CHAPTER 0
REVIEW OF ALGEBRA
02. Properties of Real Numbers

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A list of properties of the real numbers.

1. **The Transitive Property of Equality**

$$\text{If } a = b \text{ and } b = c, \text{ then } a = c$$

2. **The Closures Properties of Addition and Multiplication**

   For all real numbers a and b, there are unique real numbers $a + b$ and ab

3. **The Commutative Properties of Addition and Multiplication**

$$a + b = b + a \text{ and } ab = ba$$

4. **The Associative Properties of Addition and Multiplication**

$$a + (b + c) = (a + b) + c \text{ and } a(bc) = (ab)c$$

5. **The Identity Properties**

   There are unique real numbers denoted 0 and 1 such that, for each real number a,

$$0 + a = a \text{ and } 1a = a$$

6. **The Inverse Properties**

   For each real number a, there is unique real number denoted $-a$ such that

$$a + (-a) = 0$$

   The number $-a$ is called the **negative** of a.

   For each real number $a, except$ 0, there is a unique real number denoted a^{-1} such that

$$a \times a^{-1} = 1$$

   The number a^{-1} is called the **reciprocal** of a

7. **The Distributive Properties**

$$a(b + c) = ab + ac \text{ and } (b + c)a = ba + ca$$
$$0 \times a = 0 = a \times 0$$

1 Problems 0.2

In Problems 1 - 10, determine the truth of each statement

1. Every real number has a reciprocal.
   False. Except 0
2. The reciprocal of 6.6 is $0.1515\ldots$
   $\frac{1}{6.6} = 0.1515\ldots$. True
3. The negative of 7 is $\frac{-1}{7}$
   -(7) = -7. False. It should be -7
4. $1(x \times y) = (1 \times x)(1 \times y)$
   True. It can be simplified as xy
5. $-x + y = -y + x$
   False. $-x + y = y - x$
6. $(x + 2)(4) = 4x + 8$
   True.
7. $\frac{x+3}{5} = \frac{x}{5} + 3$
   False. $\frac{x+3}{5} = \frac{x}{5} + \frac{3}{5}$
8. $3\left(\frac{x}{4}\right) = \frac{3x}{4}$
   True.
9. $2(x \times y) = (2x) \times (2y)$
   False. $2(x \times y) = (2x) \times (2y) = 2xy$
10. $x(4y) = 4xy$
    True.

In Problems 11-20, state which properties of the real numbers are being used.

11. $2(x + y) = 2x + 2y$
    The Distributive Properties
12. $(x + 5.2) + 0.7y = x + (5.2 + 0.7y)$
    The Associative Property of Addition
13. $2(3y) = (2 \cdot 3)y$
    The Associative Property of Multiplication
14. $\frac{a}{b} = \frac{1}{b} \cdot a$
    The Inverse Property
15. $5(b - a) = (a - b)(-5)$
    The Commutative Property of Multiplication and Distributive
16. $y + (x + y) = (y + x) + y$
    The Commutative Property of Addition
17. $\frac{5x-y}{7} = 1/7(5x - y)$
    The Distributive Property
18. $5(4 + 7) = 5(7 + 4)$
    The Associative Property of Addition

19. $(2+a)b = 2b + ba$

    The Distributive Property

20. $(-1)(-3+4) = (-1)(-3) + (-1)(4)$

    The Distributive Property

In Problems 21-27, show that the statements are true by using properties of the real numbers

