This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIE.2020.3000092, IEEE Transactions on Industrial Electronics

Multiple-Target Surrounding and Collision Avoidance with Second-Order Nonlinear Multi-Agent Systems

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***Abstract*—This paper proposes an equal-distance surrounding control method for second-order nonlinear multi-agent systems (MASs) to encircle multiple moving targets with guaranteed collision avoidance. First, a distributed estimator is developed to approximate the center of moving targets. Then, an adaptive distributed control law is designed for the MASs to accomplish equal-distance surrounding collaboratively. In particular, conditions for assuring asymptotical stability for the closed-loop MAS are derived. Finally, experimental results with unmanned surface vessels are reported to substantiate the effectiveness of the proposed coordinated surrounding control method.**

***Index Terms*—Surrounding control, collision avoidance, multi-agent systems, unmanned surface vessels.**

I. Introduction

Recent years have witnessed the tremendous growing research and development of the cooperation control of multi-agent systems (MASs) [1]–[5], in areas of biology, physics, systems science, computer science, etc. The explorations of coordination mechanisms of MASs facilitate to understand the nature of biological collective motions, and also to analyze and design scientific and engineering networked systems in numerous applications such as wireless sensor networks, smart grids, wind farm networks, and unmanned systems, in terms of increased cooperation efficiency, improved adaptability and robustness to system uncertainties.

So far, significant efforts have been devoted to analyze and design both consensus and flocking protocols of MASs [6]–[11]. In recent years, containment control of MASs, for making agents to move into a convex hull formed by targets, attracted more and more attention [12]–[14]. Furthermore, surrounding control, which drives a group of agents to move around target(s) in a convex hull, emerged as the inverse problem of containment control. Both containment and surrounding control protocols are viable for applications in areas of collaborative escort, patrolling and hunting with unmanned surface vehicles (USVs), formation control of unmanned aerial vehicles (UAVs), unmanned ground vehicles (UGVs), and satellites and spacecraft clusters, etc.

Notably, due to the challenges in stabilizing the collective behaviors of agents to encircle multiple targets, most of existing works focus on the single-target scenario; e.g., [15]–[21]. Initial efforts are focused on the single static target case. As one of the representative works, Lan *et al.* [15] developed a hybrid control approach to address a static target cyclic enclosing problem. Liu & Chen [16] designed a hierarchical control structure with output regulation to surround a specified target. Afterwards, Chen [17] considered a specific surrounding problem with guaranteed collision avoidance, namely, fencing problem. Due to the sophisticated target dynamics in real marine applications, more recent efforts have been devoted to surrounding control with a motional target. For instance, Lin & Dong [18] presented a systematic design and implementation of a robust real-time embedded vision system for the target tracking. Guo *et al.* [19] developed a distributed control method to surround a moving target merely based on relative neighboring positions. Zhang *et al.* [20] introduced a game-theory framework to track a moving target and maintain the tracking distance. Yu & Liu [21] proposed a distributed dynamic control law to form circles of unicycles with switching interconnection topologies, where the target is just accessible to one unicycle. Afterwards, Li *et al.* [22] presented a cooperative surrounding scheme with guaranteed collision avoidance for networked MASs with nonlinear Lagrangian dynamics. Peng *et al.* [23] addressed the event-triggered dynamic surface control of an under-actuated autonomous surface vehicle with unknown kinetics to enclose a dynamic target with unknown velocity.

The challenge of multiple-target surrounding control problem lies mainly in the distributive estimation of the center of multiple moving targets by the agents which may have no direct access to the target information. Among the few works on encircling multiple targets, Chen & Ren [24] considered the problem of enclosing a cluster of stationary targets and proposed a surrounding control law under an estimation-and-control framework with a fixed topology. Following this line, Shoja *et al.* [25] presented an adaptive approach to address the surrounding control problem of non-identical agents with unknown system parameters. Due to the challenges in the

This work was supported in part by the National Natural Science Foundation of China (NNSFC) under Grants 61751303, U1713203, 51729501, 61673330, 61673189, and in part by the Guangdong Innovative and Entrepreneurial Research Team Program under Grant 2014ZT05G304, and in part by the Program for Core Technology Tackling Key Problems of Dongguan City under Grant 2019622101007, and in part by the Research Grants Council of the Hong Kong Special Administrative Region, China, under Grant 11202318.
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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIE.2020.3000092, IEEE Transactions on Industrial Electronics

controller design, most of previous works only consider first-order MASs. Unfortunately, more of the practical motional systems, including unmanned systems, sensor networks, etc. have at least second-order dynamics. To be more applicable, Shi *et al.* [26] investigated the collective enclosing problem with linear second-order MASs with the assistance of Lasalle's Invariance principle [27], where the targets can be either stationary or moving. However, the existing results on multiple-targets surrounding control stay just at the simulations of linear MASs and have not reached experimentation with realistic agents, which hinders the further applications.

As a representive of second-order nonlinear systems, unmanned surface vessels (USVs) play more and more important roles in marine activities, such as aquatic resource exploration, aquatic rescues, naval attacks, water area detection, and so on. Although the collective surrounding control experiments for a single target is addressed in a few works [16], [28], no experimented surrounding control for multiple targets is yet available due perhaps to either sophisticated distributed state estimator-based controller design or challenging real-world group implementation. Moreover, collision avoidance issue further amplifies the difficulties. Nevertheless, with the abruptly increasing demand from modern marine industry, national defense, and resource exploration for MAS, it now becomes highly necessary and rewarding to propose a distributed protocol for multiple moving targets surrounding control with collision avoidance capability.

Toward this end, this paper presents a collective method for second-order nonlinear MASs to encircle multiple moving targets with guaranteed collision avoidance. The technical contribution of this paper lies in developing a quasi-equal-distance surrounding protocol for a second-order nonlinear MAS to encircle multiple moving targets with guaranteed collision avoidance based on the estimation of the center of targets. Still essentially, we have established a real multi-USV platform consisting of a USV motion capture system (MCS), a control station, and six 300mm-long self-designed 3D-printed HUSTER-0.3 vessels. From a practical point of view, this platform could act as a pilot device for practical implementations of theoretical analyses and designs of multi-USV systems.

The remainder of this paper is organized as follows: Section II introduces some preliminaries and the main problem to be addressed. Section III presents a surrounding scheme for encircling multiple targets and asymptotical stability conditions are derived. Experimental results are discussed to substantiate the efficacy of the proposed method in Section IV. Conclusions are finally drawn in Section V.

II. Problem Formulation

Throughout the paper, $\mathbb{R}$ and $\mathbb{R}^+$ denote the real number and positive real number, respectively. $\|*\|$ is the Euclidean norms of a vector $*$, $*\succ 0$ and $*\prec 0$ mean positive definitive and negative definitive matrixes $*$, respectively. $\mathbf{1_m} := [1,\ldots,1]^\intercal \in \mathbb{R}^m$, $(*)_{ij}$ represents the (i,j)-th entry of a matrix $*$. $\text{sig}(*)^\mu := \text{sgn}(*)\cdot|*|^\mu$, and $\otimes$ is the Kronecker product.

Consider an MAS consisting of n follower agents; i.e., $\mathbb{I}_v := \{\varsigma_i | i=1,2,\ldots,n\}$, where ς_i denotes the follower agent i with the following second-order dynamics,

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = f_i(v_i,t) + u_i(t), \end{cases} \tag{1}$$

where $i \in \mathbb{I}_v, x_i(t) \in \mathbb{R}^2$ and $v_i(t) \in \mathbb{R}^2$ are the position and velocity of follower agent i, respectively, $u_i(t) \in \mathbb{R}^2$ is the control input, and $f_i(v_i,t) \in \mathbb{R}^2$ is a nonlinear function in form of

$$f_i(v_i,t) = \phi_i^\intercal(v_i,t)\theta_i, \tag{2}$$

where $\phi_i(\cdot,\cdot): \mathbb{R}^2\times\mathbb{R} \to \mathbb{R}^{p\times 2}$ is a basis function of the input v_i and the temporal variable t, which is generally selected as polynomial, Jacobi, Laguerre, Kautz, trigonmetric, neural network (NN) functions and so on. In view of its superior properties; e.g., [12], [14], [29], [30], we use an echo state NN with sigmoid function $1/(1+e^{-x})$ as $\phi_i(\cdot,\cdot)$ to approximate the nonlinear term f_i. Moreover, there exist m moving targets $\mathbb{I}_t := \{\tau_j | j=n+1, n+2, \ldots, n+m\}$ with set τ_j as target j in the MAS. A target moving at variable velocity a is described as follows:

$$\dot{r}_j(t) = v_j(t), \tag{3}$$

where $r_j(t), v_j(t) \in \mathbb{R}^2$ denote the position and velocity of target j, respectively. Analogously, assume that $v_j(t)$ in Eq. (3) can be described as

$$v_j(t) = \phi_L^\intercal(t)\eta_j, \tag{4}$$

where $\phi_L(\cdot): \mathbb{R} \to \mathbb{R}^{p\times 2}$ is a nonlinear vector base function of t with constant coefficients $\eta_j \in \mathbb{R}^p, j \in \mathbb{I}_t$, as well.

