More Fun With Automata
Homework 2, CS500, Fall 2014

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Ex 7

Show that if L is regular then L^* is regular. Why does it not suffice to use the fact that the regular languages are closed under concatenation and union?

Answer:

Ex 8

Given a string w , let w^R denote w written in reverse. Given a language L, let $L^R = \{w^R | w \in L\}$. Prove that L is regular if and only if L^R is regular. Why is this harder to prove with DFAs?

Answer:

Ex 12

Given a language L, let $L1/2$ denote the set of words that can appear as first halves of words in L:

$$L_{1/2} = \{x | \exists y : |x| = |y| \text{ and } xy \in L\}$$

where $|w|$ denotes the length of a word w . Prove that if L is regular, then $L_{1/2}$ is regular. Generalize this to $L_{1/3}$, the set of words that can appear as middle thirds of words in L:

$$L_{1/3} = \{y | \exists x, z : |x| = |y| = |z| \text{ and } xyz \in L\}$$

Answer:

Ex 9

A for-all NFA is one such that $L(M)$ is the set of strings where every computation path ends in an accepting state. Show how to simulate an for-all NFA with a DFA, and thus prove that a language is recognized by some for-all NFA if and only if it is regular.

Answer:

Ex 13

Show that if $u_L v$, then $ua_L va$ for any $a \in A$.

Answer:

blah blah $\wr$ blah blah blah

$$W|\star \rightarrow \wr \sum_{i}^{x}$$

Ex 14

Describe the equivalence classes of the three languages from Exercise 2. Use them to give the minimal DFA for each language, or prove that the DFA you designed before is minimal.

Answer:

Ex 20

Consider the language

$$L_{a=b,c=d} = \{w \in \{a,b,c,d\} \star \, | \#_a(w) = \#_b(w) \text{ and } \#_c(w) = \#_d(w)$$

What are its equivalence classes? What does its minimal infinite-state machine look like?

Answer: