Homework 1, Data Mining, Fall 2014

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given the following vectors:

$$x = \{1, 3, 5, 7, 6, 8, 5, 2\} \tag{1}$$
$$y = \{7, 11, 9, 6, 4, 3, 1, 3\} \tag{2}$$
$$z = \{3, 5, 7, 9, 8, 7, 6, 5\} \tag{3}$$

1 wavelet transform each vector

An un-normalized 8-point Haar matrix

$$H_8$$

is shown below

$$H_8 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

and the vector

$$\{\sqrt{8}, \sqrt{8}, 2, 2, \sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2}\}$$

and will be used to normalize each vector after transformation.

(a) vector x

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} * \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 6 \\ 8 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 1+3+5+7+6+8+5+2 \\ 1+3+5+7-6-8-5-2 \\ 1+3+5+7 \\ 6+8-5-2 \\ 1-3 \\ 5-7 \\ 6-8 \\ 5-2 \end{pmatrix}$$

$$\begin{aligned} &= \\ &\{37, -4, -8, 7, -2, -2, -2, 3\} \\ &\odot \\ &\{\sqrt{8}, \sqrt{8}, 2, 2, \sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2},\} \\ &= \\ &\{74\sqrt{2}, -8\sqrt{2}, -16, 14, -2\sqrt{2}, -2\sqrt{2}, -2\sqrt{2}, 3\sqrt{2}\} \\ &\{104.652, -11.3137, -16., 14., -2.82843, -2.82843, -2.82843, 4.24264\} \end{aligned}$$

(b) vector y

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} * \begin{pmatrix} 7 \\ 11 \\ 9 \\ 6 \\ 4 \\ 3 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 7+11+9+6+4+3+1+3 \\ 7+11+9+6-4-3-1-3 \\ 7+11-9-6 \\ 4+3-1-3 \\ 7-11 \\ 9-6 \\ 4-3 \\ 1-3 \end{pmatrix}$$

$$\begin{aligned} &= \{44, 22, 3, 3, -4, 3, 1, -2\} \\ &\odot \\ &\{\sqrt{8}, \sqrt{8}, 2, 2, \sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2},\} \\ &= \\ &\{88\sqrt{2}, 44\sqrt{2}, 6, 6, -4\sqrt{2}, 3\sqrt{2}, \sqrt{2}, -2\sqrt{2}\} \\ &\{124.451, 62.2254, 6., 6., -5.65685, 4.24264, 1.41421, -2.82843\} \end{aligned}$$

(c) vector z

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} * \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 6 \\ 8 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 3+5+7+9+8+7+6+5 \\ 3+5+7+9+-9+-7+-6+-5 \\ 3+3+-7+-9 \\ 8+7+-6+-5 \\ 3+-5 \\ 7+-9 \\ 8+-7 \\ 6+-5 \end{pmatrix}$$

$$= \{50, -2, -8, 4, -2, -2, 1, 1\}$$
$$\odot$$
$$\{\sqrt{8}, \sqrt{8}, 2, 2, \sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2},\}$$
$$=$$
$$\{100\sqrt{2}, -4\sqrt{2}, -16, 8, -2\sqrt{2}, -2\sqrt{2}, \sqrt{2}, \sqrt{2}\}$$
$$\{141.421, -5.65685, -16, 8, -2.82843, -2.82843, 1.41421, 1.41421\}$$

2 fourier transform each vector

(Defintion from Wikipedia)

The transformation W of size $N \times N$ can be defined as

$$W = \left(\frac{\omega^{jk}}{\sqrt{N}} \right)_{j,k=0,\ldots,N-1}$$

, or equivalently:

$$W = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \omega^6 & \cdots & \omega^{2(N-1)} \\ 1 & \omega^3 & \omega^6 & \omega^9 & \cdots & \omega^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \omega^{3(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{pmatrix}$$

where $\omega = e^{-\frac{2\pi i}{N}}$

In our case, with an 8×8 matrix, we get a transformation matrix of:

$$W = \frac{1}{\sqrt{8}} \begin{pmatrix} \omega^0 & \omega^0 & \omega^0 & \dots & \omega^0 \\ \omega^0 & \omega^1 & \omega^2 & \dots & \omega^7 \\ \omega^0 & \omega^2 & \omega^4 & \dots & \omega^{14} \\ \omega^0 & \omega^3 & \omega^6 & \dots & \omega^{21} \\ \omega^0 & \omega^4 & \omega^8 & \dots & \omega^{28} \\ \omega^0 & \omega^5 & \omega^{10} & \dots & \omega^{35} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^7 & \omega^{14} & \dots & \omega^{49} \end{pmatrix}$$

where $\omega = e^{-\frac{2\pi i}{8}} = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

where $\omega = e^{-2\pi i} = 1$ is Euler's identity, allowing us to reduce the 8×8 matrix as follows:

$$\frac{1}{\sqrt{8}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{\frac{-\pi i}{4}} & -i & e^{\frac{-3\pi i}{4}} & -1 & e^{\frac{-5\pi i}{4}} \\ 1 & -i & -1 & i & 1 & -i \\ 1 & e^{\frac{-3\pi i}{4}} & i & e^{\frac{-\pi i}{4}} & -1 & e^{\frac{-7\pi i}{4}} \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & e^{\frac{-5\pi i}{4}} & -i & e^{\frac{-7\pi i}{4}} & -1 & e^{\frac{-\pi i}{4}} \\ 1 & i & -1 & -i & 1 & i \\ 1 & e^{\frac{-7\pi i}{4}} & i & e^{\frac{-5\pi i}{4}} & -1 & e^{\frac{-3\pi i}{4}} \end{pmatrix}$$

(a) Vector x

$$x = \{1, 3, 5, 7, 6, 8, 5, 2\}$$

$$\frac{1}{\sqrt{8}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{\frac{-\pi i}{4}} & -i & e^{\frac{-3\pi i}{4}} & -1 & e^{\frac{-5\pi i}{4}} \\ 1 & -i & -1 & i & 1 & -i \\ 1 & e^{\frac{-3\pi i}{4}} & i & e^{\frac{-\pi i}{4}} & -1 & e^{\frac{-7\pi i}{4}} \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & e^{\frac{-5\pi i}{4}} & -i & e^{\frac{-7\pi i}{4}} & -1 & e^{\frac{-\pi i}{4}} \\ 1 & i & -1 & -i & 1 & i \\ 1 & e^{\frac{-7\pi i}{4}} & i & e^{\frac{-5\pi i}{4}} & -1 & e^{\frac{-3\pi i}{4}} \end{pmatrix} * \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 6 \\ 8 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 1+3+5+7+6+8+5+2 \\ 1 + 3e^{\frac{-\pi i}{4}} + -5i + 7e^{\frac{-3\pi i}{4}} - 6 + 8e^{\frac{-5\pi i}{4}} + 5i + 2e^{\frac{-7\pi i}{4}} \\ 1 - 3i + -5 + 7i + 6 - 8i - 5 + 2i \\ 1 + 3e^{\frac{-3\pi i}{4}} + 5i + 7e^{\frac{-\pi i}{4}} - 6 + 8e^{\frac{-7\pi i}{4}} - 5i + 2e^{\frac{-5\pi i}{4}} \\ 1 - 3 + 5 - 7 + 6 - 8 + 5 - 2 \\ 1 + 3e^{\frac{-5\pi i}{4}} - 5i + 7e^{\frac{-7\pi i}{4}} - 6 + 8e^{\frac{-\pi i}{4}} + 5 + 2e^{\frac{-3\pi i}{4}} \\ 1 + 3i - 5 - 7i + 6 + 8i - 5 - 2i \\ 1 + 3e^{\frac{-7\pi i}{4}} + 5i + 7e^{\frac{-5\pi i}{4}} - 6 + 8e^{\frac{-3\pi i}{4}} - 5i + 2e^{\frac{-\pi i}{4}} \end{pmatrix}$$

$$= \begin{pmatrix} 37 \\ -5 + 3e^{-i\pi/4} + 2e^{i\pi/4} + 7e^{-3i\pi/4} + 8e^{3i\pi/4} \\ -3 - 2i \\ -5 + 7e^{-i\pi/4} + 8e^{i\pi/4} + 3e^{-3i\pi/4} + 2e^{3i\pi/4} \\ -3 \\ -5i + 8e^{-i\pi/4} + 7e^{i\pi/4} + 2e^{-3i\pi/4} + 3e^{3i\pi/4} \\ -3 + 2i \\ -5 + 2e^{-i\pi/4} + 3e^{i\pi/4} + 8e^{-3i\pi/4} + 7e^{3i\pi/4} \end{pmatrix} = \begin{pmatrix} 37 \\ -12.07106 \\ -3 - 2i \\ 2.07106 \\ -3 \\ 7.07106 - 5i \\ -3 + 2i \\ -12.07106 \end{pmatrix} * \frac{1}{\sqrt{8}}$$

$$\{13.0815, -4.26776, -1.06066 - 0.707107i, 0.73223,$$
$$-1.06066, 2.5 - 1.76777i, -1.06066 + 0.707107i, -4.26776\}$$

(b) y vector

$$y = \{7, 11, 9, 6, 4, 3, 1, 3\}$$

$$\frac{1}{\sqrt{8}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{\frac{-\pi i}{4}} & -i & e^{\frac{-3\pi i}{4}} & -1 & e^{\frac{-5\pi i}{4}} \\ 1 & -i & -1 & i & 1 & -i \\ 1 & e^{\frac{-3\pi i}{4}} & i & e^{\frac{-\pi i}{4}} & -1 & e^{\frac{-7\pi i}{4}} \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & e^{\frac{-5\pi i}{4}} & -i & e^{\frac{-7\pi i}{4}} & -1 & e^{\frac{-\pi i}{4}} \\ 1 & i & -1 & -i & 1 & i \\ 1 & e^{\frac{-7\pi i}{4}} & i & e^{\frac{-5\pi i}{4}} & -1 & e^{\frac{-3\pi i}{4}} \end{pmatrix} * \begin{pmatrix} 7 \\ 11 \\ 9 \\ 6 \\ 4 \\ 3 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 7+11+9+6+4+3+1+3 \\ 7+11e^{\frac{-\pi i}{4}} + -9i + 6e^{\frac{-3\pi i}{4}} + -4 + 3e^{\frac{-5\pi i}{4}} + i + 3e^{\frac{-7\pi i}{4}} \\ 7 + -11i + -9 + 6i + 4 + -3i + -1 + 3i \\ 7+11e^{\frac{-3\pi i}{4}} + 9i + 6e^{\frac{-\pi i}{4}} + -4 + 3e^{\frac{-7\pi i}{4}} + -i + 3e^{\frac{-5\pi i}{4}} \\ 7 + -11 + 9 + -6 + 4 + -3 + 1 + -3 \\ 7+11e^{\frac{-5\pi i}{4}} + -9i + 6e^{\frac{-7\pi i}{4}} + -4 + 3e^{\frac{-\pi i}{4}} + 1 + 3e^{\frac{-3\pi i}{4}} \\ 7 + 11i + -9 + -6i + 4 + 3i + -1 + -3i \\ 7+11e^{\frac{-7\pi i}{4}} + 9i + 6e^{\frac{-5\pi i}{4}} + -4 + 3e^{\frac{-3\pi i}{4}} + -i + 3e^{\frac{-\pi i}{4}} \end{pmatrix}$$

$$= \begin{pmatrix} 44 \\ (3-8i) + 11e^{-(i\pi)/4} + 3e^{(i\pi)/4} + 6e^{-(3i\pi)/4} + 3e^{(3i\pi)/4} \\ 1-5i \\ (3+8i) + 6e^{-(i\pi)/4} + 3e^{(i\pi)/4} + 11e^{-(3i\pi)/4} + 3e^{3i\pi)/4} \\ -2 \\ (4-9i) + 3e^{(} - (i\pi)/4) + 6e^{(i\pi)/4} + 3e^{-(3i\pi)/4} + 11e^{(3i\pi)/4} \\ 1+5i \\ (3+8i) + 3e^{-(i\pi)/4} + 11e^{(i\pi)/4} + 3e^{-(3i\pi)/4} + 6e^{(3i\pi)/4} \end{pmatrix} = \begin{pmatrix} 44 \\ 6.53553 - 15.77817i \\ 1-5i \\ -0.53553 + 0.22182i \\ -2 \\ 0.46446 - 1.221825i \\ 1+5i \\ 6.5355 + 15.7781i \end{pmatrix} * \frac{1}{\sqrt{8}}$$

$$\approx \{15.5563, 2.31066 - 5.57843i, 0.353553 - 1.76777i,$$
$$-0.189338 + 0.0784252i, -0.707107, 0.164211 - 0.43198i,$$
$$0.353553 + 1.76777i, 2.31065 + 5.5784i\}$$

(c) z vector

$$z = \{3, 5, 7, 9, 8, 7, 6, 5\}$$

$$\frac{1}{\sqrt{8}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{\frac{-\pi i}{4}} & -i & e^{\frac{-3\pi i}{4}} & -1 & e^{\frac{-5\pi i}{4}} \\ 1 & -i & -1 & i & 1 & -i \\ 1 & e^{\frac{-3\pi i}{4}} & i & e^{\frac{-\pi i}{4}} & -1 & e^{\frac{-7\pi i}{4}} \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & e^{\frac{-5\pi i}{4}} & -i & e^{\frac{-7\pi i}{4}} & -1 & e^{\frac{-\pi i}{4}} \\ 1 & i & -1 & -i & 1 & i \\ 1 & e^{\frac{-7\pi i}{4}} & i & e^{\frac{-5\pi i}{4}} & -1 & e^{\frac{-3\pi i}{4}} \end{pmatrix} * \begin{pmatrix} 3 \\ 5 \\ 7 \\ 9 \\ 8 \\ 7 \\ 6 \\ 5 \end{pmatrix} = \begin{pmatrix} 3+5+7+9+8+7+6+5 \\ 3+5e^{\frac{-\pi i}{4}} + -7i + 9e^{\frac{-3\pi i}{4}} + -8 + 7e^{\frac{-5\pi i}{4}} + 6i + 5e^{\frac{-7\pi i}{4}} \\ 3 + -5 + -7 + 9i + 8 + -7i + -6 + 5i \\ 3+5^{\frac{-3\pi i}{4}} + 7i + 9e^{\frac{-\pi i}{4}} + -8 + 7e^{\frac{-7\pi i}{4}} + -6i + 5e^{\frac{-5\pi i}{4}} \\ 3 + -5 + 7 + -9 + 8 + -7 + 6 + -5 \\ 3+5^{\frac{-5\pi i}{4}} + -7i + 9e^{\frac{-7\pi i}{4}} + -8 + 7e^{\frac{-\pi i}{4}} + 6 + 5e^{\frac{-3\pi i}{4}} \\ 3 + 5 + -7 + -9i + 8 + 7i + -6 + -5i \\ 3+5^{\frac{-7\pi i}{4}} + 7i + 9e^{\frac{-5\pi i}{4}} + -8 + 7e^{\frac{-3\pi i}{4}} + -6i + 5e^{\frac{-\pi i}{4}} \end{pmatrix}$$

$$= \begin{pmatrix} 50 \\ (-5-i)+5e^{-(i\pi)/4}+5e^{i\pi)/4}+9e^{-(3i\pi)/4}+7e^{3i\pi)/4} \\ -7+7i \\ (-5+i)+5^{-(3i\pi)/4}+9e^{-(i\pi)/4}+7e^{(i\pi)/4}+5e^{(3i\pi)/4} \\ -2 \\ 3+5^{1/4(-5\pi i)}-7i+9e^{1/4(-7\pi i)}-8+7e^{-(\pi i)/4}+6+5e^{1/4(-3\pi i)} \\ 3-7i \\ (-5+i)+5^{-(7i\pi)/4}+5e^{-(i\pi)/4}+7e^{-(3i\pi)/4}+9e^{(3i\pi)/4} \end{pmatrix} = \begin{pmatrix} 50 \\ -9.242-2.4142i \\ -7+7i \\ 1.9824+3.7269i \\ -2 \\ 9.7774-9.1583i \\ 3-7i \\ -13.6165-1.6663i \end{pmatrix} * \frac{1}{\sqrt{8}}$$

$$\{17.6777, -3.26754-8535.49i, -2.47487+2.47487i, \\ 0.700884+1.31766i, -0.707107, 3.45683-3.23795i, \\ 1.06066-2.47487i, -4.81416-0.589126i\}$$

3 minkowski distance

The Minkowski distance of order p between two points

$$P = (x_1, x_2, \dots, x_n) \text{ and } Q = (y_1, y_2, \dots, y_n) \in \mathbb{R}^n$$

is defined as:

$$\left(\sum_{i=1}^{n} |x_i - y_i|^p \right)^{1/p}$$

(Wikipedia)
and can be written as:

$$d(i,j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

where $i = \{x_{i1}, x_{i_2}, \dots, x_{ip}\}$ and $j = \{x_{j1}, x_{j_2}, \dots, x_{jp}\}$ are two p-dimensional data objects and h is the order (from class slides).

