KEEPING UP WITH THE JONES': ELECTRIC VEHICLE ADOPTION IN THE U.S.

JEFF HANSEN, ELIZA POWELL, GWEN MARTIN, DALLIN STEWART

Abstract. In this study, we endeavor to model the production and adoption of combustion, hybrid, and electric vehicles in the United States from 2010 into the near future. We explore the dynamics influenced by societal behaviors, cultural dynamics, and social interactions due to government regulations, economic disasters, and the growth of charging infrastructure. Our analyses reveal a shifting trend towards electric vehicles, emphasizing the need for proactive infrastructure development. This study lays the groundwork for assessing the myriad of influences on the adoption of alternative vehicles, and contributes to a more informed and realistic understanding of future automotive landscapes.

1. Background and Motivation

We seek to model the production of combustion, hybrid, and electric vehicles (EVs) in the United States from 2010 into the near future. Electric vehicles are crucial in mitigating climate change by reducing greenhouse gas emissions and dependence on fossil fuels. Modeling their adoption over time aids in understanding societal transitions to cleaner and sustainable transportation, guiding policy decisions and infrastructure development toward a more sustainable future.

Automotive companies first began producing hybrid vehicles in 1999, but the first fully electric vehicles were not commercially available until 2010. Since then, demand for these types of vehicles has increased significantly. We model the adoption of hybrid and electric vehicles over time since 2010 with a modified SIR model. While there are many factors that influence which types of vehicles companies produce and sell in the United States, we rely primarily on the effect that the relative population of each type of car has on the other populations. While cars do not directly influence each other in this way, this dynamic emerges from of people's decisions, which do affect each other. For example, local cultural values transmit as people exchange ideas and share their priorities. In addition, people with more friends and associates who own hybrid or electric vehicles are more likely to make a similar purchase. By incorporating the effect of social influence and cultural dynamics within our modified SIR model, we strive to better understand the intricate interactions shaping the adoption of hybrid and

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electric vehicles. This assumption acknowledges the significance of societal behaviors in steering the future landscape of sustainable transportation.

This model is helpful for navigating the dynamics of vehicle production, adoption, and sustainability. By examining the patterns of hybrid and electric vehicle adoption, our research offers practical insights for policymakers. These insights can inform decisions on infrastructure development, economic policies, and environmental impact goals. Understanding the factors that determine the adoption of sustainable transportation fosters a more informed and effective approach in shaping the future of the automotive industry and society as a whole.

Prior research on modeling EV adoption commonly relies on statistical, econometric, and machine learning models rather than ordinary differential equations [ASK+23], [JSCZ20], [Sal22]. For example Javid and Nejat sought to measure EV adoption and its effect on greenhouse gas emissions [JN17]. Their approach relies on techniques such as mixed logit, generalized extreme value, and probit statistical models. After reviewing current research and studies on this topic, we observe that previous models primarily employed data-driven approaches like these to understand the determinants of consumer EV purchasing behavior rather than predictive dynamical models like the one we present here.

2. Modeling

We model electric vehicle adoption with a modified SIR model with three states: combustion (C), hybrid (H), and electric (E). While we constrain the total population to a constant, and model each population as a ratio of the entire population. These constraints ensure the ratios will always sum to one. The initial conditions in each version of the model presented are the ratios of car purchases in 2010, since this was the first year that EVs were commercially available [U.S22].

For more clear control over the model, the parameters that dictate the interaction and transition between states are determined by the states in question. For example, ch indicates the rate of transition from combustion to hybrid vehicles, while hc indicates hybrid to combustion transitions. See Table 1 for more details. The model computes the difference before creating the differential equation. We explore multiple scenarios in the Results section, and each will vary from the model presented below by including additional terms or modifying these parameters.

$$\begin{aligned}
\dot{C} &= (hc - ch) \cdot C(t) \cdot H(t) + (ec - ce) \cdot C(t) \cdot E(t) \\
\dot{H} &= (ch - hc) \cdot C(t) \cdot H(t) + (eh - he) \cdot H(t) \cdot E(t) \\
\dot{E} &= (ce - ec) \cdot C(t) \cdot E(t) + (he - eh) \cdot H(t) \cdot E(t)
\end{aligned}$$

TABLE 1. Model Parameters

Parameters	**Rate of Transition From**
hc	Hybrid to Combustion
ch	Combustion to Hybrid
ce	Combustion to Electric
ec	Electric to Combustion
he	Hybrid to Electric
eh	Electric to Hybrid

3. Results

We begin our investigation with a conservative model of electric vehicle adoption, assuming current conditions and perceptions do not change significantly. Since transportation industries do not directly measure or record the rate of transition between vehicles types in the United States, we must instead rely on total sales data in order to determine these coefficients. Based on vehicle proportion data [U.S22], we fine-tuned the parameters to most closely align historical proportion patterns while also yielding a conservative approach to electric adoption. Under these conditions, $hc - ch = -0.05$, $ec - ce = -0.09$, and $eh - he = -0.19$. This model indicates that people who already own a hybrid vehicle (green) transition most to a new electric vehicle, but more people transition from combustion to hybrid or electric than the opposite direction. See the Conservative Model in Figure 1.

However, a model that more closely matches the historical data implies a much more drastic shift in electric vehicle adoption. We tune the parameters of our conservative model to $hc - ch = -0.09$, $ec - ce = -0.09$, and $eh - he = -0.02$. We also incorporate an additional term to the rate of change in the population of electric vehicles, $e = 0.0007$. This addition changes the model from a pure SIR model to $\dot{E} = (ce - ec) \cdot C(t) \cdot E(t) + (he - eh) \cdot H(t) \cdot E(t) + e$.

In this projection, the total population of vehicles increases because of the rise in electric vehicles, but the transition between hybrid and electric vehicles and vice versa is much more equal. See the Progressive Model in Figure 1.

Armed with these models in isolation, we now consider how additional factors might influence the future adoption of these alternative vehicles.

3.1. **Phasing Out Combustion Vehicles: California Regulation.**

In August 2022, California approved new legislation requiring all new cars sold after 2035 to be zero emission vehicles. According to the New York Times, California's vehicle conversion "is widely expected to accelerate the global transition toward electric vehicles" [DFP22]. In an effort to analyze the production of vehicles in California, we examine in isolation the population of electric and combustion vehicles in the near future. This model reflects trends in California and predictions by Symon et al [Sym23] based on the

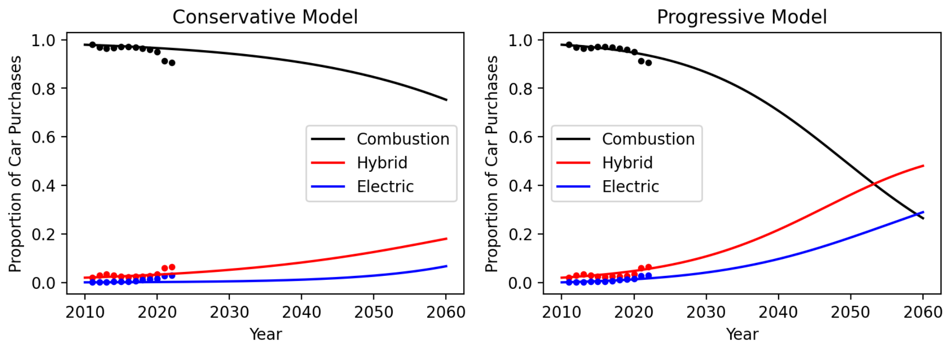

Figure 1. Model predictions, without explicitly accounting for outside influences. Scattered data represents real ratios of combustion, hybrid, and electric car sales. *Source: U.S. Department of Energy.*

current conversion around electric sales in California. Because this data only measures electric vehicle production, we determine the proportion of combustion vehicles by calculating the remaining percentage.

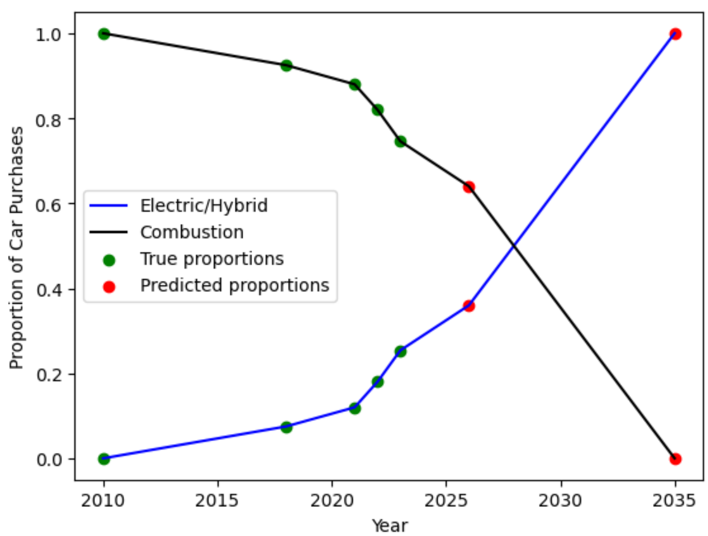

Figure 2. The following data was obtained by 'California Globe' [Sym23] and describes past proportions of vehicles in California as well as planned sales proportions for the future.

In this scenario, we do not differentiate hybrid from electric, as the regulations only prohibits combustion car sales. Instead, we assume that the electric category includes any form of a zero emission vehicle, including battery electric vehicles, plug-in hybrid electric vehicles as well as fuel cell electric vehicles. Furthermore, we restrict this model to only account for the population of cars sold within the state of California. We thus arrive at the following dynamical model for California regulation of electric vehicles:

$$\dot{C} = (-0.4) \cdot C(t) \cdot E(t) + (-0.023) \cdot C(t)$$
$$\dot{E} = (0.3) \cdot C(t) \cdot E(t) + (0.1) \cdot E(t)$$

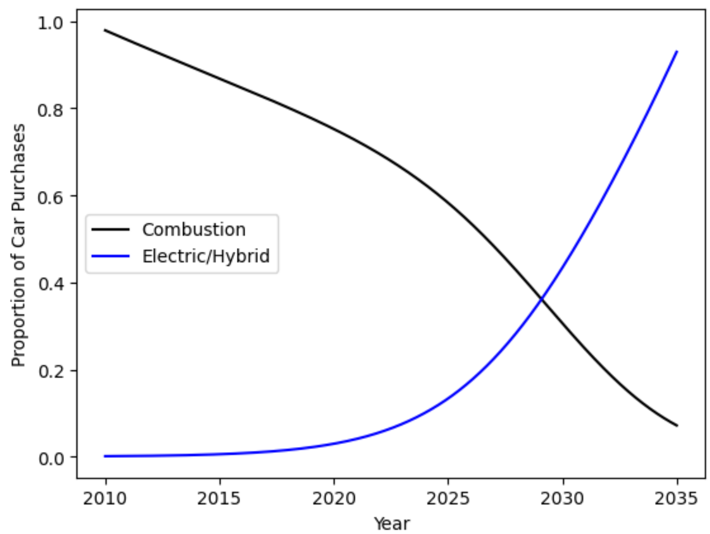

FIGURE 3. Modeled Proportion of Electric and Combustion Vehicles in California

This model is unable to perfectly match reality, as the predicted proportions range between 0.05 and 0.17 while the true data reflects significant growth from 0.10 to 0.30. However, this model resembles the general shape and structure of the true data. By 2035, we see that the proportion of combustion sales falls near zero, while electric vehicles is approximately 1.

3.2. **Disaster Effect.** Incorporating the effects of economic disasters into our model is crucial for gaining a better understanding of how adverse conditions impact consumer behavior and the dynamics of vehicle sales. During recessions, consumers tend to prioritize essential expenses and delay discretionary spending such as vehicle purchases. In order to understand the effect that a recession has on car sales, we first examine historical car sales in relation to economic recessions.

From this data, we see that as an economic disaster occurs, the total number of vehicle sales immediately decreases. After the recession has ended, the total number of car sales steadily increases back to its original equilibrium. We can predict the trajectory of car sales when an economic disaster

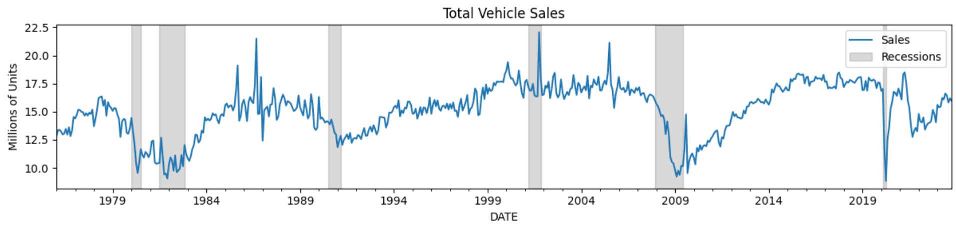

FIGURE 4. *Source: U.S. Bureau of Economic Analysis* [oEA23]

occurs based on factors such as predicted length (D) and rate of intensity of the disaster (r). In addition, our model includes an upper limit (U), a lower limit (L), and reflects the total number of car sales (S).

$$\dot{S} = (r \cdot S \cdot \left(1 - \frac{S-L}{U-L}\right) \cdot (t-D)$$

We compare our disaster model to actual car sale data from the 2008 Housing Market crash and the Covid-19 Pandemic:

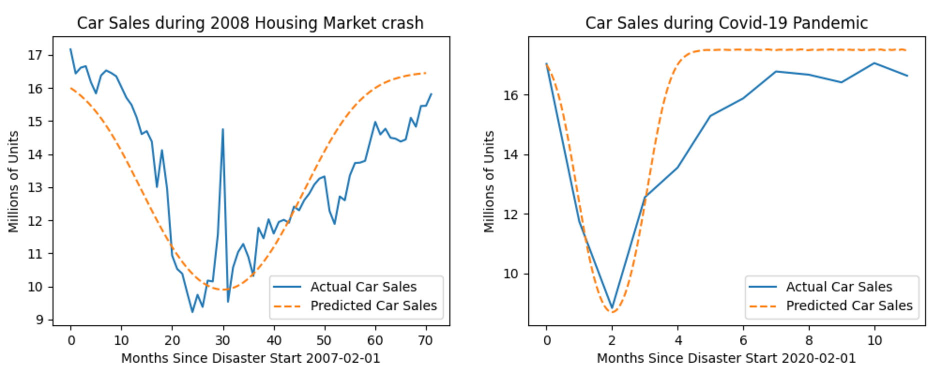

FIGURE 5. *Source: U.S. Bureau of Economic Analysis*

We now transition from modeling the effects of a disaster on total car sales to the effects of the disaster on combustion, hybrid, and electric vehicle sales individually. We adjust the original model in Figure 1 to account for the effects of an economic disaster on each car type:

$$\dot{C} = (hc - ch) \cdot C(t) \cdot H(t) + (ec - ce) \cdot C(t) \cdot E(t) + c + R_1(t)$$
$$\dot{H} = (ch - hc) \cdot C(t) \cdot H(t) + (eh - he) \cdot H(t) \cdot E(t) + h + R_2(t)$$
$$\dot{E} = (ce - ec) \cdot C(t) \cdot E(t) + (he - eh) \cdot H(t) \cdot E(t) + e + R_3(t)$$

In this system of equations, each $R_i = rS\left(1 + \frac{S-L_i}{L_i}\right)(t-D)$ reflects the effect of the disaster corresponding to each vehicle type for the duration of the disaster. L_i reflects the lower limit of each vehicle type. This value is proportional to each vehicle's initial condition, measured when the disaster strikes. We leave the upper limit as the default upper limit from the base model.

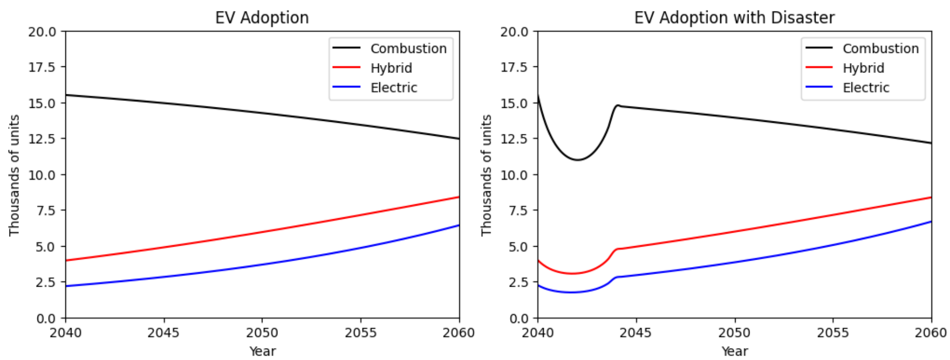

FIGURE 6. Modeling the effects of a potential economic disaster in the year of 2040. Graph has been adjusted to reflect quantities of sales instead of proportion of sales.

We can observe the adverse impact of the disaster on vehicle sales in the model, along with the subsequent rebound in sales for each vehicle type. This model aptly mirrors the overall shape and severity of genuine economic disasters, providing a solid foundation for sales predictions. However, overidealism still limits the predictive power of the model. While the predicted rate of returning to normalcy precisely matches the rate of the initial decline, the true recovery pace is typically more gradual than the initial downturn. Simply put, the recovery period following a disaster tends to be lengthier than the time it takes to reach the peak of the crisis. This discrepancy becomes evident when examining car sales during the 2008 Housing Market crash (refer to Figure 5). Our model proves conservative during the crash and overly optimistic during the recovery phase. Delving deeper into these limitations promises a more nuanced understanding and, consequently, more accurate modeling results.

3.3. **Reliance on Charging Stations.** When deciding to switch to an electric vehicle, consumers often consider the range that an EV can travel on a single charge. As companies provide more charging stations, EVs can travel farther, and this factor may appear more enticing to consumers. This relationship implies that the expansion of electric vehicle adoption is intimately linked to the availability of charging stations. To account for this relationship in our model, we include a fourth term $\dot{S}(t)$ to represent the number of charging stations in the US, and we update the $\dot{E}(t)$ term by adding $\alpha S(t)$. This term represents the number of additional EVs that people purchase because of the construction of charging stations. We also add an additional $(1 - E(t))$ factor and multiply this by the right-hand side of the $\dot{E}(t)$ equation to ensure that electric vehicles do not surpass 100% of total sales.

$$\dot{E}(t) = [(ce - ec) \cdot C(t) \cdot E(t) + (he - eh) \cdot H(t) \cdot E(t) + e + \alpha \cdot S(t)] \cdot (1 - E(t))$$

In order to create the $\dot{S}(t)$ portion of the model, we found data regarding the number of charging stations in the US at the end of each year. As demonstrated in Figure 7 the number of charging stations over time patterns exponential growth. A simple way to model this growth is $\dot{S}(t) = \beta S(t)$, but this causes the number of charging stations to grow to infinity. To combat this we add a discount factor $(1 - \frac{S(t)}{C_s})$. We claim that the number of charging stations will never surpass the number of current gas stations, which is roughly 168,000. So we set $C_s = 168,000$ when running the model.

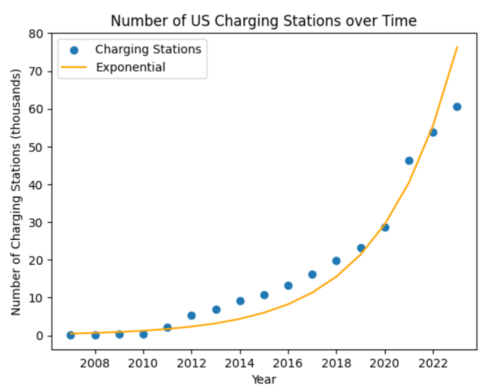

FIGURE 7. The rate of charging stations is exponential overtime
Source: U.S. Department of Energy [oE23]

Putting all of these additions together yields the system of equations:

$$\dot{C}(t) = (hc - ch) \cdot C(t) \cdot H(t) + (ec - ce) \cdot C(t) \cdot E(t)$$
$$\dot{H}(t) = (ch - hc) \cdot C(t) \cdot H(t) + (eh - he) \cdot H(t) \cdot E(t)$$
$$\dot{E}(t) = [(ce - ec) \cdot C(t) \cdot E(t) + (he - eh) \cdot H(t) \cdot E(t) + \alpha \cdot S(t)] \cdot (1 - E(t))$$
$$\dot{S}(t) = \beta S(t) \cdot (1 - \frac{S(t)}{C_s})$$

After iterative adjustments to the parameters in the above equations, we were able to fit the model's growth to the actual data between 2011 and 2022 (see Figure 8). We then extended the model to 2022-2050 while maintaining the same initial conditions and parameters to estimate the future growth of each type of vehicle and charging station. As seen in Figure 9, we project that EVs will surpass combustion vehicles in the year 2035. Examining at the red line representing the charging stations, we can see that the growth is exponential between 2011 to around 2035, but diminishes as the population approaches the critical capacity. Finally, we note that the overall system reaches a steady equilibrium where EVs represent the majority of the total purchases, hybrid vehicles represent a portion of the sales, no more charging stations are constructed, and combustion vehicle purchases tend to 0.

