MAXIMA PRACTICALS

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Subject: Maths For computing

Topic: Practical's Performed on Maxima

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- **Practical 1**: - Create and transform vectors and matrices (the transpose vector (matrix) conjugate transpose of a vector (matrix))

1. **Creating a vector(V) and matrix(M):**
   1.1. To create a vector and matrix we use 'matrix' function.

```
→ /* Practical 1 by Mozahidul Islam;;;
(%i1) V:matrix([1],[2],[3]); /* Vector V
```

$$V \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

```
(%i2) M: matrix([1,2,3],[4,5,6],[7,8,9]); /*The Matrix M*/
```

$$M \quad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

2. **Calculating transpose of vector(V) and matrix(M):**
   2.1. To calculate transpose of a vector and a matrix we use 'transpose' function.

```
→ conjugate(V);
```

$$(\%o6) \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

```
→ conjugate(M);
```

$$(\%o7) \quad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

3. **Calculating conjugate of vector(V) and matrix(M)**
   3.1. To calculate conjugate of a vector and a matrix we use 'conjugate' function.

```
→ transpose(V);
```

$$(\%o4) \quad \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$$

```
→ transpose(M);
```

$$(\%o5) \quad \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

4. **Calculating conjugate transpose of a matrix**
   4.1. To calculate conjugate transpose of a matrix we use transpose (conjugate ()) function.

```
→ transpose(conjugate(M));
```

$$(\%o8) \quad \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

- **Practical 2**: - Generate the matrix into echelon form and find its rank.
  - Creating matrix

→ /*Practical 2 by mozahidul islam*/;

(%i2) M: matrix([1,2,3],[4,5,6],[7,8,9]); /*The Matrix M*/

$$M \quad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

  - Echelon form

(%i7) echelon(M);

$$(\%o7) \quad \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

  - Finding Rank

(%i8) rank(M);

(%o8) 2

- **Practical 3:** - Find cofactors, determinant, adjoint and inverse of a matrix.

(%o9) C:/maxima-5.47.0/share/maxima/5.47.0/share/contrib/gentran/test/matrix.mac

```
(%i13) A: matrix([2,3,1],[0,-3,4],[5,2,3]); /*The Matrix A*/
```

$$A \quad \begin{pmatrix} 2 & 3 & 1 \\ 0 & -3 & 4 \\ 5 & 2 & 3 \end{pmatrix}$$

```
(%i14) determinant(A);
```

(%o14) 41

```
(%i15) adjoint(A);
```

$$(\%o15) \quad \begin{pmatrix} -17 & -7 & 15 \\ 20 & 1 & -8 \\ 15 & 11 & -6 \end{pmatrix}$$

```
(%i16) invert(A);
```

$$(\%o16) \quad \begin{pmatrix} -\left(\frac{17}{41}\right) & -\left(\frac{7}{41}\right) & \frac{15}{41} \\ \frac{20}{41} & \frac{1}{41} & -\left(\frac{8}{41}\right) \\ \frac{15}{41} & \frac{11}{41} & -\left(\frac{6}{41}\right) \end{pmatrix}$$

- **Practical 4:** - Solve a system of Homogeneous and non-homogeneous equations using Gauss elimination method.

```
→ /* practical 4 by Mozahidul Islam

(%i35) eqn1:read("equation 1 is=");
       eqn2:read("equation 2 is=");
       eqn3:read("equation 3 is=");
       eqn4:read("equation 4 is=");
       block(D:augcoefmatrix([eqn1,eqn2,eqn3,eqn4],[x,y,z,a]),
          S:echelon(D),
       [p.q],X:[x,y,z,a,1],[p,q]:matrix_size(S),
       for i:1 thru p do(for n:1 thru q do(d(i):=sum(S[i,n]−X[n],n,1,q)=0),
       s:makelist(d(i),i,1,p)),
       solve(s,[x,y,z,a]));
       /*if solve(s)= [] then print("system of given equations is inconsistent and have no solution")
       else print("system of given equations is consistent")$*/
       equation 1 is=  2·x + y + 2·z + a = 6;
eqn1   2 z+y+2 x+a=6
       equation 2 is= 6·x − 6·y + 6·z+ 12·a = 36;
eqn2   6 z−6 y+6 x+12 a=36
       equation 3 is= 4·x + 3·y + 3·z − 3·a = − 1;
eqn3   3 z+3 y+4 x−3 a=−1
       equation 4 is= 2·x + 2·y − z + a = 10;
eqn4   −z+2 y+2 x+a=10
(%o38) [[x=2,y=1,z=−1,a=3]]
```

- **Practical 5**: - Solve a system of Homogeneous equations using the Gauss Jordan method.

1. Assigning the value of matrix to a variable named Coefficient_Matrix.

```
(%i39)/* practical 5 by mozahidul islam*/
/* 5. Solve a system of Homogeneous equations using the Gauss Jordan method*/
Coefficient_Matrix: matrix([1,1,1],[2,-3,4],[3,4,5]);
```

Coefficient_Matrix $\begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

2. Assigning the value of Column matrix to a variable named Column_martix.

```
(%i2) /*Column Matrix */
Column_Matrix: matrix([9,13,40]);
```

Column_Matrix $\begin{pmatrix} 9 & 13 & 40 \end{pmatrix}$

3. We will use the function invert for inversion process of matrix.

```
(%i3) /*Finding inverse of coefficient matrix*/
Inv_of_Coefficient_Matrix: invert(Coefficient_Matrix);
```

Inv_of_Coefficient_Matrix $\begin{pmatrix} \frac{31}{12} & \frac{1}{12} & -\left(\frac{7}{12}\right) \\ -\left(\frac{1}{6}\right) & -\left(\frac{1}{6}\right) & \frac{1}{6} \\ -\left(\frac{17}{12}\right) & \frac{1}{12} & \frac{5}{12} \end{pmatrix}$

4. We will multiply the Inversed Matrix with the Column Matrix

```
(%i4) /*Solution of system of matrix*/
Solution_of_the_system_of_Equations: Inv_of_Coefficient_Matrix . Column_Matrix;
```

Solution_of_the_system_of_Equations $\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$

- **Practical 6**: -Generate basis of column space, null space, row space and left null space of a matrix space

1. Create a matrix.

```
(%i40)/* practical 6 by mozahidul islam*/
A: matrix([-1,3,1],[1,1,0],[1,1,0]);
```

$$A \quad \begin{pmatrix} -1 & 3 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

2. Basis of column space.

```
(%i4) columnspace(A);
```

$$(\%o4) \quad \text{span}\left(\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}\right)$$

3. Create null space.

```
(%i5) nullspace(A);
```

$$(\%o5) \quad \text{span}\left(\begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}\right)$$

4. Row space.

```
(%i6) rowspace(A);
```

$$(\%o6) \quad \text{rowspace}\left(\begin{pmatrix} -1 & 3 & 2 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}\right)$$

5. Left null space.

```
(%i7) leftnullspace(A);
```

$$(\%o7) \quad \text{leftnullspace}\left(\begin{pmatrix} -1 & 3 & 2 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}\right)$$

- **Practical 7:** - Check the linear dependence of vectors. Generate a linear combination of given vectors of Rn/ matrices of the same size and find the transition matrix of given matrix space.

1. Checking linear dependence of vector.
   a. Here determinant is zero, so the vectors are linearly dependent.

```
(%i40) /* practical 7 by mozahidul islam*/
(%i32) v1:[1,2,3];
       v2:[4,5,6];
       v3:[7,8,9];

v1     [1,2,3]
v2     [4,5,6]
v3     [7,8,9]

(%i36) a:transpose(matrix(v1,v2,v3));
```

$$a \quad \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

```
(%i39) determinant(a);
(%o39) 0
```

2. Generate a linear combination of given vectors of Rn/ matrices.

```
(%i9) v1: matrix([1], [2], [3]);
      v2: matrix([4], [5], [6]);
      v3: matrix([7], [8], [9]);

      c1: 2;
      c2: -1;
      c3: 3;

      V: matrix([v1, v2, v3]);
      c: matrix([c1], [c2], [c3]);

      v: V . c;
```

Output:

```
v: V . c;
```

$v1 \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$v2 \quad \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$

$v3 \quad \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$

$c1 \quad 2$

$c2 \quad -1$

$c3 \quad 3$

$V \quad \begin{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} & \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} & \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \end{pmatrix}$

$c \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$v \quad \begin{pmatrix} 19 \\ 23 \\ 27 \end{pmatrix}$

