CHAPTER 0
REVIEW OF ALGEBRA
04. Operations with Algebraic Expressions

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1 Summary

Algebraic expressions with exactly 1 term are called **monomials**.

$$3x^2$$

Algebraic expressions with exactly 2 terms are called **binomials**.

$$3x^2 + 3x$$

Algebraic expressions with exactly 3 terms are called **trinomials**.

$$3x^2 + 3x + z$$

Algebraic expressions with more terms are called **polynomials**.

$$3x^2 + 3x + z + 6 + b^3$$

Special Products

1. $x(y + z) = xy + xz$

2. $(x + a)(x + b) = x^2 + x(a + b) + ab$

3. $(ax + c)(bx + d) = abx^2 + x(ad + bc) + cd$

4. $(x + a)^2 = x^2 + 2ax + a^2$

5. $(x - a)^2 = x^2 - 2ax + a^2$

6. $(x + a)(x - a) = x^2 - a^2$

7. $(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$

8. $(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$

2 Long Division

Divide $2x^3 - 14x - 5$ by $x - 3$

$$
\begin{array}{r|l}
& 2x^2 + 6x + 4 \ \leftarrow Quotient \\
\hline
Divisor \to (x-3) & 2x^3 + 0x^2 - 14x - 5 \\
& -(2x^3 - 6x^2) \\
\hline
& 6x^2 - 14x \\
& -(6x^2 - 18x) \\
\hline
& 4x - 5 \\
& -(4x - 12) \\
\hline
& 7 \ \leftarrow Remainder
\end{array}
$$

So the result of $2x^3 - 14x - 5$ by $x - 3$ is

$$2x^2 + 6x + 4 + \tfrac{7}{x-3}$$

- $\frac{Dividend}{Divisor} = Quotient + \frac{Remainder}{Divisor}$

A way of checking a division is to verify that

- $Dividend = \left(Quotient + \frac{Remainder}{Divisor}\right) Divisor$

- $Dividend = Quotient \cdot Divisor + \frac{Remainder}{Divisor} \cdot Divisor$

- $Dividend = Quotient \cdot Divisor + \frac{Remainder}{\cancel{Divisor}} \cdot \cancel{Divisor}$

- $Dividend = Quotient \cdot Divisor + Remainder$

By using this equation, you should be able to verify the result of the example.

3   Problems 0.4

Perform the indicated operations and simplify.

1. $(8x - 4y + 2) + (3x + 2y - 5)$

   - $8x - 4y + 2 + 3x + 2y - 5$
   - $11x - 2y - 3$

2. $(4a^2 - 2ab + 3) + (5c - 3ab + 7)$

   - $4a^2 - 2ab + 3 + 5c - 3ab + 7$
   - $4a^2 - 5b + 5c + 10$

3. $(8t^2 - 6s^2) + (4s^2 - 2t^2 + 6)$

   - $8t^2 - 2t^2 - 6s^2 + 4s^2 + 6$
   - $6t^2 - 2s^2 + 6$

4. $(\sqrt{x} + 2\sqrt{x}) + (3\sqrt{x} + 4\sqrt{x})$

- $\sqrt{x} + 2\sqrt{x} + 3\sqrt{x} + 4\sqrt{x}$
- $10\sqrt{x}$

5. $(\sqrt{a} + 2\sqrt{3b}) - (\sqrt{c} - 3\sqrt{3b})$

  - $(\sqrt{a} + 2\sqrt{3b}) - \sqrt{c} + 3\sqrt{3b}$
  - $\sqrt{a} + 2\sqrt{3b} - \sqrt{c} + 3\sqrt{3b}$
  - $\sqrt{a} - \sqrt{c} + 3\sqrt{3b} + 2\sqrt{3b}$
  - $\sqrt{a} - \sqrt{c} + 5\sqrt{3b}$

6. $(3a + 7b - 9) - (5a + 9b + 21)$

  - $(3a + 7b - 9) - 5a - 9b - 21$
  - $3a + 7b - 9 - 5a - 9b - 21$
  - $3a - 5a - 9b + 7b - 9 - 21$
  - $-2a - 2b - 30$

7. $(7x^2 + 5xy + \sqrt{2}) - (2z - 2xy + \sqrt{2})$

  - $(7x^2 + 5xy + \sqrt{2}) - 2z + 2xy - \sqrt{2}$
  - $7x^2 - 2z + 2xy + 5xy + \sqrt{2} - \sqrt{2}$
  - $7x^2 - 2z + 7xy$

8. $(\sqrt{x} + 2\sqrt{x}) - (\sqrt{x} + 3\sqrt{x})$

  - $(\sqrt{x} + 2\sqrt{x}) - \sqrt{x} - 3\sqrt{x}$
  - $-\sqrt{x}$

9. $(\sqrt[2]{2x} + \sqrt[3]{3y}) - (\sqrt[2]{2x} + \sqrt[4]{4z})$

  - $(\sqrt[2]{2x} + \sqrt[3]{3y}) - \sqrt[2]{2x} - \sqrt[4]{4z}$
  - $\sqrt[3]{3y} - \sqrt[4]{4z}$

10. $4(2z - w) - 3(w - 2z)$

  - $8z - 4w - 3w + 6z$
  - $8z + 6z - 4w - 3w$
  - $14z - 7w$

11. $3(3x + 3y - 7) - 3(8x - 2y + 2)$

  - $9x + 9y - 21 - 24x + 6y - 6$
  - $9x - 24x + 9y + 6y - 6 - 21$
  - $-15x + 15y - 27$

