This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCNS.2021.3094784, IEEE Transactions on Control of Network Systems



# Distributed Moving Target Fencing in a Regular Polygon Formation

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***Abstract*—This paper proposes a distributed controller for a network of agents of second-order dynamics to fence a moving target of unknown velocity within their convex hull. Moreover, the agents form a regular polygon formation and avoid collision during the entire evolution. In the control scheme, the agents are not necessarily labelled and the nearest angle rules are applied. Each controller is composed of the functionalities of estimation of the target's velocity, regulation of distance between the agents and the target, and angle repulsion among agents resulting in an equal distribution. The latter two also guarantee collision avoidance. The asymptotical behavior of the closed-loop system is rigorously analyzed especially subject to the velocity estimation error. The effectiveness of the controller is also demonstrated by numerical simulation.**

***Index Terms*—Multi-agent systems, formation, collaborative control, target fencing, collision avoidance, autonomous agents**

## I. Introduction

Coordination of multi-agent systems (MASs) has drawn increasing attentions during the past decades due to the tremendous applications in engineering systems, such as multi-unmanned systems, distributed sensor networks, motive industrial robots and smart grid, etc. Therein, collective formation is one of the most active topics and has been widely explored by a large volume of works, among which many efforts were devoted to the leaderless framework, e.g., [1], [2], and various leader-following settings have also been extensively studied, e.g., [3]–[5]. In the latter, a typical scenario is that followers are collaboratively controlled to track the trajectory specified by a leader. It is also noted that a significant amount of works are concerned about formation of a group of agents centering on a target, which can be treated as a leader-following scenario.

In such a leader-following scenario, one interesting problem is the so-called target-enclosing problem, which aims to drive a group of the agents to encircle a target within the moving trajectories of agents. It has many potential applications in fleet convey, orbit maintenance, perimeter surveillance, collaborative escort, patrolling and hunting, etc. It has witnessed many research progresses in recent years. For instance, in [6], a distributed moving-target-enclosing method was developed based on relative neighboring positions. In [7], a distributed cyclic controller was presented to form the desired pursuit patterns and enclose a moving target in a 3D space. Afterwards, a local bearing-only protocol in [8] was proposed to achieve encirclement of a target. Technically, the target's information is available to all the agents in these works. Researchers have also considered the more practical situation where the target's state is partially known. In particular, a robust protocol in [9] was proposed to encircle a target with partial information of the target. More partially known target-enclosing methods can be found in [10] and [11] for a direct fixed topology and switching topologies, respectively. It is worth mentioning that the existing solutions for the target-enclosing problem can accommodate not only a stationary target but also a moving target.

Another interesting problem is called a target-fencing problem, or a surrounding control problem in some references. The main feature is to drive the agents to collectively fence a target within their convex hull, at every moment. It differentiates the target-fencing problem from the target-enclosing problem where the agents encircle a target within their moving trajectories. One technique for handling such a target-fencing or surrounding control problem is based on the strategies for labelled agents [12]–[17]. Moreover, the stand-off bearings and distribution are predefined for each agent in advance. In the early work [12], an estimation and control framework was proposed to surround stationary targets by multiple agents in a network with a connected topology. For linear second-order MASs, a surrounding controller was proposed in [13]; and for nonlinear and non-identical MASs, an adaptive controller was proposed in [14]. Other relevant methods based on labelled agents were investigated in [15]–[17]. However, the strategies with labelled agents have the inherent disadvantages and there is demand for strategies with unlabelled agents. Therefore, a limit-cycle-based strategy was developed to form a circular formation for a target in [18]. A hierarchical control structure with output regulation was established to surround a specified target in [19]. It is worth mentioning that a cooperative controller consisting of attractive, repulsive, rotation components was designed in [20] to fence a specified target without a predefined distance.

The algorithms developed in the aforementioned references [12]–[17] are applicable for labelled agents with fixed and con-

The first and third authors acknowledge the support from the National Natural Science Foundation of China under Grants 51729501 and U1713203, the Natural Science Foundation of Hubei Province under Grant 2019CFA005, the Program for Core Technology Tackling Key Problems of Dongguan City under Grant 2019622101007, and the Fundamental Research Funds for Central Universities, HUST: 2019KFYXMBZ032 and 2020JYCXJJ070. (Corresponding authors: Z. Chen and H.-T. Zhang)

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nected topologies, where the relative distribution and steady states are predefined in advance. However, due to its lack of flexibility, such kind of labelled agents based algorithms fail to fulfill more complicated cooperative missions. Although the methods presented in [18]–[20] consider the more practical unlabelled agents, they are applicable for first-order MASs with a static target. This motivates us to propose a distributed moving target-fencing controller for second-order MASs with a moving target in this paper.

Moreover, due to the challenge of distributed calculation of steady states, the methods in [18]–[20] or the following works in [21], [22] do not rigorously ensure a formation pattern for the agents. Another feature of this paper is that the agents equipped with the designed moving-target-fencing controllers also achieve a regular polygon formation which is an optimal formation in many senses and has attracted attention in literature [23], [24]. Collision avoidance is also guaranteed by the proposed design. Although collision avoidance has been extensively studied in many existing works, e.g., [16], [17], [20], it is usually studied associated with a specified control task. In this paper, it is studied in the new moving-target-fencing framework.

Overall, the main contribution of this paper is a new distributed controller equipping with the functionalities of estimation of the target's velocity, regulation of distance between the agents and the target, and angle repulsion among agents, which is able to achieve moving target fencing, a regular polygon formation and guaranteed collision avoidance, simultaneously, for a group of unlabelled agents. From the technical aspect, this paper handles time-varying network topologies using the nearest angle rules, achieves collision avoidance via distance regulation and angle repulsion, and resolves the impact of target velocity estimation error on the closed-loop system performance, while rigorously proving convergence of a complicated nonlinear networked system.

The remainder of this paper is organized as follows. Section II presents the problem formulation and the explicit controller with the main objectives under a certain conditions. The main theorem is stated in Section III with a rigorous proof. Numerical simulation is conducted in Section IV to verify the effectiveness of the present method. Finally, the paper is concluded in Section V.

## II. Problem Formulation and Controller

Consider an MAS consisting of $n$ agents represented by the set $\mathcal{V} = \{1, 2, \dots, n\}$. Each agent is described by the second-order dynamics in the Cartesian coordinates,

$$\begin{aligned}\dot{x}_i(t) &= v_i(t)\\ \dot{v}_i(t) &= u_i(t),\ i \in \mathcal{V},\end{aligned} \tag{1}$$

where $x_i(t), v_i(t) \in \mathbb{R}^2$ denote the position and velocity of agent $i$, respectively, and $u_i(t) \in \mathbb{R}^2$ is the control input.

