ECE 4554 / ECE 5554: Computer Vision: Homework 3

Fall 2024

Instructions

- This assignment is due at Canvas on November 3 before 11:59 PM. Late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after the deadline will cost 1 token. Please keep track of your own token count, and do not let the count fall below 0.
- Again, please review the Honor Code statement in the syllabus. As stated there, you are not allowed to copy work from another source and submit it as your own for grading.
- The assignment consists of 6 problems. Problems 1 through 3 are analytical in nature, and are presented here. Problems 4 through 6 require work using Colab. Each problem is worth 10 points.
- Problem 6 is required for 5554 students, but is optional (extra credit) for 4554 students.
- Prepare an answer sheet that contains all of your written answers in a single file named `Homework3_Problems1-3_USERNAME.pdf`. (Use your own VT username.) Handwritten solutions are permitted, but they must be easily legible to the grader. In addition, 2 more files related to Python coding must be uploaded to Canvas. Details are provided at the end of this assignment.
- For problems 4 through 6 (the coding problems), the Jupyter notebook file that you submit must be compatible with Google Colab. Your code should execute after the grader makes only 1 change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of 0 for those problems.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. The files that you submit to Canvas are the files that will be graded.

Problem 1. Consider the 2-dimensional *affine transformation*, which maps a point (x, y) to a new location (x', y') in the plane. In homogeneous form, the transformation can be represented as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \text{ where } \boldsymbol{A} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ 0 & 0 & 1 \end{bmatrix}.$$

a) Consider the 3 points $(x, y) = (0, 0)$, $(1, 0)$, and $(0, 1)$. For each of these points, write an expression for the corresponding new location (x', y') in terms of the individual a_{ij} scalar values. (Notice that these 3 points can provide insights about how one coordinate reference frame maps onto another coordinate reference frame. Another take-away is that an affine transformation can map the vertices of *any triangle* to and from the vertices of a reference triangle.)

b) Show that $\boldsymbol{A}^{-1}$ is also an affine transformation, when the inverse of $\boldsymbol{A}$ exists.

c) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are both 3×3 affine transformation matrices, show that the matrix product $\boldsymbol{AB}$ is also a 3×3 affine transformation matrix.

Problem 2. Suppose that you are given a set of points in the (x, y) plane, and you need to apply a sequence of 2D transformations to the given points. Let $\boldsymbol{v} = (x_c, y_c)$ represent some arbitrarily chosen vector. The sequence of transformations is as follows:

1) translate all of the given points by the amount $-\boldsymbol{v}$; and then
2) rotate the resulting points from the previous step by an angle θ counterclockwise about the origin; and then
3) translate the resulting points from the previous step by the amount $\boldsymbol{v}$.

a) Express this sequence of transformations as a sequence of matrix multiplications. Each matrix should be of size 3×3. Write your answer in equation form, specifying one matrix for each of the 3 steps shown above and showing all of the components within each matrix.

b) Multiply your 3 matrices from part (a) to obtain a single transformation matrix of size 3×3.

c) Let $\boldsymbol{v} = (3, 4)$, and $\theta = 30$ degrees. Using your answer from part (b), solve numerically for the final locations of these points:

$(x, y) = (3, 4)$, $(4, 4)$, and $(3, 5)$.

Problem 3. We have discussed the 2D *perspective transformation*, also known as 2D *homography*, which maps a point (x, y) to a new location (x', y') in the plane. This transformation can be represented using the equation $s\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \boldsymbol{H}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$, where $\boldsymbol{H} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix}$. When using the homogeneous coordinate representation as shown here, recall that s is simply a scalar term that is to be eliminated when solving for (x', y'), as discussed in the textbook near equations (2.20)-(2.21).

There are only 8 degrees of freedom in this equation. For this reason, some formulations set $h_{22} = 1$. It may help you when working on coding problems later if you do *not* constrain h_{22} to be 1.

a) Consider the case that you are given n corresponding pairs of points, where $n \geq 4$, and you want to use those points to determine the parameters h_{ij}. For example, assume that the following correspondences are known:

$$\begin{aligned} (x'_0, y'_0) &\leftrightarrow (x_0, y_0) \\ (x'_1, y'_1) &\leftrightarrow (x_1, y_1) \\ (x'_2, y'_2) &\leftrightarrow (x_2, y_2) \\ &\cdots \\ (x'_{n-1}, y'_{n-1}) &\leftrightarrow (x_{n-1}, y_{n-1}) \end{aligned}$$

Show how to derive one matrix equation that represents the relationship between these all of these scalar values (not including s). The form of the equation should be $\boldsymbol{Q}\,\boldsymbol{a} = \boldsymbol{0}$, where $\boldsymbol{a}$ is a 9x1 vector that contains the individual homography parameters only; $\boldsymbol{Q}$ is a matrix of size $2n$x9 that you specify containing known values; and $\boldsymbol{0}$ represents the $2n$x1 vector containing only values of 0. For this part of the problem, you do not need to solve for the parameter vector $\boldsymbol{a}$. *Hint*: you may find some inspiration in the derivation near the end of packet 15, although those lecture slides are discussing a problem that is different from 2D homography.

b) Continuing from part (a), a least-squares solution to parameter vector $\boldsymbol{a}$ is the eigenvector associated with the smallest eigenvalue of the matrix $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{Q}$. Use this approach to find a numerical solution for $\boldsymbol{H}$ for the following point correspondences:

$$\begin{aligned} (x'_i, y'_i) &\leftrightarrow (x_i, y_i) \\ (3.0, 2.0) &\leftrightarrow (0, 0) \\ (3.67, 2.0) &\leftrightarrow (1, 0) \\ (3.5, 2.5) &\leftrightarrow (1, 1) \\ (3.0, 3.0) &\leftrightarrow (0, 1) \end{aligned}$$

You may use any matrix solver to find the numerical values. For example, the NumPy functions `np.linalg.eig()` or `np.linalg.eigh()` might be used. If you use a matrix solver, cut and paste your code as part of your solution. To help with the grading, please <u>normalize your numerical solution by dividing all parameters of $\boldsymbol{H}$ by h_{22}</u>.

Problems 4, 5, and 6.

You have been given a Jupyter Notebook file `Homework3_USERNAME.ipynb` and several image files. Replace "`USERNAME`" with your Virginia Tech username. Then upload those files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, you will have created the following 3 files. Upload these 3 files to Canvas before the deadline. Do not combine them in a single ZIP file.

`Homework3_Problems1-3_USERNAME.pdf` ← Your solutions to problems 1 through 3
`Homework3_Code_USERNAME.zip` ← Your zipped Jupyter notebook file
`Homework3_Notebook_USERNAME.pdf` ← A PDF version of your Colab session
