Dynamic discrete choice models: CCP methods

Olivier De Groote

TSE

MRes: EEE, 2021-2022

Outline

CCP estimation and finite dependence

Avoid solving the model

- The computational complexity quickly increases in these models
- In contrast to reduced form methods, we need to find parameters and solve an economic model at the same time
- If solving the model takes a long time, it prevents us from enriching the model to make it more realistic
- It also requires more computing skills to program it
- Researchers used the Rust (1987) framework to develop estimation methods that do not require solving the model: CCP methods (introduced by Hotz and Miller (1993))
- We closely follow the review: Arcidiacono, P., and Ellickson, P. 2011. "Practical Methods for Estimation of Dynamic Discrete Choice Models". *Annual Review of Economics* 3: 363–94.

CCP estimation with one-period-ahead CCPs I

- Conditional value function

$$v_{ijt} = u_j(x_{it}) + \beta \int \bar{V}_{t+1}(x_{it+1}) f(x_{it+1}|x_{it}, d_{it} = j) dx_{it+1}$$

- A bit of different notation than Arcidiacono and Ellickson
  - Instead of subscripting by alternative j, v and u depend explicitly on the choice d
  - Ignore i subscript
- Agents choose the option with the highest value of $v_{ijt} + \epsilon_{ijt}$
- With ϵ_{ijt} extreme value type 1, we still have the logsum expression for the ex ante value function

$$\bar{V}_{t+1}(x_{it+1}) = \gamma + ln \sum_{j'} exp(v_{ij't+1})$$

CCP estimation with one-period-ahead CCPs II

- At the same time, we also have

$$Pr(d_{it+1} = j^* | x_{it+1}) = \frac{exp(v_{ij^*t+1})}{\sum_{j'} exp(v_{ij't+1})}$$

- Taking logs

$$lnPr(d_{it+1} = j^* | x_{it+1}) = v_{ij^*t+1} - ln \sum_{j'} exp(v_{ij't+1})$$

- Rearrange

$$ln \sum_{j'} exp(v_{ij't+1}) = v_{ij^*t+1} - lnPr(d_{it+1} = j^* | x_{it+1})$$

CCP estimation with one-period-ahead CCPs III

- Substituting this in the ex ante value function

$$\bar{V}_{t+1}(x_{it+1}) = \gamma + v_{ij^*t+1} - lnPr(d_{it+1} = j^*|x_{it+1})$$

- The expected value of behaving optimally in $t+1$ depends only on the value of an arbitrary choice j^*, and a nonnegative correction term $-lnPr(d_{it+1} = j^*|x_{it+1})$

- $Pr(d_{it+1} = j^*|x_{it+1})$ is just data (or can at least be predicted)

- So instead of solving the model, we just need to know the conditional value function of one arbitrary option j^*

CCP estimation with one-period-ahead CCPs IV

- In general, this remains complicated as this will again depend on $\bar{V}_{t+2}$ etc...
- But take for example a terminal action (e.g. a retirement decision, buying a durable good,..). In this case we can write

$$v_{ij^*t+1} = u_{ij^*t+1}$$

- (or, depending on interpretation, $\frac{u_{ij^*t+1}}{1-\beta}$)
- This can be parameterized as a function of state variable x_{it+1}, or just normalized to 0.
- We then get

$$v_{ijt} = u_j(x_{it}) + \beta \int (\gamma - lnPr(d_{it+1} = j^* | x_{it+1})) f(x_{it+1} | x_{it}, d_{it} = j) dx_{it+1}$$

CCP estimation with one-period-ahead CCPs V

- Remember that we could obtain state transitions in a first stage, now just need to get $Pr(d_{it+1} = j^*|x_{it+1})$ ("CCPs") in a first stage too

- The model essentially becomes a static model with a prespecified correction term and we do not need to solve anything

- Another example where the CCP method is useful is when there is a choice that "resets" the state

CCP estimation with one-period-ahead CCPs VI

- To see this, first note that the logit probabilities can be written using differenced value functions with some benchmark choice (e.g. 0)

$$Pr(d_{it} = j|x_{it}) = \frac{exp(v_{ijt})}{\sum_{j'} exp(v_{ij't})} = \frac{exp(v_{ijt} - v_{i0t})}{1 + \sum_{j'\setminus 0} exp(v_{ij't} - v_{i0t})}$$

- Without a terminal action, every v_{ijt} will be of the form

$$\begin{aligned} v_{ijt} &= u_j(x_{it}) \\ &+ \beta \int (\gamma + v_{ij^*t+1} - lnPr(d_{it+1} = j^*|x_{it+1}))f(x_{it+1}|x_{it}, d_{it} = j)dx_{it+1} \end{aligned}$$