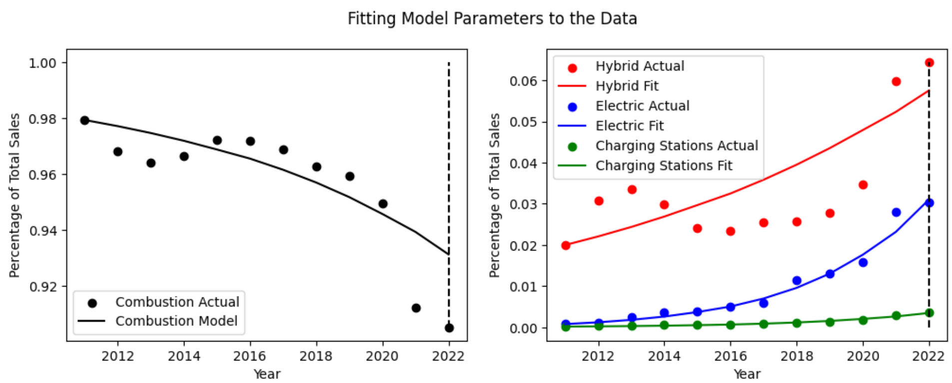

FIGURE 8. Our model's parameters are adjusted to match empirical data from 2011-2022

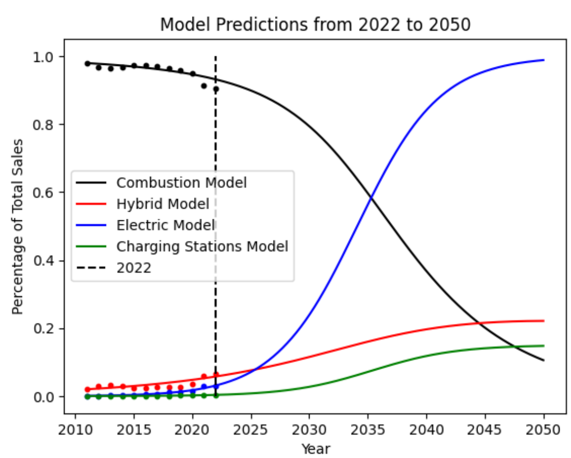

FIGURE 9. After fitting the model to 2011-2022 data, it makes predictions from 2022-2050

4. Analysis and Conclusion

Across all analyzed factors, we observe a consistent upward trend in the adoption of hybrid and electric vehicles, coupled with a decline in combustion vehicles. This trend underscores the imperative for proactive measures as the production of electric vehicles expands. Policymakers should consider strategic initiatives, such as incentivizing the widespread development of charging infrastructure to support the increasing demand for electric vehicles. Additionally, these insights highlight the importance of formulating policies that encourage sustainable transportation practices, fostering a transition toward cleaner and more environmentally friendly vehicle options. This proactive approach not only aligns with environmental goals but also supports the evolving preferences of consumers, ultimately shaping a more sustainable future for the automotive industry.

We began the process of modeling electric vehicle adoption by starting with a very simple framework based on an SIR model. We then explored various potential situations that we hypothesized would have a significant effect on people's car purchasing designs in the United States. While each of these variations reflect the changes to the model accurately, they all suffer from the same fundamental simplification. We assume that the size of each population is the primary influence on the rate of adoption, similar to the transmission of disease. However, this idea of cultural influence and local proliferation on vehicle purchases is certainly not the only factor affecting the type of car people choose to purchase.

A more appropriate model would allow the transition rate parameters to change with respect to time to incorporate more factors like changing views on electric cars, variations in relative prices, and developments in government incentive programs in a more complex and realistic manner. Furthermore, we could explore a broader range of scenarios. For instance, an examination of the production proportions of electric, hybrid, and combustion vehicles within individual car manufacturing companies may provide insights into economic trends and other behavioral patterns. Rather than addressing these factors individually, our eventual goal is to integrate them into a comprehensive model that can collectively consider multiple variables. By consolidating factors like government incentives, charging resource limitations, and the impact of natural disasters, this unified model would offer a more cohesive framework. This integrated approach not only helps in eliminating unrealistic assumptions but also ensures that the model encompasses all discussed situations, creating a more comprehensive and robust analytical tool.

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EVAdoption Code

December 7, 2023

1 Imports

[ ]:
```
# imports
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint, solve_ivp, solve_bvp
from scipy.optimize import minimize
import pandas as pd
import scipy.linalg as la

# setup the print and display options to make displaying easier
np.set_printoptions(precision=5, suppress=True)
pd.set_option("display.precision", 15)
np.set_printoptions(formatter={'float': lambda x: "{0:0.7f}".format(x)})
pd.set_option('display.float_format', lambda x: "{0:0.7f}".format(x))
pd.set_option('display.max_columns', None)
```

