Homework 1, CS550, Spring 2015

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1

Let p,q,r be proposistions:

- **p: You have the flu**
- **q: You miss the final exam**
- **r: You pass the course**

(a) $(p \implies q)$

Answer:

"I have the flu and as such, I missed the final".

(b) $(q \implies \neg r)$

Answer:

"I missed the final and as such, did not pass the course."

(c) $[(p \implies \neg r) \vee (q \implies \neg r)]$

Answer:

"I had the flu and as such I did not pass the course or I missed the final exam and did not pass the course."

(d) $[(p \wedge q) \vee (\neg q \wedge r)]$

Answer:

"I missed the final and had flu or I didn't miss the final, passed the course."

2

Let p,q,r be proposistions:

- **p: you get an A on the final exam**
- **q: You do every exercise in the book**
- **r: You get an A in this class**

(a) You get an A on the final but you do not do every exercise in the book; nevertheless, you get an aA in this class

Answer:

$(p \wedge \neg q) \implies r$

(b) Getting an A on the final and doing every exercise in teh book is sufficient for getting an A in the class

Answer:

$(p \wedge q) \implies r$

(c) You will get an A in the class if and only if you either do every exercise in the book or you get an A on the final

Answer:

$(p \vee q) \implies r$

3

Determine whether the argument is correct or incorrect and explain why

(a) Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.

Answer:

True. All students in the university have lived in dormitories and Mia has not. If she was enrolled in the university, she would live or have lived in a dorm.

(b) A convertible car is fun to drive. Isaac's car is not convertible. Therefore, Issac's car is not fun to drive.

Answer:

False. The predicate makes no mention about all cars level of fun to drive, only convertible cars. Other cars may be fun to drive as well.

(c) Quincy likes all action movies. Quincy likes the movie *Eight Men Out*. Therefore, *Eight Men Out* is an action movie.

Answer:

False. The predicate makes no mention if Quincy likes other types of movies and only establishes something for his love of action movies. *Eight Men Out* may be another type of movie that he likes.

(d) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.

Answer:

True. A basic fact of being a lobsterman is setting a dozen traps. If Hamilton violates this, he is not a lobsterman.

4

Express each of these sytem specifications using predicates, quantifies, and logical connectives:

(a) Every user has access to an electronic mailbox

Answer:

Let a user be denoted by x and access to an electronic mailbox be $P(x)$.

$$\forall x P(x)$$

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Answer:

Let x be a user *in the group* and $P(x)$ denote access to the system.

$$\forall x P(x)$$

(c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

Answer:

let $D(x)$ be the proxy server's diagnostic state and $F(x)$ be the firewall's diagnostic state.

$$D(x) \implies F(x)$$

(d) At least one router is functioning normally if the throughput is between 100kpbs and 500 kbps and the proxy server is not in diagnostic mode.

Answer:

let $R(r)$ be true if a router is functioning normally and $T(x)$ be throughput of 100 to 500 kbps.

$$(T(x) \wedge \neg D(x)) \implies R(r)$$

5

(a) Which of these compound propositions are satisfiable, falsifiable, both satisfiable and falsifiable?

- (i) both
- (ii) both

(b) Prove that $p \implies [q \implies (p \wedge q)]$ and $(p \wedge q) \implies q \implies (p \vee r)$ are tautologies or not. Is $(p \vee q) \implies [q \implies q]$ a tautology?

Answer

Truth tables omitted due to complexity of making tabular environments.

- $p \implies [q \implies (p \wedge q)]$: Yes
- $(p \wedge q) \implies q \implies (p \vee r)$: No
- $(p \vee q) \implies [q \implies q]$: Yes

(c)

- Show that $\neg p \implies (q \implies r)$ and $q \implies (p \vee r)$ are logically equivalent.
- Show that $(p \wedge q) \implies r$ and $(p \implies r) \wedge (q \implies r)$ are not equivalent.

Answer

(d) Show that $(p \implies q) \wedge (p \implies r)$ and $p \implies (q \wedge r))$ are logically equivalent.

Answer

q and r must both be true and the order of implication doesn't matter in this case.

6

(a) Write a formula that expresses that a number g is the GCD of two other numbers x, y

Answer:

$$\forall x, gcd(x, 0, x).$$
$$\forall x, \forall y, \forall g, gcd(x, y, g) \neg zero(v) gcd(y, mod(x, y), g)$$

(b) Write a formula that expresses that a number h is the least common multiple of two other numbers x, y (without using the least common multiple operation)

Answer:

Presuming we can use GCD(x,y,g) from above:

$$LCM(x, 1, 1)$$
$$LCM(x, y, h) : -habs(x * y)/gcd(x, y)$$

(c) Give an array A whose indices are natural numbers, write a formula expressing the array is sorted.

Answer:

$$\forall i A_i \leq S(A_i)$$

7

(a) What is the distinction between the following two formulas?

$$\forall x\, \exists y\, P(x, y).$$
$$\exists x\, \forall y\, P(x, y).$$

Answer:

The first example shows $P(x, y)$ is an injective function. The second example shows $P(x, y)$ is a surjection. $\mathbb{R} \to \mathbb{R} : x \to e^x$ is a canonical example of an injective function and $\mathbb{R} \to [-1, 1] : x \to sin(x)$ is a surjective function.

(b) Does $(\forall x\, P(x)) \vee (\forall x\, Q(x))$ imply $\forall x (P(x) \vee Q(x))$. What about the other way? if it does not imply, give a coutnerexample illustrating it. If it does imply, how would you convince someone about it?

Answer:

$$(\forall x\, P(x)) \vee (\forall x\, Q(x)) \implies \forall x (P(x) \vee Q(x))$$

but not the other way around.

8

In a logic programming language, define relations (i.e., write programs) to determine if a list:

(a) is a permutation of another list

Answer:

The basic process is to recusively test the list using three functions, Check, Remove, and Perm. Check tests if an item is in a list, remove removes the head of the list, and perm is only true if the the lists are permutations . It will fail if they are not.

```
Check(x,[x|L]).
Check(x, [y|l}) :-(Check(x, L)).

Remove(x, [x|L], L).
Remove(x, [y|L], z):- Remove(X,L,z).

Perm([], []).
Perm(L1, L2): -
  Check(x, L2),
  Remove(x, L2, y),
  Perm(L1,y).
```

(b) has an even number of elements in it

Answer:

We see that Even is true if x is an even natural number, built using the successor function.

```
Even(0).
Even(S(S(x)):- even(x).
```

(c) is formed by merging two other lists, i.e., successively taking one element at at time from each list and merging until one of the lists becomes empty, in which take the remaining nonempy list.

Answer: