# CS 591.03
# Introduction to Data Mining
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LECTURE 2: DATA TYPES AND SIMILARITIES

# Getting to Know Your Data

Data Objects and Attribute Types

Basic Statistical Descriptions of Data

Data Visualization

Measuring Data Similarity and Dissimilarity

Summary

# Types of Data Sets

Record

- Relational records
- Data matrix, e.g., numerical matrix, crosstabs
- Document data: text documents: term-frequency vector
- Transaction data

Graph and network

- World Wide Web
- Social or information networks
- Molecular Structures

Ordered

- Video data: sequence of images
- Temporal data: time-series
- Sequential Data: transaction sequences
- Genetic sequence data

Spatial, image and multimedia:

- Spatial data: maps
- Image data:
- Video data:

| | team | coach | play | ball | score | game | win | lost | timeout | season |
|---|---|---|---|---|---|---|---|---|---|---|
| Document 1 | 3 | 0 | 5 | 0 | 2 | 6 | 0 | 2 | 0 | 2 |
| Document 2 | 0 | 7 | 0 | 2 | 1 | 0 | 0 | 3 | 0 | 0 |
| Document 3 | 0 | 1 | 0 | 0 | 1 | 2 | 2 | 0 | 3 | 0 |

| *TID* | *Items* |
|---|---|
| **1** | **Bread, Coke, Milk** |
| **2** | **Beer, Bread** |
| **3** | **Beer, Coke, Diaper, Milk** |
| **4** | **Beer, Bread, Diaper, Milk** |
| **5** | **Coke, Diaper, Milk** |

# Important Characteristics of Structured Data

Dimensionality

- Curse of dimensionality

Sparsity

- Only presence counts

Resolution

- Patterns depend on the scale

Distribution

- Centrality and dispersion

# Data Objects

Data sets are made up of data objects.

A **data object** represents an entity.

Examples:

- sales database:  customers, store items, sales
- medical database: patients, treatments
- university database: students, professors, courses

Also called *samples , examples, instances, data points, objects, tuples*.

Data objects are described by **attributes**.

Database rows -> data objects; columns ->attributes.

# Attributes

**Attribute (**or **dimensions, features, variables**): a data field, representing a characteristic or feature of a data object.

- *E.g., customer _ID, name, address*

Types:

- Nominal
- Binary
- Ordinal
- Numeric: quantitative
  - Interval-scaled
  - Ratio-scaled

# Attribute Types

**Nominal:** categories, states, or “names of things”

- *Hair_color = {auburn, black, blond, brown, grey, red, white}*
- marital status, occupation, ID numbers, zip codes

**Binary**

- Nominal attribute with only 2 states (0 and 1)
- Symmetric binary: both outcomes equally important
  - e.g., gender
- Asymmetric binary: outcomes not equally important.
  - e.g., medical test (positive vs. negative)
  - Convention: assign 1 to most important outcome (e.g., HIV positive)

**Ordinal**

- Values have a meaningful order (ranking) but magnitude between successive values is not known.
- *Size = {small, medium, large},* grades, army rankings

# Numeric Attribute Types

Quantity (integer or real-valued)

## Interval

- Measured on a scale of **equal-sized units**
- Values have order
  - E.g., *temperature in C˚or F˚, calendar dates*
- No true zero-point

## Ratio

- Inherent **zero-point**
- We can speak of values as being an order of magnitude larger than the unit of measurement (10 K˚ is twice as high as 5 K˚).
  - e.g., *temperature in Kelvin, length, counts, monetary quantities*

# Discrete vs. Continuous Attributes

**Discrete Attribute**

- Has only a finite or countably infinite set of values
  - E.g., zip codes, profession, or the set of words in a collection of documents
- Sometimes, represented as integer variables
- Note: Binary attributes are a special case of discrete attributes

**Continuous Attribute**

- Has real numbers as attribute values
  - E.g., temperature, height, or weight
- Practically, real values can only be measured and represented using a finite number of digits
- Continuous attributes are typically represented as floating-point variables

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# Basic Statistical Descriptions of Data

Motivation

- To better understand the data: central tendency, variation and spread

Data dispersion characteristics

- median, max, min, quantiles, outliers, variance, etc.

# Measuring the Central Tendency

Mean (algebraic measure) (sample vs. population):

Note: *n* is sample size and *N* is population size.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \frac{\sum x}{N}$$

- Weighted arithmetic mean:

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

- Trimmed mean: chopping extreme values

Median:

- Middle value if odd number of values, or average of the middle two values otherwise
- Estimated by interpolation (for *grouped data*):

$$median = L_1 + \left(\frac{n/2 - (\sum freq)_l}{freq_{median}}\right) width$$

| age | frequency |
|---|---|
| 1–5 | 200 |
| 6–15 | 450 |
| 16–20 | 300 |
| 21–50 | 1500 |
| 51–80 | 700 |
| 81–110 | 44 |

Median interval → 21–50

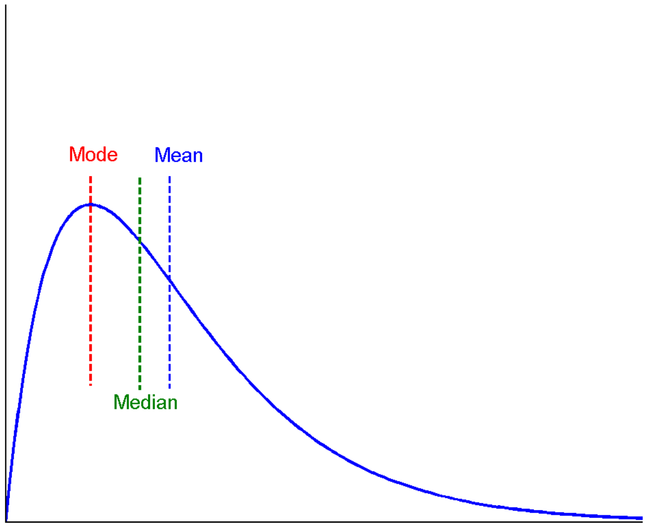
Mode

- Value that occurs most frequently in the data
- Unimodal, bimodal, trimodal
- Empirical formula:

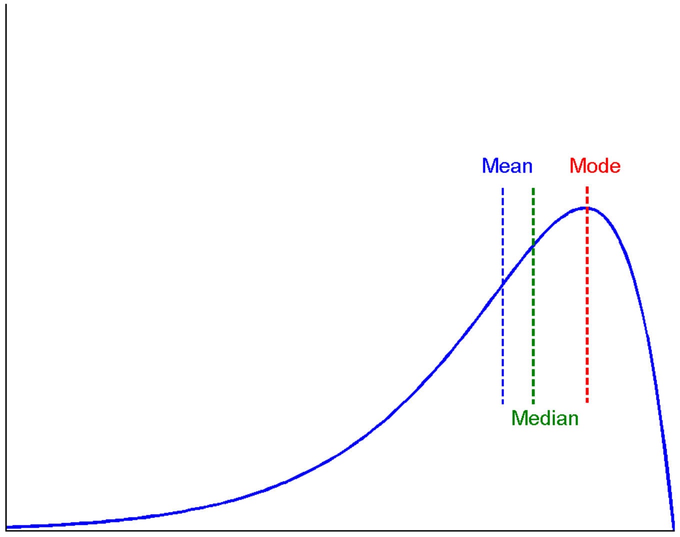
$$mean - mode = 3 \times (mean - median)$$

# Symmetric vs. Skewed Data

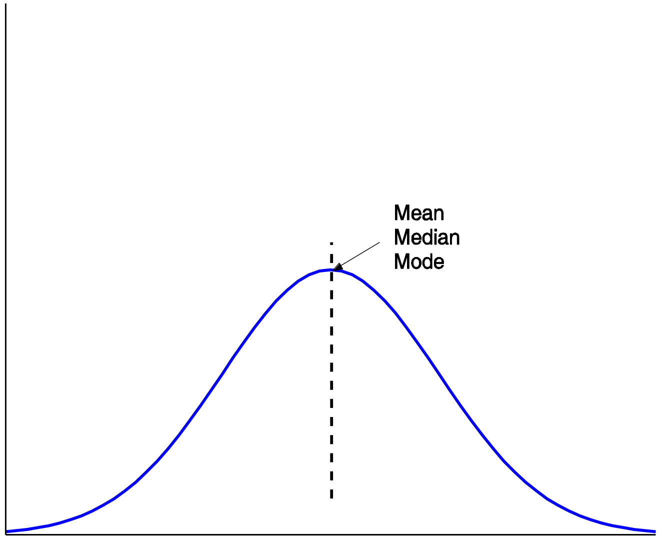
Median, mean and mode of symmetric, positively and negatively skewed data




positively skewed




negatively skewed




symmetric

# Measuring the Dispersion of Data

Quartiles, outliers and boxplots

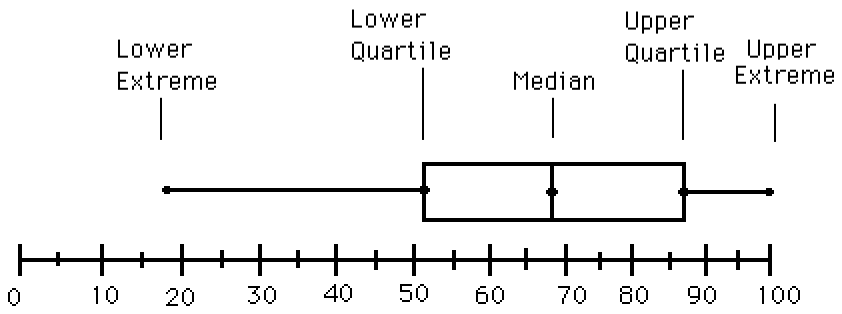
- **Quartiles**: $Q_1$ (25th percentile), $Q_3$ (75th percentile)
- **Inter-quartile range**: IQR = $Q_3 - Q_1$
- **Five number summary**: min, $Q_1$, median, $Q_3$, max
- **Boxplot**: ends of the box are the quartiles; median is marked; add whiskers, and plot outliers individually
- **Outlier**: usually, a value higher/lower than 1.5 x IQR

Variance and standard deviation (*sample: s, population: σ*)

- **Variance**: (algebraic, scalable computation)
- **Standard deviation** *s (or σ)* is the square root of variance $s^2$ (or $\sigma^2$)

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \mu)^2 = \frac{1}{N}\sum_{i=1}^{n}x_i{}^2 - \mu^2$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}[\sum_{i=1}^{n}x_i{}^2 - \frac{1}{n}(\sum_{i=1}^{n}x_i)^2]$$



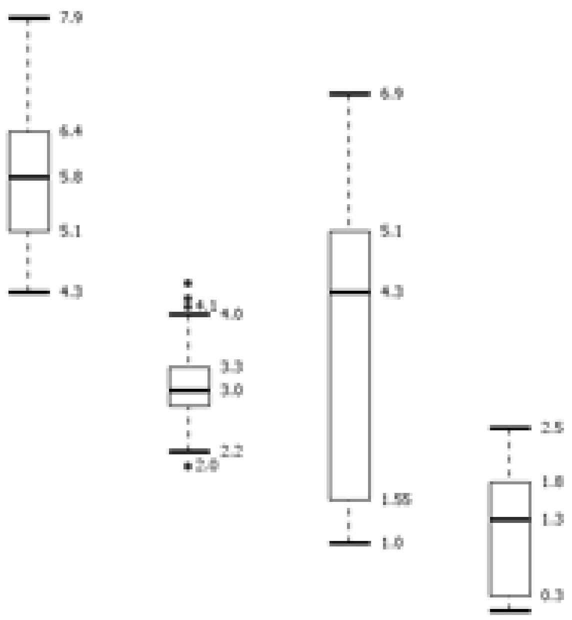

# Boxplot Analysis

**Five-number summary** of a distribution

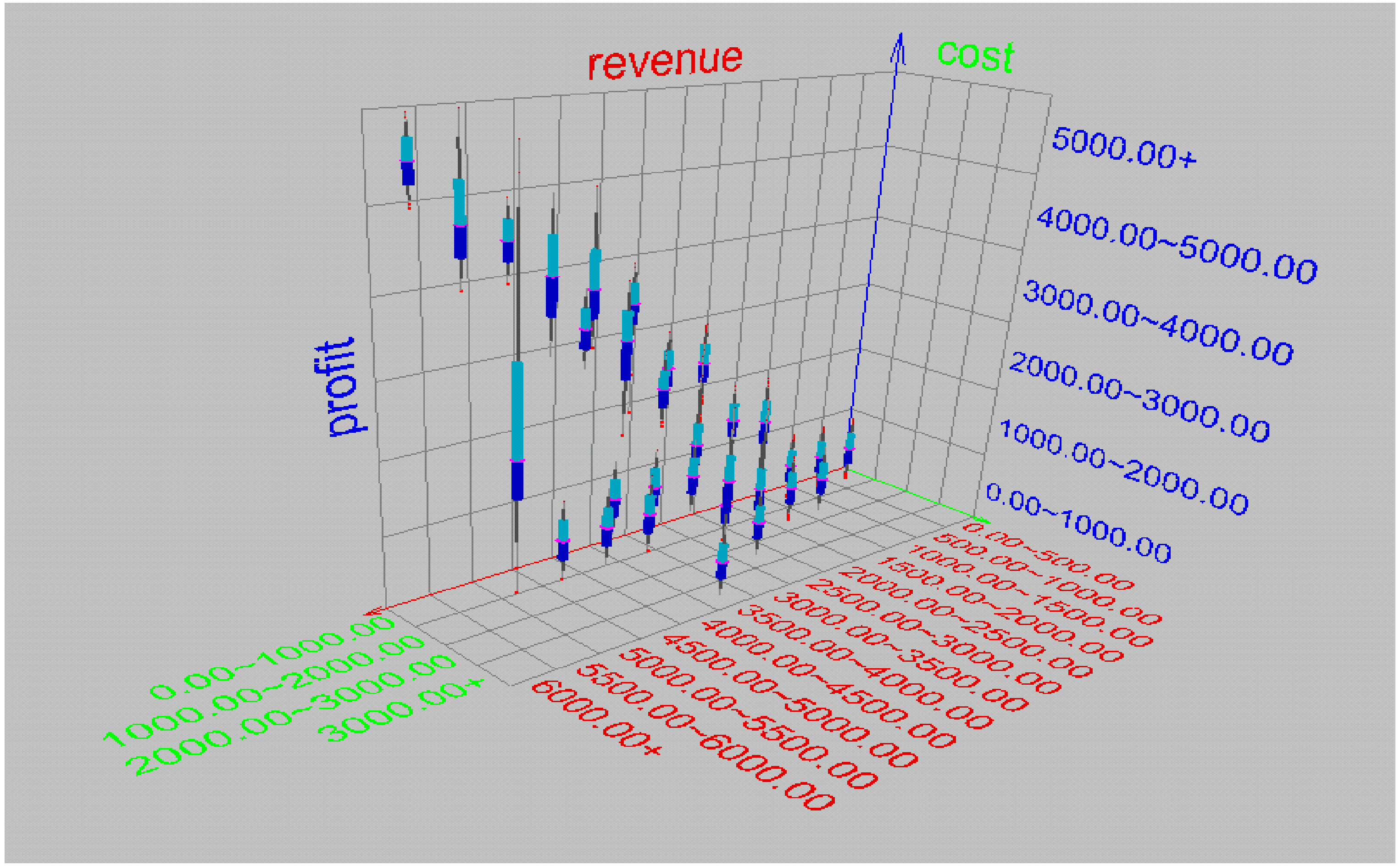
- Minimum, Q1, Median, Q3, Maximum

**Boxplot**

- Data is represented with a box
- The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
- The median is marked by a line within the box
- Whiskers: two lines outside the box extended to Minimum and Maximum
- Outliers: points beyond a specified outlier threshold, plotted individually



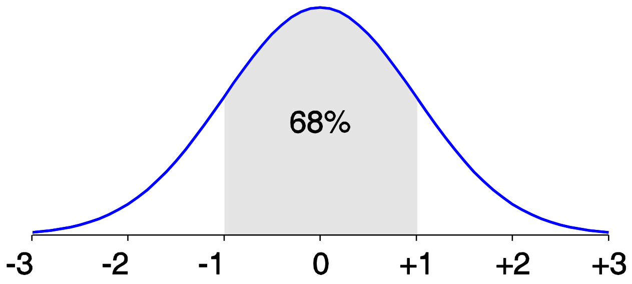
# Visualization of Data Dispersion: 3-D Boxplots

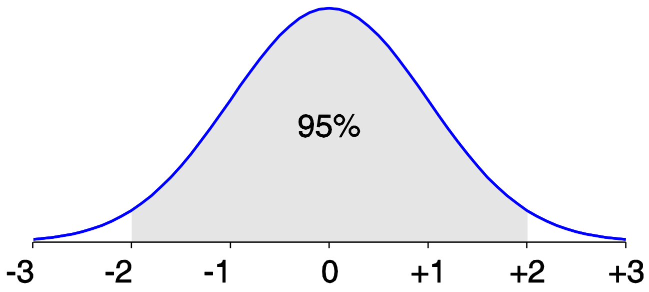


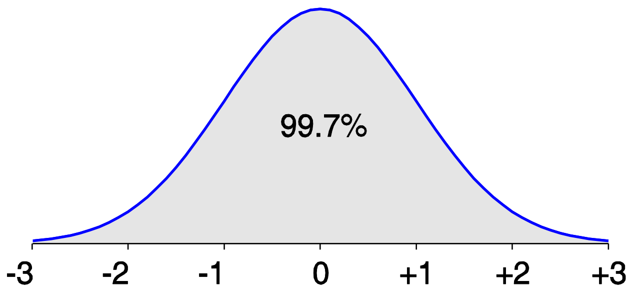
# Properties of Normal Distribution Curve

The normal (distribution) curve

- From μ–σ to μ+σ: contains about 68% of the measurements  (μ: mean, σ: standard deviation)
- From μ–2σ to μ+2σ: contains about 95% of it
- From μ–3σ to μ+3σ: contains about 99.7% of it



# Graphic Displays of Basic Statistical Descriptions

**Boxplot**: graphic display of five-number summary

**Histogram**: x-axis are values, y-axis represents frequencies

**Quantile plot**: each value $x_i$ is paired with $f_i$ indicating that approximately 100 $f_i$ % of data are $\leq x_i$

**Quantile-quantile (q-q) plot**: graphs the quantiles of one univariant distribution against the corresponding quantiles of another

**Scatter plot**: each pair of values is a pair of coordinates and plotted as points in the plane

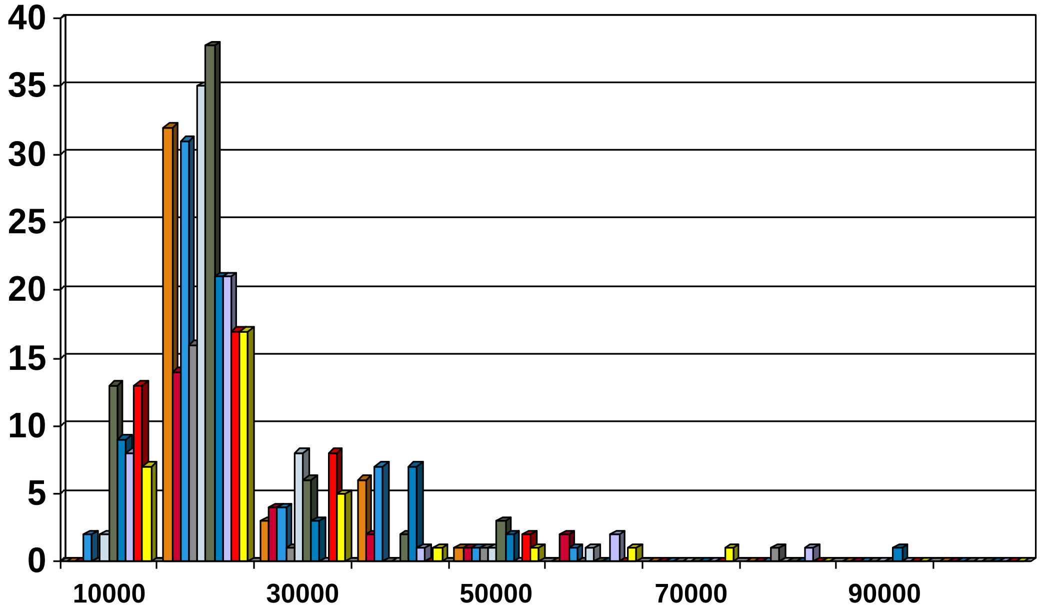
# Histogram Analysis

Histogram: Graph display of tabulated frequencies, shown as bars

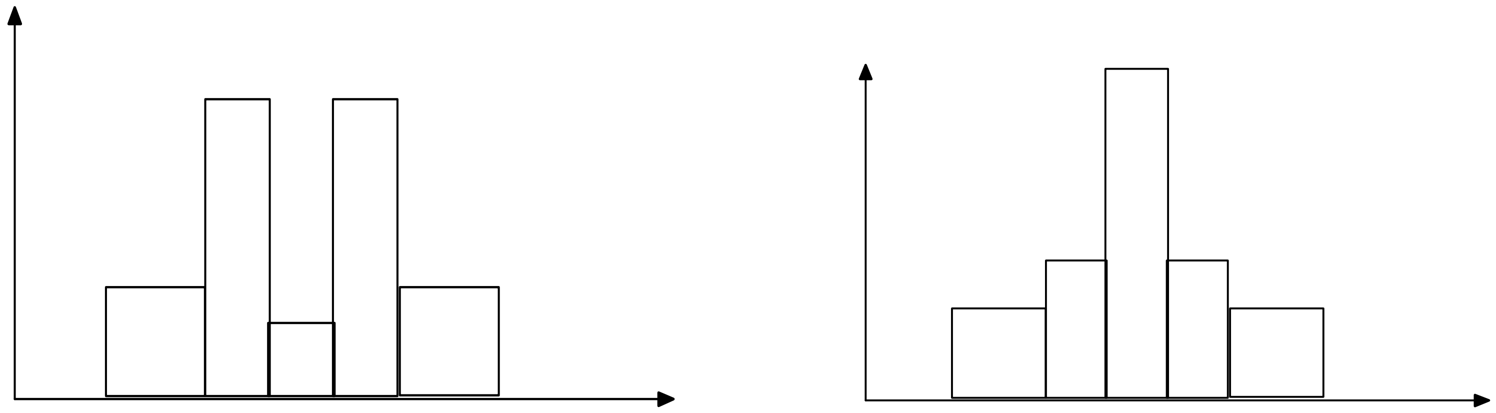
It shows what proportion of cases fall into each of several categories

Differs from a bar chart in that it is the *area* of the bar that denotes the value, not the height as in bar charts, a crucial distinction when the categories are not of uniform width

The categories are usually specified as non-overlapping intervals of some variable. The categories (bars) must be adjacent



# Histograms Often Tell More than Boxplots



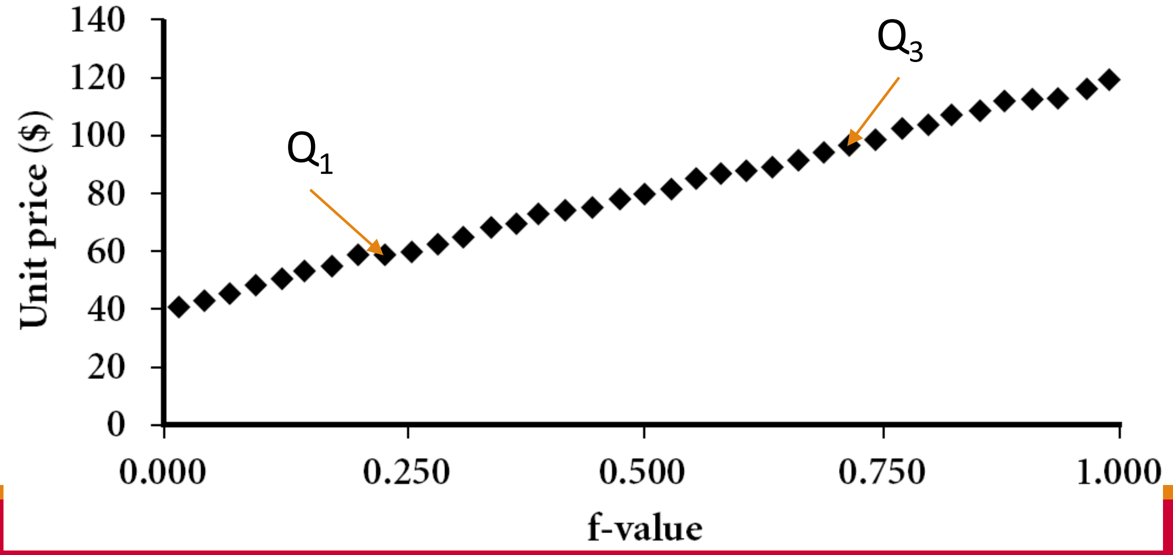
- The two histograms shown in the left may have the same boxplot representation
  - The same values for: min, Q1, median, Q3, max
- But they have rather different data distributions

