Empirical Methods for Policy Evaluation Homework 2

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Job search with formal and informal jobs

We have one value function for an unemployed individual:

$$\rho U = b + \lambda_f \int max[E_f(w), U]dG_f(w) + \lambda_i \int max[E_i(w), U]dG_i(w) - (\lambda_f + \lambda_i)U \tag{1}$$

and one equation:

$$\rho E_j(w) = w_j + \eta_j[U - E_j] \tag{2}$$

Question 1

From (2), we can get:

$$E_j(w) = \frac{w_j + \eta_j U}{\rho + \eta_j} \tag{3}$$

Next, let us define reservation wage w^* as $w^* \equiv \rho U$ and the set of wage higher than reservation wage as $A_j \equiv \{w_j : w_j > w^*\}$. By plugging w^*, A_j and (3) into (1), we can finally get the analytical expression for the reservation wage:

$$w^* = b + \frac{\lambda_f}{\rho + \eta_f} \int_{A_f} (w_f - w^*)dG_f(w) + \frac{\lambda_i}{\rho + \eta_i} \int_{A_i} (w_i - w^*)dG_i(w) \tag{4}$$

The reservation is the same for both formal and informal jobs, because individuals only care about the offered wages.

Question 2

First, let us define $\tilde{G} \equiv 1 - G_j(w^*)$. Given $G_i(w)$ and $G_j(w)$ are exogenous stationary distributions, we can get the rate of being employed (leaving unemployment):

$$h = \lambda_f \tilde{G}_f(w^*) + \lambda_i \tilde{G}_i(w^*) \tag{5}$$

Next, to calculate the distribution of formal employment, informal employment and unemployment ($p(e_f), p(e_i)$ and $p(u)$) in the steady state, we can think of the process of job change as a three-state Markov Chain in continuous time. For the ease of calculation, let us assume both the inflows of formal and informal jobs are all from people who were unemployed and the outflows of formal and informal jobs all go to unemployment. Based on this assumption, we will have a system of three equations in the steady state:

$$\begin{aligned} p(e_f)\eta_f &= p(u)\lambda_f \tilde{G}_f(w^*) \\ p(e_i)\eta_i &= p(u)\lambda_i \tilde{G}_i(w^*) \\ p(e_f) + p(e_i) + p(u) &= 1 \end{aligned} \tag{6}$$

After solving system (6), we have the following stationary distribution of $p(e_f), p(e_i)$ and $p(u)$:

$$\begin{aligned} p(e_f) &= \frac{\eta_i \lambda_f \tilde{G}_f(w^*)}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} \\ p(e_i) &= \frac{\eta_f \lambda_i \tilde{G}_i(w^*)}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} \\ p(u) &= \frac{\eta_i \eta_f}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} \end{aligned} \tag{7}$$

Question 3

At the beginning, we derive likelihood for individual i with different (un)employmenr statuses:

Unemployment,

$$L_1(\tilde{t}_u(i), u) = f_u(p_u) * p(u) = \frac{\eta_i \eta_f}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} * h * exp(-ht_u(i))$$

Formal employment,

$$L_2(w_f(i), e_f) = p(e_f) * \frac{g_f(w_f(i))}{\tilde{G}_f(w^*)} = \frac{\eta_i \lambda_f \tilde{G}_f(w^*)}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} * \frac{g_f(w_f(i))}{\tilde{G}_f(w^*)}$$

Informal employment,

$$L_3(w_i(i), e_i) = p(e_i) * \frac{g_i(w_i(i))}{\tilde{G}_i(w^*)} = \frac{\eta_f \lambda_i \tilde{G}_i(w^*)}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} * \frac{g_i(w_i(i))}{\tilde{G}_i(w^*)}$$