12. $(4s - 5t) + (-2s - 5t) + (s + 9)$

  - $4s - 2s + s - 5t - 5t + 9$
  - $3s - 10t + 9$

13. $5(x^2 - y^2) + x(y - 3x) + 4y(2x + 7y)$

- $5x^2 - 5y^2 + xy - 3x^2 + 8xy + 28y^2$

- $5x^2 - 3x^2 - 5y^2 + 28y^2 + xy + 8xy$

- $2x^2 + 23y^2 + 9xy$

14. $(7 + 3(x - 3) - (4 - 5x))$

  - $(7 + 3(x - 3) - 4 + 20x)$

  - $(7 + 3x - 9 - 4 + 20x)$

  - $20x + 3x + 7 - 9 - 4$

  - $23x - 6$

15. $2(3(3(x^2 + 2) - 2(x^2 - 5)))$

  - $2(3(3x^2 + 6 - 2x^2 + 10))$

  - $2(3(x^2 + 16))$

  - $2(3x^2 + 48)$

  - $6x^2 + 96$

16. $4(3(t + 5) - t(1 - (t + 1)))$

  - $4(3(t + 5) - t(1 - t - 1))$

  - $4(3(t + 5) - t + t^2 + t)$

  - $4(3(t + 5) + t^2)$

  - $4(3t + 15 + t^2)$

  - $4(t^2 + 3t + 15)$

  - $4t^2 + 4t + 60$

17. $-2(3u^2(2u + 2) - 2(u^2 - (5 - 2u)))$

  - $-2((6u^3 + 6u^2) - 2(u^2 - 5 + 2u))$

  - $-2((6u^3 + 6u^2) - 2u^2 + 10 + 4u)$

  - $-2(6u^3 + 6u^2 - 2u^2 + 4u + 10)$

  - $-2(6u^3 + 4u^2 + 4u + 10)$

  - $-12u^3 - 8u^2 - 8u - 20$

18. $-(-3[2a + 2b - 2] + 5(2a + 3b) - a(2(b + 5)))$

  - $-(-6a - 6b + 6 + 10a + 15b - a(2b + 10))$

  - $-(-6a - 6b + 6 + 10a + 15b - 2ab - 10a)$

  - $6a + 6b - 6 - 10a + 15b + 2ab + 10a$

  - $6a - 10a + 10a + 6b + 15b + 2ab - 6$

  - $6a + 21b + 2ab - 6$

19. $(2x + 5)(3x - 2)$

  - $6x^2 - 4x + 15x - 10$

  - $6x^2 + 11x - 10$

20. $(u+2)(u+5)$

   - $u^2 + 5u + 2u + 10$
   - $u^2 + 7u + 10$

21. $(w+2)(w-5)$

   - $w^2 - 5w + 2w - 10$
   - $w^2 - 3w - 10$

22. $(x-4)(x+7)$

   - $x^2 + 7x - 4x - 28$
   - $x^2 + 3x - 28$

23. $(2x+3)(5x+2)$

   - $10x^2 + 4x + 15x + 6$
   - $10x^2 + 19x + 6$

24. $(t^2 - 5t)(3t^2 - 7t)$

   - $3t^4 - 7t^3 - 15t^3 + 35t^2$
   - $3t^4 - 22t^3 + 35t^2$

25. $(X + 2Y)^2$

   - $X^2 + 4XY + 4Y^2$

26. $(2x - 1)^2$

   - $4x^2 - 4x + 1$

27. $(7 - X)^2$

   - $49 - 14X + X^2$
   - $X^2 - 14X + 49$

28. $(\sqrt{x} - 1)(2\sqrt{x} + 5)$

   - $2x + 5\sqrt{x} - 2\sqrt{x} - 5$
   - $2x + 3\sqrt{x} - 5$

29. $(\sqrt{5x} - 2)^2$

   - $5x - 4\sqrt{5x} + 4$

30. $(\sqrt{y} - 3)(\sqrt{y} + 3)$

   - $y + 3\sqrt{y} - 3\sqrt{y} + -9$
   - $y - 9$

31. $(2s - 1)(2s + 1)$

   - $4s^2 + 2s - 2s - 1$
   - $4s^2 - 1$

32. $(a^2 + 2b)(a^2 - 2b)$

   - $a^4 - 2a^2b + 2a^2b - 4b^2$
   - $a^4 - 4b^2$

33. $(x^2 - 3)(x + 4)$

   - $x^3 + 4x^2 - 3x - 12$

34. $(u - 1)(u^2 + 3u - 2)$

   - $u^3 + 3u^2 - 2u - u^2 - 3u + 2$
   - $u^3 + 2u^2 - 5u + 2$

35. $(x^2 - 4)(3x^2 + 2x - 1)$

   - $3x^4 + 2x^3 - x^2 - 12x^2 - 8x + 4$
   - $3x^4 + 2x^3 - 13x^2 - 8x + 4$

36. $(3y - 2)(4y^3 + 2y^2 + -3y)$

   - $12y^4 + 6y^3 - 9y^2 - 8y^3 - 4y^2 + 6y$
   - $12y^4 - 2y^3 - 13y^2 + 6y$

37. $t(3(t + 2)(t - 4) + 5(3t(t - 7)))$

   - $t((3t + 6)(t - 4) + 5(3t^2 - 21t))$
   - $t(3t^2 - 12t + 6t - 24 + 15t^2 - 105t)$
   - $3t^3 - 12t^2 + 6t^2 - 24t + 15t^3 - 105t^2$
   - $3t^3 + 15t^3 - 12t^2 + 6t^2 - 105t^2 - 24t$
   - $18t^3 - 111t^2 - 24t$

38. $((2z + 1)(2z - 1))(4z^2 + 1)$

   - $(4z^2 - 2z + 2z - 1)(4z^2 + 1)$
   - $16z^4 - 8z^3 + 8z^3 - 4z + 4z^2 - 2z + 2z - 1$
   - $16z^4 - 4z + 4z^2 - 1$
   - $16z^4 + 4z^2 - 4z - 1$

39. $(s - t + 4)(3s + 2t - 1)$

   - $3s^2 + 2ts - s - 3st - 2t^2 + t + 12s + 8t - 4$
   - $3s^2 - 2t^2 - 3st + 2st - s + 12s + t + 8t - 4$
   - $3s^2 - 2t^2 - st + 11s + 9t - 4$

40. $(x^2 + x + 1)^2$

   - $(x^2 + x + 1)(x^2 + x + 1)$
   - $x^4 + x^3 + x^2 + x^3 + x^2 + 1 + x^2 + x + 1$
   - $x^4 + x^3 + x^3 + x^2 + x^2 + x^2 + x + 1 + 1$
   - $x^4 + 2x^3 + 3x^2 + x + 2$

