CHAPTER 0
REVIEW OF ALGEBRA
03. Properties of Exponents and Radicals

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1 Summary

The product $x \cdot x \cdot x$ of 3 x's is abbreviated x^3. In general, for n a positive integer, x^n is the abbreviation product of nx's. The letter n in x^n is called the **the exponent**, and x is called the **base**. More specifically, if n is positive integers we have:

1. $x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}$
2. $\frac{1}{\underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}}$ for $x \neq 0$
3. $\frac{1}{x^{-n}} = x^n$ for $x \neq 0$
4. $x^0 = 1$

If $r^n = x$, where n is a positive integer, then r is an nth **root** of x. Second roots, the case $n = 2$, are called **squared roots**; and third roots, the case $n = 3$, are called **cube roots**.

Some numbers do not have an nth root that is a real number. For example, since the square of any real number is non-negative: there is no real number that is a square root of -4.

The principal of nth root of x is the nth root of x that is positive if x is positive and is negative if x is negative and n is odd. We denote the principal nth root of x by $\sqrt[n]{x}$:

$$\sqrt[n]{x} \text{ is } \begin{cases} \text{positive if } x \text{ is positive} \\ \text{negative if } x \text{ is negative and } n \text{ is odd} \end{cases}$$

For example, $\sqrt[2]{9} = 3$, $\sqrt[3]{-8} = -2$ and $\sqrt[3]{\frac{1}{27}} = \frac{1}{3}$ We define $\sqrt[n]{0} = 0$.
The symbol of $\sqrt[n]{x}$ is called radical.

Here are the basic laws of exponent and radicals:

Law	**Example(s)**
1. $x^m \cdot x^n = x^{m+n}$	$2^3 \cdot 2^5 = 2^8 = 256; \quad x^2 \cdot x^3 = x^5$
2. $x^0 = 1$	$2^0 = 1$
3. $x^{-n} = \frac{1}{x^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
4. $\frac{1}{x^{-n}} = x^n$	$\frac{1}{2^{-3}} = 2^3 = 8; \frac{1}{x^{-5}} = x^5$
5. $\frac{x^m}{x^n} = x^{m-n} = \frac{1}{x^{n-m}}$	$\frac{2^{12}}{2^8} = 2^4 = 16; \frac{x^8}{x^{12}} = x^{-4} = \frac{1}{x^4}$
6. $\frac{x^m}{x^m} = 1$	$\frac{2^4}{2^4} = 1$
7. $(x^m)^n = x^{mn}$	$\left(2^3\right)^5 = 2^{15}; \left(x^2\right)^3 = x^6$
8. $(xy^n) = x^n y^n$	$(2 \cdot 4)^3 = 2^3 \cdot 4^3 = 8 \cdot 64 = 512$
9. $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$
10. $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$	$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$
11. $x^{\frac{1}{n}} = \sqrt[n]{x^1}$	$3^{\frac{1}{5}} = \sqrt[5]{3^1}$
12. $x^{\frac{-1}{n}} = \frac{1}{x^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{x}}$	$4^{\frac{-1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt[2]{4}} = \frac{1}{2}$
13. $\sqrt[n]{x}\sqrt[n]{y} = \sqrt[n]{xy}$	$\sqrt[3]{9}\sqrt[3]{2} = \sqrt[3]{9 \cdot 2} = \sqrt[3]{18}$
14. $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$	$\frac{\sqrt[3]{90}}{\sqrt[3]{10}} = \sqrt[3]{\frac{90}{10}} = \sqrt[3]{9}$
15. $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$	$\sqrt[3]{\sqrt[4]{2}} = \sqrt[3 \cdot 4]{2} = \sqrt[12]{2}$
16. $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$
17. $\left(\sqrt[m]{x^m}\right) = x$	$\left(\sqrt[8]{7}\right)^8 = 7$

2 Problems 0.3

In Problems 1 - 14, simplify and express all answers in terms of positive exponent

1. $\left(2^3\right)\left(2^2\right)$
   - $2^{(3+2)}$
   - 2^5
   - 32
2. $x^6 x^9$
   - $x^{(6+9)}$
   - $x^{(15)}$
3. $17^5 \cdot 17^2$
   - $17^{(5+2)}$

- $17^{(7)}$
- $410,338,673$

4. z^3zz^2
   - $z^{(3+1+2)}$
   - z^6
5. $\frac{x^3x^5}{y^9y^5}$
   - $\frac{x^{(3+5)}}{y^{(9+5)}}$
   - $\frac{x^8}{y^{14}}$
6. $\left(x^{12}\right)^4$
   - $x^{(12\cdot4)}$
   - x^{48}
7. $\frac{\left(a^3\right)^7}{\left(b^4\right)^5}$
   - $\frac{a^{3\cdot7}}{b^{4\cdot5}}$
   - $\frac{a^{21}}{b^{20}}$
8. $\left(\frac{13^{14}}{13}\right)^2$
   - $\frac{13^{(14\cdot2)}}{13^2}$
   - $\frac{13^{28}}{13^2}$
   - $13^{(28-2)}$
   - 13^{26}
9. $\left(2x^2y^3\right)^3$
   - $2^3x^{(2\cdot3)}y^{(3\cdot3)}$
   - $8x^6y^9$
10. $\left(\frac{w^2s^3}{y^2}\right)^2$
    - $\frac{w^{(2\cdot2)}s^{(3\cdot2)}}{y^{(2\cdot2)}}$
    - $\frac{w^4s^6}{y^4}$
11. $\left(\frac{x^9}{x^5}\right)$
    - $x^{(9-5)}$
    - x^4
12. $\left(\frac{2a^4}{7b^5}\right)^6$
    - $\frac{2^6a^{(4\cdot6)}}{7^6b^{(5\cdot6)}}$
    - $\frac{2^6a^{24}}{7^6b^{30}}$
13. $\frac{(y^3)^4}{(y^2)^3y^2}$

- $\frac{y^{3\cdot 4}}{y^{2\cdot 3+2}}$
- $\frac{y^{12}}{y^8}$
- $y^{(12-8)}$
- y^4

14. $\frac{(x^2)^3(x^3)^2}{(x^3)^4}$
    - $\frac{x^{(2\cdot 3)x^{(3\cdot 2)}}}{x^{(3\cdot 4)}}$
    - $\frac{x^6x^6}{x^{12}}$
    - $\frac{x^{(6+6)}}{x^{12}}$
    - $\frac{x^{12}}{x^{12}}$
    - $x^{(12-12)}$
    - x^0
    - 1

In Problems 15 - 28, evaluate the expressions.

15. $\sqrt{25}$
    - $\sqrt[\not{2}]{5^{\not{2}}}$
    - 5
16. $\sqrt[4]{81}$
    - $\sqrt[\not{4}]{3^{\not{4}}}$
    - 3
17. $\sqrt[7]{-128}$
    - $\sqrt[7]{-2^7}$
    - $-2^{\not{7}\cdot\frac{1}{\not{7}}}$
    - -2
18. $\sqrt[5]{0.00243}$
    - $\sqrt[5]{81\cdot 3\cdot 10^{-5}}$
    - $\sqrt[5]{3^4\cdot 3\cdot 10^{-5}}$
    - $\sqrt[5]{3^5\cdot 10^{-5}}$
    - $3^{\not{5}\cdot\frac{1}{\not{5}}}10^{-\not{5}\cdot\frac{1}{\not{5}}}$
    - $3\cdot 10^{-1}$
    - 0.3
19. $\sqrt[4]{\frac{1}{16}}$
    - $\sqrt[4]{\frac{1}{2}^4}$
    - $\frac{1}{2}^{\not{4}\cdot\frac{1}{\not{4}}}$
    - $\frac{1}{2}$

20. $\sqrt[3]{-\frac{8}{27}}$

- $\sqrt[3]{-\frac{2^3}{3^3}}$
- $-\frac{2^{3\cdot\frac{1}{3}}}{3^{3\cdot\frac{1}{3}}}$
- $-\frac{2^{\cancel{3}\cdot\frac{1}{\cancel{3}}}}{3^{\cancel{3}\cdot\frac{1}{\cancel{3}}}}$
- $-\frac{2}{3}$

