Job Search with Formal and Informal Sectors

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1 Reservation Wage

Using the value from being employed in a job j, $\rho E_j(w_j) = w_j + \eta_j[U - E_j(w_j)]$, we find

$$E_j(w_j) = \frac{w_j + \eta_j U}{\rho + \eta_j}$$

which we can substitute into the value function for being unemployed:

$$\rho U = b + \lambda_f \int_{\rho U} \frac{w_f - \rho U}{\rho + \eta_f} dG_f(w_f) + \lambda_i \int_{\rho U} \frac{w_i - \rho U}{\rho + \eta_i} dG_i(w_i)$$

in this expression, we took the reservation wage to be

$$w_j^* = \rho U$$

which gives us the fixed point equation for reservation wage:

$$w_j^* = b + \lambda_f \int_{w_j^*} \frac{w_f - w_j^*}{\rho + \eta_f} dG_f(w_f) + \lambda_i \int_{w_j^*} \frac{w_i - w_j^*}{\rho + \eta_i} dG_i(w_i).$$

The reservation wage is common between jobs since conditional on being offered some wage in a sector, the worker chooses only between that job or unemployment. This means that there is a parallel between the two sector case incentives and the single sector case incentives that drives reservation wage to be the same across sectors.

2 Steady-State Proportion of Employment

As in the lectures, the probability that a randomly-sampled individual is unemployed is

$$\begin{aligned} p(u) &= \frac{Et_u}{Et_u + Et_i + Et_f} \\ &= \frac{(\lambda_i \tilde{G}_f(w^*) + \lambda_f \tilde{G}_i(w^*))^{-1}}{(\lambda_i \tilde{G}_f(w^*) + \lambda_f \tilde{G}_i(w^*))^{-1} + \eta_i^{-1} + \eta_f^{-1}} \end{aligned}$$

in addition,

$$\begin{aligned} p(i) &= \frac{\eta_i^{-1}}{(\lambda_i \tilde{G}_f(w^*) + \lambda_f \tilde{G}_i(w^*))^{-1} + \eta_i^{-1} + \eta_f^{-1}} \\ p(f) &= \frac{\eta_f^{-1}}{(\lambda_i \tilde{G}_f(w^*) + \lambda_f \tilde{G}_i(w^*))^{-1} + \eta_i^{-1} + \eta_f^{-1}} \end{aligned}$$

where $\tilde{G}_j(w^*) = Pr_j(w > w^*)$.

3 Log-likelihood Function

For an individual that is unemployed, the likelihood is

$$L(t_u, u) = \boldsymbol{G}(w^*) \exp[-\boldsymbol{G}(w^*) t_u] \times p(u)$$

where $\boldsymbol{G}(w^*) = \lambda_i \tilde{G}_f(w^*) + \lambda_f \tilde{G}_i(w^*)$, the likelihood of finding an employed individual in sector j is

$$L(w_j, j) = \frac{g_j(w_j)}{\tilde{G}_j(w^*)} \times p(j).$$

these give the empirical likelihood function for each k individual of a population size N:

$$L(\boldsymbol{w}, \boldsymbol{j}, \boldsymbol{t_u}) = \prod_{k \in j} [\frac{g_j(w_j)\boldsymbol{G}(w^*)\eta_j}{\tilde{G}_j(w^*)(\eta_i\eta_f + \boldsymbol{G}(w^*)\eta_i + \boldsymbol{G}(w^*)\eta_f)}] \times \prod_{k \in u} [\frac{\boldsymbol{G}(w^*) \exp(-\boldsymbol{G}(w^*) t_u)\eta_i\eta_f}{\eta_i\eta_f + \boldsymbol{G}(w^*)\eta_i + \boldsymbol{G}(w^*)\eta_f}]$$

giving a log-likelihood:

$$\begin{aligned}\ln L = & \sum_{k \in i} \ln(g_i(w_i)) + \sum_{k \in f} \ln(g_f(w_f)) + N \ln(\boldsymbol{G}(w^*)) + \sum_{k \in i} \ln(\eta_i) + \sum_{k \in f} \ln(\eta_f) + \sum_{k \in u} \ln(\eta_i\eta_f) \\ & - N \ln(\eta_i\eta_f + \boldsymbol{G}(w^*)\eta_i + \boldsymbol{G}(w^*)\eta_f) - \sum_{k \in i} \ln(\tilde{G}_i(w^*)) - \sum_{k \in f} \ln(\tilde{G}_f(w^*)) - \sum_{k \in u} \boldsymbol{G}(w^*) t_u.\end{aligned}$$

4 Identification

Taking the derivative of the log-likelihood with respect to the job termination rates η_j:

$$\begin{aligned}\frac{N_f}{N} &= \frac{\boldsymbol{G}(w^*)\eta_f}{\eta_i\eta_f + \boldsymbol{G}(w^*)\eta_i + \boldsymbol{G}(w^*)\eta_f} \\ \frac{N_i}{N} &= \frac{\boldsymbol{G}(w^*)\eta_i}{\eta_i\eta_f + \boldsymbol{G}(w^*)\eta_i + \boldsymbol{G}(w^*)\eta_f}\end{aligned}$$

and the first-order condition with respect to the hazard rate out of unemployment $\boldsymbol{G}(w^*)$:

5 Empirical Estimation and Results