NumCSE: Problem Sheet 7

Due on 5.11.2015

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1. Cubic Splines

(1a) α, β

To achieve smoothness of the spline, the "subfunctions" of $s_{\alpha,\beta}$ must be equal at the points they meet. This gives us the following two equations:

$$\begin{aligned} &\text{for } x = 0: && (x+1)^4 + \alpha(x-1)^4 + 1 = -x^3 - 8\alpha x + 1 && \rightarrow \alpha = -1 \\ &\text{for } x = 1, \alpha = -1: && -x^3 - 8\alpha x + 1 = \beta x^3 + 8x^2 + \frac{11}{3} && \rightarrow \beta = -\frac{11}{3} \end{aligned}$$

(1b) Matlab implementation

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function [ ] = nme7p1b( a, b )
s1=@(a,b,x)((x+1).^4 + a*(x-1).^4 + 1);
s2=@(a,b,x)(-x.^3 - 8*a*x + 1);
s3=@(a,b,x)(b * x.^3 + 8 * x.^2 + 11/3);

xs1 = linspace(-1,0,34);
xs2 = linspace(0,1,34); xs2 = xs2(1:end-1);
xs3 = linspace(1,2,34); xs3 = xs3(1:end-1);

xs = [xs1,xs2,xs3];
ys = [s1(a,b,xs1), s2(a,b,xs2), s3(a,b,xs3)];

plot(xs, ys)

end
```

2. Quadratic Splines

(2a) Subspace dimension

Counting Argument: The parametrisation of the spline function requires $2n+1$ unknowns, the 1-periodicity of the spline function is a constraint, the continuity of the derivations are n constraints, leaving us with dimension n.

(2b) Continuity

blah. continuity for free.

(2c) LSE for c_j, d_j

Plugging in $t = \frac{1}{2}(t_{j-1} + t_j)$ into (13) gives: