CHAPTER 0
REVIEW OF ALGEBRA
05. Factoring

Exercised by: Rizal Bimanto

1 Summary

Rules for Factoring

Expression	Factored Form	Type of Factoring
$xy + xz$	$x(y + z)$	*Common Factor*
$x^2 + (a + b)x + ab$	$(x + a)(x + b)$	-
$abx^2 + (ad + cb)x + cd$	$(ax + c)(bx + d)$	-
$x^2 + 2ax + a^2$	$(x + a)^2$	*Perfect Square Trinomial*
$x^2 - 2ax + a^2$	$(x - a)^2$	*Perfect Square Trinomial*
$x^2 - a^2$	$(x + a)(x - a)$	*Difference of Two Squares*
$x^3 + a^3$	$(x + a)(x^2 - ax + a^2)$	*Sum of Two Cubes*
$x^3 - a^3$	$(x - a)(x^2 + ax + a^2)$	*Difference of Two Cubes*

Always factor as completely as you can.
For example:

- $2x^2 - 8$
- $2(x^2 - 4)$
- $2(x + 2)(x - 2)$

Examples

Expression	Factored Form	Type of Factoring
$x^2 + 8x + 16$	$(x+4)^2$	*4. Perfect Square Trinomial*
$9x^2 + 9x + 2$	$(3x+1)(3x+2)$	*3. -*
$6y^3 + 3y^2 - 18y$	$3y(2y^2 + y - 6)$	*1. Common Factor*
$6y^3 + 3y^2 - 18y$	$3y(2y+3)(y-2)$	*3. -*
$x^2 - 6x + 9$	$(x-3)^2$	*5. Perfect Square Trinomial*
$z^{\frac{1}{4}} + z^{\frac{5}{4}}$	$z^{\frac{1}{4}}(1+z)$	*1. Common Factor*
$x^4 - 1$	$(x^2+1)(x^2-1)$	*6. Difference of Two Squares*
$x^4 - 1$	$(x^2+1)(x+1)(x-1)$	*6. Difference of Two Squares*
$x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 4$	$(x^{\frac{1}{3}} - 4)(x^{\frac{1}{3}} - 1)$	*2. -*
$ax^2 - ay^2 + bx^2 - by^2$	$a(x^2 - y^2) + b(x^2 - y^2)$	*1. Common Factor*
$ax^2 - ay^2 + bx^2 - by^2$	$(a+b)(x^2 - y^2)$	*1. Common Factor*
$ax^2 - ay^2 + bx^2 - by^2$	$(a+b)(x+y)(x-y)$	*6. Difference of Two Squares*
$8 - x^3$	$(2)^3 - x^3$	*8. Difference of Two Cubes*
$8 - x^3$	$(x^2 + 2x + 4)(-x+2)$	*8. Difference of Two Cubes*
$x^6 - y^6$	$(x^3)^2 - (y^3)^2$	-
$x^6 - y^6$	$(x^3 + y^3)(x^3 - y^3)$	*6. Difference of Two Squares*
$x^6 - y^6$	$(x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$	*7, 8*

2 Problems 0.5

Factor the following expressions completely

1. $5bx + 5b$
   - $5b(x+1)$
2. $6y^2 - 4y$
   - $y(6y-4)$
   - $2y(3y-2)$
3. $10xy + 5xz$
   - $5x(2y+z)$
4. $3x^2y - 9x^3y^3$

- $3(x^2y - 3x^3y^3)$
- $3(x^2y(1 - 3xy^2))$
- $3x^2y(1 - 3xy^2)$

5. $3a^3bcd^2 - 4ab^3c^2d^2 + 2a^3bc^4d^3$
   - $abcd^2(3a^2 - 4b^2c + 2a^2c^3d)$
6. $5r^2st^2 + 10r^3s^2t^3 - 15r^2t^2$
   - $5r^2t^2(s + 2rs^2t - 3)$
7. $z^2 - 49$
   - $(z + 7)(z - 7)$
8. $x^2 - x - 6$
   - $(x - 3)(x + 2)$
9. $p^2 + 4p + 3$
   - $(p + 3)(p + 1)$
10. $t^2 - t - 12$
    - $(t - 4)(t + 3)$
11. $25y^2 - 4$
    - $(5y + 2)(5y - 2)$
12. $x^2 + 2x - 24$
    - $(x + 6)(x - 4)$
13. $a^2 + 12a + 35$
    - $(a + 7)(a + 5)$
14. $4t^2 - 9s^2$
    - $(2t + 3s)(2t - 3s)$
15. $y^2 + 8y + 15$
    - $(y + 5)(y + 3)$
16. $t^2 - 18t + 72$
    - $(t - 6)(t - 12)$
17. $5x^2 + 25x + 30$
    - $5(x^2 + 5x + 6)$
    - $5(x + 3)(x + 2)$
18. $3t^2 + 12t - 15$
    - $3(t^2 + 4t - 5)$

- $3(t+5)(t-1)$

19. $3x^2 - 3$

    - $3(x^2-1)$
    - $3(x+1)(x-1)$

20. $6x^2 + 31x + 35$

    - $6x^2 + 21x + 10x + 35$
    - $(6x^2 + 21x) + (10x + 35)$
    - ~~$3x(x+7) + 5(2x+7)$~~
      wrong factoring, because of wrong grouping in second step.
    - $(6x^2 + 10x) + (21x + 35)$
    - $2x(3x+5) + 7(3x+5)$ *Factor out the common binomial factor*
    - $(2x+7)(3x+5)$

21. $5x^2 + 16x + 3$

    - $5x^2 + (x + 15x) + 3$
    - $(5x^2 + x) + (15x + 3)$
    - $x(5x+1) + 3(5x+1)$ *Factor out the common binomial factor*
    - $(x+3)(5x+1)$

22. $4x^2 - x - 3$

    - $4x^2 + (-4x + 3x) - 3$
    - $(4x^2 - 4x) + (3x - 3)$
    - $4x(x-1) + 3(x-1)$ *Factor out the common binomial factor*
    - $(4x+3)(x-1)$

23. $12s^3 + 10s^2 - 8s$

    - $12s^3 + 10s^2 - 8s$
    - $12s^3 + (16s^2 - 6s^2) - 8s$
    - $12s^3 + (-6s^2 + 16s^2) - 8s$
    - $(12s^3 - 6s^2) + (16s^2 - 8s)$
    - $6s^2(2s-1) + 8s(2s-1)$ *Factor out the common binomial factor*
    - $(6s^2 + 8s)(2s-1)$
    - $(2s(3s+4))(2s-1)$

24. $9z^2 + 30z + 25$

    - $9z^2 + (15z + 15z) + 25$
    - $(9z^2 + 15z) + (15z + 25)$
    - $3z(3z+5) + 5(3z+5)$ *Factor out the common binomial factor*

- $(3z+5)(3z+5)$
- $(3z+5)^2$

25. $a^{\frac{11}{3}}b - 4a^{\frac{2}{3}}b^3$

- $a^{\frac{2}{3}}b(a^{\frac{9}{3}} - 4b^2)$
- $a^{\frac{2}{3}}b(a^3 - 4b^2)$

26. $4x^{\frac{6}{5}} - 1$

- $(2x^{\frac{3}{5}} + 1)(2x^{\frac{3}{5}} - 1)$

27. $2x^3 + 2x^2 - 12x$

- $2x(x^2 + x - 6)$
- $2x(x+3)(x-2)$

28. $x^2y^2 - 4xy + 4$

- $x^2y^2 - (2xy + 2xy) + 4$
- $x^2y^2 - 2xy - 2xy + 4$
- $(x^2y^2 - 2xy) - (2xy + 4)$
- $xy(xy-2) - 2(xy-2)$
- $(xy-2)(xy-2)$
- $(xy-2)^2$

29. $(4x+2)^2$

- $(2(2x+1))^2$
- $4(2x+1)^2$

30. $x^2(2x^2 - 4x^3)^2$

- $x^2(2x^2(1-2x))^2$
- $x^2 4x^4(1-2x)^2$
- $4x^6(1-2x)^2$

31. $x^3y^2 - 16x^2y + 64x$

- $x(x^2y^2 - 16xy + 64)$
- $x(xy-8)(xy-8)$
- $x(xy-8)^2$

32. $(5x^2 + 2x) + (10x + 4)$

- $x(5x+2) + 2(5x+2)$
- $(x+2)(5x+2)$

33. $(x^3 - 4x) + (8 - 2x^2)$

- $(x(x^2-4)) + (-2(-4+x^2))$
- $(x(x+2)(x-2)) + (-2(x+2)(x-2))$

- $(x + (-2))(x + 2)(x - 2)$
- $(x - 2)(x + 2)(x - 2)$
- $(x + 2)(x - 2)^2$

34. $(x^2 - 1) + (x^2 - x - 2)$
    - $x^2 - 1 + x^2 - x - 2$
    - $2x^2 - 1 - x - 2$
    - $2x^2 - x - 3$
    - $(2x^2 - 3x) + (2x - 3)$
    - $x(2x - 3) + (2x - 3)$
    - $x(2x - 3) + (2x - 3)$
    - $(x + 1)(2x - 3)$
35. $4ax^2 - ay^2 + 12bx^2 - 3by^2$
    - $a(4x^2 - y^2) + 3b(4x^2 - y^2)$
    - $(a + 3b)(4x^2 - y^2)$
    - $(a + 3b)(2x + y)(2x - y)$