Autonomous Landing Allocation of Multiple Unmanned Aerial Vehicles on Multiple Unmanned Surface Vessels Subject to Energy Consumption

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Abstract. This paper proposes a distributed energy-based landing allocation method of multiple unmanned aerial vehicles (UAVs) on multiple unmanned surface vessels (USVs). First, an optimization function is established, which consists of the limited energy battery, the flight distance and the flight speed. Then, a distributed landing allocation scheme is designed to minimize the energy consumption difference. Finally, real lake experiments of multiple M-100UAVs and HUSTER-30 USVs are conducted to verify the effectiveness of the proposed method.

Keywords: Autonomous landing allocation · Unmanned aerial vehicles (UAVs) · Unmanned surface vessels (USVs) · Energy consumption

1 Introduction

Recent years have witnessed the rapid development of the unmanned systems. The initial explorations of the unmanned systems mainly focused on the coordination of unmanned surface vehicles and the unmanned aerial vehicles [1–6], which could expand the maneuverability and application scenarios of unmanned systems. However, due to the limitations of current battery technology, the flight

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time of multi-rotor drones is relatively short and the flight range is always limited, which is even more difficult for the UAVs to independently perform tasks on a wide surface of the water with installed loads. In contrast, USVs consist of better carrying performance, longer endurance, and wider mission coverage, whose sailing speed and maneuverability are however limited. To this end, it becomes a future tendency to cooperate UAVs and USVs together to expand the application scenarios of unmanned systems.

In the coordination of multi-UAV multi-USV system, the landing control is a necessary premise for the further application. In this pursuit, Xu *et al.* [7] proposed the method of landing the UAV on a moving USV through visual guidance. However, different from the single UAV-USV system, landing allocation is important in the multi-UAV multi-USV system, which implies the USV needs to choose a proper landing USV platform from multiple USVs. Additionally, it is necessary to reduce energy consumption and the energy consumption difference as much as possible during the landing process. In this way, the integrity of the UAV system is stronger. It will not happen that some drones have insufficient power while others have sufficient power. To this end, it becomes an urgent task to establish a UAV energy consumption model, and then design a reasonable task allocation plan subject to the energy constraints.

Among the construction of the UAV energy consumption model, Modares *et al.* [8] designed an experimental method to approximate the relationship between energy consumption and speed, flight distance, and turning radius. Franco *et al.* [9] further considered the impact of acceleration and deceleration on the energy consumption of the UAV in a experiment, and fitted the energy consumption curve through experiment. A similar method was also studied in [10], and the polynomial relationship between energy consumption and speed is further given. Marins *et al.* [11] analyzes the forces on the drone during the flight by means of physical modeling, and derives the power consumed by the drone during the hovering process. Uehara *et al.* [12] discussed the endurance of drones on Titan, and also obtained an energy consumption model similar to ACE by means of physical modeling.

Notably, the task distribution methods are generally divided into centralized and distributed ones [13], where the centralized task distribution method performs calculations on the designated platform, and then assigns tasks to the designated nodes. Jia *et al.* [14] proposed a genetic algorithm with multi-layer coding for the task assignment of heterogeneous UAVs under time-varying conditions. Although the heuristic algorithm fails to guarantee the global optimal solution, it can give a better allocation method within the required task window time, which is used to solve the task assignment problem of large formation and high real-time requirements. However, the distributed algorithm uses the communication negotiation method to solve each problem by sharing data with each node. Zhang *et al.* [15] designed the auction method in a task allocation process. Brunet *et al.* [16] proposed the adjust bidding strategies through the revenue function to auction the task, which has high requirements for communication and needs to share auction process information among various nodes in real time.

To this end, this paper improves a distributed auction algorithm for the mission point allocation problem of UAV based on energy consumption, making the UAV group have the smallest energy consumption while meeting the smallest difference. Meanwhile, experiments are conducted on a group of UAVs and USVs to verify the effectiveness of the proposed method.

