

# CHAPTER 0

## REVIEW OF ALGEBRA

### 03. Properties of Exponents and Radicals

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#### 1 Summary

The product  $x \cdot x \cdot x$  of 3  $x$ 's is abbreviated  $x^3$ . In general, for  $n$  a positive integer,  $x^n$  is the abbreviation product of  $nx$ 's. The letter  $n$  in  $x^n$  is called the **the exponent**, and  $x$  is called the **base**. More specifically, if  $n$  is positive integers we have:

1.  $x^n = \underbrace{x \cdot x \cdot \dots \cdot x}_{n \text{ factors}}$
2.  $\frac{1}{\underbrace{x \cdot x \cdot x \cdot \dots \cdot x}_{n \text{ factors}}}$  for  $x \neq 0$
3.  $\frac{1}{x^{-n}} = x^n$  for  $x \neq 0$
4.  $x^0 = 1$

If  $r^n = x$ , where  $n$  is a positive integer, then  $r$  is an  $n$ th **root** of  $x$ . Second roots, the case  $n = 2$ , are called **squared roots**; and third roots, the case  $n = 3$ , are called **cube roots**.

Some numbers do not have an  $n$ th root that is a real number. For example, since the square of any real number is non-negative: there is no real number that is a square root of  $-4$ .

The principal of  $n$ th root of  $x$  is the  $n$ th root of  $x$  that is positive if  $x$  is positive and is negative if  $x$  is negative and  $n$  is odd. We denote the principal  $n$ th root of  $x$  by  $\sqrt[n]{x}$ :

$$\sqrt[n]{x} \text{ is } \begin{cases} \text{positive if } x \text{ is positive} \\ \text{negative if } x \text{ is negative and } n \text{ is odd} \end{cases}$$

For example,  $\sqrt[2]{9} = 3$ ,  $\sqrt[3]{-8} = -2$  and  $\sqrt[3]{\frac{1}{27}} = \frac{1}{3}$  We define  $\sqrt[n]{0} = 0$ . The symbol of  $\sqrt[n]{x}$  is called radical.

Here are the basic laws of exponent and radicals:

Law	Example(s)
1. $x^m \cdot x^n = x^{m+n}$	$2^3 \cdot 2^5 = 2^8 = 256$ ; $x^2 \cdot x^3 = x^5$
2. $x^0 = 1$	$2^0 = 1$
3. $x^{-n} = \frac{1}{x^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
4. $\frac{1}{x^{-n}} = x^n$	$\frac{1}{2^{-3}} = 2^3 = 8$ ; $\frac{1}{x^{-5}} = x^5$
5. $\frac{x^m}{x^n} = x^{m-n} = \frac{1}{x^{n-m}}$	$\frac{2^{12}}{2^8} = 2^4 = 16$ ; $\frac{x^8}{x^{12}} = x^{-4} = \frac{1}{x^4}$
6. $\frac{x^m}{x^m} = 1$	$\frac{2^4}{2^4} = 1$
7. $(x^m)^n = x^{mn}$	$(2^3)^5 = 2^{15}$ ; $(x^2)^3 = x^6$
8. $(xy^n) = x^n y^n$	$(2 \cdot 4)^3 = 2^3 \cdot 4^3 = 8 \cdot 64 = 512$
9. $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$
10. $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$	$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$
11. $x^{\frac{1}{n}} = \sqrt[n]{x^1}$	$3^{\frac{1}{5}} = \sqrt[5]{3^1}$
12. $x^{\frac{-1}{n}} = \frac{1}{x^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{x}}$	$4^{\frac{-1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt[2]{4}} = \frac{1}{2}$
13. $\sqrt[n]{x} \sqrt[n]{y} = \sqrt[n]{xy}$	$\sqrt[3]{9} \sqrt[3]{2} = \sqrt[3]{9 \cdot 2} = \sqrt[3]{18}$
14. $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$	$\frac{\sqrt[3]{90}}{\sqrt[3]{10}} = \sqrt[3]{\frac{90}{10}} = \sqrt[3]{9}$
15. $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$	$\sqrt[3]{\sqrt[4]{2}} = \sqrt[3 \cdot 4]{2} = \sqrt[12]{2}$
16. $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$
17. $(\sqrt[n]{x^m}) = x$	$(\sqrt[8]{7})^8 = 7$

## 2 Problems 0.3

In Problems 1 - 14, simplify and express all answers in terms of positive exponent

1.  $(2^3)(2^2)$

- $2^{(3+2)}$
- $2^5$
- 32

2.  $x^6 x^9$

- $x^{(6+9)}$
- $x^{(15)}$

3.  $17^5 \cdot 17^2$

- $17^{(5+2)}$

- $17^{(7)}$
  - $410, 338, 673$
4.  $z^3 z z^2$
- $z^{(3+1+2)}$
  - $z^6$
5.  $\frac{x^3 x^5}{y^9 y^5}$
- $\frac{x^{(3+5)}}{y^{(9+5)}}$
  - $\frac{x^8}{y^{14}}$
6.  $(x^{12})^4$
- $x^{(12 \cdot 4)}$
  - $x^{48}$
7.  $\frac{(a^3)^7}{(b^4)^5}$
- $\frac{a^{3 \cdot 7}}{b^{4 \cdot 5}}$
  - $\frac{a^{21}}{b^{20}}$
8.  $\left(\frac{13^{14}}{13}\right)^2$
- $\frac{13^{(14 \cdot 2)}}{13^2}$
  - $\frac{13^{28}}{13^2}$
  - $13^{(28-2)}$
  - $13^{26}$
9.  $(2x^2 y^3)^3$
- $2^3 x^{(2 \cdot 3)} y^{(3 \cdot 3)}$
  - $8x^6 y^9$
10.  $\left(\frac{w^2 s^3}{y^2}\right)^2$
- $\frac{w^{(2 \cdot 2)} s^{(3 \cdot 2)}}{y^{(2 \cdot 2)}}$
  - $\frac{w^4 s^6}{y^4}$
11.  $\left(\frac{x^9}{x^5}\right)$
- $x^{(9-5)}$
  - $x^4$
12.  $\left(\frac{2a^4}{7b^5}\right)^6$
- $\frac{2^6 a^{(4 \cdot 6)}}{7^6 b^{(5 \cdot 6)}}$
  - $\frac{2^6 a^{24}}{7^6 b^{30}}$
13.  $\frac{(y^3)^4}{(y^2)^3 y^2}$

- $\frac{y^{3 \cdot 4}}{y^{2 \cdot 3 + 2}}$
  - $\frac{y^{12}}{y^8}$
  - $y^{(12-8)}$
  - $y^4$
14.  $\frac{(x^2)^3(x^3)^2}{(x^3)^4}$
- $\frac{x^{(2 \cdot 3)}x^{(3 \cdot 2)}}{x^{(3 \cdot 4)}}$
  - $\frac{x^6x^6}{x^{12}}$
  - $\frac{x^{(6+6)}}{x^{12}}$
  - $\frac{x^{12}}{x^{12}}$
  - $x^{(12-12)}$
  - $x^0$
  - 1

*In Problems 15 - 28, evaluate the expressions.*

15.  $\sqrt{25}$
- $\sqrt[4]{5^4}$
  - 5
16.  $\sqrt[4]{81}$
- $\sqrt[4]{3^4}$
  - 3
17.  $\sqrt[7]{-128}$
- $\sqrt[7]{-2^7}$
  - $-2^{7 \cdot \frac{1}{7}}$
  - -2
18.  $\sqrt[5]{0.00243}$
- $\sqrt[5]{81 \cdot 3 \cdot 10^{-5}}$
  - $\sqrt[5]{3^4 \cdot 3 \cdot 10^{-5}}$
  - $\sqrt[5]{3^5 \cdot 10^{-5}}$
  - $3^{\frac{5}{5} \cdot \frac{1}{5}} 10^{-\frac{5}{5} \cdot \frac{1}{5}}$
  - $3 \cdot 10^{-1}$
  - 0.3
19.  $\sqrt[4]{\frac{1}{16}}$
- $\sqrt[4]{\frac{1^4}{2^4}}$
  - $\frac{1}{2}^{\frac{4}{4} \cdot \frac{1}{4}}$
  - $\frac{1}{2}$

20.  $\sqrt[3]{-\frac{8}{27}}$

- $\sqrt[3]{-\frac{2^3}{3^3}}$
- $-\frac{2^3 \cdot \frac{1}{3}}{3^3 \cdot \frac{1}{3}}$
- $-\frac{2^{\cancel{3}} \cdot \frac{1}{\cancel{3}}}{3^{\cancel{3}} \cdot \frac{1}{\cancel{3}}}$
- $-\frac{2}{3}$

21.  $(49)^{\frac{1}{2}}$

- $(7^2)^{\frac{1}{2}}$
- $(7^{\cancel{2}})^{\frac{1}{\cancel{2}}}$
- 7

22.  $(64)^{\frac{1}{3}}$

- $(4^3)^{\frac{1}{3}}$
- $(4^{\cancel{3}})^{\frac{1}{\cancel{3}}}$
- 4

23.  $81^{\frac{3}{4}}$

- $\sqrt[4]{81^3}$
- $\sqrt[4]{(3^4)^3}$
- $\sqrt[4]{3^{12}}$
- $3^{(12 \cdot \frac{1}{4})}$
- $3^3$
- 27

24.  $(9)^{-\frac{5}{2}}$

- $(3^2)^{-\frac{5}{2}}$
- $(3^{\cancel{2}})^{-\frac{5}{\cancel{2}}}$
- $3^{-5}$
- $\frac{1}{3^5}$
- $\frac{1}{243}$

25.  $(32)^{-\frac{2}{5}}$

- $(2^5)^{-\frac{2}{5}}$
- $(2^{\cancel{5}})^{-\frac{2}{\cancel{5}}}$
- $2^{-2}$
- $\frac{1}{2^2}$
- $\frac{1}{4}$

26.  $(0.09)^{-\frac{1}{2}}$

- $(3^2 \cdot 10^{-2})^{-\frac{1}{2}}$
- $3^{(2 \cdot \frac{-1}{2})} \cdot 10^{(-2 \cdot \frac{-1}{2})}$
- $3^{-\frac{2}{2}} \cdot 10^{\frac{2}{2}}$
- $3^{-1} \cdot 10$
- $\frac{1}{3} \cdot 10$
- $\frac{10}{3}$
- $3.333 \dots$

27.  $\left(\frac{1}{32}\right)^{\frac{4}{5}}$

- $\left(\frac{1}{2^5}\right)^{\frac{4}{5}}$
- $(2^{-5})^{\frac{4}{5}}$
- $2^{-5 \cdot \frac{4}{5}}$
- $2^{-\frac{20}{5}}$
- $2^{-4}$
- $\frac{1}{2^4}$
- $\frac{1}{2^4}$
- $\frac{1}{16}$

28.  $\left(-\frac{243}{1024}\right)^{\frac{2}{5}}$

- $\left(-\frac{3^5}{2^{10}}\right)^{\frac{2}{5}}$
- $-\frac{3^{5 \cdot \frac{2}{5}}}{2^{10 \cdot \frac{2}{5}}}$
- $-\frac{3^{\frac{10}{5}}}{2^{\frac{20}{5}}}$
- $-\frac{3^2}{2^4}$
- $-\frac{9}{16}$

*In Problems 29 - 40, simplify the expressions.*

29.  $\sqrt{50}$

- $\sqrt{25 \cdot 2}$
- $5\sqrt{2}$

30.  $\sqrt[3]{54}$

- $\sqrt[3]{27 \cdot 2}$
- $3\sqrt[3]{2}$

31.  $\sqrt[3]{2x^3}$

- $x\sqrt[3]{2}$

32.  $\sqrt{4x}$

- $2\sqrt{x}$

33.  $\sqrt{49u^8}$

- $(49u^8)^{\frac{1}{2}}$
- $(7^2u^8)^{\frac{1}{2}}$
- $7^{2 \cdot \frac{1}{2}}u^{8 \cdot \frac{1}{2}}$
- $7^{\frac{2 \cdot 1}{2}}u^{\frac{8 \cdot 1}{2}}$
- $7u^4$

34.  $\sqrt[4]{\frac{x}{16}}$

- $\frac{\sqrt[4]{x}}{\sqrt[4]{2^4}}$
- $\frac{\sqrt[4]{x}}{(2^4)^{\frac{1}{4}}}$
- $\frac{\sqrt[4]{x}}{2^{4 \cdot \frac{1}{4}}}$
- $\frac{\sqrt[4]{x}}{2^{\frac{4 \cdot 1}{4}}}$
- $\frac{\sqrt[4]{x}}{2}$
- $\frac{1}{2}\sqrt[4]{x}$

35.  $2\sqrt{8} - 5\sqrt{27} + \sqrt[3]{128}$

- $2\sqrt{4 \cdot 2} - 5\sqrt{9 \cdot 3} + \sqrt[3]{64 \cdot 2}$
- $2 \cdot 2\sqrt{2} - 5 \cdot 3\sqrt{3} + 4\sqrt[3]{2}$
- $4\sqrt{2} - 15\sqrt{3} + 4\sqrt[3]{2}$

36.  $\sqrt{\frac{3}{13}}$

- $\frac{\sqrt{3}}{\sqrt{13}}$

37.  $(9z^4)^{\frac{1}{2}}$

- $9^{\frac{1}{2}}z^{4 \cdot \frac{1}{2}}$
- $\sqrt{9} \cdot z^2$
- $3z^2$

38.  $(729x^6)^{\frac{3}{2}}$

- $(81 \cdot 9x^6)^{\frac{3}{2}}$
- $81^{\frac{3}{2}}9^{\frac{3}{2}}(x^6)^{\frac{3}{2}}$
- $\sqrt{81^3}\sqrt{9^3}\sqrt{(x^6)^3}$
- $\sqrt{(9 \cdot 9)(9 \cdot 9)(9 \cdot 9)} \sqrt{(9 \cdot 9 \cdot 9)} \sqrt{x^{(6 \cdot 3)}}$
- $(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3 \cdot 3) \sqrt{x^{18}}$
- $9 \cdot 9 \cdot 9 \cdot 9 \cdot 3 (x^{18})^{\frac{1}{2}}$
- $19,683 x^{18 \cdot \frac{1}{2}}$
- $19,683x^9$

39.  $\left(\frac{27t^3}{8}\right)^{\frac{2}{3}}$

- $\frac{27^{\frac{2}{3}}(t^3)^{\frac{2}{3}}}{8^{\frac{2}{3}}}$
- $\frac{\sqrt[3]{27^2} t^{3 \cdot \frac{2}{3}}}{\sqrt[3]{8^2}}$
- $\frac{\sqrt[3]{(3^3)^2} t^{\frac{2}{3}}}{\sqrt[3]{(2^3)^2}}$
- $\frac{((3^3)^2)^{\frac{1}{3}} t^2}{((2^3)^2)^{\frac{1}{3}}}$
- $\frac{3^{3 \cdot 2 \cdot \frac{1}{3}} t^2}{2^{3 \cdot 2 \cdot \frac{1}{3}}}$
- $\frac{3^{\frac{2}{3} \cdot 2 \cdot \frac{1}{3}} t^2}{2^{\frac{2}{3} \cdot 2 \cdot \frac{1}{3}}}$
- $\frac{3^2 t^2}{2^2}$
- $\frac{9t^2}{4}$

40.  $\left(\frac{256}{x^{12}}\right)^{-\frac{3}{4}}$

- $\sqrt[4]{\left(\frac{256}{x^{12}}\right)^{-3}}$
- $\sqrt[4]{\left(\frac{1}{\frac{256}{x^{12}}}\right)^3}$
- $\sqrt[4]{\left(\frac{1 \cdot x^{12}}{\frac{256}{x^{12}} \cdot x^{12}}\right)^3}$
- $\sqrt[4]{\left(\frac{x^{12}}{\frac{256}{\cancel{x^{12}} \cdot \cancel{x^{12}}}}\right)^3}$
- $\sqrt[4]{\left(\frac{x^{12}}{256}\right)^3}$
- $\left(\frac{x^{12}}{256}\right)^{\frac{3}{4}}$
- $\frac{x^{12 \cdot \frac{3}{4}}}{256^{\frac{3}{4}}}$
- $\frac{\cancel{x}^{12 \cdot \frac{3}{4}}}{(2^8)^{\frac{3}{4}}}$
- $\frac{x^{3 \cdot 3}}{2^{8 \cdot \frac{3}{4}}}$
- $\frac{x^9}{2^{\frac{2}{3} \cdot 2 \cdot \frac{3}{4}}}$
- $\frac{x^9}{2^{2 \cdot 3}}$
- $\frac{x^9}{2^6}$
- $\frac{x^9}{64}$

In Problems 41-52, write the expressions in terms of positive exponents only. Avoid all radicals in the final form. For example:

$$y^{-1} \sqrt{x} = \frac{x^{\frac{1}{2}}}{y}$$

41.  $\frac{a^5 b^{-3}}{c^2}$

- $\frac{a^5}{c^2} \cdot \frac{1}{b^3}$
- $\frac{a^5}{c^2 b^3}$



42.  $\sqrt[5]{x^2y^3z^{-10}}$

- $\sqrt[5]{x^2y^3 \cdot \frac{1}{z^{10}}}$
- $\sqrt[5]{\frac{x^2y^3}{z^{10}}}$
- $\left(\frac{x^2y^3}{z^{10}}\right)^{\frac{1}{5}}$
- $\frac{x^{2 \cdot \frac{1}{5}}y^{3 \cdot \frac{1}{5}}}{z^{10 \cdot \frac{1}{5}}}$
- $\frac{x^{\frac{2}{5}}y^{\frac{3}{5}}}{z^{10 \cdot \frac{1}{5}}}$
- $\frac{x^{\frac{2}{5}}y^{\frac{3}{5}}}{z^2}$

43.  $3a^{-1}b^{-2}c^{-3}$

- $\frac{3}{ab^2c^3}$

44.  $x + y^{-1}$

- $x + \frac{1}{y}$

45.  $(3t)^{-2}$

- $\frac{1}{(3t)^2}$
- $\frac{1}{9t^2}$

46.  $(3 - z)^{-4}$

- $\frac{1}{(3-z)^4}$

47.  $\sqrt[5]{5x^2}$

- $(5x^2)^{\frac{1}{5}}$
- $\left(5^{\frac{1}{5}}x^{2 \cdot \frac{1}{5}}\right)$
- $\left(5^{\frac{1}{5}}x^{\frac{2}{5}}\right)$

48.  $(X^5Y^{-7})^{-4}$

- $(X^{5 \cdot -4}Y^{-7 \cdot -4})$
- $(X^{-20}Y^{28})$
- $\frac{Y^{28}}{X^{20}}$

49.  $\sqrt{x} - \sqrt{y}$

- $x^{\frac{1}{2}} - y^{\frac{1}{2}}$

50.  $\frac{u^{-2}v^{-6}w^3}{vw^{-5}}$

- $\frac{w^3}{v} \cdot \frac{1}{u^2v^6} \cdot \frac{w^5}{1}$
- $\frac{w^{3+5}}{v^{1+6}u^2}$
- $\frac{w^8}{v^7u^2}$

51.  $x^2\sqrt[4]{xy^{-2}z^3}$

- $x^2 (xy^{-2}z^3)^{\frac{1}{4}}$
- $x^2 \left( \frac{xz^3}{y^2} \right)^{\frac{1}{4}}$
- $x^2 \left( \frac{x^{\frac{1}{4}} z^{3 \cdot \frac{1}{4}}}{y^{2 \cdot \frac{1}{4}}} \right)$
- $x^2 \left( \frac{x^{\frac{1}{4}} z^{\frac{3}{4}}}{y^{\frac{2}{4}}} \right)$
- $\frac{x^{\frac{1}{4}+2} z^{\frac{3}{4}}}{y^{\frac{2}{4}}}$
- $\frac{x^{\frac{1}{4}+\frac{8}{4}} z^{\frac{3}{4}}}{y^{\frac{2}{4}}}$
- $\frac{x^{\frac{9}{4}} z^{\frac{3}{4}}}{y^{\frac{1}{2}}}$

52.  $\sqrt[4]{a^{-3}b^{-2}}a^4b^{-4}$

- $(a^{-3}b^{-2})^{\frac{1}{4}}a^4b^{-4}$
- $a^{-\frac{3}{4}}b^{-\frac{2}{4}}a^4b^{-4}$
- $\frac{a^4}{a^{\frac{3}{4}}b^{\frac{2}{4}}b^4}$
- $\frac{a^{4-\frac{3}{4}}}{b^{4+\frac{2}{4}}}$
- $\frac{a^{\frac{16}{4}-\frac{3}{4}}}{b^{\frac{16}{4}+\frac{2}{4}}}$
- $\frac{a^{\frac{13}{4}}}{b^{\frac{18}{4}}}$
- $\frac{a^{\frac{13}{4}}}{b^{\frac{9}{2}}}$

*In Problems 53-58, rewrite the exponential forms using radicals*

53.  $(a-b+c)^{\frac{3}{5}}$

- $\sqrt[5]{(a-b+c)^3}$

54.  $(ab^2c^3)^{\frac{3}{4}}$

- $\sqrt[4]{(ab^2c^3)^3}$
- $\sqrt[4]{a^3b^{(2 \cdot 3)}c^{(3 \cdot 3)}}$
- $\sqrt[4]{a^3b^6c^9}$

55.  $x^{-\frac{4}{5}}$

- $\sqrt[5]{x^{-4}}$
- $\sqrt[5]{\frac{1}{x^4}}$

56.  $2x^{\frac{1}{2}} - (2y)^{\frac{1}{2}}$

- $2\sqrt{x} - 2^{\frac{1}{2}}y^{\frac{1}{2}}$
- $2\sqrt{x} - \sqrt{2} \sqrt{y}$
- $2\sqrt{x} - \sqrt{2y}$

57.  $3w^{-\frac{3}{5}} - (3w)^{-\frac{3}{5}}$

- $3\sqrt[5]{w^{-3}} - 3^{-\frac{3}{5}}w^{-\frac{3}{5}}$

- $3\sqrt[5]{w^{-3}} - \sqrt[5]{3^{-3}}\sqrt[5]{w^{-3}}$
- $3\sqrt[5]{\frac{1}{w^3}} - \sqrt[5]{\frac{1}{3^3}}\sqrt[5]{\frac{1}{w^3}}$
- $3\sqrt[5]{\frac{1}{w^3}} - \sqrt[5]{\frac{1}{3^3}\frac{1}{w^3}}$
- $3\sqrt[5]{\frac{1}{w^3}} - \sqrt[5]{\frac{1}{27}\frac{1}{w^3}}$
- $3\frac{\sqrt[5]{1}}{\sqrt[5]{w^3}} - \frac{\sqrt[5]{1}}{\sqrt[5]{27w^3}}$
- $\frac{3}{\sqrt[5]{w^3}} - \frac{1}{\sqrt[5]{27w^3}}$

58.  $\left((y^{-2})^{\frac{1}{4}}\right)^{\frac{1}{5}}$

- $\left(y^{-2\cdot\frac{1}{4}}\right)^{\frac{1}{5}}$
- $\left(y^{\frac{-2}{4}}\right)^{\frac{1}{5}}$
- $y^{\frac{-2}{4}\cdot\frac{1}{5}}$
- $y^{\frac{-2}{20}}$
- $y^{\frac{-1}{10}}$
- $\sqrt[10]{y^{-1}}$
- $\sqrt[10]{\frac{1}{y}}$
- $\frac{1}{\sqrt[10]{y}}$