21. $2x(y-7) = 2xy - 14x$

    The Distributive Property

    - $2x(y-7)$
    - $2xy - 14x$

22. $\frac{x}{y}z = x\frac{z}{y}$

    The Commutative Property of Multiplication

    - $\frac{x}{y}z$
    - $\frac{xz}{y}$
    - $x\frac{z}{y}$

23. $(x+y)(2) = 2x + 2y$

    The Distributive Property

    - $(x+y)(2)$
    - $2x + 2y$

24. $a(b+(c+d)) = a((d+b)+c)$

    The Commutative Property of Addition and Associative

    - $a(b+c+d)$
    - $a(d+b+c)$
    - $a((d+b)+c)$

25. $x((2y+1)+3) = 2xy + 4x$

    The Commutative Property of Addition and Distributive

    - $x(2y+1+3)$
    - $x(2y+4)$
    - $2xy + 4x$

26. $(1+a)(b+c) = b + c + ab + ac$

    The Distributive Property

    - $1b + 1c + ab + ac$
    - $b + c + ab + ac$

27. Show that $(x-y+z)w = xw - yw + zw$.
    [*Hint:* $b + c + d = (b+c) + d$]

    The Distributive Property

    - $xw - yw + zw$

Simplify the following if possible

28. $-2 + (-4)$

$$-2 - 4,$$
$$-6$$

29. $-a + b$

30. $6 + (-4)$

$$6 - 4,$$
$$2$$

31. $7 - 2$

$$5$$

32. $\frac{3}{2^{-1}}$

$$\frac{3}{\frac{1}{2}}$$
$$\frac{3 \cdot 2}{\frac{1}{\cancel{2}} \cdot \cancel{2}}$$
$$\frac{}{\cancel{1}}$$
$$6$$

