Course Code: 22BM1101 / 2022 R-2022 Reg. No: 3 2 2 1 0 3 3 1 0 0 5

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING (AUTONOMOUS)

Madhurawada, Visakhapatnam

Affiliated to Andhra University, Visakhapatnam

B.Tech. I-Semester Regular Examinations, March 2023

Calculus and Linear Algebra

[Common to CH, CE, CS, EC, EE, IT, ME & ME (Robotics)]

Date: 11-03-2023 **Time: 3 Hours** **Max. Marks: 70**

1. Answer ONE Question from each UNIT
2. All parts of a Question must be answered in one place to get valued.
3. All questions carry equal marks.

UNIT-I

1. a) Discuss the nature of the convergence of the series $\sum\left(\frac{n}{n+1}\right)^n x^n, x>0$, using Cauchy's n^{th} root test. 7 Mark

   b) Verify the Cauchy's mean value theorem of the following functions 7 Mark

   $$f(x)=\sqrt{x}, g(x)=\frac{1}{\sqrt{x}} \text{ in } [a,b], 0<a<b$$

2. a) Examine the convergence of the infinite series 7 Mar

   $$\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\ldots+\left(\frac{n+1}{n+2}\right)^n x^n+\cdots,\ x>0$$

   b) Using mean value theorem show that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\left(\frac{4}{3}\right)<\frac{\pi}{4}+\frac{1}{6}$ 7 Mar

UNIT-II

3. a) If $u=x+y+z, uv=y+z, uvw=z$, show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$ 7 Mar

   b) Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 7 Mar

4. a) If $u=y^2e^{\frac{y}{x}}+x^2\tan\left(\frac{x}{y}\right)$ compute $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ 7 Mar

   b) Find the maximum and minimum values of the function 7 Mar

   $$f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$$

UNIT-III

5. a) Evaluate $\iint_R xydxdy$, where R is the region bounded by $x-$ axis, the line $x=2a$ and the curve $x^2=4ay$ 7 Ma

   b) Evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}dxdy$ by changing the order of integration. 7 Ma

6. a) Evaluate $\iint_R(x+y)^2dxdy$, Where R is the parallelogram in the $xy-plane$ with vertices (1,0), (3,1),(2,2),(0,1), using the transformation $u=x+y$ and $v=x-2y$ 7 Ma

   b) Evaluate $\int_1^e\int_1^{\log y}\int_1^{e^x}\log z\,dzdxdy$ 7 Ma

UNIT-IV

7. a) Find the values a and b for which the equations $x+y+z=3,\ x+2y+2z=6, x+ay+3z=b$ have (i) no Solution (ii) unique Solution and (iii) many solutions. 7 Marks

b) Find Eigen values and Eigen vectors of $A=\begin{bmatrix}-1 & -1 & 2\\ 0 & 5 & 3\\ 0 & 0 & 2\end{bmatrix}$ 7 Marks

8. a) Solve the system of equations $x+3y+8z=4; 4x+4y+3z=-2; x+3y+4z=1.$ 7 Marks

b) Find the rank of the matrix, $A=\begin{bmatrix}1 & -1 & 3\\ 2 & 3 & 4\\ 1 & 2 & 5\end{bmatrix}$. 7 Marks

UNIT-V

9. Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ and hence find the value of A^{-1} and A^4 14 Marks

10 Reduce the quadratic form $3x^2+2y^2+3z^2-2xy-2yz$ into canonical form. Hence find rank, index, signature and nature of the quadratic form. 14 Marks