

# Cosmology with standard sirens

Bernardo Porto Veronese\*

*PPGCOSMO, UFES*

(Dated: November 28, 2023)

# Abstract

The detection of gravitational waves has marked a new era in multi-messenger astronomy. In particular, they can serve as cosmological probes to understand the history of cosmic expansion.

## I. INTRODUCTION

The idea of using gravitational waves (GWs) from compact binary mergers to measure cosmological parameters was first introduced by Bernard Schutz in 1986 [1]. These signals directly provide a measurement of the luminosity distance measurement to the source, which is therefore independent of the cosmic distance ladder. With the addition of redshift information, measurements can therefore be made of those cosmological parameters which impact the expansion history of the Universe, such as the Hubble constant ( $H_0$ ). This approach is independent of all other local measurements to date.

The standard siren method probes the expansion history of the universe with the distance-redshift relation, with which one can infer the cosmological parameters such as  $H_0$  and the dark energy equation of state parameter  $w$ : [2]

$$D_l(z) = (1+z) \frac{c}{H_0 \sqrt{\Omega_K}} \sinh \left[ \sqrt{\Omega_K} \int_0^z \frac{H_0}{H(z') dz'} \right] \quad (1)$$
$$\frac{H(z)}{H_0} = \sqrt{\Omega_m(1+z)^3 + \Omega_K(1+z)^2 + \Omega_{de}(1+z)^{3(1+w)}}.$$

To lighten notation, we have omitted the 0-subscript next to the  $\Omega_i$ 's, although they correspond to the present day values in the above equation. Note that using Eq. (1) requires specifying a cosmological model.

The accuracy of the GW luminosity distance measurement is typically of the order of 10%. The main source of uncertainty comes from the degeneracy between the distance and inclination angle of the source. The latter is defined as the angle between the line-of-sight vector from the source to the detector and the orbital-angular momentum of the binary system.

---

\* bernardo.veronese@edu.ufes.br

### A. Bright sirens

From the GW data, it is possible to infer the luminosity distance to the binary source, but not the redshift, as the latter is degenerate with the chirp mass in the GW waveform modelling. It is therefore necessary to complement the data with another source of information that provides the redshift measurement. Multi-messenger observations, such as neutron star mergers with electromagnetic counterparts like short gamma-ray bursts or kilonovae, provide the most straight-forward measurement [3, 4]. An electromagnetic counterpart like a kilonova can typically be pinpointed to a specific galaxy, thereby identifying the host galaxy of the GW merger. The GW signal provides the distance to the host galaxy, while its electromagnetic spectrum provides the redshift. These sources are typically referred to as bright sirens. So far, the only confirmed such event has been the binary neutron star detection GW170817, which occurred so exceptionally close to our galaxy - at  $d \sim 40$  Mpc - that a direct, model-independent estimation of  $H_0$  with Hubble's law,

$$v_H = H_0 d, \quad (2)$$

could be made by measuring the Hubble flow velocity  $v_H$ , resulting in  $H_0 = 70.0^{+12.0}_{-8.0}$  km s<sup>-1</sup> Mpc<sup>-1</sup> [5].

There are systematic effects and uncertainties that need to be taken care of. From the GW side, the precision of the inferred luminosity distance value is limited by the distance-inclination degeneracy.

From the counterpart side, the redshift measurement is affected by peculiar velocity corrections, which are more important for closer sources.

### B. Where are the optical counterparts?

As stated above, almost all GW events have been detected without an EM counterpart. Selection effects play a role in this: optical transients from the CBCs are mostly expected in binary neutron star (BNS) mergers, which are more difficult to detect due to the smaller masses of the sources. At the same time, there are more galaxies at higher redshifts, where optical counterparts are more unlikely to be detected due to fainter signals and more significant noise from objects in the line-of-sight direction of the source.

These *dark sirens* can be used to probe the expansion of the universe provided that they are complemented with an external redshift measurement. In his original paper, Schutz suggested that this information could be inferred from galaxy catalogs: each galaxy’s redshift contributes to a hypothetical estimation of  $H_0$ , such that the galaxy structure within a GW event’s localisation volume is reflected in the  $H_0$  posterior it produces. How informative the individual events are will depend strongly on their localisation volumes. By combining the contributions of many events, the true value of  $H_0$  will be measured as other values will statistically average out. Such analyses have been carried out in the literature, see [6–10]. As an example, Ref. 11 applied the galaxy catalog method with the two best localized dark sirens, GW170814 and GW190814, and the photo- $z$  catalog from the Dark Energy Survey (DES) [12]. A joint analysis with the bright siren event GW170817 provided an  $\sim 18\%$  improvement on the 68% confidence interval compared to inferring  $H_0$  with GW170817 alone. Another study used 8 well-localised dark sirens alone to infer  $H_0 = 79.8^{+19.1}_{-12.8}$  [13].

At higher redshifts, galaxy-wide surveys are incomplete, and the probability that the catalog contains the merger’s host galaxy decreases. On the other hand, both the gravitational wave sources and galaxies are tracers of the matter density, and therefore, they are spatially correlated through the underlying matter field. Therefore, if the events are well localized, angular correlations between galaxy distributions in redshift and merger distributions in luminosity distance may be used to infer cosmological parameters. Some authors have explored this idea with forecasts for 3G detectors [14]. Refs. 15 and 16 used simulated data to analyse the method’s constraining power on  $H_0$  for different numbers of events. Their results show that the Hubble constant can be measured with  $2.5\%$  accuracy for  $\mathcal{O}(100)$  events. Finally, this technique is not exclusive to standard sirens, and can be applied to any redshift-free distance tracer, such as type Ia supernovae [17].

Alternative methods have been explored in the literature where the inference was set up using GW data alone. One such method consists of using the prior knowledge of the star formation rate and time delay distribution of binary mergers for modelling the prior on the merger population distribution [18–20]. In order to avoid biases by using a fixed merger distribution, it is important to jointly fit its hyperparameters with the GW data.

The different methods presented above are not at all independent. Galaxy catalogs, for instance, are most effective with well-localised, low-redshift events. Such events need to be marginalized over a smaller number of galaxies than those that encompass large volumes

so that they are more likely to provide better constraints. Better localized events also tend to be less distant, which means that a standard siren analysis would mostly be sensitive to the Hubble constant rather than to other cosmological parameters. At higher redshifts, the catalogs become increasingly uninformative due to their incompleteness.

This manuscript is organized as follows. In Sec. II, we will go over the statistical formalism for data analysis with standard sirens, and we will specialize in the galaxy catalog method.

## II. STATISTICAL FRAMEWORK

In gravitational-wave astronomy, one line of interest is extracting the distributional properties of a population of sources based on a set of observations which are drawn from that distribution. Any methodology that leads to unbiased estimates of the population parameters must simultaneously account for measurement uncertainties and selection effects. One way with which the latter affects the observed population is a Malmquist bias: the loudest or brightest sources are more likely to be detected. The standard formalism for extracting the true source population parameters by incorporating these biases in the analysis is frequently labeled as Hierarchical Bayesian inference, see [21–23].

In the discussion below, we will follow the framework outlined in Ref. 24, which is a pedagogical resource on the galaxy catalog approach.

The GW population distribution is sampled with a set of  $N_{\text{obs}}$  *observed* events with true parameters  $\{\vec{\theta}_i\}$ ,  $i \in \{1, \dots, N_{\text{obs}}\}$ . We do not have direct access to the true parameters because of noise; instead, we have a set of measured data  $\{\vec{d}_i\}$ . The  $\vec{\theta}_i$  are the individual object parameters, although we are interested in the population hyperparameters, which we call  $\vec{\Lambda}$ . We cannot determine  $\vec{\Lambda}$  directly, but we can compute the posterior probability given the observations. In the usual Bayesian formalism,

$$p(\vec{\Lambda}|\{\vec{d}_i\}) = \frac{p(\{\vec{d}_i\}|\vec{\Lambda})\pi(\vec{\Lambda})}{p(\{\vec{d}_i\})} \quad (3)$$

where  $p(\{\vec{d}_i\}|\vec{\Lambda})$  is the likelihood of observing the dataset given the population properties,  $\pi(\vec{\Lambda})$  is the prior on  $\vec{\Lambda}$  and  $p(\{\vec{d}_i\})$  is the evidence, which is the integral of the numerator over  $\vec{\Lambda}$ .

In the spirit of Ref. 22, we first start with the idealized scenario where the event parameters are perfectly measured. The total likelihood for the set of  $N_{\text{obs}}$  independent measure-

ments is then

$$p(\{\vec{\theta}_i\}|\vec{\Lambda}) = \prod_{i=1}^{N_{\text{obs}}} \frac{p_{\text{pop}}(\vec{\theta}_i|\vec{\Lambda})}{\int p_{\text{pop}}(\vec{\theta}_i|\vec{\Lambda})d\vec{\Lambda}} \quad (4)$$

where  $p_{\text{pop}}(\vec{\theta}|\vec{\Lambda})$  is related to the number density  $dN$  of objects expected to be found in the region  $[\vec{\theta}, \vec{\theta} + d\vec{\theta}]$ :

$$dN = N p_{\text{pop}}(\vec{\theta}|\vec{\Lambda})d\vec{\theta} \quad (5)$$

We shall build an incrementally more robust model than Eq. (4). Let us first consider the presence of selection effects: not all events are equally likely to be detected. We can encode this with a detection probability  $p_{\text{det}}$ . In the perfect measurement idealization, this detection probability becomes a function of the parameters  $\vec{\theta}$  only. In the general case, where noise is present, the detection probability is a function of the data. Let  $\mathcal{D}$  be the set of all data. To determine whether an event is detectable, one can use a detection statistic  $\rho_{\mathcal{D}}$ , which can be calculated for each piece of data. In practice, this statistic can be the signal-to-noise ratio (SNR), the false-alarm rate, etc. We split  $\mathcal{D}$  into two disjoint sets,  $\mathcal{D}_{<}$  and  $\mathcal{D}_{\geq}$ , according to whether  $\rho_{\mathcal{D}}$  is smaller than a threshold  $\rho_{\text{tr}}$  or not. Then

$$p_{\text{det}}(\vec{\theta}) = \int_{\mathcal{D}_{\geq}} p(\vec{d}|\vec{\theta})d\vec{d} \quad (6)$$

The probability of observing a particular dataset  $\vec{d}$  given the assumed population distribution parameterised by  $\vec{\Lambda}$  is

$$p(\vec{d}|\vec{\Lambda}) = \frac{\int p(\vec{d}|\vec{\theta})p_{\text{pop}}(\vec{\theta}|\vec{\Lambda})d\vec{\theta}}{\alpha(\vec{\Lambda})} \quad (7)$$

where  $\alpha(\vec{\Lambda})$  is a normalization factor integrated over the set of detectable data,

$$\alpha(\vec{\Lambda}) = \int_{\mathcal{D}_{\geq}} d\vec{d} \int p(\vec{d}|\vec{\theta})p_{\text{pop}}(\vec{\theta}|\vec{\Lambda})d\vec{\theta} \quad (8)$$

$$= \int \left[ \int_{\mathcal{D}_{\geq}} p(\vec{d}|\vec{\theta})d\vec{d} \right] p_{\text{pop}}(\vec{\theta}|\vec{\Lambda})d\vec{\theta} \quad (9)$$

$$= \int p_{\text{det}}(\vec{\theta})p_{\text{pop}}(\vec{\theta}|\vec{\Lambda})d\vec{\theta} \quad (10)$$

Hence, in the presence of both measurement uncertainties and selection effects, Eq. (4) becomes

$$p(\{\vec{d}_i\}|\vec{\Lambda}) = \prod_{i=1}^{N_{\text{obs}}} \frac{\int p(\vec{d}_i|\vec{\theta}) p_{\text{pop}}(\vec{\theta}|\vec{\Lambda}) d\vec{\theta}}{\int p_{\text{det}}(\vec{\theta}) p_{\text{pop}}(\vec{\theta}|\vec{\Lambda}) d\vec{\theta}} \quad (11)$$

We can also include the population rate into the framework. The probability of observing  $k$  events with an expected number of detections  $N_{\text{det}}$  is given by a Poisson distribution as

$$p(k|N_{\text{det}}) = e^{-N_{\text{det}}} (N_{\text{det}})^{N_{\text{obs}}} \quad (12)$$

The usual  $N_{\text{obs}}!$  term is absent in Eq. (12) because the events are distinguishable from the observed data. When accounting for selection effects, the expected number of detections  $N_{\text{det}}$  becomes

$$N_{\text{det}}(\vec{\Lambda}) = \int_{\mathcal{D}_{\geq}} d\vec{d} \int p(\vec{d}|\vec{\theta}) \frac{dN}{d\vec{\theta}} d\vec{\theta} \quad (13)$$

$$= \int \left[ \int_{\mathcal{D}_{\geq}} p(\vec{d}|d) d\vec{d} \right] \frac{dN}{d\vec{\theta}} d\vec{\theta} \quad (14)$$

$$= \int p_{\text{det}}(\vec{\theta}) \frac{dN}{d\vec{\theta}} d\vec{\theta} \quad (15)$$

$$= \int p_{\text{det}}(\vec{\theta}) N p_{\text{pop}}(\vec{\theta}|\vec{\Lambda}) d\vec{\theta} \quad (16)$$

$$= N \alpha(\vec{\Lambda}) \quad (17)$$

where the last two equalities are derived from Eq. (5) and Eq. (10) respectively. The full posterior with the population rate is then

$$\begin{aligned} p(\vec{\Lambda}, N|\vec{d}) &= p(N|\vec{\Lambda}, \vec{d}) p(\vec{\Lambda}|\vec{d}) \\ &= e^{-N_{\text{det}}} (N_{\text{det}})^{N_{\text{obs}}} \pi(N) \pi(\vec{\Lambda}) \alpha(\vec{\Lambda})^{-N_{\text{obs}}} \prod_{i=1}^{N_{\text{obs}}} \int p(\vec{d}_i|\vec{\theta}) p_{\text{pop}}(\vec{\theta}|\vec{\Lambda}) d\vec{\theta} \end{aligned} \quad (18)$$

If a prior  $\pi(N) \propto 1/N$  is assumed on the population rate, then the posterior can be marginalized over  $N$ :

$$\int e^{-N_{\text{det}}}(N_{\text{det}})^{N_{\text{obs}}}\frac{dN}{N} = \int e^{-N_{\text{det}}}(N_{\text{det}})^{N_{\text{obs}}-1}dN_{\text{det}} \quad (19)$$

$$= (N_{\text{obs}} - 1)! \quad (20)$$

So far, the framework we developed has been general; we now specify to the gravitational wave case. The individual event parameters  $\vec{\theta}$  describe the compact binary coalescence (CBC): the individual masses and spins, sky position, polarization, inclination angle, luminosity distance, and redshift. The population parameters  $\vec{\Lambda}$  can be split into three groups: mass, rate and cosmological parameters. The mass parameters specify the GW mass model, such as minimum and maximum mass, slopes, the positions of any features in the mass distribution function, etc. These are used in the spectral siren method. The rate parameters are used in the model to describe how the CBC merger rate evolves with redshift. Finally, the cosmological parameters are the constants which appear in Eq. (1), namely  $\{H_0, \Omega_m, \Omega_{de}, w\}$ .

### A. A simplified approach

In this section, we reproduce the formalism developed in Ref. 24. In that paper, the authors perform a mock data analysis of the dark siren approach using the galaxy catalog method to demonstrate its capability to recover an unbiased posterior for  $H_0$ . They consider that the remaining cosmological parameters, such as  $\Omega_m$  and  $w_{\text{DE}}$ , are fixed to fiducial values. They make a series of simplifying assumptions on  $p_{\text{det}}$ ,  $p_{\text{pop}}$  and  $p(\vec{d}|\vec{\theta})$ , which we discuss below.