2 Base Model

2.1 Create the base model and plot real data to show trends

[ ]:
```
# build model
def model(t, cars, ch, he, ec, hc, eh, ce, c, h, e):
    return np.array([(hc-ch)*cars[0]*cars[1] + (ec-ce)*cars[0]*cars[2] + c,
                     (ch-hc)*cars[0]*cars[1] + (eh-he)*cars[1]*cars[2] + h,
                     (ce-ec)*cars[0]*cars[2] + (he-eh)*cars[1]*cars[2] + e])

# initialize subplots
plt.figure(figsize=(10, 3))

# read in actual data
rel_sales_df = pd.read_csv('rel_sales_df.csv')

#### CONSERVATIVE MODEL ####
# initial conditions
C = 0.9792
H = 0.02
```

```
E = 0.0008

# time points
t0 = 0
tf = 50

# constants
ch = 0.15 # rate from combustion to hybrid
he = 0.1 # rate from hybrid to electric
ec = 0.01 # rate from electric to combustion
hc = 0.1 # rate from hybrid to combustion
eh = 0.01 # rate from electric to hybrid
ce = 0.1 # rate from combustion to electric
c = 0.0 # rate of new combustion
h = 0.0 # rate of new hybrid
e = 0.0 # rate of new electric

# solve ODE
sol = solve_ivp(model, [t0, tf], [C, H, E], args=(ch, he, ec, hc, eh, ce, c, h,␣
 ↪e), t_eval=np.linspace(t0, tf, 1000))


# plot actual data
plt.subplot(121)
dot_size = 10
plt.scatter(rel_sales_df['Year'], rel_sales_df['Hybrid_ratio'], color="r",␣
 ↪s=dot_size)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Combustion_ratio'], color="k",␣
 ↪s=dot_size)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Electric_ratio'], color="b",␣
 ↪s=dot_size)
plt.plot(sol.t + 2010, sol.y[0], label="Combustion", c="k")
plt.plot(sol.t + 2010, sol.y[1], label="Hybrid", c="r")
plt.plot(sol.t + 2010, sol.y[2], label="Electric", c="b")
plt.title("Conservative Model")
plt.xlabel("Year")
plt.ylabel("Proportion of Car Purchases")
plt.legend()


#### PROGRESSIVE MODEL ####
# constants for progressive model
ch = 0.19 # rate from combustion to hybrid
he = 0.2 # rate from hybrid to electric
ec = 0.01 # rate from electric to combustion
hc = 0.1 # rate from hybrid to combustion
eh = 0.18 # rate from electric to hybrid
```

```
ce = 0.1 # rate from combustion to electric
c = 0.0 # rate of new combustion
h = 0.0 # rate of new hybrid
e = 0.0007 # rate of new electric

# solve ode
sol2 = solve_ivp(model, [t0, tf], [C, H, E], args=(ch, he, ec, hc, eh, ce, c,␣
 ↪h, e), t_eval=np.linspace(t0, tf, 1000))




# plot
plt.subplot(122)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Hybrid_ratio'], color="r",␣
 ↪s=dot_size)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Combustion_ratio'], color="k",␣
 ↪s=dot_size)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Electric_ratio'], color="b",␣
 ↪s=dot_size)
plt.plot(sol2.t + 2010, sol2.y[0], label="Combustion", c="k")
plt.plot(sol2.t + 2010, sol2.y[1], label="Hybrid", c="r")
plt.plot(sol2.t + 2010, sol2.y[2], label="Electric", c="b")
plt.title("Progressive Model")
plt.xlabel("Year")
plt.ylabel("Proportion of Car Purchases")
plt.legend()

# save figure
plt.savefig('original_model.png', dpi=200, bbox_inches='tight')
plt.show()
```

3 California Regulation

3.1 Showing real data trends in the california car sale market

[ ]:
```
# real data
eh = [0,.075,.12,.18,.254, .36, 1]
c = [1, 1-.075, 1-.12, 1-.18,1-.254, 1-.36, 0]
t = [2010,2018,2021,2022,2023,2026,2035]

# plot
plt.plot(t,eh, label = "Electric/Hybrid")
plt.scatter(t,eh)
plt.plot(t,c, label = "Combustion")
plt.scatter(t,c)
plt.xlabel("Years")
```

```
plt.ylabel("Proportion of Car Purchases")
plt.legend()
plt.show()
```

3.2 Create model and fit it to California data

[ ]:
```
# build model
def model(t, cars, ec, ce, c, e):
    return np.array([(ec)*cars[0]*cars[1] + c*cars[0],
                     (ce)*cars[0]*cars[1] + e*cars[1]])

# constants
ec = -0.4 # rate from electric to combustion
ce = .3 # rate from combustion to electric
c = -0.023# rate of new combustion
e = 0.1 # rate of new electric

# initial conditions
C = 0.9792
E = 0.0008

# time points
t0 = 0
tf = 25

# solve ODE
sol = solve_ivp(model, [t0, tf], [C,E], args=(ec, ce, c,e), t_eval=np.
 ↪linspace(t0, tf, 1000))

# plot
plt.plot(sol.t, sol.y[0], label="Combustion")
plt.plot(sol.t, sol.y[1], label="Electric/Hybrid")
plt.xlabel("Years Since 2010")
plt.ylabel("Proportion of Car Purchases")
plt.legend()
plt.show()

# print values at t=10
print(len(sol.y[0]))
year = 2020
time = int((year - 2010) * 1000 / tf)
print(time)
print("Combustion: ", sol.y[0][time])
print("Electric: ", sol.y[1][time])
```