21. $(49)^{\frac{1}{2}}$

- $(7^2)^{\frac{1}{2}}$
- $(7^{\cancel{2}})^{\frac{1}{\cancel{2}}}$
- 7

22. $(64)^{\frac{1}{3}}$

- $(4^3)^{\frac{1}{3}}$
- $(4^{\cancel{3}})^{\frac{1}{\cancel{3}}}$
- 4

23. $81^{\frac{3}{4}}$

- $\sqrt[4]{81^3}$
- $\sqrt[4]{(3^4)^3}$
- $\sqrt[4]{3^{12}}$
- $3^{(12\cdot\frac{1}{4})}$
- 3^3
- 27

24. $(9)^{\frac{-5}{2}}$

- $(3^2)^{\frac{-5}{2}}$
- $(3^{\cancel{2}})^{\frac{-5}{\cancel{2}}}$
- 3^{-5}
- $\frac{1}{3^5}$
- $\frac{1}{243}$

25. $(32)^{\frac{-2}{5}}$

- $(2^5)^{\frac{-2}{5}}$
- $(2^{\cancel{5}})^{\frac{-2}{\cancel{5}}}$
- 2^{-2}
- $\frac{1}{2^2}$
- $\frac{1}{4}$

26. $(0.09)^{\frac{-1}{2}}$

- $(3^2 \cdot 10^{-2})^{\frac{-1}{2}}$
- $3^{(2\cdot\frac{-1}{2})} \cdot 10^{(-2\cdot\frac{-1}{2})}$
- $3^{\frac{-2}{2}} \cdot 10^{\frac{2}{2}}$
- $3^{-1} \cdot 10$
- $\frac{1}{3} \cdot 10$
- $\frac{10}{3}$
- $3.333\ldots$

27. $\left(\frac{1}{32}\right)^{\frac{4}{5}}$

- $\left(\frac{1}{2^5}\right)^{\frac{4}{5}}$
- $\left(2^{-5}\right)^{\frac{4}{5}}$
- $2^{-5\cdot\frac{4}{5}}$
- $2^{\frac{-20}{5}}$
- 2^{-4}
- $\frac{1}{2^4}$
- $\frac{1}{2^4}$
- $\frac{1}{16}$

28. $\left(-\frac{243}{1024}\right)^{\frac{2}{5}}$

- $\left(-\frac{3^5}{2^{10}}\right)^{\frac{2}{5}}$
- $-\frac{3^{5\cdot\frac{2}{5}}}{2^{10\cdot\frac{2}{5}}}$
- $-\frac{3^{\frac{10}{5}}}{2^{\frac{20}{5}}}$
- $-\frac{3^2}{2^4}$
- $-\frac{9}{16}$

In Problems 29 - 40, simplify the expressions.

29. $\sqrt{50}$

- $\sqrt{25 \cdot 2}$
- $5\sqrt{2}$

30. $\sqrt[3]{54}$

- $\sqrt[3]{27 \cdot 2}$
- $3\sqrt[3]{2}$

31. $\sqrt[3]{2x^3}$

- $\mathrm{x}\sqrt[3]{2}$

32. $\sqrt{4x}$

- $2\sqrt{x}$

33. $\sqrt{49u^8}$

- $(49u^8)^{\frac{1}{2}}$
- $(7^2u^8)^{\frac{1}{2}}$
- $7^{2\cdot\frac{1}{2}}u^{8\cdot\frac{1}{2}}$
- $7^{\cancel{2}\cdot\frac{1}{\cancel{2}}}u^{\cancel{8}4\cdot\frac{1}{\cancel{2}}}$
- $7u^4$

34. $\sqrt[4]{\frac{x}{16}}$

- $\frac{\sqrt[4]{x}}{\sqrt[4]{2^4}}$
- $\frac{\sqrt[4]{x}}{(2^4)^{\frac{1}{4}}}$
- $\frac{\sqrt[4]{x}}{2^{4\cdot\frac{1}{4}}}$
- $\frac{\sqrt[4]{x}}{2^{\cancel{4}\cdot\frac{1}{\cancel{4}}}}$
- $\frac{\sqrt[4]{x}}{2}$
- $\frac{1}{2}\sqrt[4]{x}$

35. $2\sqrt{8}-5\sqrt{27}+\sqrt[3]{128}$

- $2\sqrt{4\cdot 2}-5\sqrt{9\cdot 3}+\sqrt[3]{64\cdot 2}$
- $2\cdot 2\sqrt{2}-5\cdot 3\sqrt{3}+4\sqrt[3]{2}$
- $4\sqrt{2}-15\sqrt{3}+4\sqrt[3]{2}$

36. $\sqrt{\frac{3}{13}}$

- $\frac{\sqrt{3}}{\sqrt{13}}$

37. $(9z^4)^{\frac{1}{2}}$

- $9^{\frac{1}{2}}z^{4\cdot\frac{1}{2}}$
- $\sqrt{9}\cdot z^2$
- $3z^2$

38. $(729x^6)^{\frac{3}{2}}$

- $(81\cdot 9x^6)^{\frac{3}{2}}$
- $81^{\frac{3}{2}}9^{\frac{3}{2}}(x^6)^{\frac{3}{2}}$
- $\sqrt{81^3}\sqrt{9^3}\sqrt{(x^6)^3}$
- $\sqrt{(9\cdot 9)(9\cdot 9)(9\cdot 9)}\ \sqrt{(9\cdot 9\cdot 9)}\ \sqrt{x^{(6\cdot 3)}}$
- $(3\cdot 3)(3\cdot 3)(3\cdot 3)(3\cdot 3\cdot 3)\ \sqrt{x^{18}}$
- $9\cdot 9\cdot 9\cdot 9\cdot 3\ (x^{18})^{\frac{1}{2}}$
- $19,683\ x^{18\cdot\frac{1}{2}}$
- $19,683x^9$

39. $\left(\frac{27t^3}{8}\right)^{\frac{2}{3}}$

- $\frac{27^{\frac{2}{3}}(t^3)^{\frac{2}{3}}}{8^{\frac{2}{3}}}$
- $\frac{\sqrt[3]{27^2}\ t^{3\cdot\frac{2}{3}}}{\sqrt[3]{8^2}}$
- $\frac{\sqrt[3]{(3^3)^2}\ t^{\cancel{3}\cdot\frac{2}{\cancel{3}}}}{\sqrt[3]{(2^3)^2}}$
- $\frac{((3^3)^2)^{\frac{1}{3}}\ t^2}{((2^3)^2)^{\frac{1}{3}}}$
- $\frac{3^{3\cdot 2\cdot\frac{1}{3}}\ t^2}{2^{3\cdot 2\cdot\frac{1}{3}}}$
- $\frac{3^{\cancel{3}\cdot 2\cdot\frac{1}{\cancel{3}}}\ t^2}{2^{\cancel{3}\cdot 2\cdot\frac{1}{\cancel{3}}}}$
- $\frac{3^2\ t^2}{2^2}$
- $\frac{9t^2}{4}$

40. $\left(\frac{256}{x^{12}}\right)^{\frac{-3}{4}}$

- $\sqrt[4]{\left(\frac{256}{x^{12}}\right)^{-3}}$
- $\sqrt[4]{\left(\frac{1}{\frac{256}{x^{12}}}\right)^{3}}$
- $\sqrt[4]{\left(\frac{1\cdot x^{12}}{\frac{256}{x^{12}}\cdot x^{12}}\right)^{3}}$
- $\sqrt[4]{\left(\frac{x^{12}}{\frac{256}{\cancel{x^{12}}}\cdot\cancel{x^{12}}}\right)^{3}}$
- $\sqrt[4]{\left(\frac{x^{12}}{256}\right)^{3}}$
- $\left(\frac{x^{12}}{256}\right)^{\frac{3}{4}}$
- $\frac{x^{12\cdot\frac{3}{4}}}{256^{\frac{3}{4}}}$
- $\frac{x^{\cancel{12}^{3}\cdot\frac{3}{\cancel{4}}}}{(2^8)^{\frac{3}{4}}}$
- $\frac{x^{3\cdot 3}}{2^{8\cdot\frac{3}{4}}}$
- $\frac{x^9}{2^{\cancel{8}^{2}\cdot\frac{3}{\cancel{4}}}}$
- $\frac{x^9}{2^{2\cdot 3}}$
- $\frac{x^9}{2^6}$
- $\frac{x^9}{64}$