Tutorial 02

1]

$T = 8$,

$a_1 = a_{-1}^* = j \qquad a_5 = a_{-5}^* = 2$

$a_1 = j \qquad a_{-1} = -j \qquad a_5 = 2 \qquad a_{-5} = 2$

$$x(t) = a_1 e^{-j\omega_0 t} + a_{-1} e^{j\omega_0 t} + a_5 e^{-j5\omega_0 t} + \cancel{e_{-5}}\, a_{-5} e^{j5\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T} = \frac{\pi}{4}$$

$$= j \cdot e^{-j\omega_0 t} - j e^{j\omega_0 t} + \cancel{a_5}\, 2 e^{-j5\omega_0 t} + 2 e^{j5\omega_0 t}$$

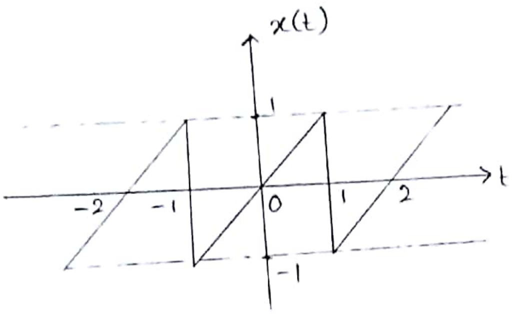
$$= -2j^2 \left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right] + 4\left[\frac{e^{j5\omega_0 t} + e^{-j5\omega_0 t}}{2}\right]$$

$$= 2\sin(\omega_0 t) + 4\cos(5\omega_0 t)$$

$$= 2\cos(\omega_0 t - \pi/2) + 4\cos(5\omega_0 t)$$

$$x(t) = 2\cos\left(\frac{\pi}{4}t - \pi/2\right) + 4\cos\left(\frac{5\pi}{4}t\right)$$

02] a) I)



$T = 2 \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$x(t) = t$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{2}\int_{-1}^{1} t\,dt = \frac{1}{2}\left|\frac{t^2}{2}\right|_{-1}^{1} = 0 \qquad \therefore a_0 = 0$$

$$a_k = \frac{1}{T}\int_T x(t)\cdot e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_{-1}^{1} t\cdot e^{-jk\omega_0 t}\,dt$$

$$a_k = \frac{1}{2}\left\{\left|\frac{t\cdot e^{-jk\omega_0 t}}{-jk\omega_0}\right|_{-1}^{1} + \frac{1}{jk\omega_0}\int_{-1}^{1} e^{-jk\omega_0 t}\,dt\right\}$$

$$= \frac{1}{2}\left\{\frac{e^{-j\omega_0 k} + e^{j\omega_0 k}}{-jk\omega_0} \cancel{+} - \frac{1}{(jk\omega_0)^2}\left|e^{-jk\omega_0 t}\right|_{-1}^{1}\right\}$$

$$= \cancel{\tfrac{1}{2}}\, \frac{j}{k\omega_0}\left[\frac{e^{-j\omega_0 k} + e^{j\omega_0 k}}{2}\right] + \frac{1}{2k^2\omega_0^2}\left[e^{-j\omega_0 k} \cancel{+} - e^{j\omega_0 k}\right]$$

$$a_k = j\cdot\frac{\cos(\omega_0 k)}{(k\omega_0)} \cancel{+} - \frac{2j}{k^2\omega_0^2}\left[\frac{e^{j\omega_0 k} - e^{-j\omega_0 k}}{2j}\right]$$

$$a_k = j\cdot\frac{\cos(\omega_0 k)}{(k\omega_0)} - \frac{j}{k^2\omega_0^2}\sin(\omega_0 k)$$

k is even,

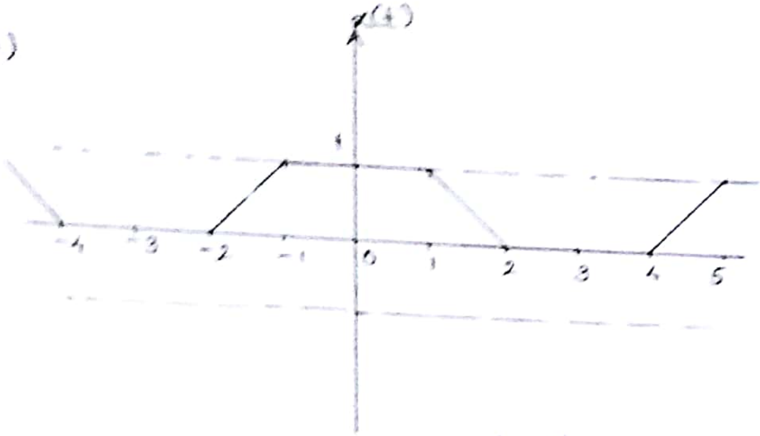
$$a_k = \frac{j}{k\omega_0} = \frac{j}{k\pi}$$

k is odd,

$$a_k = \frac{-j}{k\omega_0} = \frac{-j}{k\pi}$$

$$\therefore x(t) = \sum_{k=-\infty}^{\infty} a_k e^{+jk\pi t} \quad \text{where} \quad a_0 = 0 \text{ and } a_k = \frac{j(-1)^k}{\pi k}$$

B)



$$T = 6 \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$x(t) = \begin{cases} t+2 & ; -2 \le t \le -1 \\ 1 & ; -1 \le t \le 1 \\ 2-t & ; +1 \le t \le 2 \end{cases}$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{6}\left\{ \int_{-2}^{-1}(t+2)\,dt + \int_{-1}^{1} 1.dt + \int_{1}^{2}(2-t)\,dt \right\}$$

$$= \frac{1}{6}\left\{ \left|\frac{t^2}{2}\right|_{-2}^{-1} + 2|t|_{-2}^{-1} + |t|_{-1}^{1} + 2|t|_{1}^{2} - \left|\frac{t^2}{2}\right|_{1}^{2} \right\}$$

$$= \frac{1}{6}\left[-\frac{3}{2} + 2 + 2 + 2 - \frac{3}{2} \right]$$

$$\therefore a_0 = \frac{1}{2}$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{6}\left\{ \int_{-2}^{-1}(t+2)\,e^{-jk\omega_0 t}\,dt + \int_{-1}^{1} e^{-jk\omega_0 t}\,dt + \int_{1}^{2}(2-t).e^{-jk\omega_0 t}\,dt \right\}$$

$$= \frac{1}{6}\left\{ \frac{\left|t\, e^{-jk\omega_0 t}\right|_{-2}^{-1}}{-jk\omega_0} - \frac{\left|e^{-jk\omega_0 t}\right|_{-2}^{-1}}{(jk\omega_0)^2} + 2\frac{\left|e^{-jk\omega_0 t}\right|_{-2}^{-1}}{-jk\omega_0} + \frac{\left|e^{-jk\omega_0 t}\right|_{-1}^{1}}{-jk\omega_0} + 2\frac{\left|e^{-jk\omega_0 t}\right|_{1}^{2}}{-jk\omega_0} + \frac{\left|t\, e^{-jk\omega_0 t}\right|_{1}^{2}}{jk\omega_0} + \frac{\left|e^{-jk\omega_0 t}\right|_{1}^{2}}{(jk\omega_0)^2} \right\}$$

$$= \frac{1}{6}\left\{ \frac{\left[2e^{j2\omega_0 k} - e^{jk\omega_0}\right]}{-jk\omega_0} - \frac{\left[e^{jk\omega_0} - e^{j2k\omega_0}\right]}{(jk\omega_0)^2} - \frac{2\left[e^{jk\omega_0} - e^{j2k\omega_0}\right]}{jk\omega_0} + \dots \right.$$

$$\therefore -\frac{\left[e^{-jk\omega_0} - e^{jk\omega_0}\right]}{jk\omega_0} - \frac{2}{jk\omega_0}\left[e^{-jk2\omega_0} - e^{-jk\omega_0}\right] + \frac{\left[2e^{-j2\omega_0 k} - e^{-jk\omega_0}\right]}{jk\omega_0} + \frac{\left[e^{-j2k\omega_0} - e^{-jk\omega_0}\right]}{(jk\omega_0)^2} \Bigg\}$$

$$= \frac{1}{6}\left[\cancel{\frac{1}{jk\omega_0}} - \frac{1}{(jk\omega_0)^2} - \cancel{\frac{2}{jk\omega_0}} + \cancel{\frac{1}{jk\omega_0}}\right] e^{jk\omega_0} + \frac{1}{6}\left[-\cancel{\frac{2}{jk\omega_0}} + \frac{1}{(jk\omega_0)^2} + \cancel{\frac{2}{jk\omega_0}}\right] e^{j2k\omega_0}$$

$$+ \frac{1}{6}\left[-\cancel{\frac{1}{jk\omega_0}} + \cancel{\frac{2}{jk\omega_0}} - \cancel{\frac{1}{jk\omega_0}} - \frac{1}{(jk\omega_0)^2}\right] e^{-jk\omega_0} + \frac{1}{6}\left[-\cancel{\frac{2}{jk\omega_0}} + \cancel{\frac{2}{jk\omega_0}} + \frac{1}{(jk\omega_0)^2}\right] e^{-j2k\omega_0}$$

$$= -\frac{\cancel{2}1}{6(jk\omega_0)^2} e^{jk\omega_0} + \frac{1}{6(jk\omega_0)^2} e^{j2k\omega_0} - \frac{1}{6(jk\omega_0)^2} e^{-jk\omega_0} + \frac{1}{6(jk\omega_0)^2} e^{-j2k\omega_0}$$

$$= -\frac{1}{3(jk\omega_0)^2}\left[\frac{e^{jk\omega_0} + e^{-jk\omega_0}}{2}\right] + \frac{1}{3(jk\omega_0)^2}\left[\frac{e^{j2k\omega_0} + e^{-j2k\omega_0}}{2}\right]$$

$$= \frac{-1}{3(jk\omega_0)^2}\left\{\cos(k\omega_0) - \cos(2k\omega_0)\right\}$$

$$= \frac{1}{3k^2\omega_0^2}\left[\cos(k\omega_0) - \cos(2k\omega_0)\right]$$

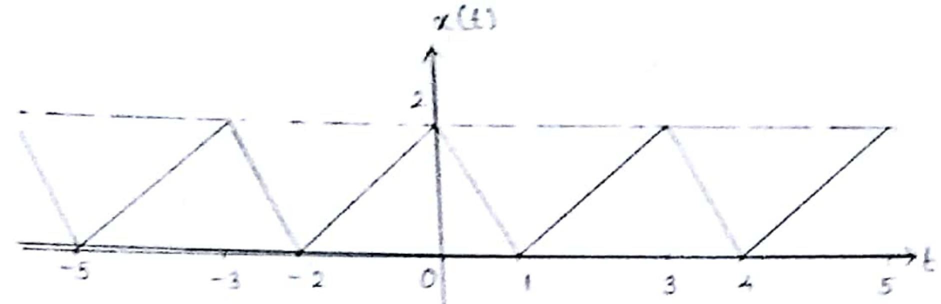
$$a_k = \frac{3}{k^2\pi^2}\left[\cos\left(\frac{\pi k}{3}\right) - \cos\left(\frac{2\pi k}{3}\right)\right] \quad ; \quad k \neq 0$$

$$\therefore x(t) = \sum_{k=-\infty}^{\infty} a_k e^{+jk(\pi/3)t} \quad \text{where } \cancel{a_0 =}\ a_k = \begin{cases} \frac{1}{2} & ; \ k = 0 \\ \frac{3}{k^2\pi^2}\left[\cos\left(\frac{\pi k}{3}\right) - \cos\left(\frac{2\pi k}{3}\right)\right] & ; \ k \neq 0 \end{cases}$$

III)

$T = 3$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$



$$x(t) = \begin{cases} t+2 & ; -2 \le t \le 0 \\ 2-2t & ; 0 \le t \le 1 \end{cases}$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{3}\left\{ \int_{-2}^{0}(t+2)\,dt + 2\int_{0}^{1}(1-t)\,dt \right\}$$

$$= \frac{1}{3}\left\{ \left|\frac{t^2}{2}\right|_{-2}^{0} + 2|t|_{-2}^{0} + 2|t|_0^1 - \cancel{2}|t^2|_0^1 \right\}$$

$$= \frac{1}{3}\left[-2 + 3(2) - 1\right]$$

$$a_0 = 1$$

$$a_k = \frac{1}{T}\int_T x(t)\,e^{-jk\omega_0 t}\,dt = \frac{1}{3}\left\{ \int_{-2}^{0}(t+2)\,e^{-jk\omega_0 t}\,dt + \int_{0}^{1}(2-2t)\,e^{-jk\omega_0 t}\,dt \right\}$$

$$= \frac{1}{3}\left\{ \left|\frac{t\,e^{-jk\omega_0 t}}{(-jk\omega_0)}\right|_{-2}^{0} - \left|\frac{e^{-jk\omega_0 t}}{(jk\omega_0)^2}\right|_{-2}^{0} + 2\left|\frac{e^{-jk\omega_0 t}}{(-jk\omega_0)}\right|_{-2}^{0} \right\}$$

$$+ \frac{2}{3}\left\{ \left|\frac{e^{-jk\omega_0 t}}{(-jk\omega_0)}\right|_0^1 + \left|\frac{t\,e^{-jk\omega_0 t}}{(jk\omega_0)}\right|_0^1 + \left|\frac{e^{-jk\omega_0 t}}{(jk\omega_0)^2}\right|_0^1 \right\}$$

$$= \frac{1}{3}\left\{ -\frac{2e^{jk\omega_0(2)}}{jk\omega_0} - \frac{[1 - e^{jk2\omega_0}]}{(jk\omega_0)^2} + 2\frac{[1 - e^{jk2\omega_0}]}{(-jk\omega_0)} \right\}$$

$$+ \frac{2}{3}\left\{ \frac{[1 - e^{-jk\omega_0}]}{jk\omega_0} + \frac{e^{-jk\omega_0}}{jk\omega_0} + \frac{[e^{-jk\omega_0} - 1]}{(jk\omega_0)^2} \right\}$$

$$= \frac{1}{3}\left[-\frac{2}{\cancel{jk\omega_0}} + \frac{1}{(jk\omega_0)^2} + \frac{2}{\cancel{jk\omega_0}} \right] e^{jk2\omega_0} + \frac{2}{3}\left[-\frac{1}{\cancel{jk\omega_0}} + \frac{1}{\cancel{jk\omega_0}} + \frac{1}{(jk\omega_0)^2} \right] e^{-jk\omega_0}$$

$$+ \frac{1}{3}\left[-\frac{1}{(jk\omega_0)^2} - \frac{2}{\cancel{jk\omega_0}} + \frac{2}{\cancel{jk\omega_0}} - \frac{2}{(jk\omega_0)^2} \right]$$

