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Cooperative Hunting Control for Multi-Underactuated Surface Vehicles

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Abstract: In this study, a control scheme based on the distance and angle between the hunters and invader is designed for the cooperative hunting behavior of the underactuated surface vehicles, in presence of small uncertain disturbance. Based on the Lyapunov synthesis, it is proven that the cooperative hunting error asymptotically converges to zero when the desired immediate control signal of the hunter vehicle is realized. Then, a PID controller is performed to achieve immediate control signal of the hunter vehicle. Several simulation results demonstrate the effectiveness of the controller.

Key Words: Cooperative hunting; Lyapunov synthesis; undercatuated surface vehicle.

1 Introduction

Over the past few decades, cooperative control of the multi-agent systems attracted great attention in system and control methods. It is partially due to the increasing need to perform more difficult and complex tasks, where it contributing to increasing efficiency, reducing the system cost and providing the redundancy against individual failure. Some relevant research has increasing in marine systems, which includes ocean exploration, harbor surveillance, hydrological survey, cooperative hunting behavior and so on. Multi-robot coordinated hunting behavior is a significant topic in the area of robotics research. In order to achieve cooperative hunting behavior of multi-agent system, several methods have been proposed, which include formation strategy [1-6,14], pursuit and capture mode strategy [7-10,13], artificial method [11], game theory method [12] and so on.

Most of cooperative hunting systems have been put forward recently. In 1998, Yamaguchi et al. firstly proposed a feedback control method making multi-robot formation control to round up the target robot into a certain area [1]. Cooperative hunting behavior were separated in four different modes to achieve the respective goal by different methods[7]. Ni et al. studied the pursuit and capture behavior in the way of neural network method and the methods of dynamic alliance and formation construction algorithms [8]. While, particle swarm optimization methods has been proposed by Nighot et al. to capture the target robot [9]. Cooperative hunting using a limit-cycle based algorithm was studied in [11], where the formation is obtained using artificial potential field and the limit circle is used to encircle the target. Further, game theory was used to achieve the goal of cooperative hunting behavior [12]. In [13], He Shen et al. presents Lyapunov based cooperative hunting method for multiple robots. A distributed consensus controller was developed for the formation patten of multi USV systems in [14].

In this paper, we develop a new approach based on the distance and angle between the hunters and the invader for cooperative hunting behavior of the underactuated surface vehicles. Compared with the existing result, the main contribution of this paper are listed as follows: (i) The cooperative hunting controller is based on the distance and angle between the hunters and invader. (ii) The cooperative hunting behavior of multi underactuated surface vehicles with small uncertain disturbance are first considered. (iii) The cooperative hunting controller is efficient and robust with small uncertain disturbance.

This paper is organized as follows: Section 2 introduces some preliminaries and gives the problem formulation of hunting behavior. Section 3 presents the cooperative hunting controller based on the distance and angle between the hunters and invader, which is proved based on the Lyapunov methods. A PID controller is utilized to perform the immediate control signal of the hunter vehicle. Some computer simulation results to illustrate the proposed method are presented in Section 4. Section 5 concludes the article.

2 Problem Formulation

Throughout the paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean Space. $\|\cdot\|$ denotes the Euclidean norms. $(\cdot)_{ij}$ denotes the element of $(\cdot)$ in row i, column j.

2.1 Vehicle Dynamics

Consider a group of N underactuated surface vehicles represented as the hunters and neglect the motion of the hunter vehicle i in heave, roll and pitch. The simplified kinematic model, which describes the relationship of the position and velocity between the earth-fixed reference frame and the body-fixed reference frame, is given as:

$$\left\{\begin{array}{lcl} \dot{x}_i &=& u_i cos(\psi_i) - v_i sin(\psi_i) \\ \dot{y}_i &=& u_i sin(\psi_i) + v_i cos(\psi_i) \\ \dot{\psi}_i &=& r_i \end{array}\right. \tag{1}$$

and assume the inertial added mass and damping matrix are diagonal, the kinetics are :

$$\left\{\begin{array}{l} m_{11}\dot{u}_i - m_{22}v_i r_i + d_{11}u_i = \tau_{1i} \\ m_{22}\dot{v}_i + m_{11}u_i r_i + d_{22}v_i = 0 \\ m_{33}\dot{r}_i + (m_{22}-m_{11})u_i v_i + d_{33}r_i = \tau_{3i} \end{array}\right. \tag{2}$$

where (x_i, y_i, ψ_i) denotes the position vector and heading angle of the i^{th} hunter vehicle in the earth-fixed reference frame. (u_i, v_i, r_i) represents the surge velocity, sway

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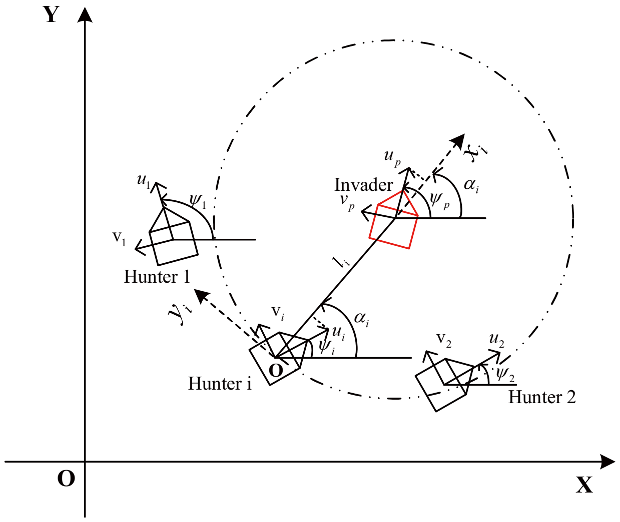

Fig. 1: Definition of the relative configuration between the hunters and invader (red represents invader vehicle).

velocity and yaw velocity respectively. (m_{11}, m_{22}, m_{33}), (d_{11}, d_{22}, d_{33}) are the added inertia and hydrodynamic damping. The available controls are the surge force τ_{1i} and the yaw moment τ_{3i} for the hunter vehicle i.

For simplicity, define the kinematic and kinetic equation of the invader vehicle which is similar to the hunter vehicle:

$$\begin{cases} \dot{x}_p = u_p\cos(\psi_p) - v_p\sin(\psi_p) \\ \dot{y}_p = u_p\sin(\psi_p) + v_p\cos(\psi_p) \\ \dot{\psi}_p = r_p \\ m_{11}\dot{u}_p - m_{22}v_p r_p + d_{11}u_p = \tau_{1p} \\ m_{22}\dot{v}_p + m_{11}u_p r_p + d_{22}v_p = 0 \\ m_{33}\dot{r}_p + (m_{22} - m_{11})u_p v_p + d_{33}r_p = \tau_{3p} \end{cases} \quad (3)$$

where (x_p, y_p, ψ_p) denotes the position vector and heading angle of the invader vehicle in the earth-fixed reference frame. (u_p, v_p, r_p) represents the surge velocity, sway velocity and yaw velocity respectively. τ_{1p} and τ_{3p} are defined as available controls of the surge force the yaw moment.

2.2 Cooperative Hunting Behavior

Fig. 1 illustrates the basic cooperative hunting structure of multi hunter vehicles and invader vehicle. The distance l_i and angle α_i between the hunter vehicle i and the invader in the earth-fixed reference frame are defined as follows:

$$\begin{cases} l_i = \sqrt{(x_p - x_i)^2 + (y_p - y_i)^2} \\ \alpha_i = atan2(y_p - y_i, x_p - x_i) \end{cases} \quad (4)$$

The control goal of the cooperative hunting behavior is to encircle the invader vehicle by multi hunter vehicles. Furthermore, hunter vehicles are evenly distributed around the invader vehicle. Specifically, the objective is to design (τ_{1i}, τ_{3i}) for the hunter i to cooperatively hunt the invader vehicle, which is described as follows:

$$\begin{cases} l_i = l_{id} \\ |\alpha_i - \alpha_{i+1}| = \frac{2\pi}{n} \\ \psi_i = \psi_j \end{cases} \quad (5)$$

where $i, j \in \mathbb{N}$ and $i \neq j$. l_{id} represents the desired distance between the hunter vehicle i and invader. n is the number of hunter vehicles.

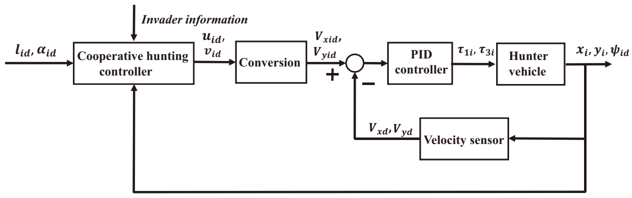

Fig. 2: Structure of cooperative hunting controller

3 Cooperative Hunting Control

Note that the cooperative hunting behavior is determined by the distance l_i and angle α_i between the hunters and invader. Fig. 2 illustrates the structure of cooperative hunting controller, which consists two parts, namely, hunting behavior control and vehicle kinetic control. Consider the desired velocity (u_{id}, v_{id}) of hunter vehicle i as immediate control signal to achieve the desired distance l_{id} and angle α_{id}, which finally form a cooperative hunting result. Then discuss the stability of the closed system. The surge force τ_{1i} and the sway force τ_{3i} of the hunter vehicle i are derived by the PID controller in Section 3.2.

