CHAPTER 0
REVIEW OF ALGEBRA
03. Properties of Exponents and Radicals

Exercised by: Rizal Bimanto

1 Summary

The product $x \cdot x \cdot x$ of 3 x's is abbreviated x^3. In general, for n a positive integer, x^n is the abbreviation product of nx's. The letter n in x^n is called the **the exponent**, and x is called the **base**. More specifically, if n is positive integers we have:

1. $x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}$
2. $\frac{1}{\underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}}$ for $x \neq 0$
3. $\frac{1}{x^{-n}} = x^n$ for $x \neq 0$
4. $x^0 = 1$

If $r^n = x$, where n is a positive integer, then r is an nth **root** of x. Second roots, the case $n = 2$, are called **squared roots**; and third roots, the case $n = 3$, are called **cube roots**.

Some numbers do not have an nth root that is a real number. For example, since the square of any real number is non-negative: there is no real number that is a square root of -4.

The principal of nth root of x is the nth root of x that is positive if x is positive and is negative if x is negative and n is odd. We denote the principal nth root of x by $\sqrt[n]{x}$:

$$\sqrt[n]{x} \text{ is } \begin{cases} \text{positive if } x \text{ is positive} \\ \text{negative if } x \text{ is negative and } n \text{ is odd} \end{cases}$$

For example, $\sqrt[2]{9} = 3$, $\sqrt[3]{-8} = -2$ and $\sqrt[3]{\frac{1}{27}} = \frac{1}{3}$ We define $\sqrt[n]{0} = 0$.
The symbol of $\sqrt[n]{x}$ is called radical.

Here are the basic laws of exponent and radicals:

Law	Example(s)
1. $x^m \cdot x^n = x^{m+n}$	$2^3 \cdot 2^5 = 2^8 = 256; \quad x^2 \cdot x^3 = x^5$
2. $x^0 = 1$	$2^0 = 1$
3. $x^{-n} = \frac{1}{x^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
4. $\frac{1}{x^{-n}} = x^n$	$\frac{1}{2^{-3}} = 2^3 = 8; \frac{1}{x^{-5}} = x^5$
5. $\frac{x^m}{x^n} = x^{m-n} = \frac{1}{x^{n-m}}$	$\frac{2^{12}}{2^8} = 2^4 = 16; \frac{x^8}{x^{12}} = x^{-4} = \frac{1}{x^4}$
6. $\frac{x^m}{x^m} = 1$	$\frac{2^4}{2^4} = 1$
7. $(x^m)^n = x^{mn}$	$\left(2^3\right)^5 = 2^{15}; \left(x^2\right)^3 = x^6$
8. $(xy^n) = x^n y^n$	$(2 \cdot 4)^3 = 2^3 \cdot 4^3 = 8 \cdot 64 = 512$
9. $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$
10. $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$	$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$
11. $x^{\frac{1}{n}} = \sqrt[n]{x^1}$	$3^{\frac{1}{5}} = \sqrt[5]{3^1}$
12. $x^{\frac{-1}{n}} = \frac{1}{x^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{x}}$	$4^{\frac{-1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt[2]{4}} = \frac{1}{2}$
13. $\sqrt[n]{x}\,\sqrt[n]{y} = \sqrt[n]{xy}$	$\sqrt[3]{9}\,\sqrt[3]{2} = \sqrt[3]{9 \cdot 2} = \sqrt[3]{18}$
14. $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$	$\frac{\sqrt[3]{90}}{\sqrt[3]{10}} = \sqrt[3]{\frac{90}{10}} = \sqrt[3]{9}$
15. $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$	$\sqrt[3]{\sqrt[4]{2}} = \sqrt[3 \cdot 4]{2} = \sqrt[12]{2}$
16. $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$
17. $\left(\sqrt[m]{x^m}\right) = x$	$\left(\sqrt[8]{7}\right)^8 = 7$

2 Problems 0.3

In Problems 1 - 14, simplify and express all answers in terms of positive exponent