33. $-5 - (-13)$

$$-5 + 13$$
$$8$$

34. $-(-a) + (-b)$

$$a - b$$

35. $(-2)(9)$

$$-18$$

36. $(7)(-9)$

$$-63$$

37. $(-1.6)(-0.5)$

$$0.8$$

38. $19(-1)$

$$19 \cdot -1$$

$$-19$$

39. $\frac{-1}{\frac{-1}{a}}$

$$\frac{\frac{-1 \cdot a}{-1 \cdot \cancel{a}}}{\cancel{a}}$$

$$\frac{\cancel{-1}a}{\cancel{-1}}$$

$$a$$

40. $-(-6 + x)$

$$6 - x$$

41. $-7(x)$

$$-7x$$

42. $-3(a - b)$

$$-3 \cdot a - -3 \cdot b$$

$$-3a - -3b$$

$$-3a + 3b$$

43. $-(-6 + (-y))$

$$-1 \cdot -6 + -1 \cdot -y$$

$$6 + y$$

44. $-3 \div 3a$

$$\frac{-3}{3a}$$

$$\frac{-\cancel{3}^{1}}{\cancel{3}^{1}a}$$

$$\frac{-1}{1a}$$

$$-a$$

45. $-9 \div (-27)$

$$\frac{-9}{-27}$$

$$\frac{\cancel{-9}1}{\cancel{-27}3}$$

$$\frac{1}{3}$$

46. $(-a) \div (-b)$

$$\frac{\cancel{-a}}{\cancel{-b}}$$

$$\frac{a}{b}$$

47. $3 + (3^{-1}9)$

$$3 + \frac{\cancel{9}^{3}}{\cancel{3}1}$$

$$3 + 3$$

$$6$$

48. $3(-2(3) + 6(2))$

$$3(-6 + 12)$$

$$3(6)$$

$$18$$

49. $(-a)(-b)(-1)$

$$(-a \cdot -b)(-1)$$

$$(ab)(-1)$$

$$-ab$$

50. $(-12)(-12)$

$$144$$

51. $X(1)$

$$X$$

52. $-71(x - 2)$

$$-71 \cdot x - -71 \cdot 2$$

$$-71x - -142$$

$$-71x + 142$$

$$71$$

53. $4(5 + x)$

$$4 \cdot 5 + 4 \cdot x$$

$$20 + 4x$$

54. $-(x - y)$

$$-1 \cdot x - -1 \cdot y$$

$$-1x - -1y$$

$$-x + y$$

55. $0(-x)$

$$0$$

56. $8\left(\frac{1}{11}\right)$

$$\frac{1 \cdot 8}{11}$$

$$\frac{8}{11}$$

57. $\frac{X}{1}$

$$X$$

58. $\frac{14x}{21y}$

$$\frac{\cancel{14}2x}{\cancel{21}3y}$$

$$\frac{2x}{3y}$$

59. $\frac{2x}{-2}$

$$\frac{\cancel{2}1x}{-\cancel{2}\cancel{1}}$$

$$-1$$

60. $\frac{2}{3} \cdot \frac{1}{x}$

$$\frac{2 \cdot 1}{3 \cdot x}$$

$\frac{2}{3x}$

61. $\frac{a}{c}(3b)$

$\frac{3ab}{c}$

62. $5a + (7 - 5a)$

7

63. $\frac{-aby}{-ax}$

$\frac{\cancel{-a}by}{\cancel{-a}x}$

$\frac{by}{x}$

64. $\frac{a}{b} \cdot \frac{1}{c}$

$\frac{a}{bc}$

65. $\frac{2}{x} \cdot \frac{5}{y}$

$\frac{10}{xy}$

66. $\frac{1}{2} + \frac{1}{3}$

$\frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2}$

$\frac{3}{6} + \frac{2}{6}$

$\frac{5}{6}$

67. $\frac{x}{3a} + \frac{y}{a}$

$\frac{x}{3a} + \frac{y \cdot 3}{a \cdot 3}$

$\frac{x}{3a} + \frac{3y}{3a}$

$\frac{x+3y}{3a}$

68. $\frac{3}{10} - \frac{7}{15}$

$\frac{3 \cdot 3}{10 \cdot 3} - \frac{7 \cdot 2}{15 \cdot 2}$

$\frac{9}{30} - \frac{14}{30}$

$\frac{-5}{30}$

69. $\frac{a}{b} + \frac{c}{b}$

$$\frac{a+c}{b}$$

70. $\frac{X}{\sqrt{5}} - \frac{Y}{\sqrt{5}}$

$$\frac{X-Y}{\sqrt{5}}$$

71. $\frac{3}{2} - \frac{1}{4} + \frac{1}{6}$

$$\frac{3\cdot 6}{2\cdot 6} - \frac{1\cdot 3}{4\cdot 3} + \frac{1\cdot 2}{6\cdot 2}$$
$$\frac{18}{12} - \frac{3}{12} + \frac{2}{12}$$
$$\frac{17}{12}$$

72. $\frac{3}{7} - \frac{5}{9}$

$$\frac{3\cdot 9}{7\cdot 9} - \frac{5\cdot 7}{9\cdot 7}$$
$$\frac{27}{63} - \frac{35}{63}$$
$$\frac{-8}{63}$$

73. $\frac{6}{\frac{x}{y}}$

$$\frac{6\cdot y}{\frac{x}{y}\cdot y}$$
$$\frac{6y}{\frac{x}{\cancel{y}}\cdot \cancel{y}}$$
$$\frac{6y}{x}$$

74. $\frac{\frac{l}{w}}{m}$

$$\frac{\frac{l}{w}\cdot m}{m}\cdot m$$
$$\frac{\frac{l}{w}\cdot m}{\cancel{m}}\cdot \cancel{m}$$
$$\frac{lm}{w}$$

75. $\frac{\frac{-x}{y^2}}{\frac{z}{xy}}$

$$\frac{\frac{-x}{y^2}\cdot\frac{xy}{z}}{\frac{z}{xy}\cdot\frac{xy}{z}}$$
$$\frac{\frac{-x}{y^2}\cdot\frac{xy}{z}}{\frac{\cancel{z}}{\cancel{xy}}\cdot\frac{\cancel{xy}}{\cancel{z}}}$$

$$\frac{-x^2 \cancel{y}}{\cancel{y^2} \cancel{y} \cdot z}$$

$$\frac{-x^2}{y \cdot z}$$

$$-\frac{x^2}{yz}$$

76. $\frac{7}{0}$

undefined

77. $\frac{0}{X}$, for $X \neq 0$

0

78. $\frac{0}{0}$

undefined