Design of a Practical Real-time Mixed-path Interpolator for 3-Axis CNC Machining

学生： 邱满满
导师： 李建刚 副教授
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Conventional approach of machining curves

CAD

CAM

Parameter curve (NURBS Curve)

Discontinuous NURBS curve
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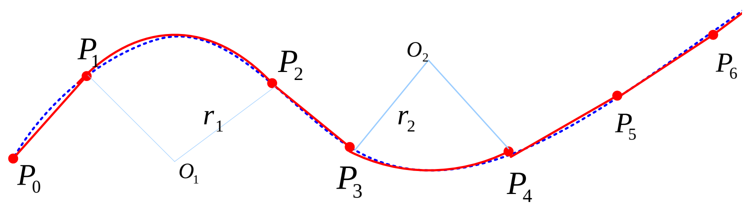
Line/Arc segments



Drawbacks：

- Contradiction between accuracy and size of data
- Deteriorate the smooth of the surface
- Velocity and acceleration discontinuity

CNC

Tool Path Optimization

Velocity Planning

Interpolation

Reference Points

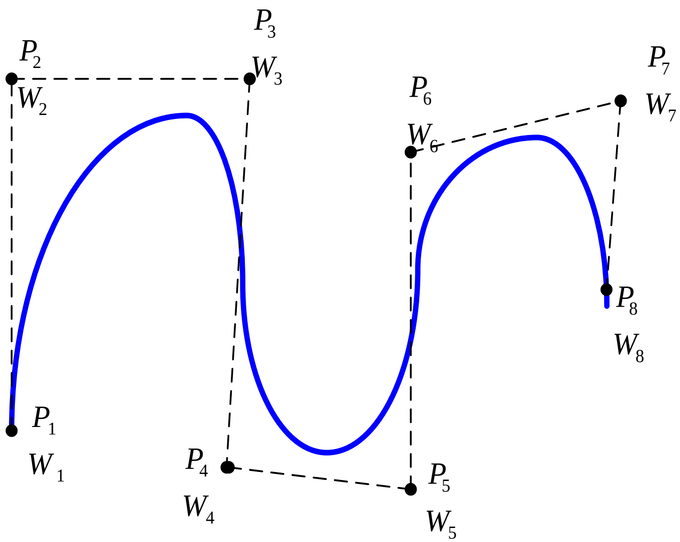
Control Loops

Machine Tools

1. Introduction

- NURBS (Non-Uniform Rational B-Spline) curve is a parameter curve
  - Represent free-form shapes

P_i : control point W_i : weight u_i : knot vector



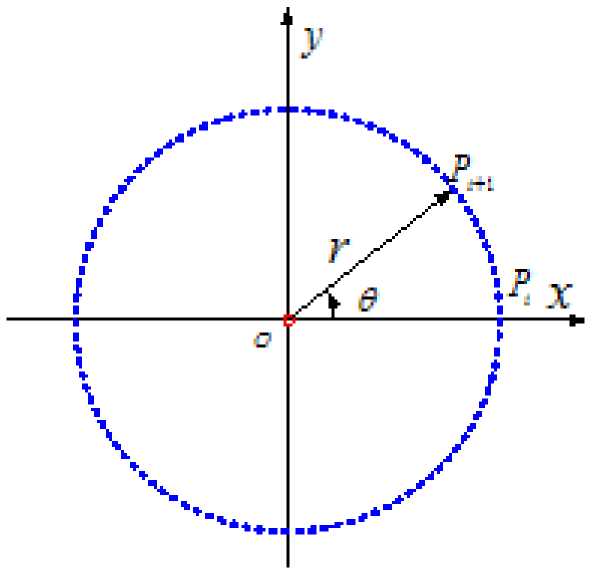
$$C(u)=\frac{\sum_{i=0}^{n} W_i N_{i,p}(u) P_i}{\sum_{i=0}^{n} W_i N_{i,p}(u)} \qquad 0 \le u \le 1$$

$$N_{i,1}(u) = \begin{cases} 1 & for \quad u_i \le u \le u_{i+1} \\ 0 & otherwise \end{cases}$$

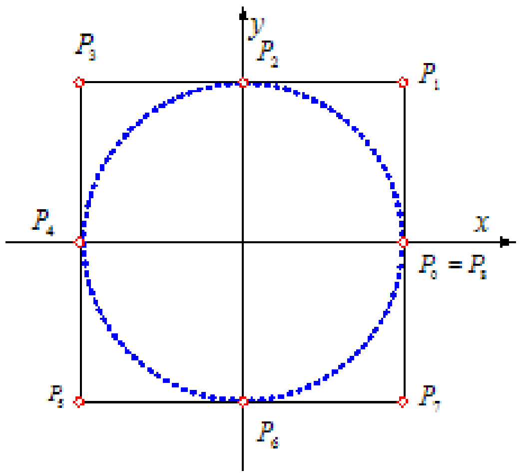
$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p-1} - u_i} N_{i,p-1}(u) + \frac{u_{i+p} - u}{u_{i+p} - u_{i+1}} N_{i+1,p-1}(u)$$

1. Introduction

- Represent standard analytical shapes



Traditional method



$$\{P_i\} = \{(1,0),(1,1),(0,1),(-1,1),$$
$$(-1,0),(-1,-1),(0,-1),(1,-1),(1,0)\}$$

$$\{W_i\} = \{1,\frac{\sqrt{2}}{2},1,\frac{\sqrt{2}}{2},1,\frac{\sqrt{2}}{2},1,\frac{\sqrt{2}}{2},1\}$$

$$U = \{0,0,0,\frac{1}{4},\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},\frac{3}{4},1,1,1\}$$

2. Real-time continuous NURBS interpolator

Off-time NURBS curve interpolator

The whole tool-path: a long NURBS curve

↓

Velocity planning of the whole tool-path

↓

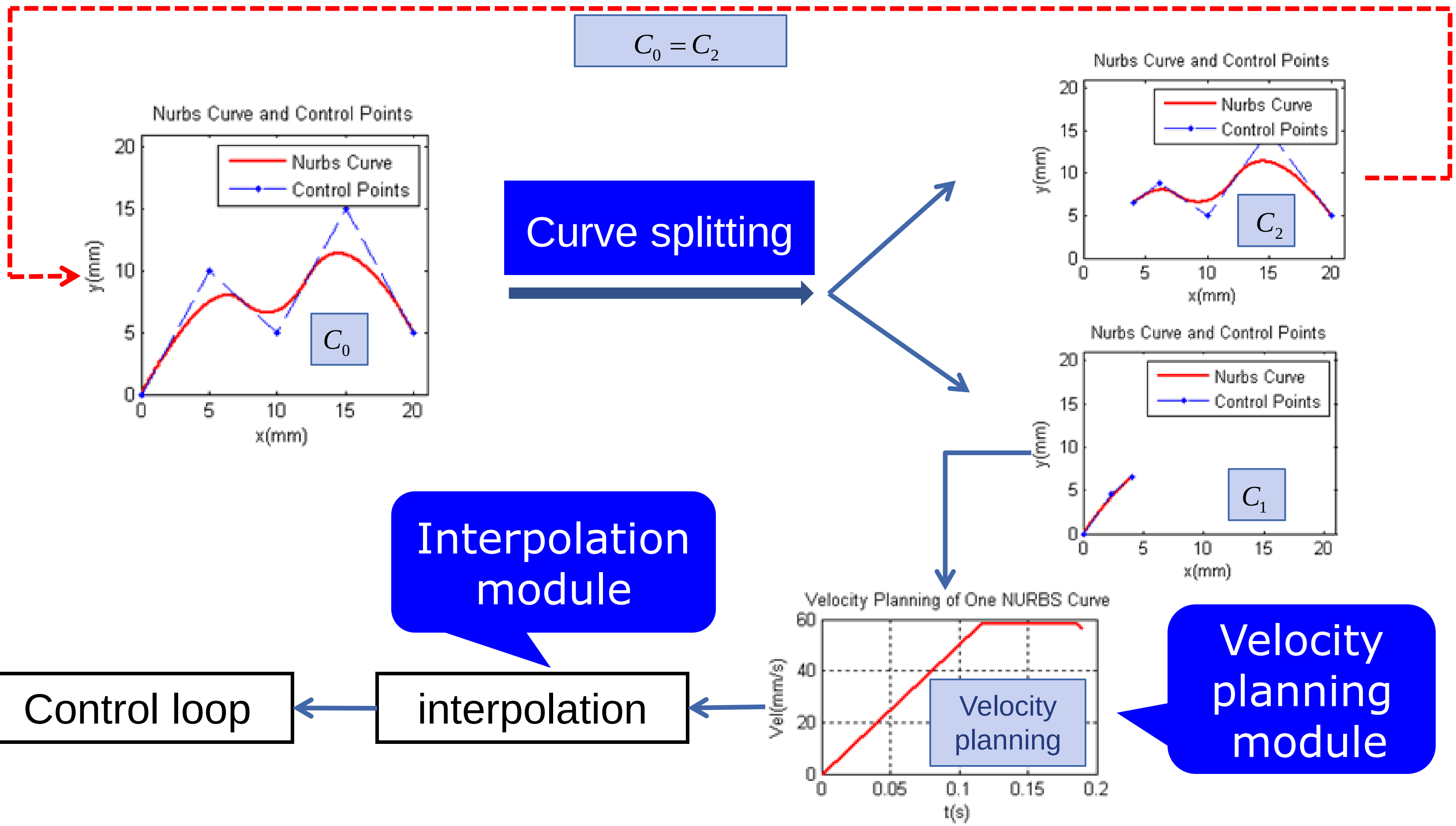
Compute the interpolation points on the tool-path

Drawbacks:

- Time-consuming
- Limiting condition can't change when machining
- Inconvenience for next machining when machining stopped for special reasons

2. Real-time continuous NURBS interpolator

- The basic idea of real-time curve interpolator



2. Real-time continuous NURBS interpolator

Four key points

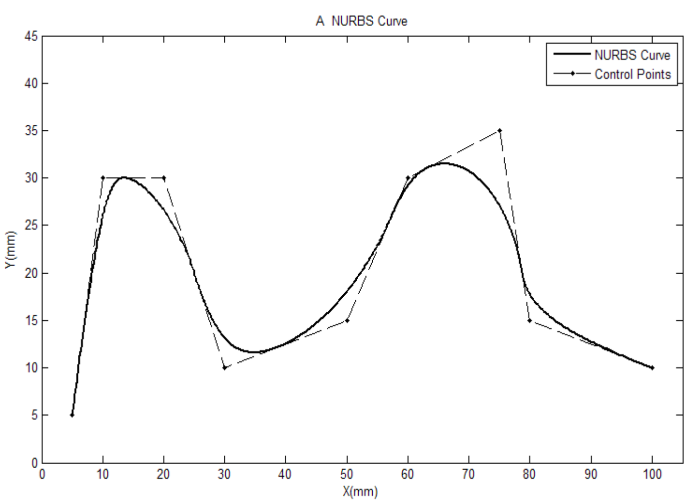
- NURBS curve <u>splitting</u>
  - How to split
  - Splitting length
- <u>Velocity planning</u> of a continuous NURBS curve
- Maximum velocity at <u>intersection point</u> and velocity curve <u>modification</u>
- Efficient <u>interpolation</u> algorithm

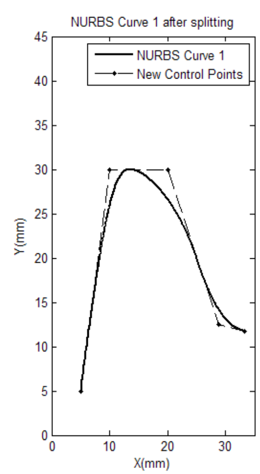
2. Real-time continuous NURBS interpolator

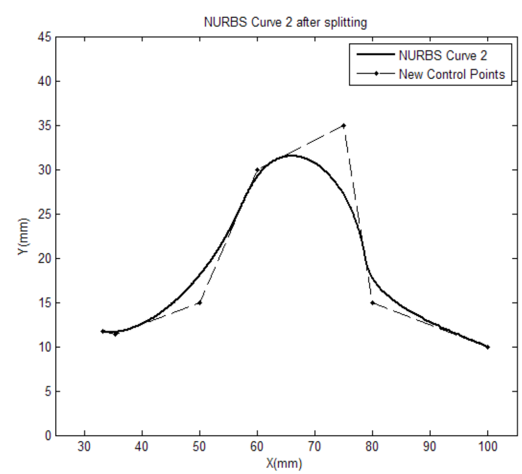
Curve splitting

Splitting method

- Determine new control points, weights and knot vectors
- NURBS curves after splitting have the same shape with original NURBS curve



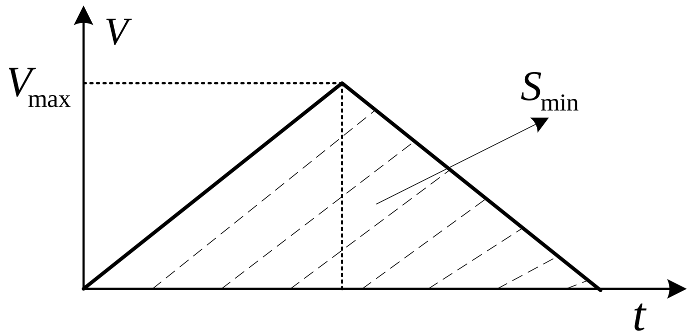




2. Real-time continuous NURBS interpolator

- Minimum splitting length

S_{min} : a minimum length for achieving once complete accelerating and decelerating process.



$$S_{min} = \frac{V_{max}^{2}}{A_{max}}$$

V_{max} : the maximum allowed velocity defined according to the machine performance.

A_{max} : the maximum allowed acceleration.

2. Real-time continuous NURBS interpolator

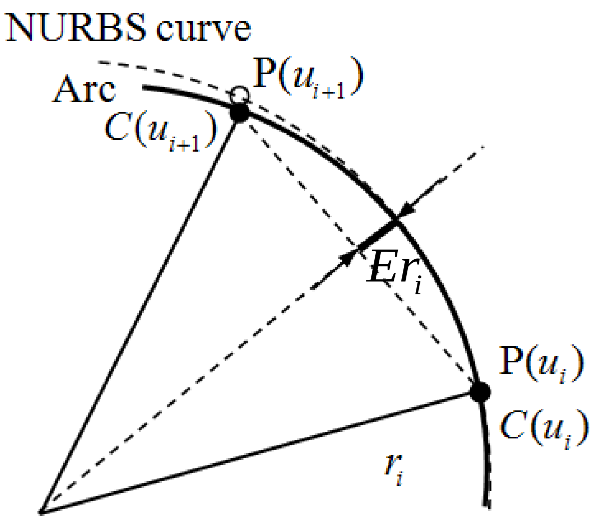
Velocity planning of continuous NURBS curve

- Step 1: By curvature limitation

$$r_i = \frac{1}{K(t_i)}$$

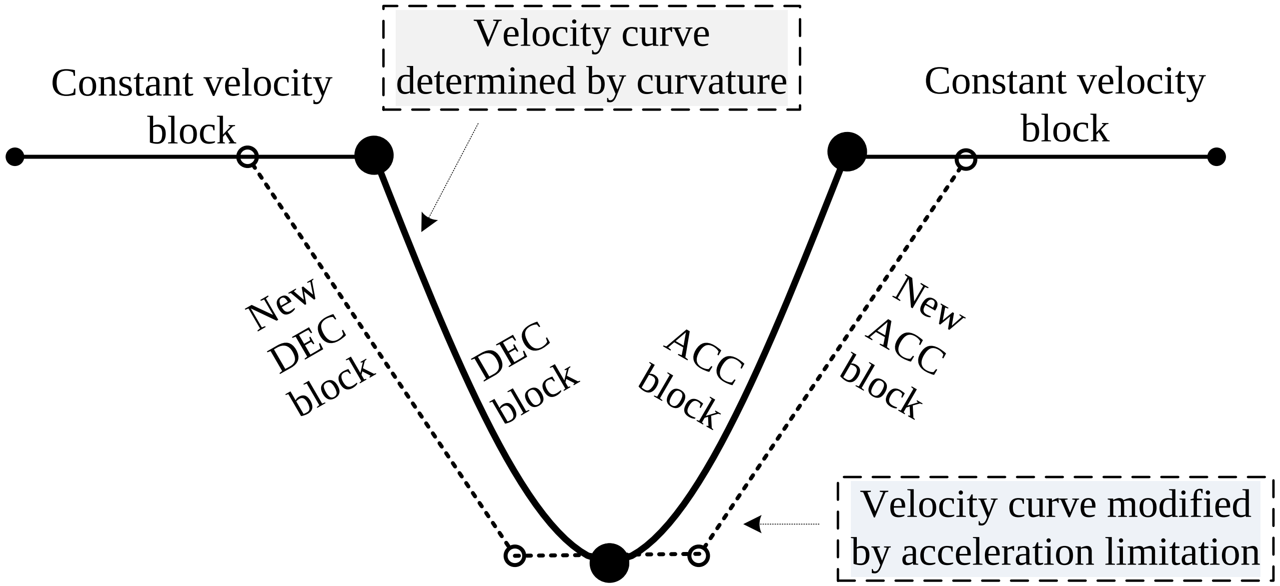
$$V_{Er} = \frac{2 \cdot \sqrt{r_i^2 - (r_i - Er_i)^2}}{T_s}$$

$$V(u_i) = \begin{cases} V_{\max}, & if \quad V_{Er} > V_{\max} \\ V_{Er}, & if \quad V_{Er} \le V_{\max} \end{cases}$$



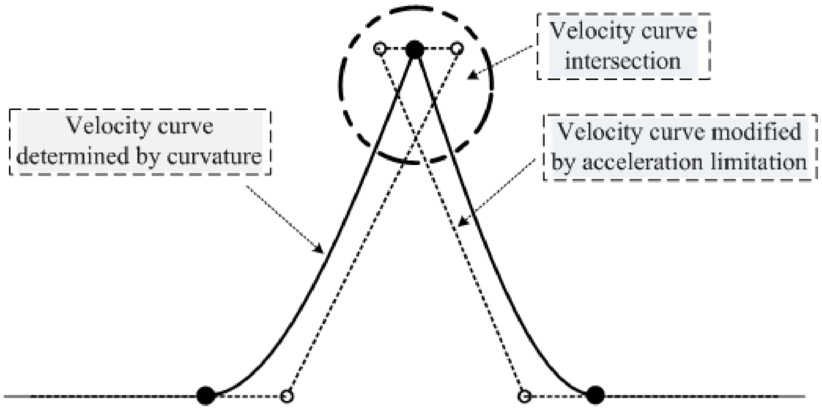
2. Real-time continuous NURBS interpolator

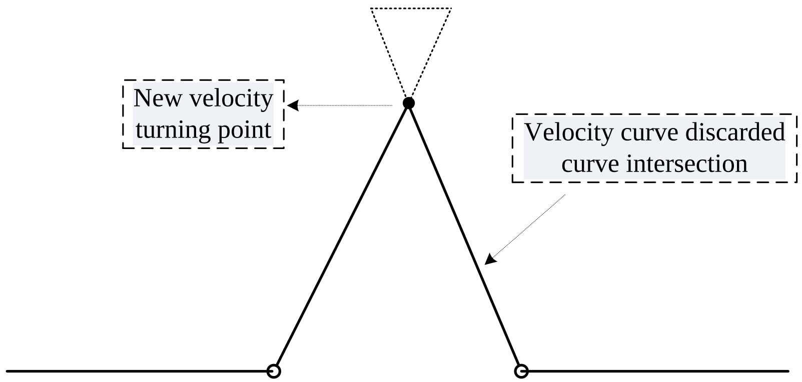
- Step 2: By acceleration limitation



2. Real-time continuous NURBS interpolator

- Step 3: Velocity curve modification

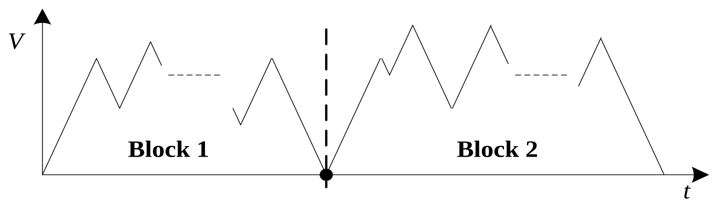




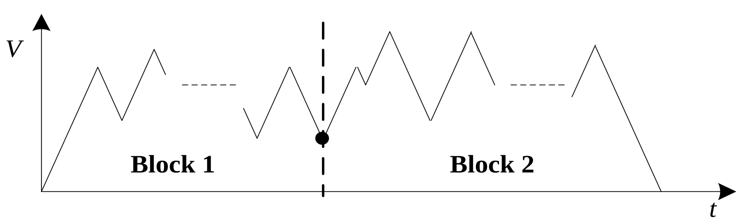
2. Real-time continuous NURBS interpolator

Velocity curve connection

Velocity connection mode

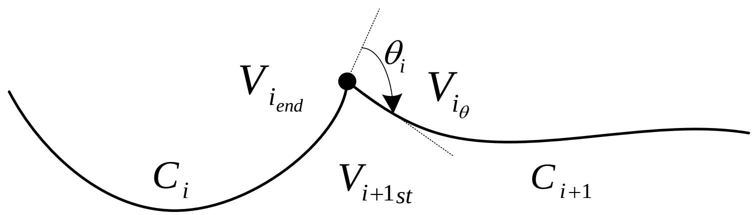


Simple connection mode



Blending connection mode

Velocity at intersect point



$$V_{i_\theta} = \frac{A_{\max} T_s}{2\sin(\theta_i / 2)}$$

$$\theta_i = \begin{cases} 0 & continuous \quad \checkmark \\ other & discontinuous \end{cases}$$

