CHAPTER 0
REVIEW OF ALGEBRA
02. Properties of Real Numbers

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A list of properties of the real numbers.

1. **The Transitive Property of Equality**

$$\text{If } a = b \text{ and } b = c, \text{ then } a = c$$

2. **The Closures Properties of Addition and Multiplication**

   For all real numbers a and b, there are unique real numbers $a + b$ and ab

3. **The Commutative Properties of Addition and Multiplication**

$$a + b = b + a \text{ and } ab = ba$$

4. **The Associative Properties of Addition and Multiplication**

$$a + (b + c) = (a + b) + c \text{ and } a(bc) = (ab)c$$

5. **The Identity Properties**

   There are unique real numbers denoted 0 and 1 such that, for each real number a,

$$0 + a = a \text{ and } 1a = a$$

6. **The Inverse Properties**

   For each real number a, there is unique real number denoted $-a$ such that

$$a + (-a) = 0$$

   The number $-a$ is called the **negative** of a.

   For each real number $a, except$ 0, there is a unique real number denoted a^{-1} such that

$$a \times a^{-1} = 1$$

   The number a^{-1} is called the **reciprocal** of a

7. **The Distributive Properties**

$$a(b + c) = ab + ac \text{ and } (b + c)a = ba + ca$$
$$0 \times a = 0 = a \times 0$$

1 Problems 0.2

In Problems 1 - 10, determine the truth of each statement

1. Every real number has a reciprocal.
   False. Except 0
2. The reciprocal of 6.6 is $0.1515\ldots$
   $\frac{1}{6.6} = 0.1515\ldots$. True
3. The negative of 7 is $\frac{-1}{7}$
   -(7) = -7. False. It should be -7
4. $1(x \times y) = (1 \times x)(1 \times y)$
   True. It can be simplified as xy
5. $-x + y = -y + x$
   False. $-x + y = y - x$
6. $(x + 2)(4) = 4x + 8$
   True.
7. $\frac{x+3}{5} = \frac{x}{5} + 3$
   False. $\frac{x+3}{5} = \frac{x}{5} + \frac{3}{5}$
8. $3\left(\frac{x}{4}\right) = \frac{3x}{4}$
   True.
9. $2(x \times y) = (2x) \times (2y)$
   False. $2(x \times y) = (2x) \times (2y) = 2xy$
10. $x(4y) = 4xy$
    True.

In Problems 11-20, state which properties of the real numbers are being used.

11. $2(x + y) = 2x + 2y$
    The Distributive Properties
12. $(x + 5.2) + 0.7y = x + (5.2 + 0.7y)$
    The Associative Property of Addition
13. $2(3y) = (2 \cdot 3)y$
    The Associative Property of Multiplication
14. $\frac{a}{b} = \frac{1}{b} \cdot a$
    The Inverse Property
15. $5(b - a) = (a - b)(-5)$
    The Commutative Property of Multiplication and Distributive
16. $y + (x + y) = (y + x) + y$
    The Commutative Property of Addition
17. $\frac{5x-y}{7} = 1/7(5x - y)$
    The Distributive Property
18. $5(4 + 7) = 5(7 + 4)$
    The Associative Property of Addition

19. $(2+a)b = 2b + ba$

    The Distributive Property

20. $(-1)(-3+4) = (-1)(-3) + (-1)(4)$

    The Distributive Property

In Problems 21-27, show that the statements are true by using properties of the real numbers

21. $2x(y-7) = 2xy - 14x$

    The Distributive Property

    - $2x(y-7)$
    - $2xy - 14x$

22. $\frac{x}{y}z = x\frac{z}{y}$

    The Commutative Property of Multiplication

    - $\frac{x}{y}z$
    - $\frac{xz}{y}$
    - $x\frac{z}{y}$

23. $(x+y)(2) = 2x + 2y$

    The Distributive Property

    - $(x+y)(2)$
    - $2x + 2y$

24. $a(b+(c+d)) = a((d+b)+c)$

    The Commutative Property of Addition and Associative

    - $a(b+c+d)$
    - $a(d+b+c)$
    - $a((d+b)+c)$

25. $x((2y+1)+3) = 2xy + 4x$

    The Commutative Property of Addition and Distributive

    - $x(2y+1+3)$
    - $x(2y+4)$
    - $2xy + 4x$

26. $(1+a)(b+c) = b + c + ab + ac$

    The Distributive Property

    - $1b + 1c + ab + ac$
    - $b + c + ab + ac$

27. Show that $(x-y+z)w = xw - yw + zw$.
    [*Hint:* $b + c + d = (b+c) + d$]

    The Distributive Property

    - $xw - yw + zw$