A MARKETING MONOTONIC

In each of Ele.me's marketing campaigns, we indicate example attributes that users will be sensitive to in Figure 1 below. Such monotonic characteristics are also different in different marketing tools. Specifically, in smart full discounts, the user's conversion rate (write-off rate) will show a monotonic downward trend with respect to the monotonic increase in the full discount threshold, while with respect to the full discount amount It shows the opposite nature; in the C configuration fine-tuning, the user's conversion rate (write-off rate) shows a monotonic decreasing trend with the increase in actual paid delivery fees; in the explosive red envelope, it is similar to the intelligent full reduction, with regard to the skyrocketing red envelope denomination. It shows a monotonically increasing trend, and the red envelope threshold shows a monotonically decreasing trend.

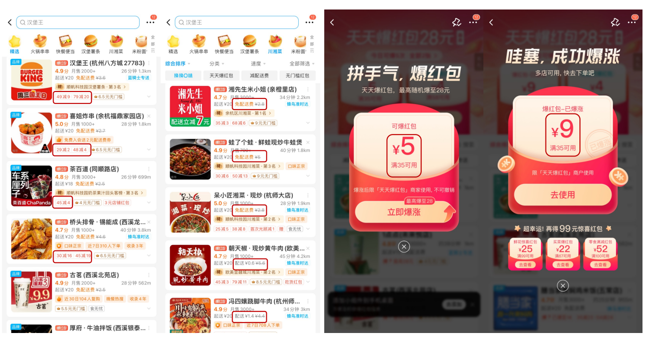

Figure 1: Monotonic Properties of Marketing Campaign. Enclosed within the red box in the figure are the monotonic characteristics of various marketing campaigns. Sequentially from left to right, the attributes represented are the discount thresholds and amounts in the tiered discount, followed by the actual and nominal delivery fees in the delivery fee waiver, and concluding with the coupon amounts and thresholds in the exploding red packets.

B LINEAR PROGRAMMING

Upon predicting the incentive sensitivity curves of users utilizing our feedback model, we formulate the time-sensitive marketing problem as a constrained linear programming problem to be solved using the optimization approach. Our objective is to devise an incentive allocation strategy that, under the constraints of a total budget B and $\mathcal{K}$ distinct business objectives encapsulated by h_k, optimally increases the sum total of user response profits to the incentives. Formally,

$$\max_{x_{i,j}} \sum_{i=1}^{|C|} \sum_{j=1}^{|\mathcal{R}|} x_{i,j} c_{i,j} \tag{1}$$

$$\begin{aligned} s.t.\ & x_{i,j} \in [0,1], for\ i = 1, ..., |C|;\ j = 1, ..., |\mathcal{R}| \\ & \sum_{j=1}^{|\mathcal{R}|} x_{i,j} \leq \mathcal{N}, for\ i = 1, ..., |C| \\ & \sum_{i=1}^{|C|} \sum_{j=1}^{|\mathcal{R}|} m_{i,j} x_{ij} \leq B \\ & \sum_{i=1}^{|C|} \sum_{j=1}^{|\mathcal{R}|} m_{i,j}^{k} x_{i,j} \leq h_k, for\ k = 1, ..., |\mathcal{K}| \end{aligned} \tag{2}$$

where $x_{i,j}$ denotes the decision vector indicating whether incentive j is allocated to user i, and $c_{i,j}$ represents the resulting revenue from assigning incentive j to user i. Each user can be offered a maximum of $\mathcal{N}$ incentives. Furthermore, $m_{i,j}$ signifies the cost incurred for allocating incentive j. By introducing dual variables λ, μ and ν corresponding to the aforementioned constraints 2, we transform the primal problem into its dual counterpart. The convex optimization problem delineated in objective function 1 is addressed through the application of the Lagrangian multiplier method, formally depicted as follows:

$$\begin{aligned} & -\sum_{i=1}^{|C|} \sum_{j=1}^{|\mathcal{R}|} x_{i,j} c_{i,j} + \sum_{i=1}^{|C|} (\mu_i (\sum_{j=1}^{|\mathcal{R}|} x_{i,j} - \mathcal{N})) \\ & + \lambda (\sum_{i,j} m_{i,j} x_{i,j} - B) + \sum_{k=1}^{|\mathcal{K}|} (\nu_k (\sum_{i,j} m_{i,j}^{k} x_{i,j} - h_k)) \end{aligned} \tag{3}$$

Given that both the objective function and the constraint functions are convex, the optimal solution is obtained by applying the KKT conditions, which yield the optimal Lagrange multipliers, denoted as λ^*. Finally, the solution can be approximated as follows:

$$x_{i,j} = \begin{cases} 1, & \text{if } j = argmax_j (c_{ij} - \sum_k \lambda_k m_{ij}^{(k)}) \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

C DETAILED PROOFS

In order to adapt to the CMNN framework, we mathematically proved the strict monotonicity of CLU. The derivation process is as follows:

$$y = \begin{cases} -\frac{\omega_0}{2} + \frac{\omega_0}{1+e^{-\omega_1 x}} & \text{if } x < 0 \\ x & \text{otherwise} \end{cases}, \omega_0 > 0, \omega_1 > 0 \tag{5}$$

,and then

$$\frac{\partial y}{\partial x} = \begin{cases} \frac{\omega_0 \omega_1 e^{-\omega_1 x}}{(1+e^{-\omega_1 x})^2} & \text{if } x < 0 \\ 1 & \text{otherwise} \end{cases} \tag{6}$$

$$
\begin{aligned}
\frac{\partial^2 y}{\partial x^2} &= \left\{ \frac{(-\omega_0\omega_1^2 e^{-\omega_1 x})(1+e^{-\omega_1 x})^2 + 2\omega_0(\omega_1 e^{-\omega_1 x})^2(1+e^{-\omega_1 x})}{(1+e^{-\omega_1 x})^4} \right. \\
&= \left\{ \frac{(1+e^{-\omega_1 x})(-\omega_0\omega_1^2 e^{-\omega_1 x} - \omega_0(\omega_1 e^{-\omega_1 x})^2 + 2\omega_0(\omega_1 e^{-\omega_1 x})^2)}{(1+e^{-\omega_1 x})^4} \right. \\
&= \begin{cases} \frac{(1+e^{-\omega_1 x})(-\omega_0\omega_1^2 e^{-\omega_1 x} + \omega_0(\omega_1 e^{-\omega_1 x})^2)}{(1+e^{-\omega_1 x})^4} & \text{if } x < 0 \\ 0 & \text{otherwise} \end{cases} \\
&= \left\{ \frac{(1+e^{-\omega_1 x})(e^{-\omega_1 x} - 1)(\omega_0\omega_1^2 e^{-\omega_1 x})}{(1+e^{-\omega_1 x})^4} \right. \qquad \geq 0
\end{aligned}
\tag{7}
$$

For a univariate function CLU, the condition that its second derivative is greater than or equal to zero entails that it is a strictly monotonic convex function.