- **Practical 8:** - Find the orthonormal basis of a given vector space using the Gram-Schmidt orthogonalization process.

```
/* practical 8 by mozahidul islam*/
```

Find The Orthonormal Basis Of A Given Vector Space Using The Gram-schmidt Orthogonalization Process.

```
(%i2) load(eigen);
      Matrix_A: matrix([2,1,0,-1],[1,0,2,-1],[0,-2,1,0]);
(%o1) C:/maxima-5.47.0/share/maxima/5.47.0/share/matrix/eigen.mac
```

$$\text{Matrix\_A}\begin{pmatrix} 2 & 1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 0 & -2 & 1 & 0 \end{pmatrix}$$

```
(%i3) Orthogonal_Basis:gramschmidt(Matrix_A);
```

$$\text{Orthogonal\_Basis}\left[[2,1,0,-1],\left[0,-\left(\frac{1}{2}\right),2,-\left(\frac{1}{2}\right)\right],\left[\frac{2}{3},-\left(\frac{2^2}{3}\right),-\left(\frac{1}{3}\right),0\right]\right]$$

```
(%i24) A:((2^2)+(1^2)+((-1)^2)+(0^2))^0.5;
       B:[2,1,0,-1]/A;
A      2.449489742783178
B      [0.8164965809277261,0.4082482904638631,0,-0.4082482904638631]

(%i26) C:((0^2)+((-1/2)^2)+(2^2)+((-1/2)^2))^0.5;
       D:[0,-1/2,2,-1/2]/C;
C      2.1213203435596424
D      [0,-0.23570226039551587,0.9428090415820635,-0.23570226039551587]

(%i28) E:((-2/3)^2+(2^2/3)^2+(1/3)^2+0^2)^0.5;
       F:[-(2/3),2^2/3,1/3,0]/E;
E      1.5275252316519468
F      [-0.4364357804719847,0.8728715609439694,0.2182178902359923 6,0]

(%i29) Orthonormal_Basis:matrix(B,D,F);
```

$$\text{Orthonormal\_Basis}\begin{pmatrix} 0.8164965809277261 & 0.4082482904638631 & 0 & -0.4082482904638631 \\ 0 & -0.23570226039551587 & 0.9428090415820635 & -0.23570226039551587 \\ -0.4364357804719847 & 0.8728715609439694 & 0.21821789023599236 & 0 \end{pmatrix}$$

- **Practical 9:** - Check the diagonalizable property of matrices and find the corresponding eigenvalue and verify the Cayley- Hamilton theorem.

```
/* practical 9 by mozahidul islam*/
(%i15) load(eigen);
(%o15) C:/maxima-5.47.0/share/maxima/5.47.0/share/matrix/eigen.mac
(%i16) load(matrix);
(%o16) C:/maxima-5.47.0/share/maxima/5.47.0/share/contrib/gentran/test/matrix.mac
(%i17) A;
```

$$(\%o17)\quad \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

```
(%i20) nondiagonalizable(A);
apply: found nondiagonalizable evaluates to false where a function was expected.
 -- an error. To debug this try: debugmode(true);
(%i18) load(nchrpl);
(%o18) C:/maxima-5.47.0/share/maxima/5.47.0/share/matrix/nchrpl.mac
(%i19) ncharpoly(A,lambda);
```

$$(\%o19)\quad lambda^3 + lambda^2 - 12\, lambda$$

- **Practical 10**: - Application of Linear algebra: Coding and decoding of messages using nonsingular matrices

  - code

```
→   /* practical 9 by mozahidul islam*/

(%i61) /*This is Practical 10 byMozahidul Islam*/
       /* Define a nonsingular matrix for encoding and decoding */
       A: matrix([2, 3], [1, 4]);

       /* Define a message vector */
       message: matrix([1], [2]);

       /* Encoding: Multiply the message by the nonsingular matrix */
       encoded_message: A. message;

       /* Decoding: Multiply the encoded message by the inverse of the matrix */
       A_inverse: invert(A);
       decoded_message: A_inverse. encoded_message;

       /* Display the results */ print("Original Message:");
       print(message); print("Encoded Message:");
       print(encoded_message); print("Decoded Message:");
       print(decoded_message);
```

