Discrete/continuous and Dynamic Demand

Pierre Dubois

Toulouse School of Economics

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Discrete/Continuous Demand

- Consumer choices are Discrete, but sometimes with a lot of modalities
- Discrete choice often criticized because observed consumers often buy several units (cans of soft drinks, ..)
- Several solutions or ways to rationalize data

Discrete/Continuous Demand

- Rationalize multiple choices assuming they are just aggregation over several choice instances
    - For example, a consumer shopping in a store for a week. Assuming each day is a choice decision means consumer who bought 5 cans of soft drinks decided to choose the outside option on two choice occasions.
    - Unappealing, because assumes choices across days are independent.
- Continuous Demand models for homogenous goods (AIDS, ..)
    - Cannot explain discrete dimension
- Discrete/continuous demand model
- Dynamic demand model with stockpiling: quantity choice is not consumption

Discrete/Continuous Demand

- Hendel (1999) studies a multi-discrete choice situation. Observes firms simultaneously buying several brands of computers and several units of each brand.
    - Non-discreteness is in two dimensions:
    - Choice of several brands as an aggregation over several tasks.
    - For each task there is an optimal brand, but observed purchases are aggregation over several tasks.
    - The purchase of several units is explained by a decreasing marginal utility from quantity.

Discrete/Continuous Demand

- Dubin and McFadden (1984) "An econometric Analysis of residential Electric Appliance Holdings and Consumption", Econometrica
- Hanemann (1984) "Discrete/Continuous Models of Consumer Demand", Econometrica

- Objective: Formulating econometric model of discrete/continuous consumer choices in which discrete and continuous choices coming both from the same underlying (random) utility maximization decision

Discrete/Continuous Demand

- Assume preferences $u(x, q, b, z, \epsilon)$ over vector of goods $x_1, .., x_J$, numeraire good q, with goods' attributes in vector b, consumer characteristics z and unobserved taste shocks ϵ
- Assume $u(.)$ is such that $\frac{\partial u}{\partial b_j} = 0$ if $x_j = 0$, such that attributes of good j don't matter if good j not consumed
- Maximize direct utility function $u(x, q, b, z, \epsilon)$ subject to budget constraint:

$$\sum_{j=1}^{J} p_j x_j + q = y$$

and exclusivity constraints

$$x_j x_{j'} = 0 \qquad \forall j \neq j'$$

and positivity constraints on x and q, where x_j is the quantity demanded of good j by the consumer with unit price p_j and q is the outside good (other purchases).

Random Utility Demand Model

- Maximization yields conditional demand functions $\overline{x}_j(p_j, y, b_j, z, \epsilon)$ and $\overline{q}(p_j, y, b_j, z, \epsilon) = y - p_j\overline{x}_j(p_j, y, b_j, z, \epsilon)$
- Conditional indirect utility functions

$$\overline{v}_j(p_j, y, b_j, z, \epsilon) = u(0, .., \overline{x}_j(p_j, y, b_j, z, \epsilon), .., 0, \overline{q}(p_j, y, b_j, z, \epsilon), b, z, \epsilon)$$

- Choice of alternative made from a choice set J, according to:

$$\overline{v}_j(p_j, y, b_j, z, \epsilon) \geq \overline{v}_k(p_j, y, b_j, z, \epsilon) \qquad \forall k \in J$$

- Random for econometrician because unobservable ϵ.
- Specification of the conditional indirect utility function: Dubin and McFadden (1984) Hanemann (1984), Smith (2004), Dubois and Jodar (2012).

Random Utility Demand Model

- Dubois and Jodar (2012) characterize indirect utility functions leading to conditional expenditure function additively separable between income and prices.
- From Roy's identity, $\overline{v}_j$ must satisfy PDE

$$\overline{x}_j(p_j, y, b_j, z, \epsilon) = -\frac{\frac{\partial \overline{v}_j(p_j, y, b_j, z, \epsilon)}{\partial p_j}}{\frac{\partial \overline{v}_j(p_j, y, b_j, z, \epsilon)}{\partial y}}$$

- Several possible specifications

Random Utility Demand Model

- If we impose a linear additive form for conditional expenditure $\overline{e}_j(p_j, y, b_j, z, \epsilon) = p_j \overline{x}_j(p_j, y, b_j, z, \epsilon)$ such as

$$\overline{e}_j(p_j, y, b_j, z, \epsilon) = h(\ln p_j) + \beta_2 y + \omega_j$$

where ω_j may depend on b_j, z, ϵ, and $h(.)$ known increasing

- From Roy's identity, $\overline{v}_j$ must satisfy PDE

$$-p_j \frac{\partial \overline{v}_j}{\partial p_j} / \frac{\partial \overline{v}_j}{\partial y} = h(\ln p_j) + \beta_2 y + \omega_j$$

Random Utility Demand Model

- Thus $\overline{v}_j(p_j, y, b_j, z, \epsilon)$ must be

$$\overline{v}_j = \Phi\left(\left[H\left(\ln p_j\right) + \beta_2 y + \psi_j\right] \exp\left(\phi_2 - \beta_2 \ln p_j\right), \omega_j\right)$$

with

$$H\left(z\right) = \beta_2 \int_z^{+\infty} h\left(x\right) \exp \beta_2 \left(z - x\right) dx$$

and where $\Phi\left(., \omega_j\right)$ is increasing in first argument.

- Can show by integration by parts that $H\left(.\right)$ is linear if $h\left(.\right)$ is affine

$$h\left(z\right) = \beta_1 \left(z - 1/\beta_2\right) \rightarrow H\left(z\right) = \beta_1 z$$

Discrete/Continuous Demand Specification

- Functional form choice (Dubin McFadden, 1984, Smith, 2004, Dubois and Jodar,2012)

$$\overline{v}_{ij} = \left[\beta_2^i y_i + \beta_1^i \ln p_j + \psi_{1j}^i\right] \exp\left[(\phi_{2j}^i - \beta_2^i \ln p_j)\zeta_i\right] + \psi_{2j}^i + \epsilon_{ij}$$

  where ζ_i is an unobservable preference shock and ϵ_{ij} is an unobserved additive preference shock.

- It implies the conditional expenditure function

$$\overline{e}_{ij}(p_j, y_i, b_{ij}, z_i, \epsilon_{ij}) = \left(\beta_1^i \ln p_j + \beta_2^i y_i + \psi_{1j}^i\right)\zeta_i - \frac{\beta_1^i}{\beta_2^i}$$

- ψ_{1j}^i and ψ_{2j}^i are good j specific tastes quality indexes. ψ_{1j}^i can contain observed characteristics z_i.

Discrete/Continuous Demand Specification

- Assuming $(\epsilon_{ij}, \zeta_i) \perp (p_j, y_i, b_j, z_i)$, distribution of $\overline{v}_{ij}$ conditional on (p_j, y_i, b_{ij}, z_i) is non parametrically identified using micro data, but not necessarily that of ζ_i and ϵ_{ij} (Berry and Haile, 2010).
- Assuming that ϵ_{ij} are i.i.d. Type-1 Extreme Value, conditional probability for i to choose j is:

$$s_{ij}(\zeta_i) = \frac{\exp\left\{\left[\beta_2^i y_i + \beta_1^i \ln p_j + \psi_{1j}^i\right] \exp\left[(\phi_{2j}^i\text{-}\beta_2^i \ln p_j)\zeta_i\right] + \psi_{2j}^i\right\}}{\sum_k \exp\left\{\left[\beta_2^i y_i + \beta_1^i \ln p_k + \psi_{1k}^i\right] \exp\left[(\phi_{2k}^i\text{-}\beta_2^i \ln p_k)\zeta_i\right] + \psi_{2k}^i\right\}}$$

- The preference variation multiplicative term $\exp \zeta_i$ gives a random coefficient structure to the random utility.
- Specifying a distribution of coefficients, we can integrate $s_{ij}(\zeta_i)$ as in random coefficient logit model (Berry 1994, Berry et al., 1994).

Discrete/Continuous Demand Specification

- β_1^i determines price elasticity of conditional demand $-1+\beta_1^i \frac{\zeta_i}{x_j p_j}$
- Unconditional demand of good j is

$$s_{ij}\left(\zeta_i\right)\overline{x}_{ij}(p_j, y_i, b_j, z_i, \epsilon_{ij})$$

  depends on all prices and characteristics
- Assumptions: (ζ_i, ϵ_{ij}) i.i.d. ζ_i assumed log-normal $LN(0,\lambda)$ and ϵ_{ij} extreme value
- Then can use Maximum Likelihood

Discrete/Continuous Demand Estimation

- Unconditional choice probability or market share of good j is

$$\pi_j = \int_\zeta s_{ij}\left(\zeta_i\right) dF\left(\zeta_i\right)$$

- The distribution of the unobserved individual characteristics determine a density $f_{\overline{e}_{ij}}\left(\overline{e}_{ij} \mid j\right)$ for this expenditure level:

$$f_{\overline{e}_{ij}}\left(\overline{e}_{ij} \mid j\right) = \frac{1}{\left[\overline{e}_{ij} + \frac{\beta_1^i}{\beta_2^i}\right]\sqrt{2\pi\lambda}} \exp \frac{-1}{2\lambda}\left[\ln\left(\frac{\overline{e}_{ij} + \left(\beta_1^i/\beta_2^i\right)}{\beta_2^i y_i + \beta_1^i \ln p_j + \psi_{1j}^i}\right)\right]^2$$

- So the Likelihood function is:

$$\ln L = \sum_{i,j} d_{ij}\left\{\ln \pi_{ij} + \ln\left[f_{\overline{e}_{ij}}\left(\overline{e}_{ij} \mid j\right)\right]\right\}$$

Discrete/Continuous Demand

- Application (Dubois and Jodar, 2012)
- Consumer decides:
  - In which retailer to buy a bundle of products (assume one-stop shopping model)
  - How much to spend in the bundle
- Dubois and Jodar (2012) analyze selection of "first-choice" retailer (as in Smith, 2004), i.e. the retailer in which most of the expenditure is made, and the conditional expenditure function

Discrete/Continuous Demand

- Firms profit maximization

$$\Pi_h = \sum_{j \in J_h} (p_j - c_j)\, q_j$$

where $q_j = \int s_{ij} x_{ij} dF(\zeta_i)$ is the total demand at retail store j.