# Quantile Plot

Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)

Plots **quantile** information

- For a data $x_i$ data sorted in increasing order, $f_i$ indicates that approximately 100 $f_i$% of the data are below or equal to the value $x_i$

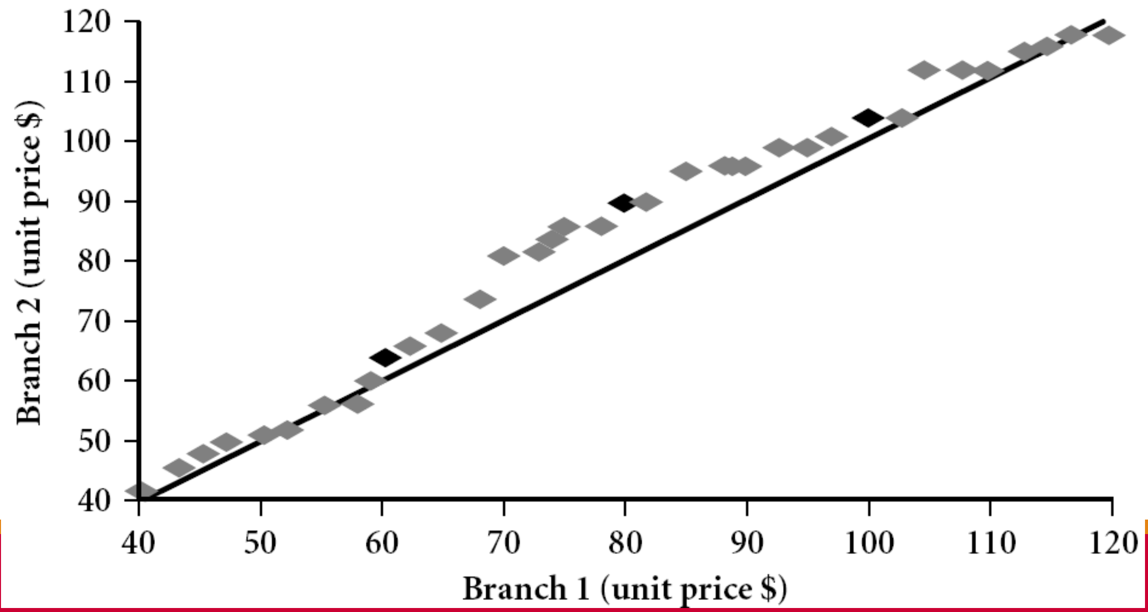


# Quantile-Quantile (Q-Q) Plot

Graphs the quantiles of one univariate distribution against the corresponding quantiles of another

View: Is there is a shift in going from one distribution to another?

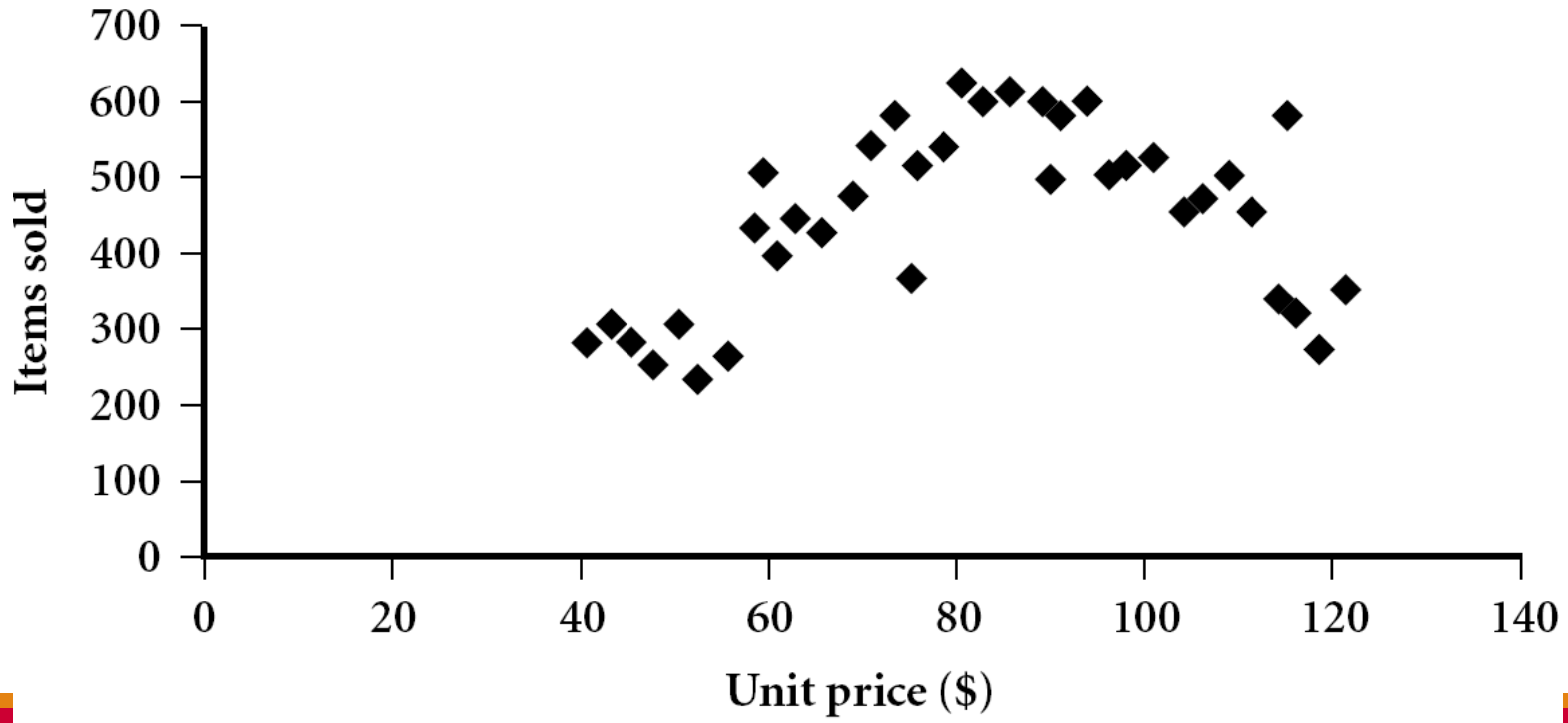
Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile.  Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.