For the multi-agent-multi-target MAS, we define $G(\nu,\boldsymbol{\varepsilon})$ as its underlying communication graph, where $\nu = \{1,2,\ldots,n+m\} = \mathbb{I}_v \cup \mathbb{I}_t$ denotes the vertex set and $\boldsymbol{\varepsilon} = \{(\nu_i,\nu_k)\} \subseteq \nu\times\nu$ the edge set. (ν_i,ν_k) denotes that the vertex ν_i is a neighbor of the vertex ν_k. The graph for follower set $\mathbb{I}_v$ is undirected if any two vertexes $i,k \in \mathbb{I}_v$ satisfy $(\nu_i,\nu_k) \in \boldsymbol{\varepsilon}, (\nu_k,\nu_i) \in \boldsymbol{\varepsilon}$. $\mathcal{N}_i(\mathcal{N}_i \subset \mathbb{I}_v)$ denotes the neighbors(s) of follower agent i. $B = [b_{ij}] \in \mathbb{R}^{n\times m}, i \in \mathbb{I}_v, j \in \mathbb{I}_t$, is the connection matrix for the follower agent set $\mathbb{I}_v$ and target set $\mathbb{I}_t$. Therein, $b_{ij} = 1$ if the target j is a neighbor of the follower agent i, and $b_{ij} = 0$, otherwise. The undirected graph for the follower set is connected if there exists at least a path between arbitrary two vertexes $\nu_i, \nu_k \in \mathbb{I}_v$.

For further derivations, the following assumptions are made.

Assumption 1. *The graph for the follower agent set $\mathbb{I}_v$ in $G(\nu,\boldsymbol{\varepsilon})$ is undirected and connected, and graph between the target set $\mathbb{I}_t$ and follower agent set $\mathbb{I}_v$ satisfies $\sum_{i=1}^{n} b_{ij} = 1, \sum_{j=n+1}^{n+m} b_{ij} \le 1, i \in \mathbb{I}_v, j \in \mathbb{I}_t$.*

Under Assumption 1, each target in $\mathbb{I}_t$ is available to only one unique follower agent in $\mathbb{I}_v$ and each follower agent in $\mathbb{I}_v$ communicates with at most one unique target in $\mathbb{I}_t$. Multiple targets connecting to a follower and multiple followers connecting with a target are not allowed, due to the reason given below. In the first case where all targets are available to a specified follower, the center of targets can be calculated directly by the follower, which implies that the problem of the multiple targets' center estimation falls into a simple problem

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of single-target estimation, similar to [21]. In the second case where only one of the followers have access to the one target, the problem degrades into another simple one, because other followers can retrieve the target information by the connected topology among follower agents. So, both of the above two cases are excluded from the present design.

Assumption 1 is reasonable and necessarily required, because the availability of the center state estimation is guaranteed to the followers, of which each just has access to a partial of the surrounding targets, as shown in Fig. 3. As no followers have access to all the targets, it poses a challenge to cooperatively estimate the center state of targets.

Lemma 1. *[25] Under Assumption 1,*

$$b_{ij}\sum_{k=n+1}^{n+m} b_{ik} = b_{ij}, j, k \in \mathbb{I}_t.$$

In addition to Assumption 1, the distance between each target and the center of $\mathbb{I}_t$ is assumed to be bounded, as given in the following assumption.

Assumption 2. *There exists $M \in \mathbb{R}^+$ such that*

$$M = sup\{\|r_j(t) - \bar{r}(t)\|\}, j \in \mathbb{I}_t,$$

where $\bar{r}(t) := \frac{1}{m}\sum_{j=n+1}^{n+m} r_j(t)$.

With Assumptions 1 and 2, we can give the following definitions.

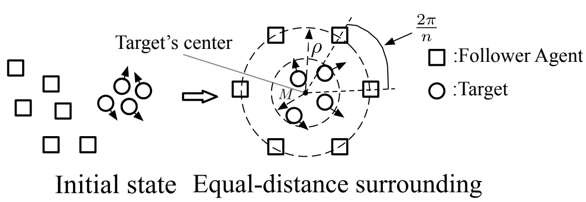

Fig. 1. An equal-distance surrounding scenario for a four-target six-agent MAS.

Definition 1. *(Equal-distance Surrounding) [16]: A set of m targets are asymptotically surrounded by n follower agents if each follower agent $i \in \mathbb{I}_v$ satisfies*

$$\lim_{t\to\infty} \|x_i(t) - \bar{r}(t) - \rho_i\| = 0, \tag{5}$$

where $\rho_i = \left[\rho\cos\frac{2\pi i}{n}, \rho\sin\frac{2\pi i}{n}\right]^\top \in \mathbb{R}^2$ *is the relative surrounding vector for follower agent $i \in \mathbb{I}_v$ with $\rho > M$ representing a constant radius.*

Definition 1 implies the follower agents space evenly around the center of targets, as illustrated in Fig. 1. Moreover, if $\rho > M$ is properly selected, the agent set $\mathbb{I}_v$ can encircle the target set $\mathbb{I}_t$, which thus solves equal-distance surrounding control problem.

Definition 2. *(Quasi-equal-distance Surrounding): A set of m targets are asymptotically surrounded by n follower agents or quasi-equal-distance surrounding is achieved, if each follower agent $i \in \mathbb{I}_v$ satisfies*

$$\lim_{t\to\infty} \|x_i(t) - \bar{r}(t) - \rho_i\| \leq \sigma, \tag{6}$$

with the threshold $\sigma > 0$.

Obviously, quasi-equal-distance surrounding in Definition 2 is weaker but more practical than equal-distance surrounding in Definition 1.

Definition 3. *(Collision avoidance) For any two follower agents $i, k \in \mathbb{I}_v$, the collision is avoided if*

$$\|x_{i,k}(t)\| > r, t \geq 0,$$

where $x_{i,k} := x_i - x_k$ and $r > 0$ is a safe distance.

The targets are assumed to move with variational velocities and to be collision-free.

Now, it is ready to introduce the main technical problems addressed in this paper.

Problem 1: Design a control law

$$u_i = f(x_i, x_k, r_j), i \in \mathbb{I}_v, k \in \mathcal{N}_i, j \in \mathbb{I}_t$$

for agent i of a multi-target-multi-agent MAS $\mathbb{I}_v \cup \mathbb{I}_t$ governed by (1) and (3) to achieve quasi-equal-distance surrounding.

Problem 2: Design a control law

$$u_i = f(x_i, x_k, r_j), i \in \mathbb{I}_v, k \in \mathcal{N}_i, j \in \mathbb{I}_t$$

to solve Problem 1 subject to guaranteed collision avoidance among the follower agents in set $\mathbb{I}_v$.

III. Surrounding Control

The present surrounding control scheme is composed of three parts: an estimator for the center of $\mathbb{I}_t$, an adaptive control law to tackle unknown dynamics f_i and moving targets, and a collision avoidance mechanism.

A. Estimation of the center of targets

Let $\widehat{x}_i(t), \widehat{v}_i(t), i \in \mathbb{I}_v$, be the estimated center position and velocity of the target set for the follower agent i, respectively, as below,

$$\begin{cases} \dot{\widehat{x}}_i(t) &= \widehat{v}_i(t), \\ \dot{\omega}_i(t) &= \beta_p \sum\limits_{j\in\mathcal{N}_i} \big(\widehat{x}_j(t) - \widehat{x}_i(t)\big), \\ \widehat{x}_i(t) &= \omega_i(t) + \widetilde{r}_i(t). \end{cases} \tag{7}$$

Here, $\omega_i(t)$ is the consensus part with initial state $\omega_i(0) = 0$, $\beta_p \in \mathbb{R}^+$ is the estimator gain, $\widetilde{r}_i(t) = \frac{n}{m}\sum_{j=n+1}^{n+m} b_{ij} r_j(t),\ i \in \mathbb{I}_v,\ j \in \mathbb{I}_t$ is derived from the target group. It follows from Eq. (7) that

$$\frac{1}{n}\sum_{i=1}^{n} \widehat{x}_i(t) = \frac{1}{n}\sum_{i=1}^{n} \widetilde{r}_i(t) = \frac{1}{m}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} b_{ij} r_j(t) = \bar{r}(t) \tag{8}$$

by virtue of Assumption 1. According to the definition of $\mathrm{sig}(\cdot) = \mathrm{sgn}(\cdot)|\cdot|$, the second term of Eq. (7) can be rewritten as $\dot{\omega}_i(t) = \beta_p \sum_{j\in\mathcal{N}_i} \mathrm{sig}\big(\widehat{x}_j(t) - \widehat{x}_i(t)\big)$, as shown in Lemma 2 given below.

Lemma 2. *[31] For the estimator governed by (7),*

$$\frac{d\|e_p\|^2}{dt} \leq \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} \{\|\widehat{x}_j - \widehat{x}_i\|((n-1)\alpha_1 - \beta_p\|\widehat{x}_j - \widehat{x}_i\|)\},$$

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where

$$\|e_p\|^2 = \sum_{i=1}^{n}(\widehat{x}_i - \bar{r})^2 \tag{9}$$

is the estimated position error and $\alpha_1 = \sup\{\|\dot{\bar{r}}_i(t)\| \; |i \in \mathbb{I}_v, t \geq 0\}$.

To estimate the center states $\bar{r}_i(t), \dot{\bar{r}}_i(t)$ of target set $\mathbb{I}_t$, we need another assumption.

Assumption 3. *There exist upper bounds of* $\|\dot{\bar{r}}_i(t)\|, \|\ddot{\bar{r}}_i(t)\|, \|\dddot{\bar{r}}_i(t)\|,\ i \in \mathbb{I}_v$ *defined in Eq. (7).*

$\|\dot{\bar{r}}_i(t)\|, \|\ddot{\bar{r}}_i(t)\|,\ \|\dddot{\bar{r}}_i(t)\|,\ i \in \mathbb{I}_v$ are available to the followers with the assistance of Assumption 1. The physical meaning of Assumption 3 is that there is no sharp brake or turn for the moving targets in practice, which corresponds to the bounded inter-distance among the targets.

Lemma 3. *Under Assumption 3, with positive real numbers* $\sigma_p,\ \sigma_a \in \mathbb{R}^+$, *the estimates of position* $\widehat{x}_i$ *and acceleration* $\dot{\widehat{v}}_i$ *approach to the center of* $\mathbb{I}_t$ *with prescribed bounds; i.e.,*

$$\sum_{i=1}^{n}(\widehat{x}_i - \bar{r})^2 \leq \sigma_p^2, \sum_{i=1}^{n}(\dot{\widehat{v}}_i - \ddot{\bar{r}})^2 \leq \sigma_a^2$$

provided that there exists an estimated gain $\beta_p > 0$ *in Eq. (7) such that*

$$\beta_p > \bar{\beta} \tag{10}$$

with $\bar{\beta} = \max\left\{2(n-1)\alpha_1, 2(n-1)\alpha_2, \frac{(n-1)^2\alpha_1}{2\sigma_p}, \frac{(n-1)^2\alpha_2}{2\sigma_a}, \sqrt[3]{\frac{(n^2-2)(n-1)^3\alpha_1^2}{4\sigma_p}}, \sqrt[3]{\frac{(n^2-2)(n-1)^3\alpha_2^2}{4\sigma_a}}\right\}$, *where* $\alpha_1 := \sup\{\|\dot{\bar{r}}_i(t)\| \; |i \in \mathbb{I}_v, t \geq 0\}$ *and* $\alpha_2 := \sup\{\|\dddot{\bar{r}}_i(t)\| \; |i \in \mathbb{I}_v, t \geq 0\}$.

Proof. Let a Lyapunov function be $V_1(t) = (\|e_p\|^2 - \sigma_p^2)^2$, whose derivative is

$$\dot{V}_1(t) = 2(\|e_p\|^2 - \sigma_p^2)\frac{d\|e_p\|^2}{dt}.$$

According to Lemma 2, one has

$$\frac{d\|e_p\|^2}{dt} \leq \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}\{\|\widehat{x}_j - \widehat{x}_i\|((n-1)\alpha_1 - \beta_p\|\widehat{x}_j - \widehat{x}_i\|)\} \tag{11}$$

with $\alpha_1 = \sup\{\|\dot{\bar{r}}_i(t)\| \; |i \in \mathbb{I}_v, t \geq 0\}$. Moreover, in accordance to Eq. (8), one has

$$\|e_p\|^2 = \sum_{i=1}^{n}(\widehat{x}_i - \bar{r})^2 = \sum_{i=1}^{n}(\widehat{x}_i - \frac{1}{n}\sum_{j=1}^{n}\widehat{x}_j)^2.$$

It can be derived that

$$\begin{aligned}\|e_p\| &= \sqrt{\sum_{i=1}^{n}(\widehat{x}_i - \tfrac{1}{n}\sum_{j=1}^{n}\widehat{x}_j)^2} \\ &\leq \sum_{i=1}^{n}\|\widehat{x}_i - \tfrac{1}{n}\sum_{j=1}^{n}\widehat{x}_j\| \\ &\leq (n-1)\max\{\|\widehat{x}_i - \widehat{x}_j\|, i,j \in \mathbb{I}_v\},\end{aligned}$$

which implies that $\max\{\|\widehat{x}_i - \widehat{x}_j\|, i,j \in \mathbb{I}_v\} > \frac{\sigma_p}{n-1}$ if $\|e_p\|^2 > \sigma_p^2$.

Furthermore, if $\frac{(n-1)\alpha_1}{2\beta_p} < \frac{\sigma_p}{n-1}$(*i.e.,* $\beta_p > \frac{(n-1)^2\alpha_1}{2\sigma_p}$), one has the following inequality

$$\begin{aligned}\frac{d\|e_p\|^2}{dt} &< (n^2-2)\frac{(n-1)^2\alpha_1^2}{4\beta_p^2} + 2\frac{\sigma_p}{n-1}((n-1)\alpha_1 - \beta_p\frac{\sigma_p}{n-1}), \\ &< -\frac{2\sigma_p}{n-1}\beta_p + \frac{(n^2-2)(n-1)^2\alpha_1^2}{4\beta_p^2} + 2\sigma_p\alpha_1\end{aligned}$$

by the virtue of Eq. (11). Note that $f_p(\beta_p) = -\frac{2\sigma_p}{n-1}\beta_p + \frac{(n^2-2)(n-1)^2\alpha_1^2}{4\beta_p^2} + 2\sigma_p\alpha_1$ is monotonically decreasing with respect to β_p, one has that

$$\begin{aligned}\frac{d\|e_p\|^2}{dt} &< -\frac{\sigma_p}{n-1}\beta_p + \frac{(n^2-2)(n-1)^2\alpha_1^2}{4\beta_p^2} - \frac{\sigma_p}{n-1}\beta_p + 2\sigma_p\alpha_1 \\ &< -\frac{2\sigma_p}{n-1}\beta_p + \frac{(n^2-2)(n-1)^2\alpha_1^2}{4\beta_p^2} + 2\sigma_p\alpha_1 \\ &< 0,\end{aligned}$$

if $\beta_p > \max\left\{2(n-1)\alpha_1, \sqrt[3]{\frac{(n^2-2)(n-1)^3\alpha_1^2}{4\sigma_p}}, \frac{(n-1)^2\alpha_1}{2\sigma_p}\right\}$ is picked. Accordingly, the derivative $\dot{V}_1(t) < 0$ if $\|e_p\|^2 > \sigma_p^2$ and $\frac{d\|e_p\|^2}{dt} < 0$, which implies that $V_1(t)$ decreases if $\|e_p\|^2 \geq \sigma_p^2$. Thereby, $\|e_p\|$ eventually converges into a set $\{e_p| \; \|e_p\|^2 \leq \sigma_p^2\}$.

Next, we will prove the boundness of $\sum_{i=1}^{n}(\dot{\widehat{v}}_i - \ddot{\bar{r}})^2$. Firstly, the dynamic of the estimated acceleration of the center of the targets is

$$\dot{\widehat{v}}_i(t) = \ddot{\omega}_i(t) + \ddot{\bar{r}}_i(t), i \in \mathbb{I}_v,$$

with $\ddot{\omega}_i(t) = \beta_p\sum_{j\in\mathcal{N}_i}(\widehat{v}_j - \widehat{v}_i)$. Note that

$$\|e_a\|^2 = \sum_{i=1}^{n}(\dot{\widehat{v}}_i - \frac{1}{n}\sum_{j=1}^{n}\dot{\widehat{v}}_j)^2,$$

one thus has $\|e_a\|^2 = \sum_{i=1}^{n}(\dot{\widehat{v}}_i - \ddot{\bar{r}})^2$ according to Eq. (8).

Then, let a Lyapunov function be $V_2(t) = (\|e_a\|^2 - \sigma_a^2)^2$ with $\sigma_a > 0$, one has

$$\dot{V}_2(t) = 2(\|e_a\|^2 - \sigma_a^2)\frac{d\|e_a\|^2}{dt},$$

and

$$\frac{d\|e_a\|^2}{dt} < \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}\{|\dot{\widehat{v}}_j - \dot{\widehat{v}}_i|((n-1)\alpha_2 - \beta_p|\dot{\widehat{v}}_j - \dot{\widehat{v}}_i|)\}$$

with $\alpha_2 = \sup\{|\dddot{\bar{r}}_i(t)| \; |i \in \mathbb{I}_v, t \geq 0\}$. If $\|e_a\|^2 > \sigma_a^2$, the following inequality holds $\frac{\sigma_a}{n-1} < \max\{|\dot{\widehat{v}}_i - \dot{\widehat{v}}_j||i,j \in \mathbb{I}_v\}$. Furthermore, if $\frac{(n-1)\alpha_2}{2\beta_p} < \frac{\sigma_a}{n-1}$ (i.e.,$\beta_p > \frac{(n-1)^2\alpha_2}{2\sigma_a}$), one has that

$$\begin{aligned}\frac{d\|e_a\|^2}{dt} &< -\frac{2\sigma_a}{n-1}\beta_p + \frac{(n^2-2)(n-1)^2\alpha_2^2}{4\beta_p^2} + 2\sigma_a\alpha_2 \\ &< 0,\end{aligned}$$

if $\beta_p > \max\left\{2(n-1)\alpha_2, \sqrt[3]{\frac{(n^2-2)(n-1)^3\alpha_2^2}{4\sigma_a}}, \frac{(n-1)^2\alpha_2}{2\sigma_a}\right\}$ is picked. It thus follows that $\dot{V}_2(t) < 0$ when $\|e_a\|^2 > \sigma_a^2$ and $\frac{d\|e_a\|^2}{dt} < 0$. Therefore, $\|e_p\|, \|e_a\|$ will eventually converge into the set $\{e_p, e_a| \; \|e_a\| \leq \sigma_a, \|e_p\| \leq \sigma_p\}$, if $\beta_p > \bar{\beta}$ is picked for $\bar{\beta} := \max\left\{2(n-1)\alpha_1, 2(n-1)\alpha_2, \frac{(n-1)^2\alpha_1}{2\sigma_p}, \frac{(n-1)^2\alpha_2}{2\sigma_a}, \sqrt[3]{\frac{(n^2-2)(n-1)^3\alpha_1^2}{4\sigma_p}}, \sqrt[3]{\frac{(n^2-2)(n-1)^3\alpha_2^2}{4\sigma_a}}\right\}$. The proof is thus completed. □

Remark 1. *The estimator gain β_p in Eq. (7) determines the convergence speed of the estimation errors e_p, e_a, which is to eliminate the influence of the upper bound of target dynamic variations if a sufficiently large estimator gain β_p is used. Moreover, $\bar{\beta}$ reflects the upper bound of target dynamic variation in practice.*

B. Equal-distance surrounding

We are now ready to propose a distributed surrounding control law to tackle Problem 1.

Before presenting the detailed controller, the following assumption concerning $\theta_i,\ \eta_j, i \in \mathbb{I}_v,\ j \in \mathbb{I}_t$, in Eqs. (2), (4) is given.

Assumption 4. *There exist $\theta_M,\ \eta_M \in \mathbb{R}^+$ such that for any follower agent $i \in \mathbb{I}_v,\ j \in \mathbb{I}_t$, $\|\theta_i\| \leq \theta_M,\ \|\eta_j\| \leq \eta_M$.*

The rationality of Assumption 4 lies in that the nonlinear function $f_i(v_i, t)$ and the velocities of targets are bounded in practice, which implies the existence of $\theta_M,\ \eta_M$. The upper bounds of the uncertain parameters in Assumption 4 form an essential part of the surrounding errors.

Let $\widehat{\theta}_i,\ \widehat{\eta}^i_j$ be the estimates of parameters θ_i, η_j for follower agent i, respectively. The estimates of $f_i(v_i, t)$ and $v_j(t)$ are hence expressed respectively as follows:

$$\widehat{f}_i(v_i, t) = \phi_i^{\mathsf{T}}(v_i, t)\widehat{\theta}_i, i \in \mathbb{I}_v, \tag{12}$$

and

$$\widehat{v}^i_j(t) = \phi_L^{\mathsf{T}}(t)\widehat{\eta}^i_j(t), i \in \mathbb{I}_v, j \in \mathbb{I}_t. \tag{13}$$

Before proposing the control protocol, let's define the equal-distance surrounding error $e_i(t)$ for the ith follower agent with the assistance of the estimator (7) as follows:

$$e_i(t) := x_i(t) - \widehat{x}_i(t) - \rho_i, \tag{14}$$

where $i \in \mathbb{I}_v$ and ρ_i is given in Definition 1. Accordingly,

$$\begin{cases} \dot{e}_i &= \delta_i, \\ \dot{\delta}_i &= u_i + \phi_i^{\mathsf{T}}\theta_i - \dot{\widehat{v}}_i, \end{cases} \tag{15}$$

where $\delta_i := v_i - \widehat{v}_i$ is the velocity error between the follower agent i and the estimated center $\widehat{v}_i$ of $\mathbb{I}_t$.

It immediately follows from Eq. (15) that

$$\begin{cases} \dot{e}_i &= -k_{i1}e_i + z_i, \\ \dot{z}_i &= u_i + \phi_i^{\mathsf{T}}\theta_i - \dot{\widehat{v}}_i + k_{i1}z_i - k_{i1}^2 e_i, \end{cases} \tag{16}$$

where $z_i := \delta_i + k_{i1}e_i$. An adaptive control law or protocol is thus proposed as below,

$$\begin{aligned} u_i &= k_{i1}^2 e_i - k_{i2}z_i - \phi_i^{\mathsf{T}}\widehat{\theta}_i + \sum_{j=n+1}^{n+m} b_{ij}\dot{\phi}_L^{\mathsf{T}}(t)\widehat{\eta}^i_j, \\ \dot{\widehat{\theta}}_i &= \phi_i z_i - k_{i3}\widehat{\theta}_i, \\ \dot{\widehat{\eta}}^i_j &= -\dot{\phi}_L z_i - k_{i4}\sum_{k=n+1}^{n+m} b_{ik}\widehat{\eta}^i_k, \end{aligned} \tag{17}$$

where $k_{i1}, k_{i2}, k_{i3}, k_{i4} \in \mathbb{R}^+,\ i \in \mathbb{I}_v$ are the control gains of agent i to be designed afterwards.

Theorem 1. *Under Assumptions 1-4, the closed-looped MAS with dynamics (1), (3), (16) and control law (17) is able to achieve quasi-equal-distance surrounding.*

Proof. Let $\widetilde{\theta}_i, \widetilde{\eta}^i_j$ be the parameter errors as

$$\widetilde{\theta}_i := \widehat{\theta}_i - \theta_i,\ \widetilde{\eta}^i_j := \sum_{k=n+1}^{n+m} b_{ik}\widehat{\eta}^i_k - \eta_j \tag{18}$$

with $i \in \mathbb{I}_v, j, k \in \mathbb{I}_t$. The first part of (18) implies that $\dot{\widetilde{\theta}}_i = \dot{\widehat{\theta}}_i$. By Assumption 1, it is obtained that $b_{ij}b_{ik} = b_{ij}$, if $j = k$, and $b_{ij}b_{ik} = 0$, otherwise, with $k, j \in \mathbb{I}_t$. Then, the derivative of $\widetilde{\eta}^i_j$ in (18) becomes

$$\dot{\widetilde{\eta}}^i_j = \sum_{k=n+1}^{n+m} b_{ik}\dot{\widehat{\eta}}^i_k, i \in \mathbb{I}_v,\ j, k \in \mathbb{I}_t. \tag{19}$$

Consider Lemma 1, one has

$$b_{ij}\dot{\widetilde{\eta}}^i_j = b_{ij}\sum_{k=n+1}^{n+m} b_{ik}\dot{\widehat{\eta}}^i_k = b_{ij}\dot{\widehat{\eta}}^i_j. \tag{20}$$

Substituting (17) and (20) into (18) yields

$$\begin{aligned} \dot{\widetilde{\theta}}_i &= \phi_i z_i - k_{i3}\widehat{\theta}_i, \\ \dot{\widetilde{\eta}}^i_j &= -\dot{\phi}_L z_i - k_{i4}\sum_{k=n+1}^{n+m} b_{ik}\widehat{\eta}^i_k. \end{aligned} \tag{21}$$

Let a Lyapunov function be

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{22}$$

where $V_1(t) := \frac{1}{2}\sum_{i=1}^{n}\{e_i^{\mathsf{T}}e_i + z_i^{\mathsf{T}}z_i\}, V_2(t) := \frac{1}{2}\sum_{i=1}^{n}\widetilde{\theta}_i^{\mathsf{T}}\widetilde{\theta}_i$, $V_3(t) := \frac{1}{2m}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m}(\widetilde{\eta}^i_j)^{\mathsf{T}}\widetilde{\eta}^i_j$. It thus follows from Eq. (18) that

$$\begin{aligned} 2\widehat{\theta}_i^{\mathsf{T}}\widetilde{\theta}_i &\geq \widetilde{\theta}_i^{\mathsf{T}}\widetilde{\theta}_i - \theta_i^{\mathsf{T}}\theta_i, \\ 2\sum_{k=n+1}^{n+m} b_{ik}(\widehat{\eta}^i_k)^{\mathsf{T}}\widetilde{\eta}^i_j &\geq (\widetilde{\eta}^i_j)^{\mathsf{T}}\widetilde{\eta}^i_j - {\eta_j}^{\mathsf{T}}\eta_j. \end{aligned}$$

Accordingly, the derivative of $V_i(t), i = 1, 2, 3$ fulfill

$$\begin{aligned} \dot{V}_1(t) \leq& \sum_{i=1}^{n}\{-(k_{i1} - \tfrac{1}{2})e_i^{\mathsf{T}}e_i + \tfrac{1}{2}z_i^{\mathsf{T}}z_i - (k_{i2} - k_{i1})z_i^{\mathsf{T}}z_i \\ &+ z_i^{\mathsf{T}}(-\phi_i^{\mathsf{T}}\widetilde{\theta}_i + \sum_{j=n+1}^{n+m} b_{ij}\dot{\phi}_L^{\mathsf{T}}\widehat{\eta}^i_j - \dot{\widehat{v}}_i\}, \\ \dot{V}_2(t) \leq& \sum_{i=1}^{n}\{z_i^{\mathsf{T}}\phi_i^{\mathsf{T}}\widetilde{\theta}_i - \tfrac{k_{i3}}{2}\widetilde{\theta}_i^{\mathsf{T}}\widetilde{\theta}_i + \tfrac{k_{i3}}{2}\theta_i^{\mathsf{T}}\theta_i\}, \\ \dot{V}_3(t) \leq& -\sum_{i=1}^{n} z_i^{\mathsf{T}}\dot{\phi}_L^{\mathsf{T}}\sum_{k=n+1}^{n+m} b_{ik}\widehat{\eta}^i_k + \tfrac{1}{m}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} z_i^{\mathsf{T}}\dot{\phi}_L^{\mathsf{T}}\eta_j \\ &- \tfrac{k_{i4}}{2m}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m}(\widetilde{\eta}^i_j)^{\mathsf{T}}\widetilde{\eta}^i_j + \tfrac{k_{i4}}{2m}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m}\eta_j^{\mathsf{T}}\eta_j, \end{aligned}$$

which thus implies that

$$\begin{aligned} \dot{V}(t) \leq& \sum_{i=1}^{n}\{-(k_{i1} - \tfrac{1}{2})e_i^{\mathsf{T}}e_i - (k_{i2} - k_{i1} - 1)z_i^{\mathsf{T}}z_i \\ &- \tfrac{k_{i3}}{2}\widetilde{\theta}_i^{\mathsf{T}}\widetilde{\theta}_i - \tfrac{k_{i4}}{2m}\sum_{j=n+1}^{n+m}(\widetilde{\eta}^i_j)^{\mathsf{T}}\widetilde{\eta}^i_j\} + \sum_{i=1}^{n}\{\tfrac{k_{i3}}{2}\theta_i^{\mathsf{T}}\theta_i \\ &+ \tfrac{k_{i4}}{2m}\sum_{j=n+1}^{n+m}\eta_j^{\mathsf{T}}\eta_j + \tfrac{1}{2}\kappa_i^{\mathsf{T}}\kappa_i\}, \end{aligned} \tag{23}$$

where $\kappa_i = \frac{1}{m}\sum_{j=n+1}^{n+m}\dot{\phi}_L^{\mathsf{T}}\eta_j - \dot{\widehat{v}}_i$.

Furthermore, it follows from Eqs. (7) that $\sum_{i=1}^{n}\beta_p\sum_{j\in N_i}\big(\widehat{x}_j(t) - \widehat{x}_i(t)\big) = 0$. Considering Eq. (8), one has $\frac{1}{n}\sum_{i=1}^{n}\widehat{v}_i = \frac{1}{m}\sum_{j=n+1}^{n+m}\phi_L^{\mathsf{T}}\eta_j$, which implies that

$\frac{1}{n}\sum_{i=1}^{n}\dot{\hat{v}}_i = \frac{1}{m}\sum_{j=n+1}^{n+m}\dot{\phi}_L^{\intercal}\eta_j$. Therefore, it follows from Lemma 3 that

$$\sum_{i=1}^{n}(\frac{1}{m}\sum_{j=n+1}^{n+m}\dot{\phi}_L^{\intercal}\eta_j-\dot{\hat{v}}_i)^2=\sum_{i=1}^{n}(\frac{1}{n}\sum_{j=1}^{n}\dot{\hat{v}}_j-\dot{\hat{v}}_i)^2\le\sigma_a^2,$$

which implies that $\sum_{i=1}^{n}\frac{1}{2}\kappa_i^{\intercal}\kappa_i\le\frac{\sigma_a^2}{2}$.

Then, Eq. (23) directly leads to that $\dot{V}(t)\le-\sum_{i=1}^{n}\{(k_{i1}-\frac{1}{2})e_i^{\intercal}e_i+(k_{i2}-k_{i1}-1)z_i^{\intercal}z_i+\frac{k_{i3}}{2}\widetilde{\theta}_i^{\intercal}\widetilde{\theta}_i+\frac{k_{i4}}{2m}\sum_{j=n+1}^{n+m}(\widetilde{\eta}_j^i)^{\intercal}\widetilde{\eta}_j^i\}+\epsilon_*$, where $\epsilon_*=\sum_{i=1}^{n}\left\{\frac{k_{i3}}{2}\theta_i^{\intercal}\theta_i+\frac{k_{i4}}{2m}\sum_{j=n+1}^{n+m}\eta_j^{\intercal}\eta_j\right\}+\frac{\sigma_a^2}{2}$.

It is clear that $\epsilon_*\le\epsilon_M=\sum_{i=1}^{n}\left\{\frac{k_{i3}}{2}\theta_M^2+\frac{k_{i4}}{2m}\sum_{j=n+1}^{n+m}\eta_M^2\right\}+\frac{\sigma_a^2}{2}$ according to Assumption 4. Let

$$\begin{array}{l} m_{i1}:=k_{i1}-\frac{1}{2}>0, m_{i2}:=k_{i2}-k_{i1}-1>0,\\ m_{i3}:=\frac{k_{i3}}{2}>0, \qquad m_{i4}:=\frac{k_{i4}}{2m}>0, \end{array} \tag{24}$$

and define γ as

$$\gamma=[e^{\intercal},z^{\intercal},\widetilde{\theta}^{\intercal},\widetilde{\eta}^{\intercal}]^{\intercal}, \tag{25}$$

where $e:=[e_1^{\intercal},e_2^{\intercal},\ldots,e_n^{\intercal}]^{\intercal}$, $z:=[z_1^{\intercal},z_2^{\intercal},\ldots,z_n^{\intercal}]^{\intercal}$, $\widetilde{\theta}:=[\widetilde{\theta}_1^{\intercal},\widetilde{\theta}_2^{\intercal},\ldots,\widetilde{\theta}_n^{\intercal}]^{\intercal}$, $\widetilde{\eta}:=[(\widetilde{\eta}_1^1)^{\intercal},(\widetilde{\eta}_1^2)^{\intercal},\ldots,(\widetilde{\eta}_1^n)^{\intercal},\ldots,(\widetilde{\eta}_m^1)^{\intercal},(\widetilde{\eta}_m^2)^{\intercal},\ldots,(\widetilde{\eta}_m^n)^{\intercal}]^{\intercal}$. It is derived that

$$\dot{V}(t)\quad\le\quad-\gamma^{\intercal}Q\gamma+\epsilon_M,$$

where $Q:=\mathrm{diag}\{m_{11},m_{21},\ldots,m_{n1},m_{12},m_{22},\ldots,m_{n2},m_{13},m_{32},\ldots,m_{n3},m_{14}\otimes 1_{\mathbf{m}}^{\intercal},m_{24}\otimes 1_{\mathbf{m}}^{\intercal},\ldots,m_{n4}\otimes 1_{\mathbf{m}}^{\intercal}\}\succ 0$. Accordingly, $\gamma^{\intercal}Q\gamma\ge\lambda_{\min}(Q)\|\gamma\|^2$ with $\lambda_{\min}(Q)>0$. Then, it is obtained that

$$\dot{V}(t)\quad\le\quad-\lambda_{\min}(Q)\|\gamma\|^2+\epsilon_M.$$

From Eq. (22), one has that $V(t)=\gamma^{\intercal}P\gamma$, where $P=\mathrm{diag}\Bigg\{\underbrace{\frac{1}{2},\frac{1}{2},\ldots,\frac{1}{2}}_{3n},\underbrace{\frac{1}{2m},\frac{1}{2m},\ldots,\frac{1}{2m}}_{n\times m}\Bigg\}\succ 0$ with $\lambda_{\max}(P)=\frac{1}{2}$ and $\lambda_{\min}(P)=\frac{1}{2m}$. It immediately leads to

$$\begin{array}{rcl}\dot{V}(t) & \le & -\lambda_{\min}(Q)\|\gamma\|^2+\epsilon_M,\\ & \le & -2\lambda_{\min}(Q)V(t)+\epsilon_M.\end{array}$$

A direct calculation gives

$$\begin{array}{rcl}V(t) & \le & V(0)e^{-\int_0^{\intercal}\xi ds}+e^{-\int_0^{\intercal}\xi ds}\int_0^t\epsilon_M e^{\xi s}ds,\\ & \le & V(0)e^{-\xi t}+\frac{\epsilon_M}{\xi}(1-e^{-\xi t}),\end{array}$$

where $\xi=2\lambda_{\min}(Q)$. In accordance to the definition of $V(t)$ in Eq. (22), one has

$$\lim_{t\to\infty}\|\gamma\|\le\lim_{t\to\infty}\sqrt{\frac{V(t)}{\lambda_{\min}(P)}}\le\sqrt{\frac{m\epsilon_M}{\lambda_{\min}(Q)}},$$

which implies that $e_i,\ z_i,\ \theta_i,\ \eta_j^i, i\in\mathbb{I}_v, j\in\mathbb{I}_t$, are all bounded by γ defined in Eq. (25).

In view of the boundness of estimated position error $\|e_p\|$ by Lemma 3, the proof is thus completed. □

Remark 2. *The equal-distance surrounding error $e_i(t)(i\in\mathbb{I}_v)$ in Eq. (14) can be reduced with an adjustable upper bound $\sqrt{\frac{m\epsilon_M}{\lambda_{\min}(Q)}}$, which is tuned by the control gains $k_{i1},k_{i2},k_{i3},k_{i4}$. Indeed, the bound satisfies Eq. (24) as well. Analogously, the estimation errors of $\widetilde{\theta}_i, i\in\mathbb{I}_v$, and $\widetilde{\eta}_j, j\in\mathbb{I}_t$, can be regulated as well, which suppresses the bound ϵ_M. The last term $\sum_{j=n+1}^{n+m}b_{ij}\dot{\phi}_L^{\intercal}(t)\widehat{\eta}_j^i$ in control law (17) aims to estimate acceleration error κ_i (i.e. $\frac{1}{m}\sum_{j=n+1}^{n+m}\dot{\phi}_L^{\intercal}\eta_j-\dot{\hat{v}}_i$) in Eq. (21) together with the adaption law $\dot{\widehat{\eta}}_j^i=-\dot{\phi}_L z_i-k_{i4}\sum_{k=n+1}^{n+m}b_{ik}\hat{\eta}_k^i$ in Eq. (17), which then forms an essential part of the bound $\sqrt{\frac{m\epsilon_M}{\lambda_{\min}(Q)}}$ of surrounding errors. Additionally, the control law (17) could be slighted modified to address more general formation control problem. For instance, by replacing the equal relative vector ρ_i in Definition 1 with specified-distance vector in advance; i.e., tuning the relative position error term e_i in (17), the proposed equal-distance surrounding method can be aligned to the pre specified group formation pattern [32].*

The procedure of the multi-target equal-distance surrounding control scheme is summarized in Algorithm 1.

Algorithm 1 Equal-distance surrounding control.

Step 1 Calculate the estimator $\widehat{x}_i,\widehat{v}_i$ in Eq. (7) for follower agent $i, i\in\mathbb{I}_v$, under Assumption 1;

Step 2 Derive the error closed-loop system of e_i,δ_i with the assistance of $\widehat{x}_i,\widehat{v}_i$;

Step 3 Design an adaptive law $u_i,\widehat{\theta}_i,\widehat{\eta}_j^i$ for follower agent $i\in\mathbb{I}_v$ with e_i,δ_i and the available target information by matrix B.

C. Equal-distance surrounding with collision avoidance

Next, we address Problem 2. Let a potential function $U_{i,k}$ between two follower agents i and k be defined as follows:

$$U_{i,k}:=\left\{\begin{array}{ll}0, & \|x_{i,k}\|>R,\\ \left(\frac{\|x_{i,k}\|^2-R^2}{\|x_{i,k}\|^2-r^2}\right)^2, & r<\|x_{i,k}\|\le R,\end{array}\right. \tag{26}$$

where R denotes the radius of detection region and r the safe radius in Definition 3. Evidently, $U_{i,k}$ approaches ∞ as $\|x_i-x_j\|\to r$. Taking the partial derivative of $U_{i,k}$ with respect to x_i, we have

$$\frac{\partial U_{i,k}}{\partial x_i}=\left\{\begin{array}{ll}0 & ,\ \|x_{i,k}\|>R,\\ x_{i,k}^{\intercal}\frac{4(R^2-r^2)(\|x_{i,k}\|^2-R^2)}{(\|x_{i,k}\|^2-r^2)^3} & ,\ r<\|x_{i,k}\|\le R.\end{array}\right. \tag{27}$$

Note that $\lim_{\|x_{i,k}\|\to R^-}U_{i,k}=\lim_{\|x_{i,k}\|\to R^+}U_{i,k}=0$, and $\lim_{\|x_{i,k}\|\to R^-}\frac{\partial U_{i,k}}{\partial x_i}=\lim_{\|x_{i,k}\|\to R^+}\frac{\partial U_{i,k}}{\partial x_i}=0$ with R^-, R^+ representing the left and right limits of R, respectively. Apparently, both $U_{i,k}$ and $\frac{\partial U_{i,k}}{\partial x_i}$ are continuous for $\|x_{i,k}\|>r$. It follows from Eq. (27) that

$$\frac{\partial U_{i,k}}{\partial x_i}=-\frac{\partial U_{i,k}}{\partial x_k}=\frac{\partial U_{k,i}}{\partial x_i}=-\frac{\partial U_{k,i}}{\partial x_k}.$$

Analogously, an adaptive surrounding control law with collision avoidance is designed as,

$$\begin{array}{rcl}u_i & = & -k_{i1}e_i-k_{i2}\delta_i-\phi_i^{\intercal}\hat{\theta}_i+\dot{\hat{v}}_i-k_{i3}\sum_{k=1}^{n}\frac{\partial U_{i,k}}{\partial x_k},\\ \dot{\hat{\theta}}_i & = & \phi_i\delta_i,\end{array} \tag{28}$$

with $k_{i1},\ k_{i2},\ k_{i3}\in\mathbb{R}^+,\ i\in\mathbb{I}_v$, being the control gains of follower agent i, e_i and δ_i given in Eq. (15), respectively,

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIE.2020.3000092, IEEE Transactions on Industrial Electronics

and the last term of first line of (28) accounts for collision avoidance.

To proceed forward, another assumption is given below.

Assumption 5. *Initially, each pair of follower agents* i, k *satisfies* $\|x_{i,k}(0)\| > r, \forall i, k \in \mathbb{I}_v$.

Assumption 5 is necessary and reasonable for the collision avoidance capability, which requires the followers start from an initial collision-free formation.

We are now ready to propose the main technical result.

Theorem 2. *Under Assumptions 1-3,5, the closed-looped MAS with dynamics (1), (3), (15), (27) and (28) is able to achieve quasi-equal-distance surrounding with guaranteed collision avoidance.*

Proof. (i) (Collision avoidance)

We will prove collision avoidance by contradiction. Let a Lyapunov function be,

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{29}$$

where $V_1(t) = \frac{1}{2}\sum_{i=1}^{n}\{k_{i1}e_i^\intercal e_i + \delta_i^\intercal \delta_i\}, V_2(t) = \frac{1}{2}\sum_{i=1}^{n}\widetilde{\theta}_i^\intercal \widetilde{\theta}_i$, $V_3(t) = \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n} k_{i3}U_{i,k}$. It follows from Eq. (27) that

$$\sum_{i=1}^{n}\sum_{k=1}^{n}\hat{v}_i^\intercal \frac{\partial U_{i,k}}{\partial x_i} = \hat{v}_i^\intercal \sum_{i=1}^{n}\sum_{k=1}^{n}\frac{\partial U_{i,k}}{\partial x_i} = 0, \tag{30}$$

and

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^{n}\{-k_{i1}\delta_i^\intercal e_i - k_{i2}\delta_i^\intercal \delta_i + \delta_i^\intercal(-\phi_i^\intercal \widetilde{\theta}_i - k_{i5} \\
&\quad \times \sum_{k=1}^{n}\frac{\partial U_{i,k}}{\partial x_k})\}, \\
\dot{V}_2(t) &= \sum_{i=1}^{n}(\phi_i \delta_i)^\intercal \widetilde{\theta}_i, \\
\dot{V}_3(t) &= \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n} k_{i3}\big(\dot{x}_i^\intercal \frac{\partial U_{i,k}}{\partial x_i} + \dot{x}_k^\intercal \frac{\partial U_{i,k}}{\partial x_k}\big), \\
&= \delta_i^\intercal \sum_{i=1}^{n}\sum_{k=1}^{n} k_{i3}\frac{\partial U_{i,k}}{\partial x_i}.
\end{aligned}$$

Accordingly, it follows from Eq. (29) that

$$\dot{V}(t) \;=\; -\sum_{i=1}^{n} k_{i2}\delta_i^\intercal \delta_i \le 0,$$

which implies that $V(t)$ is non-increasing and bounded $\forall t \in [0, \infty)$. Furthermore, one has

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n} k_{i3}U_{i,k} \le V(t),$$

which implies that $U_{i,k}, i, k \in \mathbb{I}_v$ is bounded. It contradicts $\lim_{\|x_{i,k}(t)\| \to r^+} U_{i,k} = \infty$ with r^+ the right limit of r if any collision occurs or $\|x_{i,k}\| = r$. Hence, $\|x_{i,k}\| > r$, and inter-agent collisions in $\mathbb{I}_v$ are avoided.

(ii) (Quasi-equal-distance surrounding)

$V(t)$ is proved uniformly bounded in (i), which implies that $e_i, \delta_i, \widetilde{\theta}_i, U_{i,k}, i \in \mathbb{I}_v$, are all uniformly bounded. The surrounding position error $e_i(t)$ is bounded as well. According to Lemma 3, the estimated position error $\|e_p\|$ of targets' center is also bounded, which solves Problem 2. The proof is thus completed. □

Remark 3. *The equal-distance surrounding problem can be solved as well with the availability of the acceleration* $\dot{\hat{v}}_i(t)$ *of the estimated center (7) with a proof similar to Theorem 2.*

Analogously, the multi-target equal-distance surrounding control scheme with guaranteed collision avoidance is summarized in Algorithm 2.

Algorithm 2 Equal-distance surrounding control with guaranteed collision avoidance.

Step 1 Calculate the estimator $\widehat{x}_i, \widehat{v}_i$ in Eq. (7) for follower agent $i, i \in \mathbb{I}_v$, under Assumption 1;
Step 2 Derive the error closed-loop system of e_i, δ_i with the assistance of $\widehat{x}_i, \widehat{v}_i$;
Step 3 Calculate $\sum_{k=1}^{n}\frac{\partial U_{i,k}}{\partial x_k}$ with Eq. (27) to account for collision avlidance;
Step 4 Design an adaptive law $u_i, \widehat{\theta}_i$, for follower agent $i \in \mathbb{I}_v$ with e_i, δ_i and the estimated center $\hat{\dot{v}}_i$ of multiple targets.

IV. Experiment Results

In this section, the experiment results for quasi-equal-distance surrounding with multiple moving targets and guaranteed collision avoidance based on an indoor multi-USV system are elaborated.

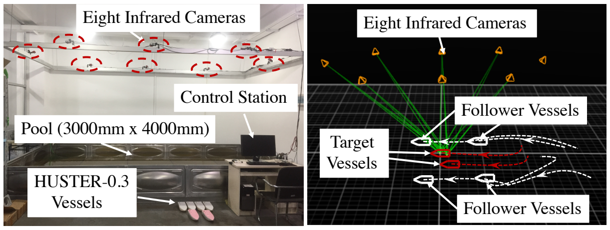
A. A Multi-USV experiments platform

As shown in Fig. 2 (a), our indoor multi-USV system platform consists of an MCS, a control station, six 300mm-long 3D-printed HUSTER-0.3 vessels and a 3000mm×4000mm pool. The MCS consists of eight infrared cameras and an optical motion capture software, which is to establish a spatial Cartesian coordinate on the pool and derive the relative positions of vessels with an accuracy of ±3mm. Each vessel is equipped with two DC motors, speed encoders, an onboard wireless device, infrared emitters, indicators and a LI-PO battery, which is managed by a micro controller unit (MCU).

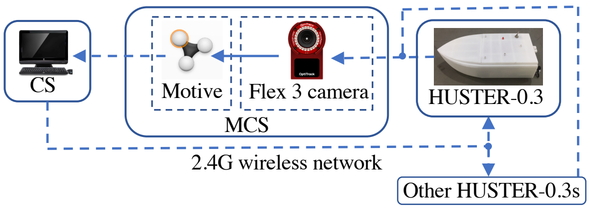
As shown in Fig. 2 (b), with an embedded infrared emitters on the vessel and the USV motion capture system, the relative position and velocity of each vessel can be measured via the control station and transmitted to each HUSTER-0.3 vessel, which communicates with other neighboring vessels via a wireless network at 100Hz. With the above information, each vessels generates its control signal to fulfill the quasi-equal-distance surrounding behavior. Moreover, the status trajectories and the controller parameters are all recorded by the control station for performance analysis. It is worth mentioning that the control law is distributed as each vessel communicates with their neighbors only.

B. Setup and results

This subsection reports surrounding control experimental results via the multi-USV system. USV dynamic control design has been widely studied in recent years [14], [16], [33]. A practical pioneering work is to simplify the USV dynamic to

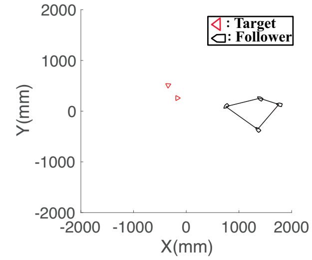


(a) Components of the indoor multi-USV system

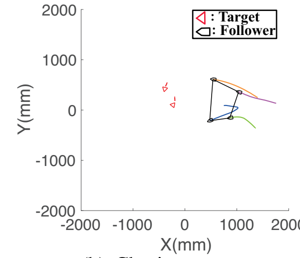


(b) The operating procedure of the indoor multi-USV system.

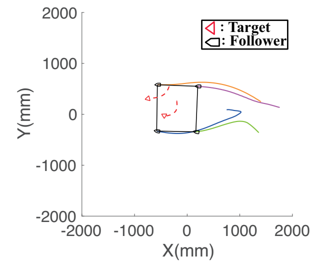
Fig. 2. An indoor multi-USV system for quasi-equal-distance surrounding experiments.



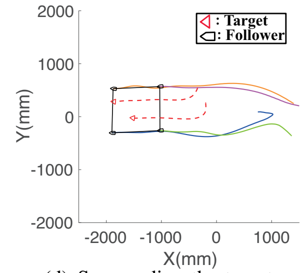
(a) Initial positions



(b) Chasing stages



(c) Catching up stage



(d) Surrounding the targets

Fig. 4. A quasi-equal-distance surrounding process of motional target USVs with the follower USVs, where the colored lines are the moving trajectories of the four vessels, while the red dashed lines are two targets.

a second-order model and then establish a two-level controller (see e.g. [16]). By this means, the control law (28) with guaranteed collision avoidance can be implemented and verified by our established 3D-printed HUSTER-0.3 vessels in practice.

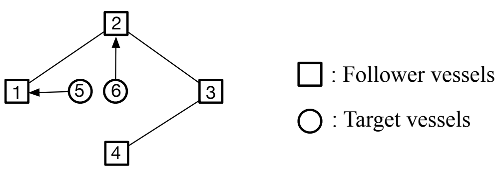


Fig. 3. The topology of the multi-agent-multi-target USVs

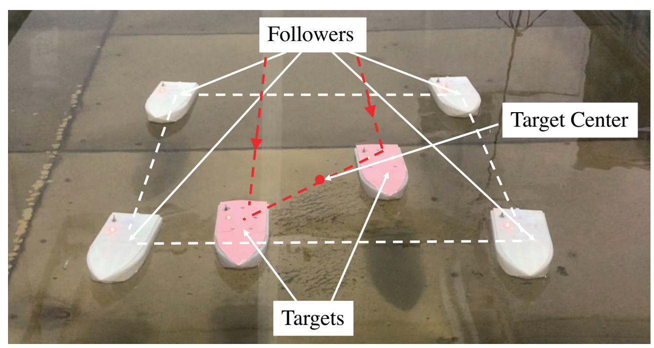


Fig. 5. A snapshot of the equally surrounding behavior with multiple vessels, which corresponds to Fig. 4 (d). The four white vessels forms a follower set and the two red vessels are the targets.

In the experiments, the multi-target-multi-agent group $V = \mathbb{I}_v \cup \mathbb{I}_v$ consist of six vessels with the sensing radius R set as 600mm and the safe radius r set as 200mm, where the first four individuals are the followers $\mathbb{I}_v$ and the last two the targets $\mathbb{I}_t$. Under Assumption 1, the inter-agent topology is set in Fig. 3, where the targets are only partially known to the follower set. Initially, four followers are randomly distributed in the pool and the initial velocities of the followers are set $v_i = 0$ mm/s, $i = 1, 2, 3, 4$, without loss of generality, while the positions for motional targets are set randomly and the velocities are set as $v_5 = [-10\sin(0.2t + 1) - 40, 0]^\intercal$mm/s, $v_6 = [-10\cos(0.4t+1)-40, 0]^\intercal$mm/s, respectively under Assumption 3. The surrounding radius ρ is set as 600mm. With the above velocities of targets, the gain of the estimator (7) is set as $\beta_p = 2.5$ in accordance to $\sigma_p = 30$mm, which satisfies the inequality (10) in Lemma 3 as well. In addition, in light of Theorem 2, the parameters of controller (28) are set as $k_1 = 0.5,\ k_2 = 5,\ k_3 = 0.01$.

Under Assumption 1, the trajectories of the surrounding process of $V = \mathbb{I}_v \cup \mathbb{I}_t$ are given in Fig. 4, where four followers $\mathbb{I}_v$ finally surround the two variable targets. To show the surrounding process vividly, Fig. 5 exhibits a snapshot corresponding to Fig. 4 (d). The transition of estimated errors $\widehat{x}_i, i = 1, 2, 3, 4$ in Fig. 6 (a) converges to the center of targets $\bar{r}$ with prescribed bound in less than 5 seconds, which reveals the feasibility of estimator (7). As shown in Fig. 6 (b), the surrounding errors $e_i, i = 1, 2, 3, 4$, approach to zero with a bound 200mm in less than 20 seconds, which shows the effectiveness of the proposed scheme (28). The inter-USV distance in Fig. 7 (a) always keeps larger than 200mm and hence collision avoidance is guaranteed, Moreover, as illustrated in Fig. 7 (b), the transition of inter-USV phase moves towards the center of targets and converges to 90°. The feasibility of Theorem 2 is substantiated in both cases.

V. Conclusions

In this paper, a distributed surrounding control protocol for second-order nonlinear MASs is proposed for encircling a group of moving targets with guaranteed collision avoidance. It is shown that by using such a protocol the agents are able to encircle multiple moving targets with evenly distributed phases around a circle. By using potential functions among agents and estimating the center of the moving targets, the

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIE.2020.3000092, IEEE Transactions on Industrial Electronics

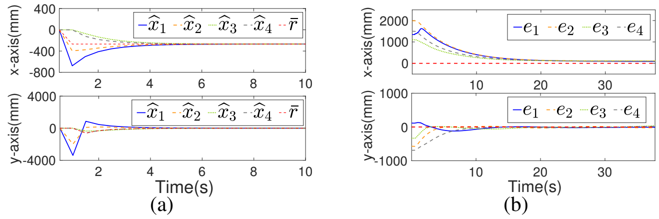


Fig. 6. (a) The transition of estimate $\widehat{x}_i, i = 1, 2, 3, 4$ of the center of target USVs with collision avoidance for follower USVs. (b) The transition of position surrounding error $e_i, i = 1, 2, 3, 4$ for follower USVs.

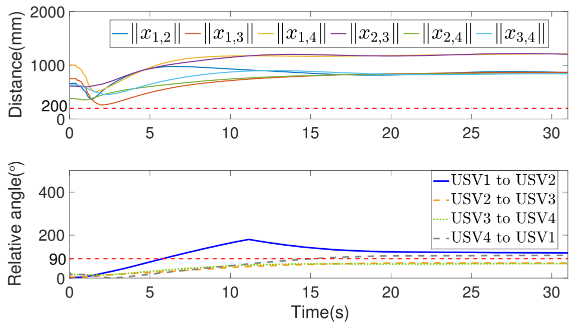


Fig. 7. (a) The transition of distance norm $\|x_{i,k}\|, i \neq k \in \mathbb{I}_t$ between each pair of follower USVs, which substantiates the collision avoidance of Algorithm 2. (b) The transition of inter-USV phase towards the center of targets finally converges to $90°$.

protocol enables the capability of obstacle avoidance. The conditions guaranteeing the stability of the closed-loop system is derived as well. Finally, experimental results based on our multi-HUSTER-0.3 USV system are discussed to substantiate the effectiveness of the proposed method. It is perceived that the presented surrounding control protocol would be viable for the applications of hunting, aquatic area patrolling, and marine resource explorations with multi-USV systems.

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