

# CHAPTER 0

## REVIEW OF ALGEBRA

### 04. Operations with Algebraic Expressions

Exercised by: Rizal Bimanto

#### 1 Summary

Algebraic expressions with exactly 1 term are called **monomials**.

$$3x^2$$

Algebraic expressions with exactly 2 terms are called **binomials**.

$$3x^2 + 3x$$

Algebraic expressions with exactly 3 terms are called **trinomials**.

$$3x^2 + 3x + z$$

Algebraic expressions with more terms are called **polynomials**.

$$3x^2 + 3x + z + 6 + b^3$$

#### Special Products

1.  $x(y + z) = xy + xz$
2.  $(x + a)(x + b) = x^2 + x(a + b) + ab$
3.  $(ax + c)(bx + d) = abx^2 + x(ad + bc) + cd$
4.  $(x + a)^2 = x^2 + 2ax + a^2$
5.  $(x - a)^2 = x^2 - 2ax + a^2$
6.  $(x + a)(x - a) = x^2 - a^2$
7.  $(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$
8.  $(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$

#### 2 Long Division

Divide  $2x^3 - 14x - 5$  by  $x - 3$

	$2x^2 + 6x + 4 \leftarrow \text{Quotient}$
$\text{Divisor} \rightarrow (x - 3)$	$2x^3 + 0x^2 - 14x - 5$
	$-(2x^3 - 6x^2)$
	$6x^2 - 14x$
	$-(6x^2 - 18x)$
	$4x - 5$
	$-(4x - 12)$
	$7 \leftarrow \text{Remainder}$

So the result of  $2x^3 - 14x - 5$  by  $x - 3$  is

$$2x^2 + 6x + 4 + \frac{7}{x-3}$$

- $\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$

A way of checking a division is to verify that

- $\text{Dividend} = \left(\text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}\right) \text{Divisor}$
- $\text{Dividend} = \text{Quotient} \cdot \text{Divisor} + \frac{\text{Remainder}}{\text{Divisor}} \cdot \text{Divisor}$
- $\text{Dividend} = \text{Quotient} \cdot \text{Divisor} + \frac{\text{Remainder}}{\text{Divisor}} \cdot \text{Divisor}$
- $\text{Dividend} = \text{Quotient} \cdot \text{Divisor} + \text{Remainder}$

By using this equation, you should be able to verify the result of the example.

### 3 Problems 0.4

Perform the indicated operations and simplify.

1.  $(8x - 4y + 2) + (3x + 2y - 5)$ 
  - $8x - 4y + 2 + 3x + 2y - 5$
  - $11x - 2y - 3$
2.  $(4a^2 - 2ab + 3) + (5c - 3ab + 7)$ 
  - $4a^2 - 2ab + 3 + 5c - 3ab + 7$
  - $4a^2 - 5b + 5c + 10$
3.  $(8t^2 - 6s^2) + (4s^2 - 2t^2 + 6)$ 
  - $8t^2 - 2t^2 - 6s^2 + 4s^2 + 6$
  - $6t^2 - 2s^2 + 6$
4.  $(\sqrt{x} + 2\sqrt{x}) + (3\sqrt{x} + 4\sqrt{x})$

- $\sqrt{x} + 2\sqrt{x} + 3\sqrt{x} + 4\sqrt{x}$
  - $10\sqrt{x}$
5.  $(\sqrt{a} + 2\sqrt{3b}) - (\sqrt{c} - 3\sqrt{3b})$
- $(\sqrt{a} + 2\sqrt{3b}) - \sqrt{c} + 3\sqrt{3b}$
  - $\sqrt{a} + 2\sqrt{3b} - \sqrt{c} + 3\sqrt{3b}$
  - $\sqrt{a} - \sqrt{c} + 3\sqrt{3b} + 2\sqrt{3b}$
  - $\sqrt{a} - \sqrt{c} + 5\sqrt{3b}$
6.  $(3a + 7b - 9) - (5a + 9b + 21)$
- $(3a + 7b - 9) - 5a - 9b - 21$
  - $3a + 7b - 9 - 5a - 9b - 21$
  - $3a - 5a - 9b + 7b - 9 - 21$
  - $-2a - 2b - 30$
7.  $(7x^2 + 5xy + \sqrt{2}) - (2z - 2xy + \sqrt{2})$
- $(7x^2 + 5xy + \sqrt{2}) - 2z + 2xy - \sqrt{2}$
  - $7x^2 - 2z + 2xy + 5xy + \sqrt{2} - \sqrt{2}$
  - $7x^2 - 2z + 7xy$
8.  $(\sqrt{x} + 2\sqrt{x}) - (\sqrt{x} + 3\sqrt{x})$
- $(\sqrt{x} + 2\sqrt{x}) - \sqrt{x} - 3\sqrt{x}$
  - $-\sqrt{x}$
9.  $(\sqrt[2]{2x} + \sqrt[3]{3y}) - (\sqrt[2]{2x} + \sqrt[4]{4z})$
- $(\sqrt[2]{2x} + \sqrt[3]{3y}) - \sqrt[2]{2x} - \sqrt[4]{4z}$
  - $\sqrt[3]{3y} - \sqrt[4]{4z}$
10.  $4(2z - w) - 3(w - 2z)$
- $8z - 4w - 3w + 6z$
  - $8z + 6z - 4w - 3w$
  - $14z - 7w$
11.  $3(3x + 3y - 7) - 3(8x - 2y + 2)$
- $9x + 9y - 21 - 24x + 6y - 6$
  - $9x - 24x + 9y + 6y - 6 - 21$
  - $-15x + 15y - 27$
12.  $(4s - 5t) + (-2s - 5t) + (s + 9)$
- $4s - 2s + s - 5t - 5t + 9$
  - $3s - 10t + 9$
13.  $5(x^2 - y^2) + x(y - 3x) + 4y(2x + 7y)$