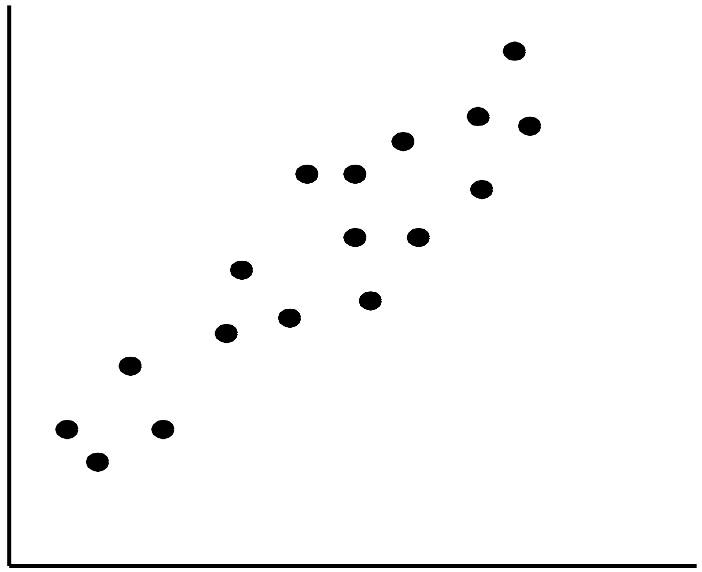
# Scatter plot

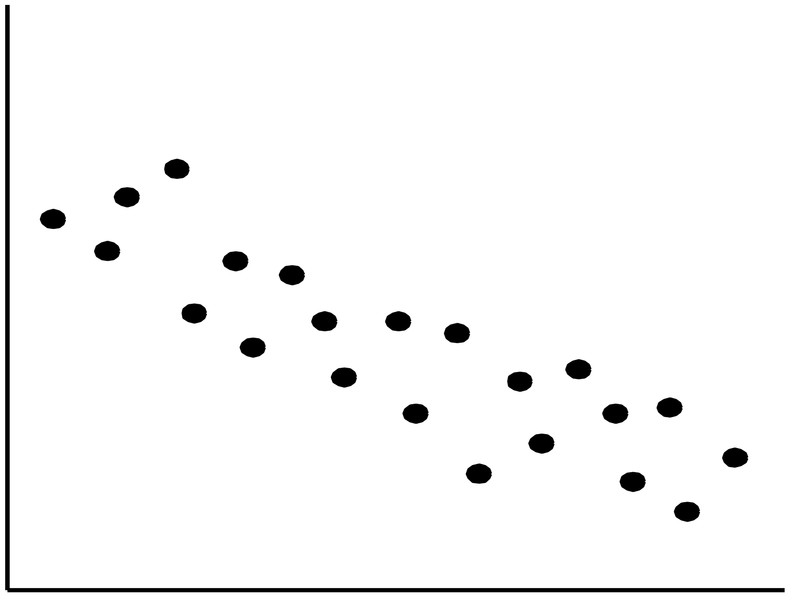
Provides a first look at bivariate data to see clusters of points, outliers, etc

Each pair of values is treated as a pair of coordinates and plotted as points in the plane

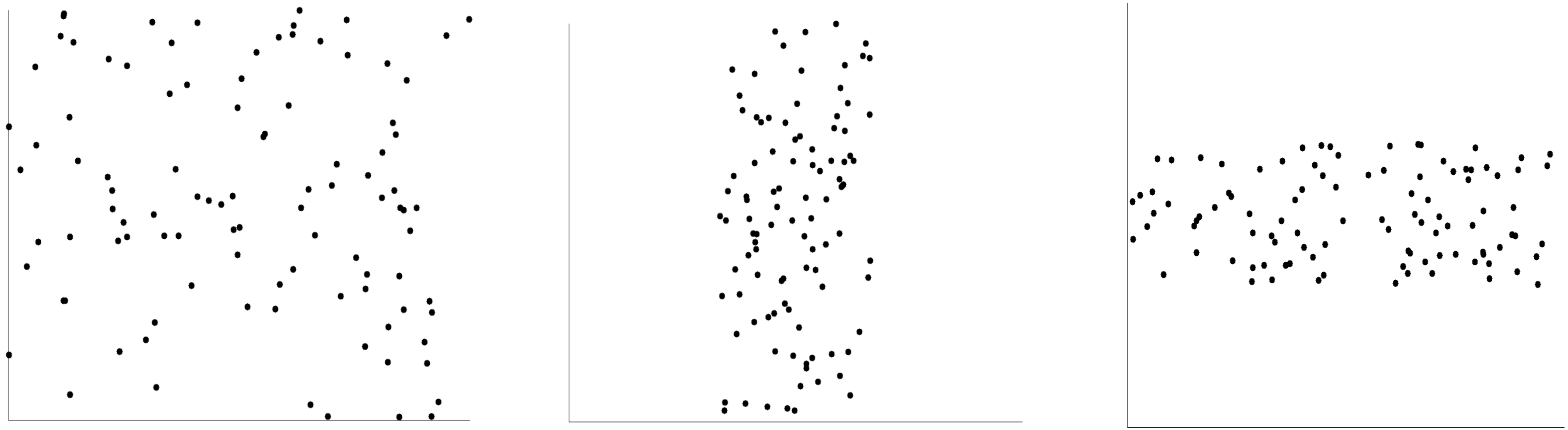


# Positively and Negatively Correlated Data


positively correlated


negative correlated

# Uncorrelated Data



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# Data Visualization

Why data visualization?

- Gain insight into an information space by mapping data onto graphical primitives
- Provide qualitative overview of large data sets
- Search for patterns, trends, structure, irregularities, relationships among data
- Help find interesting regions and suitable parameters for further quantitative analysis
- Provide a visual proof of computer representations derived

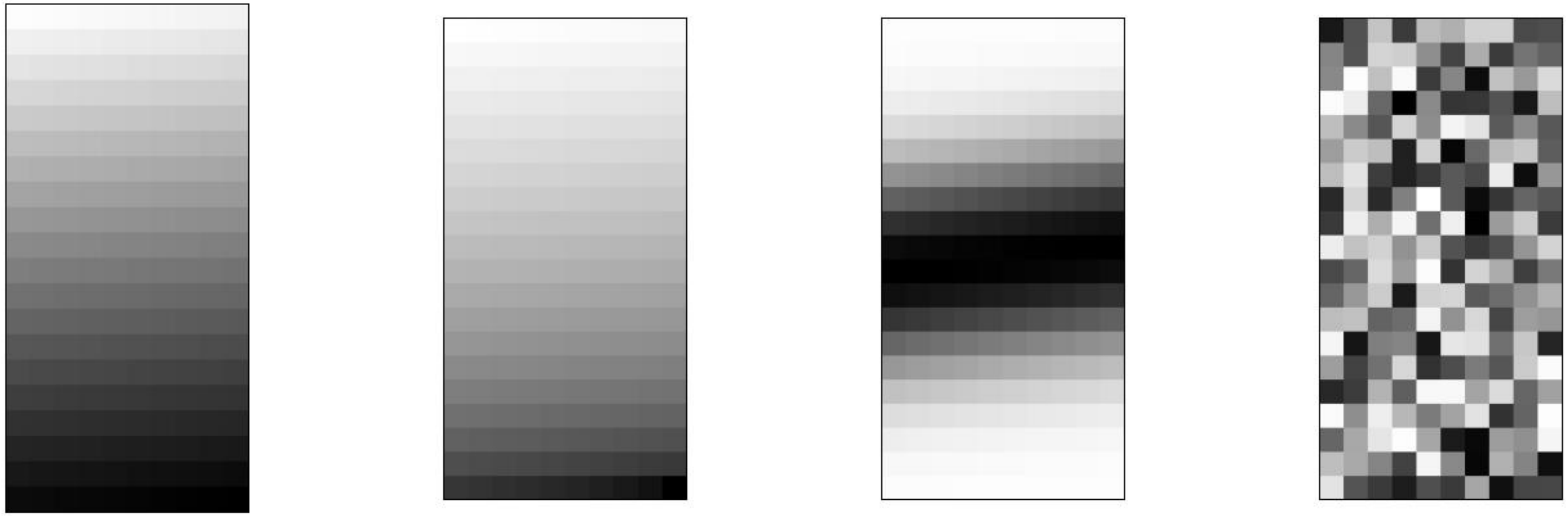
Categorization of visualization methods:

- Pixel-oriented visualization techniques
- Geometric projection visualization techniques
- Icon-based visualization techniques
- Hierarchical visualization techniques
- Visualizing complex data and relations

# Pixel-Oriented Visualization Techniques

For a data set of m dimensions, create m windows on the screen, one for each dimension

The m dimension values of a record are mapped to m pixels at the corresponding positions in the windows. The colors of the pixels reflect the corresponding values



(a) Income (b) Credit Limit (c) transaction volume (d) age

# Geometric Projection Visualization Techniques

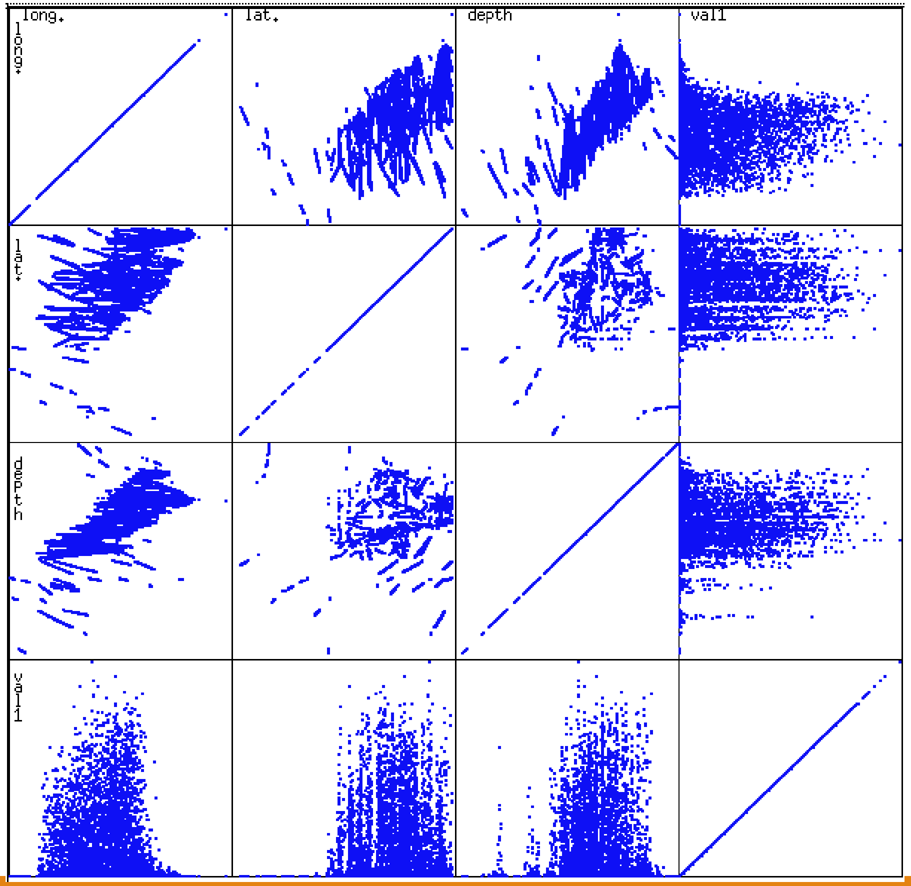
Visualization of geometric transformations and projections of the data

Methods

- Direct visualization
- Scatterplot and scatterplot matrices
- Landscapes
- Projection pursuit technique: Help users find meaningful projections of multidimensional data
- Prosection views
- Hyperslice
- Parallel coordinates

# Scatterplot Matrices

Matrix of scatterplots (x-y-diagrams) of the k-dim. data [total of (k2/2-k) scatterplots]

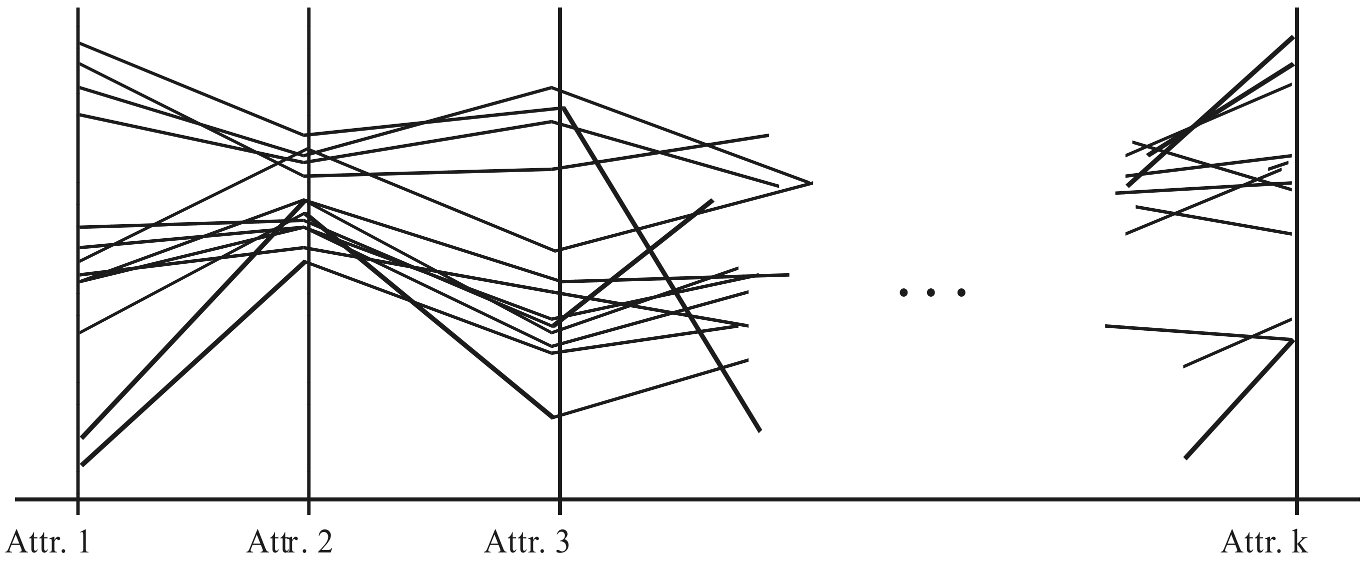


# Parallel Coordinates

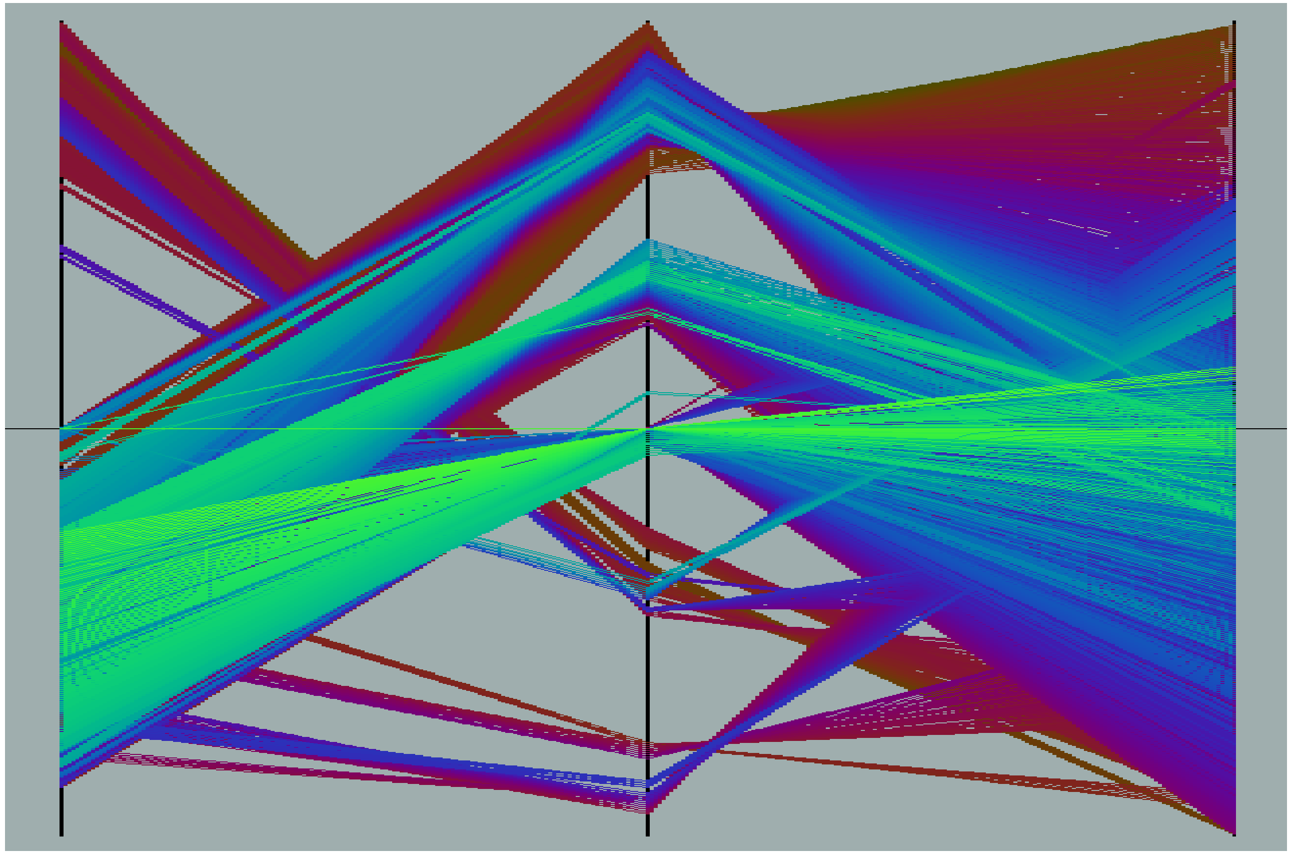
n equidistant axes which are parallel to one of the screen axes and correspond to the attributes