4 Disaster Effect

4.1 Showing historical car sales and marking recessions

[ ]:
```
# read car sales data
car_sales = pd.read_csv('TOTALSA.csv')
car_sales['DATE'] = pd.to_datetime(car_sales['DATE']) # convert to datetime␣
↪format
# read recession dates
recession = pd.read_csv('recession_dates.csv')[['Peaks', 'Troughs']]
# convert to datetime format
recession['Peaks'] = pd.to_datetime(recession['Peaks'])
recession['Troughs'] = pd.to_datetime(recession['Troughs'])

# create figure and plot car sales
fig, ax = plt.subplots(1,1,figsize=(12,3))
car_sales.plot(x='DATE', y='TOTALSA', ax=ax, c='#1f77b4')

# gray out the recession areas
for i in range(recession.shape[0]):
    ax.axvspan(recession.loc[i, 'Peaks'], recession.loc[i, 'Troughs'], alpha=0.
↪3, color='gray')

# label plot
ax.set_title('Total Vehicle Sales')
ax.set_ylabel('Millions of Units')
ax.legend(['Sales', 'Recessions'])
ax.set_xlabel('Year')
plt.tight_layout()
plt.savefig('Total Vehicle Sales')
plt.show()
```

5 Charging Ports

5.1 Setup the data

[ ]:
```
# manually create a dataframe with the data from the US Department of Energy
# regarding number of charging ports and station locations
charging_df = pd.DataFrame({
    'Year': [2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017,␣
↪2018, 2019, 2020, 2021, 2022, 2023],
    'Charging Ports': [417, 564, 771, 1256, 5248, 10726, 16619, 22470, 26532,␣
↪33165, 45789, 56842, 73838, 96190, 114451, 136513, 163753],
    'Station Locations': [139, 196, 259, 407, 2109, 5444, 6938, 9207, 10710,␣
↪13150, 16170, 19893, 23282, 28602, 46407, 53764, 60649]
})
```

```
# save the charging data to a csv in the data folder
charging_df.to_csv("data/charging.csv", index=False)
```

[ ]:
```
# load the charging data
charging_df = pd.read_csv("data/charging.csv")

# load the sales data
sales_df = pd.read_csv("data/vehicle_sales_2000_2023.csv")

# combine the charging stations data to the vehicle sales data based
# on the year column
sales_df = sales_df.merge(charging_df, on='Year', how='left')

# since charging ports did not exist in the early years of sales, fill these␣
↪with zeros
sales_df['Charging Ports'] = sales_df['Charging Ports'].fillna(0).astype(float)␣
↪/ 1000
sales_df['Station Locations'] = sales_df['Station Locations'].fillna(0).
↪astype(float) / 1000

# compute the station locations ratio
sales_df['Station Locations Ratio'] = sales_df['Station Locations'] /␣
↪sales_df['Total']

# only consider years where there were electric vehicle sales and charging␣
↪stations
rel_sales_df = sales_df.loc[sales_df['Year'] >= 2011, :].copy()
```

5.2 Find a best fit exponential for charging stations

[ ]:
```
# extract the years as a numpy array
ts = charging_df['Year'].values.astype(int) - 2000

# create a constant column for the OLS
const_col = np.ones(len(ts))

# setup the A and the the y and run OLS on this
A = np.vstack([const_col, ts]).T
sol = la.lstsq(A, np.log(charging_df['Charging Ports']/1000))[0]

# extract the coefficients for the best fit exponential
# exp(a*x + b)
b, a = sol

# display the charging location data and the best-fit exponential
plt.scatter(charging_df['Year'], charging_df['Station Locations'] / 1000,␣
↪label="Charging Stations")
```

```
plt.plot(ts + 2000, np.exp(a * ts + b), color='orange', label="Exponential")
plt.xlabel("Year")
plt.ylabel("Number of Charging Stations (thousands)")
plt.title("Number of US Charging Stations over Time")
plt.legend()
plt.show()
```