The selection effects, encoded in the  $\alpha(\Lambda)$  term from Eq. (10), not only depend on the underlying population modelling prior  $p_{\text{pop}}$ , but they are also affected by the (sky-dependent) GW detector sensitivity and detector-frame mass. A complete analytical treatment is thus intractable. Hence, the detection efficiency must be estimated by drawing synthetic objects from a fiducial distribution,  $p_{\text{draw}}(\theta)$ , drawing corresponding data from the likelihood function  $p(\vec{d}|\vec{\theta})$ , and “injecting” these data into the pipeline used to produce the GW event catalog, recording which observations are detected. The number of required injections to estimate the selection function precisely enough scales linearly with the size of the event catalog [25].



Instead, we make the following simplification: detection is assumed to happen if the *measured* luminosity distance is smaller than a threshold,  $d_L^{\text{th}}$ . If the GW likelihood is taken to be gaussian, that is,

$$\mathcal{L}(\hat{d}_L^i | d_L(z, H_0)) = \frac{1}{\sqrt{2\pi}\sigma_{d_L}} \exp \left[ -\frac{1}{2} \left( \frac{\hat{d}_L^i - d_L(z, H_0)}{\sigma_{d_L}} \right)^2 \right], \quad (21)$$

then the detection probability can be expressed analytically with Eq. (6):

$$\begin{aligned} p_{\text{det}}(z, H_0) &= \int_{-\infty}^{d_L^{\text{th}}} \mathcal{L}(\hat{d}_L^i | d_L(z, H_0)) d\hat{d}_L^i \\ &= \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\hat{d}_L^i - d_L(z, H_0)}{\sqrt{2}\sigma_{d_L}} \right) \right]. \end{aligned} \quad (22)$$

where erf is the unilateral error function of the standard normal distribution. The uncertainty  $\sigma_{d_L}$  is taken to be a constant fraction of  $d_L$ , such that  $\sigma_{d_L}/d_L = C < 1$ .

The galaxy catalog information is used to compute the population model prior  $p_{\text{pop}}(\vec{\theta} | \vec{\Lambda}) = p_{\text{pop}}(z | H_0)$ . Each galaxy in the catalog contributes with a term

$$\mathcal{L}_{\text{EM}}(\hat{z}_i | z_i) p(z_i | H_0), \quad (23)$$

where  $z_i$  and  $\hat{z}_i$  are the galaxy's true and measured values, respectively. The likelihood encodes the measurement uncertainty, while the redshift prior depends on our knowledge of the galaxy distribution on redshift. A simple choice is to pick  $p(z_i | H_0)$  to be uniform in a comoving volume,  $p(z_i | H_0) \propto dV_c/dz$ . The posterior probability becomes

$$p(H_0, \{z_g\} | d_{\text{EM}}, d_{\text{GW}}) \propto \alpha^{-1}(H_0) \left[ \sum_{i=1}^{N_{\text{gal}}} \mathcal{L}_{\text{GW}}(d_{\text{GW}} | d_L(z_i, H_0)) \right] \prod_{j=1}^{N_{\text{gal}}} \mathcal{L}_{\text{EM}}(\hat{z}_j | z_j) p(z_j | H_0). \quad (24)$$

The dependence on the true galaxy redshifts  $\{z_g\}$  can then be marginalized over to recover the posterior distribution on  $H_0$  only.

Note that in the above expression we are implicitly neglecting cross-correlations between galaxies, for instance due to clustering. On the approximation that the galaxy redshifts are measured perfectly, the likelihood  $\mathcal{L}_{\text{EM}}$  becomes a delta function, and the posterior for a single GW event reduces to a simple form:

$$p(H_0|d_{\text{EM}}, d_{\text{GW}}) \propto \frac{\sum_{i=1}^{N_{\text{gal}}} \mathcal{L}_{\text{GW}}(d_L^{\text{GW}}|d_L(\hat{z}_i, H_0))}{\sum_{i=1}^{N_{\text{gal}}} p_{\text{det}}(\hat{z}_i, H_0)}. \quad (25)$$

Alternatively, we model the photo- $z$  redshift likelihood as a gaussian,

$$\mathcal{L}_{\text{EM}}(\hat{z}_j|z_j) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp \left[ -\frac{1}{2} \left( \frac{\hat{z}_j - z_j}{\sigma_z} \right)^2 \right], \quad (26)$$

with  $\sigma_z \sim \min\{0.013(1+z)^3, 0.015\}$ , following an empirical fit described in Ref. 10.

### III. METHOD

We implement the framework developed in Sec. II A as follows. Unlike Ref. 24, we use real data from the **GLADE+** catalog [26, 27]<sup>1</sup>, which is a compilation of six different astronomical surveys, containing about 22 million galaxies. It covers the full sky with a 20% completeness up to about 800 Mpc. The gravitational wave data, on the other hand, is generated from the catalog directly:

1. A direction  $(\theta, \varphi)$  on the sky is picked, and all galaxies within an angular radius  $\alpha$  of it are selected as candidate hosts for the coalescences.
2. A sample of  $N_{\text{gw}}$  redshifts are drawn uniformly from said galaxies in the range  $0 \leq z \leq z_{\text{max}}$ , where  $z_{\text{max}}$  is a constant value fixed *a priori*.
3. The drawn redshifts are converted into true luminosity distances supposing a fiducial flat  $\Lambda$ CDM cosmology with  $\Omega_m = 0.3$  and  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Each distance value is associated to a mock gravitational wave event.
4. The *measured* luminosity distances are drawn from their respective true values consistently with the GW gaussian likelihood of Eq. (21).

Our methodology is evidently a simple proof of concept of the galaxy catalog approach to dark siren cosmography. Still, it is important to highlight the limitations to our method, as well as the approximations that are made. Firstly, the simplified data-generating process: we are generating mock events *uniformly* from the available galaxies in the catalog. The

---

<sup>1</sup> The catalog is publicly available for download at <https://glade.elte.hu/>.

resulting redshift distribution of CBCs is inevitably inaccurate for two reasons: the galaxies may not be equally likely to host these mergers; the latter may indeed depend on their masses, luminosities, metallicities, etc. Secondly, the galaxy distribution in the catalog does not reproduce the real distribution due to the catalog’s incompleteness. The selected redshifts will be artificially small because of this. This should not spoil the analysis, so long as the mock events’ generation is consistent with the likelihood model. In particular, we expect that the posterior distribution should peak around the fiducial value of  $H_0$  used in the  $z - d_L$  conversion. The galaxy distribution in the catalog, however, is not expected to reproduce this background cosmology. Moreover, the (assumed true) redshifts of the coalescences are drawn from real measurements, and therefore do not correspond to the true redshifts of their corresponding hosts. This distinction does not play a role when the spectroscopic- $z$  approximation is considered, namely the likelihood in Eq. (25), although it becomes important for the broad case where redshift uncertainty is incorporated into the analysis.

A number of approximations were also made in our likelihood model. The GW likelihood  $p(\vec{d}|\vec{\theta})$  does not, in general, follow a normal distribution. In real analyses, while a similar ansatz can be used for the conditional probability of the distance  $d$  to the event given a sky direction  $\hat{\mathbf{n}}$  [28], a more accurate procedure is to apply Bayes’s theorem and to discretely sample the posterior  $p(\vec{\theta}|\vec{d})$  reconstructed in the single-event parameter estimation with assumed prior  $\pi(\vec{\theta})$ . Likewise, the photo- $z$  PDF may be more complicated than Eq. (26), and can be estimated with more sophisticated methods, such as neural networks [13, 29]. Moreover, as already mentioned, a more realistic computation of the selection effects involves injecting artificial signals and computing the number of detections at the selected sensitivity. Finally, when using real GW data, the set of candidate host galaxies should be extracted from the credible regions estimated from the events’ skymaps - for instance, with the **BAYESTAR** package [30].

We have also explicitly ignored completeness corrections to the galaxy catalog. In our toy model, this is not as problematic since the CBCs are generated from the catalog itself. In the general case, however, not accounting for the incompleteness will make the inference far less informative for events that occurred at higher redshifts, which are commonplace for BBHs.

The implementation is available in the Github repository <https://github.com/binado/>

`standard-sirens/tree/main`. The idea is to make it a playground for exploring different strategies for performing inference with standard sirens.