- First order conditions, take into account effects of prices on conditional demand and on store choice

$$\begin{aligned} 0 &= \int s_{ij} x_{ij} dF(\zeta_i) \\ &+ (p_j - c_j) \int s_{ij} \frac{\partial x_{ij}}{\partial p_j} dF(\zeta_i) + \sum_{k \in J_h} (p_k - c_k) \int x_{ik} \frac{\partial s_{ik}}{\partial p_j} dF(\zeta_i) \end{aligned}$$

because $\frac{\partial x_{ik}}{\partial p_j} = 0$ for all $k \neq j$.

Discrete/Continuous Demand

- Model is then used to perform counterfactual simulations
- Evaluate the impact of shock in transportation cost (distance)
- Affects degree of competition between retailers because affects store choice decision and retailers equilibrium price

Demand Model in Product and Characteristic Space

- "Do Prices and Attributes Explain International Differences in Food Purchases", Dubois, Griffith, Nevo, American Economic Review 2014

Motivation

- Food purchases differ across countries, time and demographics
- Seem to be correlated with obesity and obesity-related health outcomes
- The differences are due to many factors
  - economic environment: prices, product attributes
  - other factors: culture, eating habits, preferences
- How far can prices and attributes go in explaining observed differences?

General Strategy

- Document cross-country differences
- Propose and estimate a model of demand for food (and nutrients)
  - build on Gorman (1956); nests product and characteristics models
  - estimate the model for each country separately
- Conduct counterfactuals
  - purchases (and implied nutrients) if faced with prices and attributes from other countries
- Data : France, UK and the US we have:
  - a panel of household purchases gathered using home scanners
  - product level nutritional information

Why Do We Care?

- Differences in nutritional intake mirrored in a number of health outcomes
- For example, obesity rates
  - countries: France: 14.5%, UK: 23.6% ,US: 30.0%
- A key cause of obesity is caloric intake
- Poor nutrition not just about obesity
- Just to be clear
  - other factors are at play: not just food at home
  - we will not directly look at obesity or other health outcomes

Key Findings

- Economic environment is quite important
  - American faced with French prices/attributes: prices diff explain US-FR cals diff
  - American faced with British prices/attributes: prices/attributes matter, but go the "wrong" way
- Prices/attributes are not everything
  - cannot explain composition of consumption, even when agg match
  - cannot explain US-UK differences
- Ranking "health" of the environment: $FR \succ US \succ UK$
  - France has expensive high nutrient products
  - UK has cheap low nutrient products

Related Literature

- Health Economics: diff in obesity across time and markets
  - e.g., Cutler et al. (2003), Philipson and Posner (2003)
  - we quantify the effects
- Development: Deaton (1997) Atkin (2013)
- Nutritional literature
  - e.g., Drewnowski and coauthors show that energy dense food negatively correlated with price
  - Our contribution
    1. use more detailed data
    2. account for several nutrients
    3. estimate a demand curve
    4. conduct counterfactuals

Data

- Data from US, UK and France collected using home scanning devices
  - from market research firms (TNS and Nielsen)
  - participating HH record all food purchased
  - exact date and location of purchase
  - UPC level quantity and price
  - In total hundreds of millions of transactions
- Detailed demographic information
- Nutritional information:
  - information contained on the nutritional label on the back of the package, very detailed

Data Matching: across countries

- The products, and even categories, differ widely across countries;
- We therefore classified the products into 52 categories used in the past by the USDA
- We further collapsed these into 9 broad product categories, which we focus on today
  - Fruits, Vegetables, Grains, Dairy, Meat, Fat, Sugar, Drinks, Prepared Foods
- We focus on 2005-2006

Data Caveats

- Many advantages of these data
  - detailed, comparable across countries
  - panel: observe household over time
  - exact prices and products
- But also some potential concerns about the data:
  - recording error
  - sample selection
  - consumption outside the home
- Try to make comparable as possible, performed several data quality tests when possible

Descriptive Statistics

Table 1 : Demographics

	France	UK	US
# of households	11,677	12,698	8,484
Household size	2.7	2.6	2.4
# of kids	0.7	0.6	0.5
Adult equivalent	2.2	2.1	2.0

Equivalence Scale

- Data are at the household level; household composition varies
- We therefore generate an equivalence scale
  - For each household we compute total caloric needs based on age and gender
    - girl age 4-6 requires 1545 calories per day;
    - boy age 11-14 requires 2220;
  - divide by 2500 to get "adult equivalent"
- There are alternatives that could be explored

Aggregate Purchases

Table 2 : Mean Purchases Across Countries

	FR	UK	US
calories	1776.6	1928.9	2102.7
from carbohydrates	667.4 (38%)	890.5 (47%)	1019.3 (49%)
from protein	287.9 (16%)	293.3 (16%)	264.9 (13%)
from fats	821.0 (46%)	694.5 (37%)	781.6 (37%)
carbohydrates (g)	178.0	237.5	271.8
protein (g)	72.0	73.3	66.2
fats (g)	91.2	77.2	86.8
expenditures (US$)	5.03	4.71	4.59

Average per person per day using an adult equivalent scale

Expenditure and quantity

Table 3: Expenditure and Quantity by Category

	Exp Shares (%)			Quantity			Calorie Share (%)		
Category	FR	UK	US	FR	UK	US	FR	UK	US
Fruits	6.6	9.3	8.1	14.6	14.0	17.2	4.5	4.5	5.3
Vegetables	9.7	10.4	7.9	18.2	20.2	14.0	5.3	6.0	3.0
Grains	6.0	8.4	7.8	6.7	13.4	8.8	14.3	19.8	14.3
Dairy	16.7	12.7	9.5	25.7	27.9	20.7	17.2	12.8	9.3
Meats	31.0	18.3	19.0	14.2	11.1	14.7	16.6	13.2	16.1
Oils	3.3	2.0	1.9	3.1	2.1	2.2	13.1	6.8	6.6
Sweeteners	1.4	1.1	1.4	2.4	2.4	2.6	5.1	4.9	4.4
Drinks	5.9	5.8	10.1	43.4	17.4	50.0	3.5	2.0	5.9
Prepared	21.2	32.7	36.1	16.4	26.2	30.0	22.8	31.2	38.0

Average per person per quarter using an adult equivalent scale, conditional on strictly positive expenditure

Prices

Table 4: Mean Prices by Category

	FR	UK	US
Fruits	2.09	3.21	2.12
Vegetables	2.53	2.32	2.64
Grain	**3.89**	**2.63**	**3.73**
Dairy	**3.26**	**2.22**	**2.48**
Meats	**10.33**	**7.29**	**5.88**
Oils	5.19	3.97	4.47
Sweeteners	2.79	2.38	4.61
Drinks	0.89	2.50	1.56
Prepared	**6.04**	**5.43**	**5.13**

Notes: units are US$ per 1 kilogram

Descriptive Statistics

Table 5: Nutritional Content by Category

calories from:	carbohydrates			protein			fat		
	FR	UK	US	FR	UK	US	FR	UK	US
Fruits	87	68	70	3	5	2	8	7	1
Vegetables	39	38	49	20	22	13	76	85	7
Grain	211	129	227	34	22	38	96	20	36
Dairy	18	22	29	71	57	48	188	166	130
Meats	5	21	30	76	72	66	120	129	206
Oils	2	7	6	11	3	2	678	602	671
Sweeteners	305	307	345	3	4	0	0	1	0
Drinks	27	34	69	1	4	2	1	4	5
Prepared	126	95	194	24	23	22	127	88	117

Average calories from nutrients (carbohydrates, proteins, fats) per 100 grams of food.