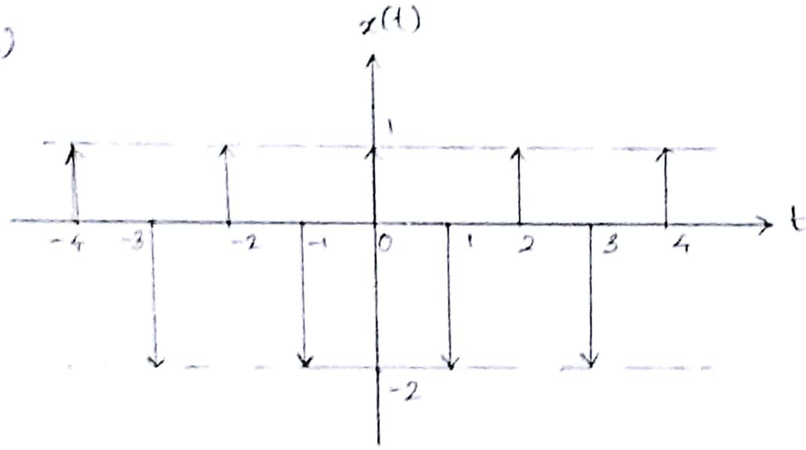
$$a_k = \frac{e^{j2k\omega_0}}{3(jk\omega_0)^2} + \frac{2}{3(jk\omega_0)^2}e^{-jk\omega_0} - \frac{1}{(jk\omega_0)^2}$$

$$a_k = \frac{1}{k^2\omega_0^2} - \frac{[\cos(2k\omega_0) + j\sin(2k\omega_0)]}{3k^2\omega_0^2} - \frac{2[\cos(k\omega_0) - j\sin(k\omega_0)]}{3k^2\omega_0^2}$$

$$\therefore a_k = \frac{9}{4\pi^2k^2} - \frac{3[\cos(2k\omega_0) + j\sin(2k\omega_0) + 2\cos(k\omega_0) - j\sin(k\omega_0)]}{\cancel{3}\,4\pi^2k^2}$$

$$\therefore x(t) = \sum_{k=-\infty}^{\infty} a_k e^{+j\frac{2\pi k}{3}t} \quad \text{where } a_k = \begin{cases} 1 & ; \ k \neq 0 \\ \frac{3}{4\pi^2k^2}\Big[3 - \cos(2k\omega_0) - j\sin(2k\omega_0) - 2\cos(k\omega_0) \\ \quad + j\sin(k\omega_0)\Big] & ; \ k \neq 0 \end{cases}$$

IV)



$T = 2$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$$x(t) = \delta(t) - 2\delta(t-1) \quad ; \ \cancel{-1 <}\ 0 \leq t < 2$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{2}\int_{-1/2}^{3/2} [\delta(t) - 2\delta(t-1)].dt$$

$$= \frac{1}{2}\left\{ \int_{-1/2}^{1/2} \delta(t).dt - 2\int_{1/2}^{3/2} \delta(t-1).dt \right\}$$

$$= \frac{1}{2}(1-2)$$

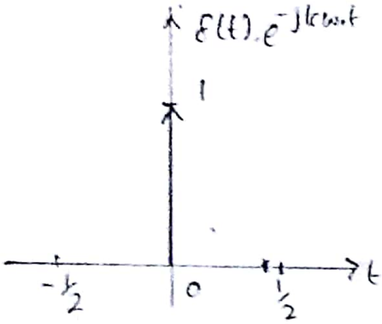
$$a_0 = -\frac{1}{2}$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_{-1/2}^{3/2} [\delta(t) - 2\,\delta(t-1)]\, e^{-jk\omega_0 t}\, dt$$

$$\therefore a_k = \frac{1}{T}\left\{ \int_{-1/2}^{1/2} \delta(t)\, e^{-jk\omega_0 t}\, dt - 2\int_{1/2}^{3/2} \delta(t-1)\, e^{-jk\omega_0 t}\, dt \right\} \quad - ①$$

$-\frac{1}{2} \le t \le \frac{1}{2}$;

$t = 0, \quad e^{-jk\omega_0 t} = e^0 = 1$



$$\therefore \int_{-1/2}^{1/2} \delta(t)\, e^{-jk\omega_0 t}\, dt = 1$$

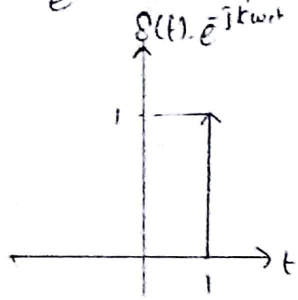
$\frac{1}{2} \le t \le \frac{3}{2}$

$t = 1, \quad e^{-jk\omega_0 t} = e^{-jk\omega_0} = e^{-jk\pi}$

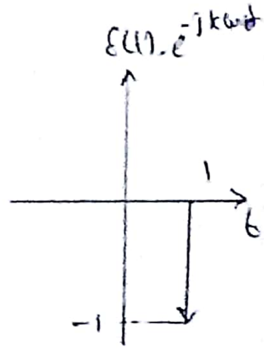
$= \cos(\pi k) - j\sin(\pi k)$

$\therefore$ k is even, $\quad e^{-jk\omega_0 t} = 1$

k is odd $\quad e^{-jk\omega_0 t} = -1$



k is even



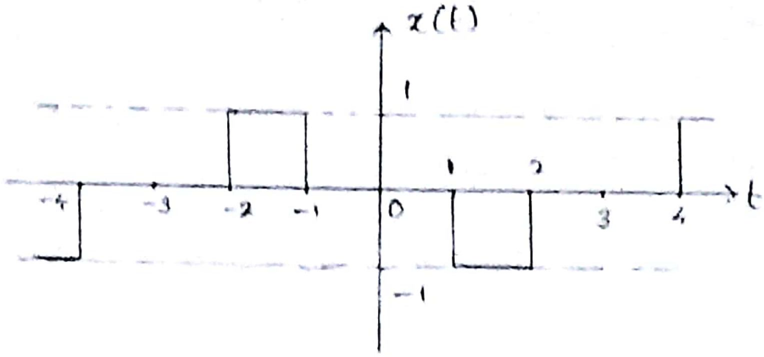
k is odd

$$\therefore \int_{1/2}^{3/2} \delta(t-1)\, e^{-jk\omega_0 t}\, dt = (-1)^k$$

$\therefore$ By ①, $\quad a_k = \frac{1}{T}\left\{1 - 2(-1)^k\right\}$ $\qquad a_k = \frac{1}{2} - (-1)^k \quad ; k \neq 0$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{+jk\pi t} \quad \text{where } a_k = \begin{cases} -\frac{1}{2} & ; k = 0 \\ \frac{1}{2} - (-1)^k & ; k \neq 0 \end{cases}$$

X)



