Q3 Given $\rho(A) < 1 \Rightarrow$ largest abs eigenvalue < 1.

want $A-I$ non-singular $\det|A-I| \neq 0$

we set $A-I$ is singular then

$(A-I)X = 0 \quad AX = X \Rightarrow 1$ is eigenvalue

$\Rightarrow \rho(A) \geq 1 \Rightarrow$ controadiction

$\Rightarrow (A-I)$ non-singular

B. $B = A[I - A^{-1}(A-B]$

$\Rightarrow \|A^{-1}(A-B)\| \leq \|A^{-1}\| \cdot \|A-B\| < \|A^{-1}\| \cdot \frac{1}{\|A^{-1}\|} = 1$

$\Rightarrow \|A^{-1}(A-B\| < 1$

$\therefore [I - A^{-1}(A-B)]$ is non-singular

A is nonsingluar $\Rightarrow$ B is non-singular

$[I - A^{-1}(A-B)]$ is non-singular

C. $\|\delta A\| \leq \frac{1}{\|A^{-1}\|}$

$A + \delta A = A + \delta A$

$$-\sigma A = A - (A+\sigma A)$$

$$\|-\sigma A\| = \|\sigma A\| = \|A-(A+\sigma A)\| < \frac{1}{\|A^{-1}\|}$$

$\because$ $\|A-B\| \le \frac{1}{\|A^{-1}\|} \Rightarrow B$ is nonsingular

$\therefore$ $\|A-(A+\sigma A)\| \le \frac{1}{\|A^{-1}\|} \Rightarrow A+\sigma A$ is nonsingular