The remainder of this paper is organized as below: Sect. 2 introduces modeling and problem statement; Sect. 3 presents the landing allocation method of the multi-UAV multi-USV system; simulation and real lake experiment are conducted in Sect. 4; finally the conclusion is drawn in Sect. 5.

2 Modeling and Problem Statement

The selection of mission points for UAVs based on energy consumption is a multi-constrained task assignment problem. First, it is necessary to analyze the energy consumption of the UAV during the flight.

2.1 UAV and Energy Consumption Modeling

Generally, multi-rotor UAVs have two layout forms, which are cross-shaped and x-shaped, and have four motors. We analyze the dynamics of the x-shaped UAV, and its structure is shown in the Fig. 1:

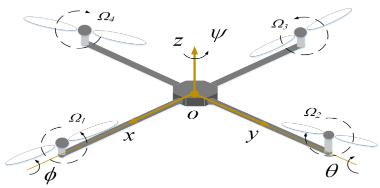

Fig. 1. Illustration of the UAV's model.

The dynamic model [17] of this structure can be written as the Eq. (1):

$$\begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}\left(\dfrac{I_y - I_z}{I_x}\right) - \dfrac{J_r}{I_x}\dot{\theta}\Omega + \dfrac{l}{I_x}U_2 - \dfrac{l}{m}\dot{\phi} \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\left(\dfrac{I_z - I_x}{I_y}\right) + \dfrac{J_r}{I_y}\dot{\phi}\Omega + \dfrac{l}{I_y}U_3 - \dfrac{l}{m}\dot{\theta} \\ \ddot{\psi} = \dot{\phi}\dot{\theta}\left(\dfrac{I_x - I_y}{I_z}\right) + \dfrac{l}{I_z}U_4 - \dfrac{1}{m}\dot{\psi} \\ \ddot{x} = (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\dfrac{1}{m}U_1 - \dfrac{1}{m}\dot{x} \\ \ddot{y} = (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\dfrac{1}{m}U_1 - \dfrac{1}{m}\dot{y} \\ \dot{z} = (\cos\phi\cos\theta)\dfrac{1}{m}U_1 - g - \dfrac{1}{m}\dot{z} \end{cases} \tag{1}$$

Among them, ϕ, θ, and ψ respectively represent the angle of counterclockwise rotation around the x, y, and z axes; I_x, I_y, and I_z respectively represent the moment of inertia of the UAV on the three coordinate axes. The moment of inertia is represented by J_r. The distance from the center of the motor to the center of mass is l; the mass of the aircraft is m; g is the acceleration of gravity.

The UAV studied in this article is the x configuration. For this type of UAV, the parameters U_1, U_2, U_3, U_4, Ω are given by the Eq. (2):

$$\begin{cases} U_1 = b\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) \\ U_2 = b\left(\Omega_4^2 - \Omega_2^2\right) \\ U_3 = b\left(\Omega_3^2 - \Omega_1^2\right) \\ U_4 = d\left(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_4^3\right) \\ \ \Omega = \Omega_2 + \Omega_4 - \Omega_1 - \Omega_3 \end{cases} \tag{2}$$

where b is the lift coefficient, and Ω_1 to Ω_4 are the speeds of motors 1 to 4 respectively.

The energy consumption of a quadrotor during flight can be divided into two parts: 1) the energy consumed during hovering, and 2) the energy consumed during forwarding. Since the energy consumption of the electronic components that control the UAV flight is small, the energy consumption of the electronic components is not considered in this article. The power of the UAV during the hovering process can be obtained by Bernoulli equation [11] and momentum theorem, and it is expressed in Eq. (3):

$$P_{\text{hov}} = \frac{(mg)^{\frac{3}{2}}}{\eta_{\text{tot}}\sqrt{2\rho A}} \tag{3}$$

where η_{tot} is the overall efficiency of the motor and the propeller, ρ is the air density, and A is the total area swept by the blades.

The process of the aircraft flying forward can be divided into acceleration, uniform motion, and deceleration phases. Assuming that the initial speed and final speed of the UAV are zero, the UAV accelerates from stationary to speed v during the acceleration phase. During the uniform movement stage, the UAV moves at a speed v. During the deceleration phase, the UAV decelerates from speed v to zero. The total energy consumption of the UAV in this case is expressed by the Eq. (4):

$$E_{\text{tot}} = \frac{1}{\eta}[(\frac{s}{v} + \frac{v}{s})P_{\text{hov}} + mv^2 + \frac{\rho}{2}sC_D A_{\text{e}} v^2] \tag{4}$$

where s is the flight distance, C_D is the coefficient of air resistance, and A_{e} is the effective windward area of the UAV.

2.2 Problem Statement

Suppose that a group of stationary UAVs on a lake are ready to return to USVs for landing. There are multiple USVs on the lake that can be selected by UAVs as landing points. But they can only choose one-to-one correspondence with each other. We assume that at the beginning of the selection, all UAVs know all the locations of all USVs that can be selected.

In this scenario, the drone needs to use the known information to select a reasonable location and ensure that the energy consumption during the flight is as small as possible by using an proper flying velocity. Since unmanned systems have high integrity demands, it is hoped that in the planning process, the difference in energy consumption of each UAV is as small as possible. This problem is essentially a task planning problem with multiple constraints, and its expression is as follows:

$$\min\{E_{\text{diff}}\} \tag{5}$$

subject to:

$$\begin{cases} \sum_{j=1}^{N_{\text{usv}}} x_{ij} \leq 1 \quad \forall i \in I \\ \sum_{i=1}^{N_{\text{uav}}} x_{ij} \leq 1 \quad \forall j \in J \\ \sum_{i=1}^{N_{\text{uav}}} (\sum_{j=1}^{N_{\text{usv}}} (x_{ij})) = \min\{N_{\text{usv}}, N_{\text{uav}}\} \\ E_{ij} = \frac{1}{\eta}[(\frac{s_{ij}}{v_i} + \frac{v_i}{s_{ij}})P_{\text{hov}} + mv_i^2 + \frac{\rho}{2} s_{ij} C_D A_e v_i^2 \\ E_{\text{diff}} = \sum_{i=1}^{N_{\text{uav}}} (\sum_{j=1}^{N_{\text{usv}}} (E_{ij}x_{ij} - \text{mean}(E_{ij}x_{ij}))) \end{cases}$$

where x_{ij} is the selection relationship between the UAV i and the USV j. $x_{ij} = 1$ means that the UAV i selects the USV j, and $x_{ij} = 0$ means it is not selected. I and J are the number sets of UAVs and USVs respectively; N_{uav} and N_{usv} represent the number of UAVs and USVs respectively; s_{ij} represents the distance from the UAV i to the USV j.

3 Allocation Algorithm Design

The distributed auction algorithm combines the characteristics of the auction algorithm and the consensus algorithm, and can complete task assignment under the condition of limited communication. This paper improves the distributed auction algorithm by introducing a new method of generating bid list of each individual. The improved algorithm makes the energy consumption difference of each individuals smaller when the lowest energy consumption constraint of the unmanned system is considered.

Before starting the algorithm, we must first preprocess the problem. In this problem, each UAV knows the location of all USVs. In the following, the position of the USV is regarded as the mission point of UAV planning. In the research, it is found that optimizing energy consumption and meeting the limit of small energy consumption difference at the same time will make the problem very complicated. In order to simplify the analysis, the optimization progress is decoupled here [11]. First, analyze the Eq. (4) and find that when the distance is constant, the velocity corresponding to the optimal energy consumption can be obtained by solving the Eq. (6):

$$(2m + s\rho C_D A)v^3 + (\frac{P_0}{a})v^2 - sP_0 = 0 \tag{6}$$

Each individual drone i can calculate the Eq. (6) to obtain the minimum energy consumption E_{ij} to reach different mission point j. This value can be used to generate the energy consumption matrix of each UAV. Assuming that the profit matrix is c, the smaller the energy consumption difference, the greater the profit. In order to achieve the average energy consumption of different UAVs in the system, the Eq. (7) is used to generate the profit matrix:

$$\begin{cases} avg = \text{mean}(E_{ij}) \\ c_{ij} = \dfrac{1}{\text{abs}(E_{ij} - avg)} \end{cases} \tag{7}$$

where avg represents the average value, abs calculates the absolute value.