- with

$$v_{ij^*t+1} = u_{j^*}(x_{it+1}) + \beta \int \bar{V}_{t+2}(x_{it+2})f(x_{it+2}|x_{it+1}, d_{it+1} = j^*)dx_{it+2}$$

CCP estimation with one-period-ahead CCPs VII

- Now assume j^*at time $t+1$ is a "renewal" action, meaning it resets the state such that

$$\int f(x_{it+1}|x_{it}, d_{it} = j) f(x_{it+2}|x_{it+1}, d_{it+1} = j^*) dx_{it+1}$$
$$= \int f(x_{it+1}|x_{it}, d_{it} = j') f(x_{it+2}|x_{it+1}, d_{it+1} = j^*) dx_{it+1}$$

- in words: the state at $t+2$ will not depend on the choice made today, if tomorrow we choose a renewal action
  - Example: in the Rust bus engine model, replacement in $t+1$ is a renewal action because it sets mileage to 0, regardless of what Harold Zurcher chooses to do today

CCP estimation with one-period-ahead CCPs VIII

- Following this assumption $\beta \int \bar{V}_{t+2}(x_{it+2}) f(x_{it+2}|x_{it+1}, d_{it+1} = j^*) dx_{it+2}$ drops out of $v_{ijt} - v_{i0t}$ and we can write the logit probabilities without a future value term

$$
\begin{aligned}
v_{ijt} - v_{i0t} = u_j(x_{it}) &+ \beta \int (\gamma + u_{ij^*t+1}(x_{it+1}) - lnPr(d_{it+1} = j^*|x_{it+1})) f(x_{it+1}|x_{it}, d_{it} = j) dx_{it+1} \\
&- u_0(x_{it}) - \beta \int (\gamma + u_{ij^*t+1}(x_{it+1}) - lnPr(d_{it+1} = j^*|x_{it+1})) f(x_{it+1}|x_{it}, d_{it} = 0) dx_{it+1}
\end{aligned}
$$

CCP estimation with more CCPs I

- If resetting the state is not possible by an action in the next period, it might work after 2,3,.. periods
- An example is Arcidiacono et al. (2016)
  - Choose between several work/college options or staying home
  - Primitives depend on accumulated work experience and past choice
  - Impossible to remove $\bar{V}_{t+2}(x_{it+2})$ in $v_{ijt} - v_{i0t}$
- Solution: solve one period further
- Consider 2 sequences of choices:
  - Choose j now, 0 in $t+1$ and 0 in $t+2$
  - Choose 0 now, j in $t+1$ and 0 in $t+2$
- In $t+3$ both have the same past choice and the same experience -> $\bar{V}_{t+3}(x_{it+3})$ is the same

CCP estimation with more CCPs II

- Need to rewrite v_{ijt} and v_{i0t} such that they both end with a term $\bar{V}_{t+3}(x_{it+3})$
- Use $\bar{V}_{t+2}(x_{it+2}) = \gamma + v_{ij^*t+2} - lnPr(d_{it+2} = j^*|x_{it+2})$ with $v_{ij^*t+2} = u_{j^*}(x_{it+2}) + \beta \int \bar{V}_{t+3}(x_{it+3}) f(x_{it+3}|x_{it+2}, d_{it+2} = j^*) dx_{it+3}$

(notation is not great here as the arbitrary choice j^* differs in each sequence and time period)

CCP estimation without finite dependence

- CCP estimation can still be useful in one of the following ways
  - In stationary problems, the $\bar{V}()$ can be written as a function of CCPs and utility parameters by inverting a matrix instead of looking for a fixed point as in Rust (1987)
  - We can write the problem using CCPs until the end (or sufficiently far into the future if infinite horizon)
- CCP estimators are less efficient (in finite samples), one can update the CCPs using the model to regain efficiency (Aguirregabiria and Mira (2002))
- Standard errors are not correct when CCPs have to be estimated first

References I

Victor Aguirregabiria and Pedro Mira.
Swapping the nested fixed point algorithm: A class of estimators for discrete Markov decision models.
Econometrica, 70(4):1519–1543, 2002.

Peter Arcidiacono, Esteban Aucejo, Arnaud Maurel, and Tyler Ransom.
College Attrition and the Dynamics of Information Revelation.
Working paper, 2016.

Peter Arcidiacono and Paul B. Ellickson.
Practical Methods for Estimation of Dynamic Discrete Choice Models.
Annual Review of Economics, 3:363–394, 2011.

References II

V. Joseph Hotz and Robert A. Miller.
Conditional choice probabilities and the estimation of dynamic models.
The Review of Economic Studies, 60(3):497–529, 1993.

John Rust.
Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher.
Econometrica, 55(5):999–1033, September 1987.