Theoretical solution (let's say that we are considering only upto 10 offsprings per object even if above distribution can generate more than this number)

For a discrete random variable X with support 0,1,2,...

We have PGF as:

$G(s)=E(s^X)$

$G(s)=\sum_{k=0}^{\infty} P(X=k)s^k$

$G(s)=P(X=0)s^0+P(X=1)s^1+P(X=2)s^2+...$

- $\frac{G^{(n)}(0)}{n!} = P(X=n)$

$E(X)=G'(1)$

$Var(X)=G''(1)+G'(1)-[G'(1)]^2$

Expectation of n^{th} generation population: $E(X_n)=m^n$

Variance of n^{th} generation population:

$Var(X_n)=m^{(n-1)}*(m^n-1)*\sigma^2/(m-1)$

where,$m=E(X_1)$ and $\sigma^2=Var(X_1)$

Above equation is valid when $m \neq 1$

When $m=1$,$Var(X_n)=n\sigma^2$

- $G_n(s)=G_{n-1}[G(s)]$

1. **Binomial Distribution:**

X~Binom(n,p) which is just a sum of n i.i.d. Bern(p)
PGF of Binomial distribution:

$G(s)=(1-p+p.s)^n$
$G(s)=(q+p.s)^n$
For 1st Generation:
$G'(s)= pn(q+p.s)^{n-1}$
$G'(1)=np=E(X)$

$G''(s)=p^2n(n-1)(q+ps)^{n-2}$
$G''(1)=n(n-1)p^2$

$Var(X)=G''(1)+G'(1)-[G'(1)]^2$
$Var(X)=n(n-1)p^2+np-(np)^2$
$Var(X_1)=npq$

For 2nd Generation:
$G_2(s)=G[G(s)]$
$G_2(s)=G[(q+p.s)^n]$
$G_2(s)=[q+p(q+ps)^n]^n$

$G_2'(s)=n^2p^2 [q+p(q+ps)^n]^{n-1} * (q+p.s)^{n-1}$
$G_2'(1)=n^2p^2$
$E(X_2)=n^2p^2$

$G_2''(s)=n^3p^3*(q+p.s)^{n-1}[q+p(q+ps)^n]^{n-2} *[q+np(q+ps)^n]$
$G_2''(1)=n^3p^3(np+q)$

$Var(X_2)=m^{(2-1)}*(m^2-1)*\sigma^2 /(m-1)$; $m=np$ & $\sigma^2=npq$
$Var(X_2)=n^2p^2q(np+1)$

After substituting the following parameters we get :
Let,n=12

$p=\frac{1}{4}$

$q=\frac{3}{4}$

For 1st generation:

$G'(1)=E(X)=np=3$

$G''(1)=n(n-1)p^2=8.25$

$Var(X_1)=\sigma^2=npq=2.25$

For 2nd generation:

$G_2'(1)=E(X_2)=n^2p^2=9$

$G_2''(1)=n^3p^3(np+q)=101.25$

$Var(X_2)=n^2p^2q(np+1)=27$

2. Poisson Distribution :

$X \sim Poisson(\lambda)$

For 1st generation :

$G(s) = e^{\lambda(s-1)}$

$G'(s) = \lambda\, e^{\lambda(s-1)}$

$G'(1) = \lambda = E(X)$

$G''(s)=\lambda^2\, e^{\lambda(s-1)}$

$G''(1)=\lambda^2$

$Var(X)=G''(1)+G'(1)-[G'(1)]^2$

$Var(X)=\lambda^2+\lambda-(\lambda)^2$

For 2nd generation:

$G_2(s)=G[G(s)]$

$G_2(s)=G[e^{\lambda(s-1)}]$

$G_2(s)=e^{\lambda[e^{\lambda(s-1)}-1]}$

$G_2(s) = e^{\lambda[e^{\lambda(s-1)}-1]}$

$G_2'(s) = \lambda^2 e^{\lambda[e^{\lambda(s-1)}-1]+\lambda(s-1)}$

$G_2'(1) = E(X_2) = \lambda^2$

$G_2''(s) = \lambda^3[\lambda e^{\lambda(s-1)} + 1] * e^{\lambda[e^{\lambda(s-1)}-1]+\lambda(s-1)}$

$G_2''(1) = \lambda^3(\lambda + 1)$

$Var(X_2) = m*(m^2-1)*\sigma^2 /(m-1)$

$Var(X_2) = \lambda^2(\lambda+1)$; $m=\lambda$ and $\sigma^2=\lambda$

After substituting the following parameters we get :

For 1st generation:

$\lambda = 5$

$G'(1) = \lambda = 5$

$G''(1) = \lambda^2 = 25$

$Var(X) = 5$

For 2nd generation:

$G_2'(1) = 25$

$G_2''(1) = 750$

$Var(X_2) = 150$

3. Geometric Distribution:

X~Geometric(p)

For 1st generation:

$G(s) = \frac{ps}{1-qs}$

$G'(s) = \frac{p}{(1-qs)^2}$

$G'(1)=E(X)=\frac{1}{p}$

$G''(s)=\frac{2pq}{(1-qs)^2}$

$G''(1) = \frac{2q}{p^2}$

$Var(X)=\sigma^2=G''(1)+G'(1)-[G'(1)]^2$

$Var(X)=\sigma^2=\frac{2q}{p^2} + \frac{1}{p}-(\frac{1}{p})^2$

$Var(X)=\sigma^2=\frac{q}{p^2}$

For 2^nd generation:

$G_2(s)=G[G(s)]$

$G_2(s)=G[\frac{ps}{1-qs}]$

$G_2(s)=\frac{p^2 s}{(1-qs)(1+p)}$

$G_2'(s)=\frac{p^2}{(qs-1)(1+p)}$

$G_2'(1)=E(X_2) =\frac{1}{1+p}$

$G_2''(s)=\frac{-2qp^2}{(qs-1)^3(1+p)}$

$G_2''(1)=\frac{2q}{p^2(1+p)}$

$Var(X_2)=m^{(2-1)}*(m^2-1)*\sigma^2 /(m-1)$

$Var(X_2)=\frac{(1+p)*q}{p^2}$

After substituting the following parameters we get :

$p = \frac{1}{8}$

$q = \frac{7}{8}$

For 1st generation:

$G'(1) = 8$

$G''(1) = 112$

$Var(X) = 56$

For 2nd generation:

$G_2'(1) = 0.88889$

$G_2''(1) = 99.555$

$Var(X_2) = 63$