A $x_{t+1} = g(x_t)$ $\qquad g(x) = x - \frac{f(x)^2}{f(x+f(x)) - f(x)}$

$$x_{t+1} = x_t - \frac{f^2(x_t)}{f(x_t+f(x_t)) - f(x_t)}$$

Taylor Expand around $\hat{x}$

$$g(x) = g(\hat{x}) + (x-\hat{x})\, g'(\hat{x}) + \frac{1}{2}(x-\hat{x})^2 g''(\hat{x}) + R_3(x)$$

$$x_{t+1} = g(x_t) = g(\hat{x}) + (x_t-\hat{x})\, g'(\hat{x}) + \frac{1}{2}(x_t-\hat{x})^2 g''(\hat{x}) + R_3(x_t)$$

$$x_{t+1} = \hat{x} + (x_t-\hat{x})\, g'(\hat{x}) \cdots + R_3(x_t)$$

$$|x_{t+1} - \hat{x}| \leq |x_t - \hat{x}|\,|g'(\hat{x})| \cdots + R_3(x_t)$$

$g(\hat{x}) = \hat{x}$ $\qquad g(x) = x - \frac{f(x)}{\left[\frac{f(x+f(x)) - f(x)}{f(x)}\right]}$

$$g(x) = x - \frac{f(x)\cdot f(x)}{f(x+f(x)) - f(x)}$$

$$g'(x) = 1 - \left[\frac{f(x)\cdot f(x)}{f(x+f(x)) - f(x)}\right]'$$

$$g'(x) = 1 - \frac{f^2(x)(1+f'(x))f'(x+f(x)) - f'(x)) - 2f'(x)f(x)(f(x+f(x)) - f(x))}{(f^2(x+f(x)) - 2f(x+f(x))f(x) + f^2(x))}$$

set $f(x) = x+1$
$f'(x) \neq 0$

$$g(x) = x - \frac{x+1}{\frac{2x+2-x-1}{x+1}}$$

$$= x - x - 1$$

$$= -1$$

$\therefore g'(\hat{x}) = 0$

$> |x_{t+1} - \hat{x}|$

$$\Rightarrow \frac{|X_{t+1}-\hat{X}|}{|X_t-\hat{X}|^{2}} \le C|X_t - \hat{X}|$$

It is Quadratic convergence

B Set $f(x) = x^2$

$$g(x) = x - \frac{x^2}{\frac{(x+x^2)^2 - x^2}{x^2}}$$

$f'(\hat{x}) = 0$

$f''(\hat{x}) \neq 0$

$$= x - \frac{x^4}{x^2 + 2x^3 + x^4 - x^2}$$

$$= x - \frac{x}{2+x}$$

$$= x - 1 + \frac{2}{2+x}$$

$$g'(x) = 1 + (2(2+x)^{-1})'$$

$$= 1 - 2(2+x)^{-2}$$

$$g'(\hat{x}) = 1 - \frac{1}{2} = \frac{1}{2}$$

$g'(\hat{x}) \neq 0 \Rightarrow$ It is linear convergency.