5.3 Run the charging station ODE to make future predictions

[ ]:
```
####################################
# SOLVING THE CHARGING STATION IVP #
####################################

# define the altered charing station base model
def ces(t, y, K_ch, K_ce, K_he, alpha, beta, C_s):
    #print(y[0], y[1], y[2], y[3], C_s, y[3] / C_s)
    return (
        -K_ch * y[0] * y[1] - K_ce * y[0] * y[2],
        K_ch * y[0] * y[1] - K_he * y[1] * y[2],
        (K_ce * y[0] * y[2] + K_he * y[1] * y[2] + alpha * y[3]) * (1-y[2]),
        beta * y[3] * (1 - y[3] / C_s)
    )

# go from 2011 to 2050
t_span = (0, 50-11)
ts = np.linspace(*t_span, 500)

# create a clean version of the years as integers
ts_int = ts.astype(int)
ts_yr_inds = np.where(ts_int[1:] != ts_int[:-1])[0] + 1
ts_yr_inds = np.concatenate([[0], ts_yr_inds])

# find which years are using historical data and which years involve future data
mn_yr, mx_yr = rel_sales_df['Year'].min(), rel_sales_df['Year'].max()
ts_2011 = (ts[ts_yr_inds] + 2011).astype(int)
inner_years = (ts_2011 >= mn_yr) & (ts_2011 <= mx_yr)
inner_ts_yr_inds = ts_yr_inds[inner_years]

# setup y0 based on the actual ratios at the start of 2011
y0 = np.array([
    rel_sales_df['Combustion_ratio'].values[0],
    rel_sales_df['Hybrid_ratio'].values[0],
    rel_sales_df['Electric_ratio'].values[0],
    rel_sales_df['Station Locations Ratio'].values[0]
])

# set constants to make the trends match the data from 2011 through 2022
```

```
K_ch = 0.10
K_ce = 0.11
K_he = 0.01
alpha = 1.7
beta = 0.28
C_s = 0.15

# solve the modified ivp
solution = solve_ivp(ces, t_span, y0, t_eval=ts, args=(K_ch, K_ce, K_he, alpha,␣
 ↪beta, C_s))

# extract the true values
C = rel_sales_df['Combustion_ratio'].values
H = rel_sales_df['Hybrid_ratio'].values
E = rel_sales_df['Electric_ratio'].values
S = rel_sales_df['Station Locations Ratio'].values

# find the predicted values in the future
model_yrs = ts + 2011
model_C = solution.y[0]
model_H = solution.y[1]
model_E = solution.y[2]
model_S = solution.y[3]

# define a color map that maps the default colors to
# our specified uniform colors
color_map = {
    'blue': 'black', # combustion
    'orange': 'red', # hybrid
    'green': 'blue', # electric
    'red': 'green' # charging stations
}

##################################################
# PLOTTING THE SOLUTION'S FIT TO THE ACTUAL DATA #
##################################################

# since the combustion ratios are a lot higher than the other ratios, plot the
# combustion fit separate from the other fits
plt.subplot(1,2,1)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Combustion_ratio'],␣
 ↪color=color_map['blue'], label='Combustion Actual')
plt.plot(rel_sales_df['Year'], model_C[inner_ts_yr_inds],␣
 ↪color=color_map['blue'], label='Combustion Model')
plt.vlines(2022, rel_sales_df['Combustion_ratio'].min(), 1,␣
 ↪linestyles='dashed', color='black')
plt.legend(loc='lower left')
```

```python
plt.ylabel("Percentage of Total Sales")
plt.xlabel("Year")

# plot the hybrid, electric and station location fits
plt.subplot(1,2,2)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Hybrid_ratio'],␣
 ↪color=color_map['orange'], label='Hybrid Actual')
plt.plot(rel_sales_df['Year'], model_H[inner_ts_yr_inds],␣
 ↪color=color_map['orange'], label='Hybrid Fit')
plt.scatter(rel_sales_df['Year'], rel_sales_df['Electric_ratio'],␣
 ↪color=color_map['green'], label='Electric Actual')
plt.plot(rel_sales_df['Year'], model_E[inner_ts_yr_inds],␣
 ↪color=color_map['green'], label='Electric Fit')
plt.scatter(rel_sales_df['Year'], rel_sales_df['Station Locations Ratio'],␣
 ↪color=color_map['red'], label='Charging Stations Actual')
plt.plot(rel_sales_df['Year'], model_S[inner_ts_yr_inds],␣
 ↪color=color_map['red'], label='Charging Stations Fit')
plt.vlines(2022, 0, rel_sales_df['Hybrid_ratio'].max(), linestyles='dashed',␣
 ↪color='black')
plt.legend()
plt.xlabel("Year")
plt.ylabel("Percentage of Total Sales")

# show the fitting plot
plt.suptitle("Fitting Model Parameters to the Data")
plt.gcf().set_size_inches(12, 4)
plt.show()

###########################
# PLOT FUTURE PREDICTIONS #
###########################

dot_size = 10

# plot the actual data from 2011 to 2022
plt.scatter(rel_sales_df['Year'], rel_sales_df['Combustion_ratio'],␣
 ↪color=color_map['blue'], s=dot_size)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Hybrid_ratio'],␣
 ↪color=color_map['orange'], s=dot_size)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Electric_ratio'],␣
 ↪color=color_map['green'], s=dot_size)
plt.scatter(rel_sales_df['Year'], rel_sales_df['Station Locations Ratio'],␣
 ↪color=color_map['red'], s=dot_size)

# plot the future predictions
plt.plot(model_yrs, model_C, label='Combustion Model', color=color_map['blue'])
```

```
plt.plot(model_yrs, model_H, label='Hybrid Model', color=color_map['orange'])
plt.plot(model_yrs, model_E, label='Electric Model', color=color_map['green'])
plt.plot(model_yrs, model_S, label='Charging Stations Model',␣
 ↪color=color_map['red'])

# plot a vertical line to demonstrate where the future predictions start
plt.vlines(2022, 0, 1, linestyles='dashed', color='black', label='2022')

# set other model parameters
plt.xlabel("Year")
plt.ylabel("Percentage of Total Sales")
plt.legend()
plt.title("Model Predictions from 2022 to 2050")
plt.show()

# print how accurate the model fits the data as of 2022
ind_2022 = np.argsort(np.abs(ts + 2011 - 2022))[0]
print("model_C in 2022:", model_C[ind_2022])
print("actualC in 2022:", C[-1])
print("model_H in 2022:", model_H[ind_2022])
print("actualH in 2022:", H[-1])
print("model_E in 2022:", model_E[ind_2022])
print("actualE in 2022:", E[-1])
print("model_S in 2022:", model_S[ind_2022])
print("actualS in 2022:", S[-1])
```