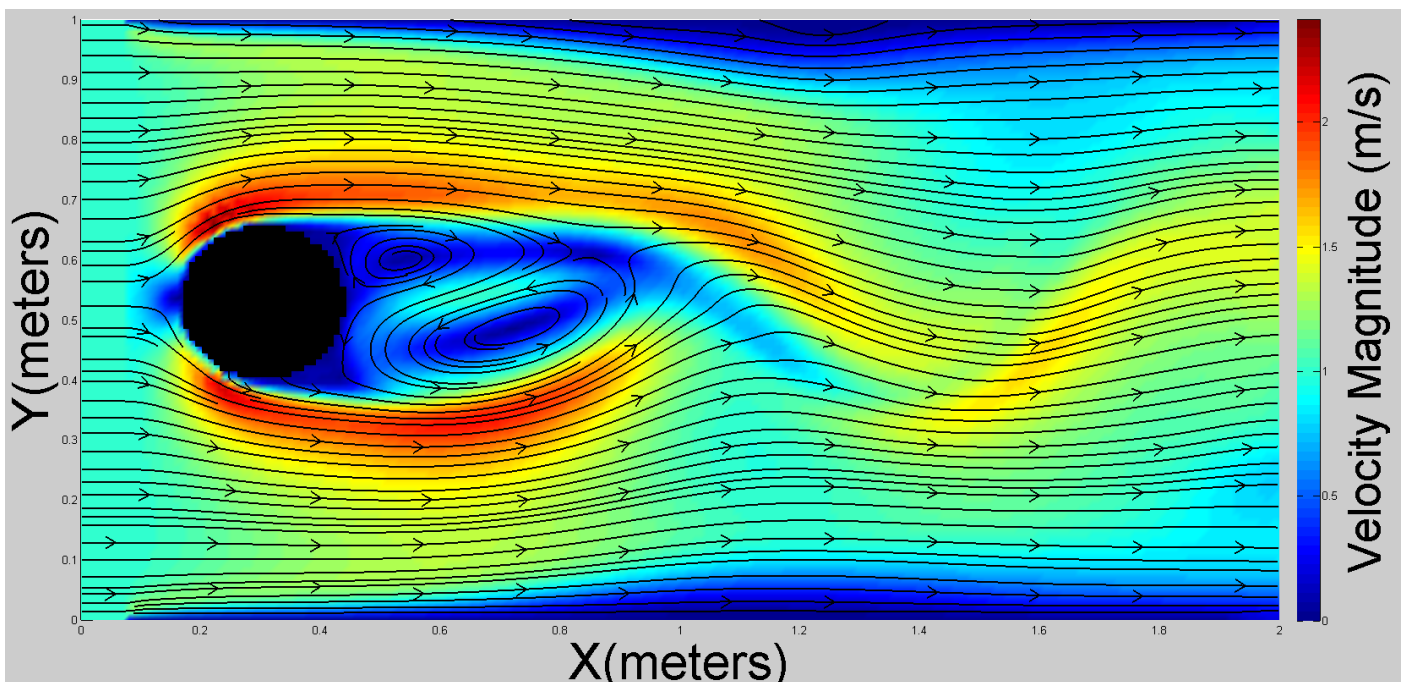


A matlab code for Numerical solution of Navier-stokes equations for two-dimensional incompressible flow (velocity-pressure formulation) along with ability for importing custom scenarios for the fluid flow.

Code Created by Jamie Johns 2018

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$
$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{dP}{dy} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$



Original source (github) of this document:

<https://github.com/JamieMJohns/Navier-stokes-2D-numerical-solve-incompressible-flow-with-custom-scenarios-MATLAB->

Note: due to work and study commitments, this document is (for now) a very brief overview of the code and will eventually be updated with further detail about the algorithms applied. [date: 23/03/2018]

Quick description of the code:

- Code applies staggered-grid scheme for velocity, following the *Marker And Cell Method (MAC Method)*. [sources of information on next pages].
- Code user can easily simulate custom scenarios of incompressible fluid flow by importing an image file (such as; jpeg, .png, .bmp etc..) along with modifying some parameters at the beginning of the code;
  - The code detects boundary conditions from the imported image (as well as what type of boundary conditions are to be used).
- The user can view resulting velocity fields in the form of still frames or an animated output, which can also be recorded as a video file (.avi file) of specified resolution, time-length and frame-rate.
- In addition, the code stores calculations for velocities (in segments of several “.mat” files) on local hard-drive as opposed to using computer memory (RAM), as calculations can be memory intensive;
  - During visualisation of calculated velocities, the code will automatically access the store files in sequence of when they were saved.
  - Provides the code user the opportunity to keep calculated velocities for later usage.

This readme provides;

- Sources of information for understanding the math/algorithm behind the code
- Demonstration of using the code (example scenarios)

Some other notes:

The code was originally created out of my own interest (rather than for the interest of a professional publication) and is subject to improvement, in particular; define a scheme (within the code) for improved handling and detection of when pressure instabilities which may occur (avoidance of divergent solutions for Pressure-Poisson equation).

For numerical solution of the pressure field (Poisson equation), a convergence criterion for the absolute difference between initial and current calculated pressure field of **0.001 is acceptable for most incompressible applications\*\*** [in the code this parameter is named “error” (i.e – error=0.001)].

**\*\*source: <https://www.flow3d.com/resources/cfd-101/numerical-issues/convergence-criteria/>**

## Outline of code algorithm

My code is inspired by MAC algorithm outlined in <https://www3.nd.edu/~qtryggva/CFD-Course2017/Lecture-5-2017.pdf>, which applied the algorithm to a standard “driven cavity” flow simulation, the algorithm which numerically solves 2D navier-stokes equations of incompressible flow in conservative form for a velocity-pressure formulation.

For my code, I have extended (and vectorised) the algorithm to handle custom (and more complex) fluid flow simulations for which the user can import the scenario as a image file and specify parameters at beginning of code such as;

- >Magnitude and direction of velocity for regions of constant velocity.
- >physical resolution of simulation (grid resolution)
- >time resolution of simulation (delta time)
- >fluid density
- >dynamic viscosity
- >and tolerance of error for solution to pressure poisson equation.

From importing an image (drawing of fluid flow scenario), the code detects conditions such as;

- > regions of free fluid flow (when a image pixel is (or close to) the white in colour).
- >region of constant velocity (when a image pixel is close to red).
- >regions of no fluid flow (“solid walls”, when a image pixel is close to black).
- >fluid flow out-lets (when an edge pixel of image is close to the colour green).

Several examples are provided on the pages below to give the user an idea about how to define a particular fluid flow simulation using an imported image as well defining parameters in the code.

The Naiver-Stokes equations for two-dimensional incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} + \vartheta \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{dP}{dy} + \vartheta \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

The above equations are solved in conservative form:]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

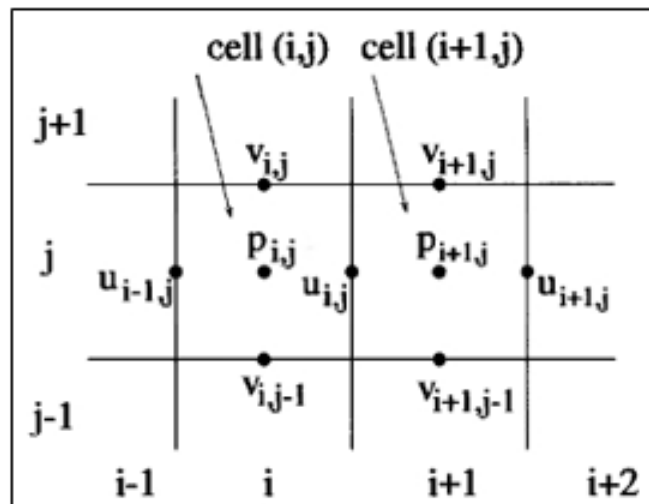
$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial vu}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} + \vartheta \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} = -\frac{1}{\rho} \frac{dP}{dy} + \vartheta \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

$(u, v) = (x, y)$  component velocity

$t = \text{time}$  |  $\rho = \text{density}$  |  $\vartheta = \text{dynamic viscosity}$  |  $P = \text{pressure}$

Staggered grid is applied:



(image source: <http://www.imrt.com.br/en/simulation-unconstrained-solidification-a356-aluminum/articulo/S2238785413001129/>)

# List of files

Below is the complete list of files required to run the code. For quality control and to ensure that you are not running any corrupt code on your computer, make sure that these files are first downloaded from the original source that this document came from:

<https://github.com/JamieMJohns/Navier-stokes-2D-numerical-solve-incompressible-flow-with-custom-scenarios-MATLAB->

## **Master file:**

**Main.m** – main matlab file

## **Dependent files [these files are automatically used when running main.m]:**

**imagegrab.m** – code that converts image files (i.e - .png,.jpg etc.) to data which is used in main.m

**progressdisp.m** – code that is used to display calculation progress in main.m

**storevar.m** – code that saves velocity data onto hardrive when running main.m (to avoid high RAM usage). The velocity data is stored as “.mat” files in directory “(matlab directory)\ temporary\_NS\_velocity” [this directory is automatically created when running .mat]/

**openvar.m** – code which retrieves (in correct sequence) files stored on to hardrive which are calculated velocity.

**visualisation\_still\_frame.m**– code for producing visualisation of velocity at single instant of time (frame); this file is used within main.m

**visualisation\_quick\_animation.m**– code for producing quick animated visualisation of velocity changing over time; this file is used within main.m

**visualisation\_record\_video.m**– code for producing animated visualisation of velocity changing over time along with the ability to record as video (.avi) file; this file is used within main.m

## **Additional files from github repository:**

**.png files** – example scenarios of fluid flow [refer to example usage in next section of this document]

## Example Usage of code

### Example 1.

This fluid simulation is commonly known 'Lid driven cavity flow' and consist of a fluid trapped inside a box with the top wall (of the box) moving at a constant velocity which moves the fluid inside the box.



.png file of resolution 1000x1000

Direction of the constant velocity specified in the code.

Pixel colour in imported diagram

Red=constant velocity region

White = region of free fluid flow

Boundary of imported diagram are handled as solid wall boundaries.

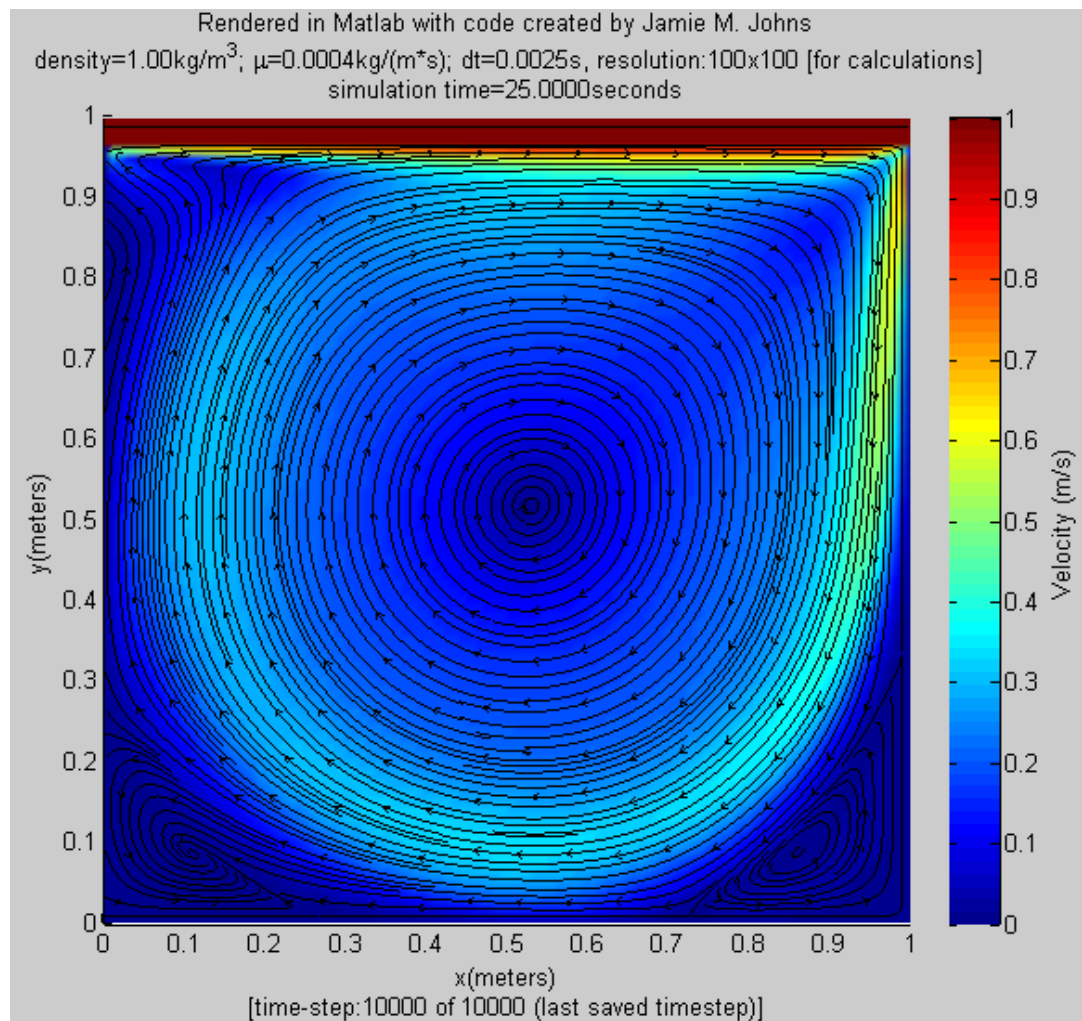
### Example settings

```
%Parameters for scenario (Modify these) #####
%set information about domain of simulation@@@@@@@@@
SCENARIO='scenario_driven_lid.png'; %<--- file (.
domainX=1; % length of domain (x-axis) [unit: me
xinc=100; %number of nodes across x-component of
dt=1/400; %set set delta time [unit: seconds]
MI=10000; %number of time steps to perform calcul
velyi=0; %y-component velocity of region with co
    %[velyi>0,velocity has vector -y with
velxi=1; %x-component velocity of region with co
    %[velxi>0,velocity has vector +x with
    %[if velxi=0.1 and velyi=-1, vector is = 0.1[X]+
dens=1; %density [unit: kg/m^3] , water(pure)=1
mu=1/2500; %0.001 %dynamic viscosity [kg/(m*s)]

%Poisson Pressure solver parameters!!!!!!!!!!!!!!
error=0.001; %set tolerance of error for converg
MAXIT=1000; %maximum number of iterations allowe
MINIT=1; %mininum number of iterations allowed f
    % Note that: MINIT should be less than MAXIT
%!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

%save parameters $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
spacelim=5; %limit for hardrive space usage (in giga
ST=[100 100 500]; % FOR variables of dimensions of S
    % save variable data for x and y co
    % in chunks of files , each with
    % size matrix.....this reduces
    %(increasing ST(3) will reduce num
    %(decreasing ST(3) will reduce num
    %[Files; openvar.m and savevar.m a
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
#####
```

### Example output from settings on previous page



## Example 2

Similar to example 1 but now the domain width is twice the length of the domain height.



Direction of the constant velocity specified in the code.

Pixel colour in imported diagram

Red=constant velocity region

White = region of free fluid flow

Boundary of imported diagram are handled as solid wall boundaries.

above: image that is imported into matlab

(.png file of resolution: 2000x1000)

### Example settings

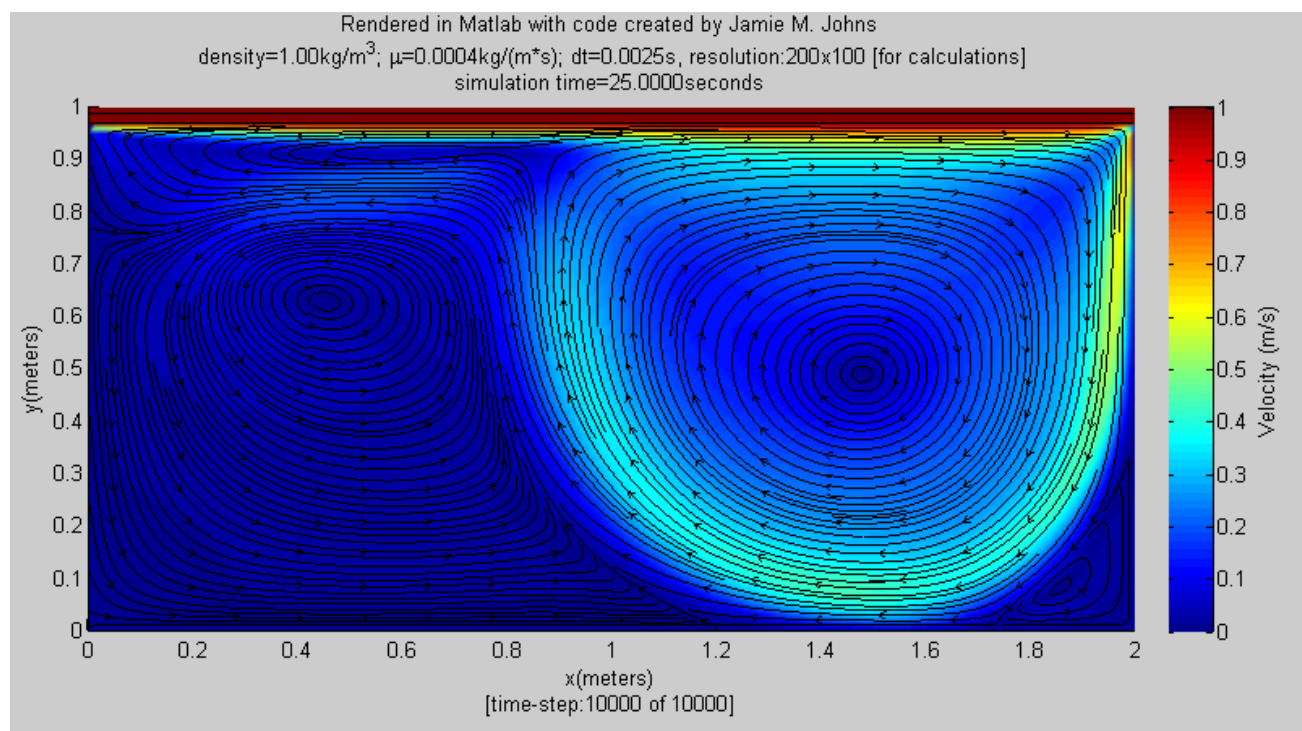
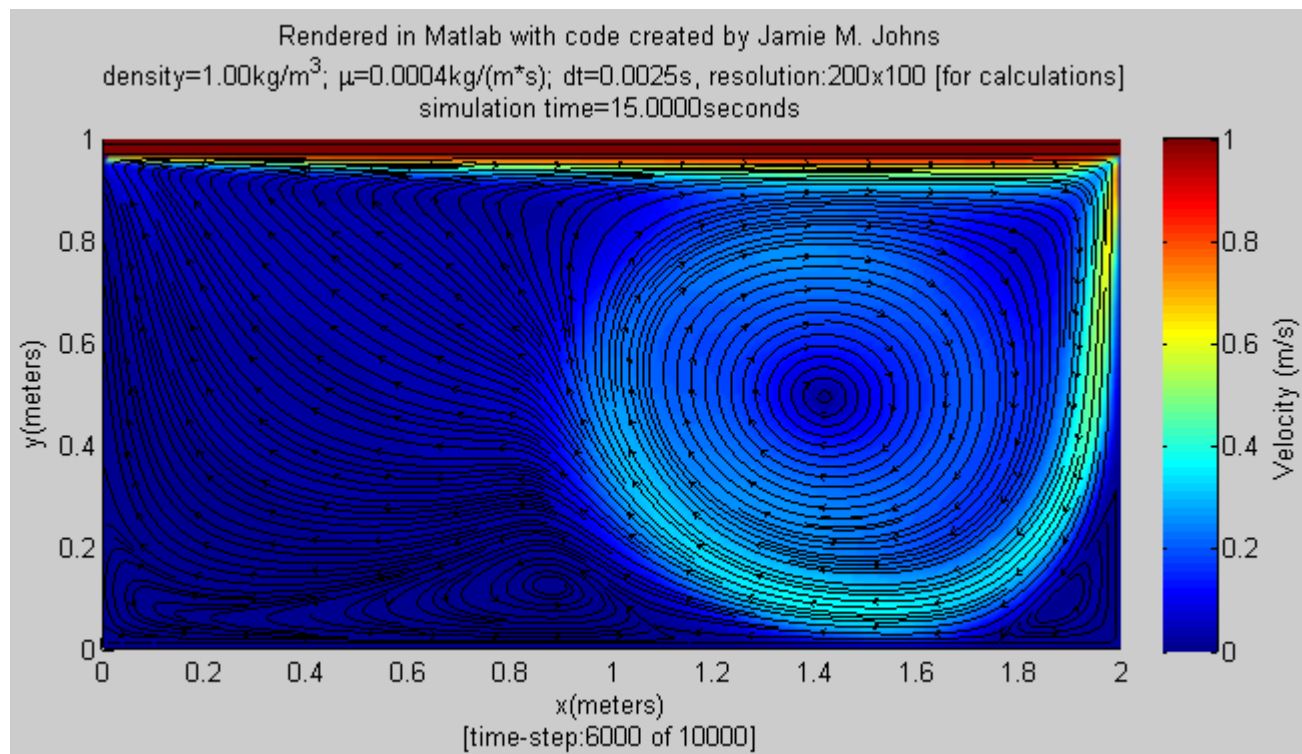
```
%Parameters for scenario (Modify these) #####
%set information about domain of simulation@@@@@
SCENARIO='scenario_driven_lid_long.png'; %<---
domainX=2; % length of domain (x-axis) [unit:
xinc=200; %number of nodes across x-component
dt=1/400; %set set delta time [unit: seconds]
MI=10000; %number of time steps to perform cal
velyi=0; %y-component velocity of region with
    %[velyi>0,velocity has vector -y wi
velxi=1; %x-component velocity of region with
    %[velxi>0,velocity has vector +x wi
    %[if velxi=0.1 and velyi=-1, vector is = 0.1[2
dens=1; %density [unit: kg/m^3] , water(pure)
mu=1/2500; %0.001 %dynamic viscosity [kg/(m*s)

%Poisson Pressure solver parameters!!!!!!!!!!!!!!
error=0.001; %set tolerance of error for conve
MAXIT=1000; %maximum number of iterations allo
MINIT=1; %mininum number of iterations allowe
    % Note that: MINIT should be less than MAXIT
%!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

%save parameters $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
spacelim=5; %limit for harddrive space usage (in g
ST=[100 100 500]; % FOR variables of dimensions of
    % save variable data for x and y
    % in chunks of files , each wit
    % size matrix.....this reduc
    %(increasing ST(3) will reduce r
    %(decreasing ST(3) will reduce r
    %[Files; openvar.m and savevar.m
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
#####
```

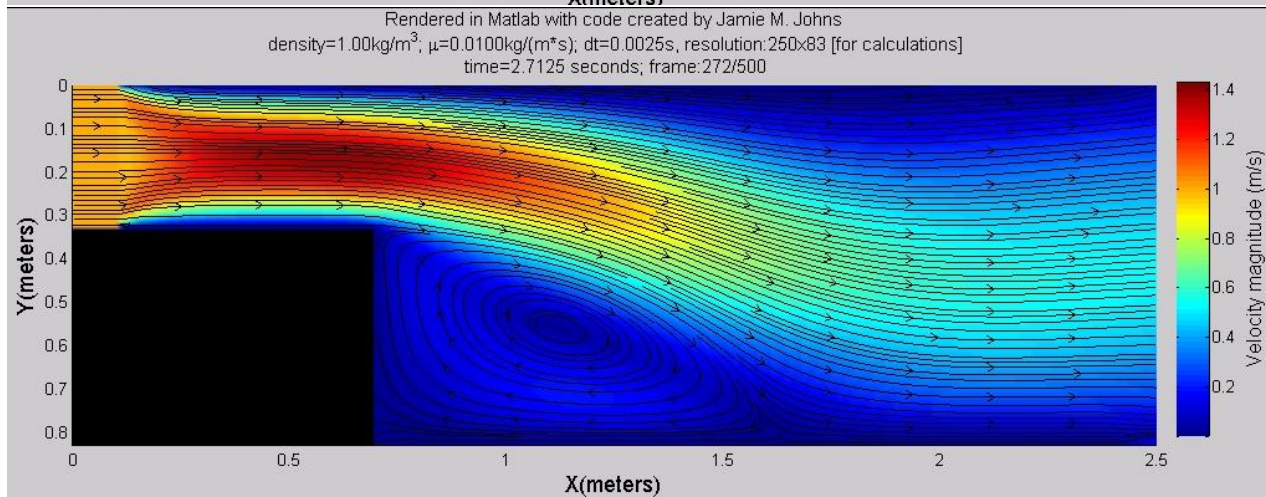
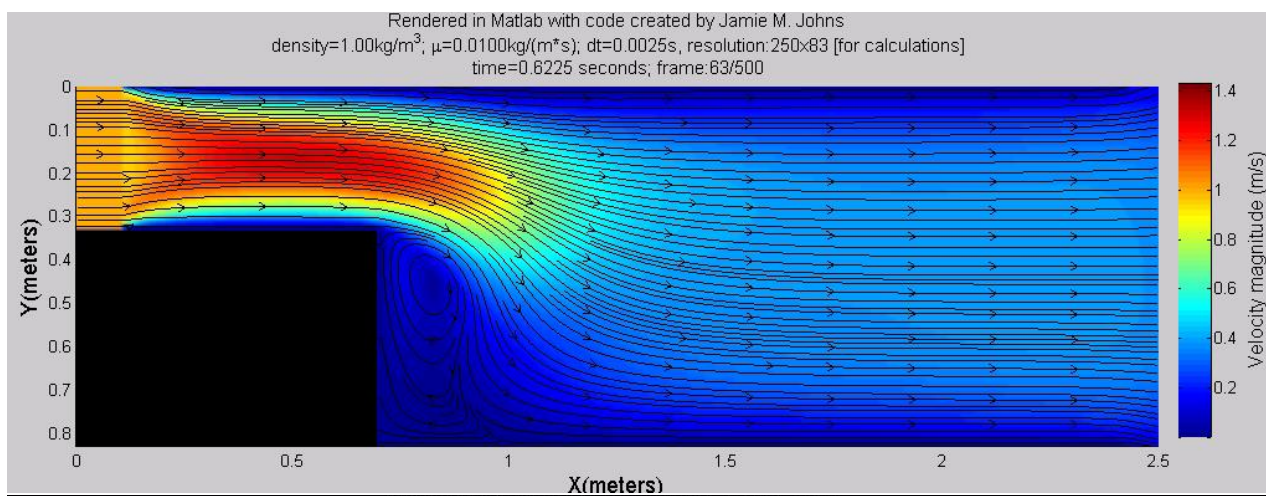
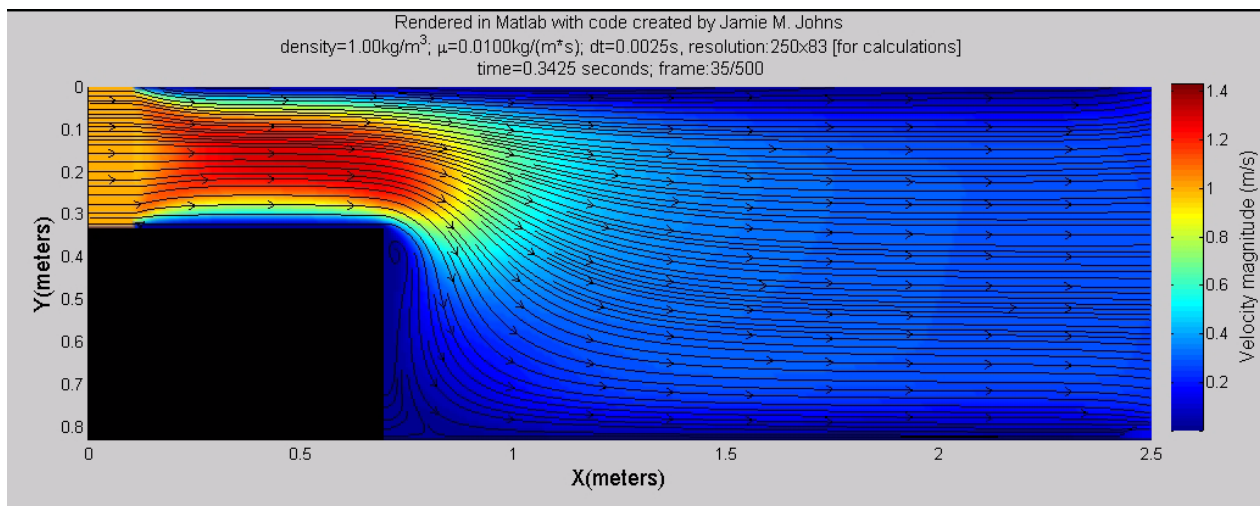


**Example output from settings on previous page**



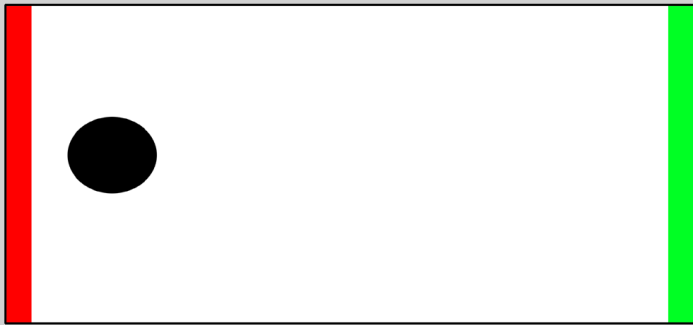


## Example output from settings on previous page



## Example 4

Simulation of fluid flow around a two-dimensional sphere.



Direction of the constant velocity specified in the code.

Pixel colour in imported diagram

Red=constant velocity region

White = region of free fluid flow

Green = fluid outlet

Black = region of no fluid flow ("solid wall")

Above: image that is imported into matlab

(.png file of resolution: 2000x1000)

The edges of the imported image are handled as solid walls, except, for where an edge pixel (of image) is coloured green. Then, the edges at these locations will be handled as fluid outlet.

### Example settings

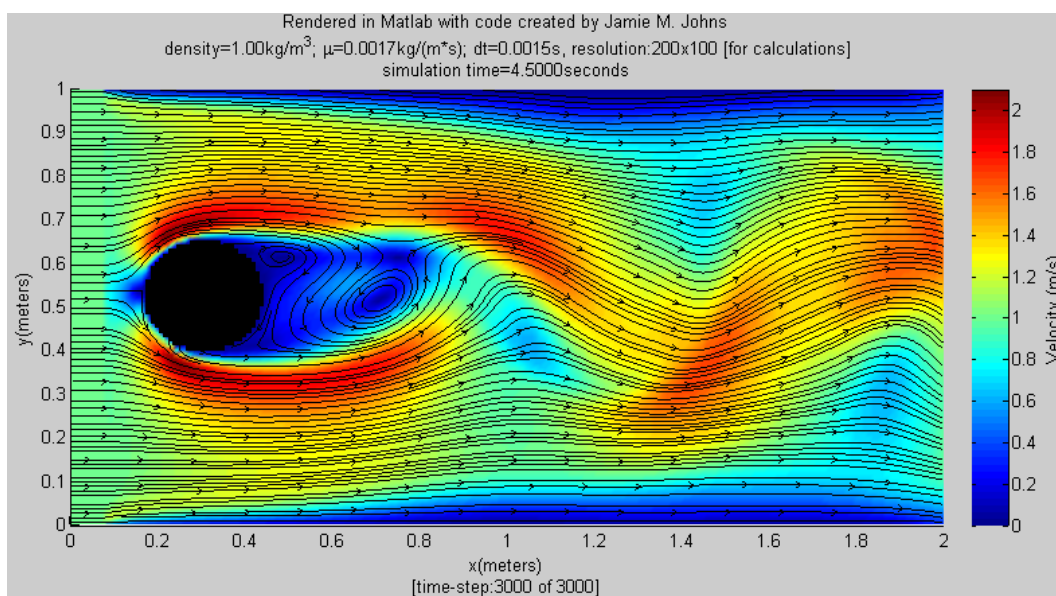
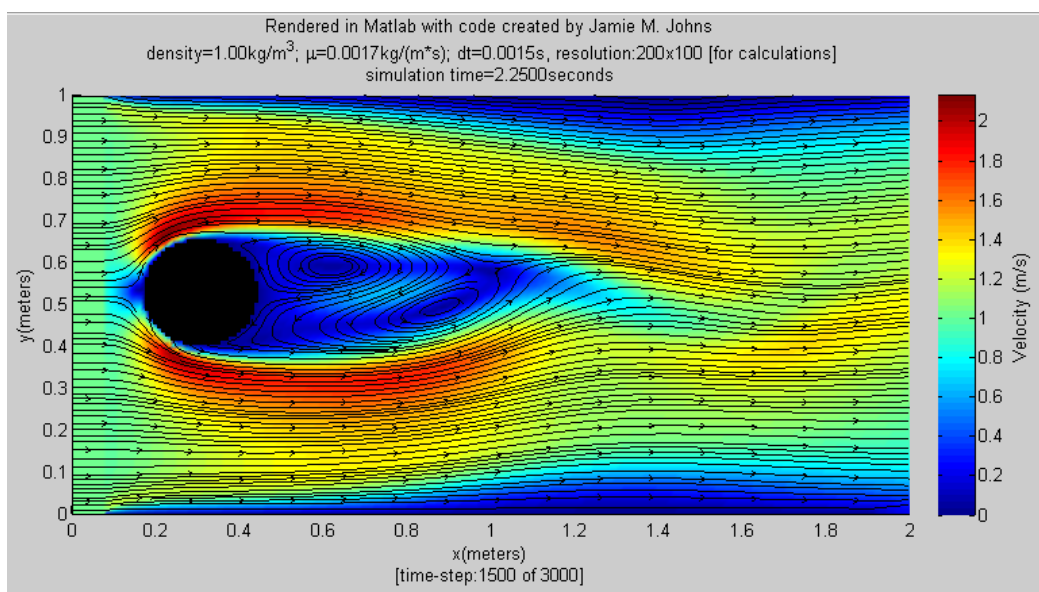
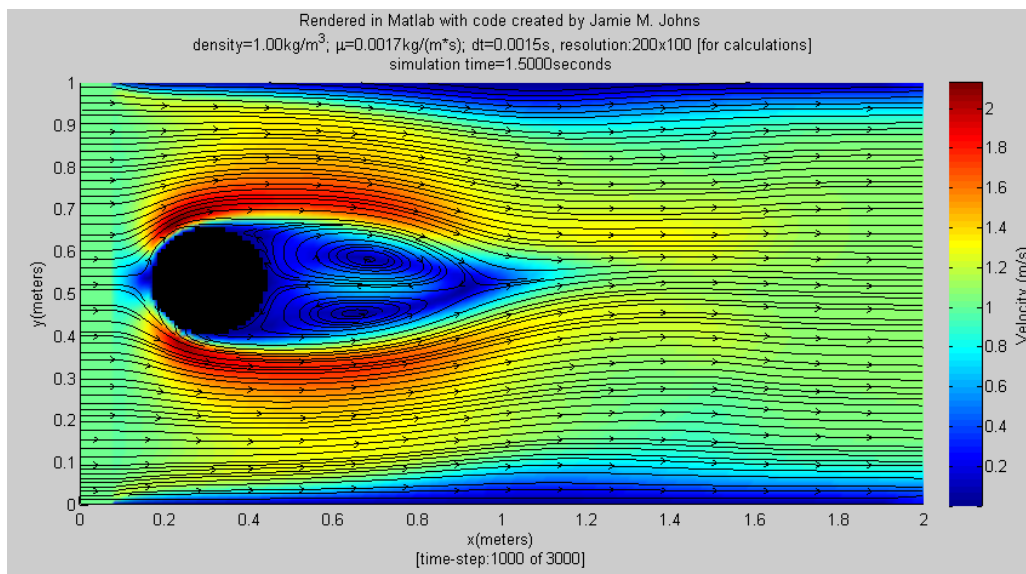
```
%Parameters for scenario (Modify these) #####
%set information about domain of simulation#####
SCENARIO='scenario_sphere.png'; %<--- file (image)
domainX=2; % length of domain (x-axis) [unit: meter
xinc=200; %number of nodes across x-component of do
dt=0.0015; %set set delta time [unit: seconds]
MI=3000; %number of time steps to perform calculati
velyi=0; %y-component velocity of region with const
    %[velyi>0,velocity has vector -y with ma
velxi=1; %x-component velocity of region with const
    %[velxi>0,velocity has vector +x with ma
    %[if velxi=0.1 and velyi=-1, vector is = 0.1[X]+(-1
dens=1; %density [unit: kg/m^3] , water(pure)=1000
mu=1/600; %0.001 %dynamic viscosity [kg/(m*s)]

%Poisson Pressure solver parameters!!!!!!!!!!!!!!
error=0.001; %set tolerance of error for convergenc
MAXIT=1000; %maximum number of iterations allowed f
MINIT=1; %mininum number of iterations allowed for
    % Note that: MINIT should be less than MAXIT
%!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

%save parameters #####
spacelim=5; %limit for harddrive space usage (in gigabyt
ST=[100 100 500]; % FOR variables of dimensions of ST(1
    % save variable data for x and y comp
    % in chunks of files , each with ST(
    % size matrix.....this reduces me
    %(increasing ST(3) will reduce number
    %(decreasing ST(3) will reduce number
    %[Files; openvar.m and savevar.m are
#####
#####
```



## Example output from settings on previous page



## Example 5

Another simulation of fluid flow around a two-dimensional sphere.



Above: image that is imported into matlab  
(.png file of resolution: 2000x500)

Direction of the constant velocity  
specified in the code.

Pixel colour in imported diagram

Red=constant velocity region

White = region of free fluid flow

Green = fluid outlet

Black = region of no fluid flow ("solid wall")

The edges of the imported image are handled as solid walls, except, for where an edge pixel (of image) is coloured green. Then, the edges at these locations will be handled as fluid outlet.

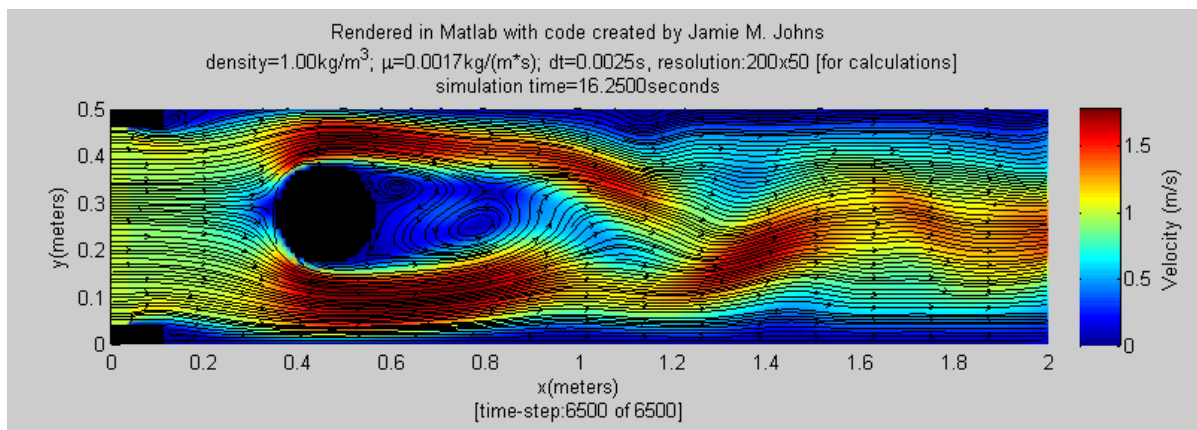
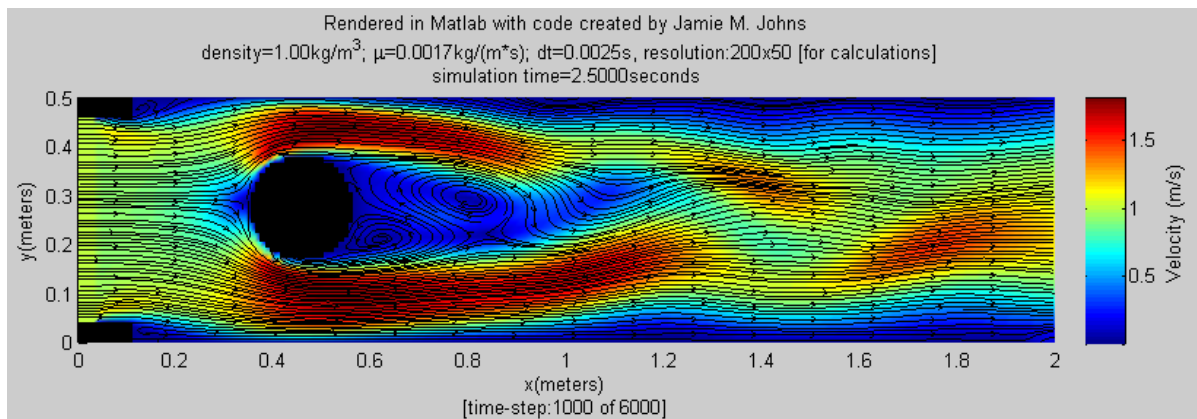
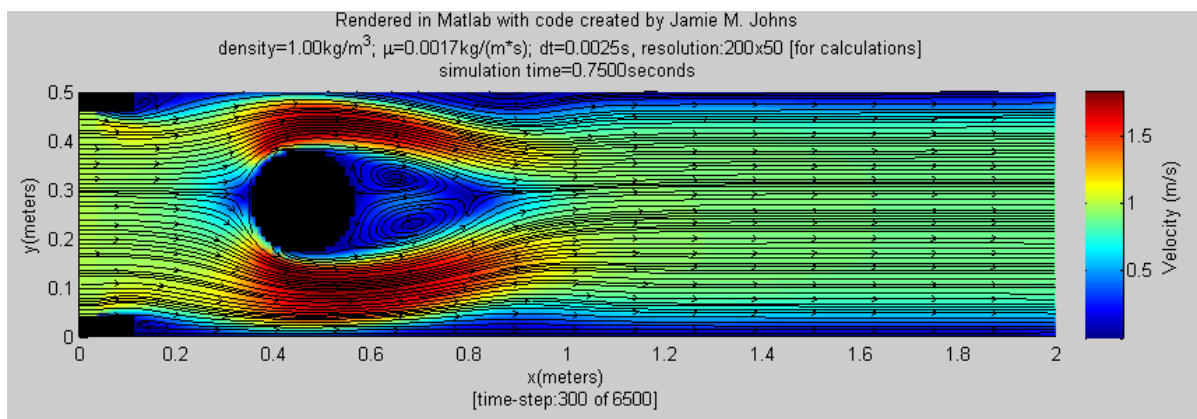
### Example settings

```
%Parameters for scenario (Modify these) #####
%set information about domain of simulation#####
SCENARIO='scenario_sphere2.png'; %<--- file (image) that
domainX=2; % length of domain (x-axis) [unit: meters]
xinc=200; %number of nodes across x-component of domain (
dt=1/400; %set set delta time [unit: seconds]
MI=6500; %number of time steps to perform calculations [t
velyi=0; %y-component velocity of region with constant ve
    %[velyi>0,velocity has vector -y with mag abs(
velxi=1; %x-component velocity of region with constant ve
    %[velxi>0,velocity has vector +x with mag abs(
    %[if velxi=0.1 and velyi=-1, vector is = 0.1[X]+(-1)[Y] a
dens=1; %density [unit: kg/m^3] , water(pure)=1000 blood
mu=1/600; %0.001 %dynamic viscosity [kg/(m*s)]

%Poisson Pressure solver parameters!!!!!!!!!!!!!!!!!!!!!!
error=0.001; %set tolerance of error for convergence pois
MAXIT=1000; %maximum number of iterations allowed for poi
MINIT=1; %minimum number of iterations allowed for poisso
    % Note that: MINIT should be less than MAXIT
%!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

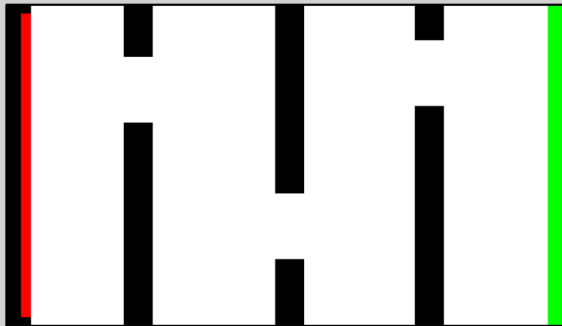
%save parameters #####
spacelim=5; %limit for harddrive space usage (in gigabytes) fo
ST=[100 100 500]; % FOR variables of dimensions of ST(1)xST(2
    % save variable data for x and y component
    % in chunks of files , each with ST(1)xST(
    % size matrix.....this reduces memory a
    %(increasing ST(3) will reduce number of ex
    %(decreasing ST(3) will reduce number of ex
    %[Files; openvar.m and savevar.m are used]
#####
#####
```

## Example outputs from settings on previous page



## Example 6

Simulation of fluid flow through a very simple maze.



Direction of the constant velocity specified in the code.

Pixel colour in imported diagram

Red=constant velocity region

White = region of free fluid flow

Green = fluid outlet

Black = region of no fluid flow ("solid wall")

Above: image that is imported into matlab  
(.png file of resolution: 1000x500)

The edges of the imported image are handled as solid walls, except, for where an edge pixel (of image) is coloured green. Then, the edges at these locations will be handled as fluid outlet.

### Example settings

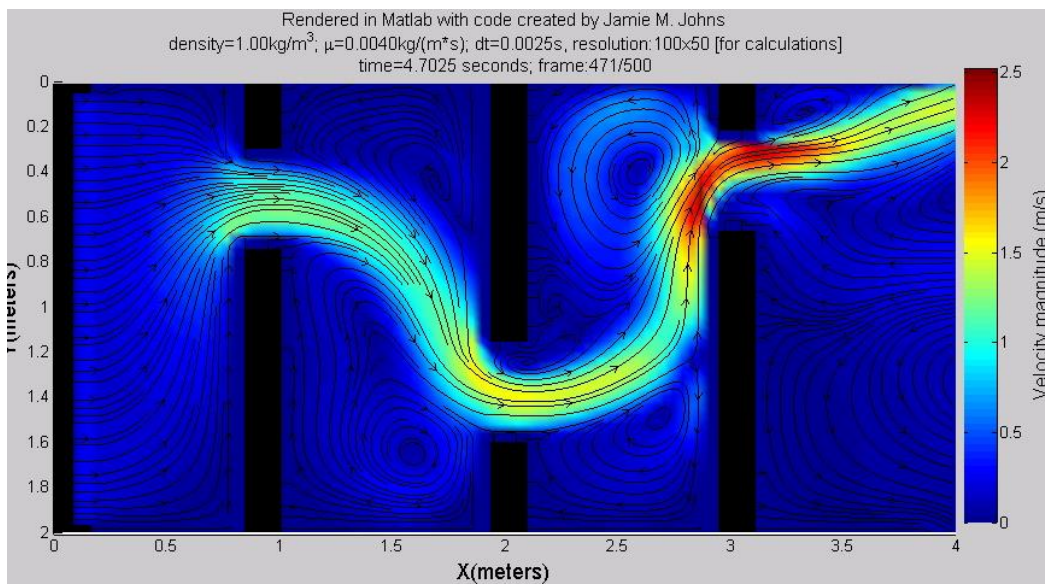
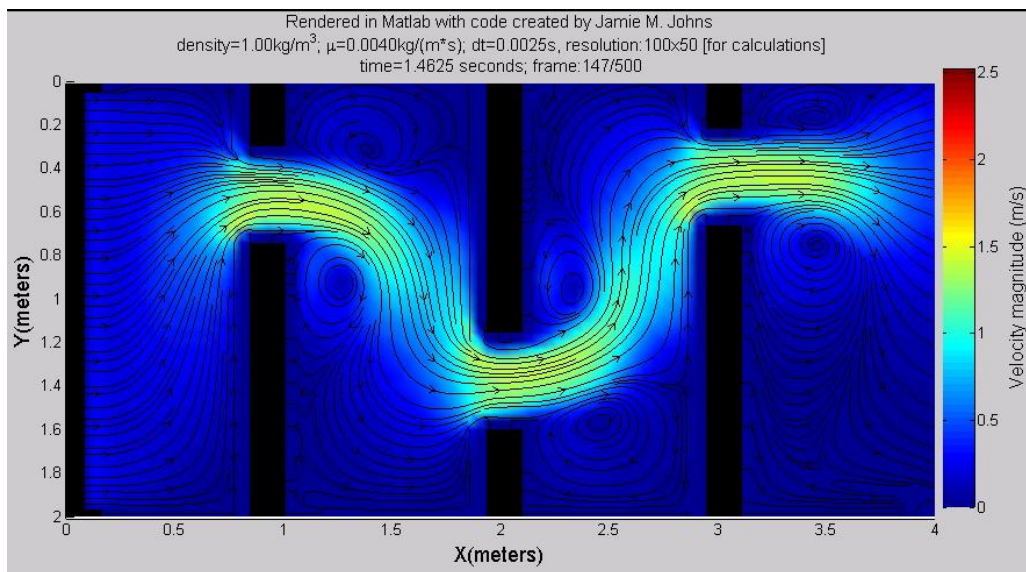
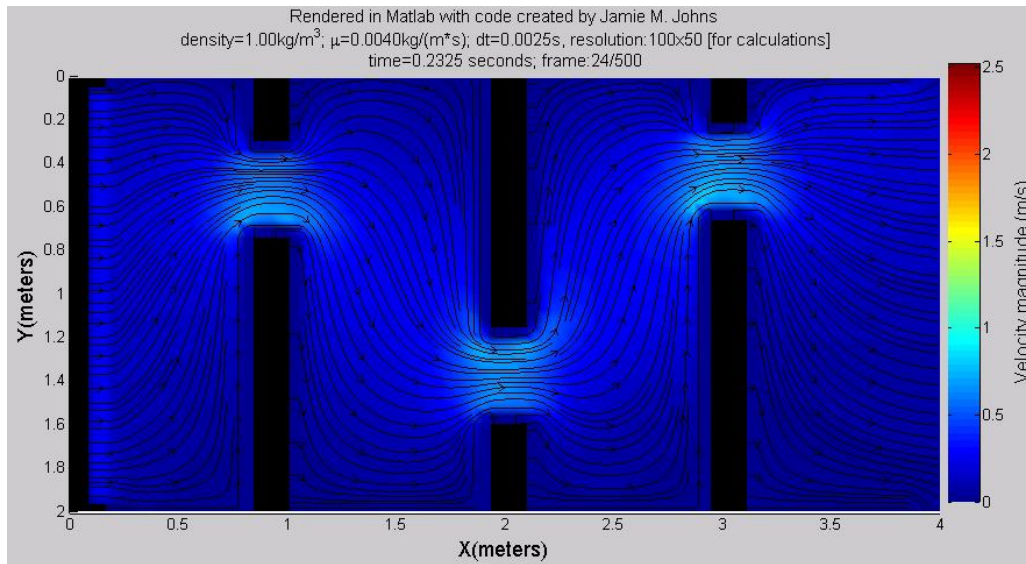
```
%Parameters for scenario (Modify these) #####
%set information about domain of simulation
SCENARIO='scenario_maze.png'; %<--- fil
domainX=4; % length of domain (x-axis)
xinc=100; %number of nodes across x-com
dt=0.0025; %set set delta time [unit: s
MI=2000; %number of time steps to perf
velyi=0; %y-component velocity of regio
    %[velyi>0,velocity has vecto
velxi=0.25; %x-component velocity of re
    %[velxi>0,velocity has vecto
    %[if velxi=0.1 and velyi=-1, vector is
dens=1; %density [unit: kg/m^3] , wate
mu=1/250; %0.001 %dynamic viscosity [kg

%Poisson Pressure solver parameters!!!!!!
error=0.001; %set tolerance of error fo
MAXIT=500; %maximum number of iteration
MINIT=1; %mininum number of iterations
    % Note that: MINIT should be less than
%!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

%save parameters $$$$$$$$$$$$$$$$$$$$$$$$
spacelim=5; %limit for hardrive space usage
ST=[100 100 500]; % FOR variables of dimens
    % save variable data for
    % in chunks of files , e
    % size matrix.....thi
    %(increasing ST(3) will r
    %(decreasing ST(3) will r
    %[Files; openvar.m and sa
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
#####
```

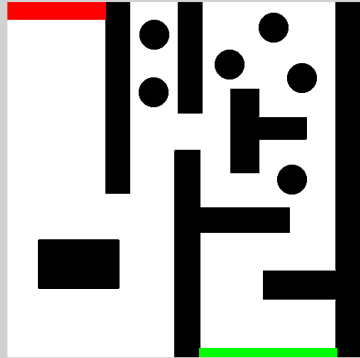


## Example outputs from settings on previous page



## Example 7

This scenario simulates fluid flowing through a random maze.



Direction of the constant velocity  
specified in the code.

Pixel colour in imported diagram

Red=constant velocity region

White = region of free fluid flow

Green = fluid outlet

Black = region of no fluid flow ("solid wall")

Above: image that is imported into matlab  
(.png file of resolution: 1000x1000)

The edges of the imported image are handled as solid walls, except, for where an edge pixel (of image) is coloured green. Then, the edges at these locations will be handled as fluid outlet.

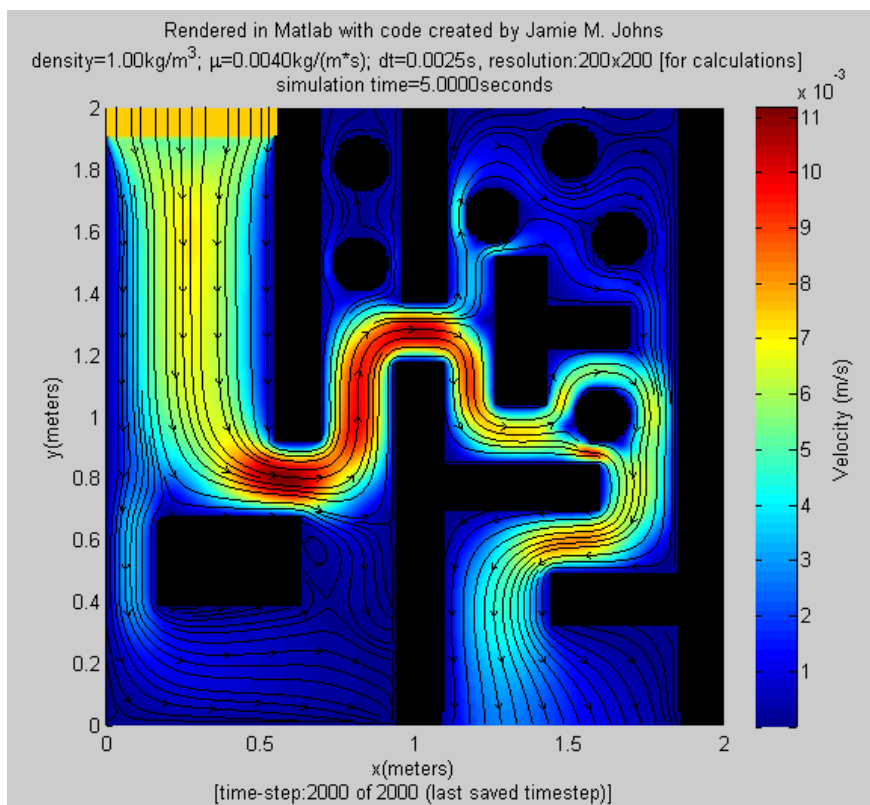
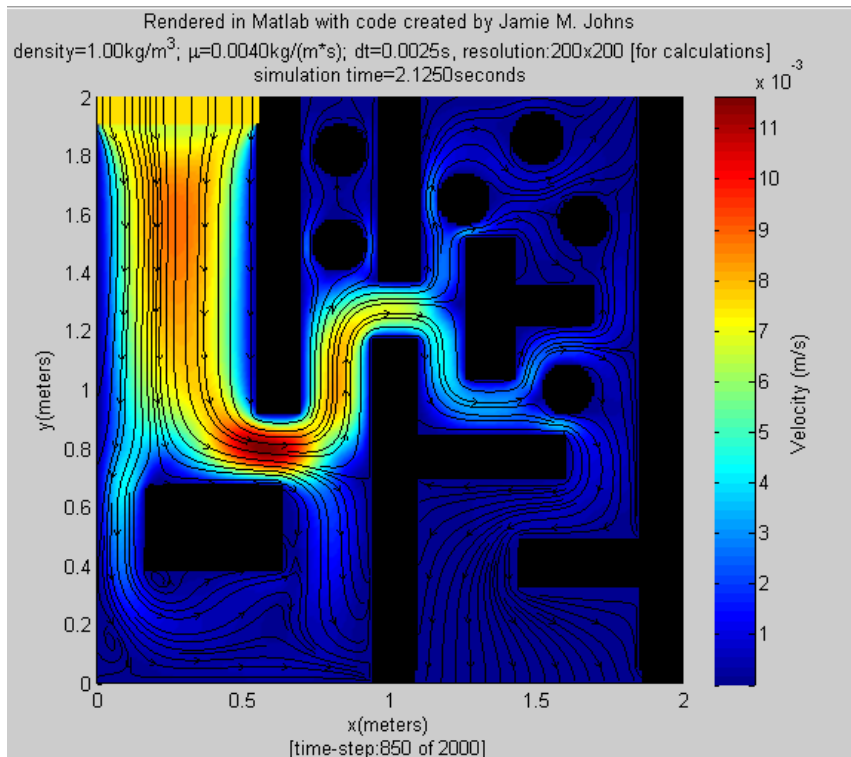
### Example settings

```
%Parameters for scenario (Modify these) #####
%set information about domain of simulation@
SCENARIO='scenario_maze_random.png'; %<-
domainX=2; % length of domain (x-axis) [
xinc=200; %number of nodes across x-comp
dt=0.0025; %set set delta time [unit: se
MI=2000; %number of time steps to perform
velyi=0.0075; %y-component velocity of r
    %[velyi>0,velocity has vector
velxi=0; %x-component velocity of region
    %[velxi>0,velocity has vector
    %[if velxi=0.1 and velyi=-1, vector is =
dens=1; %density [unit: kg/m^3] , water
mu=1/250; %0.001 %dynamic viscosity [kg/

%Poisson Pressure solver parameters!!!!!!!
error=0.001; %set tolerance of error for
MAXIT=500; %maximum number of iterations
MINIT=1; %mininum number of iterations a
    % Note that: MINIT should be less than M
%!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

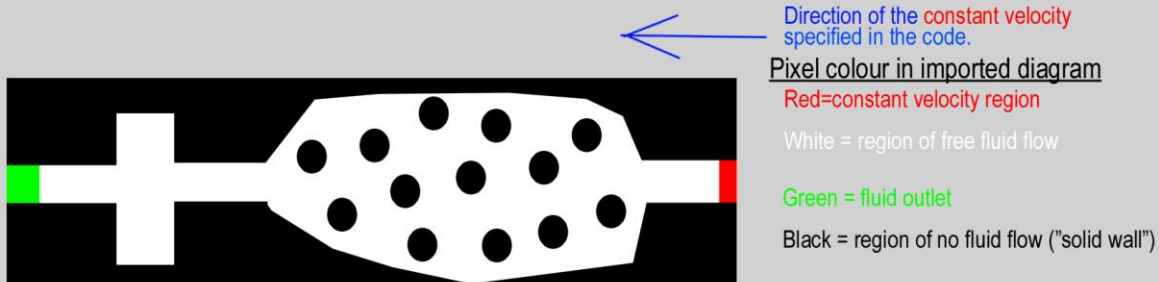
%save parameters $$$$$$$$$$$$$$$$$$$$$$$$$$
spacelim=5; %limit for hardrive space usage
ST=[100 100 500]; % FOR variables of dimensi
    % save variable data for x
    % in chunks of files , ea
    % size matrix.....this
    %(increasing ST(3) will re
    %(decreasing ST(3) will re
    %[Files; openvar.m and sav
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
#####
```

### Example output from settings on previous page



## Example 8

Another random fluid flow scenario.



Above: image that is imported into matlab  
(.png file of resolution: 2000x500)

The edges of the imported image are handled as solid walls, except, for where an edge pixel (of image) is coloured green. Then, the edges at these locations will be handled as fluid outlet.

### Example settings

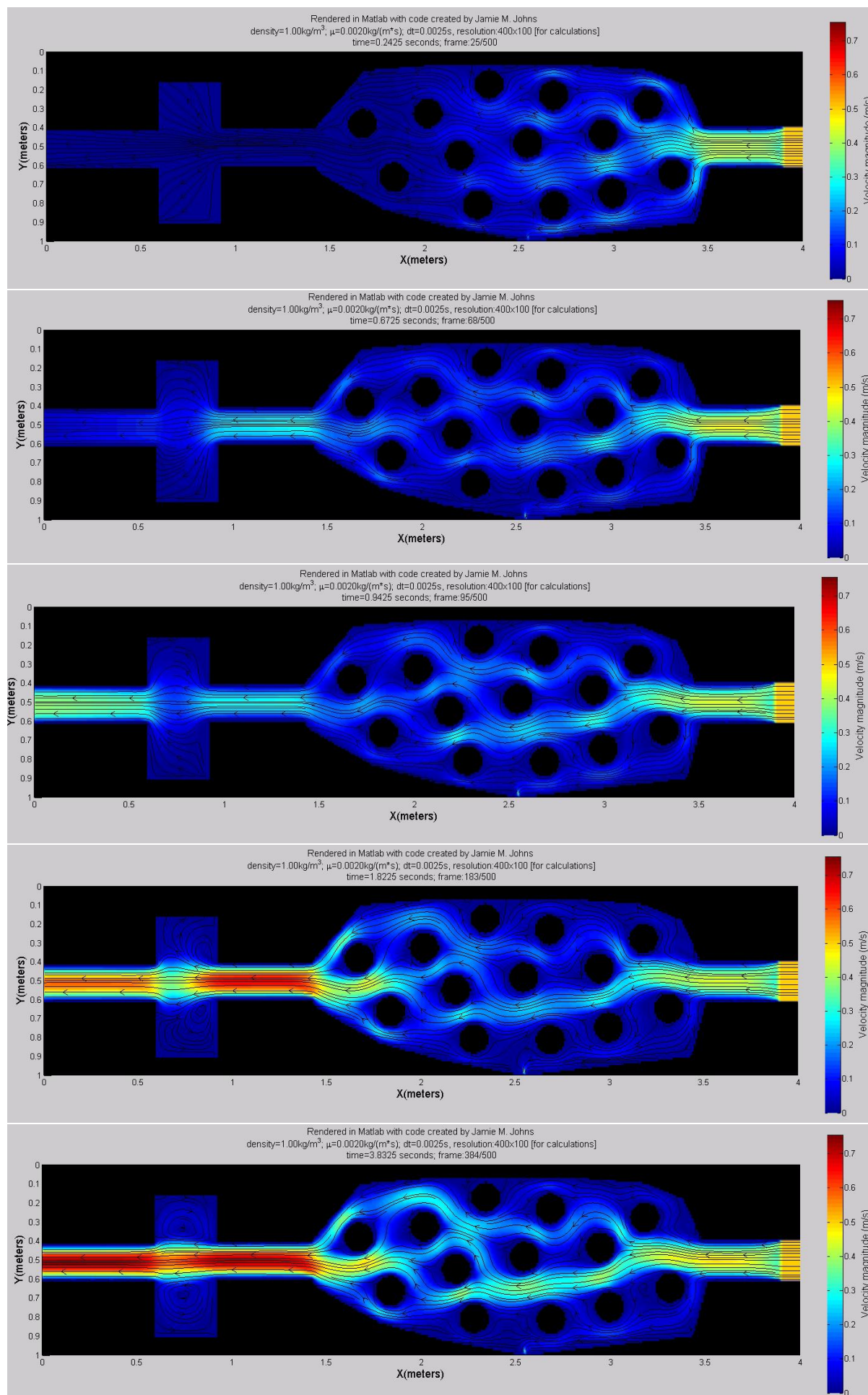
```
%Parameters for scenario (Modify these)
%set information about domain of simulation
SCENARIO='scenario_random.png';
domainX=4; % length of domain (in units)
xinc=400; %number of nodes across domain
dt=0.0025; %set set delta time
MI=2000; %number of time steps
velyi=0; %y-component velocity
%[velyi>0,velocity h
velxi=0.25; %x-component velocity
%[velxi>0,velocity h
%[if velxi=0.1 and velyi=-1, ve
dens=1; %density [unit: kg/m^3
mu=1/250; %0.001 %dynamic visco

%Poisson Pressure solver parameters
error=0.001; %set tolerance of
MAXIT=500; %maximum number of i
MINIT=1; %mininum number of ite
% Note that: MINIT should be le
%!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

%save parameters $$$$$$$$$$$$$$$$$$
spacelim=5; %limit for harddrive spa
ST=[100 100 500]; % FOR variables c
% save variable c
% in chunks of f
% size matrix....
%(increasing ST(3)
%(decreasing ST(3)
%[Files; openvar.
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
#####
```



## Example outputs from settings on previous page



## Example 9

Another random example simulating a creek/drain scenario.



above: image that is imported into matlab  
(.png file of resolution: 1000x500)

Direction of the constant velocity  
specified in the code.

Pixel colour in imported diagram  
Red=constant velocity region  
White = region of free fluid flow  
Green = fluid outlet  
Black = region of no fluid flow ("solid wall")

The edges of the imported image are handled as solid walls, except, for where an edge pixel (of image) is coloured green. Then, the edges at these locations will be handled as fluid outlet.

### Example settings

```
%Parameters for scenario (Modify these) #####
%set information about domain of simulation#####
SCENARIO='scenario_creek.png'; %<--- file (image) th
domainX=4; % length of domain (x-axis) [unit: meters
xinc=400; %number of nodes across x-component of dom
dt=0.0025; %set set delta time [unit: seconds]
MI=2000; %number of time steps to perform calculation
velyi=0; %y-component velocity of region with constan
    %[velyi>0,velocity has vector -y with mag
velxi=0.25; %x-component velocity of region with con
    %[velxi>0,velocity has vector +x with mag
    %[if velxi=0.1 and velyi=-1, vector is = 0.1[X]+(-1)
dens=1; %density [unit: kg/m^3] , water(pure)=1000
mu=1/250; %0.001 %dynamic viscosity [kg/(m*s)]

%Poisson Pressure solver parameters!!!!!!!!!!!!!!!!!!!!!!
error=0.001; %set tolerance of error for convergence
MAXIT=500; %maximum number of iterations allowed for
MINIT=1; %minimum number of iterations allowed for p
    % Note that: MINIT should be less than MAXIT
%!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

%save parameters #####
spacelim=5; %limit for hardrive space usage (in gigabyte
ST=[100 100 500]; % FOR variables of dimensions of ST(1):
    % save variable data for x and y compo
    % in chunks of files , each with ST(1
    % size matrix.....this reduces memo
    %(increasing ST(3) will reduce number o
    %(decreasing ST(3) will reduce number o
    %[Files; openvar.m and savevar.m are u
#####
%#####
```

### Example output from settings on previous page

