CHAPTER 0
REVIEW OF ALGEBRA
03. Properties of Exponents and Radicals

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1 Summary

The product $x \cdot x \cdot x$ of 3 x's is abbreviated x^3. In general, for n a positive integer, x^n is the abbreviation product of nx's. The letter n in x^n is called the **the exponent**, and x is called the **base**. More specifically, if n is positive integers we have:

1. $x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}$
2. $\frac{1}{\underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}}$ for $x \neq 0$
3. $\frac{1}{x^{-n}} = x^n$ for $x \neq 0$
4. $x^0 = 1$

If $r^n = x$, where n is a positive integer, then r is an nth **root** of x. Second roots, the case $n = 2$, are called **squared roots**; and third roots, the case $n = 3$, are called **cube roots**.

Some numbers do not have an nth root that is a real number. For example, since the square of any real number is non-negative: there is no real number that is a square root of -4.

The principal of nth root of x is the nth root of x that is positive if x is positive and is negative if x is negative and n is odd. We denote the principal nth root of x by $\sqrt[n]{x}$:

$$\sqrt[n]{x} \text{ is } \begin{cases} \text{positive if } x \text{ is positive} \\ \text{negative if } x \text{ is negative and } n \text{ is odd} \end{cases}$$

For example, $\sqrt[2]{9} = 3$, $\sqrt[3]{-8} = -2$ and $\sqrt[3]{\frac{1}{27}} = \frac{1}{3}$ We define $\sqrt[n]{0} = 0$.
The symbol of $\sqrt[n]{x}$ is called radical.

Here are the basic laws of exponent and radicals:

Law	Example(s)
1. $x^m \cdot x^n = x^{m+n}$	$2^3 \cdot 2^5 = 2^8 = 256; \quad x^2 \cdot x^3 = x^5$
2. $x^0 = 1$	$2^0 = 1$
3. $x^{-n} = \frac{1}{x^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
4. $\frac{1}{x^{-n}} = x^n$	$\frac{1}{2^{-3}} = 2^3 = 8; \frac{1}{x^{-5}} = x^5$
5. $\frac{x^m}{x^n} = x^{m-n} = \frac{1}{x^{n-m}}$	$\frac{2^{12}}{2^8} = 2^4 = 16; \frac{x^8}{x^{12}} = x^{-4} = \frac{1}{x^4}$
6. $\frac{x^m}{x^m} = 1$	$\frac{2^4}{2^4} = 1$
7. $(x^m)^n = x^{mn}$	$\left(2^3\right)^5 = 2^{15}; \left(x^2\right)^3 = x^6$
8. $(xy^n) = x^n y^n$	$(2 \cdot 4)^3 = 2^3 \cdot 4^3 = 8 \cdot 64 = 512$
9. $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$
10. $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$	$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$
11. $x^{\frac{1}{n}} = \sqrt[n]{x^1}$	$3^{\frac{1}{5}} = \sqrt[5]{3^1}$
12. $x^{\frac{-1}{n}} = \frac{1}{x^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{x}}$	$4^{\frac{-1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt[2]{4}} = \frac{1}{2}$
13. $\sqrt[n]{x}\,\sqrt[n]{y} = \sqrt[n]{xy}$	$\sqrt[3]{9}\,\sqrt[3]{2} = \sqrt[3]{9 \cdot 2} = \sqrt[3]{18}$
14. $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$	$\frac{\sqrt[3]{90}}{\sqrt[3]{10}} = \sqrt[3]{\frac{90}{10}} = \sqrt[3]{9}$
15. $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$	$\sqrt[3]{\sqrt[4]{2}} = \sqrt[3 \cdot 4]{2} = \sqrt[12]{2}$
16. $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$
17. $\left(\sqrt[m]{x^m}\right) = x$	$\left(\sqrt[8]{7}\right)^8 = 7$

2 Problems 0.3

In Problems 1 - 14, simplify and express all answers in terms of positive exponent

1. $\left(2^3\right)\left(2^2\right)$
   - $2^{(3+2)}$
   - 2^5
   - 32
2. $x^6 x^9$
   - $x^{(6+9)}$
   - $x^{(15)}$
3. $17^5 \cdot 17^2$
   - $17^{(5+2)}$

- $17^{(7)}$
- $410,338,673$

4. $z^3 z z^2$
   - $z^{(3+1+2)}$
   - z^6
5. $\frac{x^3 x^5}{y^9 y^5}$
   - $\frac{x^{(3+5)}}{y^{(9+5)}}$
   - $\frac{x^8}{y^{14}}$
6. $\left(x^{12}\right)^4$
   - $x^{(12\cdot4)}$
   - x^{48}
7. $\frac{\left(a^3\right)^7}{\left(b^4\right)^5}$
   - $\frac{a^{3\cdot7}}{b^{4\cdot5}}$
   - $\frac{a^{21}}{b^{20}}$
8. $\left(\frac{13^{14}}{13}\right)^2$
   - $\frac{13^{(14\cdot2)}}{13^2}$
   - $\frac{13^{28}}{13^2}$
   - $13^{(28-2)}$
   - 13^{26}
9. $\left(2x^2y^3\right)^3$
   - $2^3x^{(2\cdot3)}y^{(3\cdot3)}$
   - $8x^6y^9$
10. $\left(\frac{w^2s^3}{y^2}\right)^2$
    - $\frac{w^{(2\cdot2)}s^{(3\cdot2)}}{y^{(2\cdot2)}}$
    - $\frac{w^4s^6}{y^4}$
11. $\left(\frac{x^9}{x^5}\right)$
    - $x^{(9-5)}$
    - x^4
12. $\left(\frac{2a^4}{7b^5}\right)^6$
    - $\frac{2^6a^{(4\cdot6)}}{7^6b^{(5\cdot6)}}$
    - $\frac{2^6a^{24}}{7^6b^{30}}$
13. $\frac{(y^3)^4}{(y^2)^3y^2}$

- $\frac{y^{3\cdot 4}}{y^{2\cdot 3+2}}$
- $\frac{y^{12}}{y^8}$
- $y^{(12-8)}$
- y^4

14. $\frac{(x^2)^3(x^3)^2}{(x^3)^4}$

    - $\frac{x^{(2\cdot 3)x^{(3\cdot 2)}}}{x^{(3\cdot 4)}}$
    - $\frac{x^6x^6}{x^{12}}$
    - $\frac{x^{(6+6)}}{x^{12}}$
    - $\frac{x^{12}}{x^{12}}$
    - $x^{(12-12)}$
    - x^0
    - 1