Enhancing Monotonic Modeling with Spatio-Temporal Adaptive Awareness in Diverse Marketing

A MARKETING MONOTONIC

A.1 Part One

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A.2 Part Two

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B LINEAR PROGRAMMING

C PRELIMINARIES

Upon predicting the incentive sensitivity curves of users utilizing our feedback model, we formulate the time-sensitive marketing problem as a constrained linear programming problem to be solved using the optimization approach. Our objective is to devise an incentive allocation strategy that, under the constraints of a total budget B and $\mathcal{K}$ distinct business objectives encapsulated by h_k, optimally increases the sum total of user response profits to the incentives. Formally,

$$\max_{x_{i,j}} \sum_{i=1}^{|C|} \sum_{j=1}^{|\mathcal{R}|} x_{i,j} c_{i,j} \tag{1}$$

$$\begin{aligned}
s.t.\ & x_{i,j} \in [0,1], for\ i = 1, ..., |C|;\ \ j = 1, ..., |\mathcal{R}| \\
& \sum_{j=1}^{|\mathcal{R}|} x_{i,j} \leq \mathcal{N}, for\ i = 1, ..., |C| \\
& \sum_{i=1}^{|C|} \sum_{j=1}^{|\mathcal{R}|} m_{i,j} x_{ij} \leq B \\
& \sum_{i=1}^{|C|} \sum_{j=1}^{|\mathcal{R}|} m_{i,j}^{k} x_{i,j} \leq h_k, for\ k = 1, ..., |\mathcal{K}|
\end{aligned} \tag{2}$$

where $x_{i,j}$ denotes the decision vector indicating whether incentive j is allocated to user i, and $c_{i,j}$ represents the resulting revenue from assigning incentive j to user i. Each user can be offered a maximum of $\mathcal{N}$ incentives. Furthermore, $m_{i,j}$ signifies the cost incurred for allocating incentive j. By introducing dual variables λ, μ and ν corresponding to the aforementioned constraints 2, we transform the primal problem into its dual counterpart. The convex optimization problem delineated in objective function 1 is addressed through the application of the Lagrangian multiplier method, formally depicted as follows:

$$\begin{aligned}
& -\sum_{i=1}^{|C|} \sum_{j=1}^{|\mathcal{R}|} x_{i,j} c_{i,j} + \sum_{i=1}^{|C|} (\mu_i (\sum_{j=1}^{|\mathcal{R}|} x_{i,j} - \mathcal{N})) \\
& + \lambda (\sum_{i,j} m_{i,j} x_{i,j} - B) + \sum_{k=1}^{|\mathcal{K}|} (\nu_k (\sum_{i,j} m_{i,j}^{k} x_{i,j} - h_k))
\end{aligned} \tag{3}$$

Given that both the objective function and the constraint functions are convex, the optimal solution is obtained by applying the KKT conditions, which yield the optimal Lagrange multipliers, denoted as λ^*. Finally, the solution can be approximated as follows:

$$x_{i,j} = \begin{cases} 1, & \text{if } j = argmax_j (c_{ij} - \sum_k \lambda_k m_{ij}^{(k)}) \\ 0, & \text{otherwise} \end{cases} \tag{4}$$