

Descriptive Statistics

- Sample Mean:

$$\bar{y} = \frac{y_1 + y_2 + \dots + y_n}{n} = \frac{\sum y_i}{n}$$

- Sample Variance

$$s^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1} = \frac{\sum_{i=1}^n y_i^2 - \frac{\left(\sum_{i=1}^n y_i\right)^2}{n}}{n-1}$$

- Sample Standard Deviation:

$$s = \sqrt{\text{sample variance}} = \sqrt{s^2}$$

- Range = max – min
- Interquartile Range (IQR) = $Q_3 - Q_1$
- 5 # Summary: min, Q_1 , median, Q_3 , max
- Outliers:
Lower fence = $Q_1 - 1.5 \times \text{IQR}$
Upper fence = $Q_3 + 1.5 \times \text{IQR}$
- z-score: $z = \frac{y - \mu}{\sigma}$
- Empirical Rule:
Empirical rule: For a relatively bell-shaped data set, approximately 68%, 95%, 99.7% of the measurements are within one, two and three standard deviations of the mean, respectively.

Probability Theory

- Addition Rule:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Complement Rule: $P(A^c) = 1 - P(A)$

- Multiplication Rule (general):

$$\begin{aligned} P(A \text{ and } B) &= P(A \cap B) \\ &= P(A) \times P(B|A) \\ &= P(B) \times P(A|B) \end{aligned}$$

- Multiplication Rule for **Independent** Events:

If A and B are independent, then

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

- Conditional Probability of A given B, if $P(B) > 0$:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Probability of A:

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

Population Distributions

- The mean (expected value) of a discrete random variable: $\mu = E(X) = \sum xp(x)$

- The variance of a discrete random variable:

$$\sigma^2 = \text{Var}(X) = \sum (x - \mu)^2 p(x)$$

- The standard deviation of a discrete random variable: $\sigma = \sqrt{\sigma^2}$

- For two random variables X and Y, and constants a, b, and c:

$$\circ E[aX + b] = aE[X] + b$$

$$\circ V[aX + b] = a^2 V[X]$$

$$\circ E[aX + bY + c] = aE[X] + bE[Y] + c$$

$$\circ V[aX + bY + c] = a^2 V[X] + b^2 V[Y], \text{ if } X \text{ and } Y \text{ are independent.}$$

Normal Distribution

$Y \sim \text{Normal}(\mu, \sigma), \quad -\infty < y < \infty$

- $P(a < Y < b) = P\left(\frac{a - \mu}{\sigma} < Z < \frac{b - \mu}{\sigma}\right)$

Sampling Distributions

- Sampling Distribution of a Sample Proportion, \hat{p} :

- $\text{Mean}(\hat{p}) = \mu_{\hat{p}} = p$
- $\text{SD}(\hat{p}) = \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$
- For large n ($np \geq 10$ and $n(1-p) \geq 10$),

$$\hat{p} \sim N\left(\mu_{\hat{p}} = p, \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}\right)$$

- Sampling Distribution of a Sample Mean, \bar{y} :

- $\text{Mean}(\bar{y}) = \mu_{\bar{y}} = \mu$
- $\text{SD}(\bar{y}) = \sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}}$
- If $y \sim N(\mu, \sigma)$, then

$$\bar{y} \sim N\left(\mu_{\bar{y}} = \mu, \sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}}\right)$$
- CLT: If $y \sim ?(\mu, \sigma)$, then for $n \geq 30$

$$\bar{y} \approx N\left(\mu_{\bar{y}} = \mu, \sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}}\right)$$