Model Overview

- Key challenge: how do we take advantage of the richness of the data?
- Option 1: Estimate demand the "usual" way.
  - is the disaggregated choice relevant for the big picture?
  - can we generalize?
  - how do compare brands across countries?
- Option 2: Use more aggregate product definition
  - how to use nutrient information?
  - how to deal with product differences across countries?
- We follow the second approach and offer a demand system that combines models in product and characteristics space

The Consumer's Problem

- The consumer chooses from N products;
- Product n is characterized by C characteristics $\{a_{n1}, ..., a_{nC}\}$.
- The utility of consumer i with demographics η_i is $U(x_i, \mathbf{z}_i, \mathbf{y}_i; \eta_i)$
  - x_i is the numeraire; $\mathbf{z}_i$ characteristics, $\mathbf{y}_i$ quantities consumed
- Define the $N \times C$ matrix $\mathbf{A} \equiv \{a_{nc}\}_{n=1,..N,c=1,..,C}$
- $U(x, \mathbf{y}, \mathbf{z})$:
  - relies on both characteristics and "flexible" functional forms to guide substitution patterns
  - breaks assumptions of weak separability between food groups
  - control for difference across countries in products

The Consumer's Problem

The consumer's problem:

$$\max_{x_i, \mathbf{y}_i} \; U(x_i, \mathbf{z}_i, \mathbf{y}_i; \eta_i)$$

$$s.t. \quad \sum_{n=1}^{N} y_{in} p_n + p_0 x_i \leq I_i \quad ; \qquad \mathbf{z}_i = \mathbf{A}' \mathbf{y}_i; \quad x_i, \; y_{in} \geq 0,$$

Dropping i subscripts, then the FOC if $y_n > 0$

$$\sum_{c=1}^{C} a_{nc} \frac{\partial U}{\partial z_c} - \frac{\partial U}{\partial x} \frac{p_n}{p_0} + \frac{\partial U}{\partial y_n} = 0 \quad ,$$

Discussion

- Private case I: $U(x, \mathbf{z})$ – the characteristics demand model
  - With linear technology at most C products consumed
  - Nests: discrete choice, hedonics
- Private case II: $U(x, \mathbf{y})$ – product level demand
  - can be viewed as a characteristics model where:
    - each product has a *unique* characteristic
    - this characteristic is fixed over time/market
- The model $U(x, \mathbf{y}, \mathbf{z})$ can:
  - rely on both characteristics and "flexible" functional forms to guide substitution patterns
  - will break assumptions of weak separability
  - control for difference across countries in products

The Role of Hedonic Prices

- In general
$$\frac{\partial U/\partial y_n}{\partial U/\partial x} = \frac{p_n}{p_0} - \sum_{c=1}^{C} a_{nc} \frac{\partial U/\partial z_c}{\partial U/\partial x}.$$
- If $\frac{\partial U/\partial z_c}{\partial U/\partial x}$ are constants, then this defines (Marshallian) demand, by substituting the "hedonic price" for the price.
- In our model demand depends on the hedonic prices of each good instead of prices
- If two goods have the same price, but one has more of a characteristic that the consumer values positively, they will adjust downwards the hedonic price - the good is more valuable to them

Functional Forms

- Assume J categories, each with K_j products
- Functional form (for now):

$$U(x_i, \mathbf{z_i}, \mathbf{y_i}; \eta_i) = \prod_{j=1}^{J} \left(\sum_{k=1}^{K_j} f_{ikj}\left(y_{ikj}\right) \right)^{\mu_{ij}} \prod_{c=1}^{C} h_{ic}\left(z_{ic}\right) \exp\left(\gamma_i x_i\right)$$

where $z_{ic} = \sum_{k,j} a_{kj,c} y_{ikj}$, $f_{ikj}(y_{ikj}) = \lambda_{ikj} y_{ikj}^{\theta_{ij}}$ and $h_{ic}\left(z_{ic}\right) = \exp(\beta_c z_{ic})$

- Basically, Cobb-Douglas across food groups, and CES aggregator within a group
- Very rich heterogeneity; limited substitution and income effects

Functional Forms

- Maximizing utility subject to budget constraint yields first order conditions:

$$\mu_{ij} \frac{f'_{ikj}\left(y_{ikj}\right) y_{ikj}}{\sum_l f_{ijl}\left(y_{ilj}\right)} + \sum_c a_{kj,c} y_{ikj} \frac{h'_{ic}\left(z_{ic}\right)}{h_{ic}\left(z_{ic}\right)} = \gamma_i \frac{p_{kj}}{p_0} y_{ikj}.$$

  for each k, j.

- Summing over k for a given j :

$$\mu_{ij} \frac{\sum_k f'_{ikj}\left(y_{ikj}\right) y_{ikj}}{\sum_k f_{ikj}\left(y_{ikj}\right)} + \sum_c \frac{h'_{ic}\left(z_{ic}\right)}{h_{ic}\left(z_{ic}\right)} \sum_k a_{kj,c} y_{ikj} = \gamma_i \sum_k \frac{p_{kj}}{p_0} y_{ikj}.$$

- Using $f_{ikj}(y_{ikj}) = \lambda_{ikj} y_{ikj}^{\theta_{ij}}$ and $h_{ic}\left(z_{ic}\right) = \exp(\beta_c z_{ic})$, we obtain

$$\sum_k p_{kj} y_{ikj} = p_0 \frac{\mu_{ij}\theta_{ij}}{\gamma_i} + \sum_c p_0 \frac{\beta_c}{\gamma_i} \sum_k a_{kjc} y_{ikj}.$$

Defining Products

- In principle, products could be defined very narrowly
- However this creates several problems
  - Is this a good model for the choice between narrowly defined products?
  - Different characteristics can be at play at different levels
  - We need to make the estimates transferable across countries
- Therefore, focus on J "categories" (the 9 we showed above) each with K_j mutually exclusive products

Estimating Equation

- Assume one characteristic unobserved. Let

$$p_0 \frac{\mu_{ij}\theta_{ij}}{\gamma_i} + p_0 \frac{\beta_1}{\gamma_i} \sum_k a_{kj,1} \times y_{ikjt} = \delta_{ij} + \xi_{jt} + \varepsilon_{ijt}$$

- Normalize, $\gamma_i = 1$ and $p_0 = 1$

$$w_{ijt} = \sum_c \beta_c z_{ijct} + \delta_{ij} + \xi_{jt} + \varepsilon_{ijt}$$

- where
  - $w_{ijt} = \sum_k p_{ikjt} y_{ikjt}$, $z_{ijct} = \sum_k a_{kj,c} y_{ikjt}$
  - δ_{ij} HH-cat FE; ξ_{jt} cat-qtr FE

Identification

- The error term includes:
  - individual preferences for specific categories
  - category specific seasonal effects
  - promotional activities
  - random noise (to expenditure)
- The independent variable, z_{ijct}, likely correlated with these
  - e.g., positive shock ... consume more ... nutrients increase
- Include HH-category and category-quarter FE
  - absorb the mean; remaining error is deviation
  - error will be uncorrelated with regressor if it does not impact quantity choice (or does so in a way that the total nutrients are constant)

Instruments

- The independent variable, $z_{ijct} = \sum_k a_{kj,c} y_{ikjt}$, likely correlated with error term
    - e.g., positive shock ... consume more ... nutrients increase
- Basic idea: assume nutrients of **available** products are (conditionally) exogenous
- To get variation across HH and time, construct

$$\omega_{ijct} = \frac{1}{\#\mathcal{A}_{ijt}} \sum_{k \in \mathcal{A}_{ijt}} a_{kj,c}$$

where $\mathcal{A}_{ijt}$ is the choice set of products in j for type i during t and assume that

$$E\left(\varepsilon_{ijt} | \omega_{ijct}, \delta_{ij}, \xi_{jt}\right) = 0.$$

where types are by area, quarter, preferred store.
- Requires that
    - nutrients (conditional) mean ind of unobserved characteristics
    - error term correlated with intensive but not extensive margin

Demand Estimates

Table 6: IV Estimates: preferences for nutrients

	FR	UK	US
Carbohydrates	1.213**	1.716***	1.517***
Proteins			
Dairy and Meat	24.78***	18.37***	19.64***
Prepared	16.38***	19.20***	51.77***
Other	2.243	2.887	-1.088
Fats			
Dairy and Meat	1.942	1.312*	1.113
Prepared	9.237***	10.36***	-2.357***
Other	1.495***	3.750***	1.640***

Product Effects

Table 7: IV Estimates: product effects

	FR	UK	US
Fruits	26.92	38.77	31.19
Vegetables	40.77	41.03	32.93
Grains	16.49	18.41	23.54
Dairy	24.66	25.35	14.87
Meat	73.40	38.25	28.89
Oils	10.71	3.11	5.39
Sweeteners	2.53	0.50	1.77
Drinks	24.33	22.67	37.31
Prepared	54.75	73.81	71.45

Average of the household-category and category-quarter fixed effects

Counterfactuals

- Our goal is to simulate what consumers would buy if faced with prices and attributes from other countries
- Focus on US consumers (average, low income, high calorie)
- Will simulate a change in nominal and "real" prices, attributes and unobserved attributes
- Choice of products within category (i.e., average characteristics).

