for third order

Taylor around x_n

$$f(x) = f(x_n) + f'(x_n)(x - x_n) + \frac{1}{2} f''(x_n)(x - x_n)^2$$

$$f(x) = f(x_n) + f'(x_n)x - f'(x_n)x_n + \frac{1}{2} f''(x_n)(x^2 - 2x x_n + x_n^2)$$

$$= f(x_n) + f'(x_n)x - f'(x_n)x_n + \frac{1}{2} f''(x_n)x^2 - f''(x_n)x x_n + \frac{1}{2} f''(x_n) \cdot x_n^2$$

$$= \frac{1}{2} f''(x_n)x^2 + f'(x_n)x - f''(x_n)x x_n + f'(x_n)x_n + \frac{1}{2} f''(x_n)x_n^2 + f(x_n)$$

quadratic equation $\leftarrow$ $$= \frac{1}{2} f''(x_n)x^2 + (f'(x_n) - f''(x_n)x_n)x + f(x_n) + f'(x_n)x_n + \frac{1}{2} f''(x_n)x_n^2$$

$$x = \frac{f''(x_n)x_n - f'(x_n) \pm \sqrt{(f'(x_n) - f''(x_n)x_n)^2 - 2f''(x_n)(f(x_n) + f'(x_n)x_n + \frac{1}{2} f''(x_n)x_n^2)}}{f''(x_n)}$$

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)} \pm \frac{\sqrt{(f'(x_n) - f''(x_n)x_n)^2 - 2f''(x_n)(f(x_n) + f'(x_n)x_n + \frac{1}{2} f''(x_n)x_n^2)}}{f''(x_n)}$$

$\Rightarrow$ x_{n+1} has two distinct schemes

proved.

13. $\lim_{k\to\infty} \frac{|X_{k+1} - X|}{|X_k - X|^3} = \lim_{k\to\infty} \frac{|g(X_k) - X|}{|X_k - X|^3}$

$$= \lim_{k\to\infty} \frac{|\frac{1}{6} g'''(X)(X_k - X)|}{|X_k - X|^3}$$

$$= \frac{1}{6} g'''(X)$$

$\Rightarrow$ convergence is cubic