Lab Sheet

IA 3203 – DIGITAL SIGNAL PROCESSING

Department of Instrumentation and Automation Technology
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DSP 303 – Linear Time Invariant Systems

A dynamic system is time invariant if shifting the input signal on the time axis leads to an equivalent shifting of the output signal along the time axis without any other changes. A system is linear if its behavior is scalar independent. Therefore, a system is said to be a lineartime invariant system when both the time invariance and the linearity.

Linearity

A linear system is a simple system that is based on linear operations and exist within a premise that is defined by mathematical properties associated with the laws of additivity and homogeneity. Furthermore, if the system satisfies the principle of superposition, has nononlinear operators and gives a zero outputs for zero inputs, the said system could be identified as a linear system.

Superposition

In digital signal processing, linear systems decompose input signals into separatecomponents, process them individually and synthesize them together in order to get the processed output signal which would also be in the same domain. This is superposition in signal processing.

Decomposition

Decomposing a signal implies breaking of a signal into components which can be done in oneof many methods. Most common of such methods are impulse decomposition, step decomposition, even-odd decomposition and Fourier decomposition. You will be focusing onthe impulse, step and even-odd decomposition methods in this practical.

Exercises:

This practical is designed to give you a basic understanding on linear time invariant systems. You will do all the implementations for exercises using MATLAB or Octave.

Note that you are expected to give titles, label axes appropriately, and provide legends specifying results if more than one result is plotted in the same sheet.

1. Use your knowledge of basic signal shapes and their behaviors to plot and perform the following basic signal operations. Presented your results in the same panel for Parts a. through d. considering the range $-4 \leq n \leq 4$.

Hint: You may use $'stem(\ )'$ when plotting discrete signals.

   a. Plot a unit impulse signal in the given range with an impulse at 0^{th} index.
   b. Unit step signal within the range. Start from level 0 and then command the step to start at the 0^{th} index.

c. Plot the exponential signal $e^{-(3/2)n}$ that propagates in the given range.
d. Plot the unit ramp signal for the same given range.
e. Suppose $x_1[n] = [1, 2, 3, 4, 5, 6]$ and $x_2[n] = [2, 3, 4]$ are two signals. Perform addition and plot the resultant signal in the same panel with x_1 and x_2.
f. Do a multiplication on the signals given in Part e. and plot your results in the same panel with input signals.
g. For $x = [5, 4, 3]$ if $n_1 = 3$ is a positive time shift and $n_2 = 5$ is a negative time shift, plot both negative and positive time shifts in 2 separate plots in the same panel along with the original input.

2. Suppose that you are given a signal $y[n] = x[n]^2 + s$ where s is the scaling constant. If $a = 2$ and $b = 3$ are the scaling factors for $x[n] = a.\, x_1[n] + b.\, x_2[n]$ find the following.

Hint: You may use $'stem(\ )'$ when plotting discrete signals.

a. If $x_1[n] = [2, 3, 4]$ and $x_2[n] = [1, 2, 3]$ find the sequence $x[n]$. Plot all 3 signals together in the same plot.
b. Find $y_1[n]$ and $y_2[n]$ using the given equation for the signal if the scaling constant $s = 2$.
c. Using given values for scaling factors, find $y[n]$ and plot that with $y_1[n]$ and $y_2[n]$ you obtained in Part b. Note that $y[n] = a.\, y_1[n] + b.\, y_2[n]$ can be used here.
d. Plot $y[n]$ from Part c. and $x[n]$ from Part a. together in the same plot.
e. Find $y[n]$ by means of $x[n]$ directly and plot them together in the same plot. Make sure these plots are in the same panel with your result for Part d.
f. Do a comparison and state whether the system is linear. Explain your answer.

3. Suppose that you are given a signal $y[n] = x[n]$ where s is the scaling constant. If $a = 1$ and $b = 2$ are the scaling factors for $x[n]$ formulation.

Hint: You may use $'stem(\ )'$ when plotting discrete signals.

a. If $x_1[n] = [1, 2, 3]$ and $x_2[n] = [2, 3, 4]$ are the signals find the sequence $x[n]$ if $x[n] = a.\, x_1[n] + b.\, x_2[n]$.
b. Find $y_1[n]$ and $y_2[n]$ of using the given mapping equation for the signal.
c. Using given values for scaling factors and results from Part b., find $y[n]$ and plot that with $x[n]$ in the same plot. You may use $y[n] = a.\, y_1[n] + b.\, y_2[n]$ here.
d. Evaluate $y[n]$ using the mapping of $x[n]$. Plot $y[n]$ you found in this part with $x[n]$ together in the same plot.
e. Do a comparison and state whether the system is linear. Explain your answer.

4. Let $y[n] = x[n]$ and $z[n] = n.\, x[n]$ where $n_0 = 2$ is the shift along the axis. Consider the range to be the length of $x_1[n]$ if it is given that $x_1[n] = [1, 2, 3]$ and $x_2[n] = x_1[n - n_0]$.
Hint: You may use $'zeros(\ )'$ and $'stem(\ )'$ when evaluating and plotting discrete signals.

a. Find the full sequence $x[n]$ if $x[n] = x_2[n]$.

b. Evaluate the signal sequence y[n] using x[n].
c. Evaluate the signal sequence y[n] using $x_1[n]$ and $x_2[n]$ appropriately.
d. Plot the signals you obtained in Parts b. and c. for y[n] in the same plot.
e. Find z[n] using the given information for x[n] in Part a.
f. Using your knowledge on signals use $x_1[n]$ and $x_2[n]$ to find z[n].
g. Plot z[n] results you obtained in Parts e. and f together.
h. Do a brief analysis to discuss the types of time dynamics of the system.

5. Suppose that you are given a system that satisfies the equation $y[n] - 0.4\,y[n-1] + 0.75\,y[n-2] = 2.2403\,x[n] + 2.4908\,x[n-1] + 2.2403\,x[n-2]$.

Hint: You may use $'filter(\,)'$ and $'stem(\,)'$ when evaluating and plotting discrete signals, and $'round(\,)'$ in necessary calculations if needed.

a. If the sinusoidal signal is $x[n] = ax_1[n] + bx_2[n]$ where $x_1[n] = \cos(0.2\pi n)$ and $x_2[n] = \cos(0.8\pi n)$ respectively plot the signal x[n] in the range $0 \leq n \leq 40$. Take $a = 2$ and $b = -3$.
b. Note that each decomposition of x maps to y. If all initial conditions are given to be zeros, find y_1 and plot it in a separate plot with x_1.
c. For the same conditions given in Part b., find y_2. Plot y_2 in a separate plot with x_2 and make sure they are in the same panel with the result in Part b.
d. Use your knowledge on the linearity of a system and find y[n] for given $x[n] = ax_1[n] + bx_2[n]$.
e. Plot y[n] you found in Part d. in a separate plot with x[n].
f. Find y[n] if all initial conditions are given to be zeros. You may use the same method used in Parts b. and c. to find y_1 and y_2.
g. You have 2 sets of results for y[n] in Parts e. and f., take the difference of these sets. State and explain your answer.