

CS500, Theory of Computation: Final Exam

Note: Unlike the homework, you may *not* discuss this with other students, nor may you use resources on the Web. You may use statements proved in Sipser. Feel free to email me clarifying questions. Due by email in .pdf format by midnight on Wednesday, May 12th.

1. (10 points) Suppose L_1 and L_2 are regular languages recognized by DFAs M_1 and M_2 , where both M_1 and M_2 have n states. Prove that if $L_1 \neq L_2$, there is a word w of length at most n^2 which distinguishes them (i.e., $w \in L_1$ but $w \notin L_2$ or vice versa).
2. (20 points total) Consider the following problem.

Input: a nondeterministic finite automaton $M = (\Sigma, Q, \delta, q_0, F)$

Question: let L be the language accepted by M . Is $L = \emptyset$?

- (a) (10 points) This problem is complete for one of the complexity classes we discussed this semester. State which one, and prove it.
 - (b) (10 points) If we replace the NFA by a DFA, the resulting problem is also complete for one of the complexity classes we discussed. State which one, and prove it. If you think it is complete for the *same* complexity class as the problem for NFAs, describe a restricted version which is complete for a *lower* complexity class.
3. (30 points total) Consider the following model of cellphone conversations. There is an undirected graph $G = (V, E)$, where the vertices are people, and each edge indicates that two people are within range of each other so that they can potentially have a conversation. However, while two people are talking, their neighbors must stay silent (on that frequency) to avoid interference. Thus a set of conversations consists of a set of edges $S \subset E$, where vertices in different edges in S cannot be neighbors of each other. Formally:

$$(u, v), (w, t) \in S \Rightarrow (u, w) \notin E .$$

The *cellphone capacity* of G , denoted $C(G)$, is the largest number of conversations that can take place simultaneously on one frequency, i.e., the size of the largest such set S . For instance, if G is a cube (with 8 vertices and 12 edges) its cellphone capacity is 2. Now consider the following problem:

CELLPHONE CAPACITY

Input: a graph G and an integer k

Question: is $C(G) \geq k$?

Prove that CELLPHONE CAPACITY is NP-complete. You may reduce from any problem proved to be NP-complete in Sipser, but I think the easiest is to use either 3-SAT or Vertex Cover. For 10 points extra credit, provide reductions from both of these.

4. (20 points) Provide a reduction, as direct as possible, from GRAPH 3-COLORING to POSITIVE 1-IN-3 SAT. In other words, prove that if POSITIVE 1-IN-3 SAT is in P, then so is GRAPH 3-COLORING.
5. (10 points) A logspace Turing machine has two tapes, a read-only input tape of $\text{poly}(n)$ size and a read/write work tape of $O(\log n)$ size. In the original definition, it has one head for each of these tapes. However, what happens if we allow the machine to have k heads on its input tape for some constant k ? Prove that this does not make this class of machines any more powerful; i.e., any language accepted by a k -head logspace Turing machine is in L.
6. (10 points) Prove that for any NAE-3-SAT formula there exists a truth assignment that satisfies at least $3/4$ of its clauses. (Assume that no variable appears more than once in a single clause.)