer to the planet. In 3.6 billion years time Triton will drop below the **Roche limit**, and Neptune's tidal forces will tear Triton apart, forming a ring around the planet

$$\frac{\sin i}{\sin r} = n$$

If n > 1:

$$r < i$$

The refracted ray bends towards the normal

The **wave nature of light** was demonstrated convincingly for the first time in 1801 by Thomas Young by a wonderfully simple experiment. He let a ray of sunlight into a dark room, placed a dark screen in front of it, pierced with two small pinholes, and beyond this, at some distance, a white screen. He then saw two darkish lines at both sides of a bright line, which gave him sufficient encouragement to repeat the experiment, this time with spirit flame as light source, with a little salt in it to produce the bright yellow sodium light. This time he saw a number of dark lines, regularly spaced; the first clear proof that light added to light can produce darkness. This phenomenon is called *interference*. **Thomas Young** had expected it because he believed in the wave theory of light.
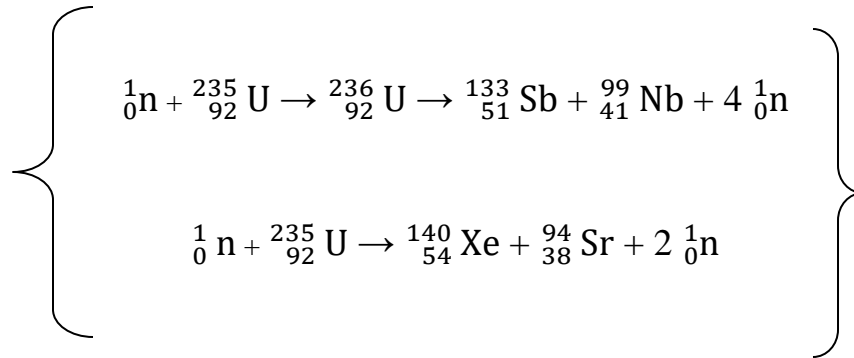
– Dennis Gabor

	Interference	Diffraction	Polarization
Longitudinal waves	✓	✓	✗

Longitudinal waves cannot be polarized because their particles vibrate in the same direction that the wave travels

$$\,^1_0n + \,^{235}_{92}U \rightarrow \,^{236}_{92}U \rightarrow \,^{144}_{56}Ba + \,^{89}_{36}Kr + 3\,^1_0n$$
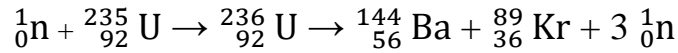
Nuclear Fission does not always produce barium and krypton.

$$\,^1_0n + \,^{235}_{92}U \rightarrow \,^{236}_{92}U \rightarrow \,^{133}_{51}Sb + \,^{99}_{41}Nb + 4\,^1_0n$$

$$\,^1_0n + \,^{235}_{92}U \rightarrow \,^{140}_{54}Xe + \,^{94}_{38}Sr + 2\,^1_0n$$

If one wants to break the oxygen atom into 8 protons, 8 neutrons and 8 electrons, this energy has to be supplied.

$\,^{16}_8O$ atom has 8 neutrons, 8 protons and 8 electrons.

- Mass of 8 neutrons = 8×1.00866 u
- Mass of 8 protons = 8×1.00727 u
- Mass of 8 electrons = 8×0.00055 u

The expected mass of $\,^{16}_8O$ atom = 8×2.01593 u = 16.12744 u.

The observed mass of $\,^{16}_8O$ atom = 15.99493 u

The mass of the $\,^{16}_8O$ atom is less than the total mass of its constituents by **0.13251u**.

Binding energy of $\,^{16}_8O$ atom = **0.13251u** $\times c^2$

Every star in the night sky is larger than the sun.

Even if you were able to travel close to the speed of light (**186000 miles per second**) you would still take 100000 years to cross the Milky Way galaxy.

Hypervelocity stars are stars that have been ejected from their systems and sent rocketing through intergalactic space at speeds up to 2 million miles per hour. Most of the hypervelocity stars that have been identified so far are of similar size and mass as the Sun, but theoretically could be bigger.

- Reynolds number $= \dfrac{\text{inertial force}}{\text{viscous force}}$

- Froude number $= \dfrac{\text{inertial force}}{\text{gravitational force}}$

- Rossby number $= \dfrac{\text{inertial force}}{\text{Coriolis force}}$

- Prandtl number $= \dfrac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$

- Mach number $= \dfrac{\text{speed of flow}}{\text{sound speed}}$

Numbers that describe the flow characteristics of a fluid

Uncertainty in number of photons

$$\Delta n \, \Delta \varphi \geq \frac{\hbar}{2}$$

$\hbar$ is so small that the uncertainties implied by the **uncertainty principle** are also too small to be observed. They are only significant in the domain of microscopic systems

Uncertainty in wave phase

A mathematical quantity called an **imaginary number**. It is equal to the square root of minus one

Called the **del-squared operator** − this quantity describes how the **wave function** ψ changes from one place to another

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \, \frac{\partial \psi}{\partial t}$$

Describes how ψ changes its shape with time

The mass of the particle being described

Describes the forces acting on the particle

- The largest known black hole (**Holmberg 15A**) has a diameter of 1 trillion km, more than 190 times the distance from the Sun to Pluto.

- The **coldest known natural place** in the universe is the **Boomerang Nebula**. At −272.15°C it is 1°C warmer than absolute zero, and 2°C colder than background radiation from the hot **Big Bang**.

Since:

$$e = 1.602192 \times 10^{-19} \text{ C}$$

There are about 6×10^{18} electrons in a charge of –1C

The accelerated motion of charge q_1 produces **electromagnetic waves**, which then propagate with the speed **c** (3×10^8 m/s), reach q_2 and cause a force on q_2

$$\text{Force on } q_2 \text{ due to } q_1 = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$q_1 = q_2 = 1\text{C}, r = 1\text{m}$

$$\text{Force on } q_2 \text{ due to } q_1 = 9 \times 10^9 \text{N}$$

Superposition Principle

The force with which two charges attract or repel each other are not affected by the presence of a third charge

A small percentage of the static shown by our Television is the Big Bang's afterglow.

Wave equation:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}\, c^2$$

The **Fourier transform** is essential to understanding more complex wave structures like **human speech**

Describes the behavior of waves — a vibrating guitar string, ripples in a pond after a stone is thrown, or light coming out of an incandescent bulb.

Schrödinger's equation:

$$i\hbar \frac{\partial}{\partial t}\, \psi = H\psi \ (\textbf{time dependent})$$

$$E\, \psi = H\psi \ (\textbf{time independent})$$

Describes the energy and position of the **electron** in space and time − taking into account the **matter wave nature** of the electron inside an atom

- Astronauts can grow up to 3% taller in outer space, or about 2 inches for a 6-foot-tall person
- **According to quantum mechanics**, every time you get confused between two things and decide to choose to go with one of them, the universe splits into two parts, where you choose the other option in another universe.
- In 2012, particles smashed together in the Large Hadron Collider's 27-kilometer circular tunnel conjured up the **Higgs boson** — the last missing particle predicted by **the Standard Model of particle physics**. The Higgs particle is supposed to be responsible for giving particles their masses (this particle would answer the basic question of why particles have the masses the do—or indeed why they have any mass at all).

Weight = mass × acceleration due to gravity

On Earth, it generally takes the value = 9.81 m/s^2 however on other planets and at extreme heights on Earth, it take other values.

An object on the Moon would have the same mass as an identical object on Earth, but its weight would be less since the Moon's gravitational field is weaker
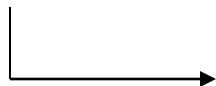
Coefficient of restitution of the collision:

$$e = \frac{\text{speed of separation of the particles}}{\text{speed of approach of the particles}}$$

- e = 1 (**elastic collision**)

  → speed of separation of the particles = speed of approach of the particles

- 0 < e < 1 (**inelastic collision**)

  → speed of separation of the particles is not zero but less than speed of approach of the particles

- e = 0 (**perfectly inelastic collision**)
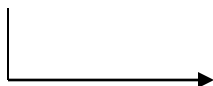
  → colliding particles do not separate out but move with same speed

1 Second to Get the Moon

8 Minutes to Get the Sun

2000 years to get out of Milky Way

46.5 Billion Years to Get the Edge of the Observable Universe

- **Coronal loops** are structured arcs of glowing, electrified plasma that flow along the powerful, curved, magnetic fields above the Sun's surface. This one is roughly **4 times** the size of Earth.
- A new star is born in our galaxy roughly every 18 day.
- Stars twinkle because we see them through the **wafting of the atmosphere**.

The color and size of a star tells astronomers its age.

- Yellow dwarfs and blue giants are young.
- Red giants and red supergiants are older.
- White dwarfs and black dwarfs are the oldest.

pH scale

Strong acids		← More acidic →			Weak acids		Neutral	Weak bases		← More basic →			Strong bases	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Most constellations are constantly shifting in the sky. However, there are some constellations that never set or rise. These constellations are known as **circumpolar constellations**. These are the constellations that make excellent reference points for locating **seasonal constellations**.

The **Apollo astronauts'** footprints on the moon will probably stay there for at least 100 million years

It would take light 650 years to travel from one end of the Tarantula Nebula to the other.

The uniform character of mathematics is the essence of science, for mathematics is the foundation of all exact scientific knowledge.

DAVID HILBERT, 1862 – 1943

Geometry is one and eternal shining in the mind of God. That men share in it is among the reasons that Man is the image of God.

JOHANNES KEPLER, 1571 – 1630

Mathematics is the language in which the gods speak to people.

PLATO, C. 427 BC – C. 347 BC

- Nebulae are **interstellar clouds** that are made of plasma, helium, hydrogen and dust
- When nuclear fusion in a star much like our Sun ceases, the star dies. It expels its outer gaseous layers and leaves behind a tiny, hot, dense core called a **white dwarf**. The cosmic result − a **planetary nebula**.
- If two pieces of the same type of metal touch in space, they will bond and be permanently stuck together. This amazing effect is called **cold welding**.

In my experience most mathematicians are intellectually lazy and especially dislike reading experimental papers.

FRANCIS CRICK, 1916 – 2004

The legend that every cipher is breakable is of course absurd, though still widespread among people who should know better.
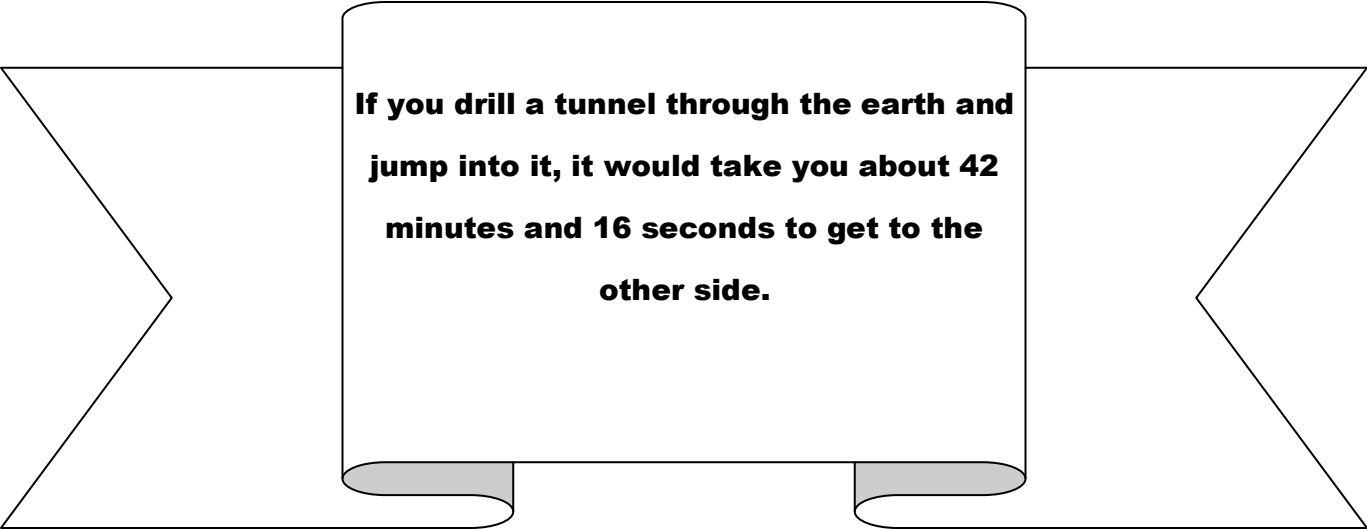
J.E. LITTLEWOOD, 1885 – 1977

Surely it is not knowledge, but learning; not owning but earning; not being there, but getting there; that gives us the greatest pleasure.

CARL FRIEDRICH GAUSS, 1777 – 1855

Actually, everything that can be known has a number; for it is impossible to grasp anything with the mind or to recognize it without this.

PHILOLAUS, C. 470 – C. 385 BC

If you drill a tunnel through the earth and jump into it, it would take you about 42 minutes and 16 seconds to get to the other side.

Euclid's work ought to have been any educationist's nightmare… it never offers any "motivations," it has no illuminating "asides," it does not attempt to make anything "intuitive," and it avoids "applications" to a fault. It is so "humorless" in its mathematical purism that… …it should have been spurned by students and "progressive" teachers in every generation. But it nevertheless survived intact all the turmoils, ravages, and illiteracies of the dissolving Roman Empire, of the early Dark Ages, of the Crusades, and of the plagues and famines of the later Middle Ages.

SALOMON BOCHNER, 1899 – 1982

- **Quantum Physics** came from the Vedas. Schrodinger, Albert Einstein and Nikola Tesla were **Vedantists.**
- **The Sun** is actually white, it just appears yellow to us through the **Earth's atmosphere**.

There may be babblers, wholly ignorant of mathematics, who dare to condemn my hypothesis, upon the authority of some part of the Bible twisted to suit their purpose. I value them not, and scorn their unfounded judgment.

NICOLAUS COPERNICUS, 1473 – 1543

Before the Copernican revolution, it was natural to suppose that God's purposes were specifically concerned with the earth, but now this has become an unplausible hypothesis. If it is the purpose of the Cosmos to evolve mind, we must regard it as rather incompetent in having produced so little in such a long time.

BERTRAND RUSSELL, 1872 – 1970

Quantum physics says reality changes with observation.

Quantum Bayesianism says reality is observation.

Instead of particles choosing a state when observed, maybe observers are just updating their knowledge.

- **Quark matter** is an extremely dense phase of matter made up of subatomic particles called **quarks**. This theoretical phase would occur at extremely high temperatures and densities. It may exist at the heart of neutron stars. It can also be created for brief moments in particle colliders on Earth, such as **CERN's Large Hadron Collider**.
- Like a **black hole**, a white hole is a prediction of Albert Einstein's theory of general relativity. It is essentially a black hole in reverse: if nothing can escape from a black hole's event horizon, then nothing can enter a white hole's event horizon.

I see some parallels between the shifts of fashion in mathematics and in music. In music, the popular new styles of jazz and rock became fashionable a little earlier than the new mathematical styles of chaos and complexity theory. Jazz and rock were long despised by classical musicians, but have emerged as art-forms more accessible than classical music to a wide section of the public. Jazz and rock are no longer to be despised as passing fads. Neither are chaos and complexity theory. But still, classical music and classical mathematics are not dead. Mozart lives, and so does Euler. When the wheel of fashion turns once more, quantum mechanics and hard analysis will once again be in style.

FREEMAN DYSON, B. 1923

Simple laws can very well describe complex structures. The miracle is not the complexity of our world, but the simplicity of the equations describing that complexity.

SANDER BAIS, B. 1945

- Rogue planets are not bound by any star and so "**free-float**" through Space.
- In April 2010, radio astronomers monitoring the **galaxy M82** spotted and unknown object that was moving 4 times faster than the speed of light and sending radio signals that had never been seen in the universe before.
- The average distance between asteroids in space is over **100,000 miles**, meaning an asteroid field would be very simple to navigate.
- The complex nuclear contents of a **neutron star** are all named after Italian food.