Simulation

- Define the "average" household

$$\overline{\sigma}_j^H = \bar{y}_j \left[\overline{p}_j^H - \sum_c \widehat{\beta}_c^H \overline{a}_{jc}^H \right] \qquad \text{for } H \in \{US, FR, UK\}$$

- 5 parameters in simulation

$$\widehat{y}_j^{(V_1 \dots V_5)} = \frac{\overline{\sigma}_j^{V_3}}{\frac{\overline{p}_j^{V_2}}{\hat{\tau}^{V_5}} - \sum_c \widehat{\beta}_c^{V_4} \overline{a}_{jc}^{V_1}} \qquad V_i \in \{FR, UK, US\}$$

$\hat{\tau}^V = p_0^V / p_0^{US}$

Simulated Scenarios

- We consider a US consumer facing UK/FR prices/attributes
- Five counterfactual scenarios

Scenario A: hold fixed US quantities; change attributes;
Scenario B: change prices; simulate choice;
Scenario C: change prices and attributes; simulate choice;
Scenario D: change "real" prices and attributes;
Scenario E: change "real" prices, attributes, and category FE

Table 8: US Consumers Facing French Prices and Attributes

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
scenario:		A	B	C	D	E	
attributes	US	FR	US	FR	FR	FR	FR
prices	US	US	FR	FR	FR	FR	FR
product effects	US	US	US	US	US	FR	FR
nutrient pref	US	US	US	US	US	US	FR
price adjustment	1	1	1	1	1.079	1.079	1
Calories	2212.3	2158.3	1884.1	1839.9	2088.5	1946.6	1873.4
Carb (cal)	1092.6	903.0	1166.6	949.4	1070.5	766.6	709.6
	49.4	41.8	61.9	51.6	51.3	39.4	37.9
Prot (cal)	279.40	326.66	171.57	213.18	243.26	287.91	299.90
	12.6	15.1	9.1	11.6	11.6	14.8	16.0
Fat (cal)	840.3	928.7	545.9	677.3	774.8	892.1	863.9
	38.0	43.0	29.0	36.8	37.1	45.8	46.1

Table 9: US Consumers Facing UK Prices and Attributes

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
scenario:		A	B	C	D	E	
attributes	US	UK	US	UK	UK	UK	UK
prices	US	US	UK	UK	UK	UK	UK
product effects	US	US	US	US	US	UK	UK
nutrient pref	US	US	US	US	US	US	UK
price adjustment	1	1	1	1	1.089	1.089	1
Calories	2212.3	2015.3	2336.4	2157.3	2567.6	2372.5	1972.8
Carb (cal)	1092.6	936.1	1269.9	1095.9	1293.3	1066.5	926.1
	49.4	46.4	54.4	50.8	50.4	45.0	46.9
Prot (cal)	279.40	313.11	270.36	299.69	359.30	387.66	306.79
	12.6	15.5	11.6	13.9	14.0	16.3	15.6
Fat (cal)	840.3	766.1	796.1	761.8	915.0	918.4	739.9
	38.0	38.0	34.1	35.3	35.6	38.7	37.5

TABLE 11—LOW INCOME US HOUSEHOLDS FACING FRENCH/UK PRICES AND ATTRIBUTES

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Scenario	US	A	B	C	D	E	French/UK
French							
Calories	2,128.9	2,105.5	1,815.1	1,778.0	2,061.7	1,800.5	1,594.1
Expenditure	4.26	4.26	3.63	3.83	4.42	3.96	3.67
Carbohydrate (percent)	50.5	42.7	64.9	53.2	52.9	42.2	39.3
Protein (percent)	12.0	14.4	8.1	10.7	10.8	13.1	15.0
Fat (percent)	37.5	42.9	27.0	36.0	36.3	44.7	45.7
UK							
Calories	2128.9	1,891.4	2,156.5	1,970.5	2,438.7	2,509.6	1,841.3
Expenditure	4.26	4.26	4.02	4.26	5.18	5.87	4.23
Carbohydrate (percent)	50.5	46.7	58.2	53.5	53.7	48.1	47.5
Protein (percent)	12.0	15.0	10.2	12.7	12.7	15.2	15.0
Fat (percent)	37.5	38.3	31.6	33.8	33.7	36.7	37.5

Notes: Figures are per adult equivalent per day. Expenditure is in US$.

TABLE 12—HIGH CALORIE US HOUSEHOLDS FACING FRENCH/UK PRICES AND ATTRIBUTES

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Scenario	US	A	B	C	D	E	French/UK
French							
Calories	3,460.0	3,239.2	2,694.3	2,476.8	2,808.4	3,185.8	2,875.0
Expenditure	7.68	7.68	6.49	6.67	7.53	9.03	8.44
Carbohydrate (percent)	49.5	41.0	62.9	51.3	51.0	38.7	36.6
Protein (percent)	12.2	15.3	8.4	11.4	11.5	14.7	16.2
Fat (percent)	38.4	43.7	28.6	37.3	37.5	46.6	47.2
UK							
Calories	3,460.0	3,105.1	3,535.2	3,132.6	3,746.0	3,608.7	2,810.7
Expenditure	7.68	7.68	7.39	7.63	9.05	9.39	7.15
Carbohydrate (percent)	49.5	46.4	55.3	51.9	51.7	47.6	47.2
Protein (percent)	12.2	15.3	11.1	13.3	13.4	15.5	15.3
Fat (percent)	38.4	38.3	33.6	34.7	34.9	36.9	37.5

Notes: Figures are per adult equivalent per day. Expenditure is in US$.

Main Findings

- US/FR difference:
  - prices/attributes explain differences in calories
  - prices seem more important
- US/UK difference:
  - preferences explain the differences in calories
  - attributes seem to push in the "right" direction
  - prices go the "wrong" way
- Source of calories quite different: it is the interaction of preference, prices and attributes that explains the cross country differences
- Can rank "healthiness" of preferences and the environment
  - French environment generally encourages healthier purchasing
  - UK environment (especially prices) generates worse outcomes
    - Note: UK consumers purchase less calories than US consumers due to their preferences and *despite* their environment;

Dynamic Demand

- Goods can be stored and consumption delayed
- Data often allow to observe consumer purchases or store sales but not necessarily consumption
- Promotions, price uncertainty give incentives to do some intertemporal optimization. Larger quantity purchased can be due to increased consumption or to storage or both.
- But most demand models are static
- What does it imply for demand estimation?

Dynamic Demand

- Demand for differentiated products: effect of dynamics depends on reasons generating dynamics:
  - Storable products (food, batteries, ..)
  - Durable Products (cars, PCs, ..)
  - Habit formation
  - Switching costs
  - Learning

Dynamic Demand

- Consider storable products:
  - if storage costs are not too large and current price is low relative to future prices (i.e., the product is on sale), incentive for consumers to store and consume in the future. Pesendorfer (2002), Hendel and Nevo (2006 Rand), Perrone (2017) present evidence that consumers store when prices are low.
  - Hendel and Nevo (2006 EMA) extends the discrete choice static models to allow for storability: find that static model overestimates price elasticity and underestimates the cross price effects.

Dynamic Demand

- Consider durable products:
  - Dynamics arise due to similar trade-offs. Transaction costs in resale market of durable goods (for example, because of adverse selection) implies consumer's decision today of whether or not to buy a durable good (and which one) is costly to change in the future and will impact future utility. Consumer makes a purchase, depending on his current holdings of the good and expectations about future prices and attributes of available products.
  - Impact of durable products on static demand estimation depends if repeat purchase or not.

Dynamic Demand

- If no repeat purchases:
  - Distribution of random coefficients likely to change over time as some consumers purchase and exit the market. If prices fall over time, likely that less price sensitive consumers purchase initially.
  - Forward looking consumers have option value to not purchasing today, reflected in value of outside option.
- If repeat purchases (Gowrisankaran and Rysman, 2009):
  - Consumers do not exit. However, consumers who previously purchased have different value of no purchase since can stay with current product. Problem with static estimation that does not account for different outside option values across consumers and over time.
  - When purchasing, consumers have endogenous duration of holding product and it changes valuation of options. Might find optimal to buy inferior option (lower flow utility) but replace quickly with better/cheaper future option.

Dynamic Demand

- Heterogeneity
  - As in static models, heterogeneity is key to explain data and have flexible demand, but some degree of unobserved heterogeneity needs be sacrificed to deal with dimensionality problem.
- Data
  - As in static models, dynamic model can be estimated using consumer level or market level data. Advantages of consumer level data obvious in dynamic setting: allow see how individual behave over time. But hard to collect for durables purchased infrequently, then need use aggregate data.

Dynamic Demand

Implications of dynamic demand

- If consumers can store, need separate short run and long run response to either a temporary or permanent price change.
- For most applications, long run changes matter.
- If permanent price changes:
  - static estimation yields consistent estimates of long run demand responses if we use only permanent price changes and ignore temporary prices changes. But temporary price change are often most or even all variation in prices.

Dynamic Demand

Implications of dynamic demand

- If temporary price changes, static demand estimates:
  - over-estimate own price effects: demand response to sale is attributed to consumption increase (which in static model equals purchase), and not to increase in storage. Decline in purchases after sale coincides with price increase, and mis-attributed to consumption decline.
  - under-estimate cross price effects: during sale, quantity sold of competing products goes down, but static estimation misses additional effect: decrease in quantity sold in the future. When a competing product was on sale in the past, consumers purchased to consume today and therefore, the relevant "effective" cross price is not the current cross price effect.