A moving target is considered with its position and velocity represented by $x_d(t), v_d(t) \in \mathbb{R}^2$ satisfying

$$\begin{aligned}\dot{x}_d(t) &= v_d(t)\\ \dot{v}_d(t) &= u_d(t).\end{aligned} \tag{2}$$

It is assumed that the position $x_d(t)$ is measurable and available to the followers, but not the velocity $v_d(t)$. Let $\widehat{v}_i(t)$ and $\widehat{u}_i(t) = \dot{\widehat{v}}_i(t)$ be the estimation of $v_d(t)$ and $u_d(t)$ by agent $i$, respectively, generated by a filter (dynamic observer) of the form

$$\begin{aligned}[\widehat{v}_i(t), \widehat{u}_i(t)] &= F(\zeta_i(t), x_d(t))\\ \dot{\zeta}_i(t) &= G(\zeta_i(t), x_d(t)),\ i \in \mathcal{V}\end{aligned} \tag{3}$$

for some functions $F, G$ to be designed. The observer (3) aims to drive the estimation error to zero in the sense that

$$\lim_{t\to\infty} \widehat{v}_i(t) - v_d(t) = 0,\ \lim_{t\to\infty} \widehat{u}_i(t) - u_d(t) = 0,\ i \in \mathcal{V} \tag{4}$$

exponentially. It is assumed that $v_d(t)$ and $u_d(t)$ are bounded, also $u_d(t)$ and $\widehat{u}_i(t)$ are uniformly continuous in $t$, which is reasonable in practical scenarios.

The relative position between each agent $i \in \mathcal{V}$ and the target is defined as

$$e_i(t) = x_i(t) - x_d(t), \tag{5}$$

which can be rewritten in the polar coordinates

$$e_i(t) = \rho_i(t) s_{\theta_i(t)}. \tag{6}$$

Also, we define $\widehat{\delta}_i(t) = v_i(t) - \widehat{v}_i(t)$ that can be rewritten in the polar coordinates in terms of $\widehat{\eta}_i(t)$ and $\widehat{\omega}_i(t)$ as follows

$$\begin{bmatrix}\widehat{\eta}_i(t)\\ \rho_i(t)\widehat{\omega}_i(t)\end{bmatrix} = J^{\intercal}_{\theta_i(t)} \widehat{\delta}_i(t). \tag{7}$$

Throughout the paper, the following notations are used,

$$s_{\theta_i} = \begin{bmatrix}\cos\theta_i\\ \sin\theta_i\end{bmatrix},\ t_{\theta_i} = \begin{bmatrix}-\sin\theta_i\\ \cos\theta_i\end{bmatrix},$$

and

$$J_{\theta_i} = \begin{bmatrix}s_{\theta_i} & t_{\theta_i}\end{bmatrix},\ J'_{\theta_i} = \frac{\partial J_{\theta_i}}{\theta_i} = \begin{bmatrix}t_{\theta_i} & -s_{\theta_i}\end{bmatrix}. \tag{8}$$

For the convenience of presentation, we denote

$$\theta_{i,k} = \theta_i - \theta_k + 2\kappa\pi \in (-\pi, \pi] \tag{9}$$

for an appropriately selected integer $\kappa$.

From the above introduction, each agent $i$ has the access to the position states $x_i, x_d$ and hence $\rho_i, \theta_i$; and the velocity states $v_i, \widehat{v}_i$ and hence $\widehat{\eta}_i(t)$, $\widehat{\omega}_i(t)$. Then, the paper is concerned about a novel distributed controller as follows, with the argument $(t)$ omitted for neatness, for $i \in \mathcal{V}$,

$$\begin{aligned}u_i =& \widehat{u}_i + J'_{\theta_i}\begin{bmatrix}\widehat{\eta}_i\widehat{\omega}_i\\ \rho_i\widehat{\omega}_i^2\end{bmatrix}\\ &+ J_{\theta_i}\begin{bmatrix}-k_1\alpha(\rho_i) - k_2\widehat{\eta}_i\\ \widehat{\eta}_i\widehat{\omega}_i + \rho_i(-k_3\sum\limits_{k\in\mathcal{N}_i}\beta(\theta_{i,k}) - k_4\widehat{\omega}_i)\end{bmatrix}.\end{aligned} \tag{10}$$

In the controller, the two functions $\alpha$ and $\beta$ are defined as

$$\begin{aligned}\alpha(\rho_i) &= \frac{(\rho_i - \rho_o)(\rho_o - M)}{(\rho_i - M)^3}\\ \beta(\theta_{i,k}) &= \frac{2\gamma(|\theta_{i,k}| - \gamma)\theta_{i,k}}{|\theta_{i,k}|^4},\end{aligned}$$

and the sensing neighborhood is

$$\mathcal{N}_i(t) = \{k \in \mathcal{V}, k \neq i \mid |\theta_{i,k}(t)| < \gamma\}. \tag{11}$$

The controller contains four positive parameters $k_1, k_2, k_3, k_4 > 0$, a sensing range $\gamma > 0$, a specified desired distance $\rho_o > 0$ between the agents and the target, and a specified minimum (safe) distance $0 < M < \rho_o$ between the agents and the target.

Now, the closed-loop system is composed of (1), (3), and (10), which aims at the following properties.

- **P1**: The agents asymptotically maintain the distance $\rho_o$ from the target, i.e., $\lim_{t\to\infty} \rho_i(t) = \rho_o, \forall i \in \mathcal{V}$.
- **P2**: The agents asymptotically maintain an equal angle distribution around the target, i.e., $\lim_{t\to\infty} |\theta_{i,k}(t)| \geq 2\pi/n,\ \forall i \neq k \in \mathcal{V}$.
- **P3**: Collision among the agents and the target is avoided in the sense of $\rho_i(t) > M,\ \theta_{i,k}(t) \neq 0,\ \forall t \geq 0,\ \forall i \neq k \in \mathcal{V}$.

**Remark 1.** *Under the properties P1 and P2, there exists a finite time $T$ such that the target-fencing property $P_{x_d(t)}(x(t)) = 0,\ \forall t \geq T$ is achieved with $x = [x_1, \cdots, x_n]^\intercal$. That is, the target $x_d$ always stays within the interior of the convex hull of the agents with $P_{x_d}(x) = \min_{s\in \mathrm{co}(x)} \|x_d - s\|$ and*

$$\mathrm{co}(x) = \left\{ \sum_{i\in\mathcal{V}} \lambda_i x_i : \lambda_i \geq 0, \forall i \text{ and } \sum_{i\in\mathcal{V}} \lambda_i = 1 \right\}. \tag{12}$$

*Various target-fencing settings can be found in literature, e.g., [12]–[20]. An interesting feature of P1 and P2 is that the agents asymptotically form a regular polygon formation around the target as the centre with the radius $\rho_o$ and an equal angle of $2\pi/n$ between every two adjacent agents. The property P3 guarantees avoidance of collision not only among the agents but also between every agent and the target.*

For achieving the aforementioned properties, the following conditions are required.

- **C1**: The observer (3) is such that (4) holds exponentially.
- **C2**: The initial positions of the agents and the target satisfy $\rho_i(0) > M,\ \theta_{i,k}(0) \neq 0,\ \forall i \neq k \in \mathcal{V}$.
- **C3**: The sensing range parameter satisfies $2\pi/n \leq \gamma \leq 4\pi/n$.

**Remark 2.** *The condition C1 is the existence of a local estimator of the moving target's velocity using its position measurement by each agent. This is a local design by an individual agent involving no network communication. Such a problem has been well studied in literature, e.g., using the output regulation theory [25], [26] when $u_d$ follows a certain specified dynamics. It is not the main scope of this paper. To make the design complete, an example for the constant target velocity scenario with $u_d = 0$ is given in this remark. With $\zeta_i = [\widehat{x}_i, \widehat{v}_i]^\intercal$, the observer* (3) *has the following structure*

$$\begin{aligned} \dot{\widehat{x}}_i &= -\phi_1(\widehat{x}_i - x_d) + \widehat{v}_i \\ \dot{\widehat{v}}_i &= \widehat{u}_i = -\phi_2\phi_1(\widehat{x}_i - x_d),\ i \in \mathcal{V} \end{aligned} \tag{13}$$

*with $\phi_1, \phi_2 > 0$. Let $\widetilde{\zeta}_i = \zeta_i - [x_d, v_d]^\intercal$. One has $\dot{\widetilde{\zeta}}_i = A\widetilde{\zeta}_i$ for a Hurwitz matrix*

$$A = \begin{bmatrix} -\phi_1 & 1 \\ -\phi_2\phi_1 & 0 \end{bmatrix},$$

*which concludes $\lim_{t\to\infty} \widetilde{\zeta}_i(t) = 0$ and hence* (4) *exponentially.*

**Remark 3.** *The condition C2 is necessary for the collision avoidance property P3. It requires the agents start from a generic distribution without collision.*

**Remark 4.** *The condition $\gamma \geq 2\pi/n$ in C3 is necessary for the equal angle distribution property P2. For $\gamma < 2\pi/n$, we can easily find a trivial steady state distribution $2\pi/n > \theta_{i,i+1} > \gamma$, for $i = 1, \cdots, n-1$, but $\theta_{n,1} > 2\pi/n > \gamma$ which makes $\mathcal{N}_i = \emptyset,\ i \in \mathcal{V}$ and violates P2. The condition $\gamma \leq 4\pi/n$ in C3 gives an upper bound of the sensing angle range which simplifies the analysis on angle distribution in the next section. It basically requires every agent does not have more than two neighbors when the regular polygon formation is achieved. This condition is not restrictive even though it is not proved to be necessary.*

The main objective of the paper is to prove that the proposed controller (10) can achieve the aforementioned three properties under the three conditions. Apart from the rigorous proof of the properties as elaborated in the next section, it is worth mentioning other two technical features of the development. (i) The controller has two functionalities in maintaining the distance between the agents and the target and the angles among the agents. The former guarantees cohesion of the group of agents with a specified radius and the nearest angle rules are applied in the latter. The topology is time-varying for the network of agents. In other words, a fixed and connected topology is not explicitly assumed and the agents are not labelled as in, e.g., [18]–[20]. (ii) Estimation of the target's velocity is implemented by every agent. The estimation error, albeit convergent to zero, brings influence on the system's behavior, especially when the closed-loop system is of hard nonlinearities. In general, a stable nonlinear system may exhibit divergence and even a finite escape time for a sufficiently small disturbance [27]. Technically, we cannot simply assume that the estimation error is zero. This brings some major challenges in the proof of the main theorem.

## III. Main Theorem

The main theorem is presented in this section with a rigorous proof.

**Theorem 1.** *The closed-loop system composed of* (1), (3), *and* (10) *achieves the properties P1, P2, and P3, under the conditions C1, C2, and C3.*

*Proof:* We divide the proof in four steps for readers' convenience. With some preliminary manipulation on the closed-loop system, we select the Lyapunov function candidates in the first step, and the proofs for P3, P1, and P2 are given in the remaining steps in order.

*Step 1: Lyapunov function candidates.*

We first denote $\delta_i = \dot{e}_i$ as the velocity difference between agent $i$ and the target. From the definition of $e_i$ in the Cartesian coordinates (5), one has $\delta_i = v_i - v_d$. In the polar coordinates (6), one has

$$\delta_i = \rho_i t_{\theta_i}\dot{\theta}_i + \dot{\rho}_i s_{\theta_i} = J_{\theta_i}\begin{bmatrix}\eta_i \\ \rho_i\omega_i\end{bmatrix} \tag{14}$$

for $\eta_i = \dot{\rho}_i$ and $\omega_i = \dot{\theta}_i$. As a result, the difference between $\widehat{\delta}_i$ in (7) and $\delta_i$ in (14) can be expressed in both coordinates as follows

$$\widehat{\delta}_i - \delta_i = v_d - \widehat{v}_i = J_{\theta_i}\begin{bmatrix}\widetilde{\eta}_i \\ \rho_i\widetilde{\omega}_i\end{bmatrix} \tag{15}$$

for $\widetilde{\eta}_i = \widehat{\eta}_i - \eta_i$ and $\widetilde{\omega}_i = \widehat{\omega}_i - \omega_i$. For the convenience of presentation, we also define $\widetilde{u}_i = \widehat{u}_i - u_d$.

Since $J_{\theta_i}$ in (8) is nonsingular, it follows from C1 and (15) that

$$\lim_{t\to\infty}\widetilde{\eta}_i(t) = 0,\ \lim_{t\to\infty}\rho_i(t)\widetilde{\omega}_i(t) = 0,\ \lim_{t\to\infty}\widetilde{u}_i(t) = 0. \tag{16}$$

Next, we calculate $\dot{\delta}_i = \ddot{e}_i$ as follows. On one hand,

$$\dot{\delta}_i = J'_{\theta_i}\begin{bmatrix}\eta_i\omega_i \\ \rho_i\omega_i^2\end{bmatrix} + J_{\theta_i}\begin{bmatrix}\dot{\eta}_i \\ \eta_i\omega_i + \rho_i\dot{\omega}_i\end{bmatrix} \tag{17}$$

from (14). On the other hand, using (10), one has

$$\begin{aligned}\dot{\delta}_i =& \dot{v}_i - \dot{v}_d = u_i - u_d \\ =& \widehat{u}_i - u_d + J'_{\theta_i}\begin{bmatrix}\widehat{\eta}_i\widehat{\omega}_i \\ \rho_i\widehat{\omega}_i^2\end{bmatrix} \\ &+ J_{\theta_i}\begin{bmatrix}-k_1\alpha(\rho_i) - k_2\widehat{\eta}_i \\ \widehat{\eta}_i\widehat{\omega}_i + \rho_i(-k_3\sum\limits_{k\in\mathcal{N}_i}\beta(\theta_{i,k}) - k_4\widehat{\omega}_i)\end{bmatrix}.\end{aligned} \tag{18}$$

Comparing (17) and (18) gives

$$\begin{aligned}\begin{bmatrix}\dot{\eta}_i \\ \rho_i\dot{\omega}_i\end{bmatrix} =& \begin{bmatrix}-k_1\alpha(\rho_i) - k_2\widehat{\eta}_i \\ \widehat{\eta}_i\widehat{\omega}_i + \rho_i(-k_3\sum\limits_{k\in\mathcal{N}_i}\beta(\theta_{i,k}) - k_4\widehat{\omega}_i) - \eta_i\omega_i\end{bmatrix} \\ &+ J_{\theta_i}^{\intercal}J'_{\theta_i}\begin{bmatrix}\widehat{\eta}_i\widehat{\omega}_i - \eta_i\omega_i \\ \rho_i\widehat{\omega}_i^2 - \rho_i\omega_i^2\end{bmatrix} + J_{\theta_i}^{\intercal}\widetilde{u}_i.\end{aligned}$$

Using the following facts

$$\begin{aligned}\widehat{\eta}_i\widehat{\omega}_i &= \eta_i\omega_i + \widetilde{\eta}_i\omega_i + \eta_i\widetilde{\omega}_i + \widetilde{\eta}_i\widetilde{\omega}_i \\ \widehat{\omega}_i^2 &= \omega_i^2 + 2\omega_i\widetilde{\omega}_i + \widetilde{\omega}_i^2,\end{aligned}$$

one has

$$\begin{bmatrix}\dot{\eta}_i \\ \dot{\omega}_i\end{bmatrix} = \begin{bmatrix}-k_1\alpha(\rho_i) - k_2\eta_i \\ -k_3\sum\limits_{k\in\mathcal{N}_i}\beta(\theta_{i,k}) - k_4\omega_i\end{bmatrix} + \xi_i$$

for

$$\begin{aligned}\xi_i = \begin{bmatrix}\epsilon_i \\ \varepsilon_i\end{bmatrix} =& \begin{bmatrix}-k_2\widetilde{\eta}_i \\ -k_4\widetilde{\omega}_i + (\widetilde{\eta}_i\omega_i + \eta_i\widetilde{\omega}_i + \widetilde{\eta}_i\widetilde{\omega}_i)/\rho_i\end{bmatrix} \\ &+ R_i^{-1}J_{\theta_i}^{\intercal}J'_{\theta_i}\begin{bmatrix}\widetilde{\eta}_i\omega_i + \eta_i\widetilde{\omega}_i + \widetilde{\eta}_i\widetilde{\omega}_i \\ 2\rho_i\omega_i\widetilde{\omega}_i + \rho_i\widetilde{\omega}_i^2\end{bmatrix} + R_i^{-1}J_{\theta_i}^{\intercal}\widetilde{u}_i \\ =& \begin{bmatrix}-k_2\widetilde{\eta}_i - 2\rho_i\omega_i\widetilde{\omega}_i - \rho_i\widetilde{\omega}_i^2 \\ -k_4\widetilde{\omega}_i + 2(\widetilde{\eta}_i\omega_i + \eta_i\widetilde{\omega}_i + \widetilde{\eta}_i\widetilde{\omega}_i)/\rho_i\end{bmatrix} + R_i^{-1}J_{\theta_i}^{\intercal}\widetilde{u}_i \\ =& \begin{bmatrix}-2\rho_i\omega_i\widetilde{\omega}_i \\ 2(\widetilde{\eta}_i\omega_i + \eta_i\widetilde{\omega}_i)/\rho_i\end{bmatrix} + \begin{bmatrix}\sigma_i \\ \varsigma_i\end{bmatrix},\end{aligned} \tag{19}$$

where $R_i = \text{diag}(1, \rho_i)$ and

$$\begin{bmatrix}\sigma_i \\ \varsigma_i\end{bmatrix} = \begin{bmatrix}-k_2\widetilde{\eta}_i - \rho_i\widetilde{\omega}_i^2 \\ -k_4\widetilde{\omega}_i + 2(\widetilde{\eta}_i\widetilde{\omega}_i)/\rho_i\end{bmatrix} + R_i^{-1}J_{\theta_i}^{\intercal}\widetilde{u}_i. \tag{20}$$

From the above analysis, it is seen that the closed-loop system is composed of

$$\begin{aligned}\dot{\rho}_i =& \eta_i \\ \dot{\eta}_i =& -k_1\alpha(\rho_i) - k_2\eta_i + \epsilon_i,\end{aligned} \tag{21}$$

and

$$\begin{aligned}\dot{\theta}_i &= \omega_i \\ \dot{\omega}_i &= -k_3\sum_{k\in\mathcal{N}_i}\beta(\theta_{i,k}) - k_4\omega_i + \varepsilon_i.\end{aligned} \tag{22}$$

For the system (21), we first define a function

$$P(\rho_i) = \left(\frac{\rho_i - \rho_o}{\rho_i - M}\right)^2,\ \rho_i \in (M, \infty), \tag{23}$$

which is continuously differentiable and pick a Lyapunov function candidate

$$V_1(\rho, \eta) = \frac{1}{2}\sum_{i=1}^{n}\left(k_1P(\rho_i) + \eta_i^2\right), \tag{24}$$

where $\rho = [\rho_1, \cdots, \rho_n]^{\intercal}$ and $\eta = [\eta_1, \cdots, \eta_n]^{\intercal}$. The time derivative of $V_1(\rho, \eta)$ along the trajectories of (21) is

$$\begin{aligned}\dot{V}_1(\rho, \eta) =& \sum_{i=1}^{n}\left(\frac{k_1}{2}\frac{\partial P(\rho_i)}{\partial\rho_i}\dot{\rho}_i + \eta_i\dot{\eta}_i\right) \\ =& \sum_{i=1}^{n}\Bigg(k_1\frac{(\rho_i - \rho_o)(\rho_o - M)}{(\rho_i - M)^3}\eta_i \\ & -\eta_i k_1\frac{(\rho_i - \rho_o)(\rho_o - M)}{(\rho_i - M)^3} - k_2\eta_i^2 + \epsilon_i\eta_i\Bigg) \\ =& \sum_{i=1}^{n}\left(-k_2\eta_i^2 + \eta_i\epsilon_i\right) \\ \leq& \sum_{i=1}^{n}\left(-\frac{k_2}{2}\eta_i^2 + \frac{1}{2k_2}\epsilon_i^2\right).\end{aligned} \tag{25}$$

For the system (22), we define a function

$$U(\theta_{i,k}) = \begin{cases}0, & |\theta_{i,k}| \in [\gamma, \pi] \\ \left(\frac{|\theta_{i,k}| - \gamma}{|\theta_{i,k}|}\right)^2, & |\theta_{i,k}| \in (0, \gamma)\end{cases}, \tag{26}$$

which is continuously differentiable and whose time derivative is

$$\dot{U}(\theta_{i,k}) = \begin{cases}0 & |\theta_{i,k}| \in [\gamma, \pi] \\ \beta(\theta_{i,k})(\dot{\theta}_i - \dot{\theta}_k), & |\theta_{i,k}| \in (0, \gamma)\end{cases}. \tag{27}$$

It is noted that $\dot{\theta}_{i,k} = \dot{\theta}_i - \dot{\theta}_k$ when $|\theta_{i,k}| \in (0, \gamma)$ with $\gamma < \pi$. Also, one has $\beta(\theta_{i,k}) = -\beta(\theta_{k,i})$. Pick a Lyapunov function candidate as follows

$$V_2(\theta, \omega) = \frac{k_3}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}U(\theta_{i,k}) + \frac{1}{2}\sum_{i=1}^{n}\omega_i^2, \tag{28}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCNS.2021.3094784, IEEE Transactions on Control of Network Systems



where $\theta = [\theta_1, \cdots, \theta_n]^\intercal$ and $\omega = [\omega_1, \cdots, \omega_n]^\intercal$. Its time derivative along the trajectories of (22) satisfies

$$\begin{aligned}\dot{V}_2(\theta,\omega) =& \frac{k_3}{2}\sum_{i=1}^{n}\sum_{k\in\mathcal{N}_i}\beta(\theta_{i,k})(\dot{\theta}_i-\dot{\theta}_k)+\sum_{i=1}^{n}\omega_i\dot{\omega}_i \\ =& k_3\sum_{i=1}^{n}\sum_{k\in\mathcal{N}_i}\beta(\theta_{i,k})\dot{\theta}_i \\ &+\sum_{i=1}^{n}\omega_i\left(-k_3\sum_{k\in\mathcal{N}_i}\beta(\theta_{i,k})-k_4\omega_i+\varepsilon_i\right) \\ =& \sum_{i=1}^{n}\omega_i\left(-k_4\omega_i+\varepsilon_i\right) \\ \leq& \sum_{i=1}^{n}\left(-\frac{k_4}{2}\omega_i^2+\frac{1}{2k_4}\varepsilon_i^2\right). \end{aligned} \tag{29}$$

Combining (25) and (29) gives

$$\begin{aligned}&\dot{V}_1(\rho,\eta)+\dot{V}_2(\theta,\omega) \\ \leq& \sum_{i=1}^{n}\left(-\frac{k_2}{2}\eta_i^2-\frac{k_4}{2}\omega_i^2+\frac{1}{2k_2}\epsilon_i^2+\frac{1}{2k_4}\varepsilon_i^2\right) \\ \leq& \sum_{i=1}^{n}\left(-a_i\eta_i^2-b_i\omega_i^2+c_i\right), \end{aligned} \tag{30}$$

for

$$\begin{aligned} a_i =& \frac{k_2}{2}-\frac{6\widetilde{\omega}_i^2}{k_4\rho_i^2} \\ b_i =& \frac{k_4}{2}-\frac{4(\rho_i\widetilde{\omega}_i)^2}{k_2}-\frac{6\widetilde{\eta}_i^2}{k_4\rho_i^2} \\ c_i =& \frac{\sigma_i^2}{k_2}+\frac{3\varsigma_i^2}{2k_4} \end{aligned}$$

on the space

$$\begin{aligned}\mathbb{S} = \{&[\rho^\intercal,\eta^\intercal,\theta^\intercal,\omega^\intercal]^\intercal\in\mathbb{R}^{4n} \mid \\ &\rho_i\in(M,\infty),|\theta_{i,k}|\in(0,\pi],\ i\neq k\in\mathcal{V}\}.\end{aligned}$$

The following facts from the definitions of $\epsilon_i$ and $\varepsilon_i$

$$\begin{aligned}\epsilon_i^2 =& (-2\rho_i\omega_i\widetilde{\omega}_i+\sigma_i)^2\leq 8(\rho_i\widetilde{\omega}_i)^2\omega_i^2+2\sigma_i^2 \\ \varepsilon_i^2 =& (2\widetilde{\eta}_i\omega_i/\rho_i+2\eta_i\widetilde{\omega}_i/\rho_i+\varsigma_i)^2 \\ \leq& 12(\widetilde{\eta}_i/\rho_i)^2\omega_i^2+12(\widetilde{\omega}_i/\rho_i)^2\eta_i^2+3\varsigma_i^2\end{aligned}$$

are used in the calculation.

*Step 2: Proof of P3.*

The property P3 is equivalent to the claim that the states always stay on the space $\mathbb{S}$ for $t\geq 0$. Under C2, the states are initially on space $\mathbb{S}$ for $t=0$. If P3 does not hold, there exists $\tau>0$ such that the states are on the space $\mathbb{S}$ for $t\in[0,\tau)$ but not at $t=\tau$. A contradiction is then found below.

For $t\in[0,\tau)$, the states stay on $\mathbb{S}$ and $\rho_i\in(M,\infty)$, then, by (16), there exist two real numbers $\nu_1\leq 0$ and $\nu_2>0$ such that $a_i,b_i\geq\nu_1/2$ and $\sum_{i=1}^n c_i\leq\nu_2$.

Let $V(t)=V_1(\rho(t),\eta(t))+V_2(\theta(t),\omega(t))$. Then, it follows from (24), (28), and (30) that the time derivative of $V(t)$ satisfies

$$\begin{aligned}\dot{V}\leq& \sum_{i=1}^{n}\left(-a_i\eta_i^2-b_i\omega_i^2+c_i\right) \\ \leq& \frac{-\nu_1}{2}\sum_{i=1}^{n}\left(\eta_i^2+\omega_i^2\right)+\nu_2\leq-\nu_1V+\nu_2.\end{aligned}$$

Based on the comparison principle [28], one has

$$V(\tau)\leq e^{-\nu_1\tau}V(0)+\nu_2\int_0^\tau e^{-\nu_1(\tau-s)}ds.$$

As $V(0)$ is bounded under C2, so is $V(\tau)$, which implies that the states stay on $\mathbb{S}$ at $t=\tau$. The property P3 is thus proved.

*Step 3: Proof of P1.*

It is known from Step 2 that $\rho_i\in(M,\infty)$ for all $t\geq 0$, which with (16) implies

$$\lim_{t\to\infty}\widetilde{\omega}_i(t)=0. \tag{31}$$

Together with the definitions of $\sigma_i,\varsigma_i$ in (20), the facts in (16) and (31) imply

$$\lim_{t\to\infty}\sigma_i(t)=0,\ \lim_{t\to\infty}\varsigma_i(t)=0. \tag{32}$$

By the virtue of (16), (31) and (32), after a finite time $T$, one has $a_i>0$, $b_i>0$, and $\lim_{t\to\infty}c_i(t)=0$. All the convergences hold exponentially.

Next, one has

$$\begin{aligned}V(t)\leq& V(T)-\sum_{i=1}^{n}\int_T^t\left(a_i(s)\eta_i^2(s)+b_i(s)\omega_i^2(s)\right)ds \\ &+\sum_{i=1}^{n}\int_T^t c_i(s)ds,\ \forall t\geq T.\end{aligned}$$

From the fact that $\lim_{t\to\infty}c_i(t)=0$ exponentially, one has that $\sum_{i=1}^n\int_T^t c_i(s)ds$ is bounded, so is $V(t)$ for $t\geq T$. It concludes that $\eta(t)$ and $\omega(t)$ are bounded. Also, it follows from (16), (31) and (32) that $\epsilon_i,\varepsilon_i$ in (19) satisfy

$$\lim_{t\to\infty}\epsilon_i(t)=0,\ \lim_{t\to\infty}\varepsilon_i(t)=0. \tag{33}$$

Since $V(t)$ is bounded, it follows from (21), (22), (24), (28) that $\eta_i,\omega_i,\dot{\eta}_i,\dot{\omega}_i$ are bounded. It is noted that $\dot{v}_d(t)-\dot{\widehat{v}}_i(t)=u_d(t)-\widehat{u}_i(t)$ is uniformly continuous in $t$. Then, it follows from (4) and (15) that

$$\lim_{t\to\infty}\left[\dot{\widehat{\delta}}_i(t)-\dot{\delta}_i(t)\right]=\lim_{t\to\infty}\left[\dot{v}_d(t)-\dot{\widehat{v}}_i(t)\right]=0 \tag{34}$$

by Barbalat's lemma. From (15) again, one has

$$\dot{\widehat{\delta}}_i-\dot{\delta}_i=J'_{\theta_i}\omega_i\begin{bmatrix}\widetilde{\eta}_i\\ \rho_i\widetilde{\omega}_i\end{bmatrix}+J_{\theta_i}\begin{bmatrix}\dot{\widetilde{\eta}}_i\\ \eta_i\widetilde{\omega}_i+\rho_i\dot{\widetilde{\omega}}_i\end{bmatrix}, \tag{35}$$

which, together with (34), implies that $\dot{\widetilde{\eta}}_i,\rho_i\dot{\widetilde{\omega}}_i$ are bounded, so is $\dot{\widetilde{\omega}}_i$. Then, $\widehat{\eta}_i,\widehat{\omega}_i,\dot{\widehat{\eta}}_i,\dot{\widehat{\omega}}_i$ are bounded.

Denote $d_i=a_i\eta_i^2+b_i\omega_i^2$. It has been proved that $\int_T^t d_i(s)ds$ is upper bounded and $\lim_{t\to\infty}\int_T^t d_i(s)ds$ exists. As $\dot{d}_i$ contains $a_i,b_i,\eta_i,\omega_i,\dot{\eta}_i,\dot{\omega}_i,\rho_i^{-1},\widetilde{\omega}_i,\dot{\widetilde{\omega}}_i,\rho_i\widetilde{\omega}_i,\rho_i\dot{\widetilde{\omega}}_i,\widetilde{\eta}_i,\dot{\widetilde{\eta}}_i$ which are all bounded, so is $\dot{d}_i$. In other words, $d_i(t)$ is uniformly continuous in $t$. Then, it follows Barbalat's lemma that $\lim_{t\to\infty} d_i(t) = 0$ and hence $\lim_{t\to\infty} \eta_i(t) = 0$ and $\lim_{t\to\infty} \omega_i(t) = 0$ as $a_i$ and $b_i$ are lower bounded away from zero.

For $\rho_i \in (M, \infty)$, one has that $\dot{\alpha}(\rho_i)$ is bounded. It follows from (19) and (21) that $\ddot{\eta}_i$ contains $\dot{\alpha}(\rho_i), \dot{\eta}_i, \eta_i, \omega_i, \dot{\omega}_i, \rho_i\widetilde{\omega}_i, \widetilde{\omega}_i, \rho_i\dot{\widetilde{\omega}}_i, \dot{\widetilde{\eta}}_i, \dot{\widetilde{u}}_i$ which are all bounded, so is $\ddot{\eta}_i$. It implies that $\dot{\eta}_i(t)$ is uniformly continuous in $t$. Also, it has been proved that $\lim_{t\to\infty} \eta_i(t) = 0$. Using Barbalat's lemma, one has $\lim_{t\to\infty} \dot{\eta}_i(t) = 0$. It, together with (21), gives $\lim_{t\to\infty} \alpha(\rho_i(t)) = 0$, which further implies $\lim_{t\to\infty} \rho_i(t) = \rho_o$, $\forall i \in \mathcal{V}$, i.e., P1.

*Step 4: Proof of P2.*

Using the similar argument, we can show that $\lim_{t\to\infty} \dot{\omega}_i(t) = 0$. It, together with (22), gives

$$\lim_{t\to\infty} \sum_{k\in\mathcal{N}_i(t)} \beta(\theta_{i,k}(t)) = 0,\ \forall i \in \mathcal{V}. \tag{36}$$

Next, we will prove the statement

$$\begin{aligned}&\sum_{k\in\mathcal{N}_i(t)} \beta(\theta_{i,k}(t)) = 0,\ \forall i \in \mathcal{V}\\ &\Rightarrow |\theta_{i,k}(t)| \geq 2\pi/n,\ \forall i \neq k \in \mathcal{V}.\end{aligned} \tag{37}$$

We introduce a simpler notation $\langle i, k\rangle := \theta_{i,k}(t)$ to avoid using multiple layers of subscripts in the remaining proof.

To prove the statement (37), it suffices to prove

$$|\langle i, k\rangle| \geq 2\pi/n,\ \forall i \neq k \in \mathcal{V} \tag{38}$$

under the condition

$$\sum_{k\in\mathcal{N}_i(t)} \beta(\langle i, k\rangle) = 0,\ \forall i \in \mathcal{V}. \tag{39}$$

If all agents have no neighbor, it happens only when $\gamma = 2\pi/n$ and every two adjacent agents have the angle of $\gamma$, which proves (38) trivially. In the remaining proof, we consider the case that as least one agent has at least one neighbor. The details are given below.

For every $i \in \mathcal{V}$, $k$ is called a front neighbor (FN) of $i$ if $\langle k, i\rangle \in (0, \gamma)$, a rear neighbor (RN) of $i$ if $\langle k, i\rangle \in (-\gamma, 0)$, and not a neighbor otherwise. Pick an arbitrary agent and label it as $s(0)$, and label the remaining agents in count clockwise order as $s(1), s(2), \cdots, s(n-1)$. It is obvious that $\{s(0), s(1), s(2), \cdots, s(n-1)\} = \mathcal{V}$. For notation complement, we denote $s(i) = s(i \mod n)$ for any integer $i$, e.g., $s(n) = s(0)$.

*Claim i: Every agent $i \in \mathcal{V}$ has at least one FN.*

If this claim does not hold, there exists at least one agent that has no FN, say $s(i)$. In the sequence of $s(i), s(i-1), s(i-2), \cdots$, pick the first agent that has no FN but at least one RN, called $\hbar = s(i-\ell)$. Such an agent always exists because we consider the case that as least one agent has at least one neighbor. As a result,

$$\sum_{k\in\mathcal{N}_\hbar(t)} \beta(\langle \hbar, k\rangle) = \sum_{k\in\mathcal{V}, \langle \hbar, k\rangle\in(0,\gamma)} \beta(\langle \hbar, k\rangle) < 0,$$

which contradicts (39). The claim is proved.

*Claim ii: For every $i \in \mathcal{V}$, $\langle s(i+1), s(i)\rangle \geq \gamma/2$.*

If this claim does not hold, suppose $\langle s(0), s(n-1)\rangle < \gamma/2$ without loss of generality. Let $s(a_0), s(a_1), \cdots, s(a_m)$ be the longest sequence satisfying $a_0 = 0 < a_1 < \cdots < a_m < a_{m+1} = n$ and $\langle s(a_i), s(a_i - 1)\rangle < \gamma/2$, $i = 0, \cdots, m$. We consider two cases.

Case 1: If $\langle s(a_{i+1}), s(a_i)\rangle < \gamma/2$, there is no agent between $s(a_{i+1})$ and $s(a_i)$, that is, $a_{i+1} - a_i = 1$.

Case 2: If $\langle s(a_{i+1}), s(a_i)\rangle \geq \gamma/2$, then $a_{i+1} - a_i \geq 2$ and $\langle s(a_i+1), s(a_i)\rangle > \gamma/2$. Due to $\langle s(a_i), s(a_i-1)\rangle < \gamma/2$ and $\langle s(a_i+1), s(a_i)\rangle > \gamma/2$, $s(a_i)$ has at least two FNs, that is, $\langle s(a_i+2), s(a_i)\rangle < \gamma$. Otherwise,

$$\begin{aligned}&\sum_{k\in\mathcal{N}_{s(a_i)}(t)} \beta(\langle s(a_i), k\rangle)\\ \leq&\beta(\langle s(a_i), s(a_i+1)\rangle) + \beta(\langle s(a_i), s(a_i-1)\rangle)\\ =&-\beta(\langle s(a_i+1), s(a_i)\rangle) + \beta(\langle s(a_i), s(a_i-1)\rangle)\\ <&-\beta(\gamma/2) + \beta(\gamma/2) = 0,\end{aligned}$$

noting that $\beta(|s|)$ is a strictly increasing function of $|s|$ for $|s| \in (0, \gamma)$. It is a contradiction to (39). From $\langle s(a_i+2), s(a_i)\rangle < \gamma$ and $\langle s(a_i+1), s(a_i)\rangle > \gamma/2$, one has $\langle s(a_i+2), s(a_i+1)\rangle < \gamma/2$. As a result, $a_{i+1} - a_i = 2$ and $\langle s(a_{i+1}), s(a_i)\rangle < \gamma$.

The conclusions in the two cases can be unified as $\langle s(a_{i+1}), s(a_i)\rangle < (a_{i+1} - a_i)\gamma/2$. As a result,

$$2\pi = \sum_{i=0}^{m}\langle s(a_{i+1}), s(a_i)\rangle < \sum_{i=0}^{m}(a_{i+1} - a_i)\gamma/2 = n\gamma/2,$$

which is a contradiction to C3. Claim ii is thus proved.

*Claim iii: Every agent $i \in \mathcal{V}$ has exactly one FN and one RN.*

Every agent $s_i$ has at least one FN by Claim i, but it cannot have two FNs because $\langle s(i+2)-s(i)\rangle = \langle s(i+2)-s(i+1)\rangle + \langle s(i+1)-s(i)\rangle \geq \gamma$ by Claim ii. Therefore, every agent has exactly one FN. It obviously implies every agent has exactly one RN.

Finally, by Claim iii, one has, for any $s_i$,

$$\begin{aligned}&\sum_{k\in\mathcal{N}_{s(i)}(t)} \beta(\langle s(i), k\rangle)\\ =&\beta(\langle s(i), s(i+1)\rangle) + \beta(\langle s(i), s(i-1)\rangle) = 0\end{aligned}$$

due to (39). It implies $\langle s(i+1), s(i)\rangle = \langle s(i), s(i-1)\rangle$ for all $i \in \mathcal{V}$, and hence $\langle s(i+1), s(i)\rangle = 2\pi/n$. It is equivalent to (38). The statement (37) is thus proved.

Since $\beta(\cdot)$ is a continuous function, due to (37), for any $\varrho_1 > 0$, there exists $\varrho_2 > 0$ such that

$$\begin{aligned}&\sum_{k\in\mathcal{N}_i(t)} \beta(\theta_{i,k}(t)) \leq \varrho_2,\ \forall i \in \mathcal{V}\\ &\Rightarrow |\theta_{i,k}(t)| - 2\pi/n \geq -\varrho_1,\ \forall i \neq k \in \mathcal{V}.\end{aligned}$$

Also, due to (36), there exists $T$ such that, for $t \geq T$, $\sum_{k\in\mathcal{N}_i(t)} \beta(\theta_{i,k}(t)) \leq \varrho_2$, $\forall i \in \mathcal{V}$ and hence $|\theta_{i,k}(t)| - 2\pi/n \geq -\varrho_1$, $\forall i \neq k \in \mathcal{V}$. It concludes P2. ■

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCNS.2021.3094784, IEEE Transactions on Control of Network Systems



**Remark 5.** *The collision avoidance in property P3 among the agents and the target implicitly shows that the solutions* $\rho_i = M$ *and* $\langle s(i+1), s(i)\rangle = 0$ *to the conditions* $\alpha(\rho_i(t)) = 0$ *and* $\sum_{k\in\mathcal{N}_i(t)} \beta(\theta_{i,k}(t)) = 0$ *are excluded, respectively. Moreover, another solution* $\langle s_{i+1}, s_i\rangle = \gamma, i \in \mathcal{V}$ *to* $\sum_{k\in\mathcal{N}_i(t)} \beta(\theta_{i,k}(t)) = 0$ *is excluded by the contradiction of* $\sum_{i=0}^{n-1}\langle s(i+1), s(i)\rangle = 2\pi$ *and C3. Thus, one concludes that* $\rho_i$ *and* $\langle s(i+1), s(i)\rangle$ *converge to the unique equilibrium point at* $\rho_o$ *and* $2\pi/n$ *asymptotically, that is,* $\lim_{t\to\infty}\rho_i(t) = \rho_o$ *and* $\lim_{t\to\infty}\langle s(i+1), s(i)\rangle = 2\pi/n$ *from* $\lim_{t\to\infty}\alpha(\rho_i(t)) = 0$ *and* $\lim_{t\to\infty}\sum_{k\in\mathcal{N}_i(t)} \beta(\theta_{i,k}(t)) = 0$.

## IV. Numerical simulation

In this section, we consider $n = 7$ agents in the closed-loop structure composed of (1), (10), and (13), and the target (2) of an unknown constant velocity. The controller parameters are selected as follows, $k_1 = 40,000$, $k_2 = 1.5$, $k_3 = 0.006$, $k_4 = 0.2$, $\rho_o = 30$, and $M = 20$. By Remark 2, the condition C1 is satisfied for the observer (13) with $\phi_1 = 1$ and $\phi_2 = 0.2$. The initial positions are $x_1(0) = [0, 30]^\intercal$, $x_2(0) = [0, 80]^\intercal$, $x_3(0) = [30, 0]^\intercal$, $x_4(0) = [0, 50]^\intercal$, $x_5(0) = [60, 0]^\intercal$, $x_6(0) = [10, 0]^\intercal$, $x_7(0) = [0, 10]^\intercal$, and $x_d(0) = [75, 85]^\intercal$, with which the condition C2 is satisfied. Also, the initial velocities of agents and target are set as $v_i(0) = [0, 0]^\intercal$, $i \in \mathcal{V}$ and $v_d = [0.7, 0.7]^\intercal$. The parameter $\gamma = 2.2\pi/7$ satisfies the condition C3. All the above parameters are given in the SI units.

As expected by Theorem 1, the objective of moving-target-fencing in a regular polygon formation and collision avoidance can be achieved in terms of the properties P1, P2, and P3. The performance is demonstrated by the simulation results. Fig. 1 illustrates the overall trajectories of the agents and the target during the simulation course of 100 seconds, where the dashed black polygon denotes the convex hull of agents and the red circles represent the positions of the moving target. It is observed that a regular polygon is formed with the target as the centre. More specifically, the distance $\rho_i(t)$ between the agents and the target and the angle $\theta_i$ (with the operation of $+2\kappa\pi$ for an integer $\kappa$ such that $\theta_i \in (-\pi, \pi]$ for better illustration) are demonstrated in Figs. 2 and 3, respectively. The figures verify the properties P1 and P2 with $\lim_{t\to\infty}\rho_i(t) = 30$ and $\lim_{t\to\infty}|\theta_{i,k}(t)| \geq 2\pi/7$, $\forall i \neq k \in \mathcal{V}$. Additionally, Figs. 2 and 3 show $\rho_i(t) > 20$ and $\theta_{i,k}(t) \neq 0, \forall i \neq k \in \mathcal{V}$ during the whole process $t \in [0, 100]$, which verifies the collision avoidance property P3. Finally, Figs. 2 and 3 show the convergence to 0 of the line velocity $\eta_i$ and the angle velocity $\omega_i$ for $i \in \mathcal{V}$ as expected in the proof of Theorem 1. As is shown in Fig. 4, the inter-agent distances satisfy $\|x_i - x_j\| > 0, i, j \in \mathcal{V}$, which verifies that the agents are indeed capable of collision avoidance.

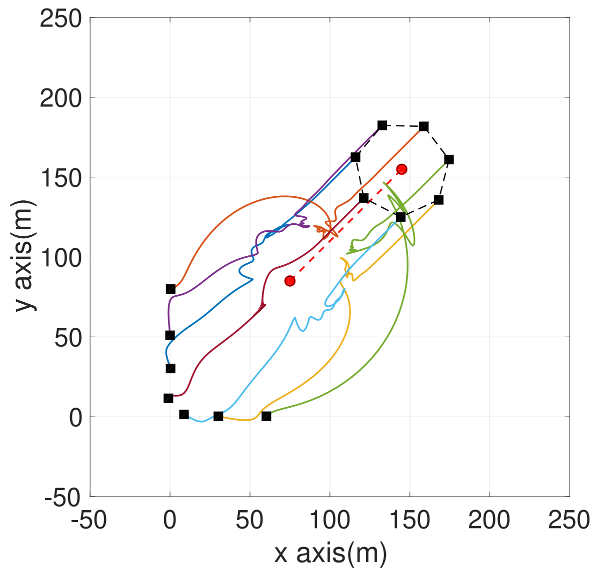


Fig. 1. Moving-target-fencing trajectories of seven agents in a two-dimensional plane. (Black rectangle: agent position with a solid line trajectory, red circle: target position with a dashed line trajectory.)

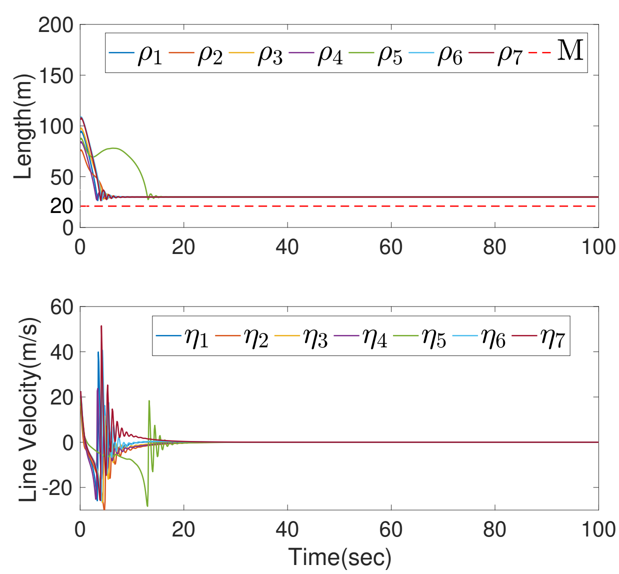


Fig. 2. Time evolution of the relative distance $\rho_i$ and the line velocity $\eta_i$ between each agent and the target.

## V. Conclusion

In this paper, we have presented a distributed controller such that a network of agents of second-order dynamics is able to fence a moving target of unknown velocity within its convex hull. The proposed control protocol consists of three functionalities: estimation of the target's velocity, regulation of distance between the agents and the target, and angle repulsion among agents. A regular polygon formation and collision avoidance are also guaranteed simultaneously. The effectiveness of the protocol has been verified in simulation. The future work may include extension of the proposed method for more complicated agent dynamics and more practical environments such as obstacle avoidance. The potential applications of the algorithm can be found in convoy protection with multi-unmanned surface vessels, motional target surrounding of unmanned systems, and collaborative monitoring of designated areas, etc., which deserves more investigation in the future work.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCNS.2021.3094784, IEEE Transactions on Control of Network Systems



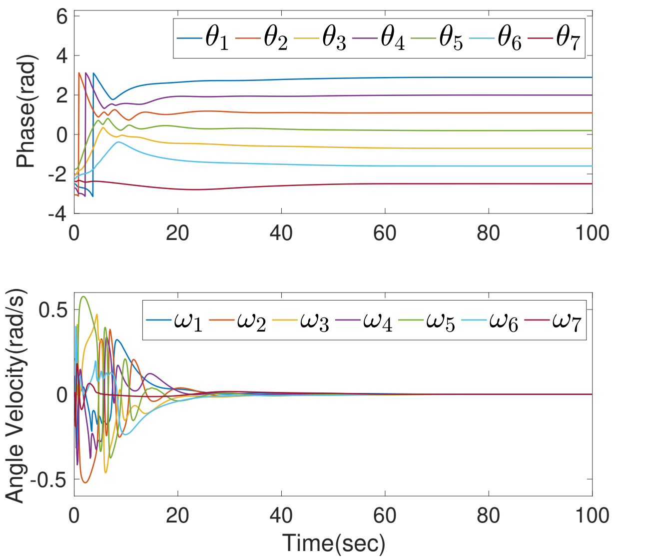


Fig. 3. Time evolution of the relative phase $\theta_i$ and the angle velocity $\omega_i$ between each agent and the target.

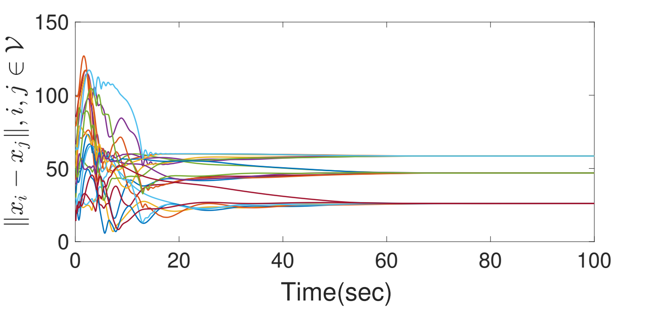


Fig. 4. Time evolution of the inter-agent distances $\|x_i - x_j\|, i, j \in \mathcal{V}$.

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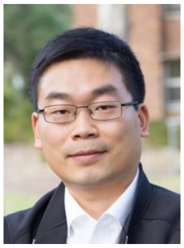

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