- $5x^2 - 5y^2 + xy - 3x^2 + 8xy + 28y^2$
  - $5x^2 - 3x^2 - 5y^2 + 28y^2 + xy + 8xy$
  - $2x^2 + 23y^2 + 9xy$
14.  $(7 + 3(x - 3) - (4 - 5x))$
- $(7 + 3(x - 3) - 4 + 20x)$
  - $(7 + 3x - 9 - 4 + 20x)$
  - $20x + 3x + 7 - 9 - 4$
  - $23x - 6$
15.  $2(3(3(x^2 + 2) - 2(x^2 - 5)))$
- $2(3(3x^2 + 6 - 2x^2 + 10))$
  - $2(3(x^2 + 16))$
  - $2(3x^2 + 48)$
  - $6x^2 + 96$
16.  $4(3(t + 5) - t(1 - (t + 1)))$
- $4(3(t + 5) - t(1 - t - 1))$
  - $4(3(t + 5) - t + t^2 + t)$
  - $4(3(t + 5) + t^2)$
  - $4(3t + 15 + t^2)$
  - $4(t^2 + 3t + 15)$
  - $4t^2 + 4t + 60$
17.  $-2(3u^2(2u + 2) - 2(u^2 - (5 - 2u)))$
- $-2((6u^3 + 6u^2) - 2(u^2 - 5 + 2u))$
  - $-2((6u^3 + 6u^2) - 2u^2 + 10 + 4u)$
  - $-2(6u^3 + 6u^2 - 2u^2 + 4u + 10)$
  - $-2(6u^3 + 4u^2 + 4u + 10)$
  - $-12u^3 - 8u^2 - 8u - 20$
18.  $-(-3[2a + 2b - 2] + 5(2a + 3b) - a(2(b + 5)))$
- $-(-6a - 6b + 6 + 10a + 15b - a(2b + 10))$
  - $-(-6a - 6b + 6 + 10a + 15b - 2ab - 10a)$
  - $6a + 6b - 6 - 10a + 15b + 2ab + 10a$
  - $6a - 10a + 10a + 6b + 15b + 2ab - 6$
  - $6a + 21b + 2ab - 6$
19.  $(2x + 5)(3x - 2)$
- $6x^2 - 4x + 15x - 10$
  - $6x^2 + 11x - 10$

20.  $(u + 2)(u + 5)$
- $u^2 + 5u + 2u + 10$
  - $u^2 + 7u + 10$
21.  $(w + 2)(w - 5)$
- $w^2 - 5w + 2w - 10$
  - $w^2 - 3w - 10$
22.  $(x - 4)(x + 7)$
- $x^2 + 7x - 4x - 28$
  - $x^2 + 3x - 28$
23.  $(2x + 3)(5x + 2)$
- $10x^2 + 4x + 15x + 6$
  - $10x^2 + 19x + 6$
24.  $(t^2 - 5t)(3t^2 - 7t)$
- $3t^4 - 7t^3 - 15t^3 + 35t^2$
  - $3t^4 - 22t^3 + 35t^2$
25.  $(X + 2Y)^2$
- $X^2 + 4XY + 4Y^2$
26.  $(2x - 1)^2$
- $4x^2 - 4x + 1$
27.  $(7 - X)^2$
- $49 - 14X + X^2$
  - $X^2 - 14X + 49$
28.  $(\sqrt{x} - 1)(2\sqrt{x} + 5)$
- $2x + 5\sqrt{x} - 2\sqrt{x} - 5$
  - $2x + 3\sqrt{x} - 5$
29.  $(\sqrt{5x} - 2)^2$
- $5x - 4\sqrt{5x} + 4$
30.  $(\sqrt{y} - 3)(\sqrt{y} + 3)$
- $y + 3\sqrt{y} - 3\sqrt{y} - 9$
  - $y - 9$
31.  $(2s - 1)(2s + 1)$
- $4s^2 + 2s - 2s - 1$
  - $4s^2 - 1$

32.  $(a^2 + 2b)(a^2 - 2b)$
- $a^4 - 2a^2b + 2a^2b - 4b^2$
  - $a^4 - 4b^2$
33.  $(x^2 - 3)(x + 4)$
- $x^3 + 4x^2 - 3x - 12$
34.  $(u - 1)(u^2 + 3u - 2)$
- $u^3 + 3u^2 - 2u - u^2 - 3u + 2$
  - $u^3 + 2u^2 - 5u + 2$
35.  $(x^2 - 4)(3x^2 + 2x - 1)$
- $3x^4 + 2x^3 - x^2 - 12x^2 - 8x + 4$
  - $3x^4 + 2x^3 - 13x^2 - 8x + 4$
36.  $(3y - 2)(4y^3 + 2y^2 + -3y)$
- $12y^4 + 6y^3 - 9y^2 - 8y^3 - 4y^2 + 6y$
  - $12y^4 - 2y^3 - 13y^2 + 6y$
37.  $t(3(t + 2)(t - 4) + 5(3t(t - 7)))$
- $t((3t + 6)(t - 4) + 5(3t^2 - 21t))$
  - $t(3t^2 - 12t + 6t - 24 + 15t^2 - 105t)$
  - $3t^3 - 12t^2 + 6t^2 - 24t + 15t^3 - 105t^2$
  - $3t^3 + 15t^3 - 12t^2 + 6t^2 - 105t^2 - 24t$
  - $18t^3 - 111t^2 - 24t$
38.  $((2z + 1)(2z - 1))(4z^2 + 1)$
- $(4z^2 - 2z + 2z - 1)(4z^2 + 1)$
  - $16z^4 - 8z^3 + 8z^3 - 4z + 4z^2 - 2z + 2z - 1$
  - $16z^4 - 4z + 4z^2 - 1$
  - $16z^4 + 4z^2 - 4z - 1$
39.  $(s - t + 4)(3s + 2t - 1)$
- $3s^2 + 2ts - s - 3st - 2t^2 + t + 12s + 8t - 4$
  - $3s^2 - 2t^2 - 3st + 2st - s + 12s + t + 8t - 4$
  - $3s^2 - 2t^2 - st + 11s + 9t - 4$
40.  $(x^2 + x + 1)^2$
- $(x^2 + x + 1)(x^2 + x + 1)$
  - $x^4 + x^3 + x^2 + x^3 + x^2 + 1 + x^2 + x + 1$
  - $x^4 + x^3 + x^3 + x^2 + x^2 + x^2 + x + 1 + 1$
  - $x^4 + 2x^3 + 3x^2 + x + 2$

41.  $(2a + 3)^3$

- $x^3 + 3ax^2 + 3a^2x + 3^3$
- $(2a)^3 + 3(3 \cdot (2a)^2) + 3(3^2 2a) + 3^3$
- $2^3 a^3 + 3(3 \cdot 2^2 a^2) + 3(18a) + 27$
- $8a^3 + 3(3 \cdot 4a^2) + 54a + 27$
- $8a^3 + 36a^2 + 54a + 27$

42.  $(2a - 3)^3$

- $x^3 - 3ax^2 + 3a^2x - a^3$
- $(2a)^3 - 3((3)(2a)^2) + 3((3^2)2a) - 3^3$
- $8a^3 - 3((3)4a^2) + 3((9)2a) - 27$
- $8a^3 - 3(12a^2) + 3(18a) - 27$
- $8a^3 - 36a^2 + 54a - 27$

43.  $(2x - 3)^3$

- $x^3 - 3ax^2 + 3a^2x - a^3$
- $(2x)^3 - 3((3)(2x)^2) + 3((3)^2(2x)) - (3)^3$
- $8x^3 - 3((3)4x^2) + 3(9(2x)) - 27$
- $8x^3 - 3(12x^2) + 3(18x) - 27$
- $8x^3 - 36x^2 + 54x - 27$

44.  $(3a + b)^3$

- $x^3 + 3ax^2 + 3a^2x + a^3$
- $(3a)^3 + 3((b)(3a^2)) + 3((b^2)3a) + b^3$
- $27a^3 + 3((b)9a^2) + 3(b^2 3a) + b^3$
- $27a^3 + 3(9a^2 b) + 3(3ab^2) + b^3$
- $27a^3 + 27a^2 b + 9ab^2 + b^3$

45.  $\frac{z^2 - 18z}{z}$

46.  $\frac{2x^3 - 7x + 4}{x}$

47.  $\frac{6u^5 + 9u^3 - 1}{3u^2}$

48.  $\frac{(3y-4)-(9y+5)}{3y}$

49.  $(x^2 + 7x - 5) \div (x + 5)$

50.  $(x^2 - 5x + 4) \div (x - 4)$

51.  $(3x^3 - 2x^2 + x - 3) \div (x + 2)$

52.  $(x^4 + 3x^2 + 2) \div (x + 1)$

53.  $x^3 \div (x + 2)$

54.  $(8x^2 + 6x + 7) \div (2x + 1)$

55.  $(3x^2 - 4x + 3) \div (3x + 2)$

56.  $(z^3 + z^2 + z) \div (z^2 - z + 1)$