- 
- [1] B. F. Schutz, Determining the Hubble Constant from Gravitational Wave Observations, *Nature* **323**, 310 (1986).
  - [2] D. W. Hogg, Distance measures in cosmology, (1999), arXiv:astro-ph/9905116.
  - [3] D. E. Holz and S. A. Hughes, Using gravitational-wave standard sirens, *Astrophys. J.* **629**, 15 (2005), arXiv:astro-ph/0504616.
  - [4] N. Dalal, D. E. Holz, S. A. Hughes, and B. Jain, Short grb and binary black hole standard sirens as a probe of dark energy, *Phys. Rev. D* **74**, 063006 (2006), arXiv:astro-ph/0601275.
  - [5] B. P. Abbott *et al.* (LIGO Scientific, Virgo, 1M2H, Dark Energy Camera GW-E, DES, DLT40, Las Cumbres Observatory, VINROUGE, MASTER), A gravitational-wave standard siren measurement of the Hubble constant, *Nature* **551**, 85 (2017), arXiv:1710.05835 [astro-ph.CO].
  - [6] W. Del Pozzo, Inference of the cosmological parameters from gravitational waves: application to second generation interferometers, *Phys. Rev. D* **86**, 043011 (2012), arXiv:1108.1317 [astro-ph.CO].
  - [7] H.-Y. Chen, M. Fishbach, and D. E. Holz, A two per cent Hubble constant measurement from standard sirens within five years, *Nature* **562**, 545 (2018), arXiv:1712.06531 [astro-ph.CO].
  - [8] M. Fishbach *et al.* (LIGO Scientific, Virgo), A Standard Siren Measurement of the Hubble Constant from GW170817 without the Electromagnetic Counterpart, *Astrophys. J. Lett.* **871**, L13 (2019), arXiv:1807.05667 [astro-ph.CO].
  - [9] R. Gray *et al.*, Cosmological inference using gravitational wave standard sirens: A mock data analysis, *Phys. Rev. D* **101**, 122001 (2020), arXiv:1908.06050 [gr-qc].
  - [10] M. Soares-Santos *et al.* (DES, LIGO Scientific, Virgo), First Measurement of the Hubble Constant from a Dark Standard Siren using the Dark Energy Survey Galaxies and the LIGO/Virgo Binary–Black-hole Merger GW170814, *Astrophys. J. Lett.* **876**, L7 (2019), arXiv:1901.01540 [astro-ph.CO].
  - [11] A. Palmese *et al.* (DES), A statistical standard siren measurement of the Hubble constant from the LIGO/Virgo gravitational wave compact object merger GW190814 and Dark Energy Survey galaxies, *Astrophys. J. Lett.* **900**, L33 (2020), arXiv:2006.14961 [astro-ph.CO].

- [12] T. D. E. S. Collaboration, The dark energy survey (2005), arXiv:astro-ph/0510346 [astro-ph].
- [13] A. Palmese, C. R. Bom, S. Mucesh, and W. G. Hartley, A standard siren measurement of the hubble constant using gravitational-wave events from the first three ligo/virgo observing runs and the desi legacy survey, *The Astrophysical Journal* **943**, 56 (2023).
- [14] M. Oguri, Measuring the distance-redshift relation with the cross-correlation of gravitational wave standard sirens and galaxies, *Physical Review D* **93**, 10.1103/physrevd.93.083511 (2016).
- [15] S. Bera, D. Rana, S. More, and S. Bose, Incompleteness matters not: Inference of  $h_0$  from binary black hole–galaxy cross-correlations, *The Astrophysical Journal* **902**, 79 (2020).
- [16] S. Mukherjee, B. D. Wandelt, S. M. Nissanke, and A. Silvestri, Accurate precision cosmology with redshift unknown gravitational wave sources, *Physical Review D* **103**, 10.1103/physrevd.103.043520 (2021).
- [17] S. Mukherjee and B. D. Wandelt, Beyond the classical distance-redshift test: cross-correlating redshift-free standard candles and sirens with redshift surveys (2018), arXiv:1808.06615 [astro-ph.CO].
- [18] X. Ding, M. Biesiada, X. Zheng, K. Liao, Z. Li, and Z.-H. Zhu, Cosmological inference from standard sirens without redshift measurements, *Journal of Cosmology and Astroparticle Physics* **2019** (04), 033–033.
- [19] C. Ye and M. Fishbach, Cosmology with standard sirens at cosmic noon, *Physical Review D* **104**, 10.1103/physrevd.104.043507 (2021).
- [20] H. Leandro, V. Marra, and R. Sturani, Measuring the hubble constant with black sirens, *Physical Review D* **105**, 10.1103/physrevd.105.023523 (2022).
- [21] T. J. Loredo, Accounting for source uncertainties in analyses of astronomical survey data, *AIP Conf. Proc.* **735**, 195 (2004), arXiv:astro-ph/0409387.
- [22] I. Mandel, W. M. Farr, and J. R. Gair, Extracting distribution parameters from multiple uncertain observations with selection biases, *Mon. Not. Roy. Astron. Soc.* **486**, 1086 (2019), arXiv:1809.02063 [physics.data-an].
- [23] S. Vitale, D. Gerosa, W. M. Farr, and S. R. Taylor, Inferring the properties of a population of compact binaries in presence of selection effects, in *Handbook of Gravitational Wave Astronomy* (Springer Singapore, 2021) pp. 1–60.
- [24] J. R. Gair, A. Ghosh, R. Gray, D. E. Holz, S. Mastroianni, S. Mukherjee, A. Palmese, N. Tamanini, T. Baker, F. Beirnaert, M. Bilicki, H.-Y. Chen, G. Dálya, J. M. Ezquiaga, W. M.

- Farr, M. Fishbach, J. Garcia-Bellido, T. Ghosh, H.-Y. Huang, C. Karathanasis, K. Leyde, I. M. Hernandez, J. Noller, G. Pierra, P. Raffai, A. E. Romano, M. Seglar-Arroyo, D. A. Steer, C. Turski, M. P. Vaccaro, and S. A. Vallejo-Peña, The hitchhiker’s guide to the galaxy catalog approach for dark siren gravitational-wave cosmology, *The Astronomical Journal* **166**, 22 (2023).
- [25] R. Essick and W. Farr, Precision requirements for monte carlo sums within hierarchical bayesian inference (2022), arXiv:2204.00461 [astro-ph.IM].
- [26] G. Dálya, G. Galgóczi, L. Dobos, Z. Frei, I. S. Heng, R. Macas, C. Messenger, P. Raffai, and R. S. de Souza, Glade: A galaxy catalogue for multimessenger searches in the advanced gravitational-wave detector era, *Monthly Notices of the Royal Astronomical Society* **479**, 2374–2381 (2018).
- [27] G. Dálya, R. Díaz, F. R. Bouchet, Z. Frei, J. Jasche, G. Lavaux, R. Macas, S. Mukherjee, M. Pálfi, R. S. de Souza, B. D. Wandelt, M. Bilicki, and P. Raffai, Glade+ : an extended galaxy catalogue for multimessenger searches with advanced gravitational-wave detectors, *Monthly Notices of the Royal Astronomical Society* **514**, 1403–1411 (2022).
- [28] L. P. Singer, H.-Y. Chen, D. E. Holz, W. M. Farr, L. R. Price, V. Raymond, S. B. Cenko, N. Gehrels, J. Cannizzo, M. M. Kasliwal, S. Nissanke, M. Coughlin, B. Farr, A. L. Urban, S. Vitale, J. Veitch, P. Graff, C. P. L. Berry, S. Mohapatra, and I. Mandel, Supplement: “going the distance: Mapping host galaxies of ligo and virgo sources in three dimensions using local cosmography and targeted follow-up” (2016, apjl, 829, l15), *The Astrophysical Journal Supplement Series* **226**, 10 (2016).
- [29] V. Alfradique *et al.*, A dark siren measurement of the hubble constant using gravitational wave events from the first three ligo/virgo observing runs and delve (2023), arXiv:2310.13695 [astro-ph.CO].
- [30] L. P. Singer and L. R. Price, Rapid bayesian position reconstruction for gravitational-wave transients, *Phys. Rev. D* **93**, 024013 (2016).