$T = 6$,

$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{3}$

$$x(t) = \begin{cases} 1 , & -2 \le t \le -1 \\ -1 , & 1 \le t \le 2 \end{cases}$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{6}\left\{\int_{-2}^{-1} 1\,dt - \int_{1}^{2} 1\,dt\right\} = \frac{1}{6}\left\{\left|t\right|_{-2}^{-1} - \left|t\right|_{1}^{2}\right\}$$

$$\therefore a_0 = 0$$

$$a_k = \frac{1}{T}\int_T x(t)\,e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{T}\left\{\int_{-2}^{-1} e^{-jk\omega_0 t}\,dt - \int_{1}^{2} e^{-jk\omega_0 t}\,dt\right\}$$

$$= \frac{1}{6}\left\{\frac{\left|e^{-jk\omega_0 t}\right|_{-2}^{-1}}{(-jk\omega_0)} - \frac{\left|e^{-jk\omega_0 t}\right|_{1}^{2}}{(-jk\omega_0)}\right\}$$

$$= \frac{1}{6jk\omega_0}\left\{e^{-2jk\omega_0} - e^{-jk\omega_0} - e^{jk\omega_0} + e^{2jk\omega_0}\right\}$$

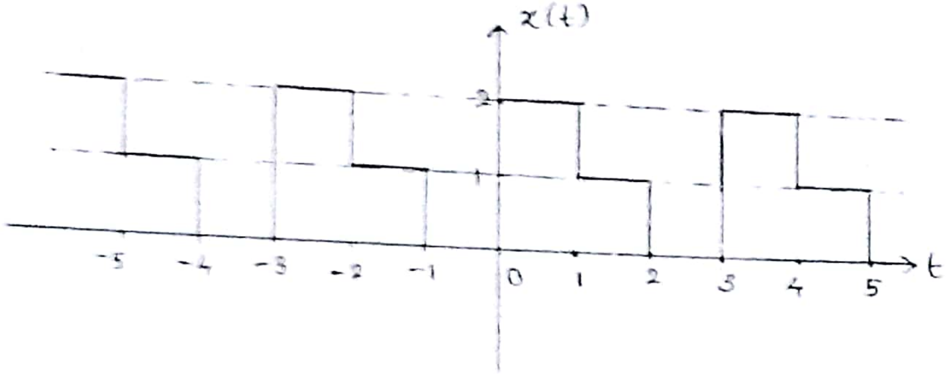
$$= \frac{1}{3jk\omega_0}\left[\frac{e^{2jk\omega_0} + e^{-2jk\omega_0}}{2} - \left(\frac{e^{jk\omega_0} + e^{-jk\omega_0}}{2}\right)\right]$$

$$= \frac{1}{3jk\omega_0}\left[\cos(2k\omega_0) - \cos(k\omega_0)\right]$$

$$\therefore a_k = \frac{\cos(2\pi k/3) - \cos(\pi k/3)}{jk\pi} \quad \&, \; k \neq 0$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{+\frac{j\pi k t}{3}} \quad \text{where} \quad a_k = \begin{cases} 0 ; & k = 0 \\ \dfrac{\cos(2\pi k/3) - \cos(\pi k/3)}{jk\pi} , & k \neq 0 \end{cases}$$

II)



$T = 3$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$

$$x(t) = \begin{cases} 2 ; & 0 \le t \le 1 \\ 1 , & 1 \le t \le 2 \end{cases}$$

$$a_0 = \frac{1}{T}\int_T x(t)\, dt = \frac{1}{3}\left\{ 2\int_0^1 1\, dt + \int_1^2 1\, dt \right\}$$

$$= \frac{1}{3}\left\{ 2|t|_0^1 + |t|_1^2 \right\}$$

$$a_0 = 1$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t} dt = \frac{1}{3}\left\{ 2\int_0^1 e^{-jk\omega_0 t} dt + \int_1^2 e^{-jk\omega_0 t} dt \right\}$$

$$a_k = \frac{1}{3}\left\{ \frac{2}{-jk\omega_0}\left| e^{-jk\omega_0 t}\right|_0^1 + \frac{1}{(-jk\omega_0)}\left| e^{-jk\omega_0 t}\right|_1^2 \right\}$$

$$= \frac{1}{3}\left\{ \frac{2}{-jk\omega_0}\left[e^{-jk\omega_0} - 1\right] - \frac{1}{jk\omega_0}\left[e^{-j2\omega_0 k} - e^{-jk\omega_0}\right] \right\}$$

$$= \frac{1}{3}\left\{ -\frac{e^{-jk\omega_0}}{jk\omega_0} + \frac{2}{jk\omega_0} - \frac{e^{-j2\omega_0 k}}{jk\omega_0} \right\}$$

$$= \frac{2 - [\cos(k\omega_0) - j\sin(k\omega_0)] - [\cos(2k\omega_0) - j\sin(2k\omega_0)]}{3jk\omega_0}$$

$$= \frac{2 - \{\cos(k\omega_0) + \cos(2k\omega_0) - j[\sin(k\omega_0) + \sin(2k\omega_0)]\}}{3jk\omega_0}$$

$$a_k = \frac{2 - 2\cos\left(\frac{k\omega_0}{2}\right)\left[\cos\left(\frac{3k\omega_0}{2}\right) - j\cdot\sin\left(\frac{3k\omega_0}{2}\right)\right]}{3jk\omega_0} = \frac{1 - (-1)^k \cos\left(\pi k/3\right)}{j\pi k} \quad k \ne 0$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{+jk(2\pi/3)t} \quad \text{where} \quad a_k = \begin{cases} 1 ; & k = 0 \\ \frac{1 - (-1)^k \cos(\pi k/3)}{j\pi k} ; & k \ne 0 \end{cases}$$

b) $T = 2, \quad x(t) = e^{-t} \; ; \; -1 < t < 1 \qquad \omega_0 = \frac{2\pi}{T} = \pi$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{2}\int_{-1}^{1} e^{-t}\,dt$$

$$= \frac{1}{2}\,\frac{\left|e^{-t}\right|_{-1}^{1}}{(-1)}$$

$$= \frac{1}{2}\left(e - e^{-1}\right)$$

$$a_0 = 1.175$$

$$a_k = \frac{1}{T}\int_T x(t)\,e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_{-1}^{1} e^{-t}\cdot e^{-jk\omega_0 t}\,dt$$

$$a_k = \frac{1}{2}\int_{-1}^{1} e^{-(1+jk\omega_0)t}\,dt$$

$$= \frac{1}{2}\,\frac{\left|e^{-(1+jk\omega_0)t}\right|_{-1}^{1}}{-(1+jk\omega_0)}$$

$$a_k = \frac{e^{(1+jk\omega_0)} - e^{-(1+jk\omega_0)}}{2(1+jk\omega_0)} = \frac{e^{(1+j\pi k)} - e^{-(1+j\pi k)}}{2(1+j\pi k)}$$

$$\therefore x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{+jk\pi t} \quad \text{where} \quad a_k = \frac{e^{(1+j\pi k)} - e^{-(1+j\pi k)}}{2(1+j\pi k)}$$

c) $T = 4 \qquad x(t) = \begin{cases} \sin(\pi t) & ; \; 0 \le t \le 2 \\ 0 & ; \; 2 < t \le 4 \end{cases} \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{4}\int_0^2 \sin(\pi t)\,dt = \frac{1}{4}\,\frac{\left|-\cos(\pi t)\right|_0^2}{\pi}$$

$$a_0 = \frac{1}{4}\cdot\frac{\left[\cos(0) - \cos(2\pi)\right]}{\pi} = 0$$

$$a_0 = 0$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{4}\int_0^2 \sin(\pi t)\, e^{-jk\omega_0 t}\, dt \quad - (1)$$

$$\cancel{\therefore a_k = \frac{1}{4}} \quad \text{Let } I = \int_0^2 \sin(\pi t)\, e^{-jk\omega_0 t}\, dt$$

$$I = -\frac{1}{jk\omega_0}\left|\sin(\pi t)\, e^{-jk\omega_0 t}\right|_0^2 + \frac{\pi}{jk\omega_0}\int_0^2 \cos(\pi t)\, e^{-jk\omega_0 t}\, dt$$

$$I = \frac{-1}{jk\omega_0}(0) + \frac{\pi}{jk\omega_0}\int_0^2 \cos(\pi t)\, \frac{d}{dt}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right] dt$$

$$I = \frac{\pi}{-(jk\omega_0)^2}\left|\cos(\pi t)\, e^{-jk\omega_0 t}\right|_0^2 - \frac{\pi^2}{(jk\omega_0)^2}\int_0^2 \sin(\pi t)\, e^{-jk\omega_0 t}\, dt$$

$$I\left[1 + \frac{\pi^2}{(jk\omega_0)^2}\right] = -\frac{\pi}{(jk\omega_0)^2}\left[e^{-jk2\omega_0} - 1\right]$$

$$\therefore I = \frac{-\pi}{\pi^2 + (jk\omega_0)^2}\left[e^{-jk2\omega_0} - 1\right]$$

By (1),

$$a_k = \frac{-\pi}{4\left[\pi^2 - k^2\omega_0^2\right]}\left(e^{-jk2\omega_0} - 1\right)$$

$$= \frac{-\pi}{4\left(\pi^2 - \frac{\pi^2 k^2}{4}\right)}\left[\cos(\pi k) - j\sin(\pi k) - 1\right]$$

$$\therefore a_k = \frac{\cos(\pi k) - j\sin(\pi k) - 1}{\pi(k^2 - 4)}$$

$$\therefore a_k = \frac{(-1)^k - 1}{\pi(k^2-4)} \quad ; \; k \neq 0$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{+jk(\pi/2)t} \quad \text{where, } a_k = \begin{cases} 0 \;; & k = 0 \\ \dfrac{(-1)^k - 1}{\pi(k^2-4)} \;; & k \neq 0 \end{cases}$$

Q3] a)

$$a_k = \begin{cases} 0 & ; \quad k = 0 \\ (j)^k \cdot \dfrac{\sin(k\pi/4)}{k\pi} & ; \quad k \neq 0 \end{cases} \qquad T = 4 \qquad \omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$$

Suppose, $x(t) \xleftrightarrow{FS} a_k$

Consider, $b_k = \dfrac{\sin(k\pi/4)}{k\pi}$; $k \neq 0$ Say $y(t) \xleftrightarrow{FS} b_k$

This is of the form $\dfrac{\sin(k\omega_0 T_1)}{k\pi}$ which is a coefficient of the following ~~to~~ signal.

$$f(t) = \begin{cases} 1 & ; \quad |t| < T_1 \\ 0 & ; \quad T_1 < |t| < T/2 \end{cases}$$

$$\omega_0 T_1 = \frac{\pi}{4} \Rightarrow \frac{\pi}{2} T_1 = \frac{\pi}{4} \Rightarrow T_1 = \frac{1}{2} \qquad b_0 = \frac{2T_1}{T} = \frac{2 \times 1/2}{4} = \frac{1}{4}$$

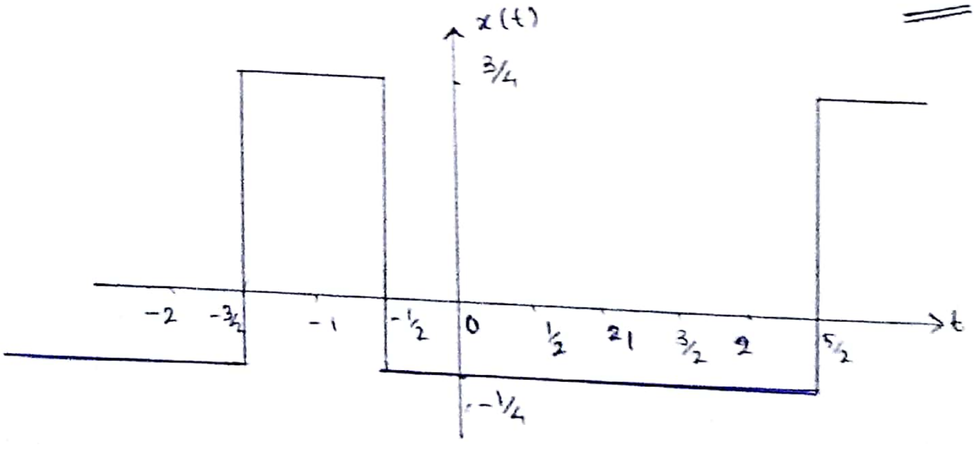
$$\therefore y(t) = \begin{cases} 1 & ; \quad |t| < 1/2 \\ 0 & ; \quad \frac{1}{2} < |t| < 2 \end{cases}$$

$$a_k = (j)^k \frac{\sin(k\pi/4)}{k\pi} = e^{j(\pi/2)k} \cdot b_k = e^{-jk\omega_0(-1)} \cdot b_k \quad \text{(time shift)}$$

$$\cancel{\therefore y} \quad \therefore x(t) = y(t+1) - \frac{1}{4}$$

$$x(t) = \begin{cases} 3/4 & ; \quad |t+1| < \frac{1}{2} \\ -1/4 & ; \quad \frac{1}{2} < |t+1| < 2 \end{cases}$$

$$\Rightarrow x(t) = \begin{cases} 3/4 & ; \quad -3/2 < t < -1/2 \\ -1/4 & ; \quad -2 < t < -3/2 \text{ or } -1/2 < t < 2 \end{cases}$$



b) $a_k = (-1)^k \dfrac{\sin(k\pi/8)}{2k\pi}$ Suppose $x(t) \xleftrightarrow{FS} a_k$

Consider, $y(t) \xleftrightarrow{FS} b_k$

Where, $b_k = \dfrac{\sin(k\pi/8)}{2k\pi}$; $k \neq 0$ then $\frac{\pi}{2} T_1 = \frac{\pi}{8} \Rightarrow T_1 = \frac{1}{4}$

$\therefore b_0 = \dfrac{2 \times 1/4}{4} = \dfrac{1}{8}$

$$\therefore y(t) = \begin{cases} 1/2 \ ; & |t| < 1/4 \\ 0 \ ; & 1/4 < |t| < 2 \end{cases}$$