Model of Consumer Stockpiling

- Consumer purchases brand j in size $x \in \{1, 2, .., X\}$: $d_{jxt} \in \{0, 1\}$
- Per-period utility from consumption

$$u_i \left(c_t, \nu_t \right) + \alpha_i m_t$$

  where c_t is vector of J quantities of each brand, ν_t is vector of J shocks that change the marginal utility of consumption and m_t is utility from outside good.
- Cost of holding vector of inventories by brand i_t is $C_i \left(i_t \right)$

Model of Consumer Stockpiling

- Purchase and consumption decisions $\{c, j, x\}$ to maximize

$$\sum_{t=1}^{\infty} \delta^{t-1} E\left[u_i\left(c_t, \nu_t\right) - C_i\left(i_t\right) + a_{jt}\beta_i - \alpha_i p_{jt} + \xi_{jxt} + \varepsilon_{ijxt} | s_1\right]$$

  s.t. $i_t \geq 0$, $c_t \geq 0$, $i_{t+1} = i_t + \sum_x d_{jxt} x_t - c_{jt}$, $\sum_{j,x} d_{jxt} = 1$, where s_t is the information set at t
- No physical depreciation of products
- Decision is made each period with perfect knowledge of current prices
- State variable $s_t = \left(i_t, p_t, a_t, \nu_t, \varepsilon_t\right)$: very large dimension
- Forward looking consumer expectations about future price and characteristics of products (could also be changing), future shocks ν_t, ε_t, using information of current and past values of all products attributes
- Without first order Markov assumption, state space even larger

Model of Consumer Stockpiling

- Value function $V_i(s_t)$ can be obtained as the unique solution of a Bellman equation:

$$\begin{aligned} V_i(s_t) &= \max_{\{c,j,x\}} \{ u_i\left(c_t, \nu_t\right) - C_i\left(i_t\right) + a_{jt}\beta_i - \alpha_i p_{jt} + \xi_{jxt} + \varepsilon_{ijxt} \\ &\quad + \delta \int V_i(s_{t+1}) dF_s\left(s_{t+1} | s_t, c, j, x\right) \} \end{aligned}$$

where F_s represents the transition probability of vector of state variables

Model of Consumer Stockpiling

- Given (ν_t, ε_t) i.i.d. over time, reduce dimensionality using integrated value function

$$EV_i\left(i_t, p_t\right) \equiv \int V_i(s_t) dF_\varepsilon\left(\varepsilon_t\right) dF_\nu\left(\nu_t\right)$$

which is unique solution to the integrated Bellman equation

$$\begin{aligned} & EV_i\left(i_t, p_t\right) \\ = \quad & \max_{\{c,x\}} \int \ln \sum_j \exp\left\{ \begin{array}{l} u_i\left(c_t, \nu_t\right) - C_i\left(i_t\right) + a_{jt}\beta_i - \alpha_i p_{jt} + \xi_{jxt} \\ +\delta E\left[EV_i\left(i_{t+1}, p_{t+1}\right) \middle| i_t, p_t, c, j, x\right] \end{array} \right\} dF(\nu_t) \end{aligned}$$

Model of Consumer Stockpiling

- Assume brands perfect substitutes in consumption and storage

$$u_i\left(c_t, \nu_t\right) = u_i\left(\sum_j c_{jt}, \sum_j \nu_{jt}\right) \quad C_i\left(i_t\right) = C_i\left(\sum_j i_{jt}\right)$$

- Reduces state space but also modify dynamic problem (modelling tension: brands differentiated at purchase but not consumption)
- Then, can show optimal consumption depends on size but not brand

$$c_k^*\left(s_t; x, k\right) = c_j^*\left(s_t; x, j\right) = c^*\left(s_t; x\right)$$

Model of Consumer Stockpiling

- Then, use inclusive value to reduce state space dimension

$$
\begin{aligned}
& EV_i\left(i_t, p_t\right) \\
= \quad & \max_{\{c,x\}} \int \ln \sum_x \exp \left\{ \begin{array}{c} u_i\left(c_t, \nu_t\right) - C_i\left(i_t\right) + \omega_{ixt} \\ +\delta E\left[EV_i\left(i_{t+1}, p_{t+1}\right) | i_t, p_t, c, x\right] \end{array} \right\} dF\left(\nu_t\right)
\end{aligned}
$$

where

$$
\omega_{ixt} = \ln\left(\sum_j \exp\left\{a_{jxt}\beta_i - \alpha_i p_{jxt} + \xi_{jxt}\right\}\right)
$$

- Problem as a choice between sizes, each with utility given by size-specific inclusive value
- Consumer just has to form expectations about the future inclusive value, or in some cases a low number of inclusive values for subsets of products, rather than expectations about the realizations of all attributes of all products.
- State space is still large and includes all prices needed to compute inclusive value

Model of Consumer Stockpiling

- Assumption: ω_{it} contains all the relevant information in s_t for ω_{it+1} conditional on s_t

$$F\left(\omega_{it+1} | s_t\right) = F\left(\omega_{it+1} | \omega_{it}\left(p_t\right)\right)$$

- Can test if other statistics of price matter
- This is consumer specific
- Then, can show expected future value only depends on lower dimensional statistic of full state

$$EV_i\left(i_t, p_t\right) = EV_i\left(i_t, \omega_{it}\left(p_t\right)\right)$$

Model of Consumer Stockpiling

- Estimation with consumer level data:
  - observe purchases over time, harder to get prices of brands not purchased
  - consumer inventory and consumption decisions not observed
- Can apply similar algorithm to Rust (1987)
  - For given parameters, solve dynamic program to obtain purchases and consumption as function of state variables including unobserved shocks
  - Assuming distribution of shocks, derive likelihood of observing consumer's decision conditional on prices and inventory
  - Search parameters values that maximize observed sample likelihood
  - Problem: state variable inventory not observed and high dimension state space

Model of Consumer Stockpiling

- Assume $\xi_{jxt} = \xi_{jx}$ (use fixed effects)
- Likelihood, given demographics D_i: $P\left(\left(j_1, x_1\right), .., \left(j_T, x_T\right)\right)$ is

$$\int \prod_{t=1}^{T} P\left(j_t, x_t | p_t, i_t\left(d_{t-1}, ., d_1, \nu_{t-1}, ., \nu_1, i_1\right), \nu_t, D_i\right)$$
$$dF\left(\nu_1, ., \nu_T\right) dF\left(i_1\right)$$

$$P\left(j_t, x_t | p_t, i_t, \nu_t, D_i\right) = \frac{\exp\left(a_{jxt}\beta_i \text{-} \alpha_i p_{jxt} + \xi_{jx} + M\left(\omega_t, i_t, \nu_t, j, x\right)\right)}{\sum_{k,y} \exp\left(a_{kyt}\beta_i \text{-} \alpha_i p_{kyt} + \xi_{ky} + M\left(\omega_t, i_t, \nu_t, k, y\right)\right)}$$

where

$$\begin{aligned} & M\left(\omega_t, i_t, \nu_t, j, x\right) \\ = \; & \max_c \left[u_i\left(c_t, \nu_t\right) \text{-} C_i\left(i_t\right) \delta E\left[EV_i\left(i_{t+1}, \omega_{t+1}\right) | i_t, \omega_t, c, j, x\right]\right] \end{aligned}$$

Model of Consumer Stockpiling

- Then, can split likelihood using brand indifference w.r.t. consumption and inventory:

$$M\left(\omega_t, i_t, \nu_t, j, x\right) = M\left(\omega_t, i_t, \nu_t, x\right)$$

then

$$\begin{aligned}
& P\left(j, x \middle| p_t, i_t, \nu_t, D_i\right) \\
= \quad & \frac{\exp\left(a_{jxt}\beta_i\text{-}\alpha_i p_{jxt} + \xi_{jx} + M\left(\omega_t, i_t, \nu_t, j, x\right)\right)}{\sum\limits_{k,y} \exp\left(a_{kyt}\beta_i\text{-}\alpha_i p_{kyt} + \xi_{ky} + M\left(\omega_t, i_t, \nu_t, k, y\right)\right)} \\
= \quad & \frac{\exp\left(a_{jxt}\beta_i - \alpha_i p_{jxt} + \xi_{jx}\right)}{\sum_k \exp\left(a_{kxt}\beta_i - \alpha_i p_{kxt} + \xi_{kx}\right)} \frac{\exp\left(\omega_{xt} + M\left(\omega_t, i_t, \nu_t, x\right)\right)}{\sum_y \exp\left(\omega_{yt} + M\left(\omega_t, i_t, \nu_t, y\right)\right)} \\
= \quad & P\left(j \middle| p_t, x_t, D_i\right) P\left(x_t \middle| \omega_t, i_t, D_i\right)
\end{aligned}$$

Model of Consumer Stockpiling

- Useful only if can assume that heterogeneity only as function of observable demographics

$$F\left(\alpha_i, \beta_i | x_t, p_t, D_i\right) = F\left(\alpha_i, \beta_i | p_t, D_i\right)$$

then

$$\begin{aligned} P\left(j | p_t, x_t, D_i\right) &= \int P\left(j | p_t, x_t, \alpha_i, \beta_i\right) dF\left(\alpha_i, \beta_i | x_t, p_t, D_i\right) \\ &= \int P\left(j | p_t, x_t, \alpha_i, \beta_i\right) dF\left(\alpha_i, \beta_i | p_t, D_i\right) \end{aligned}$$

otherwise need solve the dynamic programming to obtain

$$F\left(\alpha_i, \beta_i | x_t, p_t, D_i\right)$$

Model of Consumer Stockpiling

Likelihood in three steps (significantly reduces computational cost)

1. Estimate α_i, β_i by maximizing $P(j|p_t, x_t)$: (static) conditional logit using only options of size x_t. Can include many controls (concerns price endogeneity ..)
2. Use estimated parameters to get ω_{xit} and estimate $F(\omega_{it+1}|\omega_{it})$
3. Estimate dynamic parameters (utility from consumption, storage cost and distribution of ν_t) using $P(x_t|\omega_t, i_t)$ which require solving modified dynamic program

Model of Consumer Stockpiling

TABLE IV

FIRST STEP: BRAND CHOICE CONDITIONAL ON SIZE[a]

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Price	−0.51	−1.06	−0.49	−0.26	−0.27	−0.38	−0.38	−0.57	−1.41	−0.75
	(0.022)	(0.038)	(0.043)	(0.050)	(0.052)	(0.055)	(0.056)	(0.085)	(0.092)	(0.098
*Suburban dummy				−0.33	−0.30	−0.34	−0.33	−0.25	−0.45	−0.19
				(0.055)	(0.061)	(0.055)	(0.056)	(0.113)	(0.127)	(0.127
*Nonwhite dummy				−0.34	−0.39	−0.38	−0.33	−0.34	−0.33	−0.26
				(0.075)	(0.083)	(0.076)	(0.076)	(0.152)	(0.166)	(0.168
Large family				−0.23	−0.13	−0.21	−0.22	−0.46	−0.38	−0.43
				(0.080)	(0.107)	(0.080)	(0.082)	(0.181)	(0.192)	(0.195
Feature			1.06	1.05	1.08	0.92	0.93	1.08		1.05
			(0.095)	(0.096)	(0.097)	(0.099)	(0.100)	(0.123)		(0.126
Display			1.19	1.17	1.20	1.14	1.15	1.55		1.52
			(0.069)	(0.070)	(0.071)	(0.071)	(0.072)	(0.093)		(0.093
Brand dummy variable		✓	✓	✓	✓					
*Demographics					✓					
*Size						✓				
Brand–size dummy variable							✓			
Brand-HH dummy variable								✓		
*Size									✓	✓

[a]Estimates of a conditional logit model. An observation is a purchase instance by a household. Options include only products of the same size as the product actually purchased
Asymptotic standard errors are shown in parentheses.

Model of Consumer Stockpiling

TABLE V

SECOND STEP: ESTIMATES OF THE PRICE PROCESS[a]

	Same Process for All Types				Different Process for Each Type			
	ω_{2t}	ω_{4t}	ω_{2t}	ω_{4t}	ω_{2t}	ω_{4t}	ω_{2t}	ω_{4t}
$\omega_{1,t-1}$	0.003	−0.014	0.005	0.014	−0.023	−0.005	−0.019	0.007
	(0.012)	(0.011)	(0.014)	(0.014)	(0.017)	(0.014)	(0.019)	(0.015)
$\omega_{2,t-1}$	0.413	0.033	0.295	0.025	0.575	−0.003	0.520	0.011
	(0.007)	(0.010)	(0.008)	(0.007)	(0.013)	(0.010)	(0.016)	(0.013)
$\omega_{3,t-1}$	0.003	−0.034	0.041	−0.006	0.027	−0.072	0.051	−0.018
	(0.007)	(0.007)	(0.009)	(0.009)	(0.020)	(0.016)	(0.025)	(0.020)
$\omega_{4,t-1}$	0.029	0.249	0.026	0.236	−0.018	0.336	−0.018	0.274
	(0.008)	(0.008)	(0.008)	(0.017)	(0.020)	(0.016)	(0.021)	(0.017)
$\sum_{\tau=2}^{5} \omega_{1,t-\tau}$			−0.003	−0.012			−0.008	−0.003
			(0.005)	(0.004)			(0.006)	(0.005)
$\sum_{\tau=2}^{5} \omega_{2,t-\tau}$			0.089	0.006			0.073	−0.004
			(0.003)	(0.002)			(0.005)	(0.004)
$\sum_{\tau=2}^{5} \omega_{3,t-\tau}$			−0.008	−0.009			−0.004	−0.016
			(0.003)	(0.003)			(0.008)	(0.006)
$\sum_{\tau=2}^{5} \omega_{4,t-\tau}$			−0.013	0.018			−0.008	0.056
			(0.003)	(0.003)			(0.007)	(0.005)

Model of Consumer Stockpiling

TABLE VI

THIRD STEP: ESTIMATES OF DYNAMIC PARAMETERS[a]

Household Type:	1	2	3	4	5	6
	Urban Market			Suburban Market		
Household Size:	1–2	3–4	5+	1–2	3–4	5+
Cost of inventory						
Linear	9.24	6.49	21.96	4.24	4.13	11.75
	(0.01)	(0.02)	(0.09)	(0.01)	(0.17)	(5.3)
Quadratic	−3.82	1.80	−35.86	−8.20	−6.14	−0.73
	(29.8)	(1.77)	(0.19)	(0.03)	(1.69)	(1.53)
Utility from consumption	1.31	0.75	0.51	0.08	0.92	3.80
	(0.02)	(0.09)	(0.21)	(0.03)	(0.18)	(0.38)
Log likelihood	365.6	926.8	1,530.1	1,037.1	543.6	1,086.1

[a] Asymptotic standard errors are shown in parentheses. Also included are size fixed effects, which are allowed to vary by household type.

Ratio of Short Run to Long Run Elasticities

TABLE VIII

AVERAGE RATIOS OF ELASTICITIES COMPUTED FROM A STATIC MODEL TO LONG-RUN ELASTICITIES COMPUTED FROM THE DYNAMIC MODEL[a]

		64 oz.						128 oz.					
Brand	Size (oz.)	All[b]	Wisk	Surf	Cheer	Tide	Private Label	All[b]	Wisk	Surf	Cheer	Tide	Private Label
All[b]	64	1.03	0.13	0.14	0.12	0.13	0.15	0.14	0.17	0.17	0.18	0.21	0.34
	128	0.17	0.24	0.26	0.20	0.28	0.35	1.23	0.09	0.11	0.09	0.15	0.22
Wisk	64	0.14	1.20	0.13	0.17	0.12	0.13	0.16	0.22	0.14	0.22	0.25	0.20
	128	0.25	0.27	0.23	0.31	0.26	0.28	0.08	1.42	0.08	0.13	0.18	0.11
Surf	64	0.14	0.13	0.93	0.16	0.13	0.14	0.18	0.18	0.12	0.18	0.22	0.28
	128	0.25	0.22	0.18	0.27	0.25	0.18	0.12	0.11	1.20	0.08	0.15	0.14
Cheer	64	0.12	0.17	0.16	0.84	0.09	0.13	0.14	0.24	0.16	0.14	0.22	0.24
	128	0.25	0.26	0.26	0.12	0.23	0.22	0.09	0.12	0.06	0.89	0.15	0.07
Tide	64	0.16	0.17	0.13	0.13	1.26	0.15	0.22	0.28	0.16	0.26	0.22	0.37
	128	0.25	0.31	0.22	0.24	0.22	0.31	0.11	0.16	0.08	0.13	1.44	0.31
Solo	64	0.15	0.12	0.15	0.14	0.12	0.14	0.17	0.15	0.15	0.30	0.30	0.28
	128	0.23	0.20	0.24	0.21	0.21	0.25	0.07	0.07	0.06	0.16	0.17	0.21
Era	64	0.21	0.12	0.13	0.13	0.10	0.19	0.43	0.17	0.15	0.22	0.19	0.35
	128	0.31	0.22	0.24	0.25	0.17	0.38	0.19	0.08	0.09	0.11	0.10	0.22
Private	64	0.19	0.15	0.14	0.17	0.17	1.02	0.32	0.22	0.15	0.26	0.31	0.25
label	128	0.29	0.28	0.34	0.30	0.39	0.29	0.16	0.12	0.13	0.10	0.27	1.29
No purchase		2.12	1.13	1.15	1.40	1.27	2.39	1.80	7.60	2.26	14.11	2.38	10.86

[a]Cell entries i and j, where i indexes row and j indexes column, give the ratio of the (short-run) elasticities computed from a static model divided by the long-run elasticities computed from the dynamic model. The elasticities for both models are the percent change in market share of brand i with a 1 percent change in the price of j. The static model is identical to the model estimated in the first step, except that brands of all sizes are included as well as a no-purchase decision, not just products of the same size as the chosen option. The results from the dynamic model are based on the results presented in Tables IV–VI.

[b]Note that "All" is the name of a detergent produced by Unilever.

Elasticities

- Static own-price elasticities overestimate dynamic ones by roughly 30%
- Part of this difference is driven by the bias in estimates of static model: price coefficient estimated in static model is roughly 15 percent higher than estimated in the first stage of the dynamic model
- Static cross-price elasticities, with the exception of the no-purchase option, are smaller than the long-run elasticities
- The effect on the no-purchase option is expected because the static model fails to account for the effect of inventory. A short-run price increase is most likely to chase away consumers who can wait for a better price, namely those with high inventories. Therefore, the static model will overestimate the substitution to the no-purchase option

Elasticities

- Several effects impact cross-price elasticities to the other brands:
  - coefficients estimated in static model tend to be upward biased
  - additional effect due to difference between long-run and short-run effects: consider a reduction in price of a brand.
    - Static elasticities computed from temporary price reductions. Switchers are households willing to switch, from another brand and have a low enough inventory at the time of the price change.
    - Long-run elasticities capture those households willing to substitute at all relevant levels of inventory, because they represent reactions to a permanent change in the price of that brand
    - Which one dominates depends on relative size of both effects and whether observed price variation was temporary or more permanent.
- For own-price effect and cross-price effects towards no-purchase option: econometric bias and the difference between short- and long-run effects operate in the same direction. Both overstate price responses.