41. $(2a + 3)^3$

- $x^3 + 3ax^2 + 3a^2x + 3^3$
- $(2a)^3 + 3(3 \cdot (2a)^2) + 3(3^2 2a) + 3^3$
- $2^3 a^3 + 3(3 \cdot 2^2 a^2) + 3(18a) + 27$
- $8a^3 + 3(3 \cdot 4a^2) + 54a + 27$
- $8a^3 + 36a^2 + 54a + 27$

42. $(2a - 3)^3$

- $x^3 - 3ax^2 + 3a^2x - a^3$
- $(2a)^3 - 3((3)(2a)^2) + 3((3^2)2a) - 3^3$
- $8a^3 - 3((3)4a^2) + 3((9)2a) - 27$
- $8a^3 - 3(12a^2) + 3(18a) - 27$
- $8a^3 - 36a^2 + 54a - 27$

43. $(2x - 3)^3$

- $x^3 - 3ax^2 + 3a^2x - a^3$
- $(2x)^3 - 3((3)(2x)^2) + 3((3)^2(2x)) - (3)^3$
- $8x^3 - 3((3)4x^2) + 3(9(2x)) - 27$
- $8x^3 - 3(12x^2) + 3(18x) - 27$
- $8x^3 - 36x^2 + 54x - 27$

44. $(3a + b)^3$

- $x^3 + 3ax^2 + 3a^2x + a^3$
- $(3a)^3 + 3((b)(3a^2)) + 3((b^2)3a) + b^3$
- $27a^3 + 3((b)9a^2) + 3(b^2 3a) + b^3$
- $27a^3 + 3(9a^2 b) + 3(3ab^2) + b^3$
- $27a^3 + 27a^2 b + 9ab^2 + b^3$

45. $\frac{z^2 - 18z}{z}$

- $\frac{z^2}{z} - \frac{18z}{z}$
- $\frac{z\!\!\!/\,^1}{\not z} - \frac{18\not z}{\not z}$
- $z - 18$

46. $\frac{2x^3 - 7x + 4}{x}$

- $\frac{2x^3}{x} - \frac{7x}{x} + \frac{4}{x}$
- $\frac{2x\!\!\!/\,^2}{\not x} - \frac{7\not x}{\not x} + \frac{4}{x}$
- $2x^2 - 7 + \frac{4}{x}$

47. $\frac{6u^5 + 9u^3 - 1}{3u^2}$

- $\frac{6u^5}{3u^2} + \frac{9u^3}{3u^2} - \frac{1}{3u^2}$

- $\frac{\cancel{6}\,2\cancel{u^5}\,u^3}{\cancel{3}\,\cancel{u^2}} + \frac{9\cancel{u^3}\,3u}{3\cancel{u^2}} - \frac{1}{3u^2}$
- $2u^3 + 3u - \frac{1}{3u^2}$

48. $\frac{(3y-4)-(9y+5)}{3y}$

  - $\frac{3y-4}{3y} - \frac{9y+5}{3y}$
  - $\frac{3y}{3y} - \frac{4}{3y} - \frac{9y}{3y} - \frac{5}{3y}$
  - $1 - \frac{4}{3y} - 3 - \frac{5}{3y}$
  - $-2 - \frac{9}{3y}$
  - $-2 - \frac{\cancel{9}\,3}{\cancel{3}y}$
  - $-2 - \frac{3}{y}$
  - $-\frac{2y}{y} - \frac{3}{y}$
  - $\frac{-2y-3}{y}$

49. $(x^2 + 7x - 5) \div (x + 5)$

  - $x + 5 \;/\; x^2 + 7x - 5 - (x(x+5)) = ?$
  - $x + 5 \;/\; x^2 + 7x - 5 - (x^2 + 5x) = x$
  - $x + 5 \;/\; 2x - 5 = x$
  - $x + 5 \;/\; 2x - 5 - (2(x+5)) = x$
  - $x + 5 \;/\; 2x - 5 - (2x + 10) = x + 2$
  - $x + 5 \;/\; -15 = x + 2$
  - $x + 2 - \frac{15}{x+5}$

