APS/123-QED

Cosmology with standard sirens

Bernardo Porto Veronese*

PPGCOSMO, UFES

(Dated: October 30, 2023)

Abstract

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I. INTRODUCTION

The idea of using gravitational waves (GWs) from compact binary mergers to measure cosmological parameters was first introduced by Bernard Schutz in 1986 [1]. These signals directly provide a measurement of the luminosity distance measurement to the source, which is therefore independent of the cosmic distance ladder. With the addition of redshift information, measurements can therefore be made of those cosmological parameters which impact the expansion history of the Universe, such as the Hubble constant (H_0). This approach is independent of all other local measurements to date.

The standard siren method probes the expansion history of the universe with the distance-redshift relation, with which one can infer the cosmological parameters such as H_0 and the dark energy equation of state parameter w: [2]

$$
\begin{aligned}
D_l(z) &= (1+z)\frac{c}{H_0\sqrt{\Omega_K}}\sinh\left[\sqrt{\Omega_K}\int_0^z \frac{H_0}{H(z')dz'}\right] \\
\frac{H(z)}{H_0} &= \sqrt{\Omega_m(1+z)^3+\Omega_K(1+z)^2+\Omega_{de}(1+z)^{3(1+w)}}.
\end{aligned}
\tag{1}
$$

To lighten notation, we have omitted the 0-subscript next to the Ω_i's, although they correspond to the present day values in the above equation. Note that using Eq. (1) requires specifying a cosmological model.

* bernardo.veronese@edu.ufes.br

A. Gravitational-wave distances

The accuracy of the GW luminosity distance measurement is typically of the order of 10%. The main source of uncertainty comes from the degeneracy between the distance and inclination angle of the source. The latter is defined as the angle between the line-of-sight vector from the source to the detector and the orbital-angular momentum of the binary system.

B. Assigning redshifts to GW sources

From the GW data, it is possible to infer the luminosity distance to the binary source, but not the redshift, as the latter comes degenerate with the chirp mass in the GW waveform modelling. It is therefore necessary to complement it with another source of information that provides the redshift measurement. Multi-messenger observations, such as neutron star mergers with electromagnetic counterparts like short gamma-ray bursts or kilonovae, provide the most straight-forward measurement [3, 4]. An electromagnetic counterpart like a kilonova can typically be pinpointed to a specific galaxy, thereby identifying the host galaxy of the GW merger. The GW signal provides the distance to the host galaxy, while its electromagnetic spectrum provides the redshift. These sources are typically referred to as bright sirens. So far, the only confirmed such event has been the binary neutron star detection GW170817, which occurred so exceptionally close to our galaxy - at $d \sim 40$Mpc - that a direct, model-independent estimation of H_0 with Hubble's law,

$$v_H = H_0 d, \tag{2}$$

could be made by measuring the Hubble flow velocity v_H, resulting in $H_0 = 70.0^{+12.0}_{-8.0}$ km s^{-1} Mpc^{-1} [5].

As stated above, almost all GW events have been detected without an EM counterpart. These *dark sirens* can be used to probe the expansion of the universe provided that they are complemented with an external redshift measurement. In his original paper, Schutz suggested that this information could be inferred from galaxy catalogs: each galaxy contributes to a hypothetical measurement of H_0, such that the galaxy structure within a GW event's localisation volume is reflected in the H0 posterior it produces. How informative the individual events are will depend strongly on their localisation volumes. By combining the contributions of many events, the true value of H_0 will be measured as other values will statistically average out. Such analyses have been carried out in the literature, see [6–11].

This manuscript is organized as follows. In Sec. II, we will go over the statistical formalism to perform data analysis with standard sirens, and we will specialize in the galaxy catalog method.

II. STATISTICAL FRAMEWORK

In gravitational-wave astronomy, one subject of interest is extracting the distributional properties of a population of sources based on a set of observations which are drawn from that distribution. Any methodology that leads to unbiased estimates of the population parameters must simultaneously account for measurement uncertainties and selection effects. One way with which the latter affects the observed population is a Mamquist bias: the loudest or brightest sources are more likely to be detected. The standard formalism for extracting the true source population parameters by incorporating these biases in the analysis is frequently labeled as Hierarchical Bayesian inference, see [12–14].

In the discussion below, we will follow the framework outlined in Ref. 15, which is a pedagogical resource on the galaxy catalog approach.

The GW population distribution is sampled with a set of $N_{\rm obs}$ *observed* events with true parameters $\{\vec{\theta}_i\}$, $i \in \{1, \cdots, N_{\rm obs}\}$. We do not have direct access to the true parameters because of noise; instead, we have a set of measured data $\{\vec{d}_i\}$. The $\vec{\theta}_i$ are the individual object parameters, although we are interested in the population hyperparameters, which we call $\vec{\lambda}$. We cannot determine $\vec{\lambda}$ directly, but we can compute the posterior probability given the observations. In the usual Bayesian formalism,

$$p(\vec{\lambda}|\{\vec{d}_i\}) = \frac{p(\{\vec{d}_i\}|\vec{\lambda})\pi(\vec{\lambda})}{p(\{\vec{d}_i\})} \tag{3}$$

where $p(\{\vec{d}_i\}|\vec{\lambda})$ is the likelihood of observing the dataset given the population properties, $\pi(\vec{\lambda})$ is the prior on $\vec{\lambda}$ and $p(\{\vec{d}_i\})$ is the evidence, which is the integral of the numerator over $\vec{\lambda}$.

In the spirit of Ref. 13, we first start with the idealized scenario where the event parameters are perfectly measured. The total likelihood for the set of $N_{\rm obs}$ independent measurements is then

$$p(\{\vec{\theta}_i\}|\vec{\lambda}) = \prod_{i=1}^{N_{\rm obs}} \frac{p_{\rm pop}(\vec{\theta}_i|\vec{\lambda})}{\int p_{\rm pop}(\vec{\theta}_i|\vec{\lambda})d\vec{\lambda}} \tag{4}$$

where $p_{\rm pop}(\vec{\theta}|\vec{\lambda})$ is related to the number density dN of objects expected to be found in the region $[\vec{\theta}, \vec{\theta} + d\vec{\theta}]$:

$$dN = N p_{\rm pop}(\vec{\theta}|\vec{\lambda})d\vec{\theta} \tag{5}$$

We shall build an incrementally more robust model than Eq. (4). Let us first consider the presence of selection effects: not all events are equally likely to be detected. We can encode this with a detection probability $p_{\rm det}$. In the perfect measurement idealization, this detection probability becomes a function of the parameters $\vec{\theta}$ only. In the general case, where noise is present, the detection probability is a function of the data. Let $\mathcal{D}$ be the set of all data. To determine whether an event is detectable, one can use a detection statistic $\rho_{\mathcal{D}}$, which can be calculated for each piece of data. In practice, this statistic can be the signal-to-noise ratio (SNR), the false-alarm rate, etc. We split $\mathcal{D}$ into two disjoints sets, $\mathcal{D}_<$ and $\mathcal{D}_\geq$, according to whether $\rho_{\mathcal{D}}$ is smaller than a threshold $\rho_{\rm tr}$ or not. Then

$$p_{\rm det}(\vec{\theta}) = \int_{\mathcal{D}_\geq} p(\vec{d}|\vec{\theta})d\vec{d} \tag{6}$$

The probability of observing a particular dataset $\vec{d}$ given the assumed population distribution parameterised by $\vec{\lambda}$ is

$$p(\vec{d}|\vec{\lambda}) = \frac{\int p(\vec{d}|\vec{\theta})p_{\rm pop}(\vec{\theta}|\vec{\lambda})d\vec{\theta}}{\alpha(\vec{\lambda})} \tag{7}$$

where $\alpha(\vec{\lambda})$ is a normalization factor integrated over the set of detectable data,

$$\begin{aligned}
\alpha(\vec{\lambda}) &= \int_{\mathcal{D}_\geq} d\vec{d} \int p(\vec{d}|\vec{\theta})p_{\rm pop}(\vec{d}|\vec{\lambda})d\vec{\theta} && (8)\\
&= \int \left[\int_{\mathcal{D}_\geq} p(\vec{d}|\vec{\theta})d\vec{d}\right] p_{\rm pop}(\vec{d}|\vec{\lambda})d\vec{\theta} && (9)\\
&= \int p_{\rm det}(\vec{\theta})p_{\rm pop}(\vec{\theta}|\vec{\lambda})d\vec{\theta} && (10)
\end{aligned}$$

Hence, in the presence of both measurement uncertainties and selection effects, Eq. (4) becomes

$$p(\{\vec{d}_i\}|\vec{\lambda}) = \prod_{i=1}^{N_{\rm obs}} \frac{\int p(\vec{d}_i|\vec{\theta})p_{\rm pop}(\vec{\theta}|\vec{\lambda})d\vec{\theta}}{\int p_{\rm det}(\vec{\theta})p_{\rm pop}(\vec{\theta}|\vec{\lambda})d\vec{\theta}} \tag{11}$$

We can also include the population rate into the framework. The probability of observing k events with an expected number of detections $N_{\rm det}$ is given by a Poisson distribution as

$$p(k|N_{\rm det}) = e^{-N_{\rm det}}(N_{\rm det})^{N_{\rm obs}} \tag{12}$$

The usual $N_{\rm obs}!$ term is absent in Eq. (12) because the events are distinguishable from the observed data. When accouting for selection effects, the expected number of detections $N_{\rm det}$ becomes

$$\begin{aligned}
N_{\rm det}(\vec{\lambda}) &= \int_{\mathcal{D}_\geq} d\vec{d} \int p(\vec{d}|\vec{\theta})\frac{dN}{d\vec{\theta}}d\vec{\theta} && (13)\\
&= \int \left[\int_{\mathcal{D}_\geq} p(\vec{d}|d)d\vec{d}\right] \frac{dN}{d\vec{\theta}}d\vec{\theta} && (14)\\
&= \int p_{\rm det}(\vec{\theta})\frac{dN}{d\vec{\theta}}d\vec{\theta} && (15)\\
&= \int p_{\rm det}(\vec{\theta})Np_{\rm pop}(\vec{\theta}|\vec{\lambda})d\vec{\theta} && (16)\\
&= N\alpha(\lambda) && (17)
\end{aligned}$$

where the last two equalities are derived from Eq. (5) and Eq. (10) respectively. The full posterior with the population rate is then

$$\begin{aligned}
p(\vec{\lambda}, N|\vec{d}) &= p(N|\vec{\lambda},\vec{d})p(\vec{\lambda}|\vec{d}) && (18)\\
&= e^{-N_{\rm det}}(N_{\rm det})^{N_{\rm obs}}\pi(N)\pi(\vec{\lambda})\alpha(\vec{\lambda})^{-N_{\rm obs}}\prod_{i=1}^{N_{\rm obs}}\int p(\vec{d}_i|\vec{\theta})p_{\rm pop}(\vec{\theta}|\vec{\lambda})d\vec{\theta} && (19)
\end{aligned}$$

If a prior $\pi(N) \propto 1/N$ is assumed on the population rate, then the posterior can be marginalized over N:

$$\int e^{-N_{\text{det}}}(N_{\text{det}})^{N_{\text{obs}}}\frac{dN}{N} = \int e^{-N_{\text{det}}}(N_{\text{det}})^{N_{\text{obs}}-1}dN_{\text{det}} \tag{20}$$

$$= (N_{\text{obs}}-1)! \tag{21}$$

So far, the framework we developed has been general; we now specify to the gravitational wave case. The individual event parameters $\vec{\theta}$ describe the compact binary coalescence (CBC): the individual masses and spins, sky position, polarization, inclination angle, luminosity distance, and redshift. The population parameters $\vec{\lambda}$ can be split into three groups: mass, rate and cosmological parameters. The mass parameters specify the GW mass model, such as minimum and maximum mass, slopes, the positions of any features, etc. These are used in the spectral siren method. The rate parameters are used in the model to describe how the CBC merger rate evolves with redshift. Finally, the cosmological parameters are the constants which appear in Eq. (1), namely $\{H_0, \Omega_m, \Omega_{de}, w\}$.

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