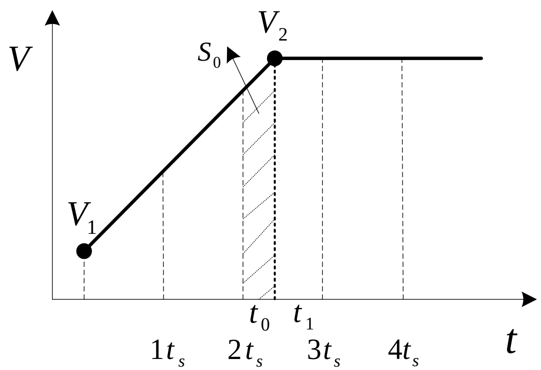
$$V_{i_{\max}} = \min(V_{i_{end}}, V_{i+1_{st}}, \cancel{V_{i_\theta}})$$

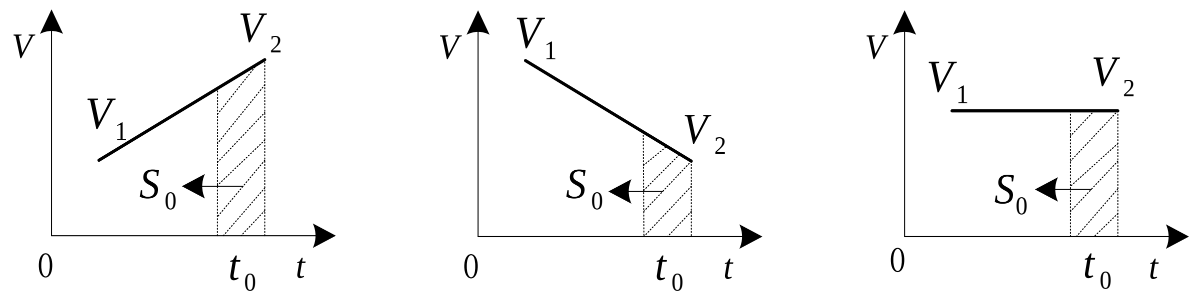
2. Real-time continuous NURBS interpolator

Interpolation module

$$S = \int_0^t V dt$$

$$P_0 = function(S, curve)$$





(a) (b) (c)

$$V_2 t_0 - A_{max} t_0 / 2 = S_0$$
$$t_0 = (-V_2 + \sqrt{V_2^2 - 2A_{max} S_0}) / (-A_{max})$$

$$V_2 t_0 + A_{max} t_0 / 2 = S_0$$
$$t_0 = (-V_2 + \sqrt{V_2^2 + 2A_{max} S_0}) / A_{max}$$

$$S_0 = V_2 t_0$$
$$t_0 = S_0 / V_2$$

2. Real-time continuous NURBS interpolator

Simulation results

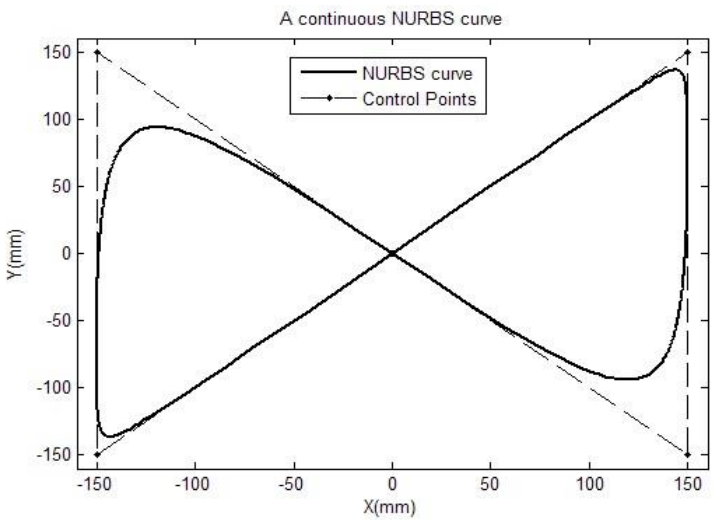
- Interpolator parameters：

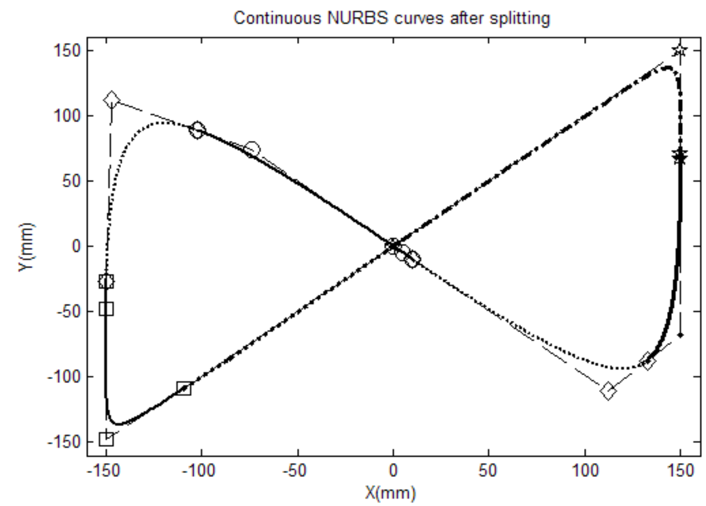
$$T_s = 0.002s \qquad V_{\max} = 200mm/s$$

$$A_{\max} = 300mm/s^2$$

$$S_{\min} = V_{\max}{}^2 / A_{\max} = 200^2 / 300 = 133.33(mm)$$

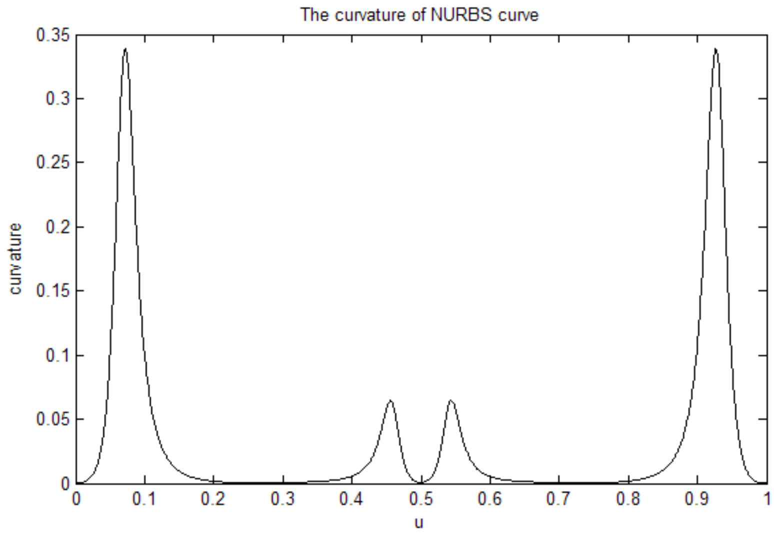
- Initial NURBS curve
- NURBS curves after splitting



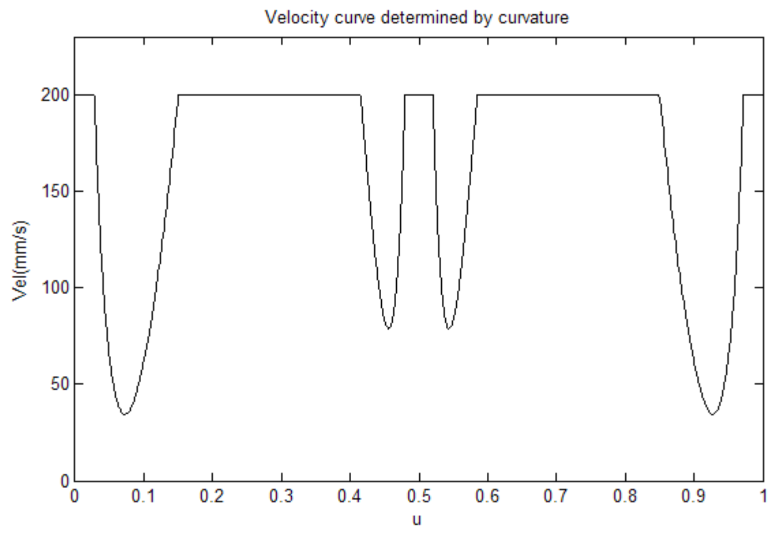


2. Real-time continuous NURBS interpolator

- Curvature

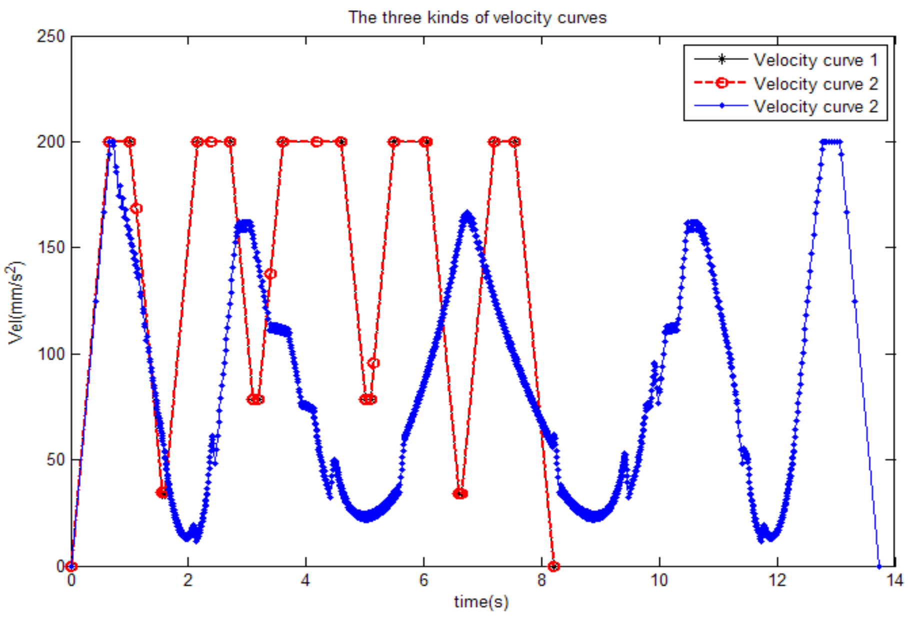


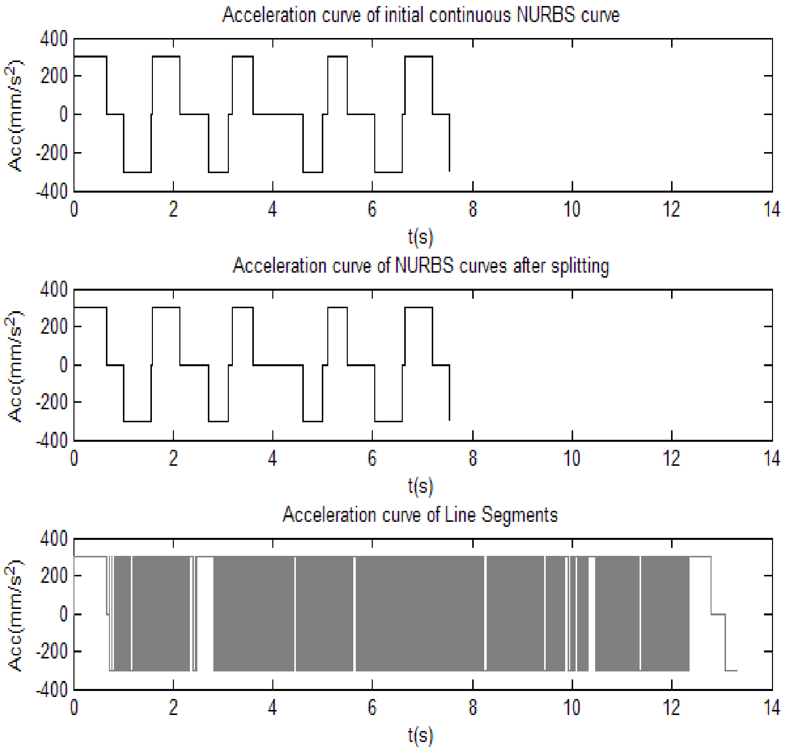
- Velocity curve



2. Real-time continuous NURBS interpolator

- Velocity curve
- Acceleration curve





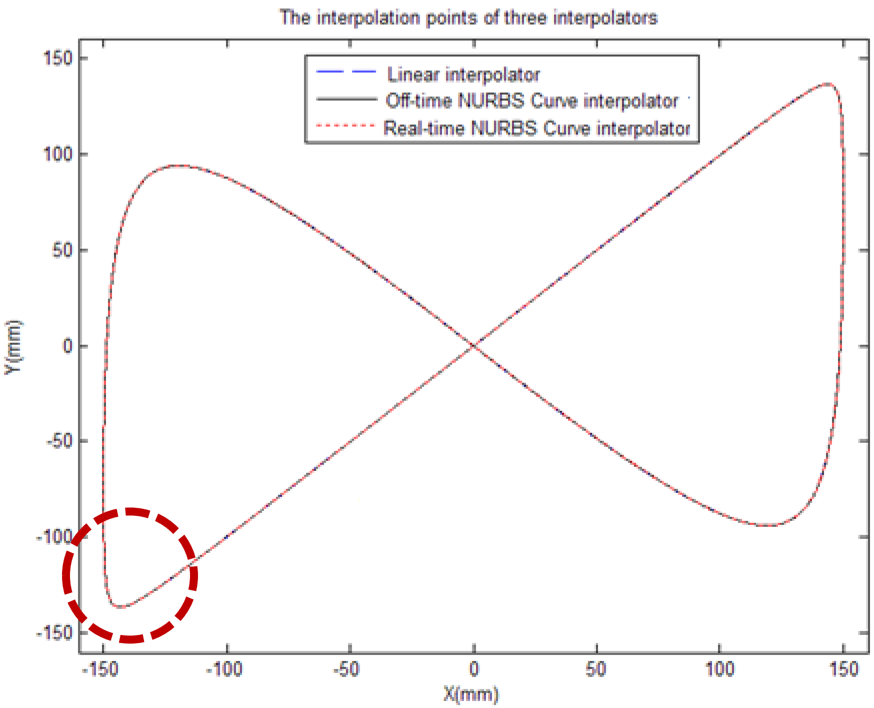
Velocity curve 1: NURBS curve & off-time

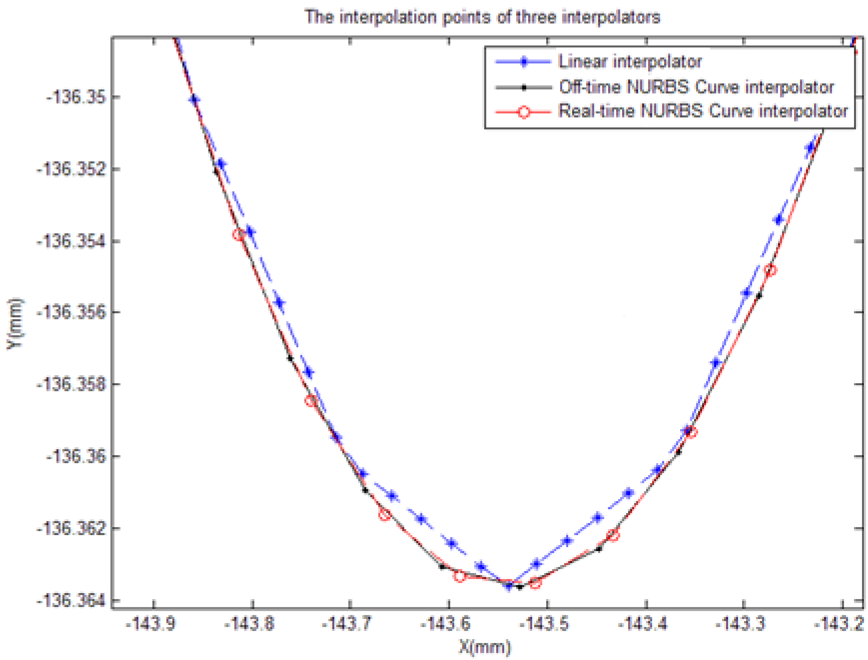
Velocity curve 2: NURBS curve & real-time

Velocity curve 3: line segments & look-ahead

2. Real-time continuous NURBS interpolator

- Interpolation points
- Magnification





2. Real-time continuous NURBS interpolator

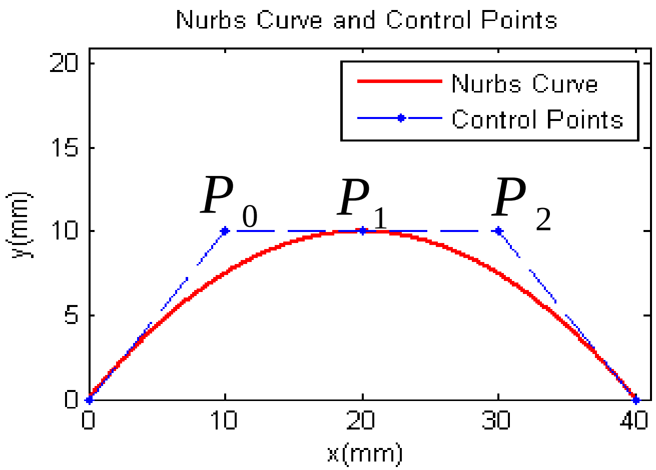
- Performance analysis

Linear interpolator		Off-time NURBS curve interpolator		Real-time NURBS curve interpolator	
Compute time(s)	Machining time(s)	Compute time(s)	Machining time(s)	Compute time(s)	Machining time(s)
0.5932	13.7408	1.8433	8.1987	1.8641	8.1987

3. Real-time discontinuous NURBS interpolator

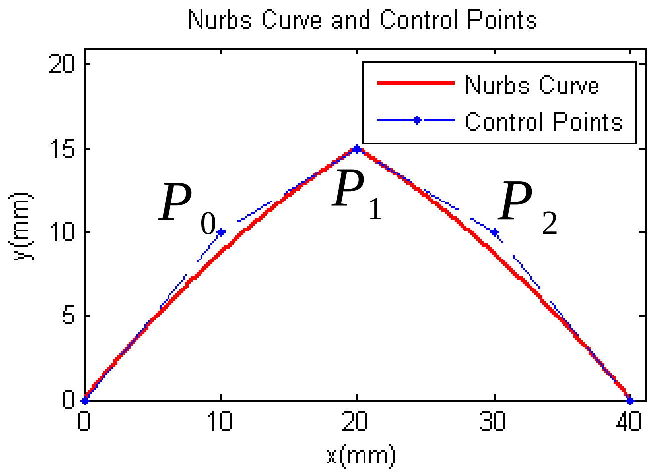
The discontinuous point on NURBS curve

Continuous NURBS curve



$$U = \{0,0,0,0.5,0.5,1,1,1\}$$
$$P = \{(0,0),(10,10),$$
$$(20,10),(30,10),(40,0)\}$$
$$W = \{1,1,1,1,1\}$$

Discontinuous NURBS curve



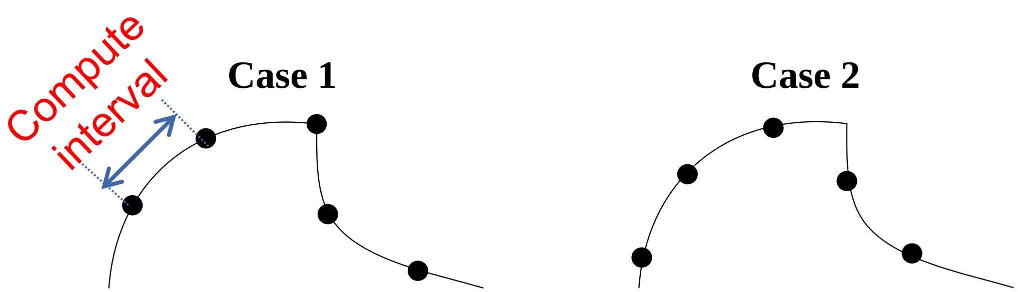
$$U = \{0,0,0,0.5,0.5,1,1,1\}$$
$$P = \{(0,0),(10,10),$$
$$(20,15),(30,10),(40,0)\}$$
$$W = \{1,1,1,1,1\}$$

3. Real-time discontinuous NURBS interpolator

The cases of cusp on a NURBS curve

	$p=2$	$p=3$
Single Knot u_j	Double control points P_i, P_i	Triply control points P_i, P_i, P_i
Double knots u_j, u_j	Single control point P_i	Double control points P_i, P_i
Triply knots u_j, u_j, u_j	-	Single control point P_i