The ultimate truths of mathematics, then, cannot be established by any experimental proof that the deductions from them are true; since the supposed experimental proof takes them for granted.

HERBERT SPENCER, 1885 – 1977

It is a platitude that pure mathematics can have unexpected consequences and affect even daily life.

J.E. LITTLEWOOD, 1885 – 1977

Pure mathematics exist by themselves; no will produces them, no power can limit them. They are eternal laws that no man can infringe, and from which it is impossible to escape.

S. SANDARAM IYER, 1883

- Saturn has a moon called **Mimas** and it has a crater in it that makes it look like the **Death Star**.
- When the **red supergiant Betelguese** explodes and becomes a **supernova**, which could happen any time between today and a million years from now, it will be visible from Earth, even during the day, and will shine brighter than a full moon for months before fading.
- In 2022, you'll be able to see two stars collide. The stars are too dim to see with the naked eye, but astronomers have been watching them slowly spiral closer and closer together. Their collision will cause a giant explosion (**AKA a Red Nova**), which will then fizzle out into a dimmer, but permanently-visible addition to the night sky.

The invention of logarithms came to the world as a bolt from the blue. No previous work had led up to it… It stands isolated, breaking in upon human thought abruptly, without borrowing from the work of other intellects or following known lines of mathematical thought.

JOHN MOULTON, 1844 – 1921

Division is esteemed one of the busiest operations of Arithmetic, and such as requireth a mind not wandering, or settled upon other matters.

THOMAS HYLLES, THE ARTE OF VULGAR ARITHMETICKE, 1600

The literary convention that numbers less than 10 should be given in words is often highly unsuitable in mathematics… The excessive use of the word forms is regrettably spreading at the present time.

J.E. LITTLEWOOD, 1885 – 1977

What exactly is mathematics? Many have tried but nobody has really succeeded in defining mathematics; it is always something else. Roughly speaking, people know that it deals with numbers, figures, with relations, operations, and that its formal procedures involving axioms, proofs, lemmas, theorems have not changed since the time of Archimedes.
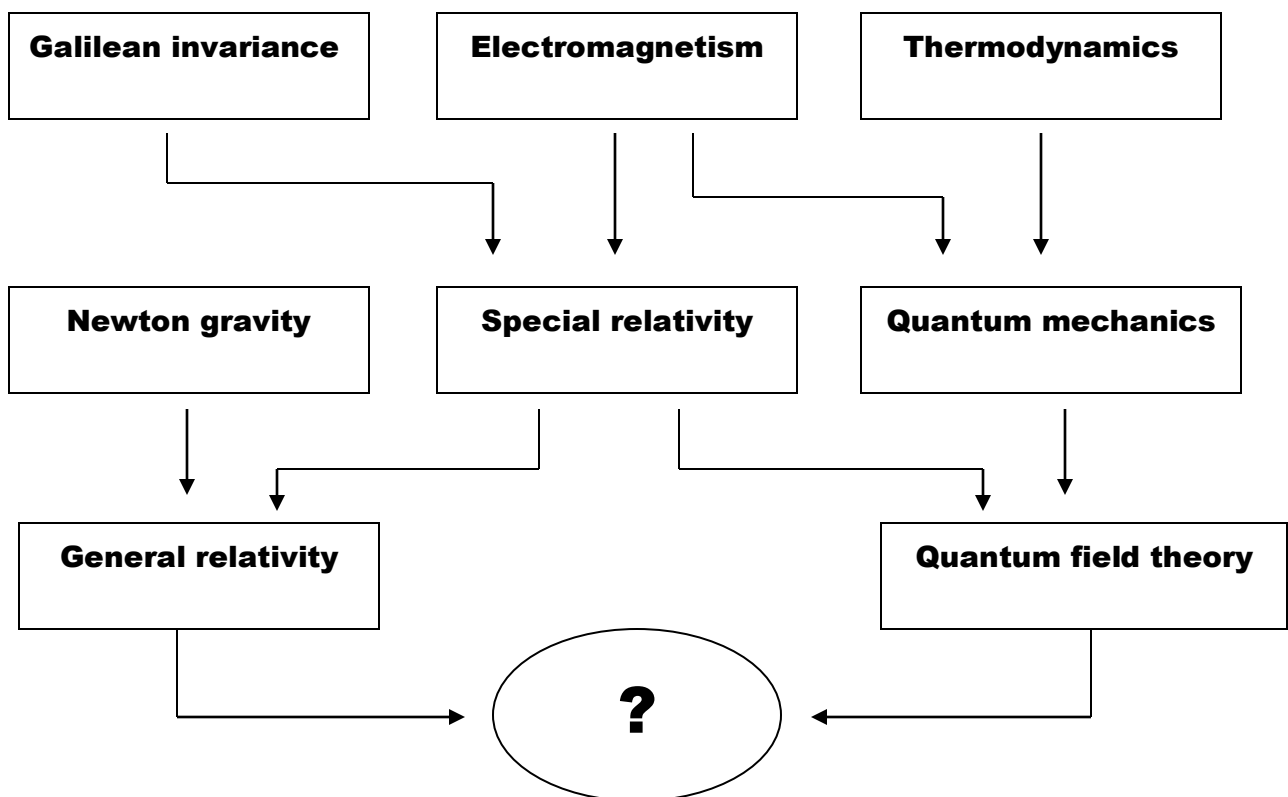
STAN ULAM, 1909 – 1984