After that, we combine them and attain the log-likelihood function:

$$\begin{aligned}
\ln L &= \sum_u \ln(L_1(\tilde{t}_u(i), u)) + \sum_f \ln(L_2(w_f(i), e_f)) + \sum_i \ln(L_3(w_i(i), e_i)) \\
&= -N \ln(\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)) + N_{e_f} \ln(\eta_i \lambda_f \tilde{G}_f(w^*)) + N_{e_i} \ln(\lambda_i \tilde{G}_i(w^*)\eta_f) + N_u \ln(\eta_i \eta_f) \\
&+ N_u \ln h - N_{e_f} \tilde{G}_f(w^*) - N_{e_i} \tilde{G}_i(w^*) + \sum_f \ln g_f(w_f(i)) + \sum_i \ln g_i(wi(i)) - h \sum_u t_u(i)
\end{aligned} \tag{8}$$

Question 4

With four FOCs of the log-likelihood function, we could identify four parameters in this model. The four equations are:

With respect to η_i,

$$-N \frac{\lambda_f \tilde{G}_f(w^*) + \eta_f}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} + \frac{N_{e_f} + N_u}{\eta_i} = 0$$

With respect to η_f,

$$-N \frac{\lambda_i \tilde{G}_i(w^*) + \eta_i}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} + \frac{N_{e_i} + N_u}{\eta_i} = 0$$

With respect to λ_i,

$$-N \frac{\eta_f \tilde{G}_i(w^*)}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} - \tilde{G}_i(w^*) \sum_u t_u(i) + N_u \frac{\tilde{G}_i(w^*)}{h} + \frac{N_{e_i}}{\lambda_i} = 0$$

With respect to λ_j,

$$-N \frac{\eta_i \tilde{G}_f(w^*)}{\lambda_i \tilde{G}_i(w^*)\eta_f + \eta_i \eta_f + \eta_i \lambda_f \tilde{G}_f(w^*)} - \tilde{G}_f(w^*) \sum_u t_u(i) + N_u \frac{\tilde{G}_f(w^*)}{h} + \frac{N_{e_f}}{\lambda_f} = 0$$

First of all, $G_j(w)$ will be parametrically estimated given the data with an assumption of log-normal distribution. The estimation of w^* is done by $\hat{w}^* = min(w_1, ..., w_N)$ as we saw in the class, thus we have the estimation of $\tilde{G}_j(w^*)$. The other variables N, N_{e_j} and $t_u(i)$ are available from the data. This leaves the four unknown parameters of $\eta_f, \eta_i, \lambda_f$ and λ_i. Given we have four FOCs with four parameters all the parameters are uniquely determined. By referring to equation (4), we will fix ρ to identify b.

Question 5

Given our log-likelihood function in Q3 and the parametric assumption on $G_j(w)$, the parameters are estimated using mle2() function in pacakge 'bbmle' in R. We set $\rho = 0.05$ and imposed the restriction that are $\lambda_f = \lambda_i = \lambda$ and $\eta_f = \eta_i = \eta$.

The estimation of $G_j(w)$ is reported in Table 1 where the first two panels show the estimated mean and standard deviation. Overall, the formal sector has a higher mean wage while there is a larger dispersion in the informal sector: μ_f and μ_i are \$1.02 and \$0.81 respectively; and σ_f and σ_i are 0.41 and 0.48 respectively.

The last panel in Table 1 summarizes the result of the MLE on the search model. The estimate of λ is 0.18 whereas η is only 0.04. This is in line with our model that in the stationary condition, $p(u)h = (p(e_f)+p(e_i))\eta$ must hold. If we look at Table 2, the percentage of unemployed is only 9.7%, thus η must be smaller than λ in order to satisfy the equality.

Lastly, The estimated reservation wage is from an individual in the informal sector. The opportunity cost of being employed or the total benefit of being unemployed, b, is negative; $-\$9.19$.

Table 1: Estimation of the job sector distributions

Unit (\$)	Estimation	Std.Error
Formal Sector		
μ_f	1.02	0.0032
σ_f	0.41	0.0023
Informal Sector		
μ_i	0.81	0.0120
σ_i	0.48	0.0085
Search Model		
λ	0.18	0.0042
η	0.04	0.0013
b	-9.19	-
w^*	0.42	-

Table 2: Size of unemployed, formal and informal sectors

	Unemployed	Formal	Informal	Total
%	9.7	82.0	8.2	-
N	1875	15802	1580	19263