Empirical IO homework

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December 28, 2020

Question 1

1.1 Demand side

The utility of a consumer i is for product j is:

$$U_{ij} = \xi_j - \alpha_i p_j + \varepsilon_{ij}$$

where ξ_j is the product-specific shock:

$$\xi_j = 4 + j/5 \tag{1.1}$$

and ε_{ij} is the indiosyncratic shock. We assume that α_i is distributed log-normal, i.e. $log(\alpha) \sim N(1,1)$, and ε_{ij} is distributed type I extreme value.

The probability of buying product j for individual i is:

$$s_{ij}(p, \alpha_i) = \frac{\exp\left(\xi_j - \alpha_i p_j\right)}{1 + \sum_{k\in\mathcal{J}} \exp\left(\xi_k - \alpha_i p_k\right)}$$

Then we aggregate individual probabilities to get market shares:

$$s_j(p) = \int \frac{\exp\left(\xi_j - \alpha p_j\right)}{1 + \sum_{k\in\mathcal{J}} \exp\left(\xi_k - \alpha p_k\right)} d\phi(\alpha)$$

To approximate this integral, we draw $ns = 200,000$ observations of α and compute:

$$s_j(p) = \frac{1}{ns} \sum_{n=1}^{ns} \frac{\exp\left(\xi_j - \alpha_i p_j\right)}{1 + \sum_{k\in\mathcal{J}} \exp\left(\xi_k - \alpha_i p_k\right)}$$

The derivative of market shares with respect to price is:

$$\frac{\partial s_j(p)}{\partial p_j} = \frac{1}{ns} \sum_{n=1}^{M} (-\alpha_i)(s_{ij} - s_{ij}^2) \tag{1.2}$$

$$\frac{\partial s_k(p)}{\partial p_j} = \frac{1}{ns} \sum_{n=1}^{ns} \alpha_i s_{ik} s_{ij} \quad \forall k \neq j \tag{1.3}$$

The consumer surplus (in money value) is computed by the log-sum formula as usual. Denote by M the market size ($M = 2000$). The consumer surplus for all consumers is:

$$CS = M \times \frac{1}{ns} \sum_{n=1}^{ns} \frac{1}{\alpha_i} log\left(1 + \sum_{k\in\mathcal{J}} \exp\left(\xi_k - \alpha_i p_k\right)\right) \tag{1.4}$$

1.2 Supply side

Each firm j maximizes their profit:

$$\pi_j = (p_j - c_j)s_j(p)M \quad [1.5]$$

We assume that $c_j = 1 + j/8$. Denote by Ω^1 the **diagonal** matrix such that $\Omega^1_{jj} = \frac{\partial s_j(p)}{\partial p_j}$. The first order condition, written for all firms, is:

$$s(p) + \Omega^1(p)(p - c) = 0 \quad [1.6]$$

where s, p, c are vectors of market shares, prices and marginal costs.

1.3 Bertrand-Nash equilibrium

Solving the system of non-linear equations [1.6] gives us the equilibrium prices. For our case with 5 products:

	prices	market share	Producer profit
Product 1	1.8055	0.1199	163.1640
Product 2	1.9711	0.1099	158.5455
Product 3	2.1460	0.1015	156.5384
Product 4	2.3311	0.0944	156.9363
Product 5	2.5273	0.0884	159.6063

The Consumer Welfare is (by [1.4]):

$$CS_1 = 4278.3 \quad [1.7]$$

Question 2

2.1 Exogenous retailer's outside option

The producers set the retail prices p_j, the wholesale prices w_j and the franchise fees F_j to maximize:

$$\pi_j = (w_j - c_j)s_j(p)M + F_j \qquad [2.1]$$

subject to participation constraint:

$$\pi_r = \sum_{j \in S_r} \left[\left(p_j - w_j\right) s_j(p)M - F_j \right] \geq 0 \qquad [2.2]$$

The constraint must be binding so the objective function is rewritten as:

$$\pi_j = (p_j - c_j)s_j(p)M + \sum_{k \neq j}(p_k - w_k)s_k(p)M - \sum_{k \neq j} F_k \qquad [2.3]$$

Assume that $w_k = c_k$ for all k. The first order condition for each firm j is therefore only with respect to p_j:

$$(p_j - c_j)\frac{\partial s_j(p)}{\partial p_j} + s_j(p) + \sum_{k \neq j}(p_k - c_k)\frac{\partial s_k(p)}{\partial p_j} = 0 \qquad [2.4]$$

Denote by Ω^2 the matrix of market share derivatives, i.e. $\Omega^2_{j,k} = \dfrac{\partial s_k(p)}{\partial p_j}$. The First order conditions are written in matrix form:

$$s(p) + \Omega^2(p - c) = 0 \qquad [2.5]$$

Solving the system of nonlinear equations [2.5] gives us the equilibrium prices. For our case:

	prices	market share	Producer profit
Product 1	4.6609	0.0445	314.4107
Product 2	4.9738	0.0415	308.8996
Product 3	5.2762	0.0395	308.4593
Product 4	5.5688	0.0384	312.5785
Product 5	5.8522	0.0380	320.9722

The Consumer Welfare is (by [1.4]):

$$CS_1 = 3824 \qquad [2.6]$$

2.2 Endogenous retailer's outside option

We now endogenize the retailers' outside option. The producer j maximizes his profit

$$\pi_j = (w_j - c_j)s_j(p)M + F_j \qquad [2.7]$$

subject to the participation constraint:

$$\pi_r = \sum_{s \in S_r} \left[(p_s - w_s)\, s_s(p)M - F_s \right] \geq \sum_{s \in S_r \setminus \{j\}} \left[(\tilde{p}_s^{jr} - w_s)s_s\left(\tilde{p}_s^{jr}\right) M - F_s \right] \qquad [2.8]$$

where $\tilde{p}_s^{jr}$ and $s_s\left(\tilde{p}_s^{jr}\right)$ are the prices and market shares of other products $s \neq j$ when product j does not exist (i.e. is rejected by retailer r). With RPM, $\tilde{p}_i^{jr} = p_i$ if $i \neq j$, and with the constraint binding, we can write the profit as:

$$\left(p_j - c_j\right) s_j(p)M + \sum_{s \neq j} (p_s - w_s)\, s_s(p)M - \sum_{s \neq j} (p_s - w_s)\, s_s\left(\tilde{p}^{jr(s)}\right) M \qquad [2.9]$$

where $r(s)$ is the retailer who sells product s. Clearly, p_j is the only choice variable left and the last term is not dependent on p_j. Assume $w_j = c_j$ for all j. Therefore, the First order condition is exactly the same as the exogenous retailers' outside option case, giving us the same equilibrium prices, market shares, consumer surplus and total profits. However, the distribution of total profits between producers and retailers is different now that retailers require strictly positive profit.

Formula [2.9] gives us the profit of each producer, which is equal to the fixed fee F_j that they charge their retailer. The profit of each retailer is simply the difference between total margin and fixed fees:

$$\pi_r = \sum_{s \in S_r} [(p_s - c_s)\, s_s(p)M] - \sum_{s \in S_r} F_s \qquad [2.10]$$

The results are in the table below:

	prices	market share	Producer profit
Product 1	4.6609	0.0445	255.1872
Product 2	4.9738	0.0415	253.0026
Product 3	5.2762	0.0395	184.3583
Product 4	5.5688	0.0384	189.1047
Product 5	5.8522	0.0380	197.6587

and the profit of Retailer 1 and 2 are 115.1205 and 370.8882, respectively. This table is different from the previous one only in the last column where the producer profits are smaller.

Remark: If we allow the retailers to set the prices, then manufacturers and retailers will have to share the bilateral profit. With this solution concept we will find the prices by using a vertical merger (manufacture 1 and 2 merge, and manufactures 3,4, 5 merge). In this case prices are the below:

	prices
Product 1	2.1388
Product 2	2.3229
Product 3	2.6685
Product 4	2.8970
Product 5	3.1270

Hence, the consumer surplus is 3863.8, total profit is 1076.1, welfare is 4939.9, and the welfare change is -150.8994.

Question 3

Prices are higher and Total welfare is lower in the vertical non-linear pricing (with RPM) case than the horizontal price competition case.

	Horizontal price competition (Q1)	Vertical nonlinear pricing with RPM (Q2)	Difference (Q2-Q1)
Price			
—Product 1	1.8055	4.6609	2.8554
—Product 2	1.9711	4.9738	3.0027
—Product 3	2.1460	5.2762	3.1302
—Product 4	2.3311	5.5688	3.2377
—Product 5	2.5273	5.8522	3.3249
Total welfare	5073.091	3824	-1249.09
—consumer surplus	4278.3	2258.7	-2019.6
—Total profit	794.7905	1565.3	770.5095