

## Basic Operations

Addition:

$$\vec{v} + \vec{w}$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

Subtraction:

$$\vec{w} + (\vec{v} - \vec{w}) = \vec{v}$$

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \left( \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Scalar Multiplication

$$(\vec{v} \cdot \vec{w})$$

$$2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$-1/2 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3/2 \\ -1 \\ -1/2 \end{bmatrix}$$

## Magnitude & Direction

2 Dimensions

$$\|\vec{v}\|^2 = v_x^2 + v_y^2$$

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

3 Dimensions

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$n$  Dimensions

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$$

**Normalization** - Process of finding a unit vector in the same direction as a given vector

Unit vector in direction of  $\vec{v}$

$$\frac{1}{\|\vec{v}\|} = \vec{v}$$

Normalize

$$\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Magnitude

$$\|\vec{v}\| = \sqrt{-1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\vec{u} = \frac{1}{\|\vec{v}\|} \vec{v} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

Verify unit vector of  $u$  is 1

$$\|\vec{v}\| = \sqrt{\frac{-1^2}{\sqrt{3}} + \frac{1^2}{\sqrt{3}} + \frac{1^2}{\sqrt{3}}} = 1$$

## Zero Vector

$$\vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Magnitude of 0 Vector is 0

$$\|\vec{0}\| = 0$$

Cannot be normalized, has no direction

$$\frac{1}{\|\vec{0}\|} = \text{undefined}$$

## Dot Products

*Inner product*: angle between 2 vectors

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos \theta$$

Solve for the angle using inverse function

$$\begin{aligned} \theta &= \arccos\left(\frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \cdot \|\vec{w}\|}\right) \\ &= \arccos\left(\frac{1}{\|\vec{v}\|} \vec{v} \cdot \frac{1}{\|\vec{w}\|} \vec{w}\right) \end{aligned}$$

Shorter formula:

$$\begin{aligned} (\vec{v} \cdot \vec{w}) &= v_1 w_1 + v_2 w_2 + \dots + v_n w_n \\ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ 10 \end{bmatrix} &= 1 \cdot 3 + 2 \cdot 1 + (-1) \cdot 0 = 5 \end{aligned}$$

Angle between 2 vectors:

$$\begin{aligned} \theta &= \arccos\left(\frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \cdot \|\vec{w}\|}\right) \\ \theta &= \arccos\left(\frac{5}{\sqrt{6} \sqrt{10}}\right) \\ &\approx .87 \text{ radians} \\ &\approx 50^\circ \end{aligned}$$

Same direction:

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\|$$

$$\cos \theta = 1$$

$$\theta = 0 \text{ \& radians, 0 degrees}$$

Opposite direction:

$$\vec{v} \cdot \vec{w} = -\|\vec{v}\| \cdot \|\vec{w}\|$$

$$\cos \theta = -1$$

$$\theta = \pi \text{ radians, 180 degrees}$$

Right angle:

$$\vec{v} \cdot \vec{w} = 0$$

$$\cos \theta = 0$$

$$\theta = \frac{\pi}{2} \text{ radians, 90 degrees}$$

Of a vector:

$$\begin{aligned} \vec{v} \cdot \vec{v} &= \|\vec{v}\| \cdot \|\vec{v}\| \cos 0 \\ &= \|\vec{v}\|^2 \\ \|\vec{v}\| &= \sqrt{\vec{v} \cdot \vec{v}} \end{aligned}$$

$$\cos 0 = 1$$

