

$$A. f(a+h) = f(a) + f'(a)h + \frac{f''(a)}{2}h^2 + \frac{f'''(a)}{3!}h^3 + O(h^4)$$

$$f(a-h) = f(a) + f'(a)(-h) + \frac{f''(a)}{2}(-h)^2 + \frac{f'''(a)}{3!}(-h)^3 + O(h^4)$$

$$f(a+h) - f(a-h) = f'(a) \cdot 2h + \frac{f'''(a)}{3!} 2 \cdot h^3 + O(h^4)$$

$$\frac{f(a+h) - f(a-h)}{2h} = f'(a) + \frac{f'''(a)}{3!}h^2 + O(h^4)$$

$$\begin{aligned} \text{Approximation error} &= f'(a) + \frac{f'''(a)h^2}{3!} + O(h^4) - f'(a) \\ &= \frac{f'''(a)h^2}{3!} + O(h^4) = O(h^2) + O(h^4) \\ &= O(h^2) \end{aligned}$$

$$B. \text{ if } f'''(a) = 0$$

$$\text{then Approximation error} = O(h^4)$$

the Approximation error is smaller