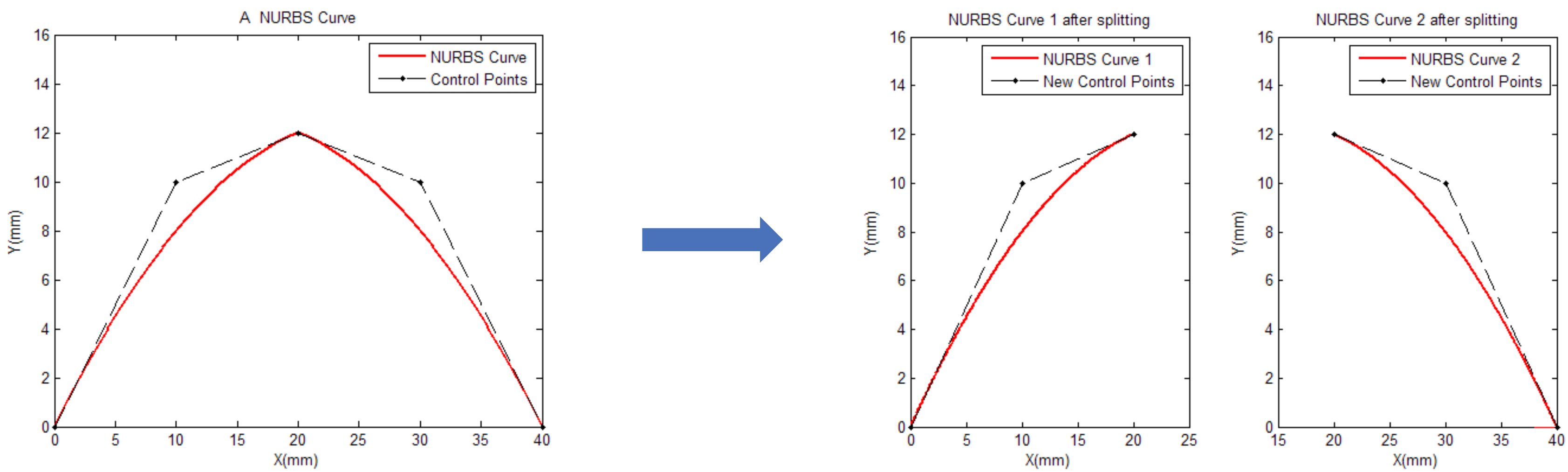
Curvature computation

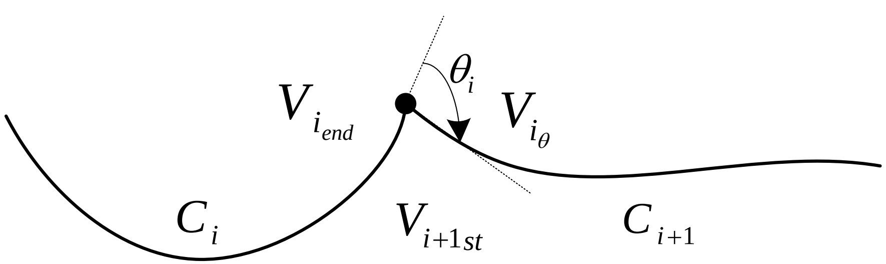


$$K(t)=\frac{|\dot{P}(t)\times\ddot{P}(t)|}{|\dot{P}(t)|^3}$$

3. Real-time discontinuous NURBS interpolator

Splitting at cusp





$$V_{i_\theta} = \frac{A_{\max} T_s}{2\sin(\theta_i / 2)}$$

$$\theta_i = \begin{cases} 0 & continuous \\ other & discontinuous \ \checkmark \end{cases}$$

$$V_{i_{\max}} = \min(V_{i_{end}}, V_{i+1_{st}}, V_{i_\theta})$$

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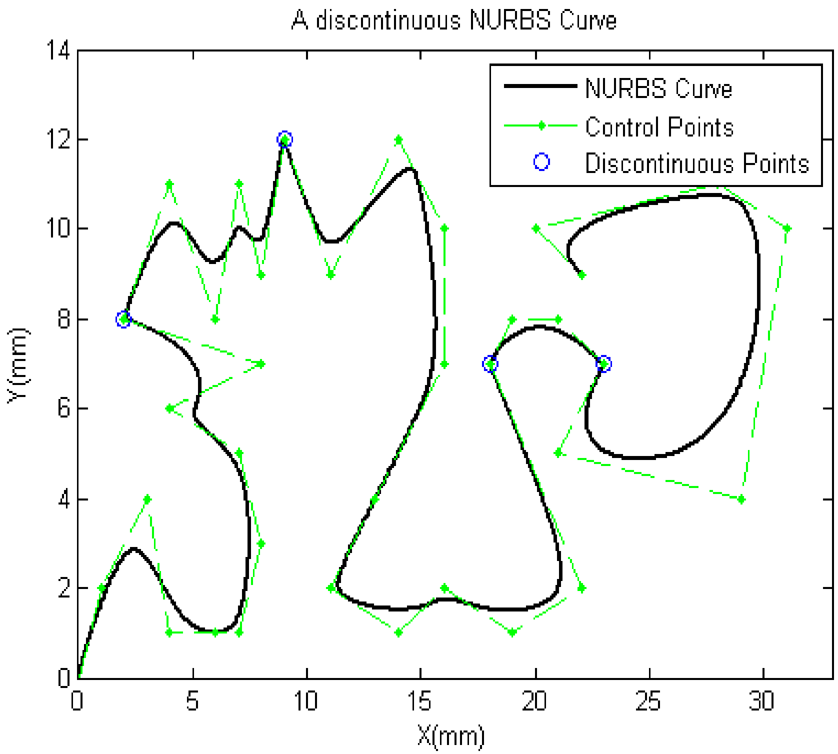
3. Real-time discontinuous NURBS interpolator

2-D discontinuous NURBS curve



- 4 discontinuous points
- The curve length between point 3 and 4 is short
- This part will be written as small line segments block
- Degree: 2

$$T_s = 0.002s$$

$$V_{\max} = 58mm/s$$

$$A_{\max} = 500mm/s^2$$

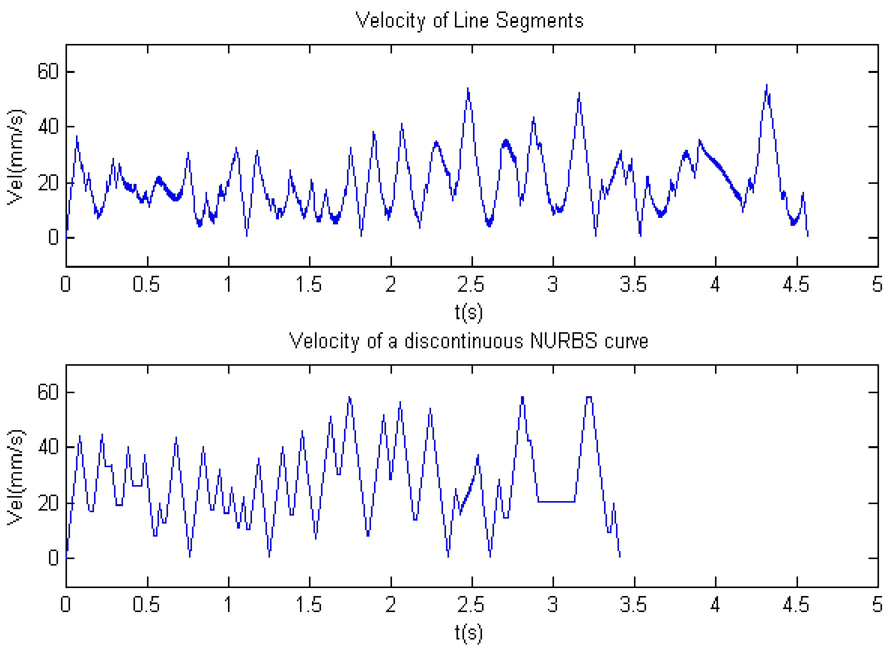
$$N_t = 10$$

$$S_{\min} = V_{\max}{}^2 / A_{\max} = 58^2 / 500mm = 6.728mm$$

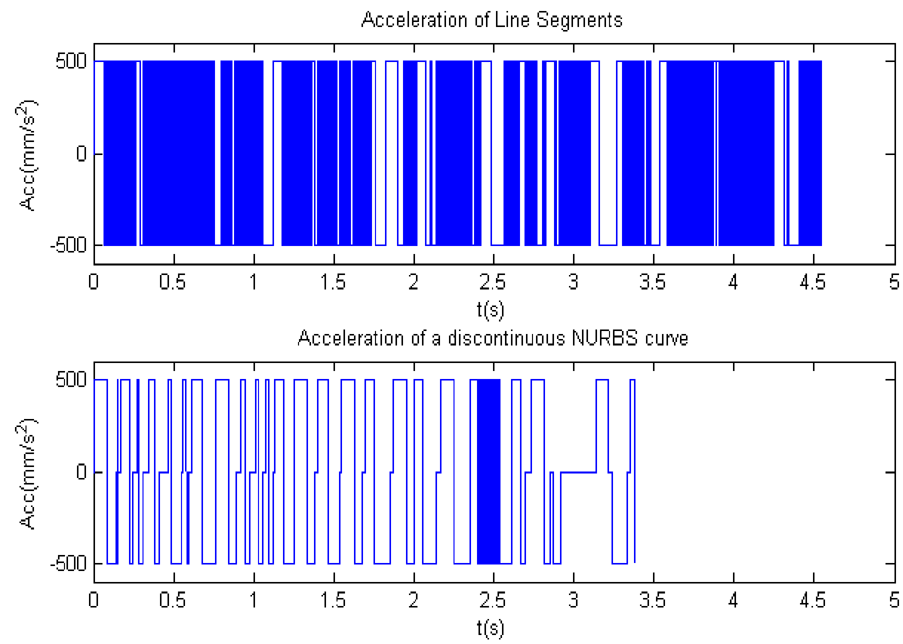
3. Real-time discontinuous NURBS interpolator

Results comparison

- Velocity curve

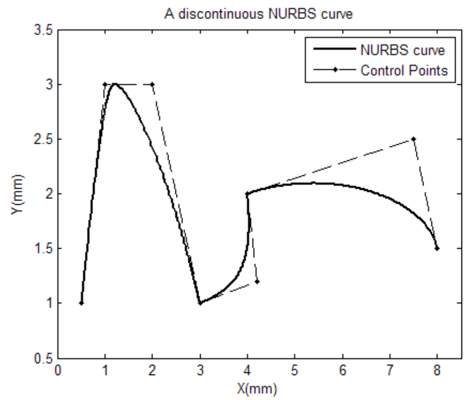


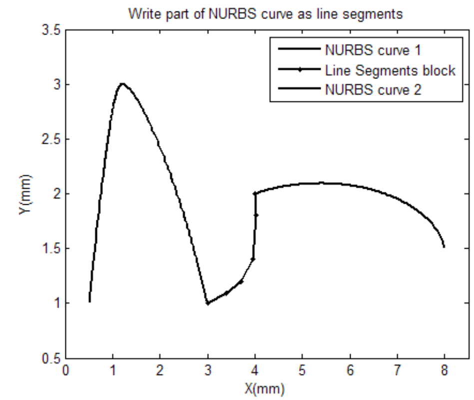
- Acceleration curve



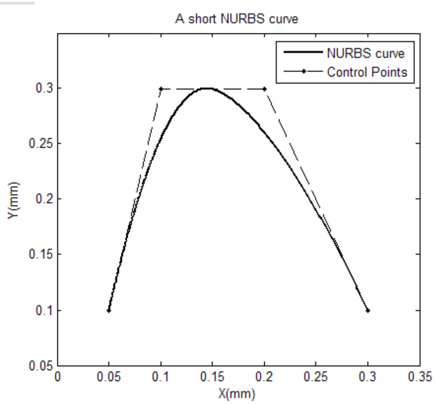
4. Real-time mixed-path interpolator

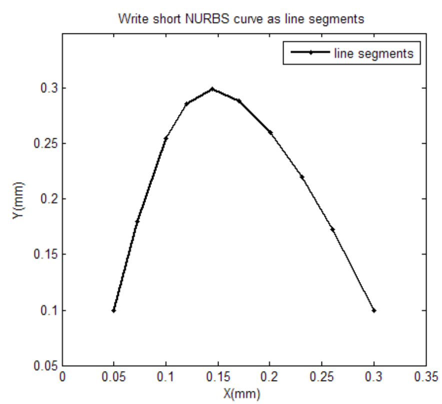
Three kinds of mix-path



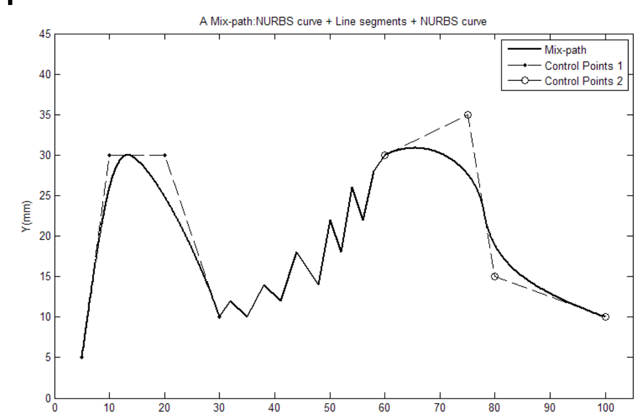


The length between two cusp is short





The length of NURBS curve is short



The original curve is mix-path

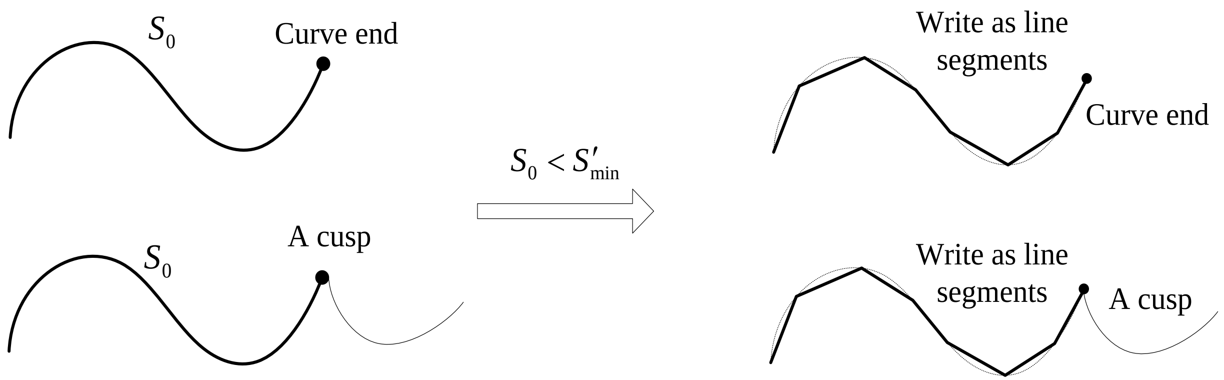
4. Real-time mixed-path interpolator

Splitting case

$S'_{\min}$ is the splitting length

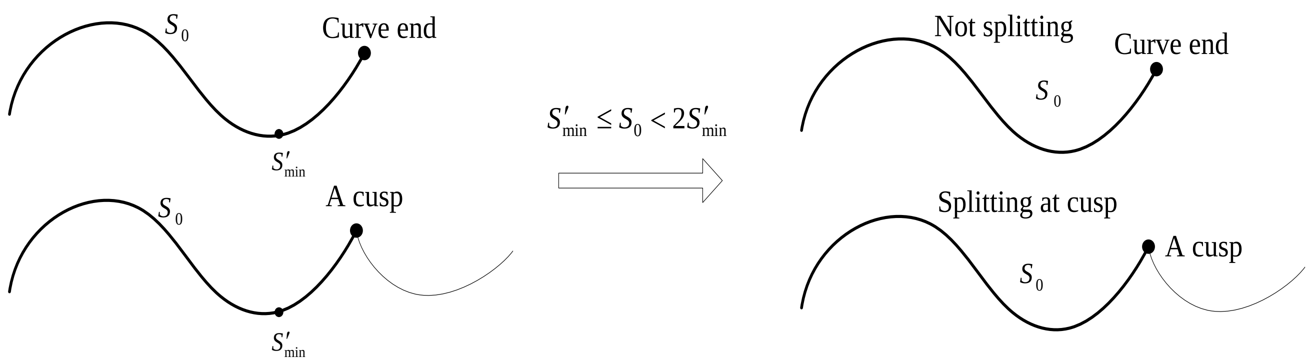
which is a value great than or equal to the minimum splitting length $S_{\min}$

- Case 1: Write NURBS curve as line segments

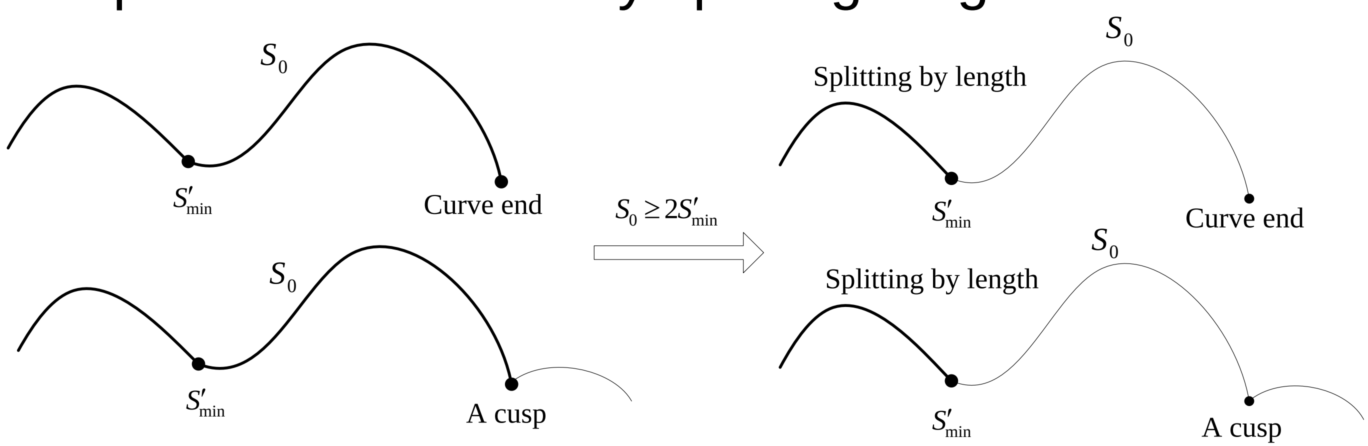


4. Real-time mixed-path interpolator

- **Case 2**: Not split NURBS curve or split at the first cusp

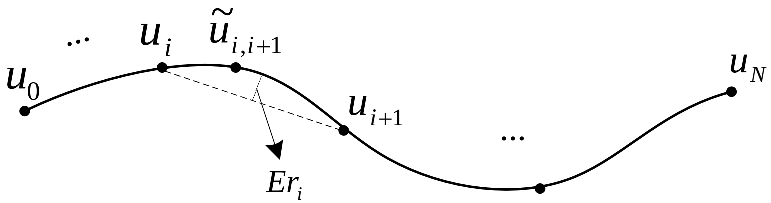


- **Case 3**: Split NURBS curve by splitting length



4. Real-time mixed-path interpolator

Continuous line segments

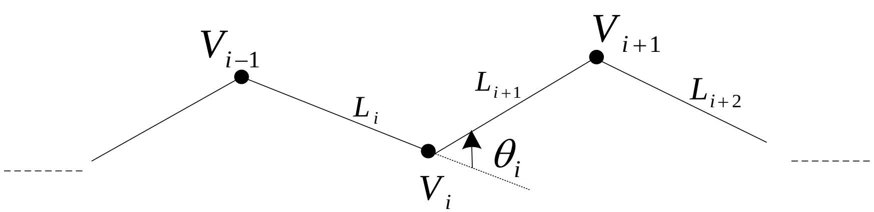


First insert:

$$U=\{u_0,u_1,\cdots u_i,\cdots u_N\}=\{u_0,1d,2d,3d,\cdots,u_N\}$$

If $Er_i > Er_{\max}$, insert $\tilde{u}_{i,i+1}=(u_i+u_{i+1})/2$

If $Er_i > Er_{\max}$, no insert



Velocity limitation condition:

$$\begin{cases} V_i^2 \le \begin{cases} {V_{i-1}}^2 + 2A_{\max} L_i \\ {V_{i+1}}^2 + 2A_{\max} L_{i+1} \end{cases} \\ 0 \le V_i \le V_{i_\theta}, V_i \le V_{\max} \end{cases}$$

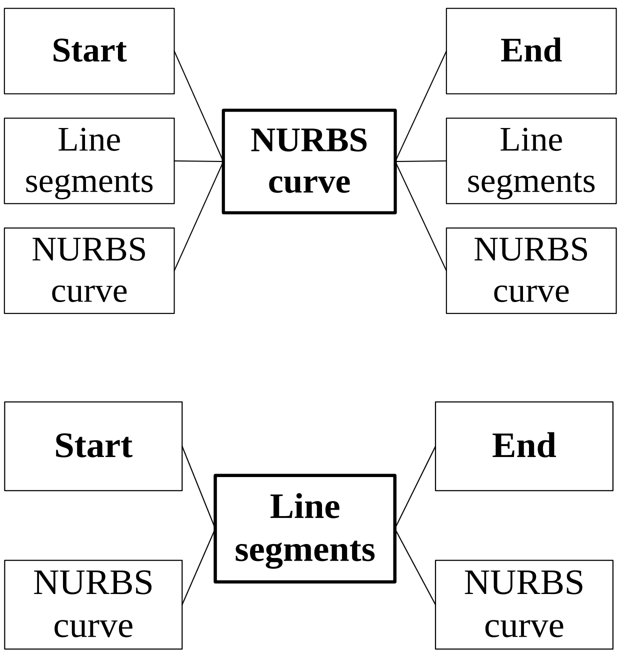
Start & end velocity limitation condition:

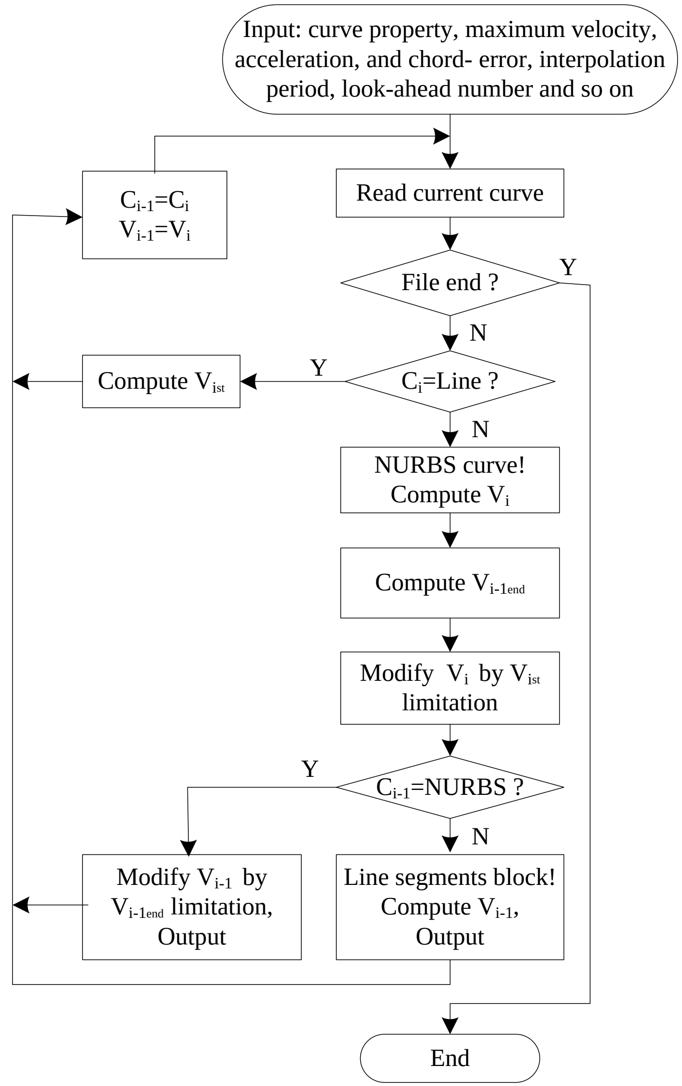
$$V_{i_{st}} = \min(V_{i-1_{end}}, V_{i_\theta})$$
$$V_{i_{end}} = \min(V_{i+1_{st}}, V_{i+1_\theta})$$

4. Real-time mixed-path interpolator

Program flow chart of velocity planning for mixed-path

The combination forms of the tool-path





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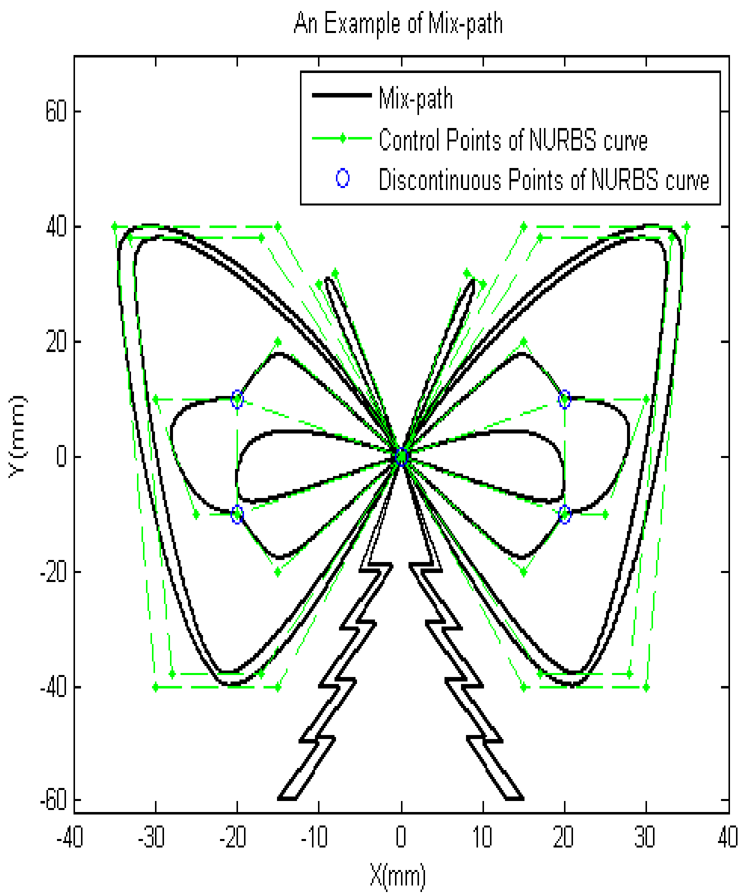
4. Real-time mixed-path interpolator

2-D mixed-path



- 5 NURBS curves & 1 line segments block
- 4 discontinuous points
- NURBS curve degree: 2

$$T_s = 0.002s$$

$$V_{\max} = 25mm/s$$

$$A_{\max} = 300mm/s^2$$

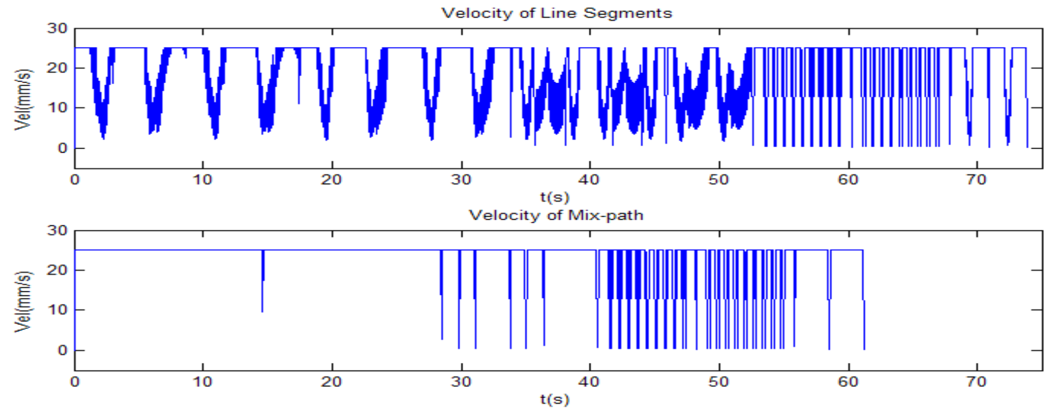
$$N_t = 10$$

$$S_{\min} = V_{\max}{}^2 / A_{\max} = 25^2 / 300mm = 2.083mm$$

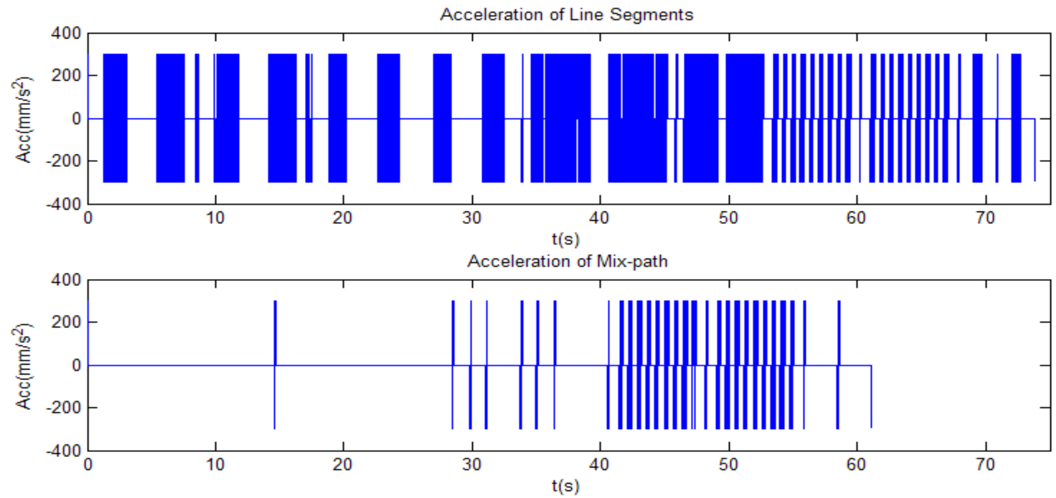
4. Real-time mixed-path interpolator

Results comparison

- Velocity curve

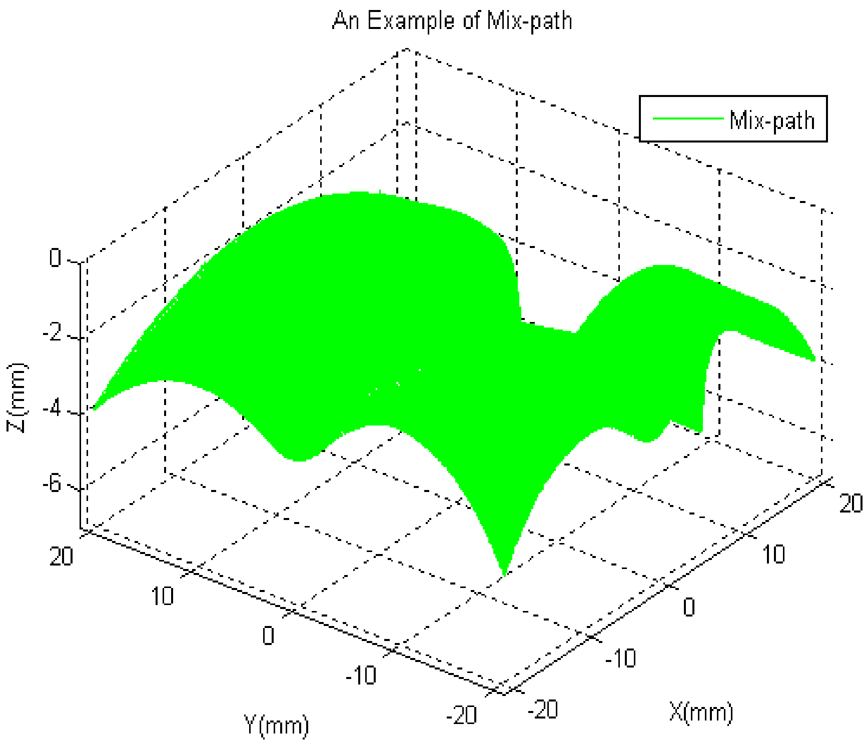


- Acceleration curve



4. Real-time mixed-path interpolator

3-D mixed-path



- Actual tool-path
- Path length: 33670.62 mm
- Intercepted path length: 315.97 mm
- NURBS curve degree: 3

$$T_s = 0.002s$$

$$V_{\max} = 58mm/s$$

$$A_{\max} = 500mm/s^2$$

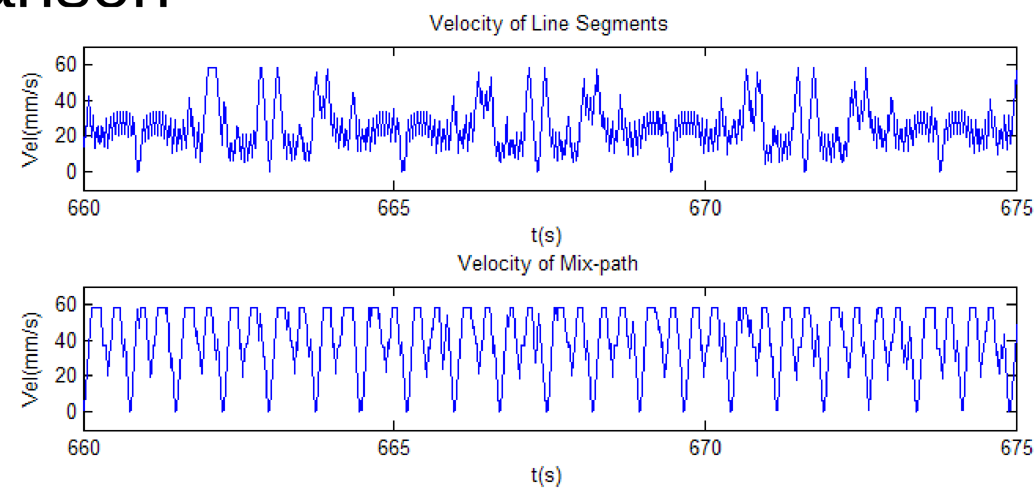
$$N_t = 10$$

$$S_{\min} = V_{\max}{}^2 / A_{\max} = 58^2 / 500mm = 6.728mm$$

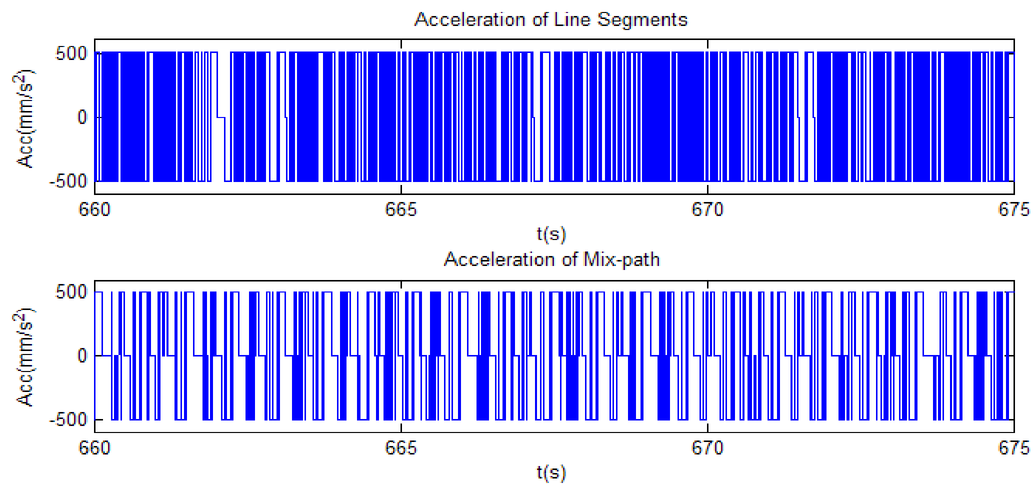
4. Real-time mixed-path interpolator

Results comparison

- Velocity curve



- Acceleration curve



4. Real-time mixed-path interpolator

Performance analysis

	Mixed-path interpolator		Linear interpolator	
	Compute time(s)	Machining time(s)	Compute time(s)	Machining time(s)
Ex 1	1.3590	3.415	0.5160	4.5740
Ex 2	2.0780	61.208	0.6090	73.860
Ex 3	198.7660	800.698	58.4990	1262.455

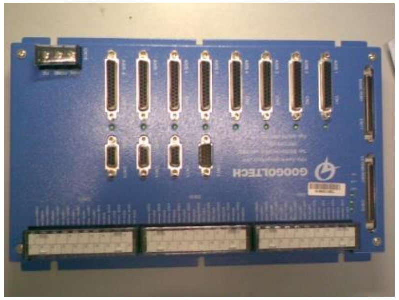
5. Experiment results

Experiment platform

Machine tool

GUC of Googoltech

- X86 architecture(600MHz) processor
- PCI communication
- 8-axis controller



Terminal Board

- I/O management
- A/D transformation

5. Experiment results

Video for machining

Linear interpolator

$T_s = 0.002s$

$V_{\max} = 25mm/s$

$A_{\max} = 300mm/s^2$

Machining time

73.860 s

Real-time mix-path interpolator

$T_s = 0.002s$

$V_{\max} = 25mm/s$

$A_{\max} = 300mm/s^2$

Machining time

61.208 s

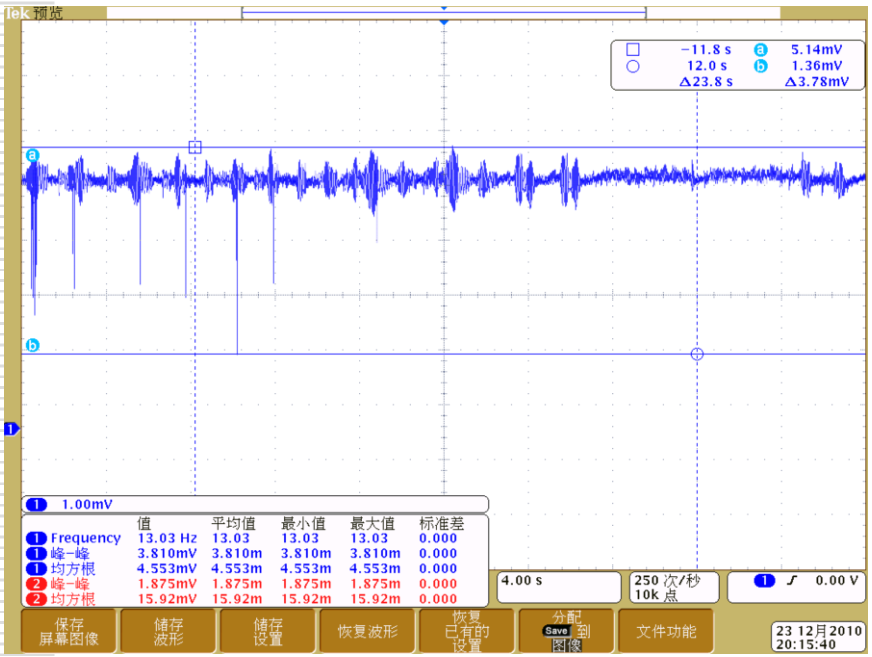
5. Experiment results

Laser Scanning Viberometer

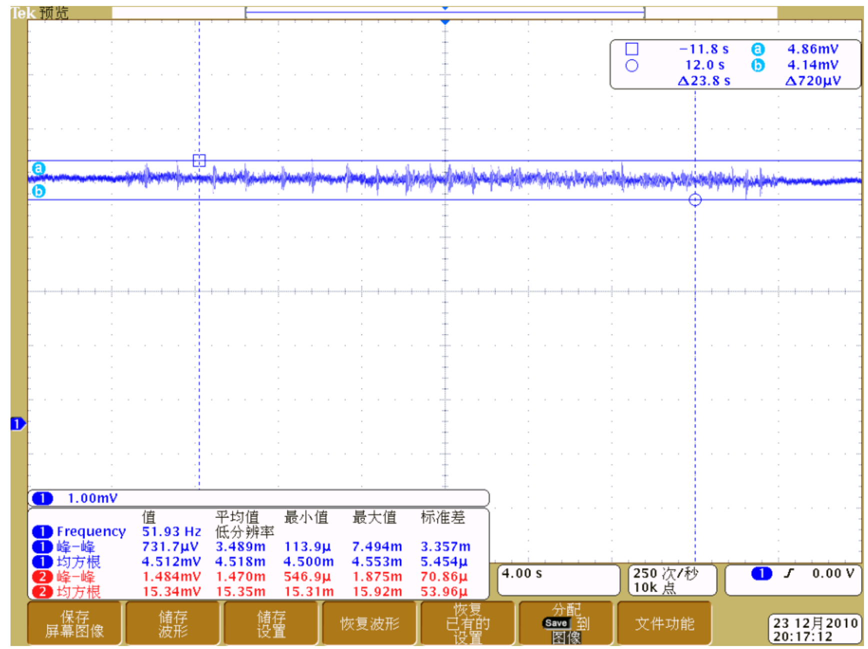
5. Experiment results

vibration waveform

Linear interpolator



Real-time mix-path interpolator

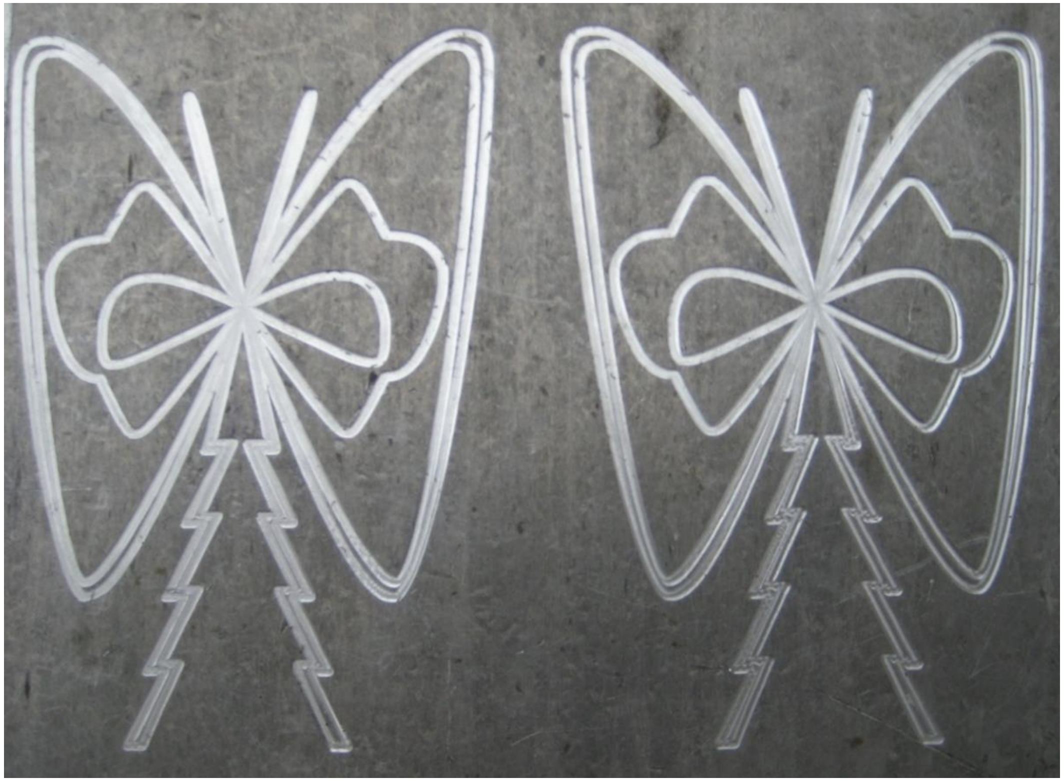


5. Experiment results

- Workpieces after machining

Linear interpolator

Real-time mix-path interpolator



6. Conclusions

Conclusions

- New idea of real-time (two disposing module)
- Can dispose discontinuous NURBS curve, short NURBS curve, mixed-path (NURBS curves & line segments blocks)
- Velocity curves connected smoothly; Both acceleration and deceleration are limited in the capability of machining tool
- Has a higher performance than linear interpolator (machined surface is smoother, velocity fluctuation is smaller and the machining time is shorter)

Thank you!