More Fun With Automata
Homework 3, CS500, Fall 2014

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February 26, 2015

1 Ex 23, Automata Notes

Given a language L, the language $sort(L)$ consists of the words in L with their characters sorted in alphabetical order. For instance, if

$$L = \{bab, cca, abc\}$$

then

$$sort(L) = \{abb, acc, abc\}.$$

Give an example of a regular langauge L_1 such that $sort(L_1)$ is nonregular and a nonregular language $sort(L_2)$ such that $sort(L_2)$ is regular. You may use any technique you like to prove that the languages are regular.

(a) Regular to Nonregular

Let

$$L_1 = \text{palindrome over } \{a, b\}$$

This implies that

$$sort(L_1) = \{a^m b^n | \text{either m or n is odd, not both}, |m|, |n|, \geq 0\}$$

as an example, the palindrome *abababa* when sorted becomes *aaaabbb*.

(b) Nonregular to regular

Let

$$L_2 = (abaa^*)^*$$

. Sorting L_2 gives us

$$a^m b^n | m > n \geq 0$$

.

2 Infinite sequences of languages

Find an infinite sequence of languages $A_0 \subset A_1 \subset A_2 \subset \cdots \subset A_k \subset \ldots$ such that for each even n, A_n is regular, and for each odd n, A_n is non-regular. Prove your solution is correct.

Answer:

3 Regex Golf

Go to and solve at least 5 of the puzzles. Solving means finding a regular expression that matches a substring of every string on the "match" list, and no substring of any string on the "none of these" list. Of your solutions, submit the 5 you like best, along with the score for each. Your solutions should be proper regular expressions, defined as follows:

- You may use ranges, such as [a-z]
- You may use the start-of-string character: ând the end-of-string character: $.
- You may use the OR character: `|`; the Kleene star operator: *; and parentheses: (,).
- You may NOT use backrefs or other constructs that allow the construction of expressions that match non-regular languages. (The server allows some of these, despite calling the game "regexp golf," but this assignment does not.)

Answer:

- level: warmup; foo : 207 pts
- level: anchors; k\$: 208 pts
- level: ranges;

  ```
  /^[a-f].*[a-f][abd-f]$/
  ```

  : 179 pts
- level: long count;

  ```
  / 0.*1 0010 0011 0100 0101 0110 01+ 10+ 1001 1010 1011 1100 1101 1+0 1+$ /
  ```

  : 200 pts

4 Context-Free Grammars

Give Context-Free Grammars that generate the following languages over alphabet $\{0,1\}$. Also say whether each language is regular.

(a) $\{w : w$ **contains at least two 1's**$\}$

Answer:

(b) $\{w : w$ **starts and ends with the same symbol, and has odd length**$\}$

Answer:

(c) $\{wx : x$ **is a substring of the reverse of** $w\}$

Answer:

5 Grammar and language

What language is generated by the following grammar? Prove whether it is a regular language or not. THere are 3 variables: S, A, B **and two terminals** $\{0, 1\}$

$$S \rightarrow AA, B$$
$$A \rightarrow 0A, A0, 1$$
$$B \rightarrow 0B00, 1$$

Answer

$$L = 0*10*10*|0^n10^2n|w\epsilon 0, 1*$$

6 Exercise 36, Automata notes

Show that a 1-DCA can be simulated by a DPDA, and similarly for 1-NCAs and NPDAs. Do you think this is true for two-counter automata as well?

Answer: