

More Fun With Automata  
Homework 3, CS500, Fall 2014

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## 1 Ex 23, Automata Notes

Given a language  $L$ , the language  $\text{sort}(L)$  consists of the words in  $L$  with their characters sorted in alphabetical order. For instance, if

$$L = \{bab, cca, abc\}$$

then

$$\text{sort}(L) = \{abb, acc, abc\}.$$

Give an example of a regular language  $L_1$  such that  $\text{sort}(L_1)$  is nonregular and a nonregular language  $\text{sort}(L_2)$  such that  $\text{sort}(L_2)$  is regular. You may use any technique you like to prove that the languages are regular.

Answer:

## 2 Infinite sequences of languages

Find an infinite sequence of languages  $A_0 \subset A_1 \subset A_2 \subset \dots \subset A_k \subset \dots$  such that for each even  $n$ ,  $A_n$  is regular, and for each odd  $n$ ,  $A_n$  is non-regular. Prove your solution is correct.

Answer:

## 3 Regex Golf

Go to and solve at least 5 of the puzzles. Solving means finding a regular expression that matches a substring of every string on the "match" list, and no substring of any string on the "none of these" list. Of your solutions, submit the 5 you like best, along with the score for each. Your solutions should be proper regular expressions, defined as follows:

- You may use ranges, such as  $[a-z]$
- You may use the start-of-string character:  $\hat{}$  and the end-of-string character:  $\$$ .

- You may use the OR character: `|`; the Kleene star operator: `*`; and parentheses: `(, )`.
- You may NOT use backrefs or other constructs that allow the construction of expressions that match non-regular languages. (The server allows some of these, despite calling the game "regex golf," but this assignment does not.)

**Answer:**

## 4 Context-Free Grammars

Give Context-Free Grammars that generate the following languages over alphabet  $\{0,1\}$ . Also say whether each language is regular.

**(a)**  $\{w : w \text{ contains at least two 1's}\}$

**Answer:**

**(b)**  $\{w : w \text{ starts and ends with the same symbol, and has odd length}\}$

**Answer:**

**(c)**  $\{wx : x \text{ is a substring of the reverse of } w\}$

**Answer:**

## 5 Grammar and language

What language is generated by the following grammar? Prove whether it is a regular language or not.  
There are 3 variables:  $S, A, B$  and two terminals  $\{0, 1\}$

(a)  $S \rightarrow AA, B$

Answer:

(b)  $A \rightarrow 0A, A0, 1$

Answer:

(c)  $B \rightarrow 0B00, 1$

Answer:

## 6 Exercise 36, Automata notes

Show that a 1-DCA can be simulated by a DPDA, and similarly for 1-NCAs and NPDAs. Do you think this is true for two-counter automata as well?

**Answer:**