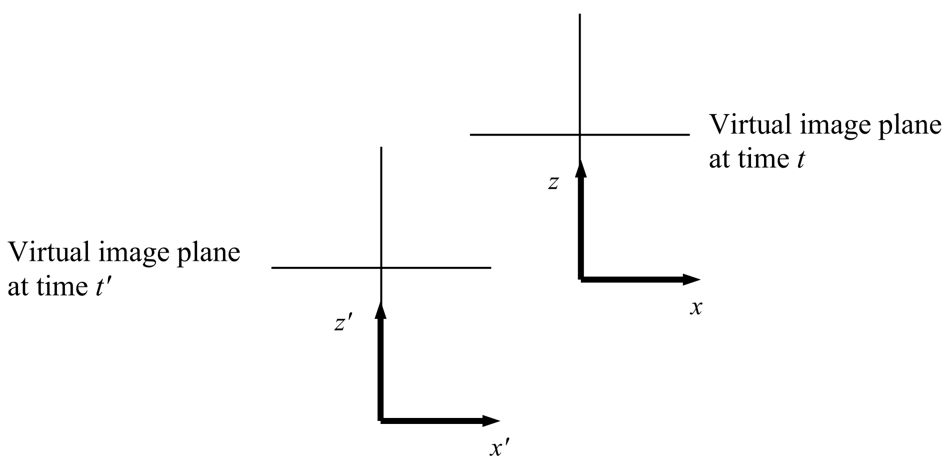
ECE 4554 / ECE 5554: Computer Vision: Homework 4

Fall 2024

Instructions

- This assignment is due at Canvas on Friday, November 22, before 11:59 PM. Late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after the deadline will cost 1 token. Please keep track of your own token count, and do not let the count fall below 0.
- Please review the Honor Code statement in the syllabus. As described there, you are not allowed to copy work from another source and submit it as your own for grading.
- The assignment consists of 5 problems. Provide solutions for all of the problems. (There are no optional or extra-credit problems.)
- Problems 1 through 3 are analytical in nature, and are presented here. Problems 4 and 5 require work using Colab. Each problem is worth 10 points.
- Prepare an answer sheet that contains all of your written answers in a single file named `Homework4_Problems1-3_USERNAME.pdf`. (Use your own VT username.) Handwritten solutions are permitted, but they must be easily legible to the grader. In addition, 2 more files related to Python coding must be uploaded to Canvas. Details are provided at the end of this assignment.
- For problems 4 and 5 (the coding problems), the Jupyter notebook file that you submit must be compatible with Google Colab. Your code should execute after the grader makes only 1 change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of 0 for those problems.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. The files that you submit to Canvas are the files that will be graded.

Problem 1. Assume that 2 images have been captured using a single camera at 2 different time instants, t' and t. Between these 2 points in time, the camera has translated (but not rotated) as illustrated in the birds-eye view that appears below: in the direction x' by the distance f (the focal length), and also in the direction z' by the distance f. There is no translation in the direction y'.



a) Find the essential matrix that relates these two camera positions.

b) Solve for the epipole in the image at time t. Clearly explain your choice of image coordinates.

Problem 2. Consider a calibrated stereo camera arrangement, and assume that you have been given the translation vector t and rotation matrix R that relates the two cameras:

$$t = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad R = \begin{bmatrix} \cos(30°) & -\sin(30°) & 0 \\ \sin(30°) & \cos(30°) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

a) Solve for the essential matrix.

b) Compute the rank of your answer to part (a) using singular value decomposition, SVD. If you use computational tools, cut-and-paste your code as part of your answer. (If you encounter very small values such as 10^{-15} in your calculations, consider those values to be zero.)

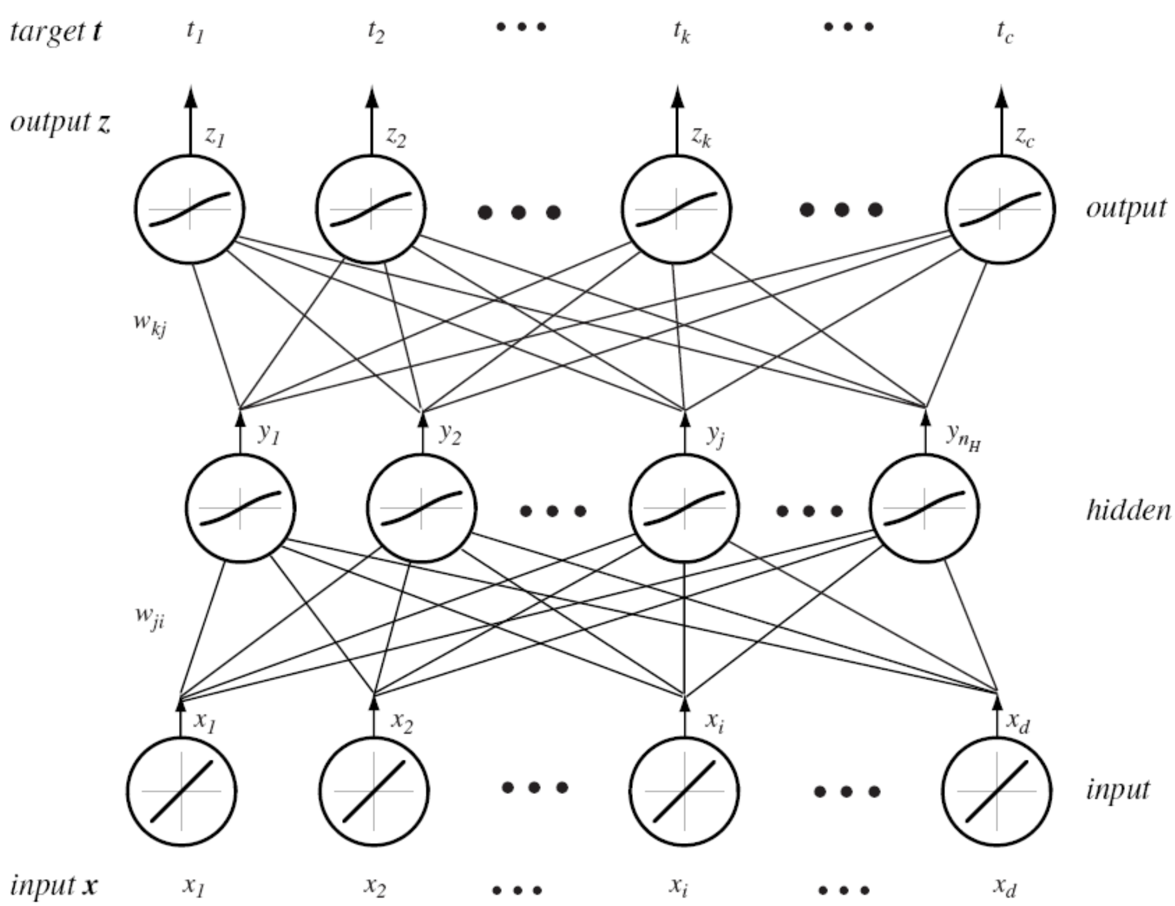
Problem 3. Consider the fully-connected artificial neural network that is shown on the next page. (The figure was taken from Duda, Hart, and Stork, 2012.) Assume that we want to train this network using the backpropagation algorithm. Assume that the goal will be to minimize L_2 loss, which was defined in a recent lecture as $J(\boldsymbol{w}) = \frac{1}{2}\sum_{k=1}^{c}(t_k - z_k)^2$. Signal z_k is the computed output from neuron k in the output layer; t_k is the desired output from that neuron; c is the number of network outputs; and $\boldsymbol{w}$ represents the set of all weights/parameters in the network. Also assume that the activation function for each neuron is the *sigmoid* function $f(s) = \frac{1}{1+e^{-s}}$.

Next, consider the particular parameter w_{kj}, which is the weight associated with the signal that passes from neuron j in the *hidden* layer to neuron k in the *output* layer. Let y_j be the output from neuron j. In a recent lecture we showed that the derivative needed to update w_{kj} during backpropagation is given by

$$\frac{\partial J}{\partial w_{kj}} = (z_k - t_k)(z_k)(1 - z_k)(y_j)$$

Let x_i represent input i, and let w_{ji} represent the weight that is applied to x_i by neuron j in the *hidden* layer. Show the derivative that is needed to update w_{ji} during backpropagation is given by the following expression:

$$\frac{\partial J}{\partial w_{ji}} = \left(-\sum_{k=1}^{c}(t_k - z_k)z_k(1 - z_k)w_{kj}\right)y_j(1 - y_j)x_i$$



Problems 4 and 5.
You have been given a Jupyter Notebook file `Homework4_USERNAME.ipynb`. Replace "`USERNAME`" with your Virginia Tech username. Then upload those files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, you will have created the following 3 files. Upload these 3 files to Canvas before the deadline. Do not combine them in a single ZIP file.

`Homework4_Problems1-3_USERNAME.pdf`	← Your solutions to problems 1 through 3
`Homework4_Code_USERNAME.zip`	← Your zipped Jupyter notebook file
`Homework4_Notebook_USERNAME.pdf`	← A PDF version of your Colab session
