CHAPTER 0
REVIEW OF ALGEBRA
04. Operations with Algebraic Expressions

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1 Summary

Algebraic expressions with exactly 1 term are called **monomials**.

$$3x^2$$

Algebraic expressions with exactly 2 terms are called **binomials**.

$$3x^2 + 3x$$

Algebraic expressions with exactly 3 terms are called **trinomials**.

$$3x^2 + 3x + z$$

Algebraic expressions with more terms are called **polynomials**.

$$3x^2 + 3x + z + 6 + b^3$$

Special Products

1. $x(y + z) = xy + xz$
2. $(x + a)(x + b) = x^2 + x(a + b) + ab$
3. $(ax + c)(bx + d) = abx^2 + x(ad + bc) + cd$
4. $(x + a)^2 = x^2 + 2ax + a^2$
5. $(x - a)^2 = x^2 - 2ax + a^2$
6. $(x + a)(x - a) = x^2 - a^2$
7. $(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$
8. $(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$

2 Long Division

Divide $2x^3 - 14x - 5$ by $x - 3$

$$
\begin{array}{r|l}
 & 2x^2+6x+4 \ \leftarrow \textit{Quotient} \\
\hline
\textit{Divisor} \rightarrow (x-3) & 2x^3+0x^2-14x-5 \\
 & -(2x^3-6x^2) \\
\hline
 & \quad 6x^2-14x \\
 & -(6x^2-18x) \\
\hline
 & \quad\quad 4x-5 \\
 & -(4x-12) \\
\hline
 & \quad\quad\quad 7 \ \leftarrow \textit{Remainder}
\end{array}
$$

So the result of $2x^3-14x-5$ by $x-3$ is

$$2x^2+6x+4+\tfrac{7}{x-3}$$

- $\frac{Dividend}{Divisor} = Quotient + \frac{Remainder}{Divisor}$

A way of checking a division is to verify that

- $Dividend = \left(Quotient + \frac{Remainder}{Divisor}\right) Divisor$
- $Dividend = Quotient \cdot Divisor + \frac{Remainder}{Divisor} \cdot Divisor$
- $Dividend = Quotient \cdot Divisor + \frac{Remainder}{\cancel{Divisor}} \cdot \cancel{Divisor}$
- $Dividend = Quotient \cdot Divisor + Remainder$

By using this equation, you should be able to verify the result of the example.

3 Problems 0.4

Perform the indicated operations and simplify.

1. $(8x-4y+2)+(3x+2y-5)$
2. $(4a^2-2ab+3)+(5c-3ab+7)$
3. $(8t^2-6s^2)+(4s^2-2t^2+6)$
4. $(\sqrt{x}+2\sqrt{x})+(3\sqrt{x}+4\sqrt{x})$
5. $(\sqrt{a}+2\sqrt{3b})-(\sqrt{c}-3\sqrt{3b})$
6. $(3a+7b-9)-(5a+9b+21)$
7. $(7x^2+5xy+\sqrt{2})-(2z-2xy+\sqrt{2})$
8. $(\sqrt{x}+2\sqrt{x})-(\sqrt{x}+3\sqrt{x})$
9. $(\sqrt[2]{2x}+\sqrt[3]{3y})-(\sqrt[2]{2x}+\sqrt[4]{4z})$
10. $4(2z-w)-3(w-2z)$

11. $3(3x + 3y - 7) - 3(8x - 2y + 2)$
12. $(4s - 5t) + (-2s - 5t) + (s + 9)$
13. $5(x^2 - y^2) + x(y - 3x) + 4y(2x + 7y)$
14. $(7 + 3(x - 3) - (4 - 5x))$
15. $2(3(3(x^2 + 2) - 2(x^2 - 5)))$
16. $4(3(t + 5) - t(1 - (t + 1)))$
17. $-2(3u^2(2u + 2) - 2(u^2 - (5 - 2u)))$
18. $-(-3[2a + 2b - 2] + 5(2a + 3b) - a(2(b + 5)))$
19. $(2x + 5)(3x - 2)$
20. $(u + 2)(u + 5)$
21. $(w + 2)(w - 5)$
22. $(x - 4)(x + 7)$
23. $(2x + 3)(5x + 2)$
24. $(t^2 - 5t)(3t^2 - 7t)$
25. $(X + 2Y)^2$
26. $(2x - 1)^2$
27. $(7 - X)^2$
28. $(\sqrt{x} - 1)(2\sqrt{x} + 5)$
29. $(\sqrt{5x} - 2)^2$
30. $(\sqrt{y} - 3)(\sqrt{y} + 3)$
31. $(2s - 1)(2s + 1)$
32. $(a^2 + 2b)(a^2 - 2b)$
33. $(x^2 - 3)(x + 4)$
34. $(u - 1)(u^2 + 3u - 2)$
35. $(x^2 - 4)(3x^2 + 2x - 1)$
36. $(3y - 2)(4y^3 + 2y^2 + -3y)$
37. $t(3(t + 2)(t - 4) + 5(3t(t - 7)))$
38. $((2z + 1)(2z - 1))(4z^2 + 1)$
39. $(s - t + 4)(3s + 2t - 1)$
40. $(x^2 + x + 1)^2$
41. $(2a + 3)^3$

42. $(2a-3)^3$

43. $(2x-3)^3$

44. $(3a+b)^3$

45. $\frac{z^2-18z}{z}$

46. $\frac{2x^3-7x+4}{x}$

47. $\frac{6u^5+9u^3-1}{3u^2}$

48. $\frac{(3y-4)-(9y+5)}{3y}$

49. $(x^2+7x-5) \div (x+5)$

50. $(x^2-5x+4) \div (x-4)$

51. $(3x^3-2x^2+x-3) \div (x+2)$

52. $(x^4+3x^2+2) \div (x+1)$

53. $x^3 \div (x+2)$

54. $(8x^2+6x+7) \div (2x+1)$

55. $(3x^2-4x+3) \div (3x+2)$

56. $(z^3+z^2+z) \div (z^2-z+1)$