CHAPTER 0
REVIEW OF ALGEBRA
04. Operations with Algebraic Expressions

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1 Summary

Algebraic expressions with exactly 1 term are called **monomials**.

$$3x^2$$

Algebraic expressions with exactly 2 terms are called **binomials**.

$$3x^2 + 3x$$

Algebraic expressions with exactly 3 terms are called **trinomials**.

$$3x^2 + 3x + z$$

Algebraic expressions with more terms are called **polynomials**.

$$3x^2 + 3x + z + 6 + b^3$$

Special Products

1. $x(y + z) = xy + xz$

2. $(x + a)(x + b) = x^2 + x(a + b) + ab$

3. $(ax + c)(bx + d) = abx^2 + x(ad + bc) + cd$

4. $(x + a)^2 = x^2 + 2ax + a^2$

5. $(x - a)^2 = x^2 - 2ax + a^2$

6. $(x + a)(x - a) = x^2 - a^2$

7. $(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$

8. $(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$

2 Long Division

Divide $2x^3 - 14x - 5$ by $x - 3$

$$
\begin{array}{r|l}
& 2x^2 + 6x + 4 \ \leftarrow Quotient \\
\hline
Divisor \to (x-3) & 2x^3 + 0x^2 - 14x - 5 \\
& -(2x^3 - 6x^2) \\
\hline
& 6x^2 - 14x \\
& -(6x^2 - 18x) \\
\hline
& 4x - 5 \\
& -(4x - 12) \\
\hline
& 7 \ \leftarrow Remainder
\end{array}
$$

So the result of $2x^3 - 14x - 5$ by $x - 3$ is

$$2x^2 + 6x + 4 + \tfrac{7}{x-3}$$

- $\frac{Dividend}{Divisor} = Quotient + \frac{Remainder}{Divisor}$

A way of checking a division is to verify that

- $Dividend = \left(Quotient + \frac{Remainder}{Divisor}\right) Divisor$

- $Dividend = Quotient \cdot Divisor + \frac{Remainder}{Divisor} \cdot Divisor$

- $Dividend = Quotient \cdot Divisor + \frac{Remainder}{\cancel{Divisor}} \cdot \cancel{Divisor}$

- $Dividend = Quotient \cdot Divisor + Remainder$

By using this equation, you should be able to verify the result of the example.

3   Problems 0.4

Perform the indicated operations and simplify.

1. $(8x - 4y + 2) + (3x + 2y - 5)$

   - $8x - 4y + 2 + 3x + 2y - 5$
   - $11x - 2y - 3$

2. $(4a^2 - 2ab + 3) + (5c - 3ab + 7)$

   - $4a^2 - 2ab + 3 + 5c - 3ab + 7$
   - $4a^2 - 5b + 5c + 10$

3. $(8t^2 - 6s^2) + (4s^2 - 2t^2 + 6)$

   - $8t^2 - 2t^2 - 6s^2 + 4s^2 + 6$
   - $6t^2 - 2s^2 + 6$

4. $(\sqrt{x} + 2\sqrt{x}) + (3\sqrt{x} + 4\sqrt{x})$

- $\sqrt{x} + 2\sqrt{x} + 3\sqrt{x} + 4\sqrt{x}$
- $10\sqrt{x}$

5. $(\sqrt{a} + 2\sqrt{3b}) - (\sqrt{c} - 3\sqrt{3b})$

  - $(\sqrt{a} + 2\sqrt{3b}) - \sqrt{c} + 3\sqrt{3b}$
  - $\sqrt{a} + 2\sqrt{3b} - \sqrt{c} + 3\sqrt{3b}$
  - $\sqrt{a} - \sqrt{c} + 3\sqrt{3b} + 2\sqrt{3b}$
  - $\sqrt{a} - \sqrt{c} + 5\sqrt{3b}$

6. $(3a + 7b - 9) - (5a + 9b + 21)$

  - $(3a + 7b - 9) - 5a - 9b - 21$
  - $3a + 7b - 9 - 5a - 9b - 21$
  - $3a - 5a - 9b + 7b - 9 - 21$
  - $-2a - 2b - 30$

7. $(7x^2 + 5xy + \sqrt{2}) - (2z - 2xy + \sqrt{2})$

  - $(7x^2 + 5xy + \sqrt{2}) - 2z + 2xy - \sqrt{2}$
  - $7x^2 - 2z + 2xy + 5xy + \sqrt{2} - \sqrt{2}$
  - $7x^2 - 2z + 7xy$

8. $(\sqrt{x} + 2\sqrt{x}) - (\sqrt{x} + 3\sqrt{x})$

  - $(\sqrt{x} + 2\sqrt{x}) - \sqrt{x} - 3\sqrt{x}$
  - $-\sqrt{x}$

9. $(\sqrt[2]{2x} + \sqrt[3]{3y}) - (\sqrt[2]{2x} + \sqrt[4]{4z})$

  - $(\sqrt[2]{2x} + \sqrt[3]{3y}) - \sqrt[2]{2x} - \sqrt[4]{4z}$
  - $\sqrt[3]{3y} - \sqrt[4]{4z}$

10. $4(2z - w) - 3(w - 2z)$

  - $8z - 4w - 3w + 6z$
  - $8z + 6z - 4w - 3w$
  - $14z - 7w$

11. $3(3x + 3y - 7) - 3(8x - 2y + 2)$

  - $9x + 9y - 21 - 24x + 6y - 6$
  - $9x - 24x + 9y + 6y - 6 - 21$
  - $-15x + 15y - 27$

12. $(4s - 5t) + (-2s - 5t) + (s + 9)$

  - $4s - 2s + s - 5t - 5t + 9$
  - $3s - 10t + 9$

13. $5(x^2 - y^2) + x(y - 3x) + 4y(2x + 7y)$

- $5x^2 - 5y^2 + xy - 3x^2 + 8xy + 28y^2$

- $5x^2 - 3x^2 - 5y^2 + 28y^2 + xy + 8xy$

- $2x^2 + 23y^2 + 9xy$

14. $(7 + 3(x - 3) - (4 - 5x))$

   - $(7 + 3(x - 3) - 4 + 20x)$

   - $(7 + 3x - 9 - 4 + 20x)$

   - $20x + 3x + 7 - 9 - 4$

   - $23x - 6$

15. $2(3(3(x^2 + 2) - 2(x^2 - 5)))$

   - $2(3(3x^2 + 6 - 2x^2 + 10))$

   - $2(3(x^2 + 16))$

   - $2(3x^2 + 48)$

   - $6x^2 + 96$

16. $4(3(t + 5) - t(1 - (t + 1)))$

   - $4(3(t + 5) - t(1 - t - 1))$

   - $4(3(t + 5) - t + t^2 + t)$

   - $4(3(t + 5) + t^2)$

   - $4(3t + 15 + t^2)$

   - $4(t^2 + 3t + 15)$

   - $4t^2 + 4t + 60$

17. $-2(3u^2(2u + 2) - 2(u^2 - (5 - 2u)))$

18. $-(-3[2a + 2b - 2] + 5(2a + 3b) - a(2(b + 5)))$

19. $(2x + 5)(3x - 2)$

20. $(u + 2)(u + 5)$

21. $(w + 2)(w - 5)$

22. $(x - 4)(x + 7)$

23. $(2x + 3)(5x + 2)$

24. $(t^2 - 5t)(3t^2 - 7t)$

25. $(X + 2Y)^2$

26. $(2x - 1)^2$

27. $(7 - X)^2$

28. $(\sqrt{x} - 1)(2\sqrt{x} + 5)$

29. $(\sqrt{5x} - 2)^2$

30. $(\sqrt{y} - 3)(\sqrt{y} + 3)$

31. $(2s - 1)(2s + 1)$

32. $(a^2 + 2b)(a^2 - 2b)$

33. $(x^2 - 3)(x + 4)$

34. $(u - 1)(u^2 + 3u - 2)$

35. $(x^2 - 4)(3x^2 + 2x - 1)$

36. $(3y - 2)(4y^3 + 2y^2 + -3y)$

37. $t(3(t + 2)(t - 4) + 5(3t(t - 7)))$

38. $((2z + 1)(2z - 1))(4z^2 + 1)$

39. $(s - t + 4)(3s + 2t - 1)$

40. $(x^2 + x + 1)^2$

41. $(2a + 3)^3$

42. $(2a - 3)^3$

43. $(2x - 3)^3$

44. $(3a + b)^3$

45. $\frac{z^2 - 18z}{z}$

46. $\frac{2x^3 - 7x + 4}{x}$

47. $\frac{6u^5 + 9u^3 - 1}{3u^2}$

48. $\frac{(3y - 4) - (9y + 5)}{3y}$

49. $(x^2 + 7x - 5) \div (x + 5)$

50. $(x^2 - 5x + 4) \div (x - 4)$

51. $(3x^3 - 2x^2 + x - 3) \div (x + 2)$

52. $(x^4 + 3x^2 + 2) \div (x + 1)$

53. $x^3 \div (x + 2)$

54. $(8x^2 + 6x + 7) \div (2x + 1)$

55. $(3x^2 - 4x + 3) \div (3x + 2)$

56. $(z^3 + z^2 + z) \div (z^2 - z + 1)$