In the distributed auction algorithm, all UAVs do not have any mission point in the initial state. Each individual i has its own profit list c_{ij}, a bid list y_i and a mission vector x_i. The position j of the vector x_i indicates the selection status of the task, $x_i(j) = 0$ means not selected, and $x_i(j) = 1$ means selected. For example, $x_i = [0, 0, 0, 0, 1, 0]$ means individual i has selected mission point 5. The bid list y_i indicates the desire degree of each node i for mission point j. The higher the value of its position j, the higher its bid for mission point j. In the process of each auction, first find mission point that can still be profitable by comparing the profit list c_{ij} and bid list y_i. Then calculate the optimal profit t_{ij^*}, and suboptimal profit $t_{ij'}$, where j^* and j' respectively represent the optimal and suboptimal mission points. Finally calculate the new bid list $y_{ij_{new}}$ and new profit list $c_{ij_{new}}$. Equations can be expressed as the following [18]:

$$\begin{cases} H_{ij} = f(c_{ij} > y_{ij}) \\ t_{ij^*} = \max\{H_{ij}\} \\ t_{ij'} = \max\limits_{k \neq j^*}\{H_{ij}\} \\ y_{ij_{\text{new}}} = y_{ij^*} + t_{ij^*} - t_{ij'} + \varepsilon \\ c_{ij_{\text{new}}} = c_{ij^*} - (t_{ij^*} - t_{ij'} - \varepsilon) \end{cases} \tag{8}$$

where the $f(c, y)$ function can find the profitable mission points collection H_{ij} and ε is the update range of each auction, usually taken as $\varepsilon < \frac{1}{N_{\text{uav}}}$.

The second stage of the algorithm is the consensus process [19]. First define the undirected graph g to represent the connectivity of the communication network. Assume that the matrix is a symmetric matrix, and there is a communication relationship between two connected points. If two points are connected to each other, then $g_{mn} = g_{nm} = 1$; if there is no interconnection between two points, then $g_{mn} = g_{nm} = 0$. In g, each node is considered to be self-connected, that is, $g_{nn} = 1$.

In the process of consensus, each individual i and the connected individual i' in this iteration will share their bid list information y_i and $y_{i'}$. Individual i compares y_i with $y_{i'}$, and updates the highest bid into all y_i and $y_{i'}$. At the same time, if it is found that there is a higher bid in $y_{i'}$ than the task selected by itself, individual i will lose the task selected in this iteration [16].

4 Simulation and Experiment

In order to test the robustness and effectiveness of the algorithm, methods of simulation and experiment are used for verification in this section. More UAVs are set up in the simulation to increase the intensity of competition in the allocation process; finally, the usability of the algorithm is tested in actual experiments.

50 UAVs and 50 USVs are set up in the simulation. The UAVs are numbered 1 to 50. Adjacent UAVs will exchange the current bid list y_{ij} with each other. Take the mass of the UAV as $m = 2.5\,\text{kg}$. Acceleration $a = 1\,\text{m/s}^2$. $C_D \times A_e = 0.01547\,\text{m}$. Hovering power $P_{\text{hov}} = 572.7\,\text{W}$. Efficiency $\eta = 0.9$. The positions of UAVs and USVs are randomly initialized in the range of 0 to 50 and 50 to 100 on the x and y axes, and the limit of simulation iterations is set to 10,000.