- The **Dragonfly 44 galaxy** is as big as the Milky Way, and is 99.99% dark matter.
- In order for **astronauts** to live for a year in space they have to drink 730 liters of their own recycled sweat and urine because water is extremely limited.

Earth has a powerful magnetic field − this phenomenon is caused by nickel-iron core of the planet

If you were to compress the entire Earth down to the size of a **peanut**, you would have a black hole.

special relativity + quantum mechanics	Relativistic quantum electrodynamics **(very precise and highly successful)**
quantum mechanics + gravity	Theories of quantum gravity (**no data to test them**)

Electroweak force splits into electromagnetic and weak forces (around 10^{-6} seconds after the big bang)

Galilean invariance	Electromagnetism	Thermodynamics

Newton gravity	Special relativity	Quantum mechanics

General relativity	Quantum field theory

?

NASA has discovered a "water world" planet 40 light-years away that may contain exotic materials such as "hot ice" and "superfluid water".

Theory of relativity	Quantum mechanics
• Removes inconsistencies in the classical theory. • Describes the behavior of matter at high energies and high speeds	• Removes disagreements between theory and experiments • Describes the behavior of microscopic particles.

At small distances large 'quantum fluctuations' in the gravitational field make gravity very strongly-interacting.

The effort to understand the universe is one of the very few things that lifts human life a little above the level of farce, and gives it some of the grace of tragedy.

From the properties of subatomic particles and the realm of quantum physics to the formation of a giant mathematical object (universe), math proves unquestionably effective in describing and predicting their physical reality. However, a question that lies at the intersection of philosophy and science arises: **Is Math the Language of the Universe?**

References:

- **Light** – The Physics of the Photon, by Ole Keller.
- University Physics with Modern Physics by Hugh D. Young.
- Isaac Newton and the Laws of Motion by Andrea Gianopoulos.
- **Our Mathematical Universe:** My Quest for the Ultimate Nature of Reality by Max Tegmark.

Black holes do not suck in objects. They have an extremely strong gravitational force which is almost impossible to overcome by any object that comes too close to it. If our Sun was replaced with a black hole of equal mass, the planets would orbit around the black hole as they orbit the Sun today.

According to the **NASA**, the speed of Earth rotation is gradually slowing and it's happening at a rate of 1.4 milliseconds per 100 years. You may think it's not a big deal. But if you add up that small discrepancy every day for years and years, it can make a very big difference indeed. At this speed, the day may become 25 hours after 140 million years.

Source of information:

- https://www.wikipedia.org/
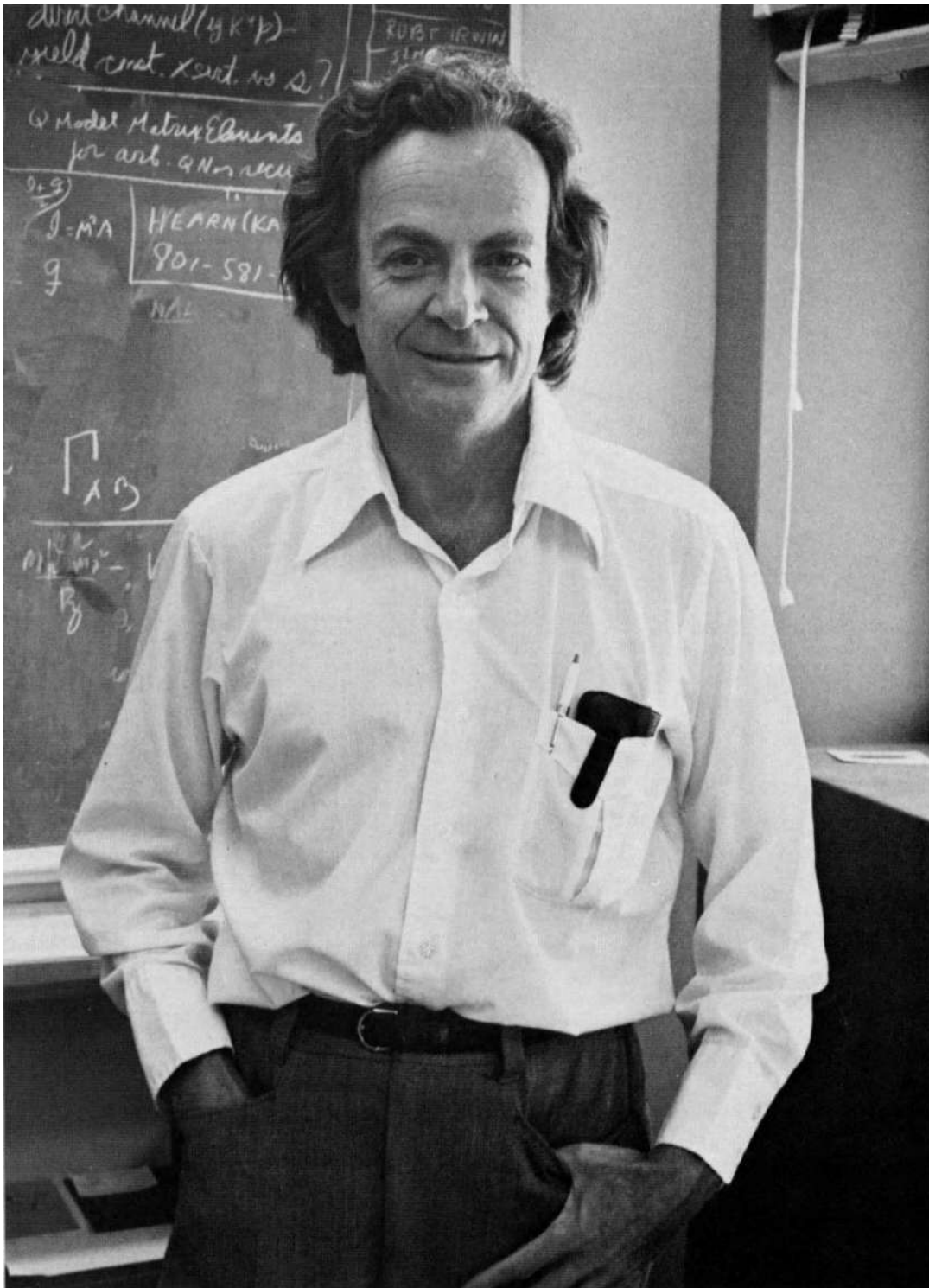
I... a universe of atoms, an atom in the universe.