3.1 Hunting Behavior Control

Consider the distance l_i between the hunter i and invader, which is decided by the velocity of the hunter i and the invader. Furthermore, if the invader vehicle is regarded as the center of a circle, whose radius can be defined as the distance l_i. It is obvious that α_i is changed if wire speed exists on the circle. By taking the derivative of l_i and α_i, then we obtain the following equation:

$$\begin{cases} \dot{l}_i = (\dot{x}_i - \dot{x}_p) * \cos(\alpha_i) + (\dot{y}_i - \dot{y}_p) * \sin(\alpha_i) \\ l_i\dot{\alpha}_i = -(\dot{x}_i - \dot{x}_p) * \sin(\alpha_i) + (\dot{y}_i - \dot{y}_p) * \cos(\alpha_i) \end{cases} \quad (6)$$

where $(\dot{x}_i, \dot{y}_i), (\dot{x}_p, \dot{y}_p)$ represent the velocity vector of the hunter i and invader in the earth-fixed reference frame respectively. α_i is defined as the angle between the hunter i and invader.

Substitute $(\dot{x}_i, \dot{y}_i), (\dot{x}_p, \dot{y}_p)$ with the Eq.(1) and (3), we obtain

$$\begin{cases} \dot{l}_i = u_p\cos(\psi_p - \alpha_i) - v_p\sin(\psi_p - \alpha_i) \\ \quad\quad -u_i\cos(\psi_i - \alpha_i) - v_i\sin(\psi_i - \alpha_i) \\ l_i\dot{\alpha}_i = u_p\sin(\psi_p - \alpha_i) + v_p\cos(\psi_p - \alpha_i) \\ \quad\quad -u_i\sin(\psi_i - \alpha_i) - v_i\cos(\psi_i - \alpha_i) \end{cases} \quad (7)$$

Rewrite the equation (7) in the form of matrix as follows:

$$\begin{bmatrix} \dot{l}_i \\ \dot{\alpha}_i \end{bmatrix} = \begin{bmatrix} \cos(\psi_p - \alpha_i) & -\sin(\psi_p - \alpha_i) \\ \frac{\sin(\psi_p - \alpha_i)}{l_i} & \frac{\cos(\psi_p - \alpha_i)}{l_i} \end{bmatrix} \begin{bmatrix} u_p \\ v_p \end{bmatrix} + \begin{bmatrix} -\cos(\psi_i - \alpha_i) & \sin(\psi_i - \alpha_i) \\ \frac{-\sin(\psi_i - \alpha_i)}{l_i} & \frac{-\cos(\psi_i - \alpha_i)}{l_i} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \end{bmatrix} \quad (8)$$

Here, define the desired distance as l_{id} and the desired angle as α_{id}, which are normally a constant. In order to perform the cooperative hunting perfectly, introduce the hunting error:

$$\begin{cases} e_{l_i} = l_i - l_{id} \\ e_{\alpha_i} = \alpha_i - \alpha_{id} \end{cases} \quad (9)$$

Taking the derivative of e_{l_i} and e_{α_i} , then by (6), we obtain the following error dynamics:

$$\begin{bmatrix} \dot{e}_{l_i} \\ \dot{e}_{\alpha_i} \end{bmatrix} = \begin{bmatrix} \cos(\psi_p - \alpha_i) & -\sin(\psi_p - \alpha_i) \\ \frac{\sin(\psi_p-\alpha_i)}{l_{id}+e_{l_i}} & \frac{\cos(\psi_p-\alpha_i)}{l_{id}+e_{l_i}} \end{bmatrix} \begin{bmatrix} u_p \\ v_p \end{bmatrix} + \begin{bmatrix} -\cos(\psi_i - \alpha_i) & \sin(\psi_i - \alpha_i) \\ \frac{-\sin(\psi_i-\alpha_i)}{l_{id}+e_{l_i}} & \frac{-\cos(\psi_i-\alpha_i)}{l_{id}+e_{l_i}} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \end{bmatrix} \tag{10}$$

where (u_i, v_i) represents the the immediate control signal of the hunter i. In order to simplify the dynamic of the cooperative hunting error, we define:

$$\begin{aligned} \dot{E}_i &= \begin{bmatrix} \dot{e}_{l_i} \\ \dot{e}_{\alpha_i} \end{bmatrix}, U_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} \\ F_i &= \begin{bmatrix} \cos(\psi_p - \alpha_i) & -\sin(\psi_p - \alpha_i) \\ \frac{\sin(\psi_p-\alpha_i)}{l_{id}+e_{l_i}} & \frac{\cos(\psi_p-\alpha_i)}{l_{id}+e_{l_i}} \end{bmatrix} \times \begin{bmatrix} u_p \\ v_p \end{bmatrix} \\ B_i &= \begin{bmatrix} -\cos(\psi_i - \alpha_i) & \sin(\psi_i - \alpha_i) \\ \frac{-\sin(\psi_i-\alpha_i)}{l_{id}+e_{l_i}} & \frac{-\cos(\psi_i-\alpha_i)}{l_{id}+e_{l_i}} \end{bmatrix} \end{aligned} \tag{11}$$

where $\dot{E}_i$ represents the hunting error and U_i is the desired immediate control signal. Then rewrite the dynamic equation as follows:

$$\dot{E}_i = F_i + B_i U_i \tag{12}$$

Theorem 1 *The dynamic hunting error system given in Eq.(12) is asymptotically converges to zero when the controller in the form:*

$$U_i = B_i^{-1}(-F_i - kE_i) \tag{13}$$

Proof: First, we choose a Lyapunov function for the dynamic of the cooperative hunting error as:

$$V_i = \tfrac{1}{2} E_i^T E_i \tag{14}$$

then take the derivative of the V_i and substitute U_i for Eq. (12), we get:

$$\dot{V}_i = E_i^T \dot{E}_i = E_i^T (F_i + B_i U_i) = E_i^T(-kE_i) \leqslant 0 \tag{15}$$

where k is positive constant parameter. Note that V_i is equal to zero only when the error E_i is zero. Then we know, the error E_i asymptotically converges to zero with the controller in the form of Eq. (13). ■

Then substitute B_i, F_i, E_i in (12), we obtain the controller U_i of the hunter i:

$$\begin{aligned} \begin{bmatrix} u_i \\ v_i \end{bmatrix} &= \begin{bmatrix} -\cos(\psi_i - \alpha_i) & \sin(\psi_i - \alpha_i) \\ \frac{-\sin(\psi_i-\alpha_i)}{l_{id}+e_{l_i}} & \frac{-\cos(\psi_i-\alpha_i)}{l_{id}+e_{l_i}} \end{bmatrix}^{-1} \times \\ & \left(\begin{bmatrix} -\cos(\psi_p - \alpha_i) & \sin(\psi_p - \alpha_i) \\ \frac{-\sin(\psi_p-\alpha_i)}{l_{id}+e_{l_i}} & \frac{-\cos(\psi_p-\alpha_i)}{l_{id}+e_{l_i}} \end{bmatrix} \begin{bmatrix} u_p \\ v_p \end{bmatrix} \right. \\ & \left. - k \begin{bmatrix} e_{l_i} \\ e_{\alpha_i} \end{bmatrix} \right) \\ &= \begin{bmatrix} -\cos(\psi_i - \alpha_i) & -(l_{id} + e_{l_i})\sin(\psi_i - \alpha_i) \\ \sin(\psi_i - \alpha_i) & -(l_{id} + e_{l_i})\cos(\psi_i - \alpha_i) \end{bmatrix} \\ & \times \left(\begin{bmatrix} -\cos(\psi_p - \alpha_i) & \sin(\psi_p - \alpha_i) \\ \frac{-\sin(\psi_p-\alpha_i)}{l_{id}+e_{l_i}} & \frac{-\cos(\psi_p-\alpha_i)}{l_{id}+e_{l_i}} \end{bmatrix} \begin{bmatrix} u_p \\ v_p \end{bmatrix} \right. \\ & \left. - k \begin{bmatrix} e_{l_i} \\ e_{\alpha_{hi}} \end{bmatrix} \right) \end{aligned} \tag{16}$$

It is noted that if the desired immediate control signal (u_i, v_i) are chosen as the equation above, the cooperative hunting behavior is finished perfectly. Then rewrite it as (u_{id}, v_{id}):

$$\begin{cases} u_{id} = u_p \cos(\psi_p - \psi_i) - v_p \sin(\psi_p - \psi_i) + ke_{l_i} \\ \quad \times \cos(\psi_i - \alpha_i) + ke_{\alpha_i}(l_i + e_{l_i})\sin(\psi_i - \alpha_i) \\ v_{id} = u_p \sin(\psi_p - \psi_i) + v_p \cos(\psi_p - \psi_i) - ke_{l_i} \\ \quad \times \sin(\psi_i - \alpha_i) + ke_{\alpha_i}(l_i + e_{l_i})\cos(\psi_i - \alpha_i) \end{cases} \tag{17}$$

Theorem 2 *The heading angle of the hunter vehicles converge to consensus when the desired immediate control signal (u_{id}, v_{id}) is in the form of (17):*

$$\psi_i = \psi_j \tag{18}$$

where $i, j \in \mathbb{N}$ and $i \neq j$. i, j represent the heading angle of the hunter vehicle i and hunter vehicle j respectively.

Proof: Consider the hunting error e_{li}, e_{α_i} of the hunter vehicle i converges to zero since the theorem 1. The equation of (17) is in the form:

$$\begin{cases} u_{id} = u_p \cos(\psi_p - \psi_i) - v_p \sin(\psi_p - \psi_i) \\ v_{id} = u_p \sin(\psi_p - \psi_i) + v_p \cos(\psi_p - \psi_i) \end{cases} \tag{19}$$

Transform it to the control signal defined as V_{xid}, V_{yid} in the earth-fixed reference frame,

$$\begin{cases} V_{xid} & = u_{id}\cos(\psi_i) - v_{id}\sin(\psi_i) \\ & = u_p \cos(\psi_p) - v_p \sin(\psi_p) \\ & = V_{xp} \\ V_{yid} & = u_{id}\sin(\psi_i) + v_{id}\cos(\psi_i) \\ & = u_p \sin(\psi_p) + v_p \cos(\psi_p) \\ & = V_{yp} \end{cases} \tag{20}$$

where V_{xid} and V_{yid} are the velocity of hunter vehicle i which along the horizontal x-axis and the vertical y-axis in the earth-fixed reference frame. V_{xp}, V_{yp} are the velocity of invader vehicle defined in the earth-fixed reference frame. Then ψ_{id} defined as the desired phase angle of hunter vehicle i, is calculated as:

$$\psi_{id} = atan2(V_{yid}, V_{xid}) = atan2(V_{yp}, V_{xp}) \tag{21}$$

■

Then we conclude the heading angle of hunter vehicles converge to be same when the cooperative hunting behavior is realized.

3.2 Vehicle Kinetic Control

The following work is to design a control law τ_{1i} and τ_{3i} to make the surge velocity u_i and the sway velocity v_i of the hunter i converges to the desired immediate control signal u_{id} and v_{id}. By analysing the kinetic equation of the underactuated hunter vehicles, the surge velocity u_i and the heading angle ψ_i are mainly influenced by control input τ_{1i} and τ_{3i} respectively. For the simplicity to identify efficiency of the desired immediate control signal, A PID controller to make the surge velocity u_i and phase angle ψ_i converge to the desired surge velocity u_{id} and desired phase angle ψ_{id}.

Define the surge velocity error and the phase angle error as:

$$\begin{cases} e_{u_i} = u_i - u_{id} \\ e_{\psi_i} = \psi_i - \psi_{id} \end{cases} \tag{22}$$

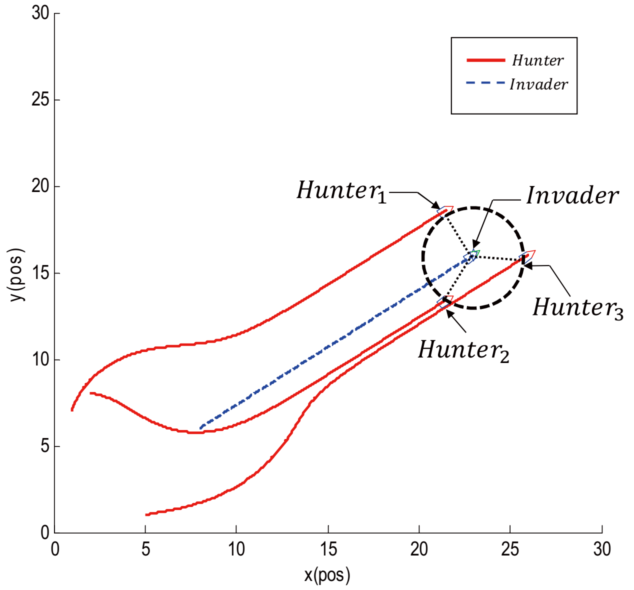

Fig. 3: Cooperative hunting of the invader with a constant velocity

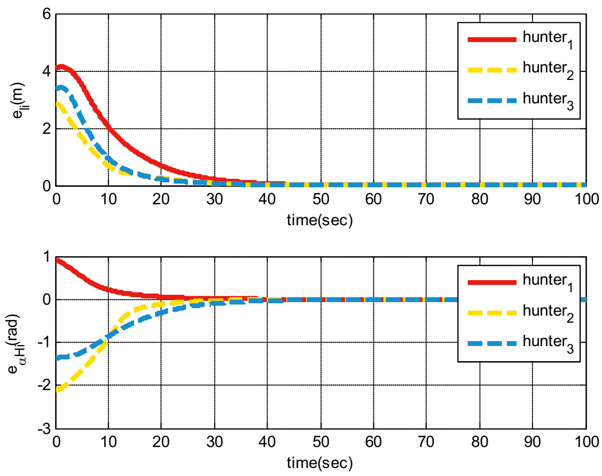

Fig. 4: Hunting error e_{li} and e_{α_i} of the invader with a constant velocity

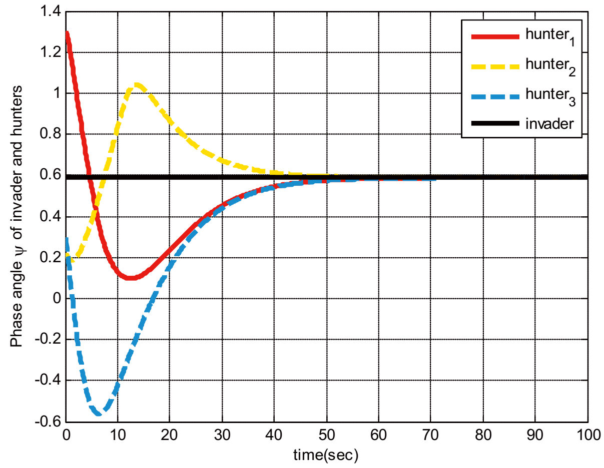

Fig. 5: Phase angle ψ_p and ψ_i of the hunter with a constant velocity

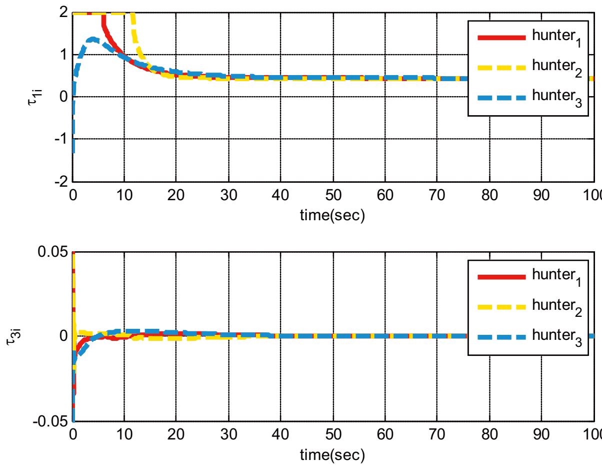

Fig. 6: Control input τ_{li} and τ_{3i} of the invader with a constant velocity

where e_{u_i} and e_{ψ_i} are the surge velocity error and heading angle error between vehicle variables and the desired immediate control signal. Based on the PID controller scheme, we obtain the control law of hunter vehicle i as follows:

$$\begin{cases} \tau_{1i} = k_{pui} e_{u_i} + k_{iui} \int_0^t e_{u_i}\, dt + k_{dui} \dot{e}_{u_i} \\ \tau_{3i} = k_{ppi} e_{\psi_i} + k_{ipi} \int_0^t e_{\psi_i}\, dt + k_{dpi} \dot{e}_{\psi_i} \end{cases} \tag{23}$$

where $(k_{pui}, k_{iui}, k_{dui})$ and $(k_{ppi}, k_{ipi}, k_{dpi})$ are positive constant, which represent proportionality coefficient, integral coefficient and differential coefficient of the surge velocity and phase angle of hunter vehicle i.

4 A Numerical Example

In this section, we carry out some computer simulations to demonstrate the performance of our controller for the cooperative hunting of the underactuated surface vehicles. An underactuated surface vehicle with model parameters are given as: $m_{11} = 1.956, m_{22} = 2.405, m_{33} = 0.043, d_{11} = 2.436, d_{22} = 12.992, d_{33} = 0.0564$. For simulation use, we define the velocity of the invader in two forms, which are a constant and sinusoidal variable. Then we define one invader and three hunters, making the following choice of the desired distance and angle between the invader and hunters are $(l_{1d}, l_{2d}, l_{3d}) = (3m, 3m, 3m)$, $(\alpha_{1d}, \alpha_{2d}, \alpha_{2d}) = (\frac{-\pi}{3}, -\pi, \frac{\pi}{3})$. Then we give the initial conditions for the cooperative hunting system: $x_1 = 1m, y_1 = 7m, \psi_1 = atan(4), u_1 = 0m/s, v_1 = 0m/s, r_1 = 0rad/s, x_2 = 5m, y_2 = 1m, \psi_2 = atan(\frac{1}{4}), u_2 = 0m/s, v_2 = 0m/s, r_2 = 0rad/s, x_3 = 2m, y_3 = 8m, \psi_3 = atan(\frac{1}{3}), u_3 = 0m/s, v_3 = 0m/s, r_3 = 0rad/s$ for the hunters. $x_p = 8m, y_p = 6m, \psi_p = atan(\frac{1}{5}), V_{px} = 0.2m/s, V_{py} = 0.1m/s$ for the line trajectory of the invader. $x_p = 8m, y_p = 6m, \psi_p = atan(\frac{1}{5}), V_{px} = 0.5m/s, V_{py} = 0.5sin(0.01t)m/s$ for the sinusoidal velocity of the invader. The control parameter of hunter i are taken as $k1 = 0.1, k_{pui} = 12, k_{iui} = 0.7, k_{dui} = 0, k_{ppi} = 0.1, k_{ipi} = 0, k_{dpi} = 5$. Simulation results are shown in the following figures, and the simulation time is set 100s.