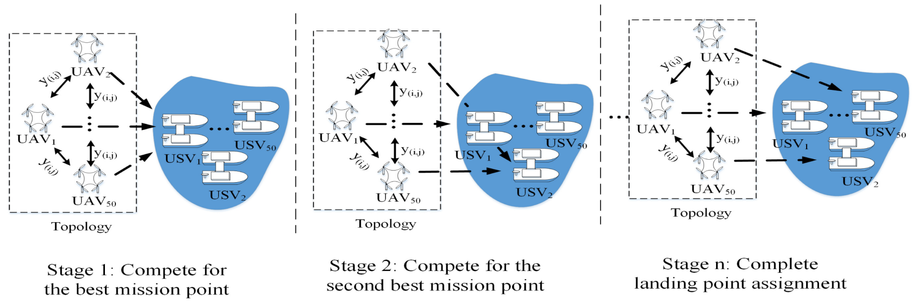

Fig. 2. Competition progress in the algorithm.

In the simulation, since the UAV knows the position of all mission points at the beginning, the optimal speed to reach each mission point can be solved by Eq. (6). In the unimproved algorithm, each UAV brings the optimal speed into Eq. (4) to obtain the optimal energy consumption to reach each mission point. Then the algorithm will make each UAV preferentially select the mission point with the lowest energy consumption. Under normal circumstances, the shorter the flight distance, the lower the energy consumption, so the drone group will give priority to the closest point, which will cause fierce competition. The process is shown in Fig. 2. Figure 3 (a) shows the competitive process at a certain mission point in the unimproved algorithm. It can be seen that the number of UAVs participating in the competition is large, and it takes multiple rounds of iterations to end the competition. From the perspective of system energy consumption, the energy consumed by each UAV is quite different, as shown in Fig. 4 (a).

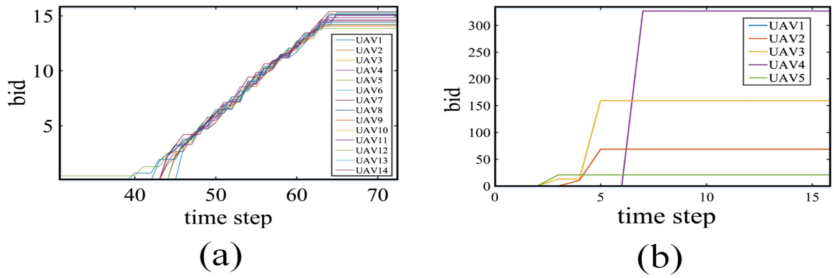

Fig. 3. (a) UAVs compete for a certain mission point in the unimproved algorithm. 14 drones participated in the bidding process, and finally UAV4 obtained the mission point; (b)UAVs compete for a certain mission point in the improved algorithm. The number of UAVs participating in the competition has decreased significantly. The UAV that most want to get this mission point will give a higher bid, and the mission point will be selected after fewer iterations.

Using Eq. (7) to improve the algorithm can make the optimal mission points of each UAV more dispersed and reduce the number of UAVs participating in the competition. Figure 3 (b) shows the competitive process at a certain mission point after the algorithm is improved. It can be seen that the number of UAVs participating in the competition for this point has decreased significantly. At the same time, this improved method can also make the energy consumption of each UAV more even, as shown in Fig. 4 (b). This can improve the integrity of the whole unmanned system.

In order to test this algorithm, an UAV experimental platform is built, and actual experiments are completed on a lake in Dongguan, Guangdong, China as shown in Fig. 5.

In this experiment, dji M100 series unmanned aerial (wheelbase 650mm) vehicles and huster30 (300 cm in length) and huster12 (120 cm in length) series USVs are used as the experimental platform. The M100 is equipped with a manifold onboard computer for control, and uses vonets 5G module for communication. The position of the unmanned boat is forwarded to the drone through the base station, so that the drone can obtain the positions of all mission points. The architecture of the experimental platform is shown in the Fig. 6.

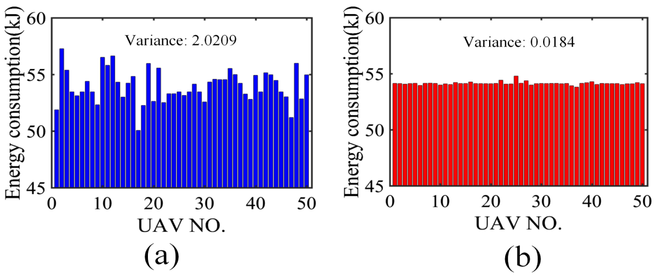

Fig. 4. (a) UAV energy consumption in the unimproved algorithm. (b) UAV energy consumption int the improved algorithm.

