CS500, Theory of Computation
Homework #5

Note: You may discuss this homework with others in the class. However, you must do your own writeup, and you must clearly state on your homework who you worked with. Due by email in .pdf format by midnight on Friday, May 7th.

1. Show that the bracket language $\{\epsilon, (), (()), ()(), \ldots\}$ is in L.

2. Show that the language of palindromes over the alphabet $\{a, b\}$ is in L.

3. Let L_2 be the two-type bracket language $\{\epsilon, (), [], [()], ([]), ()[], \ldots\}$. Note that the brackets must be properly nested, so words like ([)] are not allowed. Show that $L_2 \in$ L. This is tricky! Hint: consider an inductive strategy that checks a property of every substring.

4. A directed graph G is *strongly connected* if for every pair of vertices u, v there is a directed path from u to v. Show that, given G as input, Strong Connectedness is NL-complete. Hint: show how to modify the graph to reduce Reachability (where two particular vertices are given as input) to Strong Connectedness.

5. Show that Graph 2-Colorability (also known as Bipartiteness) is in NL.

6. I made a comment in class that NL is the set of problems for which a certificate exists that can be verified by a Turing machine with $O(\log n)$ workspace, with the additional restriction that we only read this certificate left-to-right (i.e., the certificate is given on an additional read-only tape on which the TM can only move to the right). Formalize and justify this comment.

7. In the previous problem, what happens if we remove the restriction that the certificate is read left-to-right? What complexity class do we get? (What's the hardest problem you can think of where the certificate can be verified with $O(\log n)$ workspace, if we can go back and look at the certificate many times?)

8. Remember our old friend

$$L = \{w \in \{0,1\}^* \mid w \text{ does not contain } 11\} \ .$$

   Give an NC^1 family of circuits, consisting of binary AND and OR gates, for membership in L. If we are allowed to use AND or OR gates of arbitrary *fan-in* (i.e., which take arbitrary numbers of inputs) instead of binary gates, how shallow can the circuit be?