A simple model of demand with inventories

- A simple model to provide structural estimates of long run price elasticities
- No need to solve dynamic program
- Flexible with respect to unobserved heterogeneity
- Identification of beginning of the period inventory
- No restriction on price process
- Flexible on consumer expectation

A simple model of demand with inventories

- Identification: uses periods when prices are high and there is no incentive to stock
- Results: short run elasticities overestimates long run elasticities by 40%-60%

Consumer Model

- Consumption c_{it}, purchase q_{it}, inventories y_{it}
- Consumer:

$$\max_{c_{it},q_{it},y_{it}} E_t \left\{ \sum_{t=\tau}^{\infty} \delta^t \left[u\left(c_{it}\right) - \alpha_i p_t q_{it} - \Phi\left(y_{it}\right) \right] \right\}$$

$$\begin{aligned} s.t. \quad y_{it} = q_{it} - c_{it} + y_{it-1} \quad & \left(\lambda_{it}\right) \\ q_{it} \geqslant 0 \quad & \left(\Psi_{it}\right) \\ y_{it} \geqslant 0 \quad & \left(\mu_{it}\right) \end{aligned}$$

with

$$u'\left(0\right) = \infty \tag{A1}$$

$$\Phi\left(y_{it}\right) = \frac{\phi_i}{2} y_{it}^2 \tag{A2}$$

Consumer Model

- First Order Conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_{it}} &= 0 \Rightarrow u'\left(c_{it}\right) = \lambda_{it} \\
\frac{\partial \mathcal{L}}{\partial q_{it}} &= 0 \Rightarrow \alpha_i p_t = \lambda_{it} + \Psi_{it} \\
\frac{\partial \mathcal{L}}{\partial y_{it}} &= 0 \Rightarrow y_{it} = \frac{\delta E_t\left(\lambda_{it+1}\right)}{\phi_i} - \frac{\lambda_{it}}{\phi_i} + \frac{\mu_{it}}{\phi_i}
\end{aligned}$$

Consumer Model

- Main proposition: If $q_{it} > 0$ then

$$p_t > \delta E_t\left[p_{t+1}\right] \Leftrightarrow y_{it} = 0$$

  - Build beginning of the period inventory
  - Separate periods with and without stocks
    $\longrightarrow$ identification of parameters in purchase decision when no stock
- Purchase decision when there's no stockpiling is independent of forward looking variables
- Need identify model parameters to calculate price elasticities
- Need utility function, price expectation and an assumption on consumption at periods without purchases

Consumer Model

- Purchase decision: using Main Prop. + FOC: if $q_{it} > 0$ & $p_t > \delta E_t\left(p_{t+1}\right)$:
$$q_{it} = h\left(\alpha_i p_t\right) - y_{it-1}$$
where $h = u'^{-1}$
- Add measurement error
$$q_{it}^* = q_{it} + v_{it} \tag{A3}$$
with v_{it} $\sim$symmetrically $\left[-q_{it}, q_{it}\right]$ so $q_{it} > 0 \Leftrightarrow q_{it}^* > 0$

Consumer Model

- Inventories

$$y_{it-1} = y_{i0} + \sum_{n=1}^{t-1} q_{in}^{*} - \sum_{n=1}^{t-1} c_{in} - \sum_{n=1}^{t-1} v_{in}$$

- $t_0(i) \equiv 1^{st}$ period right after purchase at high price
- Then, main proposition $\Rightarrow y_{t_0(i)} = 0$
- Hence:

$$y_{it-1} = \sum_{n=t_1'(i)}^{t-1} q_{in}^{*} - \sum_{n=t_1'(i)}^{t-1} c_{in} - \sum_{n=1}^{t-1} v_{in}$$

where $\sum_{n=t_1'(i)}^{t-1} q_{in}^{*}$ is observable

Consumer Model

- Assumption on consumption without purchases

$$q_{it} = 0 \Rightarrow c_{it} = h\left(\alpha_i p_r\right)$$

where p_r is expected regular price (needs sensitivity analysis)

- Utility Specification

$$u\left(c_{it}\right) = -\frac{1}{\rho}\exp\left(-\rho c_{it}\right) \quad \text{((A4a))}$$

$$u\left(c_{it}\right) = \chi\left(c_{it} - \gamma c_{it}^2\right) \quad \text{((A4b))}$$

Estimable Equation

- if $q_{it} > 0$ & $p_t > \delta E_t\left(p_{t+1}\right)$ and (A4a):

$$\begin{aligned} Q_{it}^{*} &= -\frac{1}{\rho}\ln\left(\alpha_i\right) T_i^{t-1} - \frac{1}{\rho}\left[\ln p_t + \sum_{n\in T_{i1}^{t-1}} \ln p_n + T_{i0}^{t-1}\ln p_r\right] \\ &\quad + \sum_{n=t_1(i)}^{t} v_{in} \end{aligned}$$

where $Q_{it}^{*} = \sum_{n=t_1'(i)}^{t} q_{in}^{*}$ and

T_i^{t-1} = nb of periods between $t_1(i)$ and $t-1$

T_{i0}^{t-1} = nb of periods between $t_1(i)$ and $t-1$ without purchases

- Identification with variation over time of prices:

$$E_t\left(\sum_{n=t_1(i)}^{t} v_{in} | q_{it} > 0, p_t > \delta E_t\left(p_{t+1}\right)\right) = 0$$

Data

- Representative survey french households (1999, 2000, 2001)
- Home scan registering of every food product purchased
- Product and household characteristics
- Products: milk, butter, pasta, yogurt, coffee, tuna
    - regular consumption, storage costs, storability

Data

Descriptive stats for all t and i such that $q_{it} > 0$ and $p_t > p_r$

	All	$p_{it} > Ep^{(0)}_{it+1}$		$p_{it} > Ep^{(1)}_{it+1}$		$p_{it} > Ep^{(2)}_{it+1}$	
Product	Freq	Freq	%	Freq	%	Freq	%
Butter	146,079	61,066	41.80	75,403	51.62	71,198	48.74
Coffee	97,884	56,539	57.76	64,671	66.07	58,014	59.27
Pasta	255,521	102,623	40.16	113,651	44.48	117,692	46.06
Milk	137,261	60,576	44.13	57,341	41.78	57,602	41.97
Yogurt	109,097	46,338	42.47	48,157	44.14	53,461	49.00

Price Elasticities

TABLE 3 Estimated Long-run Price Elasticities at the Average Price and Ratio of Short- to Long-run Price Elasticities Considering Different Price Expectation Hypotheses

		Long-run Price Elasticities			Ratio Short-run to Long-run Price Elasticities		
Products	Estimation Method	ϵ^{lr0}	ϵ^{lr1}	ϵ^{lr2}	$\epsilon^{sr0}/\epsilon^{lr0}$	$\epsilon^{sr1}/\epsilon^{lr1}$	$\epsilon^{lr2}/\epsilon^{lr2}$
Butter	fixed effect	−1.488	−1.376	−1.354	2.083	2.193	2.197
	random coefficient	−1.312	−1.183	−1.383	2.371	2.463	2.119
Coffee	fixed effect	−1.457	−0.998	−1.012	3.181	4.229	4.198
	random coefficient	−1.333	−1.032	−1.078	2.742	3.439	3.280
Milk	fixed effect	−2.243	−2.035	−2.201	1.955	2.749	2.644
	random coefficient	−2.404	−1.900	−2.085	1.952	2.716	2.575
Pasta	fixed effect	−1.143	−1.056	−1.133	3.406	3.085	2.887
	random coefficient	−1.038	−0.983	−1.088	3.362	3.073	2.862
Yogurt	fixed effect	−2.239	−1.815	−1.922	1.920	1.951	1.942
	random coefficient	−1.879	−1.732	−1.828	1.899	1.972	1.837

Notes: (i) Averages across households computed using elasticities in between the 1st and the 99th percentiles evaluated at the average price; (ii) ϵ^{lrk} is the long-run price elasticity under price expectation hypothesis (Ek), k = 0,1,2; (iii) *sr* denotes short run.

Usage-Based Pricing and Demand for Residential Broadband

- Increased usage of internet has led to congestion.
- Usage based pricing (three-part tariffs) is a proposed way to fix this problem.
- Welfare implications of this pricing is only theoretical.
- Demand estimation of broadband services to evaluate welfare implications of alternatives to address network congestion.
- Detailed (high frequency) data of a North American ISP.

Usage-Based Pricing and Demand for Residential Broadband

- Three part tariffs: monthly fee for a monthly data allowance. Price per Gigabyte if allowance exceeded.
- Marginal price is 0 until allowance is used.
- Forward looking users realize shadow price is not 0.
- Shadow price depends on days left on the billing cycle and fraction of allowance already used.
- Different plans offered: from almost linear prices to very high allowances.
- These features imply the need for a dynamic model of consumers decisions throughout the billing cycle.

Data

- 54.801 subscribers from a North American ISP. Alternative is a slow DSL connection.
- Features of plan offered: monthly fees, maximum download and upload speeds, usage allowance and overage price per GB.
- ISP offers only usage based plans to new subscribers. Some old subscribers have unlimited plans.
- Monthly usage from may 2011 to may 2012.
- 15-minutes intervals from may 10th to June 30th, 2012.
- Consumers can carefully track their usage data

Data

- Rapid growth in usage. Median subscriber's usage more than doubles.
- Cyclical pattern during the day. Peak usage between 10 and 11pm.
- On average unlimited plan users use more internet and pay less per GB.
- Subscribers choose optimal plans:
  - 10% of consumers exceeded their allowance in June 2012.
  - Dominated plans: could have paid less for service no slower.
  - Considering 13 months, only 0.13% of consumers chose dominated plan.

Consumers

- Identification depends on consumers responding to shadow prices.
- Marginal cost for consumers exceeding allowance: discounted overage price times the probability of exceeding the allowance.
- Heavy users: behave as shadow price equals to overage price.
- Consumers not exceeding allowance: behavior should not vary during cycle.
- Consumers in between: usage should vary depending on the day and the cumulative usage so far.
- Estimate regression of usage on allowance used so far and the current day of the billing cycle:
  - Current usage is responsive to past usage (within billing cycle).
  - Discrete change in the shadow price when usage is refreshed (across billing cycle).

Model

- Plans are indexed by k, have speed s_k, allowance $\overline{C}_k$, fixed fee F_k and per-GB overage price p_k.
- Consumers have quasi-linear utilities, where subscriber of type h on plan k utility is:

$$u_h(c_t, y_t; v_t; k) = v_t \left(\frac{c_t^{1-\beta_h}}{1-\beta_h} \right) - c_t \left(\kappa_{1h} + \frac{\kappa_{2h}}{\ln(s_k)} \right) + y_t$$

- The first term is the gross utility from content. v_t is a time-varying unobservable not known by subscriber until period t. For each type h, v_t is i.i.d $LN\left(\mu_h, \sigma_h\right)$.
- The second term is the non-price cost of consuming online content.
- κ_{1h} is opportunity cost of content (wait time), and the ratio of κ_{2h} is preference for speed.
- The specification implies a satiation point.

Model

- A vector of parameters $(\beta_h, \kappa_{1h}, \kappa_{2h}, \mu_h, \sigma_h)$ describes a subscriber of type h
- Conditional on choosing plan k, subscriber's problem is:

$$\max_{c_1,..,c_T} \sum_{t=1}^{T} E\left[u_h\left(c_t, y_t, v_t; k\right)\right]$$

$s.t.$

$$F_k + p_k \max\left(C_T - \overline{C}_k, 0\right) + Y_T \leq I$$

$$C_T = \sum_{t=1}^{T} c_t \quad , \quad Y_T = \sum_{t=1}^{T} y_t$$

- Assume that I is large enough so wealth don't constrain consumption of content.
- Stochastic finite horizon dynamic program.

Model

- Value function and optimal policy function:

$$V_{hkt}\left(C_{t-1}, v_t\right) = \left\{ v_t \left(\frac{c_t^{1-\beta_h}}{1-\beta_h} \right) - c_t \left(\kappa_{1h} + \frac{\kappa_{2h}}{\ln(s_k)} \right) + y_t \right.$$
$$\left. -p_k O_{tk}(c_t) + E\left[V_{hk(t+1)}\left(C_{t-1} + c^*_{hkt}\right) \right] \right\}$$

where overage is $O_{tk}(c_t) = \max(c_t - \overline{C}_{kt})$ and $\overline{C}_{kt} = \max(\overline{C}_k - C_{t-1}, 0)$ and

$$c^*_{hkt}\left(C_{t-1}, v_t\right) = \arg\max V_{hkt}\left(C_{t-1}, v_t\right)$$

- Define the shadow price:

$$\widetilde{p}_k(c_t, C_{t-1}) = \begin{cases} p_k & \text{if } O_{tk}(c_t) > 0 \\ \frac{dE\left[V_{hk(t+1)}(C_{t-1}+c_t)\right]}{dc_t} & \text{if } O_{tk}(c_t) = 0 \end{cases}$$

Model

- Solve for the consumer's optimal choice in period t as a function of the shadow price and the parameters

$$c^*_{hkt} = \left(\frac{v_t}{\kappa_{1h} + \frac{\kappa_{2h}}{\ln(s_k)} + \widetilde{p}_k(c_t, C_{t-1})} \right)^{\frac{1}{\beta_h}}$$

- Then, the expected value function:

$$E\left[V_{hkt}\left(C_{t-1}\right)\right] = \int_0^{\overline{v}_h} V_{hkt}\left(C_{t-1}, v_t\right) dG_h\left(v_t\right)$$

- And the mean subscriber's usage at each state:

$$E\left[c^*_{hkt}\left(C_{t-1}\right)\right] = \int_0^{\overline{v}_h} c^*_{hkt}\left(C_{t-1}, v_t\right) dG_h\left(v_t\right)$$

- Optimal plan choice:

$$k^*_h = \arg\max_k \left\{E\left[V_{hk1}\left(0\right) - F_k\right]\right\}$$

Estimation

- First step: solve the model.
  - Solve the model for 18,144 types and store the optimal policy.
  - Solution can be characterized by the expected value functions and policy functions (because subscriber does not now v_t prior to period t).
  - Compute transition probabilities between possible states.
- Second step: empirical and predicted moments.
  - Choose weights minimizing the distance between moments in the data and average predicted moments (what mixture of types results in predicted behavior best matching the data).
  - Two sets of moments: mean usage at each state and mass of subscribers at a particular state.

Results

- Most common type accounts for 43% of total mass, top 10 for 83% and top 30 for 96%.
- Correlation between empirical moments and fitted moments is above 0.99. This means that model replicates average usage and density of subscribers at each state successfully.
- Allowing many types is important.
- Majority of high volume subscribers have highly elastic demand.
- Overall value placed in improving speed is substantial.
- Under linear tariff: average willingness to pay is roughly 280 dollars per month.
- Waiting costs are important: traffic is likely to increase in the future.

Counterfactuals

- Usage based pricing compared to unlimited allowances. Same plans but with no overage prices.
  - Subscriber welfare decreases with UBP.
  - Effect on total welfare is less clear.
  - 20% of consumers choose same plan, 74% choose different plans, 1% purchase only when service is unlimited.
  - Some consumers increase usage under UBP.
  - UBP modestly increases total surplus generated from usage, while transferring some surplus from subscribers to the ISP.

Counterfactuals

Table 7: *Usage-Based Pricing Counterfactual, Usage and Welfare*

	Same Plan		Switch Plan		Only Unlimited	
	Unlimited (1)	**UBP (2)**	**Unlimited (3)**	**UBP (4)**	**Unlimited (5)**	**UBP (6)**
Percent of Types (%)	20.23		73.53		1.31	0
Usage and Surplus						
Speed (Mb/s)	16.23	16.23	10.43	13.05	10	–
Usage (GBs)	77.76	62.57	36.47	34.74	23.74	–
Consumer Surplus ($)	189.25	177.40	153.96	138.13	4.84	–
Revenue ($)	81.07	81.14	43.25	64.48	39.99	–
Total Surplus ($)	270.32	258.54	197.21	202.61	44.83	–
Δ in Total Surplus ($)	-11.78		5.40		44.83	–

	Same Plan		Switch Plan		Only Unlimited	
	↑ Usage (1)	**↓ Usage (2)**	**↑ Usage (3)**	**↓ Usage (4)**	**Unlimited (5)**	**UBP (6)**
Percent of Types (%)	0	20.23	65.63	7.90	1.31	0
Mean Type						
mean of shocks ($\bar{\mu}_h$)	–	0.91	1.28	0.49	0.24	–
s.d. of shocks ($\bar{\sigma}_h$)	–	0.75	0.82	0.74	0.78	–
opp cost of content ($\bar{\kappa}_{1h}$)	–	4.88	6.67	2.54	2.35	–
pref for speed ($\bar{\kappa}_{2h}$)	–	7.04	5.12	3.56	0.70	–
curvature ($\bar{\beta}_h$)	–	0.44	0.41	0.38	0.41	–

Counterfactuals

- Economic viability of Next Generation Networks.
- Single plan with unlimited usage and one GB/s connection.
- Assume fixed fee of 100 dollars.
- Total surplus is 87 dollars higher per subscriber.
- ISP only realized 22 dollars of additional revenue.
- This implies gap between social and private incentives to invest.
- Socially the investment is recovered in 37 months, but for the ISP in 150 months.

Counterfactuals

Table 8: *Next-Generation Network Counterfactual, Usage and Welfare*

	Both Unlimited (1)	Both UBP (2)	Single Unlimited (3)	Single UBP (4)
Percent of Types (%)	84.40		0	9.36
Usage and Surplus				
Speed (Mb/s)	1,024.00	13.97	–	11.74
Usage (GBs)	102.24	41.80	–	31.30
Consumer Surplus ($)	235.97	160.37	–	22.64
Revenue ($)	100.00	70.03	–	50.46
Total Surplus ($)	335.97	230.40	–	73.10
Δ in Total Surplus ($)	-105.57		–	73.10

	Both ↑ Usage (1)	Both ↓ Usage (2)	Single Unlimited (3)	Single UBP (4)
Percent of Types (%)	0	84.40	0	9.36
Mean Type				
mean of shocks ($\bar{\mu}_h$)	–	1.26	–	-0.01
s.d. of shocks ($\bar{\sigma}_h$)	–	0.82	–	0.55
opp cost of content ($\bar{\kappa}_{1h}$)	–	6.17	–	3.82
pref for speed ($\bar{\kappa}_{2h}$)	–	5.69	–	2.86
curvature ($\bar{\beta}_h$)	–	0.40	–	0.56

Conclusion

- Analyze empirically two alternatives to deal with congestion: usage based pricing and high speed next generation networks.
- USB pricing increases total welfare. Higher speed decreases waiting costs. Usage only falls slightly.
- What would happen with plans is important to the welfare implications.
- For the next generation networks, there is a large gap between private and social incentives to invest in those networks.