The axes are scaled to the [minimum, maximum]: range of the corresponding attribute

Every data item corresponds to a polygonal line which intersects each of the axes at the point which corresponds to the value for the attribute



# Parallel Coordinates of a Data Set



# Hierarchical Visualization Techniques

Visualization of the data using a hierarchical partitioning into subspaces

Methods

- Dimensional Stacking
- Worlds-within-Worlds
- Tree-Map
- Cone Trees
- InfoCube

# Dimensional Stacking

Partitioning of the n-dimensional attribute space in 2-D subspaces, which are ‘stacked’ into each other

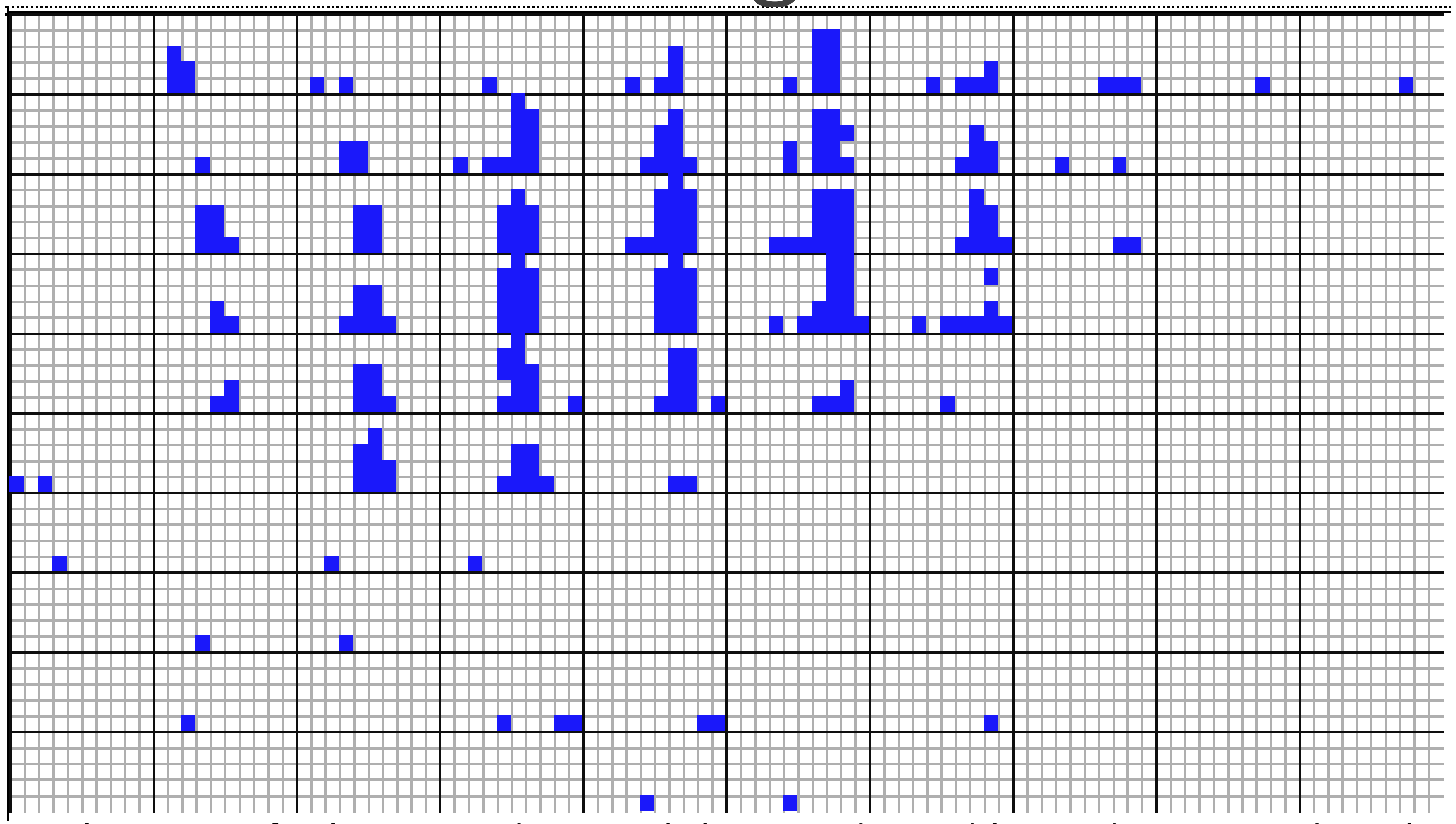
Partitioning of the attribute value ranges into classes.  The important attributes should be used on the outer levels.

Adequate for data with ordinal attributes of low cardinality

But, difficult to display more than nine dimensions

Important to map dimensions appropriately

# Dimensional Stacking



Visualization of oil mining data with longitude and latitude mapped to the outer x-, y-axes and ore grade and depth mapped to the inner x-, y-axes

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# Similarity and Dissimilarity

**Similarity**

- Numerical measure of how alike two data objects are
- Value is higher when objects are more alike
- Often falls in the range [0,1]

**Dissimilarity** (e.g., distance)

- Numerical measure of how different two data objects are
- Lower when objects are more alike
- Minimum dissimilarity is often 0
- Upper limit varies

**Proximity** refers to a similarity or dissimilarity

# Data Matrix and Dissimilarity Matrix

## Data matrix

- n data points with p dimensions

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

## Dissimilarity matrix

- n data points, but registers only the distance
- A triangular matrix

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \cdots & \cdots & 0 \end{bmatrix}$$

# Proximity Measure for Nominal Attributes

Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)

Method 1: Simple matching

- *m*: # of matches, *p*: total # of variables/features

$$d(i,j) = \frac{p-m}{p}$$

Method 2: Use a large number of binary attributes

- creating a new binary attribute for each of the *M* nominal states

# Proximity Measure for Binary Attributes

A contingency table for binary data

| | | Object $j$ | | |
|---|---|---|---|---|
| | | 1 | 0 | sum |
| Object $i$ | 1 | $q$ | $r$ | $q+r$ |
| | 0 | $s$ | $t$ | $s+t$ |
| | sum | $q+s$ | $r+t$ | $p$ |

Distance measure for symmetric binary variables:

$$d(i, j) = \frac{r+s}{q+r+s+t}$$

Distance measure for asymmetric binary variables:

$$d(i, j) = \frac{r+s}{q+r+s}$$

Jaccard coefficient (*similarity* measure for *asymmetric* binary variables):

$$sim_{Jaccard}(i, j) = \frac{q}{q+r+s}$$

# Dissimilarity between Binary Variables

Example

| Name | Gender | Fever | Cough | Test-1 | Test-2 | Test-3 | Test-4 |
|---|---|---|---|---|---|---|---|
| Jack | M | Y | N | P | N | N | N |
| Mary | F | Y | N | P | N | P | N |
| Jim | M | Y | P | N | N | N | N |

- Gender is a symmetric attribute
- The remaining attributes are asymmetric binary
- Let the values Y and P be 1, and the value N 0

$$d(jack, mary) = \frac{0+1}{2+0+1} = 0.33$$

$$d(jack, jim) = \frac{1+1}{1+1+1} = 0.67$$

$$d(jim, mary) = \frac{1+2}{1+1+2} = 0.75$$

# Standardizing Numeric Data

Z-score:

- X: raw score to be standardized, μ: mean of the population, σ: standard deviation
- the distance between the raw score and the population mean in units of the standard deviation
- negative when the raw score is below the mean, "+" when above

$$z = \frac{x - \mu}{\sigma}$$

An alternative way: Calculate the mean absolute deviation

$$s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + \ldots + |x_{nf} - m_f|)$$

where

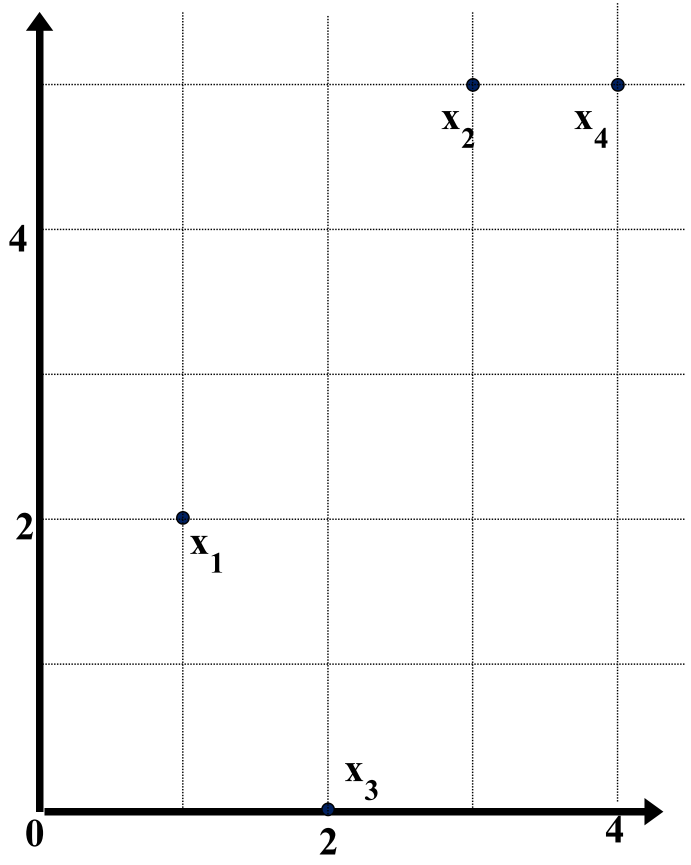
$$m_f = \frac{1}{n}(x_{1f} + x_{2f} + \ldots + x_{nf}).$$

- standardized measure (*z-score*):

$$z_{if} = \frac{x_{if} - m_f}{s_f}$$

- Using mean absolute deviation is more robust than using standard deviation

# Example:
# Data Matrix and Dissimilarity Matrix




**Data Matrix**

| point | attribute1 | attribute2 |
|---|---|---|
| *x1* | 1 | 2 |
| *x2* | 3 | 5 |
| *x3* | 2 | 0 |
| *x4* | 4 | 5 |

**Dissimilarity Matrix**

**(with Euclidean Distance)**

| | *x1* | *x2* | *x3* | *x4* |
|---|---|---|---|---|
| *x1* | 0 | | | |
| *x2* | 3.61 | 0 | | |
| *x3* | 2.24 | 5.1 | 0 | |
| *x4* | 4.24 | 1 | 5.39 | 0 |

# Distance on Numeric Data: Minkowski Distance

*Minkowski distance*: A popular distance measure

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

where $i = (x_{i1}, x_{i2}, \ldots, x_{ip})$ and $j = (x_{j1}, x_{j2}, \ldots, x_{jp})$ are two *p*-dimensional data objects, and *h* is the order (the distance so defined is also called L-*h* norm)

Properties

- $d(i, j) > 0$ if $i \neq j$, and $d(i, i) = 0$ (Positive definiteness)
- $d(i, j) = d(j, i)$ (Symmetry)
- $d(i, j) \leq d(i, k) + d(k, j)$ (Triangle Inequality)

A distance that satisfies these properties is a metric

# Special Cases of Minkowski Distance

$h$ = 1: Manhattan (city block, $L_1$ norm) distance

- E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i,j) = |x_{i_1} - x_{j_1}| + |x_{i_2} - x_{j_2}| + \ldots + |x_{i_p} - x_{j_p}|$$

$h$ = 2: ($L_2$ norm) Euclidean distance

$$d(i,j) = \sqrt{(|x_{i_1} - x_{j_1}|^2 + |x_{i_2} - x_{j_2}|^2 + \ldots + |x_{i_p} - x_{j_p}|^2)}$$

$h \to \infty$. "supremum" ($L_{max}$ norm, $L_\infty$ norm) distance.

- This is the maximum difference between any component (attribute) of the vectors

$$d(i,j) = \lim_{h \to \infty} \left( \sum_{f=1}^{p} |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_{f}^{p} |x_{if} - x_{jf}|$$

# Example: Minkowski Distance

| point | attribute 1 | attribute 2 |
|---|---|---|
| x1 | 1 | 2 |
| x2 | 3 | 5 |
| x3 | 2 | 0 |
| x4 | 4 | 5 |

**Euclidean ($L_2$)**

| L2 | x1 | x2 | x3 | x4 |
|---|---|---|---|---|
| x1 | 0 | | | |
| x2 | 3.61 | 0 | | |
| x3 | 2.24 | 5.1 | 0 | |
| x4 | 4.24 | 1 | 5.39 | 0 |

**Manhattan ($L_1$)**

| L | x1 | x2 | x3 | x4 |
|---|---|---|---|---|
| x1 | 0 | | | |
| x2 | 5 | 0 | | |
| x3 | 3 | 6 | 0 | |
| x4 | 6 | 1 | 7 | 0 |

**Supremum**

| $L_\infty$ | x1 | x2 | x3 | x4 |
|---|---|---|---|---|
| x1 | 0 | | | |
| x2 | 3 | 0 | | |
| x3 | 2 | 5 | 0 | |
| x4 | 3 | 1 | 5 | 0 |

# Ordinal Variables

An ordinal variable can be discrete or continuous

Order is important, e.g., rank

Can be treated like interval-scaled

$$r_{if} \in \{1, \ldots, M_f\}$$

- replace $x_{if}$ by their rank
- map the range of each variable onto [0, 1] by replacing *i*-th object in the *f*-th variable by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

- compute the dissimilarity using methods for interval-scaled variables

# Attributes of Mixed Type

A database may contain all attribute types

- Nominal, symmetric binary, asymmetric binary, numeric, ordinal

One may use a weighted formula to combine their effects

$$d(i,j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}$$

- $f$ is binary or nominal:

  $d_{ij}^{(f)} = 0$ if $x_{if} = x_{jf}$ , or $d_{ij}^{(f)} = 1$ otherwise

- $f$ is numeric: use the normalized distance
- $f$ is ordinal
  - Compute ranks $r_{if}$ and
  - Treat $z_{if}$ as interval-scaled

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

# Cosine Similarity

A **document** can be represented by thousands of attributes, each recording the *frequency* of a particular word (such as keywords) or phrase in the document.

| *Document* | *team* | *coach* | *hockey* | *baseball* | *soccer* | *penalty* | *score* | *win* | *loss* | *season* |
|---|---|---|---|---|---|---|---|---|---|---|
| Document1 | 5 | 0 | 3 | 0 | 2 | 0 | 0 | 2 | 0 | 0 |
| Document2 | 3 | 0 | 2 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| Document3 | 0 | 7 | 0 | 2 | 1 | 0 | 0 | 3 | 0 | 0 |
| Document4 | 0 | 1 | 0 | 0 | 1 | 2 | 2 | 0 | 3 | 0 |

Other vector objects: gene features in micro-arrays, …

Applications: information retrieval, biologic taxonomy, gene feature mapping, ...

Cosine measure: If $d_1$ and $d_2$ are two vectors (e.g., term-frequency vectors), then

$$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1|| \, ||d_2|| ,$$

where $\bullet$ indicates vector dot product, $||d||$: the length of vector $d$

# Example: Cosine Similarity

$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1||\ ||d_2||$ ,
where $\bullet$ indicates vector dot product, $||d||$: the length of vector $d$

Ex: Find the **similarity** between documents 1 and 2.

$d_1$ **=** (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)
$d_2$ **=** (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)

$d_1 \bullet d_2 = 5*3+0*0+3*2+0*0+2*1+0*1+0*1+2*1+0*0+0*1 = 25$
$||d_1|| = (5*5+0*0+3*3+0*0+2*2+0*0+0*0+2*2+0*0+0*0)^{0.5} = (42)^{0.5} = 6.481$
$||d_2|| = (3*3+0*0+2*2+0*0+1*1+1*1+0*0+1*1+0*0+1*1)^{0.5} = (17)^{0.5} = 4.12$
$\cos(d_1, d_2) = 0.94$

# Summary

Data attribute types: nominal, binary, ordinal, interval-scaled, ratio-scaled

Many types of data sets, e.g., numerical, text, graph, Web, image.

Gain insight into the data by:

- Basic statistical data description: central tendency, dispersion, graphical displays
- Data visualization: map data onto graphical primitives
- Measure data similarity

Above steps are the beginning of data preprocessing

Many methods have been developed but still an active area of research

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