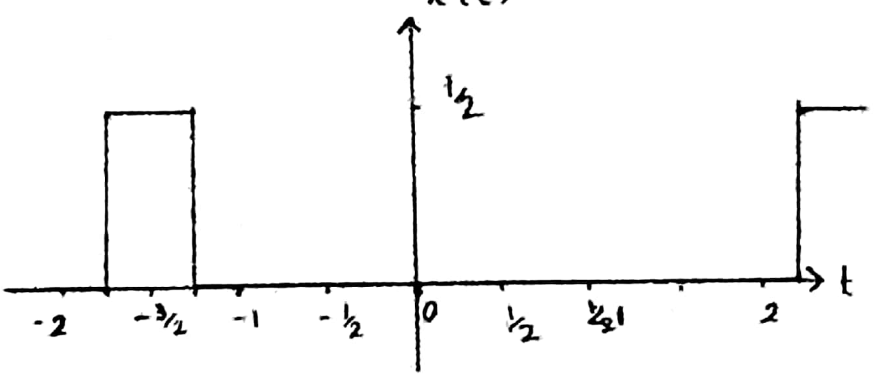
$$a_k = (-1)^k b_k = [\cos(\pi k) + j\sin(\pi k)]\, b_k = e^{j\pi k} b_k$$

$$\Rightarrow a_k = e^{-j(\pi/2)(-2)k} \cdot b_k \qquad \therefore t_0 = -2$$

$$x(t) = y(t+2)$$

$$\therefore x(t) = \begin{cases} 1/2 \ ; & |t+2| < 1/4 \\ 0 \ ; & 1/4 < |t+2| < 2 \end{cases}$$

$$\Rightarrow x(t) = \begin{cases} 1/2 \ ; & -9/4 < t < -7/4 \\ 0 \ ; & -2 < t < -9/4 \text{ or } -7/4 < t < 2 \end{cases}$$



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c) $a_k = \begin{cases} jk \; ; & |k| < 3 \\ 0 \; ; & \cancel{3} \text{ otherwise} \end{cases}$

$$\therefore x(t) = \sum_{k=-2}^{2} jk \, e^{jk(\pi/2)t}$$

$$= -2j\,e^{-j\pi t} - j\,e^{-j(\pi/2)t} + j\,e^{j(\pi/2)t} + 2j\,e^{j\pi t}$$

$$= \cancel{2} j \left[e^{j(\pi/2)t} - e^{-j(\pi/2)t}\right] + 2j \left[e^{j\pi t} - e^{-j\pi t}\right]$$

$$= j \cdot 2j \sin(\pi/2\, t) + 2j \cdot 2j \sin(\pi t)$$

$$\therefore x(t) = -2 \sin(\tfrac{\pi}{2} t) - 4 \sin(\pi t)$$

d) $a_k = \begin{cases} 1 \; , & k \text{ is even} \\ 2 \; ; & k \text{ is odd} \end{cases}$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{jk(\pi/2)t}$$

$$= \dots + e^{-j2\pi t} + 2\,e^{-j(3\pi/2)t} + e^{-j\pi t} + 2\,e^{-j(\pi/2)t} + 1 + 2e^{j(\pi/2)t} + e^{j\pi t} + 2e^{j(3\pi/2)t} + e^{j(2\pi)t} + \dots$$

$$= 1 + 2\left[e^{j(\pi/2)t} + e^{-j(\pi/2)t}\right] + \left[e^{j\pi t} + e^{-j\pi t}\right] + 2\left[e^{j(3\pi/2)t} + e^{-j(3\pi/2)t}\right] + \left[e^{j(2\pi t)} + e^{-j(2\pi t)}\right] + \dots$$

$$\therefore x(t) = 1 + 4\cos(\tfrac{\pi}{2} t) + 2\cos(\pi t) + 4\cos(\tfrac{3\pi}{2} t) + 2\cos(2\pi t) + \dots$$

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Q4]

$$x(t) = \begin{cases} t \; ; & 0 \leq t \leq 1 \\ 2-t \; ; & 1 \leq t \leq 2 \end{cases} \qquad T = 2 \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$$x(t) \xleftrightarrow{FS} a_k$$

a)

$$a_0 = \frac{1}{T}\int_T x(t).dt = \frac{1}{2}\left\{ \int_0^1 t\,dt + \int_1^2 (2-t).dt \right\}$$

$$= \frac{1}{2}\left\{ \left|\frac{t^2}{2}\right|_0^1 + 2|t|_1^2 - \left|\frac{t^2}{2}\right|_1^2 \right\}$$

$$= \frac{1}{2}\left\{ \frac{1}{2} + 2 - 2 + \frac{1}{2} \right\}$$

$$\underline{\underline{a_0 = \frac{1}{2}}}$$

$$\cancel{a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}.dt}$$

$$\cancel{= \frac{1}{2}\left\{ \int_0^1 t.\, e^{-jk\omega_0 t} \right.}$$

b)

$$y(t) = \frac{d}{dt}[x(t)] = \begin{cases} 1 \; ; & 0 \leq t \leq 1 \\ -1 \; ; & 1 \leq t \leq 2 \end{cases}$$

$$y(t) \xleftrightarrow{FS} b_k$$

$$b_0 = 0$$

$$b_k = \frac{1}{T}\int_T y(t).\, e^{-jk\omega_0 t}.dt$$

$$= \frac{1}{2}\left\{ \int_0^1 e^{-jk\omega_0 t}\,dt - \int_1^2 e^{-jk\omega_0 t}\,dt \right\}$$

$$= \frac{1}{2}\left\{ \frac{\left|e^{-jk\omega_0 t}\right|_0^1}{-jk\omega_0} + \frac{\left|e^{-jk\omega_0 t}\right|_1^2}{jk\omega_0} \right\}$$

$$= \frac{1}{2}\left\{ \frac{1 - e^{-j\omega_0 k}}{jk\omega_0} + \frac{e^{-j2\omega_0 k} - e^{-j\omega_0 k}}{jk\omega_0} \right\}$$

$$\therefore b_k = \frac{\left[1 - 2e^{-j\pi k} + e^{-j2\pi k}\right]}{2jk\pi}$$

$$b_k = \frac{1 - 2\left[\cos(\pi k) - j\sin(\pi k)\right] + \cos(2\pi k) - j\sin(2\pi k)}{j(2\pi k)}$$

$$= \frac{1 - 2\cos(\pi k) + 1}{j(2\pi k)}$$

$$\therefore b_k = \frac{1 - \cos(\pi k)}{j\pi k} \; ; \; k \neq 0$$

$$\therefore b_k = \begin{cases} 0 \; ; & k = 0 \\ \dfrac{1 - \cos(\pi k)}{j\pi k} \; ; & k \neq 0 \end{cases}$$

c)

$$x(t) \xleftrightarrow{FS} a_k$$

$$y(t) \xleftrightarrow{FS} b_k$$

By differentiation property, $b_k = jk\omega_0 a_k$

$$\Rightarrow a_k = \frac{1}{jk\omega_0} \cdot b_k$$

$$\therefore a_k = \frac{1}{jk\pi} \cdot \frac{[1-\cos(\pi k)]}{j\pi k}$$

$$\therefore a_k = \frac{\cos(\pi k) - 1}{k^2\pi^2} \quad ; k \neq 0$$

$$a_k = \begin{cases} 1/2 \ ; & k = 0 \\ \dfrac{\cos(\pi k) - 1}{k^2\pi^2} \ ; & k \neq 0 \end{cases}$$

