

$$\text{data } \text{TIO} \langle l_{\text{can}}, l_{\text{do}} \rangle a = \\ \text{TIO} \left(l: \{ l_{\text{do}} \subseteq l \} \rightarrow \left(\{ l': l \cap l_{\text{can}} \subseteq l' \}, a \right) \right)$$

$$\text{get} :: \text{Field} \langle l_f \rangle v \rightarrow \text{TIO} \langle l_f, \perp \rangle a$$

$$\text{get } (\text{Field } l_f a) = \text{TIO} \$ \setminus W \, l \, \text{sto} \rightarrow \\ \text{let } l' = l \cap l_f \text{ in}$$

$$(\text{W } \text{l' sto}, \text{ sel sto } a)$$

$$\frac{\text{by def of } \cap}{l \cap l_f \subseteq l'}$$

set :: Field $\langle l_f \rangle v \rightarrow v \rightarrow \text{TIO} \langle T, \underline{l_f} \rangle ()$

set (Field $l_f a$) $v = \text{TIO } \$ \setminus W \ L \ \text{sto} \rightarrow$

if $l_f \subseteq L$ then

$(W \underline{L} (\text{upd sto } a \ v), ())$

else

assert false "IFC EXCEPTION"

$\overline{L \cap T \subseteq L}^2$

← dead code as (1)

bind :: $\forall l_2' \subseteq l_1.$ ⁽⁰⁾

$\text{TIO} \langle l_1, l_1' \rangle a$

$\rightarrow (a \rightarrow \text{TIO} \langle l_2, l_2' \rangle b)$

$\rightarrow (\text{TIO} \langle l_1 \cap l_2, l_1' \cup l_2' \rangle b)$

$\frac{l_1' \cup l_2' \subseteq L}{l_1' \subseteq L}$

(1) pre

$l_1' \cup l_2' \subseteq L$

$\therefore l_2' \subseteq l_1$

$\therefore l_1 \cap l_2' = l_2'$ (*)

as $l_2' \subseteq l_1$ (0)

$l_1 \cap l_1 \subseteq l_1$ (1) post

$\therefore l_1 \cap l_2' \subseteq l_1$

by (*)

$\frac{l_1' \subseteq l_1}{l_2' \subseteq l_1'} (2) \text{ pre}$

$l_1 \cap l_1 \subseteq l_1' (f_1 l)$

$l_1' \cap l_2 \subseteq l_2' (f_2 l')$

$\frac{l_1 \cap l_1 \subseteq l_1' \quad l_1' \cap l_2 \subseteq l_2'}{l_1 \cap (l_1 \cap l_2) \subseteq l_2'} (3)$

bind (TIO f_1) $k_2 = \text{TIO } \$ \setminus l \rightarrow$

let $(l', v_1) = \underline{f_1 l}_1 \xrightarrow{\quad} l$

$\text{TIO } f_2 = k_2 v_1 \xrightarrow{\quad} l \cap l_1$

$(l'', v_2) = \underline{f_2 l'}_2 \xrightarrow{\quad} l \cap l_1 \cap l_2$

in

$\underline{(l'', v_2)}_3$