

C. I think node set 2 is the best.

Because . My criteria is the error should not be too large node set 2

The error range of node set 2 is much smaller than node set 1

$$D \quad w_j = \frac{1}{\prod_{\substack{i=0 \\ i \neq j}}^n (x_j - x_i)}$$

$$\text{want } w_j = (-1)^j \binom{n}{j} = \frac{\left(\frac{n}{2}\right)^n (-1)^{n-j}}{n!} \binom{n}{j}$$

$$\binom{n}{j} = \frac{n!}{j! (n-j)!}$$

$$\Rightarrow \frac{\left(\frac{n}{2}\right)^n (-1)^{n-j}}{n!} \frac{n!}{j! (n-j)!}$$

$$= \frac{\left(\frac{n}{2}\right)^n (-1)^{n-j}}{j! (n-j)!}$$

$$\text{set } n-j=j$$

$$\frac{\left(\frac{n}{2}\right)^n (-1)^{n-j}}{j! (n-j)!} = (-1)^j \frac{j^{2j}}{(n-j)! j!} = (-1)^j \frac{n!}{(n-j)! j!}$$

$$w_j = \left( \prod_{\substack{i=0 \\ i \neq j}}^n \left( -1 + \frac{2j}{n} - x_i \right) \right)^{-1}$$

$$= \left( \left( \frac{2j}{n} - \frac{2x_0}{n} \right) \times \left( \frac{2j}{n} - \frac{2x_1}{n} \right) \cdots \left( \frac{2j}{n} - \frac{2x_n}{n} \right) \right)^{-1}$$

$$= \left( \left( \frac{2j}{n} \right) \times \frac{2j-2}{n} \times \frac{2j-2n}{n} \right)^{-1}$$

$$= \left( \frac{2^n j! (n-j)!}{n^n n!} \right)^{-1}$$

$$= \frac{\binom{n}{2}^n \frac{n!}{j! (n-j)!}}{n!} = \frac{\left(\frac{n}{2}\right)^n (-1)^{n-j}}{n!} \binom{n}{j}$$

proved