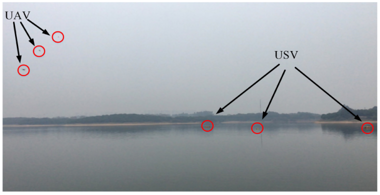

Fig. 5. Experiment on lake.

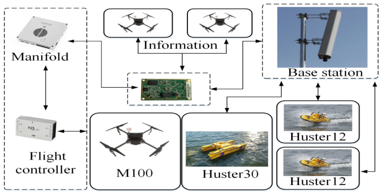

Fig. 6. Structure of the multi-UAV multi-USV system.

Three UAVs and three USVs are deployed scattered on the lake in this experiment. After the experiment starts, the UAVs will compete with each other for three USVs, which is regarded as the mission point, and then fly to them. The algorithm first initializes the minimum energy consumption and corresponding velocity of each UAV to each mission point through the model in Eq. (4) and Eq. (6). The energy consumption and corresponding velocity of the UAV arriving at different mission points is shown in Fig. 7. Then, through experiment, the task allocation of the algorithm before and after the improvement is compared.

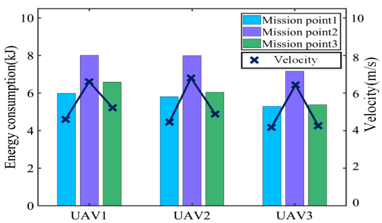

Fig. 7. Energy consumption and velocities of UAVs to various points.

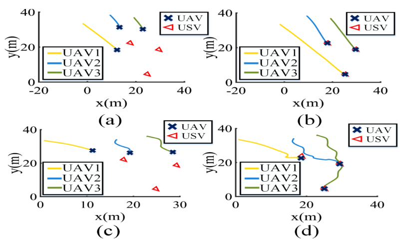

Fig. 8. (a)–(b) are experimental routes of the unimproved algorithm; (c)–(d) are improved algorithm.

Figure 8 are the results of the assignment of task points by the two algorithms. Figure 9 shows the velocity changes of the two algorithms in the process which corresponds to the velocity data in the Fig. 7. What needs to be pointed out is that since the flight path may cross, we have added a collision avoidance item to the control algorithm. When the distance between two aircraft is less than the safety limit, evasive action will be taken, so the speed in the Fig. 9 (b) fluctuates. Figure 10 shows the distribution result of the final energy consumption of the UAVs. It can be seen that when the algorithm is improved, the energy consumption of the unmanned system is more even.

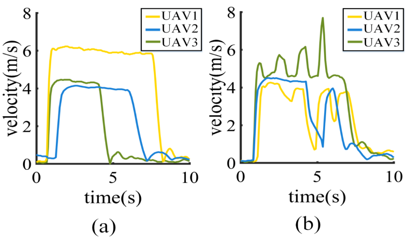

Fig. 9. (a)Velocities of UAVs in the unimproved algorithm; (b) Velocities of UAVs in the improved algorithm.

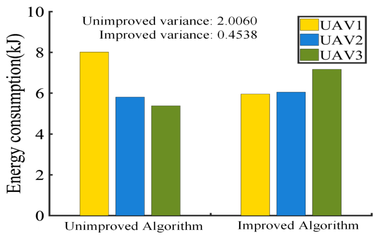

Fig. 10. Experimental results.

5 Conclusion

In this paper, a distributed allocation method of multiple unmanned aerial vehicles (UAVs) on multiple unmanned surface vessels (USVs) is proposed for the UAVs landing allocation with minimum energy consumption. With such a proposed method, the multiple UAVs land on the multiple USVs with the minimized energy consumption difference. Finally, experiments based on our multi-UAV multi-USV system are conducted to substantiate the proposed method. It is viable that the presented allocation protocol would be applicable in the further cooperation of multi-UAV multi-USV systems.

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