Fig. 3-Fig. 6 show the cooperative hunting of the invader

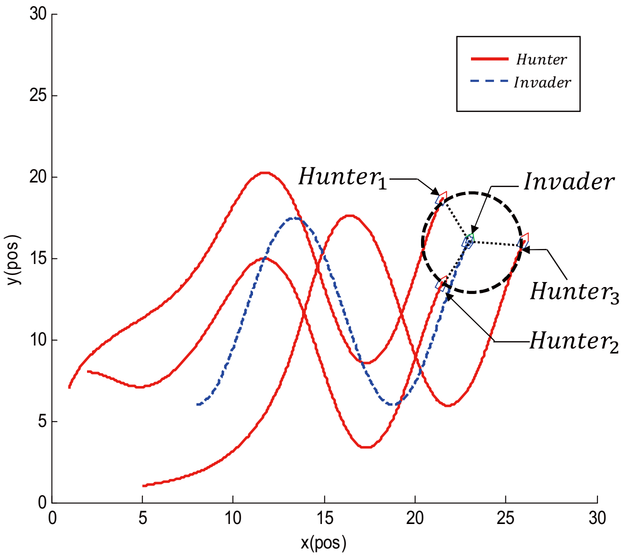

Fig. 7: Cooperative hunting of the invader with a sinusoidal variable velocity

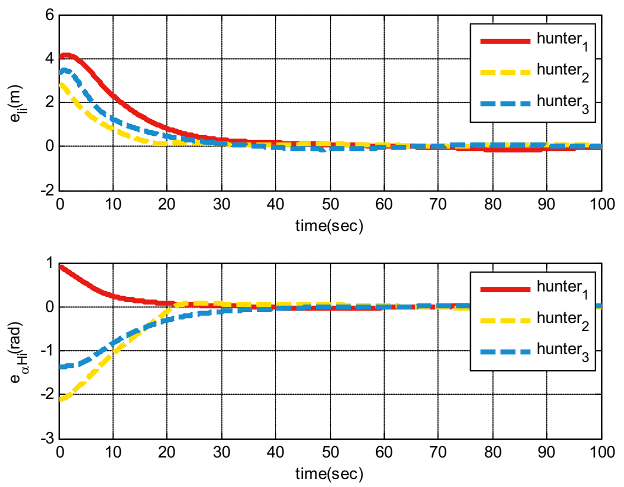

Fig. 8: Hunting error e_{li} and e_{α_i} of the invader with a sinusoidal variable velocity

with a invariant velocity. Fig. 7-Fig. 10 give the results of the cooperative hunting of the invader with sinusoidal velocity. The robustness of the controller is illustrated in Fig. 11 and Fig. 12. As we see, Fig. 3 displaces the final results of cooperative hunting based on the control law given in this paper. It is clear that three hunter vehicles successfully enclose the invader in the circle while are evenly distributed around the invader vehicle. Fig. 4 shows the error of the cooperative hunting asymptotically converges to zero in the finite time when the invader's velocity is invariant. Fig. 5 shows the phase angle between the hunters and the invader asymptotically converges to be same. The forces applied to the underactuated surface vehicle are shown in Fig. 6. From the simulation result of the invader with a constant velocity, we conclude the effectiveness of the controller to cooperatively hunt the invader.

Furthermore, when it comes to the invader with variable velocity, it become more practical in life. Fig. 7 shows the

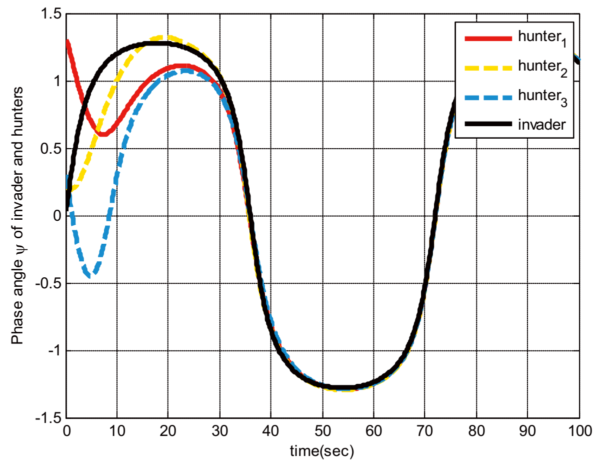

Fig. 9: Phase angle ψ_p and ψ_i of the hunters with a sinusoidal variable velocity

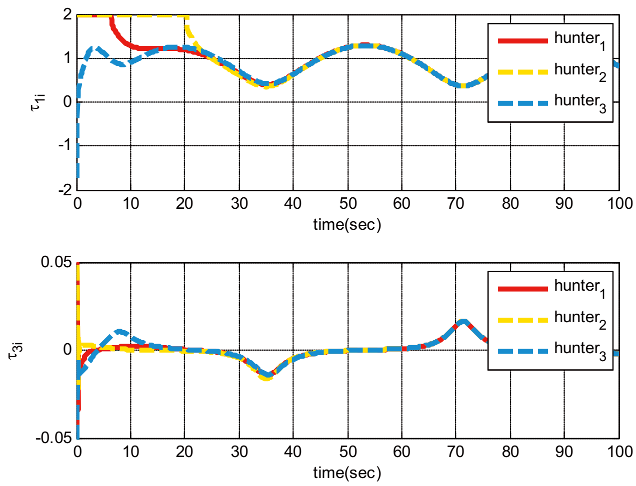

Fig. 10: Control input τ_{1i} and τ_{3i} of the hunters with a sinusoidal variable velocity

effectiveness of the controller when the invader's velocity is variable. Fig. 8-Fig. 9 show the error of the cooperative hunting asymptotically converges to zero and the phase angle between the hunters and the invader asymptotically converges to be consensus.

We put some uncertain disturbance in the u_i, v_i, r_i, ψ_i to test the robust ability of controller. The simulation results are shown in Fig. 11-12. Fig. 11 illustrates the cooperative hunting error also asymptotically converges to zero with some uncertain disturbance. However, the phase angle shown in Fig. 12 is disturbed by some disturbance, but it also gets close to the desired value. Finally, from three cases above, the controller is proven to successfully realize the cooperative hunting problem. And it is robust with some small uncertain disturbance.

5 Conclusions

In this paper, we have propose a new controller to address the cooperative hunting problem of underactuated surface vehicles, in particular, when the invader's velocity is invariant or variable with some uncertain disturbance. By giving a

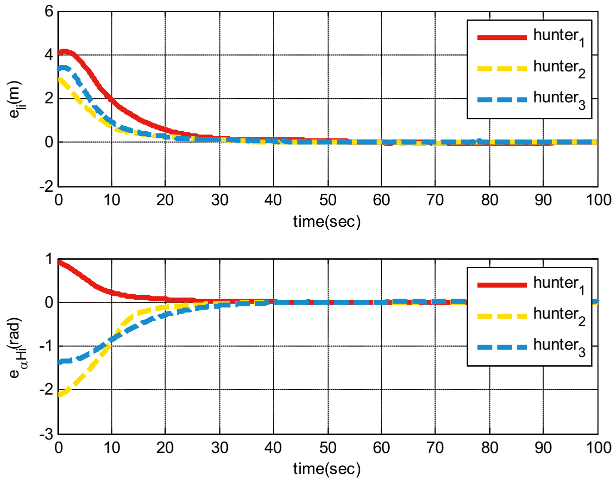

Fig. 11: Hunting error e_{li} and e_{α_i} with uncertain disturbance

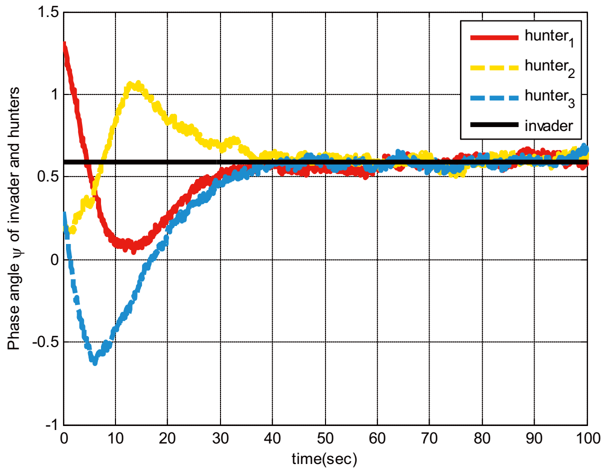

Fig. 12: Phase angle ψ_p and ψ_i of the hunters with a sinusoidal variable velocity

Lyapunov's function, we conclude the errors of the cooperative hunting behavior asymptotically converge to zero. Invader with constant velocity and sinusoidal variable velocity hunting problem are included to demonstrate the effectiveness of the suggested approach.

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