Policy Evaluation Takehome

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Question 1: Reservation Wage

We start by formulating the value of employment as a function of U:

$$\rho E_j(w) = w_j + \eta_j[U - E_j] <=> E_j(w) = \frac{w_j + \eta_j U}{\rho + \eta_j}$$

Then

$$max[E_j(w), U] = max[\frac{w_j + \eta_j U}{\rho + \eta_j}, U] = \frac{1}{\rho + \eta_j} max[U(\rho + \eta_j), w_j + \eta_j U] = U + \frac{max[0, w_j - \rho U]}{\rho + \eta_j}$$

We thus get that $w_i^* = w_f^* = w^* = \rho U$. This makes sense because at the reservation value of the wage, worker is indifferent between staying unemployed and accepting the offer, thus (also due to the poisson properties of the separation process) the separation rates have no effect on the possible difference in reservation wages. Job offer rates and wage distributions trivially also have no effect.
Thus, by substituting new formula of $max[E_j(w), U]$ into the equation of value of unemployment and keeping in mind that $w^* = \rho U$, we get:

$$w^* = \rho U = b + \sum_{j=\{i,f\}} \frac{\lambda_j}{\rho + \eta_j} \int_{w^*} (w - w^*) dG_j(w)$$

Question 2: Steady-state employment proportions

To derive the probabilities $p(u), p(e_f), p(e_i)$, we find find the ergodic distributions of the corresponding variables using continous Markov Chain transition matrix:

$$AP = P$$

,where $P = (p(u), p(e_f), p(e_i))^T$ and the transition matrix is

$$A = \begin{bmatrix} 1 - \sum_{j \in \{f,i\}} \lambda_j \tilde{G}_j & \eta_f & \eta_i \\ \lambda_f \tilde{G}_f & 1 - \eta_f & 0 \\ \lambda_i \tilde{G}_i & 0 & 1 - \eta_i \end{bmatrix}$$

Intepretation for the transition matrix is the following: using $p(e_f)$ as the example, proportion of people employed in the formal market in the "next period" is equal to proportion of unemployed people multiplied by the rate of getting a good offer from the formal market plus the proportion of people who are already employed in the formal market and will not have their jobs terminated.
Solving this system of linear equations, we get the following stationary unemployment and employment values:

$$p(u) = \frac{1}{1 + \lambda_f \tilde{G}_f \eta_f^{-1} + \lambda_i \tilde{G}_i \eta_i^{-1}}$$

$$p(e_k) = \frac{\lambda_k \tilde{G}_k \eta_k^{-1}}{1 + \lambda_f \tilde{G}_f \eta_f^{-1} + \lambda_i \tilde{G}_i \eta_i^{-1}}, k \in \{i, f\}$$

Question 3: Likelihood Function

Starting with the likelihood of a single observation being unemployed with the duration t_u or employed in formal/informal market with wage w, we have:
$L(u, t_u) = \tilde{f}_u(t_u)p(u) = \frac{(\lambda_f \tilde{G}_f + \lambda_i \tilde{G}_i)exp(-(\lambda_f \tilde{G}_f + \lambda_i \tilde{G}_i)t_u)}{1+\lambda_f \tilde{G}_f \eta_f^{-1} + \lambda_i \tilde{G}_i \eta_i^{-1}}$
$L(w, e_k) = \frac{g_k(w)}{\tilde{G}_k(w^*)}p(e_k) = \frac{g_k(w)\lambda_k \eta_k^{-1}}{1+\lambda_f \tilde{G}_f \eta_f^{-1} + \lambda_i \tilde{G}_i \eta_i^{-1}}$
Then the likelihood function of a sample is:
$L(w_{f1}, ..., w_{fN_f}, w_{i1}, ..., w_{iN_i}, t_1, ..., t_{N_u}) =$
$\prod_{j \in e_f}[\frac{g_f(w_{fj})}{\tilde{G}_f(w^*)}p(e_f)] \prod_{j \in e_i}[\frac{g_i(w_{ij})}{\tilde{G}_i(w^*)}p(e_i)] \prod_{j \in u}[(\lambda_f \tilde{G}_f(w^*) + \lambda_i \tilde{G}_i(w^*))exp(-(\lambda_f \tilde{G}_f(w^*) + \lambda_i \tilde{G}_i(w^*))t_u(j))p(u)]$

Loglikelihood is then:
$lnL = -Nln[1 + \lambda_f \tilde{G}_f(w^*)\eta_f^{-1} + \lambda_i \tilde{G}_i(w^*)\eta_i^{-1}] + \sum_{j \in e_f} ln[g_f(w_{fj})] + \sum_{j \in e_i} ln[g_i(w_{ij})] + N_f ln[\lambda_f \eta_f^{-1}] + N_i ln[\lambda_i \eta_i^{-1}] + N_u ln[\lambda_f \tilde{G}_f(w^*) + \lambda_i \tilde{G}_i(w^*)] - (\lambda_f \tilde{G}_f(w^*) + \lambda_i \tilde{G}_i(w^*)) \sum_{j \in u} t_u(j)$

Question 4: First-order Conditions and Identification

First, set $\hat{w}^* = min(w_{f1}, ..., w_{fN_f}, w_{i1}, .., w_{iN_i})$ as $w_f^* = w_i^*$
Now we will calculate the first-order conditions with respect to offer rates λ and termination rates η:
$\frac{\partial lnL}{\partial \lambda_k} = -N\frac{\tilde{G}_k(\hat{w}^*)\eta_k^{-1}}{1+\lambda_f \tilde{G}_f(\hat{w}^*)\eta_f^{-1} + \lambda_i \tilde{G}_i(\hat{w}^*)\eta_i^{-1}} + N_k \frac{1}{\lambda_k} + N_u \frac{\tilde{G}_k(\hat{w}^*)}{\lambda_f \tilde{G}_f(\hat{w}^*) + \lambda_i \tilde{G}_i(\hat{w}^*)} - \tilde{G}_k(\hat{w}^*) \sum_{j \in u} t_u(j) = 0$ for $k = \{f, i\}$

$\frac{\partial lnL}{\partial \eta_k^{-1}} = -N\frac{\tilde{G}_k(\hat{w}^*)\lambda_k}{1+\lambda_f \tilde{G}_f(\hat{w}^*)\eta_f^{-1} + \lambda_i \tilde{G}_i(\hat{w}^*)\eta_i^{-1}} + N_k \frac{1}{\eta_k^{-1}} = 0 <=> \frac{N_k}{N} = (\frac{\tilde{G}_k(\widehat{w^*})\lambda_k \eta_k^{-1}}{1+\lambda_f \tilde{G}_f(\hat{w}^*)\eta_f^{-1} + \lambda_i \tilde{G}_i(\hat{w}^*)\eta_i^{-1}})$ for $k = \{f, i\}$
Plugging the 2nd equation into the 1st, we get:
$-N_k \frac{1}{\lambda_k} + N_k \frac{1}{\lambda_k} + N_u \frac{\tilde{G}_k(\hat{w}^*)}{\lambda_f \tilde{G}_f(\hat{w}^*) + \lambda_i \tilde{G}_i(\hat{w}^*)} - \tilde{G}_k(\hat{w}^*) \sum_{j \in u} t_u(j) = 0 <=> \frac{\sum_{j \in u} t_u(j)}{N_u} = [\lambda_f \tilde{G}_f(\hat{w}^*) + \lambda_i \tilde{G}_i(\hat{w}^*)]^{-1}$
This last equation is independent of k and thus, out of 4 FOCs calculated above, only 3 of them are linearly independent. Hence, we cannot separately identify all of these parameters without additional assumptions.

Question 5: Estimation

Assuming $\eta_f = \eta_i, \lambda_f = \lambda_i, w_j \sim N(\mu_j, \sigma_j)$, we can use some of the FOCs inspected before to simplify the calculations.
Using the very last equation:
$\lambda = \frac{N_u}{\sum_{j \in u} t_u(j)} \frac{1}{(1-G_f(w^*)) + (1-G_i(\hat{w}^*)}$
Using the two FOCs with respect to $\eta_k, k \in \{f, i\}$, we also have
$\eta \lambda^{-1} = \frac{N}{N_f}(1 - G_f(\hat{w}^*)) - (1 - G_f(\hat{w}^*)) - (1 - G_i(\hat{w}^*)) = \frac{N}{N_i}(1 - G_i(\hat{w}^*)) - (1 - G_f(\hat{w}^*)) - (1 - G_i(\hat{w}^*))$
This thus both gives us η and λ as functions of G_i, G_f and an additional condition on the wage distributions:
$\frac{(1-G_f(\hat{w}^*))}{N_f} = \frac{(1-G_i(\hat{w}^*))}{N_i}$
We can use the first two equations to maximize the loglikelihood functions with respect only to wage distribution parameters, which simplifies the computation. The last optimality condition cannot be used to concentrate out any of the parameters in the numerical estimation step, because the Log-Normal distribution does not allow an analytical expression. Still, the optimality condition turns out to be extremely useful to check the consistency of the results.

We estimate parameters numerically optimizing via R's built-in optim package using the BFGS algorithm as well as a box-constrained version called L-BFGS-B. We find that the former gives more stable results and converges to the same parameter values for different starting values. We further validate that the L-BFGS-B algorithm converges to the same parameters when feeding it with the optimal parameter values found for the BFGS algorithm. We then validate the estimated parameters by checking the last optimality condition mentioned above, which we did not enforce at this point of the estimation. We find that the estimated parameters fulfill this condition very well; the two sides of the equation only differ by a factor of around 0.003%.

To estimate standard errors, we turn to the maxLik package in R, which is a wrapper function that takes a slightly different optimization routine and makes the computation of standard errors much easier. Interestingly, while this package is the standard in R, the optimization algorithm of this package does not converge to our estimated parameters for quite a few starting values. To validate that our previous estimates are indeed correct, we perform two exercises. First, we show that the maxLik estimator indeed converges to our parameter estimates if we change the starting parameter values. Secondly, we show that the estimated parameter values based on the maxLik package that differ from our estimates do not fulfill the optimality condition that we use as a consistency check. In the end, we thus feel confident about the estimates we obtain and we report these results in Table 1. We suspect that the reason why the process converged to the different estimates was local flatness of the likelihood function.
Results are the following:

Table 1: Maximum Likelihood results

μ_f	1.024535
	(0.003239)***
σ_f	0.407075
	(0.002290)***
μ_i	-4.894144
	(0.093319)***
σ_i	3.136670
	(0.072597)***
λ	0.3326796
η	0.03961811
w^*	0.4169447
b	-55.49957

Note: *p$<$0.1; **p$<$0.05; ***p$<$0.01

Note that the wage distribution parameters are in the log form. The missing standard errors are not calculated as these parameters were not part of the maximum likelihood estimation and instead would require delta method to be estimated.
Looking at the results, we can see that the informal market has lower expected wages but higher variance which makes sense given that there are fewer workers employed in the informal sector (thus indicating lower expected value), but still some are and those workers may still enjoy high wages, which requires higher variance. Estimated benefit from unemployment b is negative due to low reservation wage, but this may be a byproduct of the random search model and homogenous workers: within this model, worker loses an opportunity to earn much higher wages later on by refusing the current job offer, thus indicating low b, while in reality these opportunities may simply not exist. The job offer rate is estimated to be 10 times larger than the job termination rate.