  - Output

A $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

message $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

encoded_message $\begin{pmatrix} 8 \\ 9 \end{pmatrix}$

A_inverse $\begin{pmatrix} \frac{4}{5} & -\left(\frac{3}{5}\right) \\ -\left(\frac{1}{5}\right) & \frac{2}{5} \end{pmatrix}$

decoded_message $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Original Message:

(%o56) Original Message:

$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(%o57) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Encoded Message:

(%o58) Encoded Message:

$\begin{pmatrix} 8 \\ 9 \end{pmatrix}$

(%o59) $\begin{pmatrix} 8 \\ 9 \end{pmatrix}$

Decoded Message:

(%o60) Decoded Message:

$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(%o61) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

- **Practical 11:** - Compute Gradient of a scalar field.

  - Code

(%i66)
```
/* Practical 11 By Mozahidul Islam*/;
/* Defining variables*/

load(vector);

/* Define the scalar field in therm of variables x, y, z */
scalar_field: x^2 + 2·y −3·z;

/* Compute the gradient of the scalar field */
gradient_field: gradient(scalar_field,[x,y,z]);

/* Display the gradient field */
print("gradient of the Scalar Field:");
print(gradient_field);
```

  - Output

(%o62) C:/maxima−5.47.0/share/maxima/5.47.0/share/vector/vector.mac

scalar_field $-(3\,z)+2\,y+x^{2}$

gradient_field $$[[\frac{d^{2}}{d\,x^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,x\,d\,y}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,x\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)],[\frac{d^{2}}{d\,x\,d\,y}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)],[\frac{d^{2}}{d\,x\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,z^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)]]$$

gradient of the Scalar Field:

(%o65) gradient of the Scalar Field:

$$[[\frac{d^{2}}{d\,x^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,x\,d\,y}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,x\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)],[\frac{d^{2}}{d\,x\,d\,y}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)],\left[\frac{d^{2}}{d\,x\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,z^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)\right]]$$

(%o66) $$[[\frac{d^{2}}{d\,x^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,x\,d\,y}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,x\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)],[\frac{d^{2}}{d\,x\,d\,y}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)],\left[\frac{d^{2}}{d\,x\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,z^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)\right]]$$

(%o66) $$[[\frac{d^{2}}{d\,x^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,x\,d\,y}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,x\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)],[\frac{d^{2}}{d\,x\,d\,y}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)],\left[\frac{d^{2}}{d\,x\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,y\,d\,z}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right),\frac{d^{2}}{d\,z^{2}}\mathrm{ient}\left(-(3\,z)+2\,y+x^{2},[x,y,z]\right)\right]]$$

- **Practical 12:** - Compute Divergence of a vector field.

```
(%i34) /* This practical 12 is done by Mozahidul Islam*/
       /* Load vector*/
       load("vector");

       /* Define the variables */
       [wx,wy,wz]:[x,y,z];

       /*Define the vector field in terms of variables x, y and z */
       vector_field: [2·x·y, x^2−z,−y^2·z];

       /* Compute the divergence of the vector field */
       divergence: Div(vector_field, wx, wy, wz);

       print("Divergence of  the Vector field: ");
       print(divergence);
(%o29) C:/maxima−5.47.0/share/maxima/5.47.0/share/vector/vector.mac
(%o30) [x,y,z]
```

vector_field $\left[2\,x\,y, x^2 - z, -\left(y^2\,z\right)\right]$

```
divergence 0
       Divergence of  the Vector field:
(%o33) Divergence of  the Vector field:
       0
(%o34) 0
```

- **Practical 13:** - Compute Divergence of a vector field.

```
(%i99) /* This practical 13 is done by Mozahidul Islam*/
       /* Load vector*/

       /* Define the variables */
       [wx,wy,wz]:[x,y,z];

       /* Define the vector field in terms of variables x, y and z */
       vector_filed: [2·y, 3·x·z, x^2−y];
       /* Compute the curl of the vector field */
       c: curl(vector_field,wx,wy,wz);

       /* Display the curl vector */
       print("Curl of the Vector Field:");
       print(c);
```

(%o95) [x,y,z]

vector_filed $\left[2\,y, 3\,x\,z, x^2 - y\right]$

c $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$

Curl of the Vector Field:

(%o98) Curl of the Vector Field:

$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$

(%o99) $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$