ECE 4554 / ECE 5554: Computer Vision: Homework 2

Fall 2024

Instructions

- This assignment is due at Canvas on Sept. 22 before 11:59 PM. Late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after the deadline will cost 1 token. Please keep track of your own token count, and do not let the count fall below 0.
- Again, please review the Honor Code statement in the syllabus. As stated there, you are not allowed to copy work from another source and submit it as your own for grading.
- The assignment consists of 6 problems. Problems 1 through 4 are analytical in nature, and are presented here. Problems 5 and 6 require work using Colab. Each problem is worth 10 points.
- One of the problems is required for 5554 students, but is optional (extra credit) for 4554 students.
- Prepare an answer sheet that contains all of your written answers in a single file named `Homework2_Problems1-4_USERNAME.pdf`. (Use your own VT username.) Handwritten solutions are permitted, but they must be easily legible to the grader. In addition, 2 more files related to Python coding must be uploaded to Canvas. Details are provided at the end of this assignment.
- For problems 5 and 6 (the coding problems), the Jupyter notebook file that you submit must be compatible with Google Colab. Your code should execute after the grader makes only 1 change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of 0 for problems 5 and 6.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. The files that you submit to Canvas are the files that will be graded.

Problem 1. Consider two kernels g and h, which are shown below. Let I represent an arbitrary image, and let "*" represent 2D <u>convolution</u>.

$$g = \begin{bmatrix} 1 & -4 & -7 \\ 2 & 5 & -8 \\ 0 & 6 & 0 \end{bmatrix} \qquad h = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$$

Instead of computing $(I * g) * h$, we wish to obtain the same result using a single filter f and the computation $I * f$. What is f?

Problem 2. In a recent lecture, the following template was presented as an approximation of a 2-dimensional Gaussian function for $\sigma = 1$:

$\frac{1}{273} \times$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

How good is this approximation? Solve numerically for the mean squared error (MSE) of this 5×5 template, as it compares to a true Gaussian function. If you use a computational tool, cut and paste your code as part of your solution.

Problem 3. Consider an arbitrary function of 2 variables in the continuous domain, such as $f(x, y)$. The *Laplacian* of f is defined as $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$.

The following kernel is often used as a linear filter that approximates the Laplacian operation:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Provide an analytical derivation to explain why this kernel provides a reasonable approximation to the Laplacian operation.

Problem 4. (10 points) For <u>5554</u> students, this problem is <u>required</u>. For <u>4554</u> students, this problem is optional and can be submitted for <u>extra credit</u>.

a) A 2-dimensional filter is said to be *separable* if it can be replaced by a sequence of two 1-dimensional filters. For example, let I represent a 2-dimensional image in the continuous domain, and let f represent a 2-dimensional filter in the continuous domain. We say that f is separable if 2 functions f_1 and f_2 exist such that $I(x, y) * f(x, y) = I(x, y) * f_1(x) * f_2(y)$.

Consider the two-dimensional, continuous-domain Gaussian function, which is given by

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

Show, analytically, that G is a separable filter.

b) The textbook (equation 3.26) states that the Laplacian-of-Gaussian filter satisfies the following equality:

$$\nabla^2 G(x, y) = \left(\frac{x^2 + y^2}{\sigma^4} - \frac{2}{\sigma^2}\right) G(x, y)$$

Show, analytically, that the above equation is correct.

Problems 5 and 6.
You have been given a Jupyter Notebook file `Homework2_USERNAME.ipynb` and an image file `zebra.png`. Replace "`USERNAME`" with your Virginia Tech username. Then upload both files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, you will have created the following 3 files. Upload these 3 files to Canvas before the deadline. Do not combine them in a single ZIP file.

`Homework2_Problems1-4_USERNAME.pdf` ← Your solutions to problems 1 through 4

`Homework2_Code_USERNAME.zip` ← Your zipped Jupyter notebook file

`Homework2_Notebook_USERNAME.pdf` ← A PDF version of your Colab session