50. $(x^2 - 5x + 4) \div (x - 4)$

  - $x - 4 \;/\; x^2 - 5x + 4 - (x(x-4)) = ?$
  - $x - 4 \;/\; x^2 - 5x + 4 - (x^2 - 4x) = x$
  - $x - 4 \;/\; -x + 4 = x$
  - $x - 4 \;/\; -x + 4 - (-1(x-4)) = x$
  - $x - 4 \;/\; -x + 4 - (-x + 4) = x - 1$
  - $x - 4 \;/\; -x + 4 + x - 4 = x - 1$
  - $x - 1$

51. $(3x^3 - 2x^2 + x - 3) \div (x + 2)$

  - $x + 2 \;/\; 3x^3 - 2x^2 + x - 3 - (3x^2(x+2)) = 3x^2$
  - $x + 2 \;/\; 3x^3 - 2x^2 + x - 3 - (3x^3 + 6x^2) = 3x^2$
  - $x + 2 \;/\; -8x^2 + x - 3 = 3x^2$
  - $x + 2 \;/\; -8x^2 + x - 3 - (-8x(x+2)) = 3x^2$
  - $x + 2 \;/\; -8x^2 + x - 3 - (-8x^2 - 16x) = 3x^2 - 8x$
  - $x + 2 \;/\; -8x^2 + x - 3 + 8x^2 + 16x = 3x^2 - 8x$
  - $x + 2 \;/\; 17x - 3 = 3x^2 - 8x$
  - $x + 2 \;/\; 17x - 3 - (17(x+2)) = 3x^2 - 8x$

- $x + 2 \ / \ 17x - 3 - (17x + 34) = 3x^2 - 8x + 17$
- $x + 2 \ / \ 17x - 3 - 17x - 34 = 3x^2 - 8x + 17$
- $x + 2 \ / \ -37 = 3x^2 - 8x + 17$
- $3x^2 - 8x + 17 + \frac{-37}{x+2}$
- $3x^2 - 8x + 17 - \frac{37}{x+2}$

52. $(x^4 + 3x^2 + 2) \div (x + 1)$

- $x + 1 \ / \ x^4 + 3x^2 + 2 = ?$
- $x + 1 \ / \ x^4 + 3x^2 + 2 - (x^3(x + 1)) = x^3$
- $x + 1 \ / \ x^4 + 3x^2 + 2 - (x^4 + x^3) = x^3$
- $x + 1 \ / \ x^4 + 3x^2 + 2 - x^4 - x^3 = x^3$
- $x + 1 \ / \ -x^3 + 3x^2 + 2 = x^3$
- $x + 1 \ / \ -x^3 + 3x^2 + 2 - (-x^2(x + 1)) = x^3 - x^2$
- $x + 1 \ / \ -x^3 + 3x^2 + 2 - (-x^3 - x^2) = x^3 - x^2$
- $x + 1 \ / \ -x^3 + 3x^2 + 2 + x^3 + x^2 = x^3 - x^2$
- $x + 1 \ / \ 4x^2 + 2 = x^3 - x^2$
- $x + 1 \ / \ 4x^2 + 2 - (4x(x + 1)) = x^3 - x^2 + 4x$
- $x + 1 \ / \ 4x^2 + 2 - (4x^2 + 4x) = x^3 - x^2 + 4x$
- $x + 1 \ / \ -4x + 2 = x^3 - x^2 + 4x$
- $x + 1 \ / \ -4x + 2 - (-4(x + 1)) = x^3 - x^2 + 4x + 4$
- $x + 1 \ / \ -4x + 2 - (-4x - 4) = x^3 - x^2 + 4x + 4$
- $x + 1 \ / \ -4x + 2 + 4x + 4 = x^3 - x^2 + 4x - 4$
- $x + 1 \ / \ 6 = x^3 - x^2 + 4x - 4$
- $x^3 - x^2 + 4x - 4 + \frac{6}{x+1}$

53. $x^3 \div (x + 2)$

- $x + 2 \ / \ x^3 = ?$
- $x + 2 \ / \ x^3 - (x^2(x + 2)) = x^2$
- $x + 2 \ / \ x^3 - (x^3 + 2x^2) = x^2$
- $x + 2 \ / \ x^3 - x^3 - 2x^2 = x^2$
- $x + 2 \ / \ -2x^2 = x^2$
- $x + 2 \ / \ -2x^2 - (-2x(x + 2)) = x^2 - 2x$
- $x + 2 \ / \ -2x^2 - (-2x^2 - 4x) = x^2 - 2x$
- $x + 2 \ / \ -2x^2 + 2x^2 + 4x = x^2 - 2x$
- $x + 2 \ / \ 4x = x^2 - 2x$
- $x + 2 \ / \ 4x - (4(x + 2)) = x^2 - 2x + 4$
- $x + 2 \ / \ 4x - (4x + 8) = x^2 - 2x + 4$
- $x + 2 \ / \ 4x - 4x - 8 = x^2 - 2x + 4$
- $x + 2 \ / \ -8 = x^2 - 2x + 4$

- $x^2 - 2x + 4 + \frac{-8}{x+2}$
- $x^2 - 2x + 4 - \frac{8}{x+2}$

54. $(8x^2 + 6x + 7) \div (2x + 1)$

  - $2x + 1 \;/\; 8x^2 + 6x + 7 = ?$
  - $2x + 1 \;/\; 8x^2 + 6x + 7 - (4x(2x + 1)) = 4x$
  - $2x + 1 \;/\; 8x^2 + 6x + 7 - (8x^2 + 4x) = 4x$
  - $2x + 1 \;/\; 8x^2 + 6x + 7 - 8x^2 - 4x = 4x$
  - $2x + 1 \;/\; 2x + 7 = 4x$
  - $2x + 1 \;/\; 2x + 7 - (1(2x + 1)) = 4x + 1$
  - $2x + 1 \;/\; 2x + 7 - (2x + 1) = 4x + 1$
  - $2x + 1 \;/\; 2x + 7 - 2x - 1 = 4x + 1$
  - $2x + 1 \;/\; 6 = 4x + 1$
  - $4x + 1 + \frac{6}{2x+1}$

55. $(3x^2 - 4x + 3) \div (3x + 2)$

  - $3x + 2 \;/\; 3x^2 - 4x + 3 = ?$
  - $3x + 2 \;/\; 3x^2 - 4x + 3 - (x(3x + 2)) = x$
  - $3x + 2 \;/\; 3x^2 - 4x + 3 - (3x^2 + 2x) = x$
  - $3x + 2 \;/\; 3x^2 - 4x + 3 - 3x^2 - 2x = x$
  - $3x + 2 \;/\; -6x + 3 = x$
  - $3x + 2 \;/\; -6x + 3 - (-2(3x + 2)) = x - 2$
  - $3x + 2 \;/\; -6x + 3 - (-6x - 4) = x - 2$
  - $3x + 2 \;/\; -6x + 3 + 6x + 4 = x - 2$
  - $3x + 2 \;/\; 7 = x - 2$
  - $x - 2 + \frac{7}{3x+2}$

56. $(z^3 + z^2 + z) \div (z^2 - z + 1)$

  - $z^2 - z + 1 \;/\; z^3 + z^2 + z = ?$
  - $z^2 - z + 1 \;/\; z^3 + z^2 + z - (z(z^2 - z + 1)) = z$
  - $z^2 - z + 1 \;/\; z^3 + z^2 + z - (z^3 - z^2 + z) = z$
  - $z^2 - z + 1 \;/\; z^3 + z^2 + z - z^3 + z^2 - z = z$
  - $z^2 - z + 1 \;/\; 2z^2 = z$
  - $z^2 - z + 1 \;/\; 2z^2 - (2(z^2 - z + 1)) = z + 2$
  - $z^2 - z + 1 \;/\; 2z^2 - (2z^2 - 2z + 2) = z + 2$
  - $z^2 - z + 1 \;/\; 2z^2 - 2z^2 + 2z - 2 = z + 2$
  - $z^2 - z + 1 \;/\; 2z - 2 = z + 2$
  - $z + 2 + \frac{2z-2}{z^2-z+1}$