Even More Fun With Automata
Homework 3, CS500, Fall 2014

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Note to teammates: I'm sorry about the low quality of this assignment. I will make myself available as much as possible to correct and compile our solution set and will continue to work on these before we meet.

1 Ex 23, Automata Notes

Given a language L, the language $sort(L)$ consists of the words in L with their characters sorted in alphabetical order. For instance, if

$$L = \{bab, cca, abc\}$$

then

$$sort(L) = \{abb, acc, abc\}.$$

Give an example of a regular langauge L_1 such that $sort(L_1)$ is nonregular and a nonregular language $sort(L_2)$ such that $sort(L_2)$ is regular. You may use any technique you like to prove that the languages are regular.

(a) Regular to Nonregular

Let

$$L_1 = \text{palindrome over } \{a, b\}$$

This implies that

$$sort(L_1) = \{a^m b^n | \text{either m or n is odd, not both}, |m|, |n|, \geq 0\}$$

as an example, the palindrome *abababa* when sorted becomes *aaaabbb*.

(b) Nonregular to regular

Let

$$L_2 = (abaa^*)^*$$

. Sorting L_2 gives us

$$a^m b^n | m > n \geq 0$$

.

2 Infinite sequences of languages

Find an infinite sequence of languages $A_0 \subset A_1 \subset A_2 \subset \cdots \subset A_k \subset \ldots$ such that for each even n, A_n is regular, and for each odd n, A_n is non-regular. Prove your solution is correct.

Answer:

This was a difficult problem.

Taking our example from problem 1, define two languages: Let below be the language when n is odd

$$L_n = \{\text{palindrome over } \{a^i, b^{i-1}\}\}$$

and below bet the language when n is even.

$$sort(L_n) = \{a^m b^p | \text{either m or p is odd, not both}, |m|, |p|, \geq 0\}$$

Sorting the palindromic language gives us regularity and the next step takes us out of it.

3 Regex Golf

Go to and solve at least 5 of the puzzles. Solving means finding a regular expression that matches a substring of every string on the "match" list, and no substring of any string on the "none of these" list. Of your solutions, submit the 5 you like best, along with the score for each. Your solutions should be proper regular expressions, defined as follows:

- You may use ranges, such as [a-z]
- You may use the start-of-string character: ând the end-of-string character: $.
- You may use the OR character: |; the Kleene star operator: *; and parentheses: (,).
- You may NOT use backrefs or other constructs that allow the construction of expressions that match non-regular languages. (The server allows some of these, despite calling the game "regexp golf," but this assignment does not.)

Answer:

- level: warmup; foo : 207 pts
- level: anchors; $k\$$: 208 pts
- level: ranges;

  ```
  /^[a-f].*[a-f][abd-f]$/
  ```

  : 179 pts
- level: long count;

  ```
  / 0.*1 0010 0011 0100 0101 0110 01+ 10+ 1001 1010 1011 1100 1101 1+0 1+$ /
  ```

  : 200 pts
- level: glob;

  ```
  /^((er|f|i|p|t|v)|b|c[^a]).*[b-jm-z]$/
  ```

  : 124 pts but i failed matching many cases.

4 Context-Free Grammars

Give Context-Free Grammars that generate the following languages over alphabet $\{0,1\}$. Also say whether each language is regular.

(a) $\{w : w$ contains at least two 1's$\}$

Answer:

$$\begin{aligned} S &\to 1A, 0A \\ A &\to 1A, 0A, B \\ B &\to 1, A, C \end{aligned}$$

(b) $\{w : w$ starts and ends with the same symbol, and has odd length$\}$

Answer:

$$
\begin{aligned}
G = V, \Sigma, \Gamma, S \\
V =& \{S, A\} \\
\Sigma =& \{0, 1\} \\
S =& S \\
S \rightarrow& A|1|0 \\
A \rightarrow& 0B0|1B1 \\
B \rightarrow& 0B0|1B1|1B0|0B1|0|1
\end{aligned}
$$

The first rule accounts for the singleton case, e.g. a string of 1 or 0. A ensures that the word starts and ends with either 1 or 0. B allows only odd filler values.

(c) $\{wx : x$ is a substring of the reverse of $w\}$

Answer:

Unanswered.

5 Grammar and language

What language is generated by the following grammar? Prove whether it is a regular language or not. THere are 3 variables: S, A, B and two terminals $\{0, 1\}$

$$
\begin{aligned}
S \rightarrow& AA, B \\
A \rightarrow& 0A, A0, 1 \\
B \rightarrow& 0B00, 1
\end{aligned}
$$

Answer

$$L = ((0^*)\,1\,(0^*)\,1(0^*)\,|0^n 10^{2n}$$

No proof. I'll leave it as an exercise for me to do after the assignment is due.

6 Exercise 36, Automata notes

Show that a 1-DCA can be simulated by a DPDA, and similarly for 1-NCAs and NPDAs. Do you think this is true for two-counter automata as well?

Answer:

For DPDA $\rightarrow$ 1–DCA The stack symbol can be thought of as an increment or decrement operation, allowing us to keep a pseudo "count" of the number of things that happen.

For NPDA $\rightarrow$ 1–NCA, the same method as above follows.

For the two counter automata, they can recognize a language that a DPDA cannot, namely

$$L = a^p b^p c^p | p > 1$$

The proof is left as an exercise for the reader... who is me.