05

$T = 1/2 \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1/2} = 4\pi$

$x(t) = \cos(4\pi t) \qquad x(t) \xleftrightarrow{FS} a_k$

$y(t) = \sin(4\pi t) \qquad y(t) \xleftrightarrow{FS} b_k$

$z(t) = x(t) \cdot y(t) \qquad z(t) \xleftrightarrow{FS} c_k$

a)

$$x(t) = \cos(4\pi t) = \frac{e^{j4\pi t} + e^{-j4\pi t}}{2} = \frac{1}{2}e^{j4\pi t} + \frac{1}{2}e^{-j4\pi t}$$

$$a_k = \begin{cases} \frac{1}{2} \ ; & |k| = 1 \\ 0 \ ; & \text{otherwise} \end{cases}$$

b)

$$y(t) = \sin(4\pi t) = \frac{e^{j4\pi t} - e^{-j4\pi t}}{2j} = \frac{1}{2j}e^{j4\pi t} - \frac{1}{2j} \cdot e^{-j4\pi t}$$

$b_1 = \frac{1}{2j} \qquad b_{-1} = -\frac{1}{2j} \qquad b_k = 0 \ ; \text{otherwise}$

c)

$$c_1 = \sum_{i=-1}^{1} a_i b_{1-i} = a_{-1}b_2 + a_0 b_1 + a_1 b_0 = 0$$

$$c_{-1} = 0$$

$$C_2 = \sum_{i=-1}^{1} a_i b_{2-i} = \cancel{C_1 + a}\ a_{-1} b_3 + a_0 b_2 + a_1 b_1 + a_2 b_0$$

$$= \frac{1}{2} \times \frac{1}{2j}$$

$$\therefore C_2 = \frac{1}{4j}$$

$$C_{-2} = \sum_{\cancel{i=-1}}^{1} a_i b_{-2-i} = a_{-1} b_{-1} + a_0 b_{-2} + a_1 b_{-3}$$

$$= \frac{1}{2} \cdot \times -\frac{1}{2j}$$

$$\therefore C_{-2} = -\frac{1}{4j}$$

$$\therefore C_k = \begin{cases} \frac{1}{4j} & : \quad k = 2 \\ -\frac{1}{4j} & : \quad k = -2 \\ 0 & : \quad \text{otherwise} \end{cases}$$

d) $$z(t) = \cos(4\pi t) \cdot \sin(4\pi t) = \frac{1}{2} \cdot \sin(8\pi t)$$

$$\therefore z(t) = \frac{1}{2} \cdot \left[\frac{e^{j8\pi t} - e^{-j8\pi t}}{2j}\right] = \frac{1}{4j} \cdot e^{j2(4\pi)t} - \frac{1}{4j} \cdot e^{j(-2)(4\pi)t}$$

$$\therefore C_2 = \frac{1}{4j}, \quad C_{-2} = -\frac{1}{4j}, \quad C_k = 0, \text{ otherwise}$$

06]

$$a_k = \begin{cases} 2 & : \quad k = 0 \\ j\left(\frac{1}{2}\right)^{|k|} & : \quad \text{otherwise} \end{cases}$$

a) $x(t)$ is real $\Leftrightarrow$ $x^*(t) = x(t)$ $\Leftrightarrow$ $a_k = a_{-k}^*$

$$a_k = j\left(\frac{1}{2}\right)^{|k|} \text{ — (1)} \qquad a_{-k} = j\left(\frac{1}{2}\right)^{|k|} \text{ — (2)} \Rightarrow a_{-k}^* = -j\left(\frac{1}{2}\right)^{|k|}$$

$$\therefore a_k \neq a_{-k}^*$$

$\therefore$ $x(t)$ is not real

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b) 并 $x(t)$ is even $\Leftrightarrow$ $x(t) = x(-t)$ $\Leftrightarrow$ $a_k = a_{-k}$

By ① & ②, $a_k = a_{-k}$

$\therefore$ $x(t)$ is even

c) Suppose, $g(t) = \frac{d[x(t)]}{dt} \xleftrightarrow{FS} b_k = jk\omega_0 a_k$

$$\therefore b_k = \begin{cases} 0 \; ; & k = 0 \\ -k\omega_0 \left(\frac{1}{2}\right)^{|k|} ; & \text{otherwise} \end{cases}$$

$b_k = -k\omega_0 \left(\frac{1}{2}\right)^{|k|}$ — ③, $\quad b_{-k} = k\omega_0 \left(\frac{1}{2}\right)^{|k|}$ — ④

$\Rightarrow$ $g(t)$ is not even

07] $\quad x(t) \xleftrightarrow{FS} a_k$

a) $x(t-t_0) + x(t+t_0) \xleftrightarrow{FS} b_k$

$x(t-t_0) \xleftrightarrow{FS} c_k$

$C_k = \frac{1}{T}\int_T x(t-t_0)\, e^{-jk\omega_0 t} . dt$ $\quad$ Substitute, $t - t_0 = \tau \Rightarrow dt = d\tau$
$t = \tau + t_0$

$= \frac{1}{T}\int_T x(\tau)\, e^{-jk\omega_0(t_0+\tau)} . d\tau$

$\therefore C_k = e^{-jk\omega_0 t_0} \frac{1}{T}\int_T x(\tau)\, e^{-jk\omega_0 \tau} . d\tau = a_k . e^{-jk\omega_0 t_0}$

$\therefore b_k = a_k e^{-jk\omega_0 t_0} + a_k e^{jk\omega_0 t_0}$

$= a_k \left[e^{jk(2\pi/T)t_0} + e^{-jk(2\pi/T)t_0} \right]$

$\therefore b_k = 2a_k \cos\left(\frac{2\pi k t_0}{T}\right)$

b) $Ev\{x(t)\} \xleftrightarrow{FS} b_k$

$$Ev\{x(t)\} = \frac{x(t) + x(-t)}{2}$$

$$x(-t) \xleftrightarrow{FS} c_k$$

$$c_k = \frac{1}{T}\int_T x(-t) \cdot e^{-jk\omega_0 t} \cdot dt \qquad -t = \tau \Rightarrow -dt = d\tau$$

$$\cancel{t \to 0} \quad t \to T, \; \tau \to -T$$

$$= \frac{1}{T}\int_T x(\tau) \cdot e^{jk\omega_0 \tau} \cdot d\tau$$

$$= \frac{1}{T}\int_T x(\tau) \cdot e^{-j(-k)\omega_0 \tau} \cdot d\tau$$

$$\therefore c_k = a_{-k}$$

$$\therefore b_k = \frac{a_k + a_{-k}}{2}$$

c) $Re\{x(t)\} \xleftrightarrow{FS} b_k \qquad Re\{x(t)\} = \frac{x(t) + x^*(t)}{2}$

Let, $x^*(t) \xleftrightarrow{FS} c_k$

$$c_k = \frac{1}{T}\int_T x^*(t) \cdot e^{-jk\omega_0 t} \cdot dt \Rightarrow c_k^* = \frac{1}{T}\int_T x(t) \cdot e^{jk\omega_0 t} \cdot dt$$

$$\Rightarrow c_k^* = a_{-k} \Rightarrow c_k = a_{-k}^*$$

$$\therefore b_k = \frac{a_k + a_{-k}^*}{2}$$

d)

$$y(t) = \frac{d^2[x(t)]}{dt^2} \xleftrightarrow{FS} b_k$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{jk\omega_0 t}$$

$$\frac{d[x(t)]}{dt} = \sum_{k=-\infty}^{\infty} jk\omega_0 \cdot a_k \cdot e^{jk\omega_0 t}$$

$$\frac{d^2[x(t)]}{dt^2} = \sum_{k=-\infty}^{\infty} (jk\omega_0)^2 \cdot a_k \cdot e^{jk\omega_0 t}$$

$$\therefore \quad b_k = (jk\omega_0)^2 a_k = -k^2 \cdot (2\pi/T)^2 \cdot a_k$$

$$\therefore \quad b_k = -\frac{4\pi^2 k^2}{T^2} \cdot a_k$$

e)

$$x(t) \xleftrightarrow{FS} a_k$$

$$x(3t-1) \xleftrightarrow{FS} b_k$$

period of $x(3t-1) = T/3$

$$x(3t) \xleftrightarrow{FS} a_k$$

$$\therefore \quad b_k = a_k \cdot e^{-jk\omega_0(1)}$$

$$\Rightarrow \quad b_k = a_k \cdot e^{-jk(2\pi/T)}$$

28] $T = 3$ $\quad x(t) \xleftrightarrow{FS} a_k$

a) $a_k = a_{k+2}$

b) $a_k = a_{-k}$

c) $\displaystyle\int_{-0.5}^{0.5} x(t)\,dt = 1$

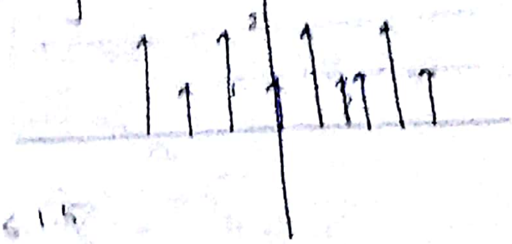
d) $\displaystyle\int_{0.5}^{1.5} x(t)\,dt = 2$

By b) $x(t)$ is an even signal

~~By a),~~ $\cancel{x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t} + \sum_{m=-\infty}^{\infty} a_{m+1} e^{jm\omega_0 t}}$

By a),

$$x(t) = \sum_{k=-\infty}^{\infty} \left[a_{2k} e^{jk\omega_0 t} + a_{2k+1} e^{jk\omega_0 t}\right]$$



By c), $x(t) = \delta(t)$, $-0.5 \le t \le 0.5$

By d), $x(t) = 2\delta(t-1)$; $0.5 \le t \le 1.5$

$$x(t) = \dots + 2\delta(t+3) + \delta(t+2) + 2\delta(t+1) + \delta(t) + 2\delta(t-1) + \delta(t-2) + 2\delta(t-3) + \dots$$

$$x(t) = \sum_{k=-\infty}^{\infty} \left\{\delta(t-2k) + 2\delta[t-(2k+1)]\right\}$$

29] $x(t) \xleftrightarrow{FS} a_k$ $\quad x(t)$ is a real valued signal

a) If $x(t)$ is real, then

$x(t) = x^*(t)$ — ①

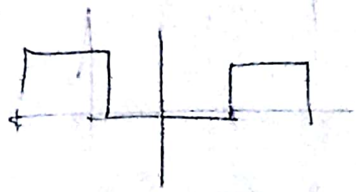
$x^*(t) \xleftrightarrow{FS} a^*_{-k}$ (conjugate property) — ②

proof

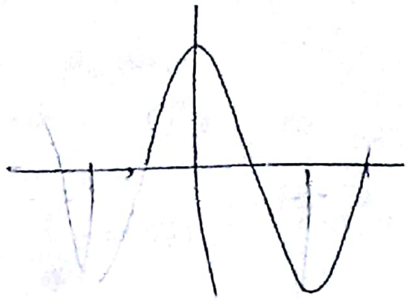
By ① & ②, $a_k = a^*_{-k}$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt$$

~~$a_k = \int_T x(t)$~~ Let $x^*(t) \xleftrightarrow{FS} b_k$ $a_k = a^*_{-k}$



$$a_k = \frac{1}{T}\int_T x(t)\, e^{jk\omega_0 t}\, dt$$



$$b_k = \frac{1}{T}\int_T x^*(t) . e$$

When $k = 0$,

$a_0 = a_0^*$

When $a_k = a^*_{-k}$, $Re\{a_k\} = Re\{a_{-k}\}$ and $Im\{a_k\} = - Im\{a_{-k}\}$

$$\therefore \sum_{k=-\infty}^{-1} a_k e^{jk\omega_0 t} + \sum_{k=1}^{\infty} a_k e^{jk\omega_0 t} \text{ is real}$$

$\Rightarrow x(t) - a_0$ is real

$\Rightarrow a_0$ is real ($\because x(t)$ is real)

b) $x(t)$ is even $\Rightarrow a_k = a_{-k}$ — ①

since $x(t)$ is real, $a_k = a_k^*$ — ②

By ① & ②, $a_k = a_k^* \Rightarrow a_k$ is real and even

c) If $x(t)$ is odd, $x(t) = -x(-t)$

~~$x(t) =$~~ $a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$ — ①

$x(t) \xleftrightarrow{FS} a_k$

~~$b_k = \frac{1}{T}\int_T$~~ ~~$x(t) = \sum a_k e^{jk\omega_0 t}$~~

$-x(-t) \xleftrightarrow{FS} -a_{-k}$

~~$-x(-t) = -\sum b_k e^{-jk\omega_0 t}$~~

$\therefore a_k = -a_{-k}$

$\Rightarrow$ Coefficients are odd.

Because $x(t)$ is real, $a_{-k} = a_k^*$

$a_k = -a_k^*$ $\Rightarrow$

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d) $Ev[x(t)] = \dfrac{x(t) + x(-t)}{2}$

$$Ev[x(t)] \xleftrightarrow{FS} \frac{a_k + a_{-k}}{2}$$

since $x(t)$ is real, $a_k^* = a_{-k}$

$$\frac{a_k + a_{-k}}{2} = \frac{a_k + a_k^*}{2} = \frac{2Re\{a_k\}}{2} = \underline{\underline{Re\{a_k\}}}$$

e) $Od[x(t)] = \dfrac{x(t) - x(-t)}{2}$

$$Od[x(t)] \xleftrightarrow{FS} \frac{a_k - a_{-k}}{2}$$

$$\frac{a_k - a_{-k}}{2} = \frac{a_k - a_k^*}{2} = \frac{2Im\{a_k\}}{2} = \underline{\underline{Im\{a_k\}}}$$

$\boxed{10}$ $$x(t) = a_0 + 2\sum_{k=1}^{\infty}\left[B_k \cos k\omega_0 t - C_k \sin k\omega_0 t\right] \quad \text{—— (1)}$$

a) $$Ev[x(t)] = \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega_0 t} \quad \text{—— (2)}$$

~~By (1)~~,

$$Ev[x(t)] = \frac{x(t) + x(-t)}{2}$$

$$= \frac{a_0 + 2\sum_{k=1}^{\infty} B_k \cos(k\omega_0 t) - C_k \sin(k\omega_0 t) + a_0 + 2\sum_{k=1}^{\infty} B_k \cos(k\omega_0 t) + C_k \sin(k\omega_0 t)}{2}$$

$$= a_0 + 2\sum_{k=1}^{\infty} B_k \cos(k\omega_0 t)$$

$$= a_0 e^{j(0)\omega_0 t} + 2\sum_{k=1}^{\infty} B_k \left[\frac{e^{jk\omega_0 t} + e^{-jk\omega_0 t}}{2}\right]$$

$=$