(a) raw vectors

Between x and y:

$$x = \{1, 3, 5, 7, 6, 8, 5, 2\}$$

$$y = \{7, 11, 9, 6, 4, 3, 1, 3\}$$

$$\sqrt{|1-7|^2 + |3-11|^2 + |5-9|^2 + |7-6|^2 + |6-4|^2 + |8-3|^2 + |5-1|^2 + |2-3|^2}$$

$$= \sqrt{163}$$
$$\approx 12.767$$

Between x and z:

$$x = \{1, 3, 5, 7, 6, 8, 5, 2\}$$

$$z = \{3, 5, 7, 9, 8, 7, 6, 5\}$$

$$\sqrt{|1-3|^2 + |3-5|^2 + |5-7|^2 + |7-9|^2 + |6-8|^2 + |8-7|^2 + |5-6|^2 + |2-5|^2}$$
$$= \sqrt{13}$$
$$\approx 5.56776$$

between y and z:

$$y = \{7, 11, 9, 6, 4, 3, 1, 3\}$$

$$z = \{3, 5, 7, 9, 8, 7, 6, 5\}$$

$$\sqrt{|7-3|^2 + |11-5|^2 + |9-7|^2 + |6-9|^2 + |4-8|^2 + |3-7|^2 + |1-6|^2 + |3-5|^2}$$

$$= 3\sqrt{14}$$
$$\approx 11.2249$$

$$\begin{pmatrix} 1 & 3 & 5 & 7 & 6 & 8 & 5 & 2 \\ 7 & 11 & 9 & 6 & 4 & 3 & 1 & 3 \\ 3 & 5 & 7 & 9 & 8 & 7 & 6 & 5 \end{pmatrix}$$

(b) wavlet coeff.

X : $\{104.652, -11.3137, -16., 14., -2.82843, -2.82843, -2.82843, 4.24264\}$
Y: $\{124.451, 62.2254, 6., 6., -5.65685, 4.24264, 1.41421, -2.82843\}$
Z: $\{141.421, -5.65685, -16, 8, -2.82843, -2.82843, 1.41421, 1.41421\}$

Between x and y:

$$\sqrt{|104.652 - 124.451|^2 + |-11.3137 - 62.2254|^2 + |-16-6|^2 + |14-6|^2 + |-2.82843 - -5.65685|^2 + |-2.82843 - 4.}$$

$$\approx 80.4612$$

between y and z Y: $\{124.451, 62.2254, 6., 6., -5.65685, 4.24264, 1.41421, -2.82843\}$
Z: $\{141.421, -5.65685, -16, 8, -2.82843, -2.82843, 1.41421, 1.41421\}$

$$\sqrt{|124.451 + 141.451|^2 + |62.2254 - -5.65685|^2 + |6 - -16|^2 + |6 - 8|^2 + |-5.65685 - 2.82843|^2 + |4.24264 - 2.82843|^2 + |1.41421 - 1.41421|^2 + |-2.82843 - 1.41421|^2}$$

$$\approx 275.485$$

X : $\{104.652, -11.3137, -16., 14., -2.82843, -2.82843, -2.82843, 4.24264\}$
Z: $\{141.421, -5.65685, -16, 8, -2.82843, -2.82843, 1.41421, 1.41421\}$

$$\sqrt{|104.652 - 141.421|^2 + |-11.3137 - -5.65658|^2 + |-16 - 16|^2 + |14 - 8|^2 + |-2.82843 - -2.82843|^2 + |-2.82843 - -2.82843|^2 + |-2.82843 - 1.41421|^2 + |4.24264 - 1.41421|^2}$$

$$\approx 49.6987$$

(c) largest three wavelet coefficients of x

(d) largest three wavelet coefficients of y

(e) which one of the above